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Understanding Qiskit

Quantum by Quantum

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- International Tech Speaker
- Distinguished Guest Lecturer
- Represented India at reputed International Hackathons
- Deep Learning and Quantum Computing Researcher
- 8+ International Publications
- Microsoft, IBM, and HP Certified Technology Associate
- ALL STACK DEVELOPER
- Mentor

Agenda

- Basics of **Quantum Computation**
- **Qubits** and the **Quantum States**
- Working with **Multiple Qubits**
- **Qiskit** and **Qubit Gates**
- Hands-on mode
- Building small **Quantum Circuits**
- Where to go from here - a **roadmap** for working with **Quantum Algorithms**
- **Applications** for the **GIS community (Spatial Algorithms and more)**
- Questions and Answers

Pre-requisites

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- Decent knowledge of Linear Algebra - the language of Quantum Computing
- Grasp over the Python programming language
- Classical Switching Theory and Logic Designing
- Inquisitiveness to dive into the world of Quantum Mechanics

Understanding Quantum Computing

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- Computation involves storing the information and manipulating it
- Classical Computers use the concept of bits - **0 and 1 binary states ?**
- Quantum Computers use Quantum Mechanical phenomena to manipulate information ~ *Quantum Mechanics + Information Theory*
- Quantum Mechanics is a mathematical framework for construction of physical theories
- These Quantum Mechanical phenomena have no Classical counterpart for computational purposes

Why Quantum Computing?

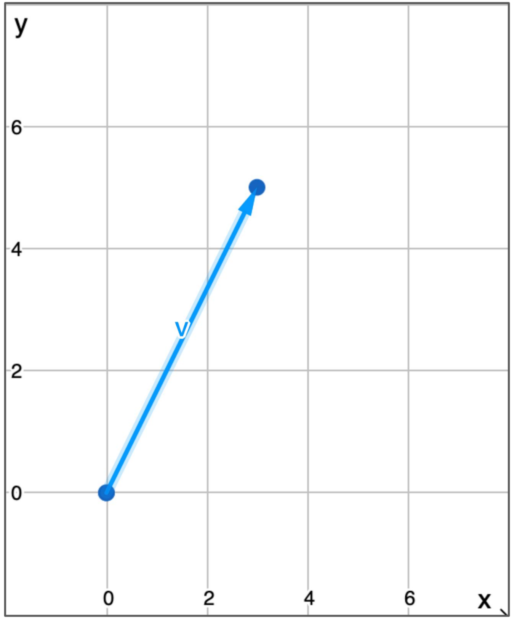
- **Simulations don't work:** Simulations of Quantum Phenomena on Classical Computers are inefficient; we need real Quantum Computers
- **Miniaturization of Computational Hardware:** Computational Hardware has been made so small, fabricated to micro-levels, where Quantum Effects occur and interfere with the functioning of the electronic devices
- **Solve certain Intractable Computational Problems:** This space is quite unknown
- **Computational Speed Up:** Leveraging Quantum Effects can speed up certain computations significantly, even exponentially in some cases
- **Nature is Quantum Mechanical:** Our understanding of Nature is Quantum Mechanical and hence Quantum Mechanical phenomena should be leveraged for computation purposes as well
- **A Hunch:** A hunch to explore and unleash something more than what we know

Vector Spaces

- A *Vector Space* V over field $\mathbb{F}$ is a set of objects, called *vectors*, satisfying the following conditions:
  - if $x, w \in V$, then $cx + dw \in V$ for all $c, d \in \mathbb{F}$
  - V is closed under addition and scalar multiplications
- A *vector* is a mathematical quantity having both direction and magnitude
- Formally, a vector v is denoted by $|v\rangle$
- The notation '$|\ \rangle$' is called the *Dirac Notation* and represents a state vector in Quantum Mechanics

- Consider a vector |v⟩ having the following form:

$$| v > = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$



Bits and Qubits

- The fundamental concept of Classical Computation and Classical Information is a *bit*
- A classical bit has a state either 0 or 1
- Quantum Computers rely on *quantum bits* or *qubits* for computational purposes
- A qubit is analogous to a classical bit
- A qubit can be thought of as a mathematical object with some special properties
- A qubit can have two *observable* states, either $|0\rangle$ or $|1\rangle$
- $|0\rangle$ and $|1\rangle$ corresponds to classical bit states 0 and 1, respectively

What's the difference then?

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- Unlike a classical bit, a qubit can be in a state *other* than $|0\rangle$ or $|1\rangle$ as well, until observed or *measured*
- A qubit can be treated as a *linear combination* of $|0\rangle$ and $|1\rangle$ states
- These linear combinations of $|0\rangle$ and $|1\rangle$ states are often called as *superpositions*
- This superposition of states $|0\rangle$ and $|1\rangle$ can be mathematically represented as:

$$|\psi> \; = \alpha|0> \; + \beta|1>$$

- $\alpha, \beta \in \mathbb{C}$ i.e. complex numbers
- Hence the state of a qubit is a vector in a 2-D complex vector space

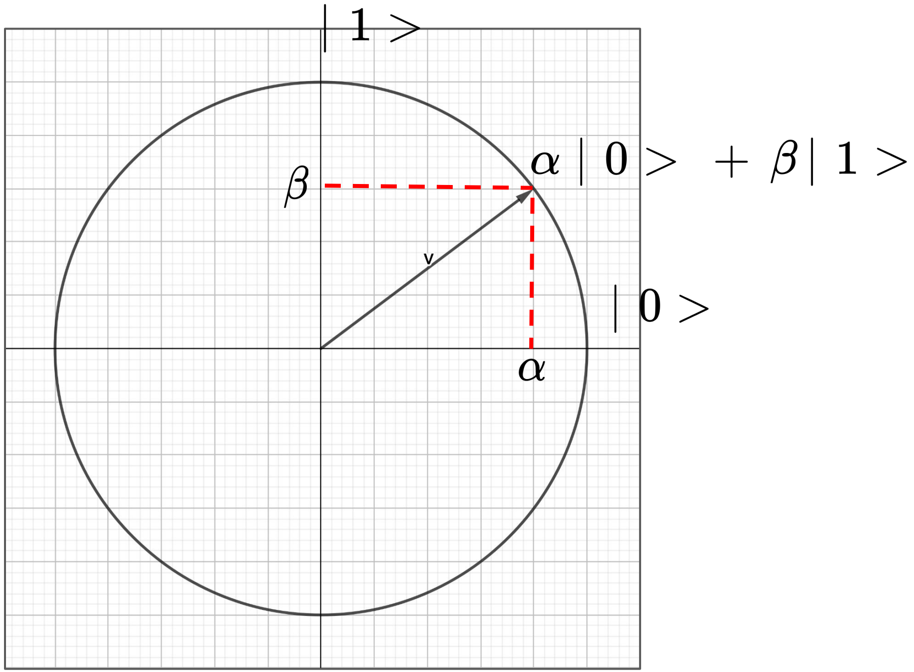
2-D Representation of a Qubit

- A qubit can be visualized as a *unit vector* in a 2-D plane
- The special condition which α and β follow is:

$$| \alpha |^2 + | \beta |^2 = 1$$

[Normalization Constraint]

- α and β are also known as *probability amplitudes*



Measurement

- A qubit cannot be examined to retrieve it's quantum state i.e. the values of α and β cannot be known
- Quantum Mechanics only allows to retrieve some *restricted information* about the quantum state
- This information retrieval process is known as *Measurement*
- Measuring a qubit yields either 0 or 1 i.e. qubit is forced to *collapse* to $|0\rangle$ or $|1\rangle$
- Probability of getting 0 on measurement is $|\alpha|^2$
- Probability of getting 1 on measurement is $|\beta|^2$
- Geometrically speaking, this can be understood as normalization of a qubit's state to length 1 (unit length)

$$|\alpha|^2 + |\beta|^2 = 1$$

More on Qubits and Quantum States

- $|0\rangle$ and $|1\rangle$ states are known as *computational basis states* as these are the two states in which a qubit can be physically when measured
- $|0\rangle$ and $|1\rangle$ states are *orthonormal* i.e. the two vectors are orthogonal to each other and are of unit length (normalized)
- A qubit represents a two-level quantum state
- **Examples of two-level quantum systems that can form a qubit in real world:**
  - Polarized Photons
  - Excited Atoms
  - Atoms in Ground State
  - ½ Spin Particles
- Though we can get only some restricted information about the quantum state using measurement, qubit states can be manipulated and transformed in ways which can tell about different distinct properties of the state
- $|0\rangle \equiv |\uparrow\rangle \equiv$ ground state
- $|1\rangle \equiv |\downarrow\rangle \equiv$ excited state

More on Superpositions

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- Let's put $\alpha = 1/\sqrt{2}$ and $\beta = 1/\sqrt{2}$

$$|\,\psi > \; = 1/\sqrt{2}\,|\,0 > \; + 1/\sqrt{2}\,|\,1 >$$

$$|\,1/\sqrt{2}\,|^2 + |\,1/\sqrt{2}\,|^2 = 1$$

$$\Rightarrow \tfrac{1}{2} + \tfrac{1}{2} = 1$$

- This means that there is 50% probability of getting $|0\rangle$ and $|1\rangle$ states, respectively
- This state is often referred to as $|+\rangle$ state or the *halfway* state

Polar Representation

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$$Consider\,|\,\psi > \;= \alpha\,|\,0 > \;+ \beta\,|\,1 >$$

$$If\ some\ c \in \mathbb{C}\, with\,|\,\mathrm{c}\,| = 1,\ then$$

$$c\,|\,\psi > \;= c\,\alpha\,|\,0 > \;+ c\,\beta\,|\,1 >$$

$$then\,|\,\mathrm{c}\ \alpha\,|^{2} = |\,\mathrm{c}\,|^{2}\,|\,\alpha\,|^{2} \;= |\,\alpha\,|^{2}$$

$$also\ |\,\mathrm{c}\ \beta\,|^{2} = |\,\mathrm{c}\,|^{2}\,|\,\beta\,|^{2} \;= |\,\beta\,|^{2}$$

- Hence probability of obtaining $|0\rangle$ or $|1\rangle$ upon measurement is not changed by multiplying a quantum state by a complex value of absolute value 1
- All complex numbers having absolute value 1 can be represented as

$$e^{\phi\, i} = \cos(\phi) + i \sin(\phi)$$

$$0 \leq \phi < 2\pi$$

- The qubit can be mathematically represented as:

$$|\,\psi\,\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = r_1\,e^{\phi_1 i}\,|\,0\,\rangle + r_2\,e^{\phi_2 i}\,|\,1\,\rangle$$

- Rearranging this equation gets the following:

$$|\,\psi\,\rangle = e^{\phi_1 i}\left(r_1\,|\,0\,\rangle + r_2\,e^{(\phi_2\,-\,\phi_1) i}\,|\,1\,\rangle\right)$$

Since $e^{\phi_1 i} \in \mathbb{C}$ and its absolute value is 1

$$|\,\psi\,\rangle = r_1\,|\,0\,\rangle + r_2\,e^{\phi i}|\,1\,\rangle$$

Since $r_1,\ r_2\ \in\ \Re$ and $0 \leq \phi < 2\pi$

$r_1\ =cos(\theta/2)$ and $r_2\ = sin(\theta/2)$ for some $0 \leq \theta \leq \pi$

Hence

$$|\,\psi\rangle\ = cos(\theta/2)\,|\,0\,\rangle + sin(\theta/2)\,e^{\phi i}\,|\,1\,\rangle$$

Bloch Sphere - 3-D Representation of Qubit

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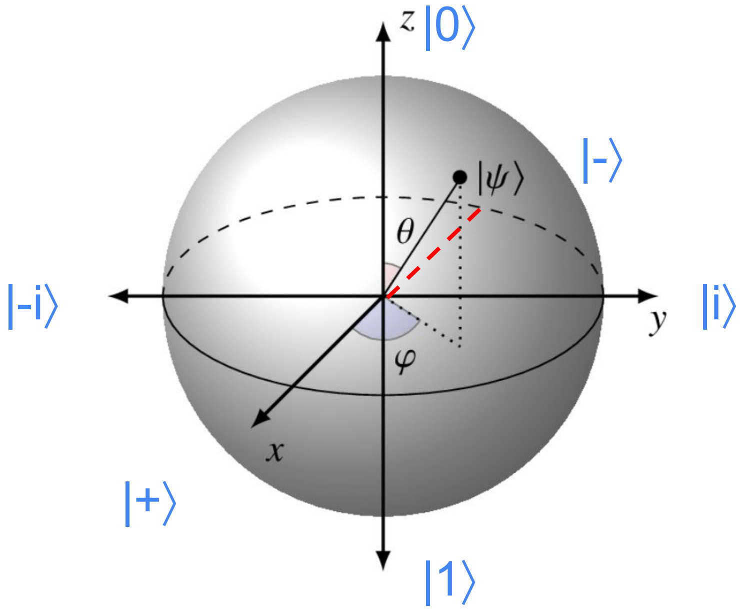
- There are infinite number of points on a unit sphere
- Each vector pointing towards such point on the unit sphere represents a quantum state which a qubit can have



$$|\,\psi > \; = \alpha\,|\,0> \; + \beta\,|\,1>$$

The vector representation for ψ looks like

$$\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

Putting $\alpha = 1$ and $\beta = 0 \implies |\,\psi > \; = |\,0>$

Putting $\alpha = 0$ and $\beta = 1 \implies |\,\psi > \; = |\,1>$

Hence the vector representation for $|\,0>$ looks like

$$\begin{bmatrix}1 \\ 0\end{bmatrix}$$

And the vector representation for $|\,1>$ looks like

$$\begin{bmatrix}0 \\ 1\end{bmatrix}$$

Linear Transformations and Unitary Matrices

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- $|0\rangle$ and $|1\rangle$ are *linearly independent* vectors as none of them can represent each other in form of some linear combination of the other
- Any set of linearly independent vectors belonging to $\mathbb{C}^2$ can serve as the computational basis but we take $|0\rangle$ and $|1\rangle$ as the basis
- Basis is determined by the measurement process or device; basically corresponds to the states in which a qubit can be physically
- Linear Transformations on qubits are *isometries* i.e. length of vectors is preserved and the vectors remain unit vectors
- Matrices of Linear Transformations are unitary
- A matrix *U* is unitary if $U^\dagger U = I$ *(dagger means conjugate transpose)*
- Unitary Matrices are invertible; hence moving a qubit from one state to another state is reversible

Quantum Computation, Gates, and Circuits

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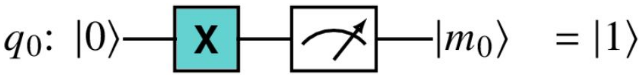
- Quantum Computation involves building Quantum Circuits using Quantum Gates
- Quantum Gates are the operations performed to manipulate the Quantum Information
- Single qubit and multiple qubit quantum gates are possible
- Single qubit quantum gates are unitary i.e. reversible
- Measurement is irreversible i.e. once a qubit collapses, its state cannot be reverted
- Since a qubit is a 2-D complex vector, all the quantum gates are 2 x 2 matrices and have complex entries according to some basis

Single Qubit Quantum Gates

- **Pauli-X Gate - Bit Flip Gate:**
  - Represented by σ_x
  - Reversible gate
  - Flips the quantum bit i.e. $|0\rangle \leftrightarrow |1\rangle$
  - Measurement returns $|0\rangle$ or $|1\rangle$
  - Rotates by π around the *x* axis
  - Poles are flipped and points in the lower hemisphere move to the upper hemisphere and vice versa



$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

such that

$$\sigma_x\,|\,0\,\rangle = |1\rangle \text{ and } \sigma_x\,|\,1\,\rangle = |0\rangle$$

Consider $|\,\psi\,\rangle = \alpha\,|\,0\,\rangle + \beta\,|\,1\,\rangle$

Applying Pauli-X gate to the qubit yields

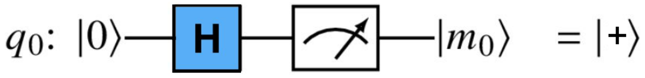
$$\sigma_x\,|\,\psi\,\rangle = \alpha\,|\,1\,\rangle + \beta\,|\,0\,\rangle$$

Single Qubit Quantum Gates

- **Hadamard Gate - H Gate:**
  - Represented by **H**
  - Reversible gate
  - Changes the basis matrix from

    $\{|0\rangle, |1\rangle\} \leftrightarrow \{|+\rangle, |-\rangle\}$

  - Takes the qubit to superposition (poles $\leftrightarrow$ equator)
  - First quantum gate used in the circuits
  - Most frequently used quantum gate



$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

such that

$$H\,|\,0\,\rangle = \frac{1}{\sqrt{2}}(|\,0\,\rangle + |\,1\,\rangle) = |\,+\,\rangle$$

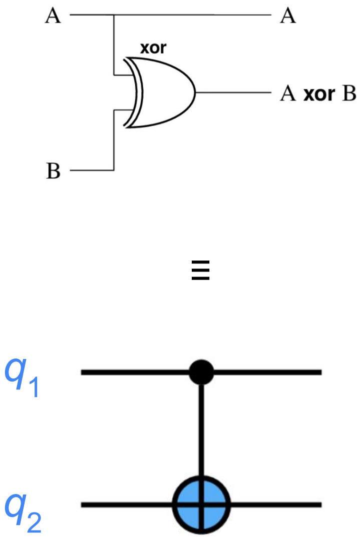
$$H\,|\,1\,\rangle = \frac{1}{\sqrt{2}}(|\,0\,\rangle - |\,1\,\rangle) = |\,-\,\rangle$$

and

$$H\,|\,+\,\rangle = |\,0\,\rangle \text{ and } H\,|\,-\,\rangle = |\,1\,\rangle$$

Two Qubit Quantum Gate

- **Controlled NOT Gate - CNOT Gate:**
  - Represented by **CX**
  - Very important
  - Creates entangled qubits
  - Conditional flipping
  - Consider two qubits q_1 and q_2 and their states $|\psi\rangle_1$ and $|\psi\rangle_2$, respectively
  - If $|\psi\rangle_1$ is $|1\rangle$, then the state of q_1 remains $|\psi\rangle_1$ but for q_2 it becomes $\boldsymbol{\sigma}_\mathbf{x}$ $|\psi\rangle_2$
  - Otherwise the states of q_1 and q_2 are not changed



CX $|00\rangle = |00\rangle$

CX $|01\rangle = |01\rangle$

CX $|10\rangle = |11\rangle$

CX $|11\rangle = |10\rangle$

Where from here?

- **Explore other qubit gates:**
  - Pauli-Y and Pauli-Z Gates
  - R_ϕ Gate
  - I, S, and T Gates
  - Toffoli Gate
  - SWAP Gate
  - Controlled Z Gate
- **Quantum Algorithms and Protocols:**
  - Deutsch-Jozsa Algorithm
  - Shor's Algorithm and Grover's Algorithm
  - Superdense Coding
- Quantum Error Correction

References and Acknowledgements

- **Quantum Computation and Quantum Information**: Textbook by Isaac Chuang and Michael Nielsen
- **Dancing with Qubits**: Textbook by Robert S. Sutor
- https://qiskit.org/

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Thanks for attending :)

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