Title

Antoine Gréa

Abstract

Table of Content

Acknowledgements 🇬🇧

Sarah Gréa helping me doing the rubber duck method.

Remerciements 🇫🇷

Photo de l'époque

Je voudrais remercier quelques personnes sans qui rien de cela n'aurait été possible.

C'était une belle matinée de juin que j'ai parcouru pour la première fois les allée boisée de la Doua afin de rencontrer mon équipe encadrante: Samir Aknine et Laetitia Matignon. Ces derniers m'ont accordé leur confiance dans la réussite de ce projet de thèse. Sans leur conseils et soutient rien de cela n'aurait été possible. Je remercie particulièrement Samir qui en tant qu'encadrant et ami a su m'écouter et me soutenir même dans les moments les plus pénibles avec compassion et compréhension.

Je souhaite également remercier Christine Gertosio, qui m'a offert une aide sans faille dans mon service d'enseignement et qui m'a donné l'opportunité d'une offre d'ATER à ↻ **POLYTECH** Lyon. C'était un véritable plaisir de donner des cours à des promotions d'élèves passionnés et enthousiastes.

Je souhaite également offrir ma gratitude au chercheurs qui m'ont aidé dans cette aventure, sans ordre particulier: Fabrice Jumel, Jacques Saraydaryan, Shirin Sohrabi, Damien Pelier. J'aimerais également remercier Linas Vepstas, qui m'a grandement aidé dans la partie mathématique de cette thèse.

Enfin je tiens à remercier ma famille pour m'avoir supporté malgré mes périodes d'avolition. Je voudrais remercier tout particulièrement ma mère Charlotte et ma sœur Sarah pour avoir fait de leurs mieux pour me tenir compagnie et faire semblant de m'écouter déblatérer des technicités de ma thèse afin de me faire avancer.

Encore merci.

Sarah Gréa m'aidant en utilisant la methode du canard en plastique.

Preface

In this section we present a quick guide to the presentation of the information in this document. This will give its global structure and each of the type of formating and its meaning.

Text format

The text can be emphasized **more** or *less* to make a key word more noticeable.

Citations

In text citations will be in this format: Author *et al.* (year) to make the author part of the text and (Author *et al.* year) when simply referencing the work.

Quotes

Sometimes, important quotes needs emphasis. They are presented as:

> *"Don't quote me on that !"*
>
> Gréa (2019)

1 Introduction

1.1 Thesis Context

The work in this thesis was started in 2014, funded at the time by an arc 2 ● (Academic Research Community) research allocation courtesy of the Auvergne-Rhône-Alpes ● region in France. This organism is acting toward an improvement of the quality of life and of aging. Hence, the original subject was oriented toward assisting dependent persons using intelligent robots. However, such assistance is often rejected by these people who are also very demanding on the social skills of caregivers. That's why the work lays on the aspect of intent recognition and collaboration in assistive robotics.

This work was done under the ● **Lyon 1** university in the ʟɪʀɪꜱ (Laboratory of computeR science in Image and Information Systems). I was supervised by Samir Aknine and Laëtitia Matignon from the **sma** ● team (Multi-Agent System). Part of the work was funded by the university ʟᴜᴍɪᴇ̀ʀᴇ ʟʏᴏɴ 2 and by the ↻ **POLYTECH** engineering school.

1.2 Motivations

The social skills of modern robot are rather poor. Often, it is that lack that inhibits human-robot communication and cooperation. Humans being a social species, they require the use of implicit social cues in order to interact comfortably with an interlocutor. The primary goal of this thesis is to address this issue and create the formal foundations of a system able to help dependent people.

In order to enhance assistance to dependent people, we need to account for any deficiency they might have. An entire field of medicine and computer sciences is already dedicated to helping with the physical deficiencies. In our case we chose to address cognitive deficiencies. The main one is anterograde amnesia and dementia caused by Alzheimer's disease. In such cases, the patient is unable to express their needs. That is a problem even with human caregivers as the information about their intent needs to be inferred from their past actions.

1.3 Issues

This is the main issue addressed in the Artificial Intelligence (AI) field of intent recognition. The problem is very simple: from observational data, we need to infer the goals and intent of an external agent. There are several ways to handle this issue.

Most approaches use trivial machine learning techniques like Bayesian Networks (BN) or Markovian Decision Processes (MDP). The issue with these common approaches is that they require an extensive amount of training data and need to be trained on each patient. This makes the practicality of such system quite limited. To address this issue, some works proposed hybrid approaches using hand-made plan libraries or enforcing logical constraints on the probabilistic method.

A work from Ramirez **CITATION**, added an interesting method to solve that kind of problems. Indeed, they noticed an interesting parallel of that problem with the field of automated planning. This analogy was made by using the Theory of mind **CITATION**, that states that any agent will infer the intent of other agents using a projection of their own expectation on the observed behavior of the external agent.

This made the use of planner possible to infer intent without the need for extensive and well-crafted plan libraries. Now only the domain of the possible actions and their effects and prerequisites is needed to infer the logical intent of an agent.

The main issue of planning for that particular use is that of computation time and search space sizes. This prevents most planner to make any decision before the intent is already realized and therefore being useless for assistance. This time constraint lead to the search of a real-time planner algorithm that is also very expressive and flexible to accommodate for dynamical changes.

1.4 Contributions

In order to achieve such a planner, the first step was to formalize what is exactly needed to express a domain. Hierarchical and lenient forms of plan ordering gave the most expressivity and flexibility but at the cost of time performance. This is why, a new formalism of knowledge representation was needed in order to increase the speed of the search space exploration while restricting it using semantic inference rules.

While searching for a knowledge representation system, some prototypes were done using standard ontology tools but all proved to be way too slow and inexpressive for that application. This made the creation of a lighter but more flexible knowledge representation system, a requirement of the project.

Then the planning formalism was to be converted into our general knowledge representation tool. Since automated planning have a very diverse ecosystem of approaches and paradigms, it was also devoid of a universal standard formalism. A new formalism is proposed and compared to the one used in some part of the planning community.

Then finally, a couple of planner were conceived to attempt answering the speed and flexibility requirement of human intent recognition. The first one is a prototype that aims to evaluate the advantages of repairing plans using several heuristics. The second is a more complete prototype derived from the first (without plan repair), that also implement a Breadth-First Search (BFS) approach to hierarchical decomposition of composite actions. This allows the algorithm to provide intermediary plan that,

while incomplete, are an abstraction of the result plan. This allows for anytime probability computation using existing techniques of invert planning.

1.5 Plan

2 Fondation and Tools

Mathematics and logic are at the heart of all formal sciences, including computer science. Some of the most important problems in mathematics are the consistency and formalization issues. Research on these issues starts at the end of the 19th century, with Cantor inventing set theory. Then after a crisis in the beginning of the 20th century with Russel's paradox and Gödel's incompletude theorem, revised versions of the set theory become one of the foundations of mathematics. The most accepted version is the Zermelo-Fraenkel axiomatic set theory with the axiom of Choice (ZFC). This effort leads to a formalization of mathematics itself, at least to a certain degree.

Cantor ([1874](#))

Fraenkel *et al.* ([1973](#), vol. 67); Ciesielski ([1997](#))

Any knowledge must be expressed using a medium like language. Natural languages are quite expressive and allows for complex abstract ideas to be communicated between individual. However, in science we encounter the first issues with such a language. It is culturally biaised and improperly convey formal notions and proof constructs. This is one of the main conclusion of the works of Korzybski ([1958](#)) on general semantics. The original goal of Korzybski was to pinpoint the errors that led humans to fight each other in WWI. He affirmed that the language is inaddapted to convey information reliably about objective facts or scientific notions. There is a decrepancy between the natural language and the underlying structure of the reality. This issue is exacerbated in mathematics as the ambiguity in the definition of a term can be the cause for a contradiction and make the entire theory inconsistent.
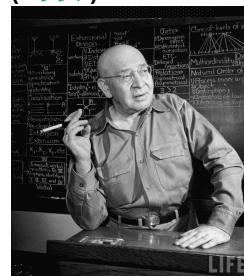
Figure 2.1: Alfred Korzybski

> *"Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true."*

Russell ([1917](#))

In this first chapter we analyse the issue and propose our solution. Then we use our fondation to build some important mathematical notions used later to formalize data structures and algorithms.

2.1 Properties

From this point we can apply some of these ideas to mathematics to analyse what properties a fondation of mathematics must hold.

2.1.1 Abstraction

> **abstraction** (n.d.): *The process of formulating generalized ideas or concepts by extracting common qualities from specific examples*

Collins English Dictionary ([2014](#))

The idea behind abstraction is to simplify the representation of (potentially infinitely) complex instances. This mechanism is at the base of any knowledge representation system. Indeed, it is unecessarily expensive to try to represent all properties of an object. An efficient way to reduce that knowledge representation is to prune away all irrelevent properties while also only keeping the one that will be used in the context. *This means that abstraction is a loosy process.* Information is lost when abstracting from an object.

Since this is done using a language as a medium, this language is a *host language*. Abstraction will refer to an instance using a *term* (also called symbol) of the host language. Since abstraction is a generalization process, if the host language is expressive enough, abstraction can be applied to already abstracted knowledge. The number of underlying abstraction needed for a term is called its *abstraction level*. Very general notions have a higher abstraction level and we usually represent reality using the null abstraction level. In practice abstraction uses terms of the host language to bind to a referenced instance in a lower abstraction level. This forms a structure that is strongly hierarchical with higher abstraction level terms on top.

Example 1. We can describe an individual organism with a name that is associated to this specific individual. If we name a dog "Rex" we abstract a lot of information about a complex dynamical living being. We can also abstract from a set of qualities of the specimen to build higher abstraction. For example its species would be *Canis lupus familiaris* from the *Canidae* family. Sometimes several terms can be used at similar abstraction level like the commonly used denomination "dog" in this case.

Terms are only a part of that structure. It is possible to combine several terms into a *formula* (also called proposition or statement).

2.1.2 Formalization

> **formal** (adj.): *Relating to or involving outward form or structure, often in contrast to content or meaning.*

Formalization is the act to make formal. The word "formal" comes from Latin *fōrmālis*, from *fōrma*, meaning form, shape or structure. This is the same base as for the word "formula". In mathematics and *formal sciences* the act of formalization is to reduce knowledge down to formula. Like stated previously, a formula combines several terms. But a formula must follow rules at different levels:

- *Lexical* by using terms belonging in the host language.
- *Syntaxic* as it must follow the grammar of the host language.
- *Semantic* as it must be internally consistant and meaningful.

The information conveied from a formula can be reduced to one element: its semantic structure. Like its ethymology suggest a formula is simply a structured statement about terms. This structure holds its meaning. Along with using abstraction, it becomes possible to abstract a formula and to therefore make a formula about other formulae should the host language allowing it.

Example 2. The formula using English "dog is man's best friend" combines terms to hold a structure between words. It is lexically correct since it uses English words and gramatically correct since it can be grammatically decomposed as (n. v. n. p. adj. n.). In that the (n.) stands for noumn, (v.) for verb, (adj.) for adjective and (p.) for possessive. Since the verb "is" is the third person singular present indicative of "be", and the adjective is the superlative of "good", this form is correct in the English language. From there the semantic aspect is correct too but that is too subjective and extensive to formalize here. We can also build a formula about a formula like "this is a common phrase" using the referencial pronoun "this" to refer to the previous formula.

However, there is a strong limitation of a formalization. Indeed, a complete formalization cannot occur about the host language. It is possible to express formulae about the host language but *it is impossible to completely describe the host language using itself*. This comes from two principal reasons. As abstraction is a loosy process one cannot completely describe a language that can evolve and be infinitely complex. And even when taking a simple language, there is a problem with describing the different levels of rules to process the language using itself. This supposes of a knowledge of the language *a priori*. This is contradictory and therefore impossible to achieve.

When abstracting a term, it may be useful to add informations about the term to define it properly. That is why most formal system require a *definition* of each term using a formula. This definition is the main piece of semantic information on a term and is used when needing to evaluate a term in a different abstraction level. However, this is causing yet another problem.

2.1.3 Circularity

> **circularity** (n.d.): *Defining one word in terms of another that is itself defined in terms of the first word.*

American Heritage Dictionary (2011b)

Defining a term requires using a formula in the host language to express the abstracted properties of the generalization. The problem is that most terms will have *circular* definitions. Like stated this means that every knowledge system will have some circularity in their definitions. This means that it is impossible to have a complete definition of a formal system without needing another formal system to describe its base.

Example 3. Using definitions from the American Heritage Dictionary (2011b), we can find that the term "word" is defined using the word "meaning" that is in turn defined using the term "word". Such circularity can happen using an arbitrarily long chain of definition that will form a cycle in the dependancies.

This problem is very important as it is overlooked in most fondations of mathematics. This also makes Russel's paradox unavoidable in such systems since there are always an infinite descent in the definition relation. Also, since a formalization cannot fully be self defined, another host language is generally used, sometimes without being acknoledged. This causes cycles in the dependancies of languages and theories in mathematics.

The only practical way to make some of this circularity disapear is to base a fondation of mathematics using natural language as host language for defining the most basic terms. This allows to aknowledge the problem in an instinctive way while being aware of it while building the theory.

2.2 Fonctional theory of mathematics

We aim to find the smallest possible set of axioms allowing to describe a fondation of mathematics. The following theory is a proposition for a possible fondation that takes into account the previously described constraints. It is inspired by category theory, and typed lambda calculus **CITATION**.

In this part, as an introduction to the fundamentals of maths and logic, we propose another view of that fondation based on functions. The unique advantage of it lays in its explicit structure that have emergent reflexive properties. It also holds a coherent algebra that have a strong expressive power. This approach is loosly based on category theory. However it differs in its priorities and formulation.

Our theory is axiomatic, meaning it is based on fundamental logical proposition called axioms. Those forms the base of the logical system and therefore are accepted without any need for proof. The following axiom is the mandatory undefinable notion created from natural language. It is the explicit base of the formalism.

Aχiom (Function). Lets be functions that associates a unique function (image) to each function (argument).

That axioms states that the formalism is based on *functions*. Any function can be used in anyway as long as it has a single argument and return only one value at a time which are also functions.

Next we need to lay off the base definitions of the formalism.

2.2.1 Formalism definition

This functional algebra at the base of our fondation is inspired by *operator algebra* **CITATION**. The problem with the operator algebra is that it supposes vectors and real numbers to work properly.

Since any formalism has circular definition we will have two way of definining each notions: using natural language as host language or by using notions that will be defined later. The main definitions we will express are using natural language but we'll add the formula using future notion as side notes. The goal is to explicitly separate fully defined notions from the circular base while giving the dependancies between notions to ease future improvements.

Here we define the basic notions of our functional algebra that dictates the rules of the formalism we are defining.

Definition 1 (Currying). Currying is the opperation named after the mathematician Haskell Brooks Curry that allows multiple arguments functions in a simpler monoidal formalism. A monome is a function that takes only one argument and has only one value, as in our main axiom.

The operation of *currying* is a function $(\!|\,|\!)$ that associates to each function f another function that recursively partially applies f with one argument at a time.

$$(\!|f|\!) = (x \to (\!|f(x)|\!))$$

If we take a function h such that when applied to x gives the function g that takes an argument y, *uncurrying* is the function $(\!|\,)$ so that $f(x,y)$ behaves the same way as $(\!|h|\!)(x)(y)$.

$$(\!|f|\!) = (\!|x,y \to f(x)(y)|\!)^+$$

Definition 2 (Application). We note the application of f with an argument x as $f(x)$. The application allows to recover the image y of x which is the value that f associates with x.

$$y = f(x)$$

We call *partial application* the application using an insufficient number of arguments to any function f. This results in a function that takes less arguments with the first ones being fixed by the partial application. It is interesting to note that currying is simply a recursion of partial applications.

From now on we will note $f(x,y,z,...)$ any function that takes multiple arguments but will suppose that they are implicitly curryied. If a function only takes two arguments, we can also use the infix notation e.g. xfy for its application.

Example 4. Applying this to basic arithmetics for illustration, it is possible to create a function that will triple its argument by doing a partial application of the multiplication function $\times(3)$ so we can write the operation to triple the number 2 as $\times(3)(2)$ or $\times(2,3)$ or with the infix notation 2×3.

$$\times(3) = x \to 3 \times x$$

Definition 3 (Association). Lets be the *association function* $(\to)$ such that, for any two function x and $f(x)$, $x \to f(x)$ has f as value.

$$(\to) = x, f(x) \to f$$

Definition 4 (Specification). The *function of specification* (noted $:$) is a function that restricts the validity of an expression given a predicate. It can be intuitively be read as "*such that*".

$$(:) = f,? \to f \triangledown (\mathcal{D}(? = \bot) \mapsto \mathcal{D}(\bullet f))$$

The specification operator is extensively used in classical mathematics but informally, it is often seen as an extension of natural language and can be quite ambiguous. In the present document any usage of $(:)$ in any mathematical formula will follow the previously discussed definition.

Definition 5 (Null). The *null function* is the function between nothing and nothing. We note it $\gg$.

$$\gg = \gg \to \gg$$

Definition 6 (Identity). The *identity function* is the function that associates any function to itself noted $(=)$. This function is therefore transparent as by definition $= (x)$ is the same as x. A more useful form of this function is as a binary predicate (see **LATER**). The predicate can work in two ways: *equation* and *affectation*.

$$(=) = x \to x$$

$$(=) = (x, x \to \top) \bowtie (x, y \to \bot)$$

- As an **equation**, the predicate tests whether two function are the same.

- The **affectation** allows to assert that two functions have the same identity making them an alias to one another. An affectation is often used to define a new function.

In the rest of the document, classical equality and the identity function will refer to the same notion.

2.2.2 Literals and Variables

In our formalism, we use the null function to define notions of variables and literals.

$l =\!\!\gg\,\to l$

Definition 7 (Literal). A literal is a function that associates nothing with itself. This consists of any fonction l writen as $\gg\,\to\ l$. This means that the function have only itself as an immutable value. We call *constants* function that have no arguments and have as value either another constant or a literal.

Example 5. A good example of that would be the yet to be defined natural numbers. We can define the literal 3 as $3 =\!\!\gg\,\to 3$.

$x = x \to\!\!\gg$

Definition 8 (Variable). A variable is a function that associates itself to nothing. This consists of any fonction x writen as $x \to\!\!\gg$. This means that the function takes itself as argument and takes the value of the argument on a specific domain (also called type).

This definition means that a variable requires an input that can then be hold as a possible value when evaluating.

An interesting property of this notation is that $\gg$ is both a variable and a constant. When defining currying, we annotated with the formalism $(\!|f|\!) = f \to (x \to (\!|f(x)|\!))$. The obvious issue is the absence of stopping condition in that recursive expression. While the end of the recursion doesn't technically happens, in practice from the way variables and literals are defined, the recursion chain either ends up becoming a variable or a constant because it is undefined when currying a nullary function.

2.2.3 Functional algebra

Inspired by relational algebra and by category theory, we present an highly expressive and compact functional algebra. In the figure 2.2 we illustrate the different operators of this algebra and their properties.

The first operator of this algebra allows to combine several functions into one.

$f_1 \bowtie f_2 = x \to y$
with $f_1(x) = y$ **or**
$f_2(x) = y$.

Definition 9 (Combination). The *combination function* $\bowtie$ is the function that associates any two function f_1 and f_2 to a new function that associates any functions associated by **either** functions.

Example 6. For two functions $f_1 = 1 \to 2$ and $f_2 = 2 \to 3$ the combination $f_3 = f_1 \bowtie f_2$ will have the following properties: $f_3(1) = 2$ and $f_3(2) = 3$.
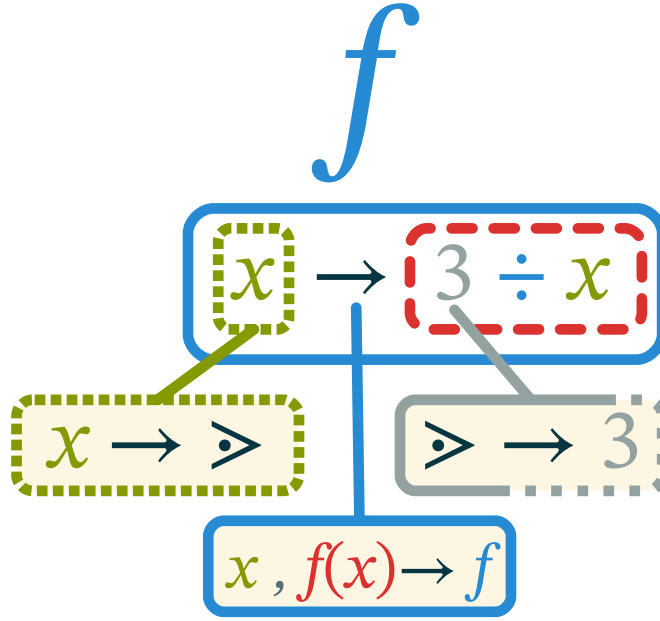
Figure 2.2: Illustration of basic functionnal operators and their properties.

Definition 10 (Superposition). The *superposition function* $\triangle$ is the function that associates any two functions f_1 and f_2 to a new function that associates any function associated by **both** functions.

$f_1 \triangle f_2 = x \to y$
with
$f_1(x) = f_2(x) = y$.

Example 7. Re-using the functions of the previous example we can note that $f_3 \triangle f_1 = f_1$ because $1 \to 2$ is the only combination that both functions associates.

Definition 11 (Subposition). The *subposition function* is the function $\triangledown$ that associates any two functions f_1 and f_2 to a new function $f_1 \triangledown f_2$ that associates anything associated by the first function f_1 but **not** by the second functionb f_2.

We can also note a few properties of these functions:

- $f \triangle f = f$
- $f \triangle \gg = \gg$
- $f_1 \triangle f_2 = r_2 \triangle f_1$
- $f \triangledown f = \gg$
- $f \triangledown \gg = f$

Intuitively these fonctions are the fonctional equivalent of the union, intersection and difference from set theory. In our formalism we will define the set operations from these.

The following operators are the classical operations on functions.

Definition 12 (Composition). The *composition function* is the function that associates any two functions f_1 and f_2 to a new function such that: $f_1 \circ f_2 = x \to f_1(f_2(x))$.

Definition 13 (Inverse). The *inverse function* is the function that associates any function to its inverse such that if $y = f(x)$ then $x = \bullet(f)(y)$.

We can also use an infix version of it with the composition of functions: $f_1 \bullet f_2 = f_1 \circ \bullet(f_2)$.

These properties are akin to multiplication and division in arithmetics.

- $f \circ \geqslant = \geqslant$ ($\geqslant$ is the absorbing element)
- $f \circ \; = = f$ (= is the neutral element)
- $\bullet(\geqslant) = \geqslant$ and $\bullet(=) ==$
- $f_1 \circ f_2 \neq f_2 \circ f_1$

From now on we will use numbers and classical arithmetics as we had defined them. However, we consider defining them from a fondation point of view, later using set theory and Peano's axioms.

In classical mathematics, the inferse of a fonction f is often wrote as f^{-1}. Therefore we can define the transitivity of the nth degree as the power of a fonction such that:

- $f^{-1} = \bullet f$
- $f^0 = (=)$
- $f^1 = f$
- $f^n = f^{n-1} \circ f$

By generalizing the formula, we can define the *transitive cover* of a function f and its inverse respectively as $f^+ = f^{+\infty}$ and $f^- = f^{-\infty}$.

We also call *arity* the number of arguments (or the currying order) of a function noted $|f|$.

2.2.4 Properties

Since our goal is to minimize the formal basis of our system as well as identifying the circularity issues, we provide a dependancy graph in figure 2.3.

The algebra formed by the previously defined operations on functions is a semiring $(\mathbb{F}, \bowtie, \circ)$ with $\mathbb{F}$ ceing the set of all functions. Indeed, $\bowtie$ is a commutative monoid having $\geqslant$ as its identity element and $\circ$ is a monoid with $(=)$ as its identity element. Also the composition of the combination is the combination of the composition and therefore $\circ$ distributes over $\bowtie$. At last, using partial application $\circ(\geqslant) = \geqslant$

This fondation is now ready to define other fields of mathematics. We start with logic as it is a very basic formalism in mathematics. Then we redefine the ZFC set theory using our formalism as a base. And finally we will present derived mathematical tools to represent data structures and their properties.
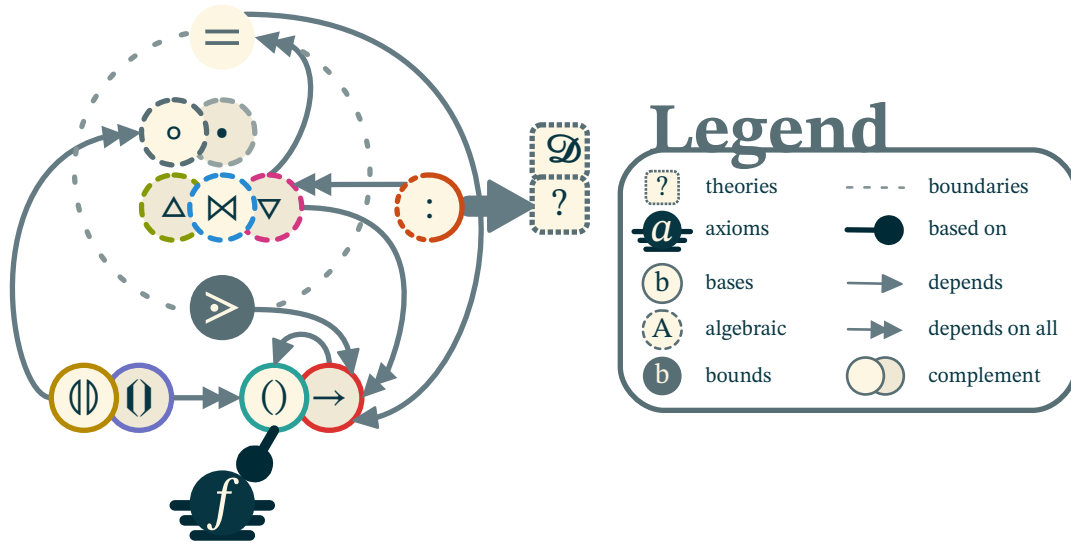
Figure 2.3: Dependancy graph of notions in the functional theory

2.3 First Order Logic

In this section, we present First Order Logic (FOL). All notations are presented in **??**. FOL is based on boolean logic with the two literals $\top$ *true* and $\bot$ *false*.

A function $?$ that have as only values either $\top$ or $\bot$ is called a **predicate**. $\mathcal{D}(\bullet?) = \{\bot, \top\}$

We define the classical logic *entailment*, the predicate that holds true when a predicate (the conclusion) is true if and only if the first predicate (the premise) is true.

$$\vdash = (\bot, x \to \top) \bowtie (\top, x \to x)$$

Then we define the classical boolean operators $\neg$ *not*, $\wedge$ *and* and $\vee$ *or* as:

- $\neg = (\bot \to \top) \bowtie (\top \to \bot)$, the negation associates true to false and false to true.
- $\wedge = x \to ((\top \to x) \bowtie (\bot \to \bot))$, the conjonction is true when all its arguments are simultaneously true.
- $\vee = x \to ((\top \to \top) \bowtie (\bot \to x))$, the disjonction is true if all its arguments are not false.

The last two operators are curried function and can take any number of arguments as necessary and recursively apply their definition.

Functions that takes an expression as parameter are called *modifiers*. FOL introduce a useful kind of modifer used to modalize expressions: *quantifiers*. Quantifiers takes an expression and a variable as arguments. Classical quantifiers are also predicates: they restrict the values that the variable can take.

The classical quantifiers are:

- The *universal quantifier* $\forall$ meaning *"for all"*. $\forall = \S(\wedge)$

$\exists = \S(\lor)$

- The *existential quantifier* $\exists$ meaning *"it exists"*.

They are sometimes extended with :

$\exists! = \S(= (1) \circ +)$
$\nexists = \S(\neg \circ \land)$

- The *uniqueness quantifier* $\exists!$ meaning *"it exists a unique"*.
- The *exclusive quantifier* $\nexists$ meaning *"it doesn't exist"*.

An another exotic quantifier that isn't a predicate can be proven useful (Hehner 2012):

$\S = f, x, ? \rightarrow$
$\{\![f(x) : ?]\!\}$

- The *solution quantifier* $\S$ meaning *"those"*.

The last three quantifiers are optional in FOL but will be conducive later on. It is interesting to note that most quantified expression can be expressed using the set builder notation discussed in the following section.

2.4 Set Theory

Since we need to represent knowledge, we will handle more complex data than simple booleans. The first theory we will define is the set theory. It is used as the classical fondation of mathematics. It is so important to mathematics that most other proposed fondation of mathematics invoke the concept of sets even before their first formula to describe the kind of notions they are introducing. The issue is then to define the sets themselves. At the beginning of his funding work on set theory, Cantor wrote:

> "*A set is a gathering together into a whole of definite, distinct objects of our perception or of our thought–which are called elements of the set.*"

Georg Cantor
(1895)

For Cantor, a set is a collection of concepts and percepts. In our case both notions are grouped in what we call *objects*, *entities* that are all ultimately *functions* in our formalism.

2.4.1 Base Definitions

We define a set using the notations in **??**.

Definition 14 (Set). A collection of *distinct* objects considered as an object in its own right. We define a set one of two ways (always using braces):

- In extension by listing all the elements in the set: $\{0, 1, 2, 3, 4\}$
- In intension by specifying the rule that all elements follow: $\{n : ?(n)\}$

Using our fonctional fondation, we can define any set as a predicate $\mathcal{S} = e \rightarrow \top$ with e being a member of $\mathcal{S}$. This allows us to define the member function noted $e \in \mathcal{S}$ to indicate that e is an element of $\mathcal{S}$.

$\in = e, \mathcal{S} \rightarrow \mathcal{S}(e)$

Another, useful definition using sets is the *domain* of a function f as the set of all arguments for which the function is defined. We call *co-domain* the domain of the inverse of a function. We can note them $f : \mathcal{D}(f) \mapsto \mathcal{D}(\bullet f)$. In the case of our functional version of sets, they are their own domain.

2.4.2 Set Operations

Along with defining the domains of functions using sets, we can use function on sets. In this section, basic set operations are presented. The first one is the subset.

Definition 15 (Subset). A subset is a part of a set that is integrally contained within it. We note $\mathcal{S} \subset \mathcal{T} \vdash ((e \in \mathcal{S} \vdash e \in \mathcal{T}) \wedge \mathcal{S} \neq \mathcal{T})$, that a set $\mathcal{S}$ is a proper subset of a more general set $\mathcal{T}$.

Definition 16 (Union). The union of two or more sets $\mathcal{S}$ and $\mathcal{T}$ is the set that contains all elements in *either* set. We can note it:

$$\mathcal{S} \cup \mathcal{T} = \{e : e \in \mathcal{S} \vee a \in \mathcal{T}\}$$

Definition 17 (Intersection). The intersection of two or more sets $\mathcal{S}$ and $\mathcal{T}$ is the set that contains only the elements member of *both* set. We can note it:

$$\mathcal{S} \cap \mathcal{T} = \{e : e \in \mathcal{S} \wedge e \in \mathcal{T}\}$$

Definition 18 (Difference). The difference of one set $\mathcal{S}$ to another set $\mathcal{T}$ is the set that contains only the elements contained in the first but not the last. We can note it:

$$\mathcal{S} \setminus \mathcal{T} = \{e : e \in \mathcal{S} \wedge e \notin \mathcal{T}\}$$

An interesting way to visualize relationsips with sets is by using Venn diagrams. In figure 2.4 we present the classical set operations.
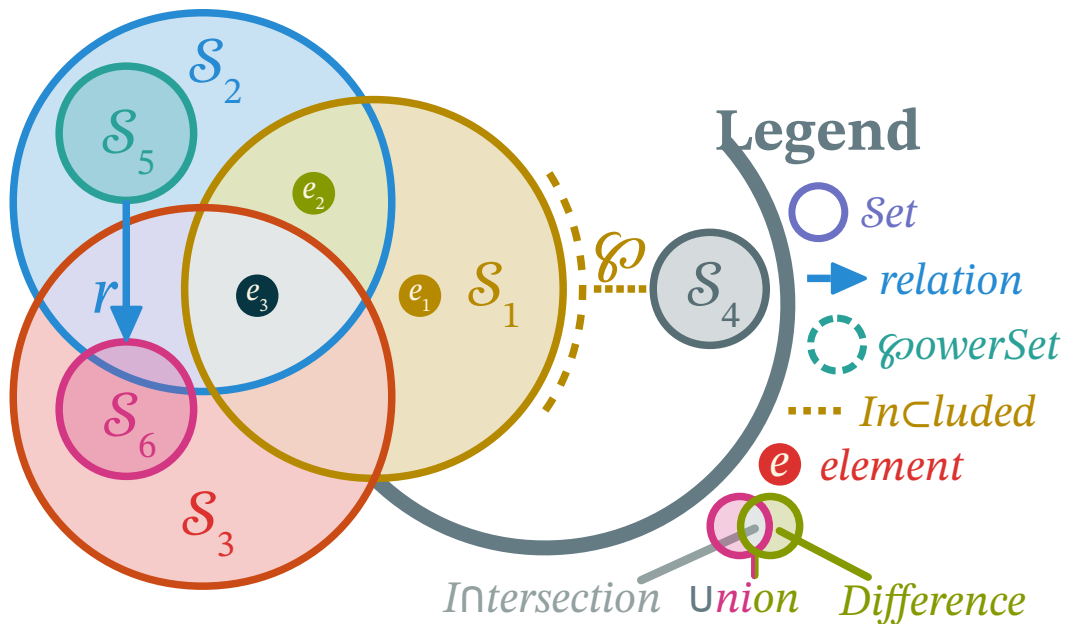
Figure 2.4: Example of an upgraded Venn diagram to illustrate operations on sets.

These diagrams have a lack of expressivity regarding complex operations on sets. Indeed, from their dimmensionality it is complicated to express numerous sets having intersection and disjunctions. For example it is difficult to represent the following notion.

Definition 19 (Cartesian product). The cartesian product of two sets $\mathcal{S}$ and $\mathcal{T}$ is the set that contains all possible combination of an element of both sets. These combination are a kind of ordered set called *tuples*. We note this product:

$$\mathcal{S} \times \mathcal{T} = \{\langle e_{\mathcal{S}}, e_{\mathcal{T}} \rangle : e_{\mathcal{S}} \in \mathcal{S} \wedge e_{\mathcal{T}} \in \mathcal{T}\}$$

From this we can also define the set power recursively by $\mathcal{S}^1 = \mathcal{S}$ and $\mathcal{S}^n = \mathcal{S} \times \mathcal{S}^{n-1}$.

The cartesian product is the set equivalent of currying as:

$$\mathcal{S} \times \mathcal{T} = e_{\mathcal{S}}, e_{\mathcal{T}} \rightarrow \mathcal{S}(e_{\mathcal{S}}) \wedge \mathcal{T}(e_{\mathcal{T}})$$

The angles $\langle \rangle$ notation is used for tuples, that are another view on currying by replacing several arguments using a single one as an ordered list. A tuple of two element is called a *pair*, of three elements a *triple*, etc. We can access elements in tuples using their index in the following way $e_2 = \langle e_1, e_2, e_3 \rangle_2$.

Definition 20 (Mapping). The mapping notation $\{\!\!\{\}\!\!\}$ is a function such that $\{\!\!\{f(x) : x \in \mathcal{S}\}\!\!\}$ will give the result of applying all elements in set $\mathcal{S}$ as arguments of the function using the uncurrying operation recursively. If the function isn't specified, the mapping will select a member of the set non deterministically. The function isn't defined on empty sets or on sets with less members than arguments of the provided function.

Example 8. The classical sum operation on numbers can be noted:

$$\sum_{i=1}^{3} 2i = \{\!\!\{+(2 * i) : i \in [1, 3]\}\!\!\} = +(2 * 1)(+(2 * 2)(2 * 3))$$

2.4.3 The ZFC Theory

The most common axiomatic set theory is ZFC. In that definition of sets there are a few notions that comes from its axioms. By being able to distinguish elements in the set from one another we assert that elements have an identity and we can derive equality from there:

Axiom (Extensionality). $\forall \mathcal{S} \forall \mathcal{T} : \forall e((e \in \mathcal{S}) = (e \in \mathcal{T})) \vdash \mathcal{S} = \mathcal{T}$

Another axiom of ZFC that is crucial in avoiding Russel's paradox ($\mathcal{S} \in \mathcal{S}$) is the following:

Axiom (Foundation). $\forall \mathcal{S} : (\mathcal{S} \neq \varnothing \vdash \exists \mathcal{T} \in \mathcal{S}, (\mathcal{T} \cap \mathcal{S} = \varnothing))$

This axiom uses the empty set $\varnothing$ (also noted $\{\}$) as the set with no elements. Since two sets are equals if and only if they have precisely the same elements, the empty set is unique.

The definition by intention uses the set builder notation to define a set. It is composed of an expression and a predicate $?$ that will make any element e in a set $\mathcal{T}$ satisfying it part of the resulting set $\mathcal{S}$, or as formulated in ZFC:

Axiom (Specification). $\forall?\forall\mathcal{T}\exists\mathcal{S} : (\forall e \in \mathcal{S} : (e \in \mathcal{T} \wedge ?(e)))$

The last axiom of ZFC we use is to define the power set $\wp(\mathcal{S})$ as the set containing all subsets of a set $\mathcal{S}$:

Axiom (Power set). $\wp(\mathcal{S}) = \{\mathcal{T} : \mathcal{T} \subseteq \mathcal{S}\}$

With the symbol $\mathcal{S} \subseteq \mathcal{T} \vdash (\mathcal{S} \subset \mathcal{T} \vee \mathcal{S} = \mathcal{T})$. These symbols have an interesting property as they are often used as a partial order over sets.

2.5 Graphs

With set theory, it is possible to introduce all of standard mathematics. A field of interest for this thesis is the study of the structure of data. Most of these structures uses graphs and isomorphic derivatives.

Definition 21 (Graph). A graph is a mathematical structure g which is defined by its *connectivity function* χ that

2.5.1 Adjacency, Incidency and Connectivity

Definition 22 (Connectivity). The connectivity function is a combination of the classical adjacency and incidency functions of the graph. It is defined using a circular definition in the following way:

- *Adjacency*: $\chi_{\circ\!\circ} = v \rightarrow \{e : v \in \chi_{\rightarrow}(e)\}$ Also: $\chi_{\circ\!\circ} = \bullet\chi_{\rightarrow}$
- *Incidency*: $\chi_{\rightarrow} = e \rightarrow \{v : e \in \chi_{\circ\!\circ}(v)\}$

Defining either function defines the graph. For convinience, the connectivity function combine the adjacency and incidency:

$$\chi = \chi_{\circ\!\circ} \bowtie \chi_{\rightarrow}$$

Usually, graphs are noted $g = (V, E)$ with the set of vertices V (also called nodes) and edges E (arcs) that links two vertices together. Each edge is classically a pair of vertices ordered or not depending on if the graph is directed or not. It is possible to go from the set based definition to the functional relation using the following equation: $\mathcal{D}(\chi_{\rightarrow}) = E$ $E \subseteq V^2$
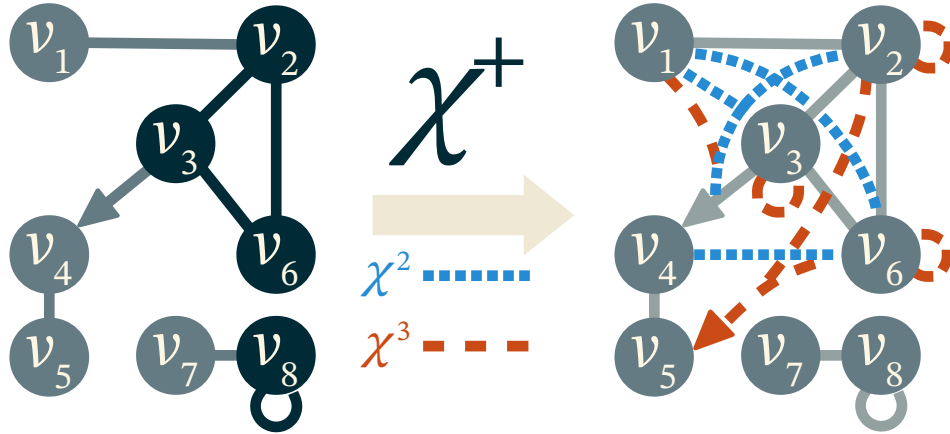
Figure 2.5: Example of the recursive application of the transitive cover to a graph.

Example 9. A graph is often represented with lines or arrows linking points together like illustrated in figure 2.5. In that figure, the vertices v_1 and v_2 are connected through an undirected edge. Similarly v_3 connects to v_4 but not the opposite since they are bonded with a directed edge. The vertex v_8 is also connected to itself.

2.5.2 Digraphs

Digraphs or *dirrectionnal graphs* are a specific case of graphs where edges have a dirrection. This means that we can have two vertices v_1 and v_2 linked by an edge and while it is possible to go from v_1 to v_2, the inverse is impossible. For such case the edges are ordered pairs and the incidency function can be decomposed into:

$$\chi_{\leftharpoonup} = \chi_{\rightharpoonup} \bowtie \chi_{\leftharpoondown}$$

We note $\chi_{\rightharpoonup}$ the **incomming relation** and $\chi_{\leftharpoondown}$ the **outgoing relation**.

In digraphs, classical edges can exist if allowed and will simply be bi-dirrectional edges.

2.5.3 Path, cycles and transitivity

Most of the intrinsic information of a graph is contained within its structure. Exploring its properties require to study the "shape" of a graph and to find relationships between vertices. That is why graph properties are easier to explain using the transitive cover χ^+ of any graph $g = (V, E)$.

This transitive cover will create another graph in which two vertices are connected through an edge if and only if it exists a path between them in the original graph g. We illustrate this process in figure 2.5. Note how there is no edge in $\chi^2(g)$ between v_5

and v_6 and the one in $\chi^3(g)$ is directed towards v_5 because there is no path back to v_6 since the edge between v_3 and v_4 is directed.

Definition 23 (Path). We say that vertices v_1 and v_2 are *connected* if it exists a path from one to the other. Said otherwise, there is a path from v_1 to v_2 if and only if $\langle v_1, v_2 \rangle \in \mathcal{D}(\chi^+(g))$.

The notion of connection can be extended to entire graphs. An undirected graph g is said to be *connected* if and only if $\forall e \in V^2(e \in \mathcal{D}(\chi^+(g)))$.

Similarly we define *cycles* as the existence of a path from a given vertex to itself. For example, in figure 2.5, the cycles of the original graph are colored in blue. Some graphs can be strictly acyclical, enforcing the absence of cycles.

2.5.4 Trees

A **tree** is a special case of a graph. A tree is an acyclical connected graph. If a special vertex called a *root* is chosen we call the tree a *rooted tree*. It can then be a directed graph with all edge pointing away from the root. When progressing away from the root, we call the current vertex *parent* of all exterior *children* vertices. Vertex with no children are called *leaves* of the tree and the rest are called *branches*.

An interesting application of trees to FOL is called *and/or trees* where each vertex has two sets of children: one for conjunction and the other for disjunction. Each vertex is a logic formula and the leaves are atomic logic propositions. This is often used for logic problem reduction. In figure 2.6 we illustrate how and/or trees are often depicted.
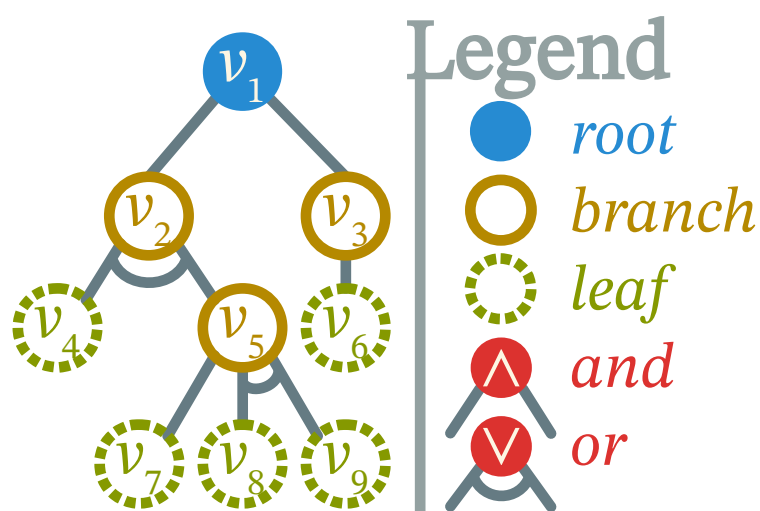
Figure 2.6: Example of and/or tree.

2.5.5 Quotient

Another notion often used for reducing big graphs is the quotiening as illustrated in figure 2.7.

Definition 24 (Graph Quotient). A quotient over a graph is the act of reducing a subgraph into a node while preserving the external connections. All internal structure becomes ignored and the subgraph now acts like a regular node. We note it $\div_f(g) = (\{f(v) : v \in V\}, \{f(e) : e \in E\})$ with f being a function that maps any vertex either toward itself or toward its quotiened vertex.
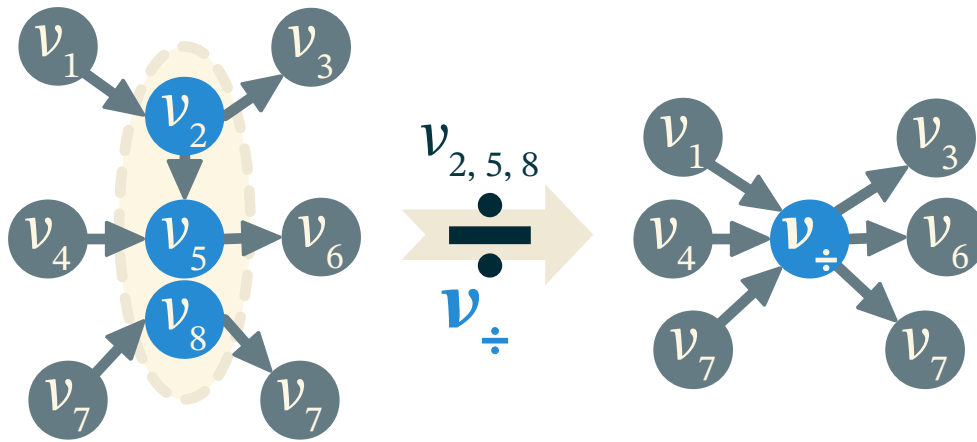
Figure 2.7: Example of graph quotient.

2.5.6 Hypergraphs

A generalization of graphs are **hypergraphs** where the edges are allowed to connect to more than two vertices. They are often represented using Venn-like representations but can also be represented with edges "gluing" several vertex like in figure 2.8.

An hypergraph is said to be *n-uniform* if the edges are restricted to connect to only n vertices together. In that regard, classical graphs are 2-uniform hypergraphs.

Hypergraphs have a special case where $E \subset V$. This means that edges are allowed to connect to other edges. In figure 2.8, this is illustrated by the edge e_3 connecting to three other edges. Information about these kinds of structures for knowledge representation is hard to come by and rely mostly on a form of "folk wisdom" within the mathematics community where knowledge is rarely published and mostly transmitted orally during lessons. One of the closest information available is this forum post (Kovitz 2018) that associated this type of graph to port graphs (Silberschatz 1981). Additional information was found in the form of a contribution of Vepstas (2008) on an encyclopedia article about hypergraphs. In that contribution, he says that a generalization of hypergraph allowing for edge-to-edge connections violate the axiom of Foundation of ZFC by allowing edge-loops. Indeed, like in figure 2.8, an edge $e_9 = \{e_{10}\}$
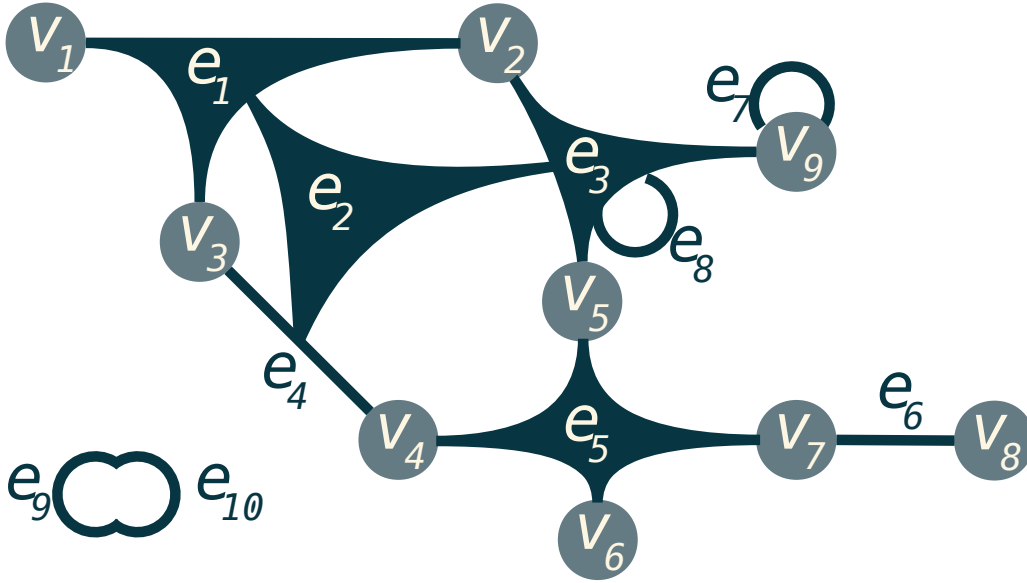
Figure 2.8: Example of hypergraph with total freedom on the edges specification.

can connect to another edge $e_{10} = \{e_9\}$ causing an infinite descent inside the $\in$ relation in direct contradiction with ZFC.

This shows the limits of standard mathematics especially on the field of knowledge representation. Some structures needs higher dimensions than allowed by the structure of ZFC and FOL. However, it is important not to be mistaken: such non-standard set theories are more general than ZFC and therefore contains ZFC as a special case. All is a matter of restrictions.

2.6 Sheaf

In order to understand sheaves, we need to present a few auxiliary notions. Most of these definitions are adapted from (Vepštas 2008). The first of which is a seed.

Definition 25 (Seed). A seed corresponds to a vertex along with the set of adjacent edges. Formally we note a seed $\star = (v, \chi_g(v))$ that means that a seed build from the vertex v in the graph g contains a set of adjacent edges $\chi_g(v)$. We call the vertex v the *germ* of the seed. The edges in a seed does not connect to the other vertices but keep the information and are able to match the correct vertices through typing (often a type of a single individual). We call the edges in a seed *connectors*.

Seeds are extracts of graphs that contains all information about a vertex. Illustrated in the figure 2.9, seeds have a central germ (represented with discs) and connectors leading to a typed vertex (outlined circles). Those external vertices are not directly contained in the seed but the information about what vertex can fit in them is kept.
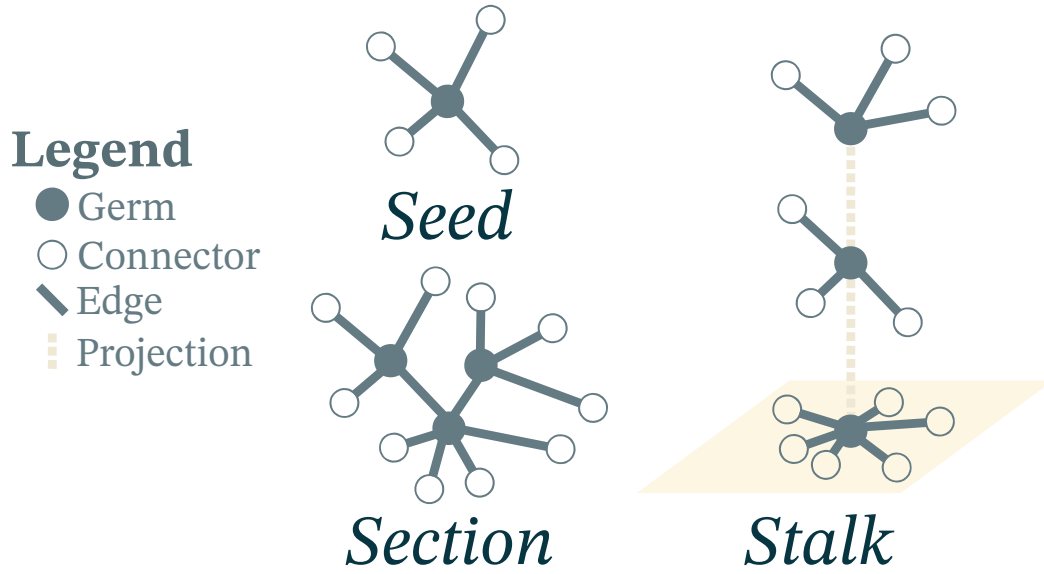
Figure 2.9: Example of a seed, a section and a stalk.

It is useful to represent connectors like jigsaw puzzle pieces: they can match only a restricted number of other pieces that match their shape.

From there, it is useful to build a kind of partial graph from seeds called sections.

Definition 26 (Section). A section is a set of seeds that have their common edges connected. This means that if two seeds have an edge in common connecting both germs, then the seeds are connected in the section and the edges are merged. We note $g_\star = (V, \{\!\{\cup : E_{section}\}\!\})$ the graph formed by the section.

In figure 2.9, a section is represented. It is a connected section composed of seeds along with the additional seeds of any vertices they have in common. They are very similar to subgraph but with an additional border of typed connectors. This tool was originally mostly meant for big data and categorization over large graphs. As graph quotient is often used in that domain, it was ported to sections instead of graphs allows us to define stalks.

Definition 27 (Stalk). Given a projection function $f : V \to V'$ over the germs of a section $\star$, the stalk above the vertex $v' \in V'$ is the quotient of all seeds that have their germ follow $f(v) = v'$.

The quotienning is used in stalks for their projection. Indeed, as shown in figure 2.9, the stalks are simply a collection of seeds with their germs quotiened into their common projection. The projection can be any process of transformation getting a set of seeds in one side and gives object in any base space called the image. Sheaves are a generalization of this concept to sections.

Definition 28 (Sheaf). A sheaf is a collection of sections, together with a projection. We note it $\mathcal{F} = \langle g_\star, glue \rangle$ with the function $glue$ being the gluing axioms that the

Stalk Field   *Sheaf*
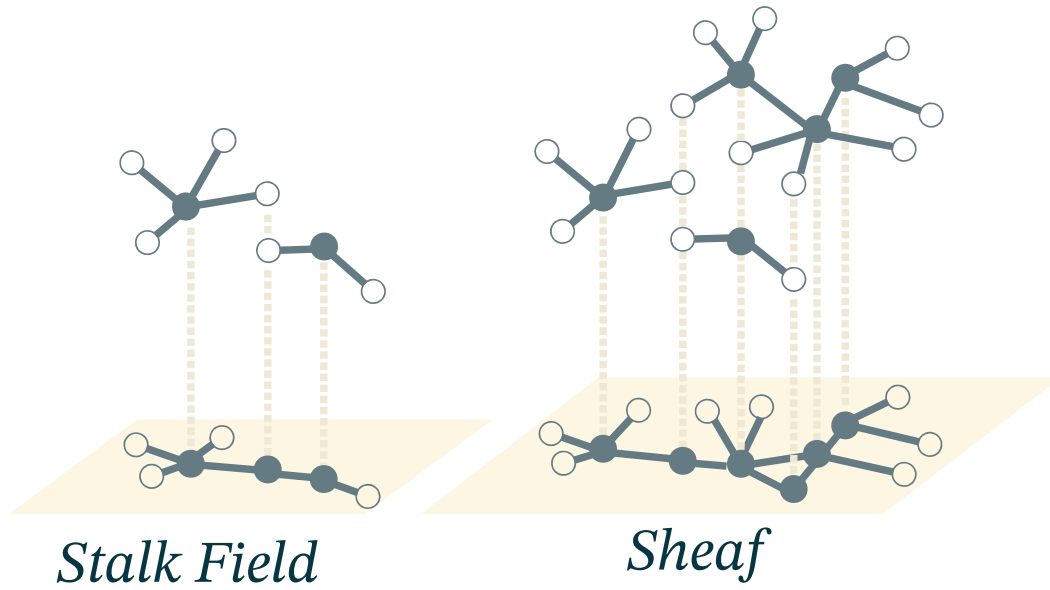
Figure 2.10: Example of sheaves.

projection should respect depending on the application. The projected sheaf graph is noted as the fusion of all quotiened sections:

$$glue_{\mathcal{F}} = \{\div_{glue_{\star}} : \{glue_{\star} \in g_{\star}\}$$

By merging common vertices into a section, we can build stack fields. These fields are simply a subcategory of sheaves. Illustrated in figure 2.10, a sheaf is a set of section with a projection relation.

3 Knowledge Representation

Figure 3.1: Noam Chomsky 2017

Knowledge representation is at the intersection of maths, logic, language and computer sciences. Knowledge description systems rely on syntax to interoperate systems and users to one another. The base of such languages comes from the formalization of automated grammars by Chomsky (1956). It mostly consists of a set of hierarchical rules aiming to deconstruct an input string into a sequence of terminal symbols. This deconstruction is called parsing and is a common operation in computer science. More tools for the characterization of computer language emerged soon after thanks to Backus (1959) while working on a programming language at IBM. This is how the Backus-Naur Form (BNF) metalanguage was created on top of Chomsky's formalization.

A similar process happened in the 1970's, when logic based knowledge representation gained popularity among computer scientists (Baader 2003). Systems at the time explored notions such as rules and networks to try and organize knowledge into a rigorous structure. At the same time other systems were built based on First Order Logic (FOL). Then, around the 1990's, the research began to merge in search of common semantics in what led to the development of Description Logics (DL). This domain is expressing knowledge as a hierarchy of classes containing individuals.

From there and with the advent of the world wide web, engineers were on the lookout for standardization and interoperability of computer systems. One such standardization took the name of "semantic web" and aimed to create a widespread network of connected services sharing knowledge between one another in a common language. At the beginning of the 21st century, several languages were created, all based on the World Wide Web Consortium (W3C) specifications called Resource Description Framework (RDF) (Klyne and Carroll 2004). This language is based on the notion of statements as triples. Each can express a unit of knowledge. All the underlying theoretical work of DL continued with it and created more expressive derivatives. One such derivative is the family of languages called Web Ontology Language (OWL) (Horrocks *et al.* 2003). Ontologies and knowledge graphs are more recent names for the representation and definition of categories (DL classes), properties and relation between concepts, data and entities.

Nowadays, when designing a knowledge representation, one usually starts with existing framework or generic knowledge database systems. The most popular in practice is certainly the classical relational database, followed closely by more novel methods for either big data or more expressive solutions like ontologies.

In our case we need a tool that is more expressive than ontologies while remaining efficient. Of course, this will lead to compromises, but can also have some interesting properties.

3.1 Grammar and Parsing

Grammar is an old tool that used to be dedicated to linguists. With the funding works by Chomsky and his Contex-Free Grammars (CFG), these tools became available to mathematicians and shortly after to computer scientists.

A CFG is a formal grammar that aims to generate a formal language given a set of hierarchical rules. Each rule is given a symbol as a name. From any finite input of text in a given alphabet, the grammar should be able to determine if the input is part of the language it generates.

3.1.1 BNF

In computer science, popular metalanguage called BNF was created shortly after Chomsky's work on CFG. The syntax is of the following form :

```
<rule> ::= <other_rule> | <terminal_symbol> | "literals"
```

A terminal symbol is a rule that does not depend on any other rule. It is possible to use recursion, meaning that a rule will use itself in its definition. This actually allows for infinite languages. Despite its expressive power, BNF is often used in one of its extended forms.

In this context, we present a widely used form of BNF syntax that is meant to be human readable despite not being very formal. We add the repetition operators * and + that respectively repeat 0 and 1 times or more the preceding expression. We also add the negation operator ~ that matches only if the following expression does not match. We also add parentheses for grouping expression and brackets to group literals.

Example 10. We can make a grammar for all sequencs of `A` using the rule `<scream> ::= "A"+`. If we want to make a rule that prevent the use of the letter `z` we can write `<no-sleep> ::= ~"z"`.

3.1.2 Dynamic Grammar

A regular grammar is static, it is set once and for all and will always produce the same language. In order to be more flexible we need to talk about dynamic grammars and their associated tools.

One of the main tools for both static and dynamic grammar is a parser. It is the program that will interpret the input into whatever usage it is meant for. Most of the time, a parser will transform the input into another similarly structured language. It can be a storage inside objects or memory, or compiled into another format, or even just for syntax coloration. Since a lot of usage requires the same kind of function, a new kind of tool emerged to make the creation of a parser simpler. We call those tools parser or compiler generators (Paulson 1982). They take a grammar description as input and gives the program of a parser of the generated language as an output.

For dynamic grammar, these tools can get more complicated. There are a few ways a grammar can become dynamic. The most straightforward way to make a parser dynamic is to introduce code in the rule handling that will tweak variables affecting the parser itself (Souto *et al.* 1998). This allows for handling context in CFG without needing to rewrite the grammar.

Another kind of dynamic grammar is grammar that can modify themselves. In order to do this a grammar is valuated with reified objects representing parts of itself (Hutton and Meijer 1996). These parts can be modified dynamically by rules as the input gets parsed (Renggli *et al.* 2010; Alessandro and Piumarta 2007). This approach uses Parsing Expression Grammars (PEG)(Ford 2004) with Packrat parsing that Packrat parsing backtracks by ensuring that each production rule in the grammar is not tested more than once against each position in the input stream (Ford 2002). While PEG is easier to implement and more efficient in practice than their classical counterparts (Loff *et al.* 2018; Henglein and Rasmussen 2017), it offset the computation load in memory making it actually less efficient in general (Becket and Somogyi 2008).

Some tools actually just infer entire grammars from inputs and software (Höschele and Zeller 2017; Grünwald 1996). However, these kinds of approaches require a lot of input data to perform well. They also simply provide the grammar after expensive computations.

3.2 Description Logics

On of the most standard and flexible way of representing knowledge for databases is by using ontologies. They are based mostly on the formalism of Description Logics (DL). It is based on the notion of classes (or types) as a way to make the knowledge hierarchically structured. A class is a set of individuals that are called instances of the classes. Classes got the same basic properties as sets but can also be constrained with logic formula. Constraints can be on anything about the class or its individuals. Knowledge is also encoded in relations that are predicates over attributes of individuals.

It is common when using DLs to store statements into three boxes (Baader 2003):

- The TBox for terminology (statements about types)
- The RBox for rules (statements about properties) (Bürckert 1994)
- The ABox for assertions (statements about individual entities)

These are used mostly to separate knowledge about general facts (intentional knowledge) from specific knowledge of individual instances (extensional knowledge). The extra RBox is for "knowhow" or knowledge about entity behavior. It restricts usages of roles (properties) in the ABox. The terminology is often hierarchically ordered using a subsumption relation noted $\subseteq$. If we represent classes or type as a set of individuals then this relation is akin to the subset relation of set theory.

There are several versions and extensions of DL. They all vary in expressivity. Improving the expressivity of DL system often comes at the cost of less efficient inference engines that can even become undecidable for some extensions of DL.

3.3 Ontologies and their Languages

Most AI problem needs a way to represent data. The classical way to represent knowledge has been more and more specialized for each AI community. Each their Domain Specific Language (DSL) that neatly fit the specific use it is intended to do. There was a time when the branch of AI wanted to unify knowledge description under the banner of the "semantic web". From numerous works, a repeated limitation of the "semantic web" seems to come from the languages used. In order to guarantee performance of generalist inference engines, these languages have been restricted so much that they became quite complicated to use and quickly cause huge amounts of recurrent data to be stored because of some forbidden representation that will push any generalist inference engine into undecidability.

The most basic of these languages is perhaps RDF Turtle (Beckett and Berners-Lee 2011). It is based on triples with an XML syntax and has a graph as its knowledge structure (Klyne and Carroll 2004). A RDF graph is a set of RDF triples $\langle sub, pro, obj \rangle$ which fields are respectively called subject, property and object. It can also be seen as a partially labeled directed graph (V, E) with V being the set of RDF nodes and E being the set of edges. This graph also comes with an incomplete label relation that associates an unique string called an Uniform Resource Identifier (URI) to most nodes. Nodes without an URI are called blank nodes. It is important that, while not named, blank nodes have a distinct internal identifier from one another that allows to differentiate them.

Built on top of RDF, the W3C recommended another standard called OWL. It adds the ability to have hierarchical classes and properties along with more advanced description of their arrity and constraints. OWL is, in a way, more expressive than RDF (Van Harmelen *et al.* 2008, 1,p825). It adds most formalism used in knowledge representation and is widely used and interconnected. OWL comes in three versions: OWL Lite, OWL DL and OWL Full. The lite version is less advanced but its inference is decidable, OWL DL contains all notions of DL and the full version contains all features of OWL but is strongly undecidable.

W3C (2004a)

```
ex:ontology rdf:type owl:Ontology .
ex:name rdf:type owl:DatatypeProperty .
ex:author rdf:type owl:ObjectProperty .
ex:Book rdf:type owl:Class .
ex:Person rdf:type owl:Class .

_:x rdf:type ex:Book .
_:x ex:author _:x1 .
_:x1 rdf:type ex:Person .
_:x1 ex:name "Fred"^^xsd:string .
```

The expressivity can also come from a lack of restriction. If we allow some freedom of expression in RDF statements, its inference can quickly become undecidable (Motik 2007). This kind of extremely permissive language is better suited for specific usage for other branches of AI. Even with this expressivity, several works still deem existing ontology system as not expressive enough, mostly due to the lack of classical constructs like lists, parameters and quantifiers that don't fit the triple representation of RDF.

One of the ways which have been explored to overcome these limitations is by adding a 4th field in RDF. This field is meant for context and annotations. This field is used for information about any statement represented as a triple, such as access rights, beliefs and probabilities, or most of the time the source of the data (Tolksdorf *et al.* 2004). One of the other uses of the fourth field of RDF is to reify statements (Hernández *et al.* 2015). Indeed by identifying each statement, it becomes possible to efficiently for statements about statements.

A completely different approach is done by Hart and Goertzel (2008) in his framework for Artificial General Intelligence (AGI) called OpenCog. The structure of the knowledge is based on a rhizome, a collection of trees, linked to one another. This structure is called Atomspace. Each vertex in the tree is an atom, leaf-vertexes are nodes, the others are links. Atoms are immutable, indexed objects. They can be given values that can be dynamic and, since they are not part of the rhizome, are an order of magnitude faster to access. Atoms and values alike are typed.

The goal of such a structure is to be able to merge concepts from widely different domains of AI. The major drawback being that the whole system is very slow compared to pretty much any domain specific software.

3.4 Self

As we have seen, most existing knowledge description systems have a common drawback: they are static. This means that they are either too inefficient or too specific. To fix this issue, a new knowledge representation system must be presented. The goal is to make a minimal language framework that can adapt to its use to become as specific as needed. If it becomes specific is must start from a generic base. Since that base language must be able to evolve to fit the most cases possible, it must be neutral and simple.

To summarize, that framework must maximize the following criteria:

1. **Neutral**: Must be independent from preferences and be localization.
2. **Permissive**: Must allow as many data representation as possible.
3. **Minimalist**: Must have the minimum number of base axioms and as little native notions as possible.
4. **Adaptive**: Must be able to react to user input and be as flexible as possible.

In order to respect these requirements, we developed a framework for knowledge description. This Structurally Expressive Language Framework (SELF) is our answer to these criteria. SELF is inspired by RDF Turtle and Description Logic.

3.4.1 Knowledge Structure

SELF extends the RDF graphs by adding another label to the edges of the graph to uniquely identify each statement. This basically turns the system into a quadruple storage even if this forth field is transparent to the user.

Axiom (Structure). A SELF graph is a set of statements that transparently include their own identity. The closest representation of the underlying structure of SELF is as follows:

$$g_{\mathbb{U}} = (\mathbb{U}, S) : S = \{s = \langle sub, pro, obj \rangle : s \in \mathcal{D} \vdash s \wedge \mathcal{D}\}$$

with:

- $sub, obj \in \mathbb{U}$ being entities representing the *subject* and *object* of the *statement s*,
- $pro \in P$ being the *property* of the statement s,
- $\mathcal{D} \subset S$ is the *domain* of the *world* $g_{\mathbb{U}}$,
- $S, P \subset \mathbb{U}$ with S the set of statements and P the set of properties,

This means that the world $g_{\mathbb{U}}$ is a graph with the set of entities $\mathbb{U}$ as vertices and the set of statements S as edges. This model also suppose that every statement s must be true if they belong to the domain $\mathcal{D}$. This graph is a directed 3-uniform hypergraph.

Since sheaves are a representation of hypergraphs, we can encode the structure of SELF into a sheaf-like form. Each seed is a statement, the germ being the statement vertex. It is always accompanied of an incoming connector (its subject), an outgoing connector (its object) and a non-directed connector (its property). The sections are domains and must be coherent. Each statement, along with its property, makes a stalk as illustrated in figure 3.2.
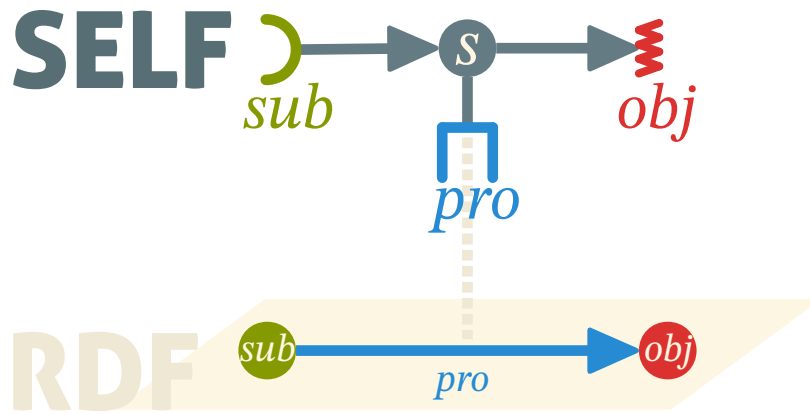
Figure 3.2: Projection of a statement from the SELF to RDF space.

The difference with a sheaf is that the projection function is able to map the pair statement-property into a labeled edge in its projection space. We map this pair into a classical labeled edge that connects the subject to the object of the statement in a directed fashion. This results in the projected structure being a correct RDF graph.

3.4.1.1 Consequences

The base knowledge structure is more than simply convenience. The fact that statements have their own identity, changes the degrees of freedom of the representa-

tion. RDF has a way to represent reified statements that are basically blank nodes with properties that are related to information about the subject, property and object of a designated statement. The problem is that such statements are very differently represented and need 3 regular statements just to define. Using the fourth field, it becomes possible to make statements about *any* statements. It also becomes possible to express modal logic about statements or to express, various traits like the probability or the access rights of a statement.

The knowledge structure holds several restrictions on the way to express knowledge. As a direct consequence, we can add several theorems to the logic system underlying SELF. The axiom of Structure is the only axiom of the system.

Theorem 1 (Identity). *Any entity is uniquely distinct from any other entity.*

This theorem comes from the axiom of Extensionality of ZFC. Indeed it is stated that a set is a unordered collection of distinct objects. Distinction is possible if and only if intrinsic identity is assumed. This notion of identity entails that a given entity cannot change in a way that would alter its identifier.

Theorem 2 (Consistency). *Any statement in a given domain is consistent with any other statements of this domain.*

Consistency comes from the need for a coherent knowledge database and is often a requirement of such constructs. This theorem also is a consequence of the axiom of Structure: $s \in \mathcal{D} \vdash s \wedge \mathcal{D}$.

Theorem 3 (Uniformity). *Any object in SELF is an entity. Any relations in SELF are restricted to $\mathbb{U}$.*

This also means that all native relations are closed under $\mathbb{U}$. This allows for a uniform knowledge database.

3.4.1.2 Native properties

In the following, we suppose all notions from previous chapter. The difference is that we define and use only a subset of the functions defined in the SELF formalism. In relation to the theory of SELF, we use the functional theory previously defined as the underlying formalism.

theorem 1 lead to the need for two native properties in the system : *equality* and *name*.

The **equality relation** $=: \mathbb{U} \to \mathbb{U}$, behaves like the classical operator. Since the knowledge database will be expressed through text, we also need to add an explicit way to identify entities. This identification is done through the **name relation** $\nu : \mathbb{U} \to L_{String}$ that affects a string literal to some entities. This lead us to introduce literals into SELF that is also entities that have a native value.

The axiom of Structure puts a type restriction on property. Since it compartments $\mathbb{U}$ using various named subsets, we must adequately introduce an explicit type system

into SELF. That type system requires a **type relation** (named using the colon) $: : \mathbb{U} \rightarrow T$. That relation is complete as all entities have a type. theorem 3 causes the set of entities to be universal. Type theory, along with Description Logic (DL), introduces a **subsumption relation** $\subseteq : T \rightarrow T$ as a partial ordering relation to the types. Since types can be seen as sets of instances we simply use the the subset relation from set theory. In our case, the entity type is the greatest element of the lattice formed by the set of types with the subsumption relation $(T, \subseteq)$.

The theorem 3 also allows for some very interesting meta-constructs. That is why we also introduce a signed **Meta relation** $\mu : \mathbb{U} \rightarrow D$ with $\mu^{\bullet} = \bullet\mu$. This allows to create domain from certain entities and to encapsulate domains into entities. $\mu^{\bullet}$ is for reification and μ is for abstraction. This Meta relation also allows to express value of entities, like lists or various containers.

To fulfill the principle of adaptability and in order to make the type system more useful, we introduce the **parameter relation** $\rho : \mathbb{U} \rightarrow \mathbb{U}$. This relation affects a list of parameters, using the meta relation, to some parameterized entities. This also allows for variables in statements.

Since axiom of Structure gives the structure of SELF a hypergraph shape, we must port some notions of graph theory into our framework. Introducing the **statement relation** $\chi : S \rightarrow \mathbb{U}$ reusing the same symbol as for the adjacency and incidence relation of graphs. This isn't a coincidence as this relation has the same properties.

Example 11. Since statements are tripples and edges, s_0 gives the subject of a statement s. Respectively, s_1 and s_2 give the property and object of any statement. For adjacencies, χ can give the set of statements any entity is the object or subject of. For any property pro, the notation $\chi(pro)$ gives the set of statements using this property.

This allows us to port all the other notions of graphs using this relation as a base.

In figure 3.3, we present all the native relations along with their domains and most subsets of $\mathbb{U}$.

3.4.2 Syntax

Since we need to respect the requirements of the problem, the RDF syntax cannot be used to express the knowledge. Indeed, RDF states native properties as English nodes with a specific URI that isn't neutral. It also isn't minimalist since it uses an XML syntax so verbose that it is not used for most examples in the documents that defines RDF because it is too confusing and complex (W3C 2004b; W3C 2004c). The XML syntax is also quite restrictive and cannot evolve dynamically to adapt to the usage.

So we need to define a new language that is minimalist and neutral. At the same time the language must be permissive and dynamic. These two goals are incompatible and will end up needing different solutions. So the solution to the problem is to actually define two languages that fit the criteria : one minimalist and one adaptive. The issue is that we need not make a user learn two languages and the second kind of language must be very specific and that violates the principle of neutrality we try to respect.

Figure 3.3: Venn diagram of subsets of $\mathbb{U}$ along with their relations. Dotted lines mean that the sets are defined a subset of the wider set.

The only solution is to make a mechanism to adapt the language as it is used. We start off with a simple framework that uses a grammar.

The description of $\mathbb{g}_0$ is pretty straightforward: it mostly is just a triple representation separated by whitespaces. The goal is to add a minimal syntax consistent with the axiom of Structure. In listing 3.1, we give a simplified version of $\mathbb{g}_0$. It is written in a pseudo-BNF fashion, which is extended with the classical repetition operators ∗ and + along with the negation operator ~. All tokens have names in uppercase. We also add the following rule modifiers:

- `<~name>` are ignored for the parsing. However, the tokens are consumed and therefore acts like separators for the other rules.
- `<?name>` are inferred rules and tokens. They play a key role for the process of derivation explained in section 3.4.3.

In order to respect the principle of neutrality, the language must not suppose of any regional predisposition of the user. There are few exceptions for the sake of convenience and performance. The first exception is that the language is meant to be read from left to right and have an occidental biased `subject verb object` triple description. Another exception is for liberals that use the same grammar as in classical Java. This means that the decimal separator is the dot (`.`). This could be fixed in later version using dynamic definitions (see section 7.2.1).

Even if sticking to the ASCII subset of characters is a good idea for efficiency, SELF can work with UTF-8 and exploits the Unicode Character Database (UCD) for its token definitions (Unicode Consortium 2018b). This means that SELF comes keywords free

Listing 3.1 Simplified pseudo-BNF description for basic SELF.

```
<~COMMENT: <INLINE: "//" (~["\n", "\r"])*>
| <BLOCK: "/*" (~["*/"])*> > //Ignored
<~WHITE_SPACE: " "|"\t"|"\n"|"\r"|"\f">
<LITERAL: <INT> | <FLOAT> | <CHAR> | <STRING>> //Java definition$\label[line]{line:literal}$
<ID: <TYPE: <UPPERCASE>(<LETTERS>|<DIGITS>)* > > $\label[line]{line:uppercase}$
| <ENTITY:  <LOWERCASE>(<LETTERS>|<DIGITS>)*>
| <SYMBOL: (~[<LITERALS>, <LETTERS>, <DIGITS>])*>>

<worselfld> ::= <first> <statement>* <EOF>
<first> ::= <subject> <?EQUAL> <?SOLVE> <?EOS> $\label[line]{line:first}$
<statement> ::= <subject> <property> <object> <EOS> $\label[line]{line:statement}$
<subject> ::= <entity>
<property> ::= <ID> | <?meta_property>
<object> ::= <entity>
<entity> ::= <ID> | <LITERAL> | <?meta_entity>
```

and that the definition of each symbol is left to the user. Each notion and symbol is inferred (with the exception of the first statement which is closer to an imposed configuration file).

In $\mathbb{g}_0$, the first two token definitions are ignored. This means that comments and white-spaces will act as separation and won't be interpreted. Comments are there only for convenience since they do not serve any real purpose in the language. It was arbitrarily decided to use Java-style comments. White-spaces are defined against UCD's definition of the separator category `Z&` (see Unicode Consortium 2018a, chap. 4).

?? uses the basic Java definition for liberals. In order to keep the independence from any natural language, boolean laterals are not natively defined (since they are English words).

Another aspect of that language independence is found starting at **??** where the definitions of `<UPPERCASE>`, `<LOWERCASE>`, `<LETTERS>` and `<DIGITS>` are defined from the UCD (respectively categories `Lu`, `Ll`, `L&`, `Nd`). This means that any language's upper case can be used in that context. For performance and simplicity reasons we will only use ASCII in our examples and application.

The rule at **??** is used for the definition of three tokens that are important for the rest of the input. `<EQUAL>` is the symbol for equality and `<SOLVE>` is the symbol for the *solution quantifier* (and also the language pendant of $\mu^\bullet$). The most useful token `<EOS>` is used as a statement delimiter. This rule also permits the inclusion of other files if a string literal is used as a subject. The underlying logic of this first statement will be presented in section 3.4.5.1. In the following examples we will consider that `<EQUAL>` ::= "=", `<SOLVE>` ::= "?" and `<EOS>` ::= ";".

At **??**, we can see one of the most defining features of $\mathbb{g}_0$: statements. The input is nothing but a set of statements. Each component of the statements are entities. We defined two specific rules for the subject and object to allow for eventual runtime

modifications. The property rule is more restricted in order to guarantee the non-ambiguity of the grammar.

3.4.3 Dynamic Grammar

The syntax we described is only valid for $\mathbb{g}_0$. As long as the input is conforming to these rules, the framework keeps the minimal behavior. In order to access more features, one needs to break a rule. We add a second outcome to handling with violations : **derivation**. There are several kinds of possible violations that will interrupt the normal parsing of the input :

- Violations of the `<first>` statement rule : This will cause a fatal error.
- Violations of the `<statement>` rule : This will cause a derivation if an unexpected additional token is found instead of `<EOS>`. If not enough tokens are present, a fatal error is triggered.
- Violations of the secondary rules (`<subject>`, `<entity>`, ...) : This will cause a fatal error except if there is also an excess of token in the current statement which will cause derivation to happen.

Derivation will cause the current input to be analyzed by a set of meta-rules. The main restriction of these rules is given in $\mathbb{g}_0$: each statement must be expressible using a triple notation. This means that the goal of the meta-rules is to find an interpretation of the input that is reducible to a triple and to augment $\mathbb{g}_0$ by adding an expression to any `<meta_*>` rules. If the input has fewer than 3 entities for a statement then the parsing fails. When there is extra input in a statement, there is a few ways the infringing input can be reduced back to a triple.

3.4.3.1 Containers

The first meta-rule is to infer a container. A container is delimited by at least a left and right delimiter (they can be the same symbol) and an optional middle delimiter. We infer the delimiters using the algorithm 1.

The function sortedDelimiters at line 1 is used to generate every ordered possibility and sort them using a few criteria. The default order is possibilities grouped from left to right. All coupled delimiters that are mirrors of each other following the UCD are preferred to other possibilities.

Checking the result of the choice is very important. At line 1 a function checks if the delimiters allow for triple reduction and enforce restrictions.

Example 12. For example, a property cannot be wrapped in a container (except if part of parameters). This is done in order to avoid a type mismatch later in the interpretation.

Once the inference is done, the resulting calls to inferDelimiter will add the rules listed in listing 3.2 to $\mathbb{g}_0$. This function will create a `<container>` rule and add it to the definition of `<meta_entity>`. Then it will create a rule for the container named after the

---

Algorithm 1 Container meta-rule

```
 1: function CONTAINER(Token current)
 2:     LOOKAHEAD(current, EOS)                       ▷ Populate all tokens of the statement
 3:     for all token in horizon do
 4:         if token is a new symbol then delimiters.APPEND(token)
 5:     if LENGTH(delimiters) <2 then
 6:         if COHERENTDELIMITERS(horizon, delimiters[0]) then
 7:             INFERMIDDLE(delimiters[0])            ▷ New middle delimiter in existing containers
 8:             return Success
 9:         return Failure
10:     while LENGTH(delimiters) > 0 do
11:         for all (left, middle, right) in SORTEDDELIMITERS(delimiters) do
12:             if COHERENTDELIMITERS(horizon, left, middle, right) then
13:                 INFERDELIMITER(left, right)
14:                 INFERMIDDLE(middle)                          ▷ Ignored if null
15:                 delimiters.REMOVE(left, middle, right)
16:                 break
17:         if LENGTH(delimiters) stayed the same  then return Success
18:     return Success
```

UCD name of the left delimiter (searching in the `NamesList.txt` file for an entry starting with "left" and the rest of the name or defaulting to the first entry). Those rules are added as a conjunction list to the rule `<container>`. It is worthy to note that the call to inferMiddle will add rules to the token `<MIDDLE>` independently from any container and therefore, all containers share the same pool of middle delimiters.

Listing 3.2 Rules added to the current grammar for handling the new container for parenthesis

---

```
<meta_entity> ::= <container>
<container> :: = <parenthesis> | …
<parentesis> ::= "(" [<naked_entity>] (<?MIDDLE> <naked_entity>)* ")"
<naked_entity> ::= <statement> | <entity>$\label{line:meta_statement}$
```

The rule at **??** is added once and enables the use of meta-statements inside containers. It is the language pendant of the μ relation, allowing to wrap abstraction in a safe way.

Example 13. If we parse the expression `a = (b,c);`, we start by tokenizing it as `<ENTITY> <EQUAL> <SYMBOL><ENTITY><SYMBOL><ENTITY><SYMBOL> <EOS>` (ignoring whitespaces and comments). This means that the statement is 4 tokens too long to form a triple. This triggers a parsing error and then an evaluation using meta-rules. All the `<ID>` tokens are new symbols, but they don't have the same subtype. This means that candidate delimiters are `(`, `,` and `)`. To Infer the inference there's only one combination and the left delimiter and right delimiters are found via their Unicode description. The comma is left to be infered as the middle delimiter. The grammar is rewriten and the statement becomes `<ENTITY> <EQUAL> <container> <EOS>` which is a valid triple statement.

3.4.3.2 Parameters

If the previous rule didn't fix the parsing of the statement, we continue with the following meta-rule. Parameters are extra containers that are used after an entity. Every container can be used as parameters. We detail the analysis in algorithm 2.

Algorithm 2 Parameter meta-rule

1: **function** PARAMETER(Entity[] statement)
2:      reduced = statement
3:      **while** LENGTH(reduced) >3 **do**
4:          **for** i from 0 to LENGTH(reduced) - 1 **do**
5:              **if** NAME(reduced[i]) not null and
6: TYPE(reduced[i+1]) = Container and
7: COHERENTPARAMETERS(reduced, i) **then**
8:                  param = INFERPARAMETER(reduced[i], reduced[i+1])
9:                  reduced.REMOVE(reduced[i], reduced[i+1])
10:                 reduced.INSERT(param, i)   ▷ *Replace parameterized entity*
11:                 **break**
12:         **if** LENGTH(statement) stayed the same **then return** Success
13:     **return** Failure

The goal is to match extra containers with the preceding named entity. The container is then combined with the preceding entity into a parameterized entity.

The call to inferParameter will add the rule in listing 3.3, replacing `<?container>` with the name of the container used.

Example 14. In this case we can have to parse `f(x) = x;`. If we already have the parentesis delimiter defined, it becomes `<ENTITY> <container> <EQUAL> <ENTITY> <EOS>`. Since the statement isn't a triple we execute the meta-rules. The container rule finds no new symbols and fails. Then the parameter meta-rule will reduce the statement by bounding the first entity to the following container and mark it as a parameterized entity. This gives `<meta_entity> <EQUAL> <ENTITY> <EOS>`.

Listing 3.3 Rules added to the current grammar for handling parameters

---
```
<meta_entity>   ::= <ID> <?container>
<meta_property> ::= <ID> <?container>
```

3.4.3.3 Operators

A shorthand for parameters is the operator notation. It allows to affect a single parameter to an entity without using a container. It is most used for special entities like quantifiers or modificators. This is why, once used, the parent entity takes a polymorphic type, meaning that type inference will not issue errors for any usage of them. Details of the way the operators are reduced is exposed in algorithm 3.

From the call of inferOperator, comes new rules explicated in listing 3.4. The call also adds the operator entity to an inferred token `<OP>`.

Algorithm 3 Operator meta-rule

```
 1: function OPERATOR(Entity[] statement)
 2:     reduced = statement
 3:     while LENGTH(reduced) >3 do
 4:         for i from 0 to LENGTH(reduced) - 1 do
 5:             if ν(reduced[i]) not null and
 6: ν(reduced[i+1]) not null and
 7: (ν(reduced[i]) is a new symbol or
 8: reduced[i] has been parameterized before) and
 9: COHERENTOPERATOR(reduced, i) then
10:                 op = INFEROPERATOR(reduced[i], reduced[i+1])
11:                 reduced.REMOVE(reduced[i], reduced[i+1])
12:                 reduced.INSERT(op, i)          ▷ Replace parameterized entity
13:                 break
14:         if LENGTH(statement) stayed the same  then return Success
15:     return Failure
```

Listing 3.4 Rules added to the current grammar for handling operators

```
<meta_entity> ::= <?OP> <ID>
<meta_property> ::= <?OP> <ID>
```

Example 15. With the input `!x = 0;` we parse `<SYMBOL> <ENTITY> <EQUAL>` `<LITERAL> <EOS>`. This cannot be a container since the new symbol is at the begining without any mirroring possible. It cannot be a parameter since no container is present. But this will conclude with the operator meta-rule as the new symbol precedes an entity. This becomes `<meta_entity> <EQUAL> <LITERAL> <EOS>` and becomes a valid statement.

If all meta-rules fail, then the parsing fails and returns an error to the user like in classical occurences.

3.4.4 Contextual Interpretation

While parsing another important part of the processing is done after the success of a grammar rule. The grammar in SELF is valuated, meaning that each rule has to return an entity. A set of functions are used to then populate the database with the right entities or retrieve an existing one that correspond to what is being parsed.

When parsing, the rules `<entity>` and `<property>` will ask for the creation or retrieval of an entity. This mechanism will use the name of the entity and its type to retrieve an entity with the same name in a given scope.

3.4.4.1 Naming and Scope

When parsing an entity by name, the system will first request for an existing entity with the same name. If such an entity is retrieved, it is returned instead of creating a

new one. The validity of a name is limited by the notion of scope.

A scope is the reach of an entity's direct influence. It affects the naming relation by removing variable's names. Scopes are delimited by containers and statements. This local context is useful when wanting to restrict the scope of the declaration of an entity. The main goal of such restriction is to allow for a similar mechanism as the RDF namespaces. This also makes the use of variable (RDF blank nodes) possible.

The scope of an entity has three special values :

- Variable: This scope restricts the scope of the entity to only the other entities in its scope.
- Local: This scope means that the parsing is still populating the scope of the entity. Its scope is limited to the currently parsing statement.
- Global: This scope means the name has no scope limitation.

The scope of an entity also contains all its parent entities, meaning all containers or statement the entity is part of. This is used when choosing between the special values of the scope. The process is detailed in algorithm 4.

Algorithm 4 Determination of the scope of an entity

1: **function** INFERSCOPE(Entity e)
2:     Entity[] reach = []
3:     **if** : $(e) = S$ **then**
4:         **for all** $i \in \chi(e)$ **do** reach.APPEND(INFERVARIABLE(i))   ▷ *Adding scopes nested in statement e*
5:     **for all** $i \in \mu^{\cdot}(e)$ **do** reach.APPEND(INFERVARIABLE(i))   ▷ *Adding scopes nested in container e*
6:     **if** $\exists \rho(e)$ **then**
7:         Entity[] param = INFERSCOPE($\rho(e)$)
8:         **for all** $i \in$ param **do** param.REMOVE(INFERSCOPE(i))   ▷ *Remove duplicate scopes from parameters*
9:         **for all** $i \in$ param **do** reach.APPEND(INFERVARIABLE(i))   ▷ *Adding scopes from paramters of e*
10:     SCOPE(e) ← reach
11:     **if** GLOBAL $\notin$ SCOPE(e) **then** SCOPE(e) ← SCOPE(e) $\cup$\{LOCAL\}
    return reach
12: **function** INFERVARIABLE(Entity e)
13:     Entity[] reach = []
14:     **if** LOCAL $\in$ SCOPE(e) **then**
15:         **for all** $i \in$ SCOPE(e) **do**
16:             **if** $\exists e_p \in \mathbb{U} : \rho(p) = i$ **then**   ▷ *e is already a parameter of another entity e_p*
17:                 SCOPE(e) ← SCOPE(e) $\backslash$\{LOCAL\}
18:                 SCOPE(e_p) ← SCOPE(e_p) $\cup$ SCOPE(e)
19:                 SCOPE(e) ← SCOPE(e) $\cup$\{VARIABLE, p\}
20:         reach.APPEND(e)
21:         reach.APPEND(SCOPE(e))
    return reach

The process happens for each entity created or requested by the parser. If a given entity is part of any other entity, the enclosing entity is added to its scope. When an entity is enclosed in any entity while already being a parameter of another entity, it becomes a variable.

3.4.4.2 Instanciation identification

When a parameterized entity is parsed, another process starts to identify if a compatible instance already exists. From theorem 1, it is impossible for two entities to share the same identifier. This makes mandatory to avoid creating an entity that is equal to an existing one. Given the order of which parsing is done, it is not always possible to determine the parameter of an entity before its creation. In that case a later examination will merge the new entity onto the older one and discard the new identifier.

3.4.5 Structure as a Definition

The derivation feature on its own does not allow to define most of the native properties. For that, one needs a light inference mechanism. This mechanism is part of the default inference engine. This engine only works on the principle of structure as a definition. Since all names must be neutral from any language, that engine cannot rely on regular mechanisms like configuration files with keys and values or predefined keywords.

To use SELF correctly, one must be familiar with the native properties and their structure or implement their own inference engine to override the default one.

3.4.5.1 Quantifiers

What are quantifiers? In SELF they differ from their mathematical counterparts. The quantifiers are special entities that are meant to be of a generic type that matches any entities including quantifiers. There are infinitely many quantifiers in SELF but they are all derived from a special one called the *solution quantifier*. We mentioned it briefly during the definition of the grammar $\mathbb{g}_0$. It is the language pendant of $\mu^\bullet$ and is used to extract and evaluate reified knowledge.

::: example For example, the statement `bob is <SOLVE>(x)` will give either a default container filled with every value that the variable `x` can take or if the value is unique, it will take that value. If there is no value it will default to `<NULL>`, the exclusion quantifier.

How are these other quantifiers defined? We use a definition akin to Lindstöm quantifiers (1966) which is a generalization of counting quantifiers (Gradel *et al.* 1997). Meaning that a quantifier is defined as a constrained range over the quantified variable. We suppose five quantifiers as existing in SELF as native entities.

- The **solution quantifier** `<SOLVE>` noted $\S$ in classical mathematics, makes a query that turns the expression into the possible range of its variable.
- The **universal quantifier** `<ALL>` behaves like $\forall$ and forces the expression to affect every possible value of its variable.
- The **existential quantifier** `<SOME>` behaves like $\exists$ and forces the expression to match *at least one* value for its variable.
- The **uniqueness quantifier** `<ONE>` behaves like $!\exists$ and forces the expression to match *exactly one* value for its variable.

- The **exclusion quantifier** `<NULL>` behaves like ¬∃ and forces the expression to match none of the value of its variable.

The last four quantifiers are inspired from Aristotle's square of opposition (D'Alfonso 2011).
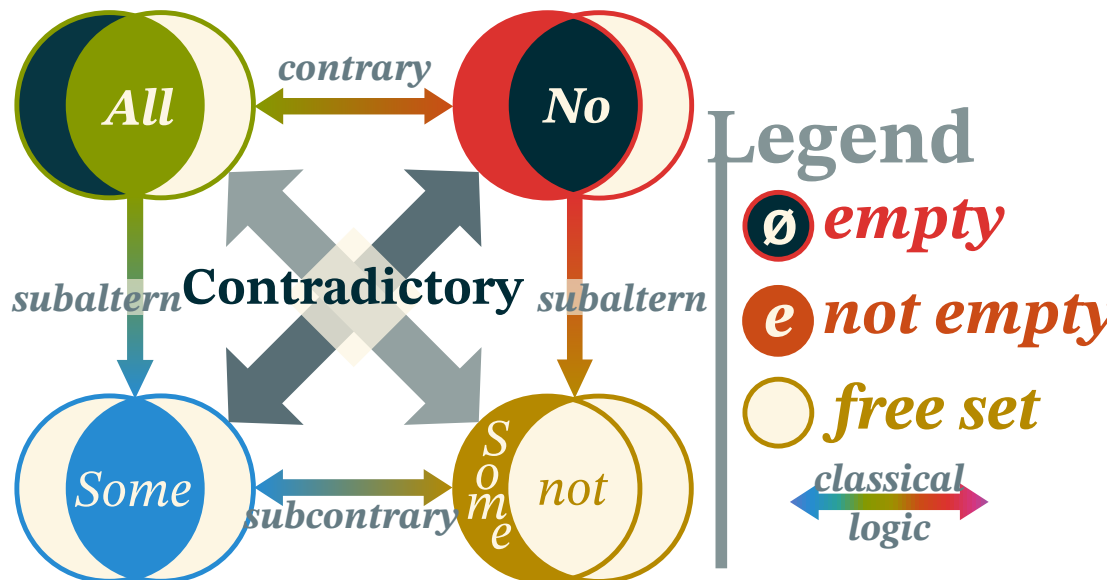
Figure 3.4: Aristotle's square of opposition

In SELF, quantifiers are not always followed by a quantified variable and can be used as a value. In that case the variable is simply anonymous. We use the exclusion quantifier as a value to indicate that there is no value, sort of like `null` or `nil` in programming languages.

In listing 3.5, we present an example file that is meant to define most of the useful native properties along with default quantifiers.

At **??**, we give the first statement that defines the solution quantifier's symbol. The reason this first statement is shaped like this is that global statements are always evaluated to be a true statement. This means that anything equaling the solution quantifier at this level will be evaluated as a domain. If it is a string literal, then it must be either a file path or URL or a valid SELF expression. If it is a single entity then it becomes synonymous to the entire SELF domain and therefore contains everything. We can infer that it becomes the universal quantifier.

All statements up to **??** are quantifiers definitions. On the left side we got the quantifier symbol used as a parameter to the solution quantifier using the operator notation. On the right we got the domain of the quantifier. The exclusive quantifier has as a range the empty set. For the existential quantifier we have only a restriction of it not having an empty range. At last, the uniqueness quantifier got a set with only one element matching its variable (noting that anonymous variables doesn't match necessarily other anonymous variables in the same statement).

Listing 3.5 The default lang.w file.

```
* =? ;$\label{line:first}$
?(x) = x; //Optional definition
?~ = { };
?_ ~(=) ~;
?!_ = {_};$\label{line:endquantifier}$

(*e, !T) : (e :: T); *T : (T :: Type);$\label{line:typing}$
*T : (Entity / T);$\label{line:subsumption}$

:: :: Property(Entity, Type);
(___) :: Statement;
(~, !, _, *) :: Quantifier;
( )::Group;
{ }::Set;
[ ]::List;
< >::Tuple;
Collection/(Set,List,Tuple);
0 :: Integer; 0.0::Float;
'\0'::Character; ""::String;
Literal/(Boolean, Integer, Float, Character, String);

(*e, !(s::String)) : (e named s);$\label{line:naming}$
(*e(p), !p) : (e param p);$\label{line:param}$
*(s p o):(((s p o) subject s),((s p o) property p),((s p o) object o));$\label{line:incidence}$
```

In that file the type hierarchy can be illustrated by the figure 3.5. It consist of entities that are either parameterized or not and that have a value or not.
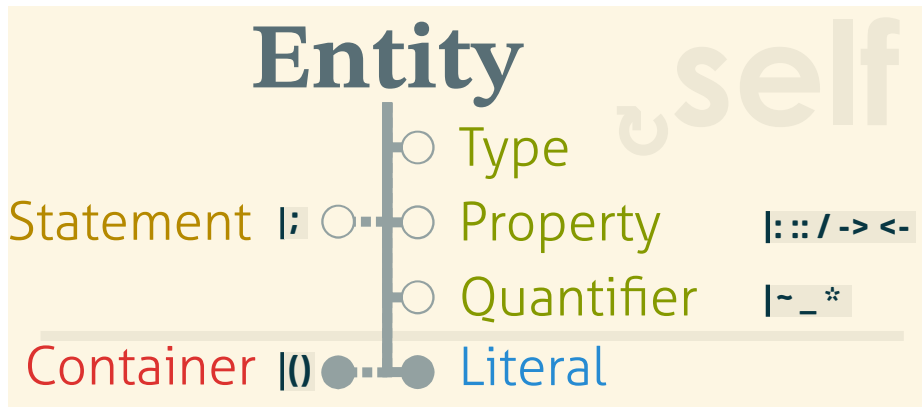
Figure 3.5: Hierarchy of types in SELF

3.4.5.2 Inferring Native Properties

All native properties can be inferred by structure using quantified statements. Here is the structural definition for each of them:

- = (at **??**) is the equality relation given in the first statement.

- $\subseteq$ (at **??**) is the first property to relate a particular type relative to all types. That type becomes the entity type.
- $\mu^\bullet$ (at **??**) is the solution quantifier discussed above given in the first statement.
- μ is represented using containers.
- ν (at **??**) is the first property affecting a string literal uniquely to each entity.
- ρ (at **??**) is the first property to effect to all entities a possible parameter list.
- : (at **??**) is the first property that matches every entity to a type.
- χ (at **??**) is the first property to match for all statements.

We limit the inference to one symbol to make it much simpler to implement and to retrieve but, except for false positives, there are no reason it should not be possible to define several notations for each relation.

3.4.6 Extended Inference Mechanisms

In this section we present the default inference engine. It has only a few functionalities. It isn't meant to be universal and the goal of SELF is to provide a framework that can be used by specialists to define and code exactly what tools they need.

Inference engines need to create new knowledge but this knowledge shouldn't be simply merged with the explicit domain. Since this knowledge is inferred, it is not exactly part of the domain but must remain consistent with it. This knowledge is stored in a special scope dedicated to each inference engine. This way, inference engines can use defeasible logic or have dynamic inference from any knowledge insertion in the system.

3.4.6.1 Type Inference

Type inference works on matching types in statements. The main mechanism consists in inferring the type of properties in a restrictive way. Properties have a parameterized type with the type of their subject and object. The goal is to make that type match the input subject and object.

For that we start by trying to match the types. If the types differ, the process tries to reduce the more general type against the lesser one (subsumption-wise). If they are incompatible, the inference uses some light defeasible logic to undo previous inferences. In that case the types are changed to the last common type in the subsumption tree.

However, this may not always be possible. Indeed, types can be explicitly specified as a safeguard against mistakes. If that's the case, an error is raised and the parsing or knowledge insertion is interrupted.

3.4.6.2 Instanciation

Another inference tool is instantiation. Since entities can be parameterized, they can also be defined against their parameters. When those parameters are variables, it allows entities to be instantiated later.

This is a complicated process because entities are immutable. Indeed, parsing happens from left to right and therefore an entity is often created before all the instantiation information are available. Even harder are completion of definition in several separate statements. In all cases, a new entity is created and then the inference realize that it is either matching a previous definition and will need to be merged with the older entity or it is a new instance and needs all properties duplicated and instantiated.

This gives us two mechanisms to take into account: merging and instanciating.

Merging is pretty straightforward: the new entity is replaced with the old one in all of the knowledge graph. containers, parameterized entities, quantifiers and statements must be duplicated with the correct value and the original destroyed. This is a heavy and complicated process but seemingly the only way to implement such a case with immutable entities.

Instanciating is similar to merging but even more complicated. It starts with computing a relation that maps each variable that needs replacing with their grounded value. Then it duplicates all knowledge about the parent entity while applying the replacement map.

3.5 Example

In the following section, a use case of the framework will be presented. First we have to explain a few notions.

3.5.1 Modal logic

In the field of logic there exists one special flavor of it called *modal logic*. It lays the emphasis upon the qualification of statements, and especially the way they are interpreted. This is a very appropriate example for SELF. The modality of a statement acts like a modifier, it specifies a property regarding its plausibility, origin or validity.

Example 16. In the figure 3.6, we present a case of three women gossiping, Alice, Becky and Carol. The presentation is inspired by the work of Schwarzentruber (2018). Here are a list of the statements in this example:

- Alice said to Becky that Carol should *probably* change her style from C_1 to C_2.
- Becky said to Alice that she finds the Carol's style *usually* good.
- Alice told Carol that Becky told her that she should *sometimes* change her style to C_2.

The following statement can be infered:

- Carol *possibly* thinks that Becky thinks that the style C_2 is *often* good.
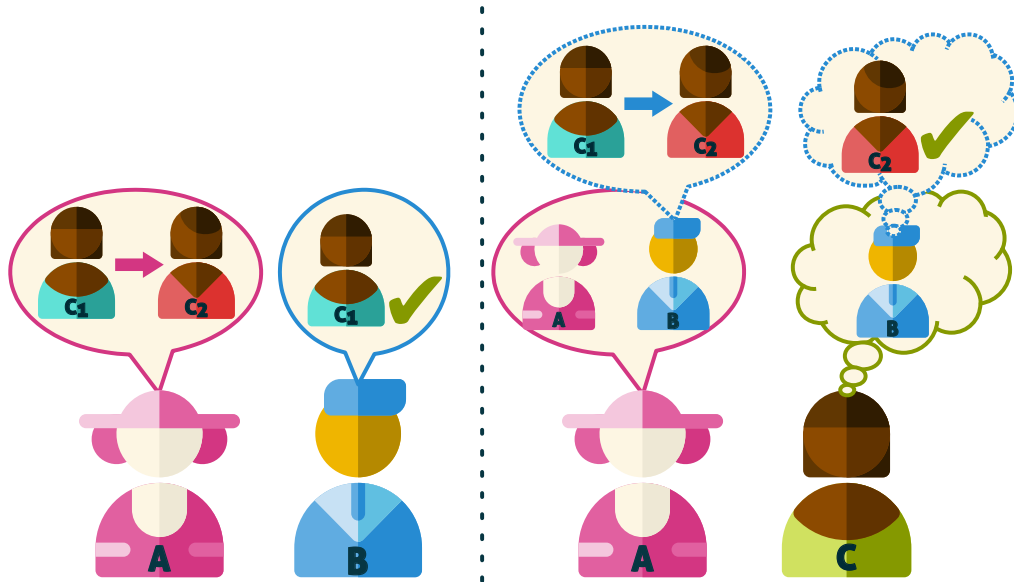
Figure 3.6: Example of modal logic propositions: Alice gossips about what Beatrice said about Claire

In the example, all modality are *emphasized*. One can notice an interesting property of these statements in that they are about other statements. This king of description is called *higher order knowledge*.

3.5.2 Higher order knowledge

SELF is based on the ability to easily process higher order knowledge. In that case the term *order* refers to the order of abstraction of a statement. For such usages, an hypergraph structure such as SELF is using is a clear advantage in term of expressivity and ease of manipulation of those statements.

In the following listings, we present the example using RDF and SELF to encode the knowledge.

```
@prefix : <http://genn.io/self/gossip#> .
@prefix owl: <http://www.w3.org/2002/07/owl#> .
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix xml: <http://www.w3.org/XML/1998/namespace> .
@prefix xsd: <http://www.w3.org/2001/XMLSchema#> .
@prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .
```

```
@base <http://genn.io/self/gossip> .

<http://genn.io/self/gossip> rdf:type owl:Ontology ;
                             owl:imports rdf: .

:modality rdf:type owl:AnnotationProperty ;
          rdfs:range :Modality .

:told_a rdf:type owl:ObjectProperty .
:told_b rdf:type owl:ObjectProperty .
:told_c rdf:type owl:ObjectProperty .
:Modality rdf:type owl:Class .
:good rdf:type owl:NamedIndividual .
:is rdf:type owl:NamedIndividual ,
             rdf:Property .
:probably rdf:type owl:NamedIndividual ,
                   :Modality .
:s1 rdf:type owl:NamedIndividual ,
             rdf:Statement ;
    rdf:object :c2 ;
    rdf:predicate :worsethan ;
    rdf:subject :c ;
    :modality :probably .
:s2 rdf:type owl:NamedIndividual ;
    rdf:object :good ;
    rdf:predicate :is ;
    rdf:subject :c ;
    :modality :usually .
:s3 rdf:type owl:NamedIndividual ,
             rdf:Statement ;
    rdf:object :s4 ;
    rdf:predicate :told_a ;
    rdf:subject :b .
:s4 rdf:type owl:NamedIndividual ,
             rdf:Statement ;
    rdf:object :c2 ;
    rdf:predicate :should ;
    rdf:subject :c ;
    :modality :sometimes .
:should rdf:type owl:NamedIndividual ,
                 rdf:Property .
:sometimes rdf:type owl:NamedIndividual ,
                    :Modality .
:told_a rdf:type owl:NamedIndividual ,
                rdf:Property .
:usually rdf:type owl:NamedIndividual ,
                  :Modality .
:worsethan rdf:type owl:NamedIndividual ,
                    rdf:Property .
:a rdf:type owl:NamedIndividual ;
   :told_b :s1 ;
   :told_c :s3 .
:b rdf:type owl:NamedIndividual ;
   :told_a :s2 .
:c rdf:type owl:NamedIndividual .
:c2 rdf:type owl:NamedIndividual .


"lang.s" = ? ;
a told(b) probably(c worsethan ctwo);
```

```
b told(a) usually(c is good);
a told(c) (b told(a) sometimes(c should ctwo));
```

It is obvious that the SELF version is an order of magnitude more concise than RDF to express modal logic. In the RDF version we use the reified statements `:s1`, `:s2`, `:s3` and `:s4` along with a `:modality` anotation to express high order knowledge and modalities. In SELF, everything is infered by structure and one can start exploiting their database right away.

4 General Planning Framework

When making artificial intelligence systems, an important feature is the ability to make decisions and act accordingly. To act, one should plan ahead. This is why the field of automated planning is being actively researched in order to find efficient algorithms to find the best course of action in any given situation. The previous chapter allowed to lay the bases of knowledge representation. How knowledge about the planning domains are represented is a main factor to take into account in order to conceive any planning algorithm.

Automated planning really started being formally investigated after the creation of the Stanford Research Institute Problem Solver (STRIPS) by Fikes and Nilsson (1971). This is one of the most influential planner, not because of its algorithm but because of its input language. Any planning system needs a way to express the information related to the input problem. Any language done for this purpose is called an *action language*. STRIPS will be mostly remembered for its eponymous action language that is at the base of any modern derivatives.

All action language is based on mainly two notions: *actions* and *states*. A state is a set of *fluents* that describe aspects of the world modeled by the domain. Each action has a logic formula over states that allows its correct execution. This requirement is called *precondition*. The mirror image of this notion are possible *effects* which are logic formula that are enforced on the current state after the action is executed. The domain is completed with a problem, most of the time specified in a separate file. The problem basically contains two states: the *initial* and *goal* states.

4.1 Illustration

To illustrate how automated planners works, we introduce a typical planning problem called **block world**.

Example 17. In this example, a robotic grabing arm tries to stack blocks on a table in a specific order. The arm is only capable of handling one block at a time. We suppose that the table is large enough so that all the blocks can be put on it without any stacks. Figure 4.1 illustrates the setup of this domain.

The possible actions are `pickup`, `putdown`, `stack` and `unstack`. There are at least three fluents needed:

- one to state if a given block is `down` on the table,
- one to specify which block is `held` at any moment and
- one to describe which block is stacked `on` which block.

Figure 4.1: The block world domain setup.

We also need a special block to state when `noblock` is held or on top of another block. This block is a constant.

The knowledge we just described is called *planning domain*.

In that example, the initial state is described as stacks and a set of blocks directly on the table. The goal state is usually the specification of one or many stacks that must be present on the table. This part of the description is called *planning problem*.

In order to solve it we must find a valid sequence of actions called a *plan*. If this plan can be executed in the initial state and result in the goal state it is called a *solution* of the planning problem. To be executed, each action must be done in a state satisfying its precondition and will alter that state according to its effects. A plan can be executed if all its action can be executed in the sequence of the plan.

Example 18. For example, in the block world domain we can have an initial state with the $blockB$ ontop of $blockA$ and the $blockC$ being on the table. In figure 4.2, we give a plan solution to the problem consisting of having the stack $\langle blockA, blockB, blockC \rangle$ from that initial state.

Every automated planner aims to find at least one such solution in any way shape or form in the least amount of time with the best plan quality. The quality of a plan is often measured by how hard it is to execute, whether by its execution time or by the resources needed to accomplish it. This metric is often called *cost* of a plan and is often simply the sum of the cost of its actions.
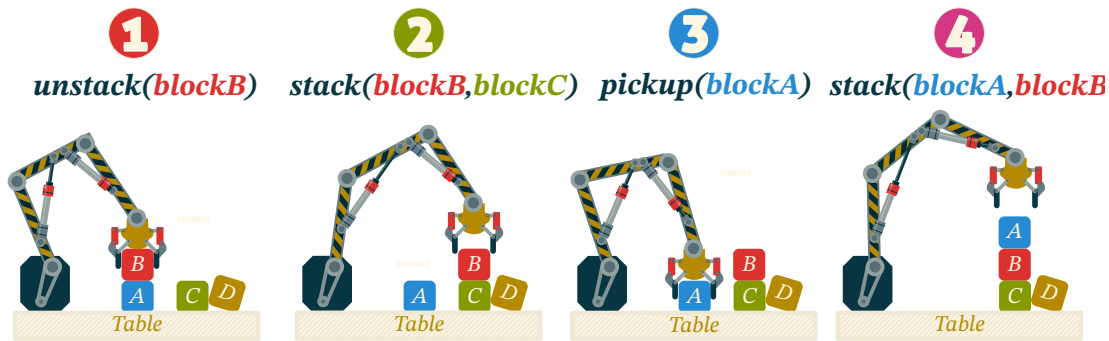
Figure 4.2: An example of a solution to a planning problem with a goal requiring three block stacked in alphabetical order.

Automated planning is very diverse. A lot of paradigms shifts the definition of domain, actions and even plan to widely varying extents. This is the reason why making a general planning formalism was deemed so hard or even impossible:

> "*It would be unreasonable to assume there is one single compact and correct syntax for specifying all useful planning problems.*" Sanner (2010)

Indeed, the block world example domain we give is mostly theoretical since there are infinitely more subtlety into this problem such as mechatronic engineering, balancing issues and partial ability to observe the environment and predict its evolution as well as failure in the execution. In our example, we didn't mentioned the misplaced $blockD$ that could very well interfere with any execution in unpredictable ways. This is why so many planning paradigms exists and why they are all so diverse: they try to address an infinitely complex problem, one sub-problem at a time. In doing so we lose the general view of the problem and by simply stating that this is the only way to resolve it we close ourselves to other approaches that can become successful. Like once said:

> "*The easiest way to solve a problem is to deny it exists.*" Asimov (1973)

However, In the next section we aim to create such a general planning formalism. The main goal is to provide the automated planning community with a general unifying framework it so badly need.

4.2 Formalism

In this section, a general formalism of automated planning is proposed. The goal is to explain what is planning and how it works. First we must express the knowledge domain formalism, then we describe how problems are represented and lastly how a general planning algorithm can be envisioned.

4.2.1 Planning domain

In order to conceive a general formalism for planning domains, we base its definition on the formalism of SELF. This means that all part of the domain must be a member of the universe of discourse $\mathbb{U}$.

4.2.1.1 Fluents

First, we need to define the smallest unit of knowledge in planning, the fluents.

Definition 29 (Fluent). A planning fluent is a predicate $f \in F$.

Fluents are signed. Negative fluents are noted $\neg f$ and behave as a logical complement. We do not use the closed world hypothesis: fluents are only satisfied when another compatible fluent is provided.

The name "fluent" comes from their fluctuating value. Indeed the truth value of a fluent is meant to vary with time and specifically by acting on it. In this formalism we represent fluents using either parameterized entities of using statements for binary fluents.

Example 19. In our example we have four predicates. They can form countless fluents like $held(no-block), on(blockA, blockB)$ or $\neg down(blockA)$. Their when expressing a fluent we suppose its truth value is $\top$ and denote falsehood using the negation $\neg$.

4.2.1.2 States

When expressing states, we need a formalism to express sets of fluents as formulas.

Definition 30 (State). A state is a logical formula of fluents. Since all logical formulas can be reduced to a simple form using only $\wedge$, $\vee$, and $\neg$, we can represent states as *and/or trees*. This means that the leaves are fluents and the other nodes are states. We note states using small squares $\square$ as it is often the symbol used in the representation of automates and grafcets.

Example 20. In the domain block world, we can express a couple of states as :

- $\square_1 = held(noblock) \wedge on(blockA, blockB) \wedge down(blockC)$
- $\square_2 = held(blockC) \wedge down(blockA) \wedge down(blockB)$

In such a case, both state $\square_1$ and $\square_2$ have their truth value being the conjunction of all their fluents. We can express a disjonction in the following way: $\square_3 = \square_1 \vee \square_2$. In that case $\square_3$ is the root of the and/or tree and all its direct children are or vertices. The states $\square_1$ and $\square_2$ have their children as *and vertices*. All the leaves are fluents.
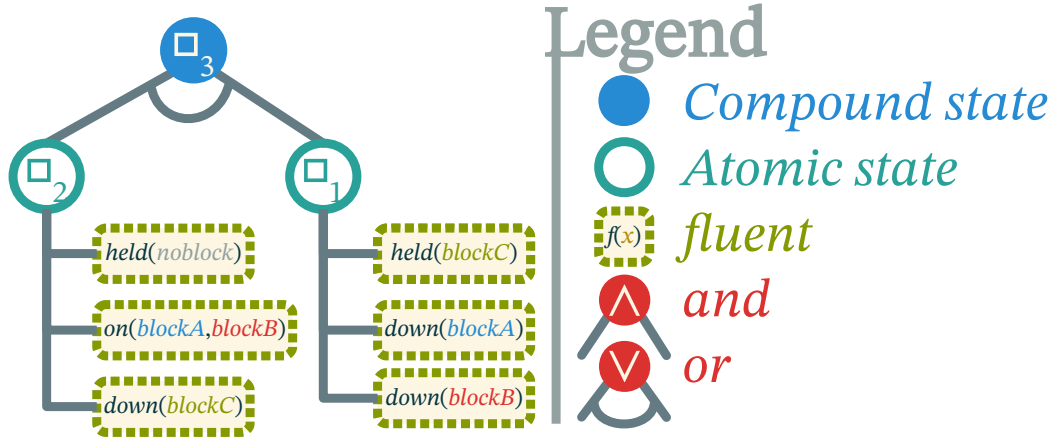
Figure 4.3: Example of a state encoded as an and/or tree.

4.2.1.3 Verification and binding constraints

When planning, there are two operations that are usually done on states: verify if precondition fits a given state and then apply the effects of an action. In our model we consider preconditions and effects as states.

The verification is the operation $\square_{pre} \vDash \square$ that has either no value when the verification fails or a binding map for variables and fluents with their respective values.

The algorithm is a regular and/or tree exploration and evaluation applied on the state $\boxplus = \square_{pre} \wedge \square$. During the valuation, if an inconsistency is found then the algorithm returns nothing. Otherwise, at each node of the tree, the algorithm will populate the binding map and verify if the truth value of the node holds under those constraints. All quantified variable is also registered in the binding map to enforce coherence in the root state. If the node is a state, the algorithm recursively applies until it reaches fluents. Once $\boxplus$ is valuated as true, the binding map is returned.
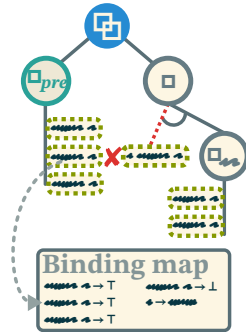
Figure 4.4: Example of verification and/or tree with binding constraints

Example 21. Using previously defined example states $\square_{1,2,3}$, and adding the following:

- $\square_4(x) = \{held(noblock), down(x)\}$ and
- $\square_5(y) = \{held(y), \neg down(y)\}$,

We can express a few example of fluent verification:

- $held(noblock) \vDash held(x) = \{x = noblock\}$
- $\neg held(x) \vDash held(x) = \varnothing$

4.2.2 Application

Once the verification is done, the binding map is kept until the planner needs to apply the action to the state. The application of an effect state is noted $\square_{eff}(\square) = \square'$ and is very similar to the verification. The algorithm will traverse the state $\square$ and use the binding

Figure 4.5: Example of application of a state with the updated binding constraints
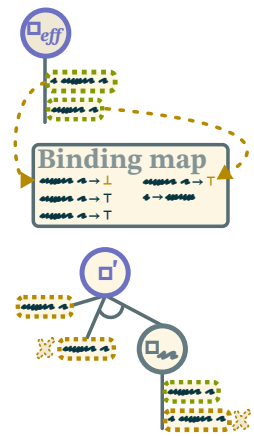
map to force the values inside it. The binding map is prealably completed using $\square_{eff}$ to enforce application of its new value in the current state. This leads to changing the state $\square$ progressively into $\square'$ and the application algorithm will return this state.

4.2.2.1 Actions

Actions are the main mechanism behind automated planning, they describe what can be done and how it can be done.

Definition 31 (Action). An action is a parameterized tuple $a(args) = \langle pre, eff, \gamma, \text{¢}, d, \mathbb{P}, \Pi \rangle$ where:

- *pre* and *eff* are states that are respectively the **preconditions and the effects** of the action.
- γ is the state representing the **constraints**.
- ¢ is the intrinsic **cost** of the action.
- d is the intrinsic **duration** of the action.
- $\mathbb{P}$ is the prior **probability** of the action succeeding.
- Π is a set of **methods** that decompose the action into smaller simpler ones.

Operators take many names in difference planning paradigm: actions, steps, tasks, etc. In our case we call operators, all fully lifted actions and actions are all the instances possibles (including operators).

In order to be more generalist, we allow in the constraints description, any time constraints, equalities or inequalities, as well as probabilistic distributions. These constraints can also express derived predicates. It is even possible to place arbitrary constraints on order and selection of actions.

Actions are often represented as state operators that can be applied in a given state to alter it. The application of actions is done by using the action as a relation on the set of states $a : \square \to \square$ defined as follow:

$$a(\square) = \begin{cases} \varnothing, & \text{if } pre \vDash \square = \varnothing \\ \mathit{eff}(\square), & \text{using the binding map otherwise} \end{cases}$$

Example 22. A useful action we can define from previously defined states is the following:

$$pickup(x) = \langle \square_4(x), \square_5(x), (x : Block), 1.0\text{¢}, 3.5s, 75\%, \varnothing \rangle$$

That action can pickup a block x in 3.5 seconds using a cost of 1.0 with a prior success probability of 75%.
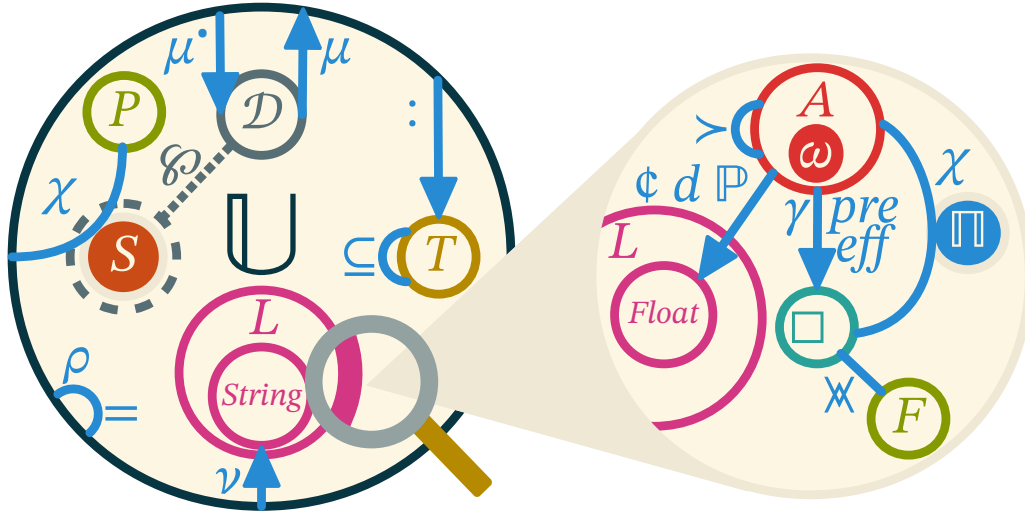
Figure 4.6: Venn diagram extended from the one from SELF to add all planning knowledge representation.

4.2.2.2 Domain

The planning domain specifies the allowed operators that can be used to plan and all the fluents they use as preconditions and effects.

Definition 32 (Domain). A planning domain $\mathcal{D}$ is a set of **operators** which are fully lifted *actions*, along with all the relations and entities needed to describe their preconditions and effects.

Example 23. In the previous examples the domain was named block world. It consists in four actions: $pickup$, $putdown$, $stack$ and $unstack$. Usually the domain is self contained, meaning that all fluents, types, constants and operators are contained in it.

4.2.3 Planning problem

The aim of an automated planner is to find a plan satisfying the goal. This plan can be of multiple forms, and there can even be multiple plans that meet the demand of the problem.

4.2.3.1 Solution

Definition 33 (Partial Plan / Method). A partially ordered plan is an *acyclic* directed graph $\pi = (A_\pi, E)$, with:

- A_π the set of **steps** of the plan as vertices. A step is an action belonging in the plan. A_π must contain an initial step a_π^0 and goal step a_π^* as convinience for certain planning paradigms.

- E the set of **causal links** of the plan as edges. We note $l = a_s \overset{\square}{\to} a_t$ the link between its source a_s and its target a_t caused by the set of fluents $\square$. If $\square = \varnothing$ then the link is used as an ordering constraint.

This definition can express any kind of plans, either temporal, fully or partially ordered or even lose hierarchical plans (using the methods of the actions). It can even express diverse planning results.

The notation can be reminicent of functional affectation and it is on purpose. Indeed, those links can be seen as relations that only affect their source to their target and the plan is a graph with its adjacence function being the combination of all links.

In our framework, *ordering constraints* are defined as the transitive cover of causal links over the set of steps. We note ordering constraints: $a_a > a_s$, with a_a being *anterior* to its *successor* a_s. Ordering constraints cannot form cycles, meaning that the steps must be different and that the successor cannot also be anterior to its anterior steps: $a_a \neq a_s \wedge a_s \not> a_a$. If we need to enforce order, we simply add a link without specifying a cause. The use of graphs and implicit order constraints help to simplify the model while maintaining its properties. Totally ordered plans are done by specifying links between all successive actions of the sequence.

Example 24. In the section 4.1, we described a classical fully ordered plan, illustrated in figure 4.2. A partially ordered plan has a tree-like structure except that it also meet in a "sink" vertex (goal step). We explicit this structure in figure 4.7.
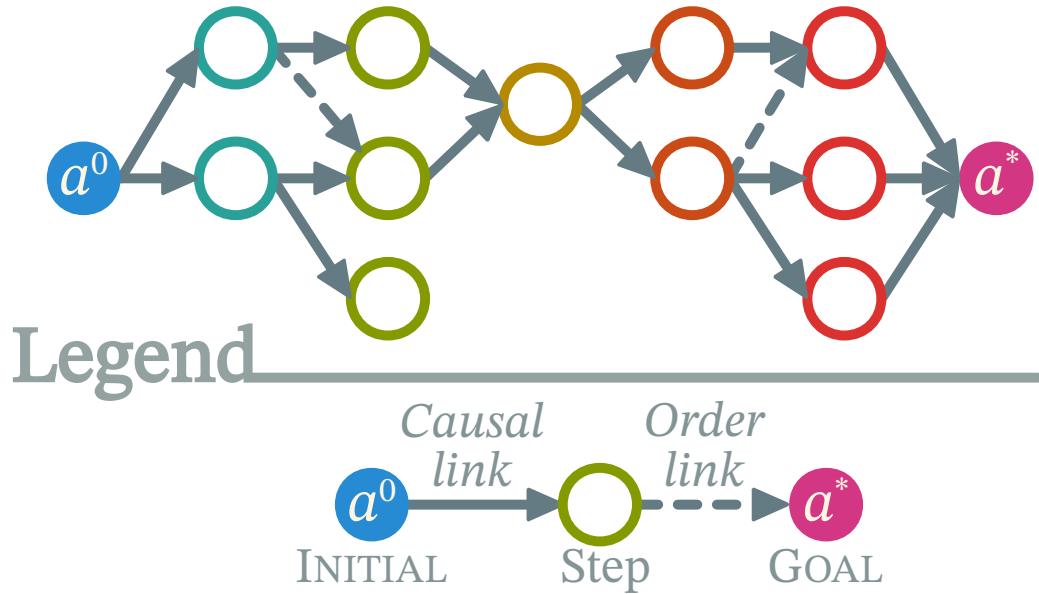
Figure 4.7: Structure of a partially ordered plan.

4.2.3.2 Problem

With this formalism, the problem is very simplified but still general.

Definition 34 (Problem). The planning problem is defined as the **root operator** ω which methods are potential solutions of the problem. Its preconditions and effects are respectively used as initial state and goal description.

As actions are very general, it is interesting to make the problem and domain space homogenous by using an action to describe any problem.

Most of the specific notions of this framework are optionnal. Any planner using it will probably define what features it supports when compiling input domains and problems.

All notions explained so far are represented in the figure 4.6 adding to the SELF Venn diagram.

4.3 Planning search

A general planning algorithm can be described as a guided exploration of a search space. The detailed structure of the search space as well as search iterators are dependent on the planning paradigm used and therefore are parameters of the algorithm.

4.3.1 Search space

Definition 35 (Planner). A planning algorithm, often called planner, is an exploration of a search space $\mathbb{S}$ partially ordered by an iterator $\chi_{\mathbb{S}}$ guided by a heuristic h. From any problem $\mathbb{p}$ every planner can derive two information immediately:

- the starting point $\mathbb{s}_0 \in \mathbb{S}$ and
- the solution predicate $?_{\mathbb{s}*}$ that gives the validity of any potential solution in the search space.

Formally the problem can be viewed as a path-finding problem in the directed graph $g_{\mathbb{S}}$ formed by the vertex set $\mathbb{S}$ and the adjacency function $\chi_{\mathbb{S}}$. The set of solutions is therefore expressed as:

$$\mathbb{S}^* = \left\{ \mathbb{s}^* : \langle \mathbb{s}_0, \mathbb{s}^* \rangle \in \chi_{\mathbb{S}}^+(\mathbb{s}_0) \wedge ?_{\mathbb{s}*} \right\}$$

We note a provided heuristic $h(\mathbb{s})$. It gives off the shortest predicted distance to any point of the solution space. The exploration is guided by it by minimizing its value.

The search in automated planning can have a lot of requirements. Often time is limited and results may need to meet a certain set of specifications.

4.3.2 Solution constraints

As finding a plan is computationally expensive, it is sometimes better to try to find either a more generally appliable plan or a set of alternatives. This is especially important in the case of execution monitoring or human interactions as proposing several relevant solutions to pick from is a very interesting feature.

For this, one will prefer either probabilistic or diverse planning. The main problem is the additional parameters required and the change of behavior of the algorithm. In order to handle such formalism, the planning algorithm needs additional specifications.

We note $\gamma_{\mathbb{S}}$ the set of constraints on the nature of the solution. This contains notably the following optionnal elements:

- Δ the plan deviation metric to compare how much two plans are different.
- k is the number of expected different solution. This simply make the process return when either it found k solutions or when it determined that $k > |\mathbb{S}^*|$.

For probabilistic planning, all elements of the probability distributions used are typically included in the domain but the solution constraints parameter can also contain all required information from any planning paradigm, present or future as long as it is expressible in SELF.

4.3.3 Temporal constraints

Another aspect of planning lies in its timming. Indeed sometimes acting needs to be done before a dealine and planning is useful only durring a finite timeframe. This is done even in optimal planning as researchers evaluating algorithms need often to set a timeout in order to be able to complete a study in a reasonable amount of time.

This time component is quite important as it oftens determine the planning paradigm used. It is expressed as two parameters:

- $t_{\mathbb{S}}$ the alloted time for the algorithm to find at least a fiting solution.
- t^* additional time for plan optimization.

This means that if the planner cannot find a fitting solution in time it will either return a timeout error or a partial or abstract solution that needs to be refined. Anytime planners will also use the extra time parameter to optimize the solution some more. If the amount of time is either unkown or unrestricted the parameters can be ommited and their value will be set to infinity.

4.4 General planner

A general planner $\mathbb{\Pi}^*(g_\mathbb{S}, \mathbb{s}_0, ?_{\mathbb{s}^*}, h, \gamma_\mathbb{S}, \mathcal{D}, t_\mathbb{S}, t^*)$ is an algorithm that can find solutions using any valid combination of planning formalism.

For the algorithm itself, we simply use a parameterized instance of the K* algorithm (Aljazzar and Leue 2011, alg. 1). This algorithm uses the classical algorithm A* to explore the graph while using Dijkstra on some sections to find the k shortest paths. The parameters are as follow: $K^*(g_\mathbb{S}, \mathbb{s}_0, ?_{\mathbb{s}^*}, h)$. In this case, the solution predicate contains the solution and time constraints.

Of course this algorithm is merely an example of a general planner algorithm. Its efficiency hasn't been tested.
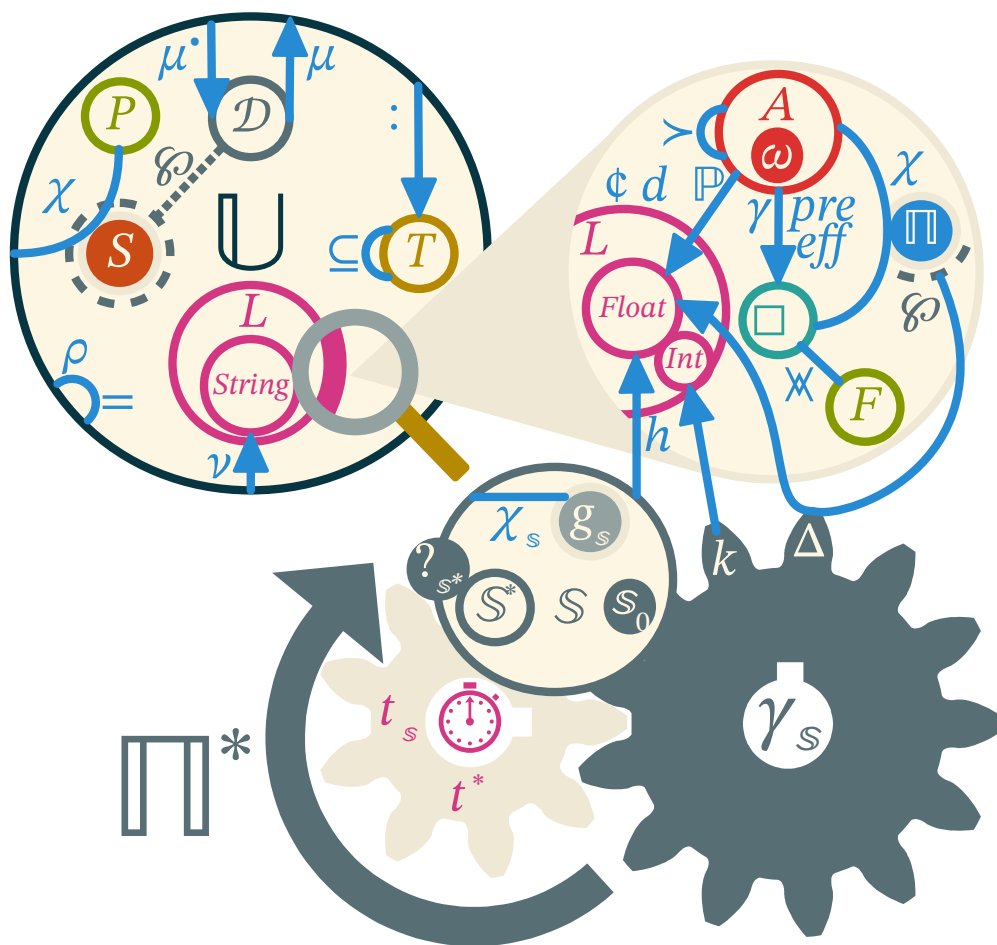
Figure 4.8: Venn diagram extended with general planning formalism.

4.5 Classical Formalisms

One of the most comprehensive work on sumarizing the automated planning domain was done by Ghallab *et al.* (2004). This book explains the different planning paradigm of its time and gives formal description of some of them. This work has been updated later (Ghallab *et al.* 2016) to reflect the changes occuring in the planning community.

4.5.1 State-transition planning

The most classical representation of automated planning is using the state transition approach: actions are operators on the set of states and a plan is a finite-state automaton. We can also see that problem description as either a graph exploration problem or even a constraint satisfaction problem. In any way that problem is isomorph to its original formulation and most efficient algorithms use a derivative of A* exploration techniques on the state space.

The parameters for state space planning are trivial:

$$\mathbb{\Pi}^*_\square = \mathbb{\Pi}^* \left((\square, A), \textit{pre}(\omega), \textit{eff}(\omega) \right)$$

This formulation takes advantage of several tools previously described. It uses the partial application of a function to omit the last parameters. It also defines the search graph using the set of all states $\square$ as the vertices set and the set of all available actions A as the set of edges while considering actions as relations that can be applied to states to make the search progress toward an eventual solution. We also use the binary nature of states to use the effects of the root operator as the solution predicate.

Usually, we would set both $t_\mathbb{S}$ and t^* to an infinite amount as it is often the case for such planners. These parameters are left to the user of the planner.

State based planning usually supose total knowledge of the state space and action behavior. No concurency or time constraints are expressed and the state and action space must be finite as well as the resulting state graph. This process is also deterministic and doesn't allow uncertainty. The result of such a planning is a totally ordered sequence of actions called a plan. The total order needs to be enforced even if it is unecessary.

All those features are important in practice and lead to other planning paradigms that are more complex than classical state based planning.

4.5.2 Plan space planning

Plan Space Planning (PSP) is a form of planning that use plan space as its search space. It starts with an empty plan and try to iteratively refine that plan into a solution.

The transformation into a general planner is more complicated than for state based planning as the progress is made through refinements. We note the set of possible

refinement of a given plan $r = \pi \rightarrow \{\!\{\odot : \otimes(\pi)\}\!\}$. Each refinement is a new plan in which we fixed a *flaw* using one of the possible *resolvers* (see **LATER**).

$$\Pi_\Pi^* = \Pi^* \left((\Pi, \{r(\pi) : \pi \in \Pi\}), (\{a^0, a^*\}, \{a^0 \rightarrow a^*\}), \otimes(\mathbb{s}) = \varnothing \right)$$

with a^0 and a^* being the intial and goal steps of the plan corresponding $\mathbb{s}_0$ such that $eff(a^0) = pre(\omega)$ and $pre(a^*) = eff(\omega)$. The iterator is all the possible resolutions of all flaws on any plan in the search space and the solution predicate is true when the plan has no more flaws.

Details about flaws, resolver and the overall Partial Order Causal Links (POCL) algorithm will be presented **LATER**.

This approach usually can give a partial plan if we set $t_\mathbb{s}$ too low for the algorithm to complete. This plan is not a solution but can eventually be engineered into having approximative properties relative to a solution.

4.5.3 Case based planning

Another plan oriented planning is called Case-Based Planning (CBP). This kind of planning relies on a library $\mathcal{C}$ of already complete plans and try to find the most appropriate one to repair.

$$\Pi_\mathcal{C} = \Pi^* \left((\mathcal{C}, \odot), (\pi : \pi \in \mathcal{C} \wedge \Delta(\pi, \omega) = \{\!\{min : \mathcal{C} \times \{\omega\}\}\!\}, \mathbb{s}(pre(\omega)) \neq \varnothing \right)$$

The planner selects efficiently a plan that fit the best with the initial and goal state of the problem. This plan is then repaired and validated iteratively. The problem with this approach is that it may be unable to find a valid plan or might need to populate and maintain a good plan library. For such case an auxiliary planner is used (preferably diverse).

4.5.4 Probabilistic planning

Probabilistic planning tries to deal with uncertainty by generating a policy instead of a plan. The initial problem holds probability laws that govern the execution of any actions. It is sometimes accompanied with a reward function instead of a deterministic goal. We use the set of pair of state with applicable actions as the search space: $\square + A = \{\langle \square, a \rangle : a \in A \wedge \square \in \square \wedge a(\square) \neq \varnothing\}$. We can also note the policy iteration $pol = \langle \square, a \rangle \rightarrow \langle a(\square), \{\!\{a' \in A \wedge a'(a(\square)) \neq \varnothing\}\!\} \rangle$

$$\Pi_\mathbb{P} = \Pi^* \left((\square + A, pol), \langle pre(\omega), \{\!\{a \in A \wedge a(pre(\omega)) \neq \varnothing\}\!\} \rangle, \forall \mathbb{s}_1 \vDash eff(\omega) \right)$$

At each iteration a state is chosen from the frontier. The frontier is updated with the application of a non-deterministically chosen pair of the last policy insertion. The search stops when all elements in the frontier are goal states.

4.5.5 Hierarchical planning

Hierarchical Task Networks (HTN) are a totally different kind of planning paradigm. Instead of a goal description, HTN uses a root task that needs to be decomposed. The task decomposition is an operation that replaces a task (action) by one of its methods Π.

$$\Pi_\omega = \Pi^* = ((\Pi, \mathbb{s} \to \{\!\!\{\pi \in \Pi(\{\!\!\{a \in A_\mathbb{s} \wedge \Pi(a) \neq \emptyset\!\!\})\}),), \omega, \forall a \in A_\mathbb{s} : \Pi(a) = \varnothing$$

TODO: Explain more

4.6 Existing Languages and Frameworks

4.6.1 Classical

After STRIPS, one of the first language to be introduced to express planning domains like ADL (Pednault 1989). That formalism adds negation and conjunction into literals to STRIPS. It also drops the closed world hypothesis for an open world one: anything not stated in conditions (initial or action effects) is unknown.

The current standard was strongly inspired by Penberthy *et al.* (1992) and his UCPOP planner. Like STRIPS, UCPOP had a planning domain language that was probably the most expressive of its time. It differs from ADL by merging the add and delete lists in effects and to change both preconditions and effects of actions into logic formula instead of simple states.

The PDDL language was created for the first major automated planning competition hosted by AIPS in 1998 (Ghallab *et al.* 1998). It came along with a syntax and solution checker written in Lisp. It was introduced as a way to standardize notation of planning domains and problems so that libraries of standard problems can be used for benchmarks. The main goal of the language was to be able to express most of the planning problems of the time.

With time, the planning competitions became known under the name of International Planning Competitions (IPC) regularly hosted by the ICAPS conference. With each installment, the language evolved to address issues encountered the previous years. The current version of PDDL is 3.1 (Kovacs 2011). Its syntax, goes similarly as described in listing 4.1.

PDDL uses the functional notation style of LISP. It defines usually two files: one for the domain and one for the problem instance. The domain describes constants, fluents and all possible actions. The problem lays the initial and goal states description.

Example 25. For example, consider the classic block world domain expressed in listing 4.2. It uses a predicate to express whether a block is on the table because several blocks can be on the table at once. However it uses a 0-ary function to describe the one block allowed to be held at a time. The description of the stack of blocks is done

Listing 4.1 Simplified explanation of the syntax of PDDL.

```
(define (domain <domain-name>)
  (:requirements :<requirement-name>)
  (:types <type-name>)
  (:constants <constant-name> - <constant-type>)
  (:predicates (<predicate-name> ?<var> - <var-type>))
  (:functions   (<function-name> ?<var> - <var-type>) - <function-type>)

  (:action <action-name>
        :parameters (?<var> - <var-type>)
        :precondition (and (= (<function-name> ?<var>) <value>) (<predicate-name> ?<var>))
        :effect
        (and (not (<predicate-name> ?<var>))
          (assign (<function-name> ?<var>) ?<var>)))
```

with an unary function to give the block that is on top of another one. To be able to express the absence of blocks it uses a constant named `no-block`. All the actions described are pretty straightforward: `stack` and `unstack` make sure it is possible to add or remove a block before doing it and `pick-up` and `put-down` manages the handling operations.

However, PDDL is far from an universal standard. Some efforts have been made to try and standardize the domain of automated planning in the form of optional requirements. The latest of the PDDL standard is the version 3.1 (Kovacs 2011). It has 18 atomic requirements as represented in figure 4.9. Most requirements are parts of PDDL that either increase the complexity of planning significantly or that require extra implementation effort to meet.

Even with that flexibility, PDDL is unable to cover all of automated planning paradigms. This caused most subdomains of automated planning to be left in a state similar to before PDDL: a zoo of languages and derivatives that aren't interoperable. The reason for this is the fact that PDDL isn't expressive enough to encode more than a limited variation in action and fluent description.

Another problem is that PDDL isn't made to be used by planners to help with their planning process. Most planners will totally separate the compilation of PDDL before doing any planning, so much so that most planners of the latest IPC used a framework that translates PDDL into a useful form before planning, adding computation time to the planning process. The list of participating planners and their use of language is presented in table 4.1.

The domain is so diverse that attempts to unify it haven't succeeded so far. The main reason behind this is that some paradigms are vastly different from the classical planning description. Sometimes just adding a seamingly small feature like probabilities or plan reuse can make for a totally different problem. In the next section we describe planning paradigms and how they differ from classical planning along with their associated languages.
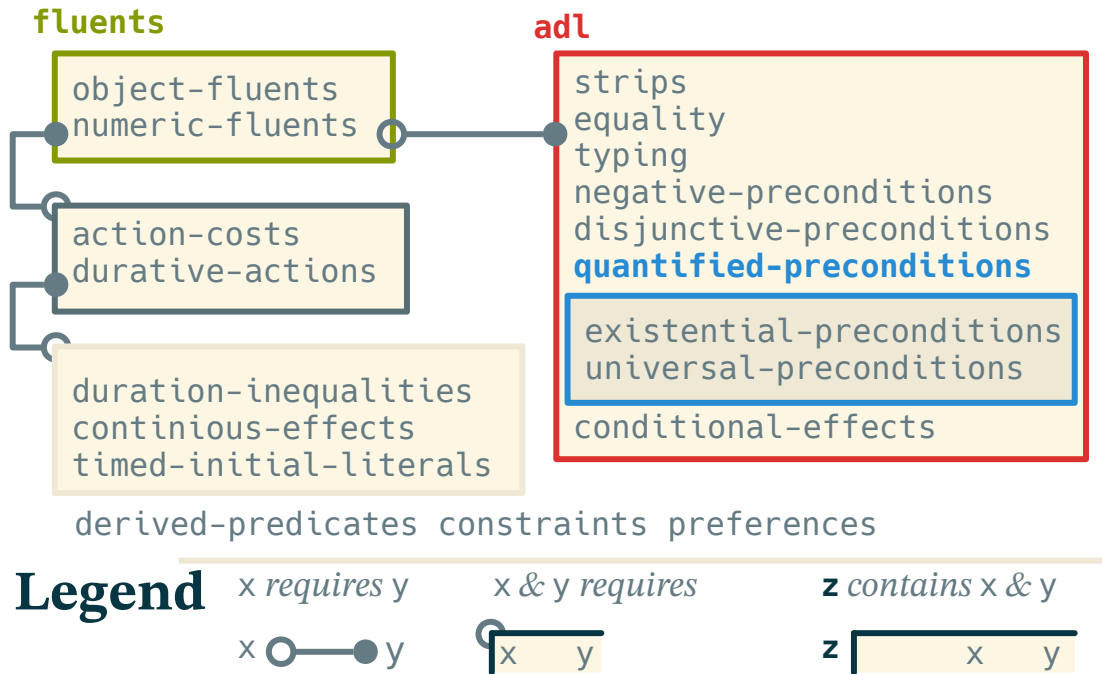
Figure 4.9: Dependancies and grouping of PDDL requirements.

Table 4.1: Planners participating in the Classic track of the 2018 International Planning Competition (IPC). The table states whether the planner used a translation and a preprocessing system to handle PDDL. Most of the planners are based on FastDownward directly.

Name	Trans	Pre	Lang	Base	Rank
Delfi	Yes	Yes	C++	FD	1
Complementary	Yes	Yes	C++	FD	2
Planning-PDBs	Yes	Yes	C++	FD	3
Scorpion	Yes	Yes	C++	FD	4
FDMS	Yes	Yes	C++	FD	5
DecStar	Yes	Yes	C++	LAMA	6
Metis	Yes	Yes	C++	FD	7
MSP	Yes	Yes	Lisp	FD	8
Symple	Yes	Yes	C++	FD	9
Ma-plan	No	Yes	C	None	10

4.6.2 Temporality oriented

When planning, time can become a sensitve constraint. Some critical tasks may required to be completed within a certain time. Actions with durations are already a feature of PDDL 3.1. However, PDDL might not provide support for external events (i.e. events occuring independant from the agent). To do this one must use another language.

4.6.2.1 PDDL+

PDDL+ is an extension of PDDL 2.1 that handles process and events (Fox and Long 2002). It can be viewed as similar to PDDL 3.1 continious effects but it differs on the expressivity. A process can have an effect on fluents at any time. They can happen either from the agent's own doing or being purely environemental. It might be possible in certain cases to modelize this using the durative actions, continious effects and timed initial literals of PDDL 3.1.

In listing 4.3, we reproduce an example from Fox and Long (2002). It shows the syntax of durative actions in PDDL+. The timed precondition are also available in PDDL 3.1, but the `increase` and `decrease` rate of fluents is an exclusive feature of PDDL+.

The main issue with durative actions is that time becomes a continous ressource that may change the values of fluents. The search for a plan in that context has a higher complexity than regular planning.

4.6.3 Probabilistic

Sometimes, acting can become unpredictable. An action can fail for many reasons, from logical errors down to physical constraints. This call for a way to plan using probabilities with the ability to recover from any predicted failures. PDDL doesn't support using probabilities. That is why all IPC's tracks dealing with it always used another language than PDDL.

4.6.3.1 PPDDL

PPDDL is such a language. It was used during the 4th and 5th IPC for its probabilistic track (Younes and Littman 2004). It allows for probabilistic effects as demonstrated in listing 4.4. The planner must take into account the probability when chosing an action. The plan must be the most likely to succeed. But even with the best plan, failure can occur. This is why probabilistic planning often gives policies instead of a plan. A policy dictates the best choice in any given state, failure or not. While this allows for much more resilient execution, computation of policies are exponentially harder than classical planning. Indeed the planner needs to take into account every outcome of every action in the plan and react accordingly.

4.6.3.2 RDDL

Another language used by the 7th IPC's uncertainty track is RDDL (Sanner 2010). This language has been choosen because of its ability to express problems that are hard to encode in PDDL or PPDDL. Indeed, RDDL is capable of expressing Partially Observable Markovian Decision Process (POMDP) and Dynamic Bayesian Networks (DBN) in planning domains. This along with complex probability laws allows for easy implementation of most probabilistic planning problems. Its syntax differs greatly from PDDL, and seems closer to scala or C++. An example is provided in listing 4.5 from Sanner (2010). In it, we can see that actions in RDDL doesn't need preconditions or effects. In that case the reward is the closest information to the classical goal and the action is simply a parameter that will influence the probability distribution of the events that conditioned the reward.

4.6.4 Multi-agent

Planning can also be a collective effort. In some cases, a system must account for other agents trying to either cooperate or compete in achieving similar goals. The problem that arise is coordination. How to make a plan meant to be executed with several agents concurently ? Several multi-agent action languages have been proposed to answer that question.

4.6.4.1 MAPL

Another extension of PDDL 2.1, MAPL was introduced to handle synchronization of actions (Brenner 2003). This is done using modal operators over fluents. In that regard, MAPL is closer to the PDDL+ extension proposed earlier. It introduce durative actions that will later be integrated into the PDDL 3.0 standard. MAPL also introduce a synchronization mechanism using speech as a comunication vector. This seems very specific as explicit comunication isn't a requirement of collaborative work. listing 4.6 is an example of the syntax of MAPL domains. PDDL 3.0 seems to share a similar syntax.

4.6.4.2 MA-PDDL

Another aspect of multi-agent planning is the ability to affect tasks and to manage interactions between agents efficiently. For this MA-PDDL seems more adapted than MAPL. It is an extension of PDDL 3.1, that makes easier to plan for a team of heteroneous agents (Kovács 2012). In the example in listing 4.7, we can see how action can be affected to agents. While it makes the representation easier, it is possible to obtain similar effect by passing an agent object as parameter of an action in PDDL 3.1. More complex expressions are possible in MA-PDDL, like referencing the action of other agents in the preconditions of actions or the ability to affect different goals to different agents. Later on, MA-PDDL was extended with probabilistic capabilities inspired by PPDDL (Kovács and Dobrowiecki 2013).

4.6.5 Hierarchical

Another approach to planning is using Hierarchical Tasks Networks (HTN) to resolve some planning problem. Instead of searching to satisfy a goal, HTNs try to find a decomposition to a root task that fit the initial state requirements and that generate an executable plan.

4.6.5.1 UMCP

One of the first planner to support HTN domains was UCMP by Erol *et al.* (1994). It uses Lisp like most of the early planning systems. Apparently PDDL was in part inspired by UCMP's syntax. Like for PDDL, the domain file describes action (called operators here) and their preconditions and effects (called postconditions). The syntax is demonstrated in listing 4.8. The interesting part of that language is the way decomposition is handled. Each task is expressed as a set of methods. Each method has an expansion expression that specifies how the plan should be constructed. It also has a pseudo precondition with modal operators on the temporality of the validity of the literals.

4.6.5.2 SHOP2

The next HTN planner is SHOP2 by Nau *et al.* (2003). It remains to this day, one of the reference implenentation of an HTN planner. The SHOP2 formalism is quite similar to UCMP's: each method has a signature, a precondition formula and eventually a decomposition description. This decomposition is a set of methods like in UCMP. The methods can be also partially ordered allowing more expressive plans. An example of the syntax of a method is given in listing 4.9.

4.6.5.3 HDDL

A more recent example of HTN formalism comes from the PANDA framework by Bercher *et al.* (2014). This framework is considered the current standard of HTN planning and allows for great flexibility in domain description. PANDA takes previous formalism and generalize them into a new language exposed in listing 4.10. That language was called HDDL after its most used file extention.

4.6.5.4 HPDDL

A very recent language proposition was done by RAMOUL (2018). He proposes HPDDL with a simple syntax similar to the one of UCMP. In listing 4.11 we give an example of HPDDL method. Its expressive power seems similar to that of UCMP and SHOP. Except for a possible commercial integration with PDDL4J (Pellier and Fiorino 2017), there doesn't seem to have any advantages compared to earlier works.

4.6.6 Ontological

Another old idea was to merge automated planning and other artificial intelligence fields with knowledge representation and more specifically ontologies. Indeed, since the "semantic web" is already widespread for service description, why not make planning compatible with it to ease service composition ?

4.6.6.1 WebPDDL

This question finds it first answer in 2002 with WebPDDL. This language, explicited in listing 4.12, is meant to be compatible with RDF by using URI identifiers for domains (McDermott and Dou 2002). The syntax is inspired by PDDL, but axioms are added as constraints on the knowledge domain. Actions also have a return value and can have variables that aren't dependant on their parameters. This allows for greater expressivity than regular PDDL, but can be partially emulated using PDDL 3.1 constraints and object fluents.

4.6.6.2 OPT

This previous work was updated by McDermott (2005). The new version is called OPT and allows for some further expressivity. It can express hierarchical domains with links between actions and even advanced data structure. The syntax is mostly an update of WebPDDL. In listing 4.13, we can see that the URI were replaced by simpler names, the action notation was simplified to make the parameter and return value more natural. Axioms were replaced by facts with a different notation.

4.7 Color and general planning representation

From the general formalism of planning proposed earlier, it is possible to create an instanciation of the SELF language for expressing planning domains. This extension was the primary goal of creating SELF and uses almost all features of the language.

4.7.1 Framework

In order to describe this planning framework into SELF, we simply put all fields of the actions into properties. Entities are used as fluents, and the entire knwoledge domain as constraints. We use parameterized types as specified **BEFORE**.

The file presented in listing 4.14, gives the definition of the syntax of fluents and actions in SELF. The first line includes the default syntax file using the first statement syntax. The fluents are simply typed as entities. This allows them to be either parameterized entities or statements. States are either a set of fluent or a logical statement between states or fluents. When a state is represented as a set, it represent the conjonction of all fluents in the set.

Then at **??**, we define the preconditions, effects and constraints formalism. They are represented as simple properties between actions and states. This allows for simple expression of commonly expressed formalism like the ones found in PDDL. **??** expresses the other attributes of actions like cost, duration and prior probability of success.

Plans are needed to be represented in the files, especially for case based and hierarchical paradims. They are expressed using statements for causal link representation. The property -> is used in these statements and the causes are either given explicitely as parameters of the property or they can be infered by the planner. We add a last property to express methods relative to their actions.

4.7.2 Example domain

Using the classical example domain used earlier we can write the following file in listing 4.15.

At line **??**, We need to include the file defined in listing 4.14. After that **??** defines the allowed arrity of each relation/function used by fluents. This restricts eventually the cardinality between parameters (one to many, many to one, etc).

?? encodes the action $pickup$ defined earlier. It is interesting to note that instead of using a constant to denote the absence of block, we can use an anonymous exclusive quantifier to make sure no block is held. This is quite useful to make concice domains that stays expressive and intuitive.

4.7.3 Differences with PDDL

SELF+Color is more consice than PDDL. It will infere most types and declaration. Variables are also infered if they are used more than once in a statement and also part of parameters.

While PDDL uses a fixed set of extensions to specify the capabilities of the domain, SELF uses inclusion of other files to allow for greater flexibility. In PDDL, everything must be declared while in SELF, type inference allows for usage without definition. It is interesting to note that the use of variables names x and y are arbitrary and can be changed for each statement and the domain will still be functionally the same. The line 3 in listing 4.2 is a specific feature of SELF that is absent in PDDL. It is possible to specify constraints on the cardinality of properties. This limits the number of different combination of values that can be true at once. This is typically done in PDDL using several predicate or constraints.

Most of the differences can be sumarized saying that 'SELF do it once, PDDL needs it twice'. This doesn't only mean that SELF is more compact but also that the expressivity allows for a drastic reduction of the search space if taken into account. Thiébaux *et al.* (2005) advocate for the recognition of the fact that expressivity isn't just a convinience but is crucial for some problem and that treating it like an obstacle by trying to compile it away only makes the problem worse. If a planner is agnostic to the domain and

problem, it cannot take advantages of clues that the instanciation of an action or even its name can hold (Babli *et al.* 2015).

Whatever the time and work that an expert spend on a planning domain it will always be incomplete and fixed. SELF allows for dynamical extension and even adresses the use of reified actions as parameters. Such a framework can be useful in multi-agent systems where agents can commnicate composite actions to instruct another agent. It can also be useful for macro-action learning that allows to learn hierarchical domains from repeating observations. It can also be used in online planning to repair a plan that failed. And at last this mechanism can be used for explanation or inference by making easy to map two similar domains together. (lots of **CITATION**).

Also another difference between SELF and PDDL is the underlying planning framework. We presented the one of SELF but PDDL seems to suppose a more classical state based formalism. For example the fluents are of two kind depending if they are used as pre-condition or effects. In the first case, the fluent is a formula that is evaluated like a predicate to know if the action can be executed in any given state. Effects are formula enforcing the values of existing fluent in the state. SELF just suppose that the new knowledge is enforcing and that the fluents are of the same kind since verification about the coherence of the actions are made prior to its application in planning.

Listing 4.2 Classical PDDL 3.0 definition of the domain Block world

```
(define (domain BLOCKS-object-fluents)
  (:requirements :typing :equality :object-fluents)
  (:types block)
  (:constants no-block - block)
  (:predicates (on-table ?x - block))
  (:functions   (in-hand) - block
        (on-block ?x - block) - block) ;;what is in top of block ?x

  (:action pick-up
        :parameters (?x - block)
        :precondition (and (= (on-block ?x) no-block) (on-table ?x) (= (in-hand) no-block))
        :effect
        (and (not (on-table ?x))
          (assign (in-hand) ?x)))

  (:action put-down
        :parameters (?x - block)
        :precondition (= (in-hand) ?x)
        :effect
        (and (assign (in-hand) no-block)
          (on-table ?x)))

  (:action stack
        :parameters (?x - block ?y - block)
        :precondition (and (= (in-hand) ?x) (= (on-block ?y) no-block))
        :effect
        (and (assign (in-hand) no-block)
         (assign (on-block ?y) ?x)))

  (:action unstack
        :parameters (?x - block ?y - block)
        :precondition (and (= (on-block ?y) ?x) (= (on-block ?x) no-block) (= (in-hand) no-
        :effect
        (and (assign (in-hand) ?x)
         (assign (on-block ?y) no-block))))
```

Listing 4.3 Example of PDDL+ durative action from Fox's paper.

```
(:durative-action downlink
    :parameters (?r - recorder ?g - groundStation)
    :duration (> ?duration 0)
    :condition (and (at start (inView ?g))
                    (over all (inView ?g))
                    (over all (> (data ?r) 0)))
    :effect (and (increase (downlinked)
                        (* #t (transmissionRate ?g)))
                (decrease (data ?r)
                        (* #t (transmissionRate ?g))))))
```

Listing 4.4 Example of PPDDL use of probabilistic effects from Younes' paper.

```
(define (domain bomb-and-toilet)
    (:requirements :conditional-effects :probabilistic-effects)
    (:predicates (bomb-in-package ?pkg) (toilet-clogged)
                (bomb-defused))
    (:action dunk-package
            :parameters (?pkg)
            :effect (and (when (bomb-in-package ?pkg)
                            (bomb-defused))
                        (probabilistic 0.05 (toilet-clogged)))))
```

Listing 4.5 Example of RDDL syntax by Sanner.

```
//////////////////////////////////////////////////////////////////////
// A simple propositional 2-slice DBN (variables are not parameterized).
//
// Author: Scott Sanner (ssanner [at] gmail.com)
//////////////////////////////////////////////////////////////////////
domain prop_dbn {

    requirements = { reward-deterministic };

    pvariables {
        p : { state-fluent,  bool, default = false };
        q : { state-fluent,  bool, default = false };
        r : { state-fluent,  bool, default = false };
        a : { action-fluent, bool, default = false };
    };

    cpfs {
        // Some standard Bernoulli conditional probability tables
        p´ = if (p ^ r) then Bernoulli(.9) else Bernoulli(.3);

        q´ = if (q ^ r) then Bernoulli(.9)
                        else if (a) then Bernoulli(.3) else Bernoulli(.8);

        // KronDelta is like a DiracDelta, but for discrete data (boolean or int)
        r´ = if (~q) then KronDelta(r) else KronDelta(r <=> q);
    };

    // A boolean functions as a 0/1 integer when a numerical value is needed
    reward = p + q - r; // a boolean functions as a 0/1 integer when a numerical value is ne
}

instance inst_dbn {

    domain = prop_dbn;
    init-state {
        p = true;  // could also just say 'p' by itself
        q = false; // default so unnecessary, could also say '~q' by itself
        r;         // same as r = true
    };

    max-nondef-actions = 1;
    horizon  = 20;
    discount = 0.9;
}
```

Listing 4.6 Example of MAPL syntax by Brenner.

```
(:state-variables
  (pos ?a - agent) - location
  (connection ?p1 ?p2 - place) - road
  (clear ?r - road) - boolean)
(:durative-action Move
  :parameters (?a - agent ?dst - place)
  :duration (:= ?duration (interval 2 4))
  :condition
    (at start (clear (connection (pos ?a) ?dst)))
  :effect (and
    (at start (:= (pos ?a) (connection (pos ?a) ?dst)))
    (at end (:= (pos ?a) ?dst))))
```

Listing 4.7 Example of MA-PDDL syntax by Kovacs.

```
(define (domain ma-lift-table)
(:requirements :equality :negative-preconditions
               :existential-preconditions :typing :multi-agent)
(:types agent) (:constants table)
(:predicates (lifted (?x - object) (at ?a - agent ?o - object))
(:action lift :agent ?a - agent :parameters ()
:precondition (and (not (lifted table)) (at ?a table)
               (exists (?b - agent)
                 (and (not (= ?a ?b)) (at ?b table) (lift ?b))))
:effect (lifted table)))
```

Listing 4.8 Example of the syntax used by UCMP.

```
(constants a b c table) ; declare constant symbols
(predicates on clear) ; declare predicate symbols
(compound-tasks move) ; declare compound task symbols
(primitive-tasks unstack dostack restack) ; declare primitive task symbols
(variables x y z) ; declare variable symbols

(operator unstack(x y)
        :pre ((clear x)(on x y))
        :post ((~on x y)(on x table)(clear y)))
(operator dostack (x y)
        :pre ((clear x)(on x table)(clear y))
        :post ((~on x table)(on x y)(~clear y)))
(operator restack (x y z)
        :pre ((clear x)(on x y)(clear z))
        :post ((~on x y)(~clear z)(clear y)(on x z)))

(declare-method move(x y z)
                :expansion ((n restack x y z))
                :formula (and (not (veq y table))
                              (not (veq x table))
                              (not (veq z table))
                              (before (clear x) n)
                              (before (clear z) n)
                              (before (on x y) n)))

(declare-method move(x y z)
                :expansion ((n dostack x z))
                :formula (and (veq y table)
                              (before (clear x) n)
                              (before (on x y) n)))
```

Listing 4.9 Example of method in the SHOP2 language.

```
(:method
  ; head
    (transport-person ?p ?c2)
  ; precondition
    (and
      (at ?p ?c1)
      (aircraft ?a)
      (at ?a ?c3)
      (different ?c1 ?c3))
  ; subtasks
    (:ordered
      (move-aircraft ?a ?c1)
      (board ?p ?a ?c1)
      (move-aircraft ?a ?c2)
      (debark ?p ?a ?c2)))
```

Listing 4.10 Example of HDDL syntax as used in the PANDA framework.

```
(define (domain transport)
  (:requirements :typing :action-costs)
  (:types
        location target locatable - object
        vehicle package - locatable
        capacity-number - object
  )
  (:predicates
     (road ?l1 ?l2 - location)
     (at ?x - locatable ?v - location)
     (in ?x - package ?v - vehicle)
     (capacity ?v - vehicle ?s1 - capacity-number)
     (capacity-predecessor ?s1 ?s2 - capacity-number)
  )

  (:task deliver :parameters (?p - package ?l - location))
  (:task unload :parameters (?v - vehicle ?l - location ?p - package))

  (:method m-deliver
    :parameters (?p - package ?l1 ?l2 - location ?v - vehicle)
    :task (deliver ?p ?l2)
     :ordered-subtasks (and
      (get-to ?v ?l1)
      (load ?v ?l1 ?p)
      (get-to ?v ?l2)
      (unload ?v ?l2 ?p))
  )
  (:method m-unload
    :parameters (?v - vehicle ?l - location ?p - package ?s1 ?s2 - capacity-number)
    :task (unload ?v ?l ?p)
    :subtasks (drop ?v ?l ?p ?s1 ?s2)
  )

  (:action drop
    :parameters (?v - vehicle ?l - location ?p - package ?s1 ?s2 - capacity-number)
    :precondition (and
        (at ?v ?l)
        (in ?p ?v)
        (capacity-predecessor ?s1 ?s2)
        (capacity ?v ?s1)
     )
    :effect (and
        (not (in ?p ?v))
        (at ?p ?l)
        (capacity ?v ?s2)
        (not (capacity ?v ?s1))
     )
  )
)
```

Listing 4.11 Example of HPDDL syntax as described by Ramoul.

```
(:method do_navigate
   :parameters(?x - rover ?from ?to - waypoint)
   :expansion((tag t1 (navigate ?x ?from ?mid))
              (tag t2 (visit ?mid))
              (tag t3 (do_navigate ?x ?mid ?to))
              (tag t4 (unvisited ?mid)))
   :constraints((before (and (not (can_traverse ?x ?from ?to)) (not (visited ?mid))
                              (can_traverse ?x ?from ?mid)) t1)))
```

Listing 4.12 Example of WebPDDL syntax by Mc Dermott.

```
(define (domain www-agents)
   (:extends (uri "http://www.yale.edu/domains/knowing")
             (uri "http://www.yale.edu/domains/regression-planning")
             (uri "http://www.yale.edu/domains/commerce"))
   (:requirements :existential-preconditions :conditional-effects)
   (:types Message - Obj Message-id - String)
   (:functions  (price-quote ?m - Money)
                (query-in-stock ?pid - Product-id)
                (reply-in-stock ?b - Boolean) - Message)
   (:predicates (web-agent ?x - Agent)
                (reply-pending a - Agent id - Message-id msg - Message)
                (message-exchange ?interlocutor - Agent
                                  ?sent ?received - Message
                                  ?eff - Prop)
                (expected-reply a - Agent sent expect-back - Message))
   (:axiom
       :vars (?agt - Agent ?msg-id - Message-id ?sent ?reply - Message)
       :implies (normal-step-value (receive ?agt ?msg-id) ?reply)
       :context (and (web-agent ?agt)
                     (reply-pending ?agt ?msg-id ?sent)
                     (expected-reply ?agt ?sent ?reply)))
   (:action send
       :parameters (?agt - Agent ?sent - Message)
       :value (?sid - Message-id)
       :precondition (web-agent ?agt)
       :effect (reply-pending ?agt ?sid ?sent))
   (:action receive
      :parameters (?agt - Agent ?sid - Message-id)
      :vars (?sent - Message ?eff - Prop)
      :precondition (and (web-agent ?agt) (reply-pending ?agt ?sid ?sent))
      :value (?received - Message)
      :effect (when (message-exchange ?agt ?sent ?received ?eff) ?eff)))
```

Listing 4.13 Example of the updated OPT syntax as described by Mc Dermott.

```
(define (domain www-agents)
  (:extends knowing regression-planning commerce)
  (:requirements :existential-preconditions :conditional-effects)
  (:types Message - Obj Message-id - String )
  (:type-fun (Key t) (Feature-type (keytype t)))
  (:type-fun (Key-pair t) (Tup (Key t) t))
  (:functions (price-quote ?m - Money)
              (query-in-stock ?pid - Product-id)
              (reply-in-stock ?b - Boolean) - Message)
  (:predicates (web-agent ?x - Agent)
               (reply-pending a - Agent id - Message-id msg - Message)
               (message-exchange ?interlocutor - Agent
                                 ?sent ?received - Message
                                 ?eff - Prop)
               (expected-reply a - Agent sent expect-back - Message))
  (:facts
    (freevars (?agt - Agent ?msg-id - Message-id
               ?sent ?reply - Message)
      (<- (and (web-agent ?agt)$<!-- -->$
               (reply-pending ?agt ?msg-id ?sent)
               (expected-reply ?agt ?sent ?reply))
               (normal-value (receive ?agt ?msg-id) ?reply))))
  (:action (send ?agt - Agent ?sent - Message) - (?sid - Message-id)
    :precondition (web-agent ?agt)
    :effect (reply-pending ?agt ?sid ?sent))
  (:action (receive ?agt - Agent ?sid - Message-id) - (?received - Message)
    :vars (?sent - Message ?eff - Prop)
    :precondition (and (web-agent ?agt)
                       (reply-pending ?agt ?sid ?sent))
    :effect (when (message-exchange ?agt ?sent ?received ?eff) ?eff)))
```

Listing 4.14 Content of the file "planning.w"

```
"lang.w" = ? ; //include default language file.
Fluent = Entity;
State = (Group(Fluent), Statement);
BooleanOperator = (&,|);
(pre,eff, constr)::Property(Action,State);$\label{line:preeff}$
(costs,lasts,probability) ::Property(Action,Float);$\label{line:attributes}$
Plan = Group(Statement);$\label{line:plan}$
-> ::Property(Action,Action); //Causal links$\label{line:causallinks}$
methods ::Property(Action,Plan);
```

Listing 4.15 Blockworld writen in SELF to work with Color

```
"planning.w" = ? ; //include base terminology $\label{line:include}$

(! on !, held(!), down(_)) :: Fluent;$\label{line:arity}$

pickUp(x) pre (~ on x, down(x), held(~));$\label{line:pickup}$
pickUp(x) eff (~(down(x)), held(~));

putDown(x) pre (held(x));
putDown(x) eff (held(~), down(x));

stack(x, y) pre (held(x), ~ on y);
stack(x, y) eff (held(~), x on y);

unstack(x, y) pre (held(~), x on y);
unstack(x, y) eff (held(x), ~ on y);
```

5 Online and Flexible Planning Algorithms

Since automated planning comes in a variety of paradigms, so does the planners algorithms. It amounts to such a number of planners that listing all those that exists would be a exceedingly long endeavor.

In that chapter, we present planners and approaches to inverted planning and intent recognition. To do that we must first have a performant online planning algorithm that can take into account observed plans or fluents and find the most likely plan being pursued by an external agent.

Classical planning can be used for such a work but lacks flexibility when needing to replan at high frequency. The planner must be either able to reuse previously found plans or be able to compute quickly plans that are good approximation of the intended goal. We could use probabilistic planning, especially Partially Observable Markovian Decision Process (POMDP) to directly encode the intent recognition problem but that approach have been explored in great detail already, including numerous Bayesian network approaches. Further discussions of inverted planning and intent recognition can be found **LATER**.

It was decided to explore more expressive and flexible approaches to use the semantics of the planning domain to attempt to guide the search to a more logical plan. This approach uses either repair heuristics or explanation to provide fast predictions of intended goals.

5.1 Existing Algorithms

In order to make a planner capable to repair plan, the most fitting paradigm is PSP as described **BEFORE**. Using the plan space for search allows to modify the refinement process into repairing existing plans.

The second approach using explanations is hierarchical. The planner will use a HTN planning domain that contains composite actions (or tasks) that have several methods (as plans) to realize them.

First, PSP will be presented in more details regarding its classical formulation and definition.

5.1.1 Plan Space Planning

Every PSP algorithms works in a similar way: their search space is the set of all plans and its iteration operation is plan refinement. This means that every PSP planner searches for *flaws* in the current plan and then computes a set of *resolvers* that potentially fix each of them. The algorithm starts usually with an empty plan only having the initial and goal step and recursively refine the plan until all flaws have been solved.

In general, PSP is faster than naive classical planning. However, with the advent of efficient state based heuristics used in Fast Forward (FF) (Hoffmann 2001) and LAMA (Richter and Westphal 2010), plan space planning has been left behind regarding raw performance. While PSP delays commitment and therefore can make very efficient choices that can be faster than classical planning, most formulation of PSP problems leads to significant increase in complexity (Tan and Gruninger 2014). The backtracking in PSP algorithm along with heavy data structure such as plans to modify at each iteration makes the approach slower by design without an excellent heuristic.

Works on PSP didn't stop at that point (Nguyen and Kambhampati 2001) since it has unique advantages over classical planning. Indeed, by using backward chaining, PSP algorithms have are sound and complete and therefore guarantee to find a solution if it exists (Sjöberg and Nissar 2015).

As PSP finds partially ordered plans, it is also by nature more flexible. Indeed, multiple totally ordered plans are contained within a partially ordered one, they are called linearizations. So, when wanting to have several plans with low diversity, PSP is the way to go.

5.1.1.1 Definitions

In **BEFORE** we have formalized how PSP works in the general planning formalism. However, since this formalism is being introduced in the present document, it isn't used by the rest of the community. This means that we still need to define the classical Partially Ordered with Causal Link (POCL algorithm. In order to define this algorithm we need to explain the notions of flaws and resolvers.

Definition 36 (Flaws). Flaws are constraints violations within a plan. The set of flaws in a plan π is noted $\otimes_\pi$. There are different kind of flaws in classical PSP and additional ones can be defined depending on the application.

Classical flaws often have a few common features. They are often **constructive** since they require an *addition* of causal links and steps in a plan to be fixed. They have a *proper fluent* f that is the cause of the violation in the plan the flaw is representing and a *needer* a_n that is the action requiring the proper fluent be fulfilled. In classical PSP flaws are either:

- **Subgoals**, also called *open condition* that are yet to be supported by a *provider* a_p. We note subgoals $\otimes_{a_n}^{\ddagger}(f)$.

- **Threats** are caused by steps that can break a causal link with their effects. They are called *breakers* of the threatened link. A step a_b threatens a causal link $l_t = a_p \xrightarrow{f} a_n$ if and only if $(eff(a_b) \not\models f) \wedge (a_b \not\succ a_p \wedge a_n \not\succ a_b)$. Said otherwise, the breaker can potentially cancel an effect of a providing step a_p, before it gets used by its needer a_n. We note threats $\otimes^{\dagger}_{a_n}(f, a_b)$.
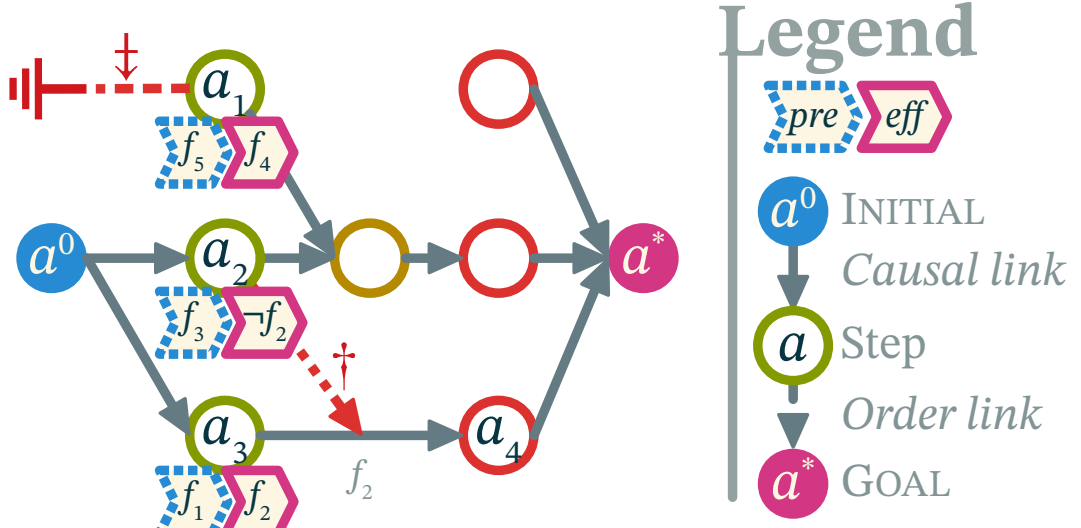
Figure 5.1: Example of partial plan having flaws

Example 26. In figure 5.1 we present a partially ordered plan with two typical flaws. The first is a subgoal missing from the plan to fullfil the f_5 precondition of a_1.

The second flaw in the figure 5.1 is the threat between a_2 and a causal link outgoing from a_3. This happens because nothing prevents a_2 to be executed after a_3 and negate the fluent f_2 needed by the next step.

These flaws need to be fixed in order for the plan to be valid. In POCL it is done by finding their resolvers.

Definition 37 (Resolvers). A resolver is a plan refinement that attempts to solve a flaw $\otimes_{a_n}$. Since classical flaws are constructive, the classical resolvers are called *positive*. They are defined as follow:

- *For subgoals*, the resolvers are a potential causal link containing the proper fluent f of a given subgoal in their causes while taking the needer step a_n as their target and a **provider** step a_p as their source. They are noted $\odot^{+}_{a_p}(\otimes^{\dagger}_{a_n}(f)) = a_p \xrightarrow{f} a_n$.
- *For threats*, we usually consider only two resolvers: **demotion** ($a_b \succ a_p$) and **promotion** ($a_n \succ a_b$) of the breaker relative to the threatened link. We call the added causeless causal link a **guarding** link. The resolvers for threats are noted $\odot^{+}_{\succ}(\otimes^{\dagger}_{a_n}(f, a_b)) = a_p \to a_b$ for promotion and $\odot^{+}_{\prec}(\otimes^{\dagger}_{a_n}(f, a_b)) = a_b \to a_n$ for demotion.

It is possible to introduce extra resolvers to fix custom flaws. In such a case we call positive resolvers, those which adds causal links and steps to the plan and negative those that removes causal links and steps. It is preferable to engineer flaws and resolver to not mix positive and negative aspect at once because of the complicated side effects that might result from it.
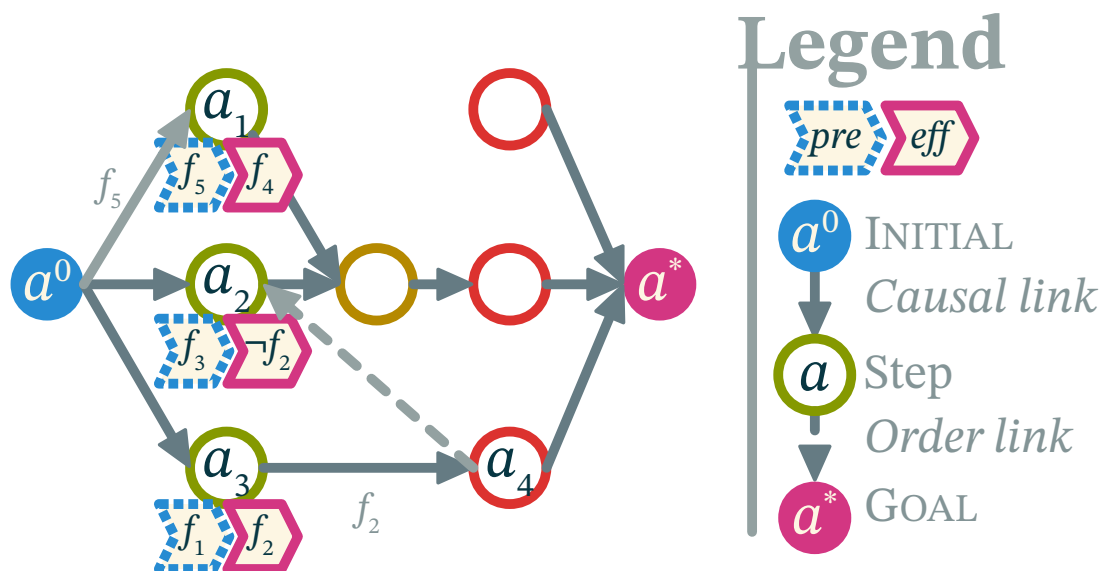
Figure 5.2: Example of resolvers that fixes the previously illustrated flaws

Example 27. From our previous example, we present the complete plan in figure 5.2. The subgoal needs to be fixed by inserting another causal link to provide the missing fluent and inserting any necessary steps to do so. In that case the initial state happens to provide the necessary fluent so we simply add a causal link for it.

For the threat, the solution is to either promote or demote a_2 so that it doesn't interfere with the causal link between a_3 and a_4. We chose here to demote a_2 so it requires a_4 to be executed before it.

The application of a resolver does not necessarily mean progress. It can have consequences that may require reverting its application in order to respect the backtracking of the POCL algorithm.

Definition 38 (Side effects)**.** Flaws that are caused by the application of a resolver are called *related flaws*. They are inserted into the *agenda* with each application of a resolver:

An agenda is a flaw container used for the flaw selection of POCL.

- *Related subgoals* are all the new open conditions inserted by new steps.
- *Related threats* are the causal links threatened by the insertion of a new step or the deletion of a guarding link.

Flaws can also become irrelevant when a resolver is applied. It is always the case for the targeted flaw, but this can also affect other flaws. Those *invalidated flaws* are removed from the agenda upon detection:

- *Invalidated subgoals* are subgoals satisfied by the new causal links or the removal of their needer.
- *Invalidated threats* happen when the breaker no longer threatens the causal link because the order guards the threatened causal link or either of them has been removed.

Figure 5.3: Example of the side effects of the application of a resolver

Example 28. In our example, by adding the step a_2 to fix a unsupposted subgoal needed by a_1, we introduced another subgoal to support the new step that also threatens the causal link between a_3 and a_4.

5.1.1.2 Classical Aglorithm

In algorithm 5 we present a generic version of POCL inspired by Ghallab *et al.* (2004, sec. 5.4.2).

Algorithm 5 POCL Algorithm

```
 1: function POCL(Agenda 𝒜, Action ω)
 2:     if 𝒜 = ∅ then                                          ▷ Populated agenda needs to be provided
 3:         return Success                                                       ▷ Stops all recursion
 4:     Flaw ⊗ ← ⟦𝒜⟧                                                    ▷ Heuristically chosen flaw
 5:     Resolvers ⊙ ← SOLVE(⊗, ⟦Π(ω)⟧)                   ▷ The root operator has only one method for PSP
 6:     for all ⊙ ∈ ⊙ do                                          ▷ Non-deterministic choice operator
 7:         APPLY(⊙, π)                                              ▷ Apply resolver to partial plan
 8:         Agenda 𝒜' ← UPDATE(𝒜)
 9:         if POCL(𝒜', ω) = Success then                                    ▷ Refining recursively
10:             return Success
11:         REVERT(𝒜, π)                                        ▷ Failure, undo resolver application
12:     𝒜 ← 𝒜 ∪ {⊗}                                                      ▷ Flaw was not resolved
13:     return Failure                                      ▷ Revert to last non-deterministic choice
```

For our version of POCL we follow a refinement procedure that works in several generic steps. In **??** we detail the resolution of a subgoal as done in the algorithm 5.

The first is the search for resolvers. It is often done in two separate steps: first, select the candidates and then check each of them for validity. This is done using the polymorphic function `solve` at algorithm 5.

In the case of subgoals, variable unification is performed to ensure the compatibility of the resolvers. Since this step is time-consuming, the operator is instantiated accordingly at this step to factories the computational effort. Composite operators have also all their methods instantiated at this step if they are selected as a candidate.

Then a resolver is picked non-deterministically for applications (this can be heuristically driven). At algorithm 5 the resolver is effectively applied to the current plan. All side effects and invalidations are handled during the update of the agenda at algorithm 5. If a problem occurs, algorithm 5 backtracks and tries other resolvers. If no resolver fits the flaw, the algorithm backtracks to previous resolver choices to explore all the possible plans and ensure completeness.

5.1.1.3 Existing PSP planners

Related works already tried to explore new ideas to make PSP an attractive alternative to regular state-based planners like the appropriately named "Reviving partial order planning" (Nguyen and Kambhampati 2001) and VHPOP (Younes and Simmons 2003). More recent efforts (Coles *et al.* 2011; Sapena *et al.* 2014) are trying to adapt the powerful heuristics from state-based planning to PSP's approach. An interesting approach of these last efforts is found in (Shekhar and Khemani 2016) with meta-heuristics based on offline training on the domain. Yet, we clearly note that only a few papers lay the emphasis upon plan quality using PSP (Ambite and Knoblock 1997; Say *et al.* 2016).

5.1.2 Plan Repair & Reuse

In online planning, the plan is computed frequently from a changing initial state. This means that the previous plan is very often available. In order to take advantage of the effort invested in previous plans, it is tempting to simply reuse the existing plan instead of re-planning from scratch. Most work on the field focus on monitoring execution and find ways to make resilient plans.

In such a case, a lot of the planning model can be subject to uncertainty. Indeed, the execution of an action can fail because an external event changed a precondition required to do it or the model itself can be inaccurate.

The idea of reusing plan emerged early on (Nebel and Koehler 1995) but with a caveat: often repairing needed more effort than re-planning. So plan repair became more of a gamble and needed incentives to reuse an existing plan given an application. For example, such process are useful for multi-agent planning where a significant change of plan is expansive among agents (Ephrati and Rosenschein 1993; Alami *et al.* 1995; Sugawara 1995; Borrajo 2013; Luis and Borrajo 2014). This motivation for plan repair comes from plan merging and is needed in cooperative environments.

The question of the efficiency of re-planning vs. repairing has been since studied extensively under several aspects (Van Der Krogt and De Weerdt 2005; Fox *et al.* 2006). The emergence of diverse planning and requirement on plan stability of execution

Table 5.1

Restrictions	Complexity
Classical	PSPACE
Task Insertion	NEXPTIME
Totally Ordered	EXPTIME
Regular	PSPACE
Acyclic	NEXPTIME
Tail-recursive	EXPSPACE

monitoring gave new research on the subject. At this point, most of the literature focuses on case based planning, where a plan library is already provided and the planner must select a plan and repair it to fit a given case (Gerevini *et al.* 2013; Borrajo *et al.* 2015).

A recent work of Zhuo and Kambhampati (2017) gives an interesting approach to the problem by questioning the domain. Indeed, the need to re-plan can be an opportunity to revised issues in the current model and to improve it by adding newly found solution to complete the given planning model.

In our case, we focus on PSP and how to make plan repair efficiently using that technique. Classical PSP algorithms don't take as an input an existing plan but can be enhanced to fit plan repairing, as for instance in (Van Der Krogt and De Weerdt 2005). Usually, PSP algorithms take a problem as an input and use a loop or a recursive function to refine the plan into a solution. We can't solely use the refining recursive function to be able to use our existing partial plan. This causes multiples side effects if the input plan is sub-optimal. This problem was already explored as of LGP-adapt (Borrajo 2013) that explains how re-using a partial plan often implies re-planning parts of the plan.

5.1.3 HTN

> "HTN planners differ from classical planners in what they plan for and how they plan for it. In an HTN planner, the objective is not to achieve a set of goals but instead to perform some set of tasks."

Ghallab *et al.* (2004)

Planning using HTN gives a completely different approach to the problem and its formulation. In this formalism, actions are composite tasks and there is no goal other than to complete the root task. One can find similarities with our general planning formalism and it isn't a coincidence. Indeed, HTN is more general than planning and therefore one need to be able to allow for this level of expressivity.

This expressivity comes at a cost. HTN problems are on a complexity category that is significantly harder than regular STRIPS planning:

From Bercher and Höller (2018) HTN Tutorial at ICAPS 2018

HTN is often combined with classical approaches since it allows for a more natural expression of domains making expert knowledge easier to encode. These kinds of planners are named **decompositional planners** when no initial plan is provided (Fox 1997). Most of the time the integration of HTN simply consists in calling another algorithm
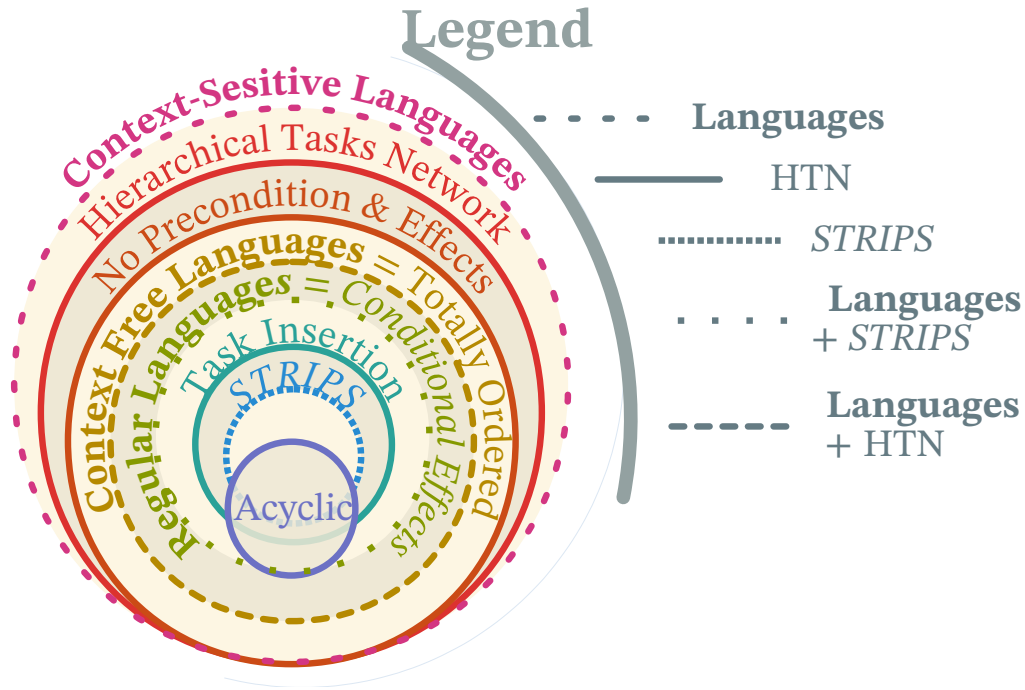
Figure 5.4: Venn diagram of the exressivity class of HTN paradigms.

when introducing a composite operator during the planning process. The DUET planner by Gerevini *et al.* (2008) does so by calling an instance of a HTN planner based on task insertion called SHOP2 (Nau *et al.* 2003) to decompose composite actions. Some planners take the integration further by making the decomposition of composite actions into a special step in their refinement process. Such works include the discourse generation oriented DPOCL (Young and Moore 1994) and the work of Kambhampati *et al.* (1998) generalizing the practice for decompositional planners.

In our case, we chose a class of hierarchical planners based on Plan-Space Planning (PSP) algorithms (Bechon *et al.* 2014; Dvorak *et al.* 2014; Bercher *et al.* 2014) as a reference approach. The main difference here is that the decomposition is integrated into the classical POCL algorithm by only adding new types of flaws. This allows keeping all the flexibility and properties of POCL while adding the expressivity and abstraction capabilities of HTN.

5.2 LOLLIPOP

That first planner is a prototype destined to test the feasibility of a plan repairing approach using PSP. This is done through the addition of new flaw types in a classical POCL algorithm.

Another issue is caused by the need to get an arbitrary partial plan as an input. This plan can contain arbitrary problems and inconsistency since there are trivial ways to ensure consistence between the plan given and the new initial state or goal.

5.2.1 Operator Graph

In order to make POCL faster we experimented a mean to build a dependency graph for operators in the domain, computed at domain compilation time. The idea is to extract a partial plan from this graph by pruning it based off the provided goal and use that plan to quickstart the planning process. First we need to define the operator graph.

Definition 39 (Operator Graph). An operator graph g_O of a set of operators O is a labeled directed graph that binds two operators with the causal link $o_1 \xrightarrow{f} o_2$ if and only if there exists at least one fluent so that $(f \in \mathit{eff}(o_1)) \land f \vDash \mathit{pre}(o_2)$.

This definition was inspired by the notion of domain causal graph as explained in (Göbelbecker *et al.* 2010) and originally used as a heuristic in (Helmert *et al.* 2011). Causal graphs have fluents as their nodes and operators as their edges. Operator graphs are the opposite: an *operator dependency graph* for a set of actions. A similar structure was used in (Peot and Smith 1994) that builds the operator dependency graph of goals and uses precondition nodes instead of labels. We call *co-dependent* operators that form a cycle. If the cycle is made of only one operator (self-loop), then it is called *auto-dependent*.

While building this operator graph, we need a **providing map** that indicates, for each fluent, the list of operators that can provide it. This is a simpler version of the causal graphs that is reduced to an associative table easier to maintain. The list of providers can be sorted to drive resolver selection (as detailed in **??**). A **needing map** is also built but is only used for operator graph generation. We note $g_\mathcal{D}$ the operator graph built with the set of operators in the domain $\mathcal{D}$.

Example 29. In the figure 5.5, we illustrate the application of this mechanism on our example from **??**. Continuous lines correspond to the *domain operator graph* computed during domain compilation time.
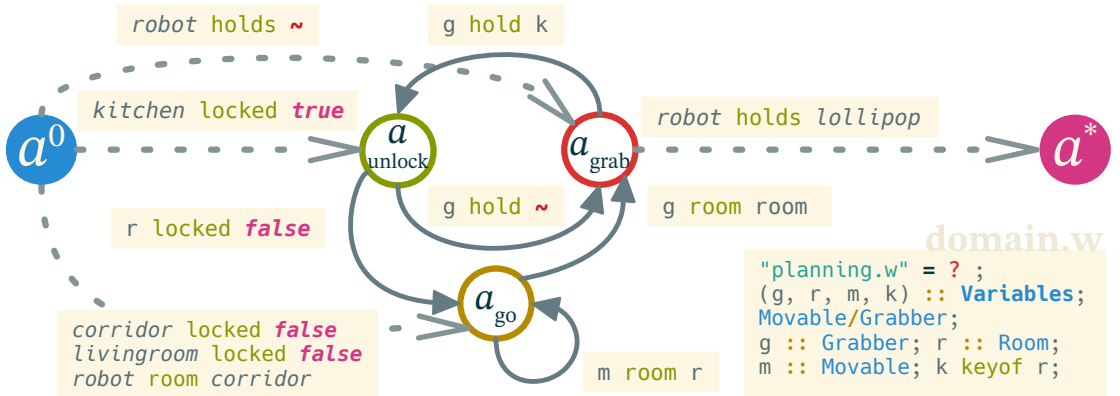

Figure 5.5: Diagram of the operator graph of example domain. Full arrows represent the domain operator graph and dotted arrows the dependencies added to inject the initial and goal steps.

The generation of the operator graph is detailed in algorithm 6. It explores the operators space and builds a providing and a needing map that gives the provided and

---
Algorithm 6 Operator graph generation and update algorithm
---

function ADDVERTEX(Action a)
    CACHE(a)   ▷ *Update of the providing and needing map*
    if binding **then**   ▷ *boolean that indicates if the binding was requested*
        BIND(a)

function CACHE(Action a)
    for all $f \in \textit{eff}(a)$ **do**   ▷ *Adds a to the list of providers of f*
        ADD($providing, f, a$)
    ...   ▷ *Same operation with needing and preconditions*

function BIND(Action a)
    for all $f \in \textit{pre}(a)$ **do**
        if $f \in providing$ **then**
            for all $\pi \in$ GET($providing, f$) **do**
                Link $l \leftarrow$ GETEDGE(π, a)   ▷ *Create the link if needed*
                $l \leftarrow l \cup \{f\}$ ▷ *Add the fluent as a cause*
    ...   ▷ *Same operation with needing and effects*

---

needed fluents for each operator. Once done it iterates on every precondition and searches for a satisfying cause to add the causal links to the operator graph.

To apply the notion of operator graphs to planning problems, we just need to add the initial and goal steps to the operator graph. In figure 5.5, we depict this insertion with our previous example using dotted lines. However, since operator graphs may have cycles, they can't be used directly as input to POCL algorithms to ease the initial backchaining. Moreover, the process of refining an operator graph into a usable one could be more computationally expensive than POCL itself.

In order to give a head start to the LOLLIPOP algorithm, we propose to build operator graphs differently with the algorithm detailed in algorithm 7. A similar notion was already presented as "basic plans" in (Sebastia *et al.* 2000). These "basic" partial plans use a more complete but slower solution for the generation that ensures that each selected steps are *necessary* for the solution. In our case, we built a simpler solution that can solve some basic planning problems but that also make early assumptions (since our algorithm can handle them). It does a simple and fast backward construction of a partial plan driven by the providing map. Therefore, it can be tweaked with the powerful heuristics of state search planning.

This algorithm is useful since it is specifically used on goals. The result is a valid partial plan that can be used as input to POCL algorithms.

5.2.2 Negative Refinements

The classical POCL algorithm works upon a principle of positive plan refinements. The two standard flaws (subgoals and threats) are fixed by *adding* steps, causal links, or variable binding constraints to the partial plan. Online planning needs to be able to *remove* parts of the plan that are not necessary for the solution. Since we assume that the input partial plan is quite complete, we need to define new flaws to optimize

Algorithm 7 Safe operator graph generation algorithm

```
function SAFE(Action ω)
    Stack<Action> open ← [a*]
    Stack<Action> closed ← ∅
    while open ≠ ∅ do
        Action a ← POCL(open)                              ▷ Remove a from open
        PUSH(closed, a)
        for all f ∈ pre(a) do
            Actions Aₚ ← GETPROVIDING(ᴨ, f)                ▷ Sorted by usefulness
            if Aₚ = ∅ then                                         ▷ (see section ??)
                S ← S \ {ᴨ}
                continue
            Action a' ← GETFIRST(ᴨ)
            if a' ∈ closed then
                continue
            if a' ∉ S then
                PUSH(open, a')
            S ← S ∪ {a'}
            Link l ← GETEDGE(a', a)                          ▷ Create the link if needed
            l ← l ∪ {f}                                       ▷ Add the fluent as a cause
```

and fix this plan. These flaws are called *negative* as their resolvers apply subtractive refinements on partial plans.

Definition 40 (Alternative). An alternative is a negative flaw that occurs when there is a better provider choice for a given link. An alternative to a causal link $a_p \xrightarrow{f} a_n$ is a provider a_b that has a better *utility value* than a_p.

The **utility value** of an operator is a measure of usefulness at the heart of our ranking mechanism detailed in **??**. It uses the incoming and outgoing degrees of the operator in the domain operator graph to measure its usefulness.

Finding an alternative to an operator is computationally expensive. It requires searching a better provider for every fluent needed by a step. To simplify that search, we select only the best provider for a given fluent and check if the one used is the same. If not, we add the alternative as a flaw. This search is done only on updated steps for online planning. Indeed, the safe operator graph mechanism is guaranteed to only choose the best provider (algorithm 7 at algorithm 7). Furthermore, subgoals won't introduce new fixable alternative as they are guaranteed to select the best possible provider.

Definition 41 (Orphan). An orphan is a negative flaw that occurs when a step in the partial plan (other than the initial or goal step) is not participating in the plan. Formally, a_o is an orphan if and only if $a_o \neq a^0 \wedge a_o \neq a^* \wedge (|\chi_{\prec}(a_o)| = 0) \vee \{= (\varnothing) : \chi_{\prec}(a_o)\}$.

With $d_\pi^+(a_o)$ being the *outgoing degree* of a_o in the directed graph formed by π and $L_\pi^+(a_o)$ being the set of *outgoing causal links* of a_o in π. This last condition checks for

hanging orphans that are linked to the goal with only bare causal links (introduced by threat resolution).

The introduction of negative flaws requires modifying the resolver definition (cf. **??**).

Definition 42 (Signed Resolvers). A signed resolver is a resolver with a notion of sign. We add to the resolver tuple the sign of the resolver noted $s \in \{+, -\}$.

The solution to an alternative is a negative refinement that simply removes the targeted causal link. This causes a new subgoal as a side effect, that will focus on its resolver by its rank (explained in **??**) and then pick the first provider (the most useful one). The resolver for orphans is the negative refinement that is meant to remove a step and its incoming causal link while tagging its providers as potential orphans.
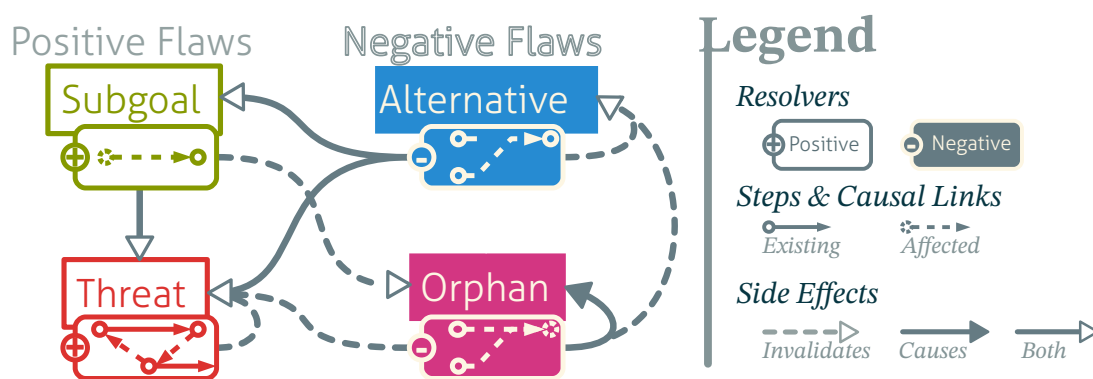

Figure 5.6: Schema representing flaws with their signs, resolvers and side effects relative to each other

The side effect mechanism also needs an upgrade since the new kind of flaws can interfere with one another. This is why we extend the side effect definition (cf. **??**) with a notion of sign.

Definition 43 (Signed Side Effects). A signed side effect is either a regular *causal side effect* or an *invalidating side effect*. The sign of a side effect indicates if the related flaw needs to be added or removed from the agenda.

The figure 5.6 exposes the extended notion of signed resolvers and side effects. When treating positive resolvers, nothing needs to change from the classical method. When dealing with negative resolvers, we need to search for extra subgoals and threats. Deletion of causal links and steps can cause orphan flaws that need to be identified for removal.

In the method described in (Peot and Smith 1993), a **invalidating side effect** is explained under the name of *DEnd* strategy. In classical POCL, it has been noticed that threats can disappear in some cases if subgoals or other threats were applied before them. For our mechanisms, we decide to gather under this notion every side effect that removes the need to consider a flaw. For example, orphans can be invalidated if a subgoal selects the considered step. Alternatives can remove the need to compute further subgoal of an orphan step as orphans simply remove the need to fix any flaws that concern the selected step.

These interactions between flaws are decisive for the validity and efficiency of the whole model, that is why we aim to drive flaw selection in a rigorous manner.

5.2.3 Usefullness Heuristic

Resolvers and flaws selection are the keys to improving performances. Choosing a good resolver helps to reduce the branching factor that accounts for most of the time spent on running POCL algorithms (Kambhampati 1994). Flaw selection is also important for efficiency, especially when considering negative flaws which can conflict with other flaws.

Conflicts between flaws occur when two flaws of opposite sign target the same element of the partial plan. This can happen, for example, if an orphan flaw needs to remove a step needed by a subgoal or when a threat resolver tries to add a promoting link against an alternative. The use of side effects will prevent most of these occurrences in the agenda but a base ordering will increase the general efficiency of the algorithm.

Based on the figure 5.6, we define a base ordering of flaws by type. This order takes into account the number of flaw types affected by causal side effects.

1. **Alternatives** will cut causal links that have a better provider. It is necessary to identify them early since they will add at least another subgoal to be fixed as a related flaw.
2. **Subgoals** are the flaws that cause most of the branching factor in POCL algorithms. This is why we need to make sure that all open conditions are fixed before proceeding on finer refinements.
3. **Orphans** remove unneeded branches of the plan. Yet, these branches can be found out to be necessary for the plan to meet a subgoal. Since a branch can contain many actions, it is preferable to leave the orphan in the plan until they are no longer needed. Also, threats involving orphans are invalidated if the orphan is resolved first.
4. **Threats** occur quite often in the computation. Searching and solving them is computationally expensive since they need to check if there are no paths that fix the flaw already. Many threats are generated without the need of resolver application (Peot and Smith 1993). That is why we rank all related subgoals and orphans before threats because they can add causal links or remove threatening actions that will fix the threat.

Resolvers need to be ordered as well, especially for the subgoal flaws. Ordering resolvers for a subgoal is the same operation as choosing a provider. Therefore, the problem becomes "how to rank operators?". The most relevant information on an operator is its usefulness and hurtfulness. These show how much an operator will help and how much it may cause branching after selection.

Definition 44 (Degree of an operator). Degrees are a measurement of the usefulness of an operator. Such a notion is derived from the incoming and outgoing degrees of a node in a graph.

We note $d_\pi^+(o)$ and $d_\pi^-(o)$ respectively the outgoing and incoming degrees of an operator in a plan π. These represent the number of causal links that goes out or toward the operator. We call proper degree of an operator $d^+(o) = |eff(o)|$ and $d^-(o) = |pre(o)|$ the number of preconditions and effects that reflect its intrinsic usefulness.

There are several ways to use the degrees as indicators. *Utility value* increases with every d^+, since this reflects a positive participation in the plan. It decreases with every d^- since actions with higher incoming degrees are harder to satisfy. The utility value bounds are useful when selecting special operators. For example, a user-specified constraint could be laid upon an operator to ensure it is only selected as a last resort. This operator will be affected with the smallest utility value possible. More commonly, the highest value is used for initial and goal steps to ensure their selection.

Our ranking mechanism is based on scores noted $s^\pm(o)$. A score contains two components: a positive subscore array that acts as a participation measurement and a negative subscore array that represents the dependencies of the operator. Each component of the score is an array of *subscores*.

The first step is the computation of the **base scores**. They are computed according to the following:

- $s^+(o)$ contains only $d_{\mathcal{D}^\Pi}^+(o)$, the positive degree of o in the domain operator graph. This will give a measurement of the predicted usefulness of the operator.
- $s^-(o)$ containing the following subscores:

  1. $d^-(o)$ the proper negative degree of o. Having more preconditions can lead to a potentially higher need for subgoals.
  2. $\sum_{c \in C_o(\mathcal{D}^\Pi)} |c|$ with $C_o(\mathcal{D}^\Pi)$ being the set of cycles where o participates in the domain operator graph. If an action is co-dependent (cf. **??**) it may lead to a dead-end as its addition will cause the formation of a cycle.
  3. $|C_o^\mathcal{S}(\mathcal{D}^\Pi)|$ is the number of self-cycle (0 or 1) o participates in. This is usually symptomatic of a *toxic operator* (cf. definition 46). Having an operator behaving this way can lead to backtracking because of operator instantiation.
  4. $|pre(o) \setminus L_{\mathcal{D}^\Pi}^-(o)|$ with $L_{\mathcal{D}^\Pi}^-(o)$ being the set of incoming causal links of o in the domain operator graph. This represents the number of open conditions. This is symptomatic of action that can't be satisfied without a compliant initial step.

Once these subscores are computed, the ranking mechanism starts the second phase, which computes the **realization scores**. These scores are potential bonuses given once the problem is known. It first searches the *inapplicable operators* that are all operators in the domain operator graph that have a precondition that isn't satisfied with a causal link. Then it searches the *eager operators* that provide fluents with no corresponding causal link (as they are not needed). These operators are stored in relation with their inapplicable or eager fluents.

The third phase starts with the beginning of the solving algorithm, once the problem has been provided. It computes the **effective realization scores** based on the initial and goal steps. It will increment $s_1^+(o)$ for each realized eager links (if the goal contains

the related fluent) and decrement $s_4^-(o)$ for each inapplicable preconditions realized by the initial step.

Last, the **final score** of each operator o, noted $h(o)$, is computed from positive and negative scores using the following formula:

$$h(o) = \sum_{n=1}^{|s^\pm(o)|} \pm p_n^\pm s_n^\pm(o)$$

A parameterized coefficient is associated to each subscore. It is noted $p_n^\pm$ with n being the index of the subscore in the array $s^\pm$. This respects the criteria of having a bound for the *utility value* as it ensures that it remains positive with 0 as a minimum bound and $+\infty$ for a maximum. The initial and goal steps have their utility values set to the upper bound to ensure their selections over other steps.

Choosing to compute the resolver selection at operator level has some positive consequences on the performances. Indeed, this computation is much lighter than approaches with heuristics on plan space (Shekhar and Khemani 2016) as it reduces the overhead caused by real time computation of heuristics on complex data. In order to reduce this overhead more, the algorithm sorts the providing associative array to easily retrieve the best operator for each fluent. This means that the evaluation of the heuristic is done only once for each operator. This reduces the overhead and allows for faster results on smaller plans.

5.2.4 Algorithm

The LOLLIPOP algorithm uses the same refinement algorithm as described in algorithm 5. The differences reside in the changes made on the behavior of resolvers and side effects. In algorithm 5 of algorithm 5, LOLLIPOP algorithm applies negative resolvers if the selected flaw is negative. In **??**, it searches for both signs of side effects. Another change resides in the initialization of the solving mechanism and the domain as detailed in algorithm 8. This algorithm contains several parts. First, the domainInit function corresponds to the code computed during the domain compilation time. It will prepare the rankings, the operator graph, and its caching mechanisms. It will also use strongly connected component detection algorithm to detect cycles. These cycles are used during the base score computation (algorithm 8). We add a detection of illegal fluents and operators in our domain initialization (algorithm 8). Illegal operators are either inconsistent or toxic.

Definition 45 (Inconsistent operators). An operator a is contradictory iff $\exists f\{f, \neg f\} \in eff(o) \vee \{f, \neg f\} \in pre(o)$

Definition 46 (Toxic operators). Toxic operators have effects that are already in their preconditions or empty effects. An operator o is toxic iff $pre(o) \cap eff(o) \neq \varnothing \vee eff(o) = \varnothing$

Algorithm 8 LOLLIPOP initialization mechanisms

---

function DOMAININIT(Operators O)
    operatorgraph $\mathcal{D}^\Pi$
    Score S
    for all Operator $o \in O$ **do**
        if ISILLEGAL(o) **then**   ▷ *Remove toxic and useless fluents*
            $O \leftarrow O \setminus \{o\}$ ▷ *If entirely toxic or useless*
            continue
        ADDVERTEX($o, \mathcal{D}^\Pi$)   ▷ *Add and bind all operators*
        CACHE(p, o)   ▷ *Cache operator in providing map*
    Cycles $C \leftarrow$ STRONGLYCONNECTEDCOMPONENT($\mathcal{D}^\Pi$)   ▷ *Using DFS*
    $S \leftarrow$ BASESCORES($O, \mathcal{D}^\Pi$)
    $i \leftarrow$ INAPPLICABLES($\mathcal{D}^\Pi$)
    $e \leftarrow$ EAGERS($\mathcal{D}^\Pi$)

function LOLLIPOPINIT(Problem $\mathcal{P}$)
    REALIZE($S, \mathcal{P}$)   ▷ *Realize the scores*
    CACHE($providing, I$)   ▷ *Cache initial step in providing ...*
    CACHE($providing, G$)   ▷ *... as well as goal step*
    SORT($providing, S$)   ▷ *Sort the providing map*
    if $L = \varnothing$ **then**
        $\mathcal{P}^\Pi \leftarrow$ SAFE($\mathcal{P}$)   ▷ *Computing the safe operator graph if the plan is empty*
    POPULATE($a, \mathcal{P}$)   ▷ *populate agenda with first flaws*

function POPULATE(Agenda a, Problem $\mathcal{P}$)
    for all Update $u \in U$ **do**   ▷ *Updates due to online planning*
        Fluents $F \leftarrow eff(u.new) \setminus eff(u.old)$ ▷ *Added effects*
        for all Fluent $f \in F$ **do**
            for all Operator $o \in$ BETTER($providing, f, o$) **do**
                for all Link $l \in L^+(o)$ **do**
                    if $f \in l$ **then**
                       ADDALTERNATIVE($a, f, o, l_\leftarrow, \mathcal{P}$)   ▷ *With $l_\leftarrow$ the target of l*
        $F \leftarrow eff(u.old) \setminus eff(u.new)$ ▷ *Removed effects*
        for all Fluent $f \in F$ **do**
            for all Link $l \in L^+(u.new)$ **do**
                if ISLIAR(l) **then**
                    $L \leftarrow L \setminus \{l\}$
                    ADDORPHANS($a, u, \mathcal{P}$)
        ...   ▷ *Same with removed preconditions and incomming liar links*
    for all Operator $o \in S$ **do**
        ADDSUBGOALS($a, o, \mathcal{P}$)
        ADDTHREATS($a, o, \mathcal{P}$)

Toxic actions can damage a plan as well as make the execution of POCL algorithm longer than necessary. This is fixed by removing the toxic fluents ($pre(a) \not\subseteq eff(a)$) and by updating the effects with $eff(a) = eff(a) \setminus pre(a)$. If the effects become empty, the operator is removed from the domain.

The lollipopInit function is executed during the initialization of the solving algorithm. We start by realizing the scores, then we add the initial and goal steps in the providing map by caching them. Once the ranking mechanism is ready, we sort the providing map. With the ordered providing map, the algorithm runs the fast generation of the safe operator graph for the problem's goal.

The last part of this initialization (algorithm 8) is the agenda population that is detailed in the populate function. During this step, we perform a search of alternatives based on the list of updated fluents. Online updates can make the plan outdated relative to the domain. This forms liar links :

Definition 47 (Liar links). A liar link is a link that doesn't hold a fluent in the preconditions or effect of its source and target. We note:

$$a_i \xrightarrow{f} a_j | f \notin eff(a_i) \cap pre(a_j)$$

A liar link can be created by the removal of an effect or preconditions during online updates (with the causal link still remaining).

We call lies the fluents that are held by links without being in the connected operators. To resolve the problem, we remove all lies. We delete the link altogether if it doesn't bear any fluent as a result of this operation. This removal triggers the addition of orphan flaws as side effects.

While the list of updated operators is crucial for solving online planning problems, a complementary mechanism is used to ensure that LOLLIPOP is complete. User provided plans have their steps tagged. If the failure has backtracked to a user-provided step, then it is removed and replaced by subgoals that represent each of its participation in the plan. This mechanism loops until every user provided steps have been removed.

5.2.5 Theoretical and Empirical Results

As proven in (Penberthy *et al.* 1992), the classical POCL algorithm is *sound* and *complete*.

First, we define some new properties of partial plans. The following properties are taken from the original proof. We present them again for convenience.

::: {#def:fullsupport .definition name="Full Support" } A partial plan π is fully supported if each of its steps $o \in S$ is fully supported. A step is fully supported if each of its preconditions $f \in pre(o)$ is supported. A precondition is fully supported if there exists a causal link l that provides it. We note:

$$\Downarrow \pi \equiv \begin{array}{l} \forall o \in S \ \forall f \in pre(o) \ \exists l \in L_{\pi}^{\rightarrow}(o) : \\ (f \in l \wedge \nexists t \in S(l_{\rightarrow} > t > o \wedge \neg f \in eff(t))) \end{array}$$

with $L_\pi^-(o)$ being the incoming causal links of o in π and $l_\rightarrow$ being the source of the link.

Definition 48 (Partial Plan Validity). A partial plan is a **valid solution** of a problem $\mathcal{P}$ iff it is *fully supported* and *contains no cycles*. The validity of π regarding a problem $\mathcal{P}$ is noted $\pi \vDash (\mathcal{P} \equiv \Downarrow \pi \wedge (C(\pi) = \varnothing))$ with $C(\pi)$ being the set of cycles in π.

5.2.5.1 Proof of Soundness

In order to prove that this property applies to LOLLIPOP, we need to introduce some hypothesis:

- operators updated by online planning are known.
- user provided steps are known.
- user provided plans don't contain illegal artifacts. This includes toxic or inconsistent actions, lying links and cycles.

Based on the definition 48 we state that:

$$\left(\begin{array}{l} \forall pre \in pre(G) : \\ \Downarrow pre \wedge \begin{array}{l} \forall o \in L_\pi^-(G)_\rightarrow \ \forall pre' \in pre(o) : \\ (\Downarrow pre' \wedge C_o(\pi) = \varnothing) \end{array} \end{array} \right) \implies \pi \vDash \mathcal{P} \qquad (5.1)$$

where $L_\pi^-(G)_\rightarrow$ is the set of direct antecedents of G and $C_o(\pi)$ is the set of fluents containing o in π.

This means that π is a solution if all preconditions of G are satisfied. We can satisfy these preconditions using operators iff their preconditions are all satisfied and if there is no other operator that threatens their supporting links.

First, we need to prove that equation (5.1) holds on LOLLIPOP initialization. We use our hypothesis to rule out the case when the input plan is invalid. The algorithm 7 will only solve open conditions in the same way subgoals do it. Thus, safe operator graphs are valid input plans.

Since the soundness is proven for regular refinements and flaw selection, we need to consider the effects of the added mechanisms of LOLLIPOP. The newly introduced refinements are negative, they don't add new links:

$$\forall f \in \mathcal{F}(\pi) \ \forall r \in r(f) : C_\pi(f.n) = C_{f(\pi)}(f.n) \qquad (5.2)$$

with $\mathcal{F}(\pi)$ being the set of flaws in π, $r(f)$ being the set of resolvers of f, $f.n$ being the needer of the flaw and $f(\pi)$ being the resulting partial plan after application of the flaw. Said otherwise, an iteration of LOLLIPOP won't add cycles inside a partial plan.

The orphan flaw targets steps that have no path to the goal and so can't add new open conditions or threats. The alternative targets existing causal links. Removing a causal link in a plan breaks the full support of the target step. This is why an alternative will always insert a subgoal in the agenda corresponding to the target of the removed

causal link. Invalidating side effects also don't affect the soundness of the algorithm since the removed flaws are already solved. This makes:

$$\forall f \in \mathcal{F}^-(\pi) : \Downarrow \pi \implies \Downarrow f(\pi) \qquad (5.3)$$

with $\mathcal{F}^-(\pi)$ being the set of negative flaws in the plan π. This means that negative flaws don't compromise the full support of the plan.

Equations (5.2) and (5.3) lead to equation (5.1) being valid after the execution of LOL-LIPOP. The algorithm is sound.

5.2.5.2 Proof of Completeness

The soundness proof shows that LOLLIPOP's refinements don't affect the support of plans in term of validity. It was proven that POCL is complete. There are several cases to explore to transpose the property to LOLLIPOP:

Lemma (Conservation of Validity). *If the input plan is a valid solution, LOLLIPOP returns a valid solution.*

Proof. With equations (5.2) and (5.3) and the proof of soundness, the conservation of validity is already proven. □

Lemma (Reaching Validity with incomplete partial plans). *If the input plan is incomplete, LOLLIPOP returns a valid solution if it exists.*

Proof. Since POCL is complete and the equation (5.3) proves the conservation of support by LOLLIPOP, then the algorithm will return a valid solution if the provided plan is an incomplete plan and the problem is solvable. □

Lemma (Reaching Validity with empty partial plans). *If the input plan is empty and the problem is solvable, LOLLIPOP returns a valid solution.*

Proof. This is proven using (**???**) and POCL's completeness. However, we want to add a trivial case to the proof: $pre(G) = \emptyset$. In this case the **??** of the algorithm 5 will return a valid plan.

□

Lemma (Reaching Validity with a dead-end partial plan). *If the input plan is in a dead-end, LOLLIPOP returns a valid solution.*

Proof. Using input plans that can be in an undetermined state is not covered by the original proof. The problem lies in the existing steps in the input plan. Still, using our hypothesis we add a failure mechanism that makes LOLLIPOP complete. On failure, the needer of the last flaw is deleted if it wasn't added by LOLLIPOP. User defined steps are deleted until the input plan acts like an empty plan. Each deletion will cause corresponding subgoals to be added to the agenda. In this case, the backtracking is preserved and all possibilities are explored as in POCL. □

Since all cases are covered, this proves the property of completeness.

5.2.5.3 Experimental Results

The experimental results focused on the properties of LOLLIPOP for online planning. Since classical POCL is unable to perform online planning, we tested our algorithm considering the time taken for solving the problem for the first time. We profiled the algorithm on a benchmark problem containing each of the possible issues described earlier.
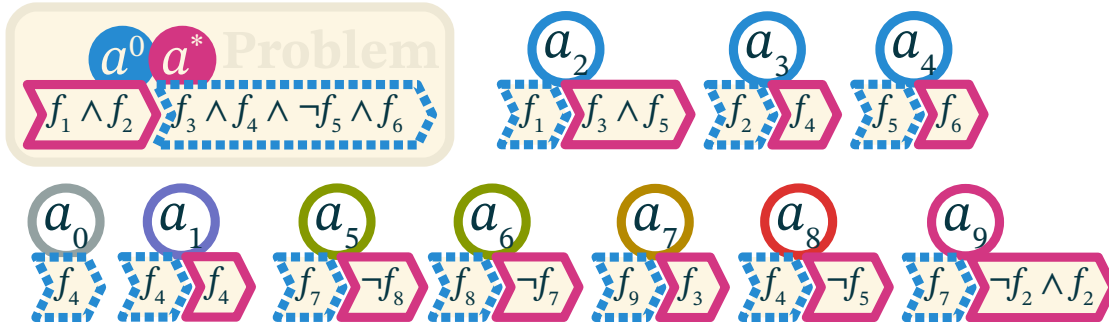

Figure 5.7: Domain used to compute the results. First line is the initial and goal step along with the useful actions. Second line contains a threatening action t, two co-dependent actions n and l, a useless action u, a toxic action v, a deadend action w and an inconsistent action x

In figure 5.7, we expose the planning domain used for the experiments. During the domain initialization, the actions u and v are eliminated from the domain since they serve no purpose in the solving process. The action x is stripped of its negative effect because it is inconsistent with the effect 2.

As the solving starts, LOLLIPOP computes a safe operator graph (full lines in figure 5.8). As we can see this partial plan is nearly complete already. When the main refining function starts it receives an agenda with only a few flaws remaining.
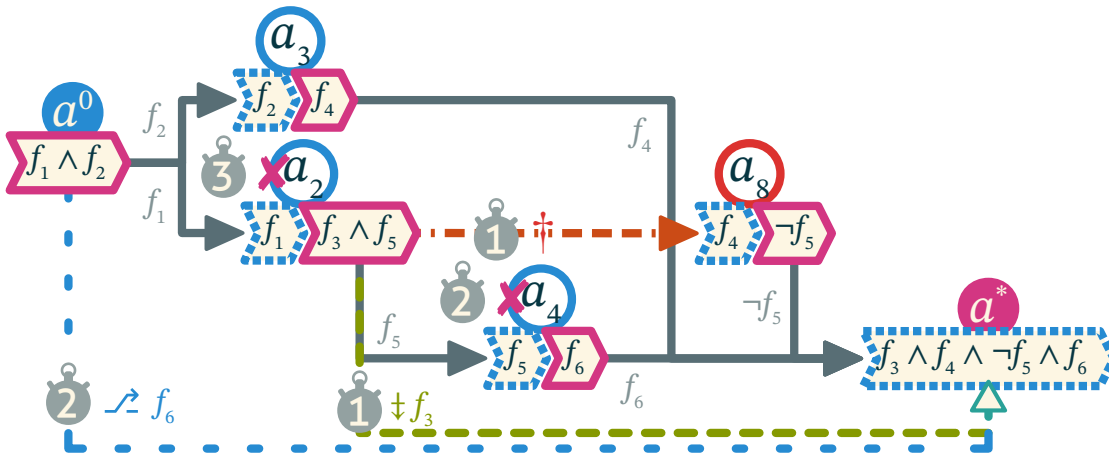

Figure 5.8: In full lines the initial safe operator graph. In thin, sparse and irregularly dotted lines a subgoal, alternative and threat caused causal link.

FIXME: names to indexes

Then the main refinement function starts (time markers **1**). LOLLIPOP selects as resolver a causal link from a to satisfy the open condition of the goal step. Once the

Table 5.2: Average times of 1.000 executions on the problem. The first column is for a simple run on the problem. Second and third columns are times to replan with one and two changes done to the domain for online planning.

Experiment	Single	Online 1	Online 2
Time (ms)	0.86937	0.38754	0.48123

first threat between a and t is resolved the second threat is invalidated. On a second execution, the domain changes for online planning with 6 added to the initial step. This solving (time markers **2**) add as flaw an alternative on the link from c to the goal step. A subgoal is added that links the initial and goal step for this fluent. An orphan flaw is also added that removes c from the plan. Another solving takes place as the goal step doesn't need 3 as a precondition (time markers **3**). This causes the link from a to be cut since it became a liar link. This adds a as an orphan that gets removed from the plan even if it was hanging by the bare link to t.

The measurements exposed in table 5.2 were made with an Intel® Core™ i7-4720HQ with a 2.60GHz clock. Only one core was used for the solving. The same experiment done only with the chronometer code gave a result of $70ns$ of error. We can see an increase of performance in the online runs because of the way they are conducted by LOLLIPOP.

5.3 HEART

5.3.1 Domain Compilation

In order to simplify the input of the domain, the causes of the causal links in the methods are optional. If omitted, the causes are inferred by unifying the preconditions and effects with the same mechanism as in the subgoal resolution in our POCL algorithm. Since we want to guarantee the validity of abstract plans, we need to ensure that user provided plans are solvable. We use the following formula to compute the final preconditions and effects of any composite action a: $pre(a) = \bigcup_{l\in L^+(I_m)}^{m\in methods(a)} causes(l)$ and $eff(a) = \bigcup_{l\in L^-(G_m))}^{m\in methods(a)} causes(l)$. An instance of the classical POCL algorithm is then run on the problem $\mathcal{P}_a = \langle \mathcal{D}, C_{\mathcal{P}}, a \rangle$ to ensure its coherence. The domain compilation fails if POCL cannot be completed. Since our decomposition hierarchy is acyclic ($a \notin A_a$, see definition 50) nested methods cannot contain their parent action as a step.

5.3.2 Abstraction in POCL

In order to properly introduce the changes made for using HTN domains in POCL, we need to define a few notions.

Transposition is needed to define decomposition.

Definition 49 (Transposition). In order to transpose the causal links of an action a' with the ones of an existing step a in a plan π, we use the following operation:

$$a \vartriangleright^-_\pi a' = \left\{ \phi^-(l) \xrightarrow{\ causes(l)\ } a' \ :\ l \in L^-_\pi(a) \right\}$$

It is the same with $a' \xrightarrow{\ causes(l)\ } \phi^+(l)$ and L^+ for $a \vartriangleright^+ a'$. This supposes that the respective preconditions and effects of a and a' are equivalent. When not signed, the transposition is generalized: $a \vartriangleright a' = a \vartriangleright^- a' \cup a \vartriangleright^+ a'$.

Example 30. $a \vartriangleright^- a'$ gives all incoming links of a with the a replaced by a'.

Definition 50 (Proper Actions). Proper actions are actions that are "contained" within an entity (either a domain, plan or action). We note this notion $A_a = A_a^{lv(a)}$ for an action a. It can be applied to various concepts:

- For a *domain* or a *problem*, $A_{\mathcal{P}} = A_{\mathcal{D}}$.
- For a *plan*, it is $A_\pi^0 = S_\pi$.
- For an *action*, it is $A_a^0 = \bigcup_{m \in methods(a)} S_m$. Recursively: $A_a^n = \bigcup_{b \in A_a^0} A_b^{n-1}$. For atomic actions, $A_a = \varnothing$.

Example 31. The proper actions of $make(drink)$ are the actions contained within its methods. The set of extended proper actions adds all proper actions of its single proper composite action $infuse(drink, water, cup)$.

Definition 51 (Abstraction Level). This is a measure of the maximum amount of abstraction an entity can express, defined recursively by:

$$lv(x) = \left(\max_{a \in A_x}(lv(a)) + 1 \right) [A_x \neq \varnothing]$$

We use Iverson brackets here, see notations in **??**.

Example 32. The abstraction level of any atomic action is 0 while it is 2 for the composite action $make(drink)$. The example domain (in **??**) has an abstraction level of 3.

The most straightforward way to handle abstraction in regular planners is illustrated by Duet (Gerevini *et al.* 2008) by managing hierarchical actions separately from a task insertion planner. We chose to add abstraction in POCL in a manner inspired by the work of Bechon *et al.* (2014) on a planner called HiPOP. The difference between the original HiPOP and our implementation of it is that we focus on the expressivity and the ways flaw selection can be exploited for partial resolution. Our version is lifted at runtime while the original is grounded for optimizations. All mechanisms we have implemented use POCL but with different management of flaws and resolvers. The original algorithm 5 is left untouched.

One of those changes is that resolver selection needs to be altered for subgoals. Indeed, as stated by the authors of HiPOP: the planner must ensure the selection of high-level operators in order to benefit from the hierarchical aspect of the domain. Otherwise, adding operators only increases the branching factor. Composite actions are not usually meant to stay in a finished plan and must be decomposed into atomic steps from one of their methods.

Definition 52 (Decomposition Flaws). They occur when a partial plan contains a non-atomic step. This step is the needer a_n of the flaw. We note its decomposition $a_n \oplus$.

- *Resolvers:* A decomposition flaw is solved with a **decomposition resolver**. The resolver will replace the needer with one of its instantiated methods $m \in methods(a_n)$ in the plan π. This is done by using transposition such that: $a_n \oplus_\pi^m = \langle S_m \cup (S_\pi \setminus \{a\}), a_n \rhd^- I_m \cup a_n \rhd^+ G_m \cup (L_\pi \setminus L_\pi(a_n))$.
- *Side effects:* A decomposition flaw can be created by the insertion of a composite action in the plan by any resolver and invalidated by its removal:

$$\bigcup_{a_m \in S_m}^{f \in pre(a_m)} \pi' \updownarrow_f a_m \bigcup_{a_b \in S_{\pi'}}^{l \in L_{\pi'}} a_b \otimes l \bigcup_{a_c \in S_m}^{lv(a_c) \neq 0} a_c \oplus$$

Example 33. When adding the step $make(tea)$ in the plan to solve the subgoal that needs tea being made, we also introduce a decomposition flaw that will need this composite step replaced by its method using a decomposition resolver. In order to decompose a composite action into a plan, all existing links are transposed to the initial and goal step of the selected method, while the composite action and its links are removed from the plan.

The main differences between HiPOP and HEART in our implementations are the functions of flaw selection and the handling of the results (one plan for HiPOP and a plan per cycle for HEART). In HiPOP, the flaw selection is made by prioritizing the decomposition flaws. Bechon *et al.* (2014) state that it makes the full resolution faster. However, it also loses opportunities to obtain abstract plans in the process.

5.3.3 Planning in cycle

The main focus of our work is toward obtaining **abstract plans** which are plans that are completed while still containing composite actions. In order to do that the flaw selection function will enforce cycles in the planning process.

Definition 53 (Cycle). A cycle is a planning phase defined as a triplet $c = \langle lv(c), agenda, \pi_{lv(c)} \rangle$ where: $lv(c)$ is the maximum abstraction level allowed for flaw selection in the $agenda$ of remaining flaws in partial plan $\pi_{lv(c)}$. The resolvers of subgoals are therefore constrained by the following: $a_p \downarrow_f a_n : lv(a_p) \leq lv(c)$.

During a cycle all decomposition flaws are delayed. Once no more flaws other than decomposition flaws are present in the agenda, the current plan is saved and all remaining decomposition flaws are solved at once before the abstraction level is lowered for the next cycle: $lv(c') = lv(c) - 1$. Each cycle produces a more detailed abstract plan than the one before.

Abstract plans allow the planner to do an approximate form of anytime execution. At any given time the planner is able to return a fully supported plan. Before the first cycle, the plan returned is $\pi_{lv(a_0)}$.

Example 34. In our case using the method of intent recognition of Sohrabi *et al.* Sohrabi *et al.* (2016), we can already use $\pi_{lv(a_0)}$ to find a likely goal explaining an observation (a set of temporally ordered fluents). That can make an early assessment of the probability of each goal of the recognition problem.

For each cycle c, a new plan $\pi_{lv(c)}$ is created as a new method of the root operator a_0. These intermediary plans are not solutions of the problem, nor do they mean that the problem is solvable. In order to find a solution, the HEART planner needs to reach the final cycle c_0 with an abstraction level $lv(c_0) = 0$. However, these plans can be used to derive meaning from the potential solution of the current problem and give a good approximation of the final result before its completion.
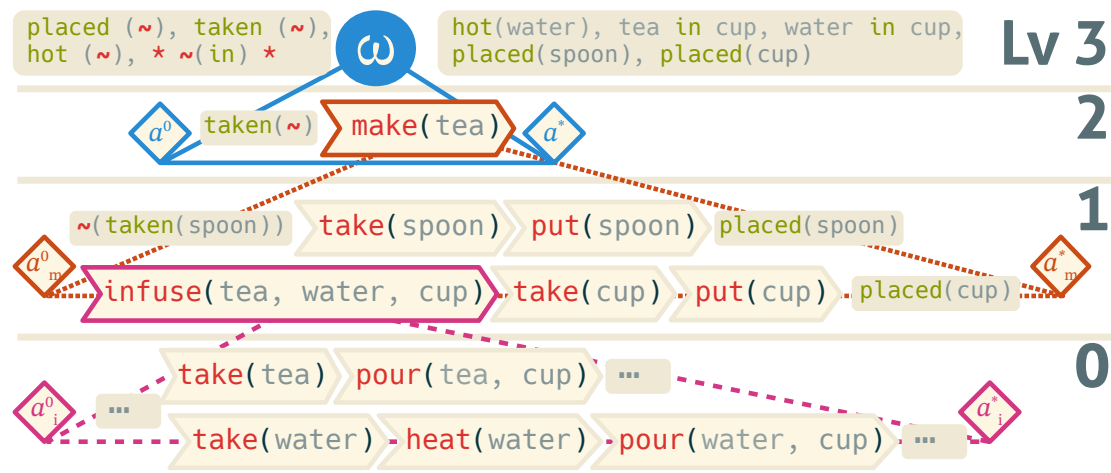

Figure 5.9: Illustration of how the cyclical approach is applied on the example domain. Atomic actions that are copied from a cycle to the next are omitted.

Example 35. In the figure 5.9, we illustrate the way our problem instance is progressively solved. Before the first cycle c_2, all we have is the root operator and its plan π_3. Then within the first cycle, we select the composite action $make(tea)$ instantiated from the operator $make(drink)$ along with its methods. All related flaws are fixed until all that is left in the agenda is the abstract flaws. We save the partial plan π_2 for this cycle and expand $make(tea)$ into a copy of the current plan π_1 for the next cycle. The solution of the problem will be stored in π_0 once found.

5.3.4 Properties of Abstract Planning

In this section, we prove several properties of our method and resulting plans: HEART is complete, sound and its abstract plans can always be decomposed into a valid solution.

The completeness and soundness of POCL has been proven in (Penberthy *et al.* 1992). An interesting property of POCL algorithms is that flaw selection strategies do not impact these properties. Since the only modification of the algorithm is the extension of the classical flaws with a decomposition flaw, all we need to explore, to update the proofs, is the impact of the new resolver. By definition, the resolvers of decomposition

flaws will take into account all flaws introduced by its resolution into the refined plan. It can also revert its application properly.

Lemma (Decomposing preserves acyclicity). *The decomposition of a composite action with a valid method in an acyclic plan will result in an acyclic plan. Formely,* $\forall a_s \in S_\pi$: $a_s \not\succ_\pi a_s \implies \forall a_s' \in S_{a \oplus_\pi^m} : a_s' \not\succ_{a \oplus_\pi^m} a_s'$.

Proof. When decomposing a composite action a with a method m in an existing plan π, we add all steps S_m in the refined plan. Both π and m are guaranteed to be cycle free by definition. We can note that $\forall a_s \in S_m$: $(\nexists a_t \in S_m : a_s > a_t \wedge \neg f \in \mathit{eff}(a_t)) \implies f \in \mathit{eff}(a)$. Said otherwise, if an action a_s can participate a fluent f to the goal step of the method m then it is necessarily present in the effects of a. Since higher level actions are preferred during the resolver selection, no actions in the methods are already used in the plan when the decomposition happens. This can be noted $\exists a \in \pi \implies S_m \uplus S_\pi$ meaning that in the graph formed both partial plans m and π cannot contain the same edges therefore their acyclicity is preserved when inserting one into the other.

□

Lemma (Solved decomposition flaws cannot reoccur). *The application of a decomposition resolver on a plan π, guarantees that $a \notin S_{\pi'}$ for any partial plan refined from π without reverting the application of the resolver.*

Proof. As stated in the definition of the methods (definition 31): $a \notin A_a$. This means that a cannot be introduced in the plan by its decomposition or the decomposition of its proper actions. Indeed, once a is expanded, the level of the following cycle $c_{lv(a)-1}$ prevents a to be selected by subgoal resolvers. It cannot either be contained in the methods of another action that are selected afterward because otherwise following definition 51 its level would be at least $lv(a) + 1$.

□

Lemma (Decomposing to abstraction level 0 guarantees solvability). *Finding a partial plan that contains only decomposition flaws with actions of abstraction level 1, guarantees a solution to the problem.*

Proof. Any method m of a composite action $a : lv(a) = 1$ is by definition a solution of the problem $\mathcal{P}_a = \langle \mathcal{D}, C_\mathcal{P}, a \rangle$. By definition, $a \notin A_a$, and $a \notin A_{a \oplus_\pi^m}$ (meaning that a cannot reoccur after being decomposed). It is also given by definition that the instantiation of the action and its methods are coherent regarding variable constraints (everything is instantiated before selection by the resolvers). Since the plan π only has decomposition flaws and all flaws within m are guaranteed to be solvable, and both are guaranteed to be acyclical by the application of any decomposition $a \oplus_\pi^m$, the plan is solvable.

□

Lemma (Abstract plans guarantee solvability). *Finding a partial plan π that contains only decomposition flaws, guarantees a solution to the problem.*

Proof. Recursively, if we apply the previous proof on higher level plans we note that decomposing at level 2 guarantees a solution since the method of the composite actions are guaranteed to be solvable.

□

From these proofs, we can derive the property of soundness (from the guarantee that the composite action provides its effects from any methods) and completeness (since if a composite action cannot be used, the planner defaults to using any action of the domain).

5.3.5 Computational Profile

In order to assess its capabilities, HEART was evaluated on two criteria: quality and complexity. All tests were executed on an Intel® Core™ i7-7700HQ CPU clocked at 2.80GHz. The Java process used only one core and was not limited by time or memory (32 GB that wasn't entirely used up). Each experiment was repeated between 700 and 10 000 times to ensure that variations in speed were not impacting the results.
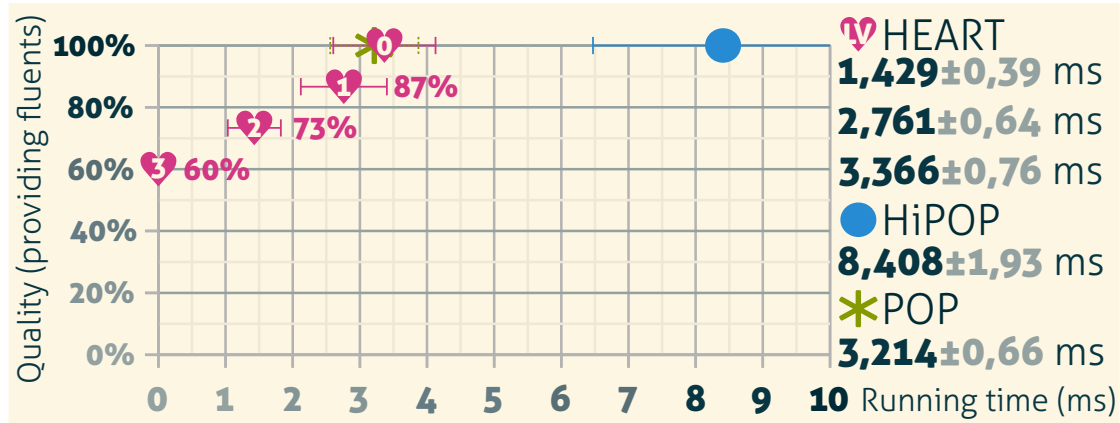

Figure 5.10: Evolution of the quality with computation time.

Figure 5.10 shows how the quality is affected by the abstraction in partial plans. The tests are made using our example domain (see **??**). The quality is measured by counting the number of providing fluents in the plan $\left| \bigcup_{a \in S_\pi} \textit{eff}(a) \right|$. This metric is actually used to approximate the probability of a goal given observations in intent recognition ($P(G|O)$ with noisy observations, see (Sohrabi *et al.* 2016)). The percentages are relative to the total number of unique fluents of the complete solution. These results show that in some cases it may be more interesting to plan in a leveled fashion to solve HTN problems. For the first cycle of level 3, the quality of the abstract plan is already of 60%. This is the quality of the exploitation of the plan *before any planning*. With almost three-quarters of the final quality and less than half of the complete computation time, the result of the first cycle is a good quality/time compromise.

In the second test, we used generated domains. These domains consist of an action of abstraction level 5. This action has a single method containing a number of actions of level 4. We call this number the width of the domain. All needed actions are built
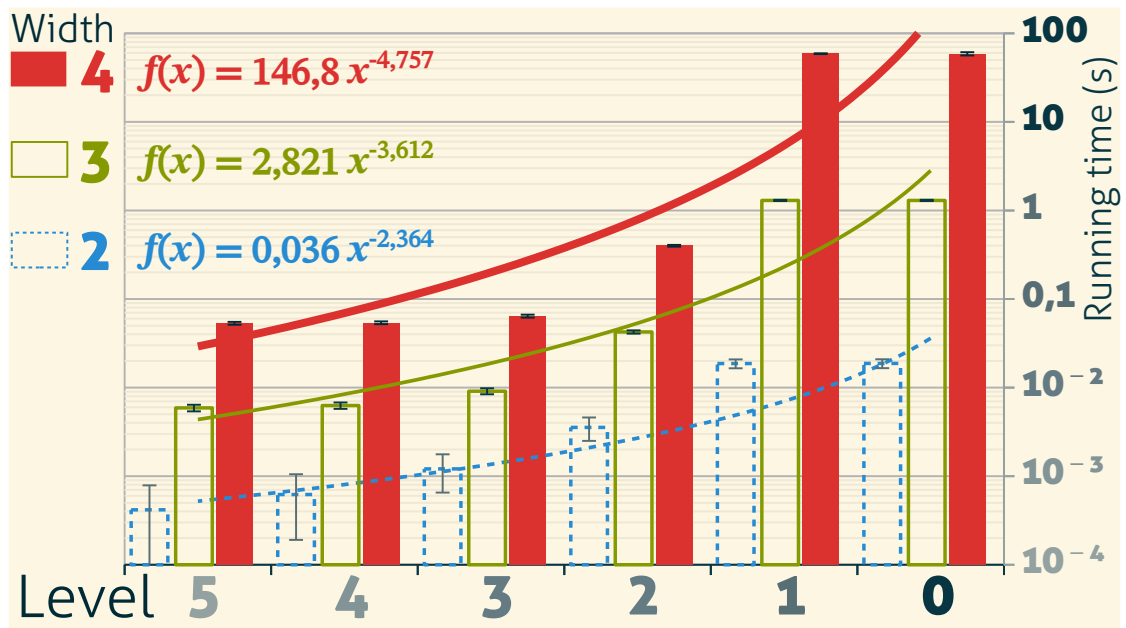
Figure 5.11: Impact of domain shape on the computation time by levels. The scale of the vertical axis is logarithmic. Equations are the definition of the trend curves.

recursively to form a tree shape. Atomic actions only have single fluent effects. The goal is the effect of the higher level action and the initial state is empty. These domains do not contain negative effects. Figure 5.11 shows the computational profile of HEART for various levels and widths. We note that the behavior of HEART seems to follow an exponential law with the negative exponent of the trend curves seemingly being correlated to the actual width. This means that computing the first cycles has a complexity that is close to being *linear* while computing the last cycles is of the same complexity as classical planning which is at least *P-SPACE* (depending on the expressivity of the domain) (Erol *et al.* 1995).

6 Toward intent recognition

Since the original goal of this thesis was on intent recognition of dependant persons, we need to explain some more about that specific domain. The problem is to infer the goals of an external agent through only observations without intervention. In the end, the idea is to infer that goal confidently enough to start assisting the other agent without explicit instructions.

6.1 Domain problems

This field was widely studied. Indeed, at the end of the last century, several works started using *abduction* to infer intents from observational data. Of course this comes as a chalenge since there is a lot of uncertainty involved. We have no reliable information on the possible goals of the other agent and we don't have the same knowledge of the world. Also, observations can sometimes be incomplete or misleading, the agent can abandon goals or pursue several goals at once while also doing them suboptimally or even fail at them. To finish, someimes agents can actively try to hide their intents to the viewer.

6.1.1 Observations and inferences

As explained above, we can only get close to reality and any progress in relevance and detail is exponentially expensive in terms of computing and memory resources. That is why any system will maintain a high degree of abstraction that will cause errors inherent in this approximation.

This noise phenomenon can impact the activity and situation recognition system and therefore seriously impact the intention recognition and decision making system with an amplification of the error as it is processed. It is also important to remember that this phenomenon of data noise is also present in inhibition and that the lack of perception of an event is as disabling as the perception of the wrong event.

It is possible to protect recognition systems in an appropriate way, but this often implies a restriction on the levels of possibilities offered by the system such as specialized recognition or recognition at a lower level of relevance.

6.1.2 Cognitive inconsistencies

In the field of personal assistance, activity recognition is a crucial element. However, it happens that these events are very difficult to recognize. The data noise mentioned above can easily be confused with an omission on the part of the observed agent. This dilemma is also present during periods of inactivity, the system can start creating events from scratch to fill what it may perceive as inhibition noise.

These problems are accompanied by others related to the behaviour of the observed agent. For example, they may perform unnecessary steps, restart ongoing activities or suddenly abandon them. It is added that other aspects of observations can make automated inferences such as ambiguous actions or the agent performing an action that resembles another complicated.

However, some noise problems can be easily detected by simple cognitive processes such as impossible sequences (e. g. closing a closed door). Contextual analyses provide a partial solution to some of these problems.

6.1.3 Sequentiality

Since our recognition is based on a highly temporal planning aspect, we must take into account the classic problems of sequentiality.

A first problem is to determine the end of one plan and the beginning of another. Indeed, it is possible that some transitions between two planes may appear to be a plane in itself and therefore may cause false positives. Another problem is that of intertwined planes. A person can do two things at once, such as answering the phone while cooking. An action in an intertwined plan can then be identified as a discontinuation of activity or a logical inconsistency. A final problem is that of overloaded actions. Not only can an agent perform two tasks simultaneously, but also perform an action that contributes to two activities. These overloaded actions make the process of intention recognition complex because they are close to data noise.

6.2 Existing approaches

The problem of intention recognition has been strongly addressed from many angles. It is therefore not surprising that there are many paradigms in the field. The first studies on the subject highlight the fact that intention recognition problems are problems of abductive logic or graph coverage. Since then, many models have competed in imagination and innovation to improve the field. These include constraint system-based models that provide a solution based on pre-established rules and compiled plan libraries, those that use state or action networks that then launch algorithms on these data, and reverse planning systems.

116

6.2.1 Constraint

One of the approaches to intention recognition is the one that builds a system around a strong logical constraint. There is often a time constraint system that is complemented by various extensions to cover as many sequential problems as possible.

In order to solve a problem of intention recognition, abductive logic can be used. Contrary to deductive logic, the goal is to determine the objective from the observed actions. Among the first models introduced is Goldman *et al.* (1999)'s model, which uses the principle of action to construct a logical representation of the problem. This paradigm consists in creating logical rules as if the action in question were actually carried out, but in hypothesizing the predicates that concretize the action and thus being able to browse the research space thus created in order to find all the possible plans containing the observed actions and concretizing defined intentions. This model is strongly based on first-order logic and SWI Prolog logic programming languages. Although revolutionary for the time, this system pale in comparison to recent systems, particularly in terms of prediction accuracy.

Some paradigms use algebra to determine possible plans from observed actions. In particular, we find the model of Bouchard *et al.* (2006) which extends the subsumption relationship from domain theory to the description of action and sequence of action in order to introduce it as an order relationship in the context of the construction of a lattice composed of possible plans considering the observed actions. This model simply takes into account the observed actions and selects any plan from the library of plans that contains at least one observed action. Then this paradigm will construct all the possible plans that correspond to the Cartesian product of the observed actions with the actions contained in the selected plans (while respecting their order). This system makes it possible to obtain a subsumption relationship that corresponds to the fact that the plans are more or less general. Unfortunately, this relationship alone does not provide any information on which plan is most likely.

That is why Roy *et al.* (2011) created a probabilistic extension of this model. This uses frequency data from a system learning period to calculate the influence probabilities of each plane in the recognition space. This makes it possible to calculate probabilistic intervals for each plan, action as well as for a plan knowing a given action. In order to determine the probability of each plane knowing the upper bound of the lattice (plane containing all observed actions) the sum of the conditional probabilities of the plane for each observed action divided by the number of observed actions is made. This gives a probability interval for each plane allowing the ordinates. This model has the advantage of considering many possible plans but has the disadvantage of seeing a computational explosion as soon as the number of observed actions increases and the context is not taken into account.

Another approach is that of grammar. Indeed, we can consider actions as words and sequences as sentences and thus define a system that allows us to recognize shots from incomplete sequences. Vidal *et al.* (2010) has therefore created a system of intention recognition based on grammar. It uses the evaluated grammar system to specify measurements from observations. These measures will make it possible to select specific plans and thus return a hierarchical hypothesis tree with the actions already carried out, the future and the plans from which they are derived. This model

is very similar to first order logic-based systems, and uses a SWI Prolog type logic language programming system. Given the scope of maritime surveillance, this model, although taking very well into account the context and the evolution of the measures, is only poorly adapted to an application in assistance, particularly in the absence of a system for discriminating against results plans.

Another class of approaches is that of diverting standard problem-solving tools to solve the problem of intention recognition. It is therefore possible, by modifying traditional algorithms or by transforming a problem, to ensure that the solution of the tool corresponds to the one sought.

Inoue and Inui (2011) develops the idea of a model that uses linear programming to solve the recognition problem. Indeed, observations are introduced in the form of causes in relation to hypotheses, in a first-order logic predicate system. Each atom is then weighted and introduced into a process of problem transformation by feedback and the introduction of order and causality constraints in order to force the linear program towards optimal solutions by taking into account observations. Although ingenious, this solution does not discriminate between possible plans and is very difficult to apply to real-time recognition situations, mainly because of the problem transformation procedure required each time the problem is updated.

Another constraint paradigm is the one presented by Raghavana *et al.* using a Markovian extension of first-order logic. The model consists of a library of plans represented in the form of Horn clauses indicating which actions imply which intentions. The aim is therefore to reverse the implications in order to transform the deduction mechanism into an abduction mechanism. Exclusionary constraints and a system of weights acquired through learning must then be introduced to determine the most likely intention. Once again, despite the presence of a system of result discrimination, there is no consideration of context and abductive transformation remains too cumbersome a process for real-time recognition.

6.2.2 Networks

As in its early days, intention recognition can still be modeled in the form of graphs. Very often in intention recognition, trees are used to exploit the advantages of acyclicity in resolution and path algorithms. In the prolific literature of Geib et al. we find the model at the basis of PHATT (Geib 2002) which consists of an AND/OR tree representing a HTN that contains the intentions as well as their plans or methods. A prior relationship is added to this model and it is through this model that constraints are placed on the execution of actions. Once an action is observed, all the successors of the action are unlocked as potential next observed action. We can therefore infer by hierarchical path the candidate intentions for the observed sequence.

Since this model does not allow discrimination of results, Geib and Goldman (2005) then adds probabilities to the explanations of the observations. The degree of coverage of each possible goal is used to calculate the probability of each goal. That is, the goal with the plan containing the most observed action and the least unobserved action will be the most likely. This is very ingenious, as the coverage rate is one of

the most reliable indicators. However, the model only takes into account temporality and therefore has no contextual support. The representation in the form of a tree also makes it very difficult to be flexible in terms of the plans, which are then fixed a priori.

The RTH model is often used in the field, such as the hierarchical tree form used by Avrahami-Zilberbrand *et al.* (2005). The tree consists of nodes that represent various levels of action and intent. A hierarchical relationship links these elements together to define each intention and its methods. To this tree is added an anteriority relationship that constrains the execution order. This paradigm uses time markers that guarantee order using an actualisation algorithm that also updates a hypothesis tree containing possible intentions for each observation.

A probabilistic extension of the Avrahami-Zilberbrand and Kaminka (2006) applies a hierarchical hidden Markov model to the action tree. Using three types of probability, that of plan tracking, execution interleaving and interruption, we can calculate the probability of execution of each plan according to the observed sequence. The logic and contextual model filtered on the possible plans upstream leaving us with few calculations to order these plans.

This contextual model uses a decision tree based on a system of world properties. Each property has a finite (and if possible very limited) number of possible values. This allows you to create a tree containing for each node a property and an arc for each value. This is combined with other nodes or leaves that are actions. While running through the tree during execution, the branches that do not correspond to the current value of each property are pruned. Once a leaf is reached, it is stored as a possible action. This considerably reduces the research space but requires a balanced tree that is not too large or restrictive.

When we approach stochastic models, we very often find Markovian or Bayesian models. These models use different probabilistic tools ranging from simple probabilistic inference to the fusion of stochastic networks. It can be noted that probabilities are often defined by standard distributions or are isomorphic to weighted systems.

A stochastic model based on THRs is the one presented by Blaylock and Allen (2006). This creates hierarchical stacks to categorize abstraction levels from basic actions to high level intentions. By chaining a hidden Markov model to these stacks, the model is able to affect a probability of intention according to the observed action.

Another stochastic paradigm is the one of Han and Pereira (2013). It uses Bayesian networks to define relationships between causes, intentions and actions in a given field. Each category is treated separately in order to reduce the search space. The observed actions are then selected from the action network and extracted. The system then uses the intention network to build a temporary Bayesian network using the NoisyOR method. The network created is combined in the same way with the network of causes and makes it possible to give the intention as well as the most probable cause according to the observations.

The model of Kelley *et al.* (2012) (based on (Hovland *et al.* 1996)) is a model using hidden Markov networks. This stochastic network is built here by learning data from robotic perception systems. The goal is to determine intent using past observations.

This model uses the theory of mind by invoking that humans infer the intentions of their peers by using a projection of their own.

Another contextual approach is the one developed for robotics by Hofmann and Williams (2007). The stochastic system is completed by a weighting based on an analysis of vernacular corpuses. We can therefore use the context of an observation to determine the most credible actions using the relational system built with corpus analysis. This is based on the observation of the objects in the scene and their condition. This makes common sense actions much more likely and almost impossible actions leading to semantic contradictions.

This principle is also used as the basis of the paradigm of Baker and Tenenbaum (2014) which forms a Bayesian theory of the mind. Using a limited representation of the human mind, this model defines formulas for updating beliefs and probabilities a posteriori of world states and actions. This is constructed with sigmoid distributions on the simplex of inferred belief. Then the probabilities of desire are calculated in order to recover the most probable intention. This has been validated as being close to the assessment of human candidates on simple intention recognition scenarios.

6.3 Inverted planning

Another way to do intent recognition is to use the *theory of mind*. This theory states that an agent recognizes other agent's behavior by projecting its own beliefs, desires and intentions on the other agent. This particular model is called BDI fo Belief, Desire and Intention. In our case the belief part is the knowledge database and planning domain. The desires are the set of available goals and the intent is the plan to pursue a set of goals.

This way of doing intent recognition is called "inverse planning" because it is the inversed problem of planning. In planning we get a goal and we need to find a plan to achieve it, and in intent recognition, we got an observed plan and we search for the goals.

This approach first emerged with the work of Ramırez and Geffner (2009). This article uses a classical planner to compute the most probable goal given a sequence of actions. This uses constraints to make the cost of a plan correlate with its likelyhood.

6.3.1 Probabilities and approximations

In that section we explain how this operation is done.

For any set of observations $\mathcal{O}$ the probability of the set is the product of the probability of any observation $o \in \mathcal{O}$. We can then note $\mathbb{P}(\mathcal{O}) = \prod_{o \in \mathcal{O}} \mathbb{P}(o)$.

We assume that the observed agent is pursuing one of the knowns goals. The event of an agent pursuing a goal is noted g. This means that $\mathbb{P}(\mathcal{G}) = \sum_{g \in \mathcal{G}} \mathbb{P}(\mathcal{G}) = 1$ because the event is certain.

Using conditional probabilities we can also note $\mathbb{P}(\mathcal{G}|\pi) = 1$ for a valid plan π that achieves G.

From dirrect application of Bayes theorem and the previous assomptions, we have :

$$P(\pi|O) = \frac{P(O|\pi)P(\pi)}{P(O)} = \frac{P(O|\pi)P(\pi|G)P(G)}{P(O)} \tag{6.1}$$

$$P(G|O) = \frac{P(O|G)P(G)}{P(O)} \tag{6.2}$$

And from the total probability formula :

$$P(O|G) = \sum_{\pi \in \Pi_G} P(O|\pi)P(\pi|G) \tag{6.3}$$

7 Conclusion and Perspectives

7.1 Fondation

7.1.1 Issues with current fondations

Like stated, these three aspect of knowledge representation makes defining a fondation of mathematics way more difficult than it may appear. In this section, we analyse some of the existing fondation of mathematics and their dependancies.

7.1.1.1 Set theory

From the book releasing the complete formulation of the ZFC theory, we can find that:

> "*the discipline underlying ZF will be the **first-order predicate calculus**. The primitive symbols of this set theory, taken from logic, are the connectives, quantifiers and variables mentioned above and, possibly, also the symbol of equality (discussed below), as well as such auxiliary symbols as commas, parentheses and brackets.*"

Fraenkel *et al.* (1973, 67,22)

This means that ZFC is using FOL as a host language. In turn FOL is using formal grammar as its host language. And at last, formal grammar are defined using set theory. This means that there is a dependancy cycle in the definition of ZFC.

This is significative as this theory is the ground work for most of the classical mathematics still used to this day. Morever, several assumptions are made in the theory that are not explicitly stated. For example, when naming an element or a set we supose the principle of identity as we can distinguish them from one another. Also, the notion of set isn't properly defined as well as the classical FOL quantifiers that are used in even the very first formula of the theory.

7.1.1.2 Type theory

Russell and Whitehead (1978)

TODO? Read the book since typed Lambda calculus is posterior to type theory

Type theory -> Typed Lambda calculus (no definition of type) -> Lambda claculus -> set theory or formal grammar

7.1.1.3 Category theory

Awodey (2010)

TODO? Is that what I found ?

Category theory -> Typed Lamda calculus

7.1.1.4 Proof theory

TODO?

Proof Theory -> Hilbert calculus -> Mathematic logic -> (FOL | Set | etc)

7.1.2 Reverse mathematics

Another radical way to approach the fondations of mathematics is called reverse mathematics. In this field, the goal is to find the minimal set of axioms needed to prove any theorem. This is not directly meant to be a fondation of mathematics but helps approaching it by finding groups of axioms that prove all theorems of a given field of mathematics.

This is the approach that should be used to find the most adequate set of axioms while keeping in mind the constraints any fondation of mathematics has.

7.1.3 Possible paradox in proposed theory

TODO: Speak about Russel paradox in fonctional form: A fonction that associates any function that doesn't associate itself. This arrise from the complement operation.

7.2 Knowledge representation

Listing the contributions there are a couple that didn't make the cut. It is mainly ideas or projects that were too long to check or implement and needed more time to complete. SELF is still a prototype, and even if the implementation seemed to perform well on a simple example, no benchmarks have been done on it. It might be good to make a theoretical analysis of OWL compared to SELF along with some benchmark results.

On the theoretical parts there are some works that seems worthy of exposure even if unfinished.

7.2.1 Literal definition using Peano's axioms

The only real exceptions to the axioms and criteria are the first statement, the comments and the liberals.

For the first statement, there is yet to find a way to express both inclusion, the equality relation and solution quantifier. If such a convenient expression exists, then the language can become almost entirely self described.

Comments can be seen as a special kind of container. The difficult part is to find a clever way to differentiate them from regular containers and to ignore their content in the regular grammar. It might be possible to at first describe their structure but then they become parseable entities and fail at their purpose.

Lastly, and perhaps the most complicated violation to fix: laterals. It is quite possible to define literals by structure. First we can define boolean logic quite easily in SELF as demonstrated by listing 7.1.

Listing 7.1 Possible definition of boolean logic in SELF.

```
~(false) = true;$\label{line:negation}$
( false, true ) :: Boolean;$\label{line:boolean}$
true =?; //conflicts with the first statement!$\label{line:true}$
*a : ((a | true) = true);$\label{line:logic}$
*a : ((false | a) = a);
*a : ((a & false) = false);
*a : ((true & a) = a);
```

Starting with **??**, we simply define the negation using the exclusive quantifier. From there we define the boolean type as just the two truth values. And now it gets complicated. We could either arbitrarily say that the false literal is always parameters of the exclusion quantifier or that it comes first on either first two statements but that would just violate minimalism even more. We could use the solution quantifier to define truth but that collides with the first statement definition. There doesn't seem to be a good answer for now.

From **??** going on, we state the definition of the logical operators $\land$ and $\lor$. The problem with this is that we either need to make a native property for those operators or the inference to compute boolean logic will be terribly inefficient.

We can use Peano's axioms (1889) to define integers in SELF. The attempt at this definition is presented in listing 7.2.

We got several problems doing so. The symbols $*$ and $/$ are already taken in the default file and so would need replacement or we should use the non-ASCII $\times$ and $\div$ symbols for multiplication and division. Another more fundamental issue is as previously discussed for booleans: the inference would be excruciatingly slow or we should revert to a kind of parsing similar to what we have already under the hood. The last problem is the definition of digits and bases that would quickly become exceedingly complicated and verbose.

Listing 7.2 Possible integration of the Peano axioms in SELF.

```
0 :: Integer;
*n : (++(n) :: Integer);
(*m, *n) : ((m=n) : (++m = ++n));
*n : (++n ~= 0);
*n : ((n + 0) = n);
(*n, *m) : ((n + ++m)= ++(n + m));
*n : ((n × 0) = 0);
(*n, *m) : ((n × ++m) = (n + (n × m)));
```

For floating numbers this turns out even worse and complicated and such a description wasn't even attempted for now.

The last part concerns textual laterals. The issue is the same as the one with comments but even worse. We get to interpret the content as literal value and that would necessitate a similar system as we already have and wouldn't improve the minimalist aspect of things much. Also we should define ways to escape characters and also to input escape sequences that are often needed in such case. And since SELF isn't meant for programming that can become very verbose and complex.

7.2.2 Advanced Inference

The inference in SELF is very basic. It could be improved a lot more by simply checking the consistency of the database on most aspects. However, such a task seems to be very difficult or very slow. Since that kind of inference is undecidable in SELF, it would be all a research problem just to find a performant inference algorithm.

Another kind of inference is more about convenience. For example, one can erase singlets (containers with a single value) to make the database lighter and easier to maintain and query.

7.2.3 Queries

We haven't really discussed quarries in SELF. They can be made using the main syntax and the solution quantifiers but efficiency of such queries is unknown. Making an efficient query engine is a big research project on its own.

For now a very simplified query API exists in the prototype and seems to perform well but further tests are needed to assess its scalability capacities.

7.3 General Automated Planning

7.4 Planning Improvements

7.4.1 Heuristics using Semantics

7.4.2 Macro-Action learning

8 References

Alami, R., F. Robert, F. Ingrand, and S. Suzuki
Multi-robot cooperation through incremental plan-merging, *Robotics and Automation, 1995. Proceedings., 1995 IEEE International Conference on*, IEEE, 1995, 3,2573–2579.

Alessandro, W., and I. Piumarta
OMeta: An object-oriented language for pattern matching, *Proceedings of the 2007 symposium on Dynamic languages*, 2007.

Aljazzar, H., and S. Leue
K*: A heuristic search algorithm for finding the k shortest paths, *Artificial Intelligence*, 175 (18), 2129–2154, 2011.

Ambite, J. L., and C. A. Knoblock
Planning by Rewriting: Efficiently Generating High-Quality Plans., DTIC Document, 1997.

American Heritage Dictionary
Formal (adj.), *American Heritage Dictionary of the English Language*, 2011a.

American Heritage Dictionary
Circularity (n.d.), *American Heritage Dictionary of the English Language*, 2011b.

Asimov, I.
The gods themselves, Greenwich, Connecticut: Fawcett Crest, 1973.

Avrahami-Zilberbrand, D., and G. A. Kaminka
Hybrid symbolic-probabilistic plan recognizer: Initial steps, *Proceedings of AAAI workshop on modeling others from observations (MOO-06)*, 2006.

Avrahami-Zilberbrand, D., G. A. Kaminka, and H. Zarosim
Fast and complete plan recognition: Allowing for duration, interleaved execution, and lossy observations, *IJCAI Workshop on Modeling Others from Observations*, 2005.

Awodey, S.
Category theory, 2nd ed. Oxford logic guides 52Oxford ; New York: Oxford University Press, 2010.

Baader, F.
The description logic handbook: Theory, implementation and applications, Cambridge university press, 2003.

Babli, M., E. Marzal, and E. Onaindia
On the use of ontologies to extend knowledge in online planning, *KEPS 2018*, 54, 2015.

Backus, J. W.
The syntax and semantics of the proposed international algebraic language of the Zurich ACM-GAMM conference, *Proceedings of the International Comference on Information Processing, 1959*, 1959.

Baker, C. L., and J. B. Tenenbaum
Modeling Human Plan Recognition using Bayesian Theory of Mind, 2014.

Bechon, P., M. Barbier, G. Infantes, C. Lesire, and V. Vidal
HiPOP: Hierarchical Partial-Order Planning, *European Starting AI Researcher Symposium*, IOS Press, 2014, 264,51–60.

Becket, R., and Z. Somogyi
DCGs+ memoing= packrat parsing but is it worth it?, *International Symposium on Practical Aspects of Declarative Languages*, Springer, 2008, 182–196.

Beckett, D., and T. Berners-Lee
Turtle - Terse RDF Triple Language, W3C Team Submission W3C, March 2011.

Bercher, P., and D. Höller
Tutorial: An Introduction to Hierarchical Task Network (HTN) Planning 2018.

Bercher, P., S. Keen, and S. Biundo
Hybrid planning heuristics based on task decomposition graphs, *Seventh Annual Symposium on Combinatorial Search*, 2014.

Blaylock, N., and J. Allen
Fast hierarchical goal schema recognition, *Proceedings of the National Conference on Artificial Intelligence*, Menlo Park, CA; Cambridge, MA; London; AAAI Press; MIT Press; 1999, 2006, 21,796.

Borrajo, D.
Multi-agent planning by plan reuse, *Proceedings of the 2013 international conference on Autonomous agents and multi-agent systems*, International Foundation for Autonomous Agents and Multiagent Systems, 2013, 1141–1142.

Borrajo, D., A. Roubíčková, and I. Serina
Progress in Case-Based Planning, *ACM Computing Surveys*, 47 (2), 1–39, January 2015. doi:10.1145/2674024.

Bouchard, B., S. Giroux, and A. Bouzouane
A Smart Home Agent for Plan Recognition of Cognitively-impaired Patients., *Journal of Computers*, 1 (5), 53–62, 2006.

Brenner, M.
A multiagent planning language, *Proc. Of the Workshop on PDDL, ICAPS*, 2003, 3,33–38.

Bürckert, H.-J.
Terminologies and rules, *Workshop on Information Systems and Artificial Intelligence*, Springer, 1994, 44–63.

Cantor, G.
On a Property of the Class of all Real Algebraic Numbers., *Crelle's Journal for Mathematics*, 77, 258–262, 1874.

Cantor, G.
Beiträge zur Begründung der transfiniten Mengenlehre, *Mathematische Annalen*, 46 (4), 481–512, 1895.

Chomsky, N.
Three models for the description of language, *IRE Transactions on information theory*, 2 (3), 113–124, 1956.

Ciesielski, K.
Set Theory for the Working Mathematician, Cambridge University Press, August 1997.

Coles, A., A. Coles, M. Fox, and D. Long
Popf2 : A forward-chaining partial order planner, *IPC*, 65, 2011.

Collins English Dictionary
Abstraction (n.d.), *Collins English Dictionary Complete and Unabridged*, 2014.

D'Alfonso, D.
Generalized Quantifiers: Logic and Language, 2011.

Dvorak, F., A. Bit-Monnot, F. Ingrand, and M. Ghallab
A flexible ANML actor and planner in robotics, *Planning and Robotics (PlanRob) Workshop (ICAPS)*, 2014.

Ephrati, E., and J. S. Rosenschein
Multi-agent planning as the process of merging distributed sub-plans, *In Proceedings of the Twelfth International Workshop on Distributed Artificial Intelligence (DAI-93*, Citeseer, 1993.

Erol, K., J. A. Hendler, and D. S. Nau
UMCP: A Sound and Complete Procedure for Hierarchical Task-network Planning, *Proceedings of the International Conference on Artificial Intelligence Planning Systems*, University of Chicago, Chicago, Illinois, USA: AAAI Press, June 1994, 2,249–254.

Erol, K., D. S. Nau, and V. S. Subrahmanian
Complexity, decidability and undecidability results for domain-independent planning, *Artificial intelligence*, 76 (1-2), 75–88, 1995.

Fikes, R. E., and N. J. Nilsson
STRIPS: A new approach to the application of theorem proving to problem solving, *Artificial intelligence*, 2 (3-4), 189–208, 1971.

Ford, B.
Packrat parsing:: Simple, powerful, lazy, linear time, functional pearl, *ACM SIGPLAN Notices*, ACM, 2002, 37,36–47.

Ford, B.
Parsing expression grammars: A recognition-based syntactic foundation, *ACM SIGPLAN Notices*, ACM, 2004, 39,111–122.

Fox, M.
Natural hierarchical planning using operator decomposition, *European Conference on Planning*, Springer, 1997, 195–207.

Fox, M., A. Gerevini, D. Long, and I. Serina
Plan Stability: Replanning versus Plan Repair., *ICAPS*, 2006, 6,212–221.

Fox, M., and D. Long
PDDL+: Modeling continuous time dependent effects, *Proceedings of the 3rd International NASA Workshop on Planning and Scheduling for Space*, 2002, 4,34.

Fraenkel, A. A., Y. Bar-Hillel, and A. Levy
Foundations of set theory, vol. 67Elsevier, 1973.

Geib, C. W.
Problems with intent recognition for elder care, *Proceedings of the AAAI-02 Workshop "Automation as Caregiver*, 2002, 13–17.

Geib, C. W., and R. P. Goldman
Partial observability and probabilistic plan/goal recognition, *Proceedings of the International workshop on modeling other agents from observations (MOO-05)*, 2005.

Gerevini, A. E., A. Roubíčková, A. Saetti, and I. Serina
On the plan-library maintenance problem in a case-based planner, *International Conference on Case-Based Reasoning*, Springer, 2013, 119–133.

Gerevini, A., U. Kuter, D. S. Nau, A. Saetti, and N. Waisbrot
Combining Domain-Independent Planning and HTN Planning: The Duet Planner, *Proceedings of the European Conference on Artificial Intelligence*, 2008, 18,573–577.

Ghallab, M., C. Knoblock, D. Wilkins, A. Barrett, D. Christianson, et al.
PDDL – The planning domain definition language, 1998.

Ghallab, M., D. Nau, and P. Traverso
Automated planning: Theory & practice, Elsevier, 2004.

Ghallab, M., D. Nau, and P. Traverso
Automated Planning and Acting, Cambridge University Press, 2016.

Goldman, R. P., C. W. Geib, and C. A. Miller
A new model of plan recognition, *Proceedings of the Fifteenth conference on Uncertainty in artificial intelligence*, Morgan Kaufmann Publishers Inc., 1999, 245–254.

Göbelbecker, M., T. Keller, P. Eyerich, M. Brenner, and B. Nebel
Coming Up With Good Excuses: What to do When no Plan Can be Found, *Proceedings of the International Conference on Automated Planning and Scheduling*, AAAI Press, May 2010, 20,81–88.

Gradel, E., M. Otto, and E. Rosen
Two-variable logic with counting is decidable, *Logic in Computer Science, 1997. LICS'97. Proceedings., 12th Annual IEEE Symposium on*, IEEE, 1997, 306–317.

Grünwald, P.
A minimum description length approach to grammar inference, in S. Wermter, E. Riloff, and G. Scheler (eds.), *Connectionist, Statistical and Symbolic Approaches to Learning for Natural Language Processing*, Lecture Notes in Computer Science; Springer Berlin Heidelberg, 1996, 203–216.

Han, T. A., and L. M. Pereira
Context-dependent incremental decision making scrutinizing the intentions of others via Bayesian network model construction, *Intelligent Decision Technologies*, 7 (4), 293–317, 2013.

Hart, D., and B. Goertzel
Opencog: A software framework for integrative artificial general intelligence, *AGI*, 2008, 468–472.

Hehner, E. C.
A practical theory of programming, Springer Science & Business Media, 2012.

Helmert, M., G. Röger, and E. Karpas
Fast downward stone soup: A baseline for building planner portfolios, *ICAPS 2011 Workshop on Planning and Learning*, Citeseer, 2011, 28–35.

Henglein, F., and U. T. Rasmussen
PEG parsing in less space using progressive tabling and dynamic analysis, *Proceedings of the 2017 ACM SIGPLAN Workshop on Partial Evaluation and Program Manipulation*, ACM, 2017, 35–46.

Hernández, D., A. Hogan, and M. Krötzsch
Reifying RDF: What works well with wikidata?, *SSWS@ ISWC*, 1457, 32–47, 2015.

Hoffmann, J.
FF: The fast-forward planning system, *AI magazine*, 22 (3), 57, 2001.

Hofmann, A. G., and B. C. Williams
Intent Recognition for Human-Robot Interaction., *Interaction Challenges for Intelligent Assistants*, 2007, 60–61.

Horrocks, I., P. F. Patel-Schneider, and F. V. Harmelen
From SHIQ and RDF to OWL: The Making of a Web Ontology Language, *Journal of Web Semantics*, 1, 2003, 2003.

Hovland, G. E., P. Sikka, and B. J. McCarragher
Skill acquisition from human demonstration using a hidden markov model, *Robotics and Automation, 1996. Proceedings., 1996 IEEE International Conference on*, Ieee, 1996, 3,2706–2711.

Höschele, M., and A. Zeller
Mining input grammars with AUTOGRAM, *Proceedings of the 39th International Conference on Software Engineering Companion*, IEEE Press, 2017, 31–34.

Hutton, G., and E. Meijer
Monadic parser combinators, 1996.

Inoue, N., and K. Inui
ILP-Based Reasoning for Weighted Abduction., *Plan, Activity, and Intent Recognition*, 2011.

Kambhampati, S.
Design Tradeoffs in Partial Order (Plan space) Planning., *AIPS*, 1994, 92–97.

Kambhampati, S., A. Mali, and B. Srivastava
Hybrid planning for partially hierarchical domains, *AAAI/IAAI*, 1998, 882–888.

Kelley, R., A. Tavakkoli, C. King, A. Ambardekar, and M. Nicolescu
Context-based bayesian intent recognition, *Autonomous Mental Development, IEEE Transactions on*, 4 (3), 215–225, 2012.

Klyne, G., and J. J. Carroll
Resource Description Framework (RDF): Concepts and Abstract Syntax, Language Specification W3C RecommendationW3C, 2004.

Korzybski, A.
Science and sanity; an introduction to non-Aristotelian systems and general semantics., Lakeville, Conn.: International Non-Aristotelian Library Pub. Co.; distributed by Institute of General Semantics, 1958.

Kovacs, D. L.
BNF Description of PDDL 3.1, Unpublished manuscript from the IPC-2011 websiteIPC, 2011.

Kovács, D. L.
A multi-agent extension of PDDL3. 1, 2012.

Kovács, D. L., and T. P. Dobrowiecki
Converting MA-PDDL to extensive-form games, *Acta Polytechnica Hungarica*, 10 (8), 27–47, 2013.

Kovitz, B.
Terminology - What do you call graphs that allow edges to edges?, *Mathematics Stack Exchange*, January 2018.

Lindström, P.
First Order Predicate Logic with Generalized Quantifiers, 1966.

Loff, B., N. Moreira, and R. Reis
The Computational Power of Parsing Expression Grammars, *International Conference on Developments in Language Theory*, Springer, 2018, 491–502.

Luis, N., and D. Borrajo
Plan merging by reuse for multi-agent planning, *Distributed and Multi-Agent Planning*, 38, 2014.

McDermott, D.
OPT Manual Version 1.7. 3 (Reflects Opt Version 1.6. 11) DRAFT*, 2005.

McDermott, D., and D. Dou
Representing disjunction and quantifiers in RDF, *International Semantic Web Conference*, Springer, 2002, 250–263.

Motik, B.
On the properties of metamodeling in OWL, *Journal of Logic and Computation*, 17 (4), 617–637, 2007.

Nau, D. S., T.-C. Au, O. Ilghami, U. Kuter, J. W. Murdock, et al.
SHOP2: An HTN planning system, *J. Artif. Intell. Res.(JAIR)*, 20, 379–404, 2003.

Nebel, B., and J. Koehler
Plan reuse versus plan generation: A theoretical and empirical analysis, *Artificial Intelligence*, 76 (1), 427–454, 1995.

Nguyen, X., and S. Kambhampati
Reviving partial order planning, *IJCAI*, 2001, 1,459–464.

Paulson, L.
A semantics-directed compiler generator, *Proceedings of the 9th ACM SIGPLAN-SIGACT symposium on Principles of programming languages - POPL '82*, Albuquerque, Mexico: ACM Press, 1982, 224–233. doi:10.1145/582153.582178.

Peano, G.
Arithmetices principia: Nova methodo exposita, Fratres Bocca, 1889.

Pednault, E. P.
ADL: Exploring the Middle Ground Between STRIPS and the Situation Calculus., *Kr*, 89, 324–332, 1989.

Pellier, D., and H. Fiorino
PDDL4J: A planning domain description library for java, December 2017. doi:10.1080/0952813X.2017.1409

Penberthy, J. S., D. S. Weld, and others
UCPOP: A Sound, Complete, Partial Order Planner for ADL, *Kr*, 92, 103–114, 1992.

Peot, M. A., and D. E. Smith
Threat-removal strategies for partial-order planning, *AAAI*, 1993, 93,492–499.

Peot, M. A., and D. E. Smith
Postponing Threats in Partial-Order Planning 1994.

Raghavana, S., P. Singlab, and R. J. Mooneya
Plan Recognition using Statistical Relational Models.

Ramırez, M., and H. Geffner
Plan recognition as planning, *Proceedings of the International Conference on International Conference on Automated Planning and Scheduling*, AAAI Press, 2009, 19,1778–1783.

RAMOUL, A.
Mixed-initiative planning system to assist the management of complex IT systems 2018.

Renggli, L., S. Ducasse, T. Gîrba, and O. Nierstrasz
Practical Dynamic Grammars for Dynamic Languages, *Workshop on Dynamic Languages and Applications*, Malaga, Spain, 2010, 4.

Richter, S., and M. Westphal
The LAMA planner: Guiding cost-based anytime planning with landmarks, *Journal of Artificial Intelligence Research*, 39 (1), 127–177, 2010.

Roy, P. C., A. Bouzouane, S. Giroux, and B. Bouchard
Possibilistic activity recognition in smart homes for cognitively impaired people, *Applied Artificial Intelligence*, 25 (10), 883–926, 2011.

Russell, B.
Mysticism and Logic and Other Essays, London: George Allen and Unwin, 1917.

Russell, B., and A. N. Whitehead
Principia mathematica, Cambridge University Press, 1978.

Sanner, S.
Relational dynamic influence diagram language (rddl): Language description Unpublished ms. Australian National University Unpublished ms. Australian National University, 2010.

Sapena, O., E. Onaindıa, and A. Torreno
Combining heuristics to accelerate forward partial-order planning, *CSTPS*, 25, 2014.

Say, B., A. A. Cire, and J. C. Beck
Mathematical programming models for optimizing partial-order plan flexibility, *22nd European Conference of Artificial Intelligence*, 2016.

Schwarzentruber, F.
Hintikka's World: Agents with Higher-order Knowledge, *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence*, Stockholm, Sweden: International Joint Conferences on Artificial Intelligence Organization, July 2018, 5859–5861. doi:10.24963/ijcai.2018/862.

Sebastia, L., E. Onaindia, and E. Marzal
A Graph-based Approach for POCL Planning, *ECAI*, 2000, 531–535.

Shekhar, S., and D. Khemani
Learning and Tuning Meta-heuristics in Plan Space Planning, *arXiv preprint arXiv:1601.07483*, 2016.

Silberschatz, A.
Port directed communication, *The Computer Journal*, 24 (1), 78–82, January 1981. doi:10.1093/comjnl/24.1.78.

Sjöberg, J., and H. Nissar
Automated scheduling: Performance in different scenarios, 2015.

Sohrabi, S., A. V. Riabov, and O. Udrea
Plan Recognition as Planning Revisited, *Proceedings of the International Joint Conference on Artificial Intelligence*, Vol. 25, 2016.

Souto, D. C., M. V. Ferro, and M. A. Pardo
Dynamic Programming as Frame for Efficient Parsing, *Proceedings SCCC'98. 18th International Conference of the Chilean Society of Computer Science (Cat. No.98EX212)(SCCC)*, November 1998, 68. doi:10.1109/SCCC.1998.730784.

Sugawara, T.
Reusing Past Plans in Distributed Planning., *ICMAS*, 1995, 360–367.

Tan, X., and M. Gruninger
The Complexity of Partial-Order Plan Viability Problems., *ICAPS*, 2014.

Thiébaux, S., J. Hoffmann, and B. Nebel
In defense of PDDL axioms, *Artificial Intelligence*, 168 (1-2), 38–69, 2005.

Tolksdorf, R., L. Nixon, F. Liebsch, D. Minh Nguyen, and E. Paslaru Bontas
Semantic web spaces, 2004.

Unicode Consortium
The Unicode Standard, Version 11.0, Core Specification 11.0Mountain View, CA, June 2018a.

Unicode Consortium
Unicode Character Database, *About the Unicode Character Database*, June 2018b.

Van Der Krogt, R., and M. De Weerdt
Plan Repair as an Extension of Planning., *ICAPS*, 2005, 5,161–170.

Van Harmelen, F., V. Lifschitz, and B. Porter
Handbook of knowledge representation, vol. 1Elsevier, 2008.

Vepstas, L.
Hypergraph edge-to-edge, *Wikipedia*, May 2008.

Vepštas, L.
Sheaves: A Topological Approach to Big Data, 2008.

Vidal, N., P. Taillibert, and S. Aknine
Online behavior recognition: A new grammar model linking measurements and intents, *2010 22nd IEEE International Conference on Tools with Artificial Intelligence*, IEEE, 2010, 2,129–137.

W3C
Examples for OWL. Eds. P. F. Patel-Schneider and I. Horrocks 2004a.

W3C
RDF Semantics, W3C, 2004b.

W3C
RDF Vocabulary Description Language 1.0: RDF Schema February 2004c.

Younes, H. akan L., and M. L. Littman
PPDDL 1.0: An extension to PDDL for expressing planning domains with probabilistic effects, *Techn. Rep. CMU-CS-04-162*, 2004.

Younes, H. akan L., and R. G. Simmons
VHPOP : Versatile heuristic partial order planner, *JAIR*, 405–430, 2003.

Young, R. M., and J. D. Moore
DPOCL: A principled approach to discourse planning, *Proceedings of the Seventh International Workshop on Natural Language Generation*, Association for Computational Linguistics, 1994, 13–20.

Zhuo, H. H., and S. Kambhampati
Model-lite planning: Case-based vs. Model-based approaches, *Artificial Intelligence*, 246, 1–21, 2017.

Apendix

8.1 Chapter 1

SAMIR:

- Cite slow start 1.2
- System : implentation -> Model : theory
- Examples
- theorems or not
- s : D -> D notation in table
- Double arrow too for ±
- to port : porter une notion in smth
- 1.2.2 : criterion
- Example of parsing
- 1.2.6.1 : light defeasible logic
- lacks properties and proofs: what are the advantages

Table 8.1

Symbol	Name	Arity	Arguments	Type	Definition
$=$	Equal	2	expr, expr	boolean	$x = x : \top$
$\neq$	Not Equal	2	expr, expr	boolean	$x \neq x : \bot$
$:$	Such that	2	element, predicate	element	axiom of Specification
$?$	Predicate	n	expr	boolean	arbitrary
$\top$	True	0	NA	boolean	
$\bot$	False	0	NA	boolean	
$\neg$	Negation	1	boolean	boolean	$\neg\top = \bot$
$\wedge$	Conjunction	n	boolean	boolean	$a \wedge b \vdash (a = b = \top)$
$\vee$	Disjunction	n	boolean	boolean	$\neg(a \vee b) \vdash (a = b = \bot)$
$\unicode{x22BB}$	Logic operator	n	boolean	boolean	arbitrary
$\vdash$	Entailment	2	boolean	boolean	
$\forall$	Universality	1	var	modifier	
$\exists$	Existentiality	1	var	modifier	
$\exists!$	Unicity	1	var	modifier	
$\nexists$	Exclusivity	1	var	modifier	$\lneq \exists$
$\S$	Solution	1	var	modifier	
$[\,]$	Iverson's brackets	1	boolean	int	$[\bot] = 0 \wedge [\top] = 1$
$\{\}$	Set	n	elements	set	
$\varnothing$	Empty set	0	NA	set	$\varnothing = \{\}$
$\in$	Member	2	element, set	boolean	
$\subset$	Subset	2	set	boolean	$\mathcal{S} \subset \mathcal{T} \vdash ((e \in \mathcal{S} \vdash e \in \mathcal{T}) \wedge \mathcal{S} \neq \mathcal{T})$
$\cup$	Union	n	set	set	$\mathcal{S} \cup \mathcal{T} = \{e : e \in \mathcal{S} \vee e \in \mathcal{T}\}$
$\cap$	Intersection	n	set	set	$\mathcal{S} \cap \mathcal{T} = \{e : e \in \mathcal{S} \wedge e \in \mathcal{T}\}$
$\setminus$	Difference	2	set	set	$\mathcal{S} \setminus \mathcal{T} = \{e : e \in \mathcal{S} \wedge e \notin \mathcal{T}\}$
$\times$	Cartesian product	n	set	set	$\mathcal{S} \times \mathcal{T} = \{\langle e_s, e_{\mathcal{T}} \rangle : e_s \in \mathcal{S} \wedge e_{\mathcal{T}} \in \mathcal{T}\}$
$\|\|$	Cardinal	1	set	int	
$\wp$	Powerset	1	set	set	axiom of Power set
$\circ$	Function composition	n	function	function	
σ	Selection	2	predicate, set	set	$\sigma_?(\mathcal{S}) = \{e : ?(e) \wedge e \in \mathcal{S}\}$
π	Projection	2	function, set	set	$\pi_f(\mathcal{S}) = \{f(e) : e \in \mathcal{S}\}$
$\mapsto$	Substitution	2	var, function, expr	expr	$(e \mapsto f(e))(\unicode{x2119}(e)) = \unicode{x2119}(f(e))$
$\langle\rangle$	Tuple	n	elements	tuple	
$\rightarrow$	Association	2	elements	tuple	
ϕ	Incidence/Adjacence	n	various	various	
χ	Transitivity	1	relation	relation	
$\div$	Quotient	2	function, graph	graph	
μ	Meta	1	entity	entity	
ν	Name	1	entity	string	
ρ	Parameter	1	entity	list	
$\otimes$	Flaws	1	plan	set	
$\odot$	Resolvers	1	flaw	set	
$\downdownarrows$	Partial support	2	link, action	boolean	
$\downarrow$	Full support	2	plan, action	boolean	
$\prec$	Precedance	2	action	boolean	
$\succ$	Succession	2	action	boolean	
$\mapsto^*$	Shortest path	?			
h	Heuristic	1	element	float	
γ	Constraints	1	element	set	
$\text{\textcent}$	Cost	1	element	float	
d	Duration	1	element	time	