Title

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Abstract

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2 Knowledge representation

Knowledge representation is at the intersection of maths, logic, language and computer sciences. Its research begins in the end of the 19th century, with Cantor inventing set theory (Cantor 1874). Then after a crisis in the begining of the 20th century with Russel's paradox and Gödel's incompletude theorem, revised versions of the set theory become one of the fundations of mathematics. The most accepted version is the Zermelo–Fraenkel axiomatic set theory with the axiom of Choice (ZFC) (Fraenkel *et al.* 1973, vol. 67; Ciesielski 1997). This effort was to represent mathematics in itself as a way to formalize its own expression.

A similar process happened in the 1970's, when logic based knowledge representation gained popularity among computer scientists (Baader 2003). Systems at the time explored notions such as rules and networks to try and organize knowledge into a rigourous structure. At the same time other systems were built arround First Order Logic (FOL). Then, around the 1990's, the research began to merge in search of common semantics in what led to the developpement of Description Logics (DL). This domain is expressing knowledge as a hierarchy of classes containing individuals.

From there and with the advent of the world wide web, engineers were on the lookout for standardization and interoperability of computer systems. One such standardization took the name of "semantic web" and aimed to created a widespread network of connected services sharing knowledge between one another in a common language. At the begining of the 21st century, several languages were created, all based around W3C specifications called Resource Description Framework (RDF) (Klyne and Carroll 2004). This language is based around the notion of statements as triples. Each can store a unit of knowledge at a time. All the underlying theoritical work of DL continued with it and created more efficient and lighter derivatives. One such derivative is the family of languages called Web Ontology Language (OWL) (Horrocks *et al.* 2003). Ontologies and knowledge graphs are more recent names for the representation and definition of categories (DL classes), properties and relation between concepts, data and entities.

All these knowledge description systems rely on a syntax to interoperate systems and users to one another. The base of such language comes from the formalization of automated grammars by Chomsky (1956). It mostly consist in a set of hierarchical rules aiming to deconstruct an input string into a sequence of terminal symbols. This deconstruction is called parsing and is a common operation in

computer science. More tools for the formalization of computer language emerged soon after thanks to Backus (1959) while working on a programming language at IBM. This is how the Backus–Naur Form (BNF) meta-language was created ontop of Chomsky's formalization.

All these tools are the base for all modern knowledge representations. In the rest of this chapter, we discuss the fundamentals of each of the aspects of knowledge description, then we propose a knowledge description framework that is able to adapt to its usage.

2.1 Fundamentals

First and foremost, we present the list of notations in this document. While trying to stick to traditional notations, we also aim for an unambiguious notation across several domains while remaining concise and precise. In table 2.1 we present the list of all symbols and relations. When possible we use the classical mathematical operator and otherwise we prefer lower case greek letters for relations. Table 2.2 lists all sets and general notions that we represent with uppercase letters.

2.1.1 Fundation of maths and logic systems

In order to understand knowledge representation, some mathematical and logical tools need to be presented.

2.1.1.1 Set theory

To start with the base, we need to explain the notion of set.
Definition 1 (Set). A collection of *distinct* objects, considered as an object in its own right. We define a set one of two way (always using braces):

- In extension by listing all the elements in the set: $\{1, 2, 4\}$
- In intension by specifying the rule that all elements follow: $\{n \; : \; n \in \mathbb{N} \wedge (n \mod 2 = 0)\}$

This definition alone gives some properties of sets. By being able to distinguish elements in the set from one another we assert that elements have an identity and we can derive equality from there. We can also give the empty set $\varnothing$ as the set with no elements. Since two sets are equals if and only if they have precisely the same elements, the empty set is unique. The member relation is noted $e \in E$ to indicate that e is an element of E. We note $S \subset G \; : \; \forall e \in S \vDash e \in G \wedge S \neq G$, that a set S is a proper subset of a more general set G.

Table 2.1: List of symbols and relations. The symbol $\pm$ shows when the notation has signed variants.

Symbol	Description
$var : exp$	The colon is a separator to be read as "such that".
$[exp]$	Iverson's brackets: $[false] = 0 \wedge [true] = 1$.
$\{e : exp(e)\}$	Set builder notation, all e such that $exp(e)$ is true.
$\langle e_1, e_2, e_n \rangle$	n-Tuple of various elements.
$e_s \xrightarrow{e_p} e_o$	Link or edge from e_s to e_o having e_p.
$l \downarrow a$	Link l partially supports step a.
$\pi \Downarrow a$	Plan π fully supports a.
$\varnothing$	Empty set, also notted $\{\}$.
$e \in E$	Element e belongs in set E also called choice operator.
$=$	Equality relation.
$\neg, \wedge, \vee$	Negation (not), conjonction (and), disjonction (logical or).
$\|E\|$	Cardinal of set E.
$t_1 \prec t_2$	t_1 subsumes t_2. Also used for order : t_1 precedes t_2
$\top, \bot$	Top and bottom symbols used as true and false respectively.
$\vDash$	Entails, used for logical implication.
$\otimes$	Flaws in a partial plan. Several variants :
$\otimes_{\updownarrow}$	• unsupported subgoal.
$\otimes_{\dagger}$	• causal threat to an existing causal link.
$\otimes_{\circlearrowleft}$	• cycle in the plan.
$\otimes_{*}$	• decomposition of a compound action.
$\otimes_{\rightarrowtail}$	• alternative to an existing action.
$\otimes_{\odot}$	• orphan action in the plan.
$\oplus, \ominus$	Positive and negative resolvers.
$\cent$	Cost of an action.
$\mathbb{i}, \mathbb{g}$	Initial and Goal step.
$\mathbb{P}(E)$	Powerset, set of all subsets of E.
$\mathcal{P}(X)$	Probability of event X.
pre, eff	Preconditions and effects of an action.
δ	Duration of an action.
$\mu^{\pm}(e)$	Signed meta-relation. μ^+ abstracts and μ^- reifies. μ alone gives the abstraction level.
$\nu(e)$	Name of entity e.
π	Plan or method of compound action.
$\rho(e)$	Parameters of entity e.
$\sigma(e)$	Scope of entity e.
$\tau(e)$	Type of entity e.
$\phi^{\pm}(e)$	Signed incidence (edge) and adjacence (vertice) function for graphs. ϕ^- gives the anterior (subject), ϕ^+ the posterior (object) and ϕ^0 gives the property of statement or cause of a causal link. ϕ alone gives a triple corresponding to the edge.

Table 2.2: List of important named sets.

Name	Description
A	Actions.
C	Causal links.
D	Domain of knowledge.
E	Entities.
F	Fluents.
L	Literals.
O	Observations.
P	Properties.
Q	Quantifiers.
S	Statements.
T	Types.
Π	Plans or method of compound action.

2.1.1.2 Logic

Another mathematical notion is logic. More precisely First Order Logic (FOL) in the context of DL.

Introducing the two boolean values $\top$ *true* and $\bot$ *false* along with the classical boolean operators $\neg$ *not*, $\wedge$ *and* and $\vee$ *or*. These are defined the following way :

- $\neg\top = \bot$
- $a \wedge b \vDash a = b = \top$
- $\neg(a \vee b) \vDash a = b = \bot$

2.1.1.3 Graphs and HyperGraphs

Next in line, we need to define a few notion of graph theory.
Definition 2 (Graph). A graph is a mathematical structure $g = (V, E)$ consisting of vertices V (also called nodes) and edges E (arcs) that links two vertices together. Each edge is basically a pair of vertices ordered or not depending if the the graph is directed or not.

A generalization of graphs are **hypergraphs** where the edges are allowed to connect more than two edges. An hypergraph is *n-uniform* when the edges are restricted to connect only n vertices together. In that regard, classical graphs are 2-uniform hypergraphs.

Another variant of graphs are graphs with the special case where $E \subset V$. This means that edges are allowed to connect to other edges. These kind of graphs are a generalization of hypergraphs. Informations about these kind of structures for knowledge reare hard to come by and rely mostly on

a form of "folk wisdom" where knowledge is rarely published and mostly transmitted orally during lessons. One of the closest information is this forum post (Kovitz 2018) that associated this type of graph to port graphs (Silberschatz 1981). Additional information was found in the form of a contribution of Vepstas (2008) on an encyclopedia article about hypergraphs. In that contribution, he says that a generalization of hypergraph allowing for edge-to-edge connections violates the axiom of fundation of ZFC by allowing edge-loops.

This shows the limits of standard mathematics especially on the field of knowledge representation. Some strucutres needs higher dimensions than allowed by the one dimensional structure of ZFC and FOL.

However, it is important to not be mistaken: such non-standard set theories are more general than ZFC and therefore contains ZFC as a subset. All is a matter of restrictions.

2.1.1.4 Sheaf

2.1.2 Grammar and Parsing

(BNF, Dynamic Grammar, Context free)

2.1.3 Ontologies and their Languages

(RDF, OWL, and Their limitations)

2.2 WORLD

2.2.1 Knowledge Structure

2.2.2 Dynamic Grammar

2.2.3 Contextual Interpretation

2.2.4 Structure as a Definition

2.2.5 Extended Inference Mechanisms

2.3 Perpectives

2.3.1 Literal definition using Peano's axioms

2.3.2 Advanced Inference

3 General Planning Framework

3.1 Existing Languages and Frameworks

État de l'art

3.2 Taxonomy

3.2.1 Action type

(Définition)

3.2.2 Plan type

(Définition)

3.2.3 Problem type

(Définition)

3.3 Color

(Framework)

4 Online and Flexible Planning Algorithms

4.1 Existing Algorithms

État de l'art

4.2 Lollipop

4.2.1 Operator Graph

4.2.2 Negative Refinements

4.2.3 Usefullness Heuristic

4.2.4 Algorithm

4.2.5 Theoretical and Empirical Results

4.3 HEART

4.3.1 Domain Compilation

4.3.2 Abstraction in POP

4.3.3 Planning in cycle

4.3.4 Properties of Abstract Planning

4.3.5 Computational Profile

4.4 Planning Improvements

4.4.1 Heuristics using Semantics

4.4.2 Macro-Action learning

4.5 Recognition

4.5.1 Existing approcahes

4.5.2 Rico

4.5.2.1 Probabilities and approximations

We define a probability distribution over dated states of the world. That distribution is in part given and fixed and the rest needs computation. **TODO : that's super bad...**

Here is the list of given prior probabilities and asumptions :

- $P(O) = \prod_{o \in O} P(o)$
- $P(\mathcal{G}) = \sum_{G \in \mathcal{G}} P(G) = 1$ since we assume that the agent must be pursuing one of the goals.

- $P(G|\pi) = 1$ for a plan π appliable in I that achieves G.

From dirrect application of Bayes theorem and the previous assomptions, we have :

$$P(\pi|O) = \frac{P(O|\pi)P(\pi)}{P(O)} = \frac{P(O|\pi)P(\pi|G)P(G)}{P(O)} \tag{4.1}$$

$$P(G|O) = \frac{P(O|G)P(G)}{P(O)} \tag{4.2}$$

From the total probability formula :

$$P(O|G) = \sum_{\pi \in \Pi_G} P(O|\pi)P(\pi|G) \tag{4.3}$$

$$P(O|G) = \sum_{\pi \in \Pi_G} P(O|\pi)P(\pi|G) \tag{4.4}$$

5 Conclusion

Apendix

References

Baader, F.
The description logic handbook: Theory, implementation and applications, Cambridge university press, 2003.

Backus, J. W.
The syntax and semantics of the proposed international algebraic language of the Zurich ACM-GAMM conference, *Proceedings of the International Comference on Information Processing, 1959*, 1959.

Cantor, G.
On a Property of the Class of all Real Algebraic Numbers., *Crelle's Journal for Mathematics*, 77, 258–262, 1874.

Chomsky, N.
Three models for the description of language, *IRE Transactions on information theory*, 2 (3), 113–124, 1956.

Ciesielski, K.
Set Theory for the Working Mathematician, Cambridge University Press, August 1997.

Fraenkel, A. A., Y. Bar-Hillel, and A. Levy
Foundations of set theory, vol. 67Elsevier, 1973.

Horrocks, I., P. F. Patel-Schneider, and F. V. Harmelen
From SHIQ and RDF to OWL: The Making of a Web Ontology Language, *Journal of Web Semantics*, 1, 2003, 2003.

Klyne, G., and J. J. Carroll
Resource Description Framework (RDF): Concepts and Abstract Syntax, Language Specification. Ed. B. McBride W3C, 2004.

Kovitz, B.
Terminology - What do you call graphs that allow edges to edges?, *Mathematics Stack Exchange*, January 2018.

Silberschatz, A.
Port directed communication, *The Computer Journal*, 24 (1), 78–82, January 1981. doi:10.1093/comjnl/24.1.78.

Vepstas, L.
Hypergraph edge-to-edge, *Wikipedia*, May 2008.