- The standard normal critical value for a 95% interval: 1.96
- Confidence interval formula:

$$\bar{x} \pm Z_{(1-\frac{\alpha}{2})} \times \frac{\sigma_X}{\sqrt{n}}$$

estimate for the mean  $\pm$  multiplier  $\pm$  standard error for the mean

- The standard normal critical value for a 99% interval: 2.58
- Multiplier formula:

$$z = \frac{x - \mathbf{mean}}{\mathbf{sd}}$$

- What is the  $\alpha$  in the multiplier: tail probability
- Multiplier pattern: when CI is bigger (e.g., 95% to 99%), the multiplier will be bigger
- $s_X$ : sample standard deviation
- In practice the true standard deviation  $\sigma_X$  is not known: We estimate it with the sample standard deviation.
- This means our critical values must now come from the 't' distribution, not the standard normal.
- t distribution CI:

$$\bar{x} \pm t_{(1-\frac{\alpha}{2},\nu)} \times \frac{sX}{\sqrt{n}}$$

- $\nu$  (degree of freedom) for t-distribution:  $\nu = n 1$
- The t-distribution will be the correct sampling distribution if: either the underlying distribution of X is normal, and/or the sample size is sufficiently large (Central Limit Theorem holds).
- What is the degree of freedom in t-distribution: to replace the mean and sd in a normal distribution (because t-distribution is always standardised)
- When to use t-distribution: when the sample size is small
- Calculate the estimate sample size when knows the CI: assuming knows the sd and mean (normally given in the question), solve the equation, rounding UP
- Comparing means with CI:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{(1-\frac{\alpha}{2},\nu)} \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

• Using CLT to test appropriate normal distribution:

$$n\pi \pm 3\sqrt{n\pi(1-\pi)}$$

gives two values between 0 and n, if not, then fails the test. This approximation is good only when: n is large,  $\pi$  is not close to 0 or 1 (this increases symmetry)

• Formula for estimating  $\pi$ :

$$P = \frac{X}{n}$$

,

$$p = \frac{x}{n}$$

, x is the observed value of X. (and more) Using the Central Limit Theorem, the resulting distribution of these proportions is approximately normal if, n is large enough,  $\pi$  far enough from 0 or 1. As before, we judge this using:

$$n\pi \pm \sqrt{n\pi(1-\pi)}$$

gives values between 0 and n.

- Derivation of the mean of the sampling distribution: If  $P = \frac{X}{n}$ , then  $\mu_P = \pi$ , so  $\mu_P = \sqrt{\frac{\pi(1-\pi)}{n}}$
- 95% confidence interval for  $\pi$  (use the sample proportion (p) to estimate the unknown true population proportion  $(\pi)$ ):

$$p \pm 1.96\sqrt{\frac{p(1-p)}{n}}$$

• Margin of error:

$$multipliers*sd$$

• Note for CI: This confidence interval (and margin of error) is correct only if the normal approximation to the binomial is appropriate. In practice, bias due to non-response should also be considered in our interpretation of an estimate.