Question 2

Find asymptotic upper and lower bound for the following recurrence relation using master theorem:

$$T(n) = 3T(\frac{n}{2}) + \frac{n}{\log n}$$

Solution

Given the recurrence:

$$T(n) = 3\,T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

Rewrite in standard form:

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

where

$$a = 3, \quad b = 2, \quad f(n) = \frac{n}{\log n}$$

Compute the exponent $\log_b(a)$:

$$\log_b a = \frac{\ln a}{\ln b} \approx 1.58496$$

Check Theorem Case 1: If $f(n) = O(n^{\log_b a - \epsilon})$ for some $\epsilon > 0$
if we divide $f(n)$ by $n^{\log_b a}$, we should get a polynomial:

$$\frac{n^{\log_b a}}{\frac{n}{logn}} \approx n^{0.58496} \log n$$

And by picking a small ϵ, it is clear that

$$\frac{n}{\log n} = O(n^{\log_b a - \epsilon})$$

So case 1 does apply. Therefore, the solution is:

$$T(n) = \Theta(n^{\log_b a}) = \Theta(n^{1.58496})$$