

Given the following numbers:

- Prime number $q = 13$
- Generator $\alpha = 2$
- Alice's private key $X_a = 5$
- Bob's private key $X_b = 4$

The following steps illustrate how Alice and Bob can establish a shared secret key K using the Diffie-Hellman key exchange protocol:

1. Alice computes $Y_a = \alpha^{X_a} \bmod q = 2^5 \bmod 13 = 6$ and sends Y_a to Bob.
2. Bob computes $Y_b = \alpha^{X_b} \bmod q = 2^4 \bmod 13 = 3$ and sends Y_b to Alice.
3. Alice computes $K = Y_b^{X_a} \bmod q = 3^5 \bmod 13 = 9$.
4. Bob computes $K = Y_a^{X_b} \bmod q = 6^4 \bmod 13 = 9$.
5. Alice and Bob now share a secret key $K = 9$, they can use the number to encrypt and decrypt message using another algorithm they agreed on.