1 Question 2-2

Given $f(n) = \frac{n^2}{log(n)}$ and $g(n) = n\log(n)^2$, determine the relationship between $f(n)$ and $g(n)$.

2 Answer

We will first assume that $f(n) = O(g(n))$. This means that there exists a constant $c > 0$ and $n_0 > 0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$.

$$\begin{aligned} f(n) &\leq c \cdot g(n) \\ \frac{n^2}{log(n)} &\leq c \cdot n\log(n)^2 \\ \frac{n}{log(n)^3} &\leq c \end{aligned}$$

Then we will can find the limit of the right hand side of the inequality to be infinity, as n approaches infinity.

$$\begin{aligned} \lim_{n\to\infty} \frac{n}{log(n)^3} &= \lim_{n\to\infty} \frac{1}{3log(n)^2} && \text{(L'Hopital)} \\ &= \lim_{n\to\infty} \frac{1}{6log(n)} \\ &= \lim_{n\to\infty} \frac{n}{6} \\ &= \infty \end{aligned}$$

However, such a constant c does not exist, as the limit of the right hand side of the inequality is infinity. Therefore, it contradicts the assumption that $f(n) = O(g(n))$.

Next we will assume that $f(n) = \Omega(g(n))$. This means that there exists a constant $c > 0$ and $n_0 > 0$ such that $f(n) \geq c \cdot g(n)$ for all $n \geq n_0$.

$$\begin{aligned} f(n) &\geq c \cdot g(n) \\ \frac{n^2}{log(n)} &\geq c \cdot n\log(n)^2 \\ \frac{n}{log(n)^3} &\geq c \end{aligned}$$

Such a constant c exists, as shown by the limit of the right hand side of the inequality.

then we can pick $c = \frac{10^{10}}{1000(log10)^3}$ and $n_0 = 10^{10}$ to satisfy the inequality.

In conclusion, $f(n) = \Omega(g(n))$. Since $f(n) = \Omega(g(n))$ and $f(n) \neq O(g(n))$ we can conclude that $f(n) \neq \Theta(g(n))$.