

1 Question 1.2

$$5^{4358} \bmod 10$$

2 Solution

from simple calculation, we suspect $5^n \bmod 10$ is always 5, so $5^{4358} \bmod 10 = 5$.

2.1 Proof

Base case:

$$5^1 \bmod 10 = 5$$

Inductive step: assume when $n = k$, the statement holds, then consider $n = k + 1$

$$\begin{aligned} 5^{k+1} \bmod 10 &= 5^k \times 5 \bmod 10 \\ &= (5^k \bmod 10 \times 5 \bmod 10) \bmod 10 \\ &= 25 \bmod 10 \\ &= 5 \end{aligned}$$

So the statement holds for $n = k + 1 \forall k \in \mathbb{Z}^+$, and by the principle of mathematical induction, $5^n \bmod 10 = 5$.