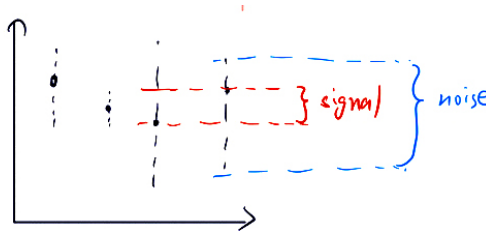


- **ANOVA:** Abbreviation of **Analysis of Variance**.
- **Methods for comparing means** of continuous responses between multiple groups.
- **F-ratio:** Signal/noise



- **Reasons why using "2+2+2=3" is undesirable:**
  - It's more work than we need to do. Three tests may not seem too bad, but to compare 10 groups we would have to do 45 different pairwise t-tests.
  - It can lead to lots of false positive results. Every test has the potential to incorrectly reject  $H_0$ ; i.e. falsely identify a difference between a pair of groups. If we do lots of tests then we risk generating lots of false positives.

- **ANOVA model:**

$$Y_{ij} = \mu_i + e_{ij}$$

$\mu_i$  is the true mean response for the  $i$ th group at the population level.  $e_{ij}$  is the error term for the  $j$ th response in the  $i$ th group. The error terms are assumed to be independent and to follow a  $N(0, \sigma^2)$  with constant variance. The number of different groups is denoted  $K$ , and the number of responses in the  $i$ th group is denoted  $n_i$ .

- **Est. mean for ANOVA:** The "." = est. Value.

$$\hat{\mu}_i = \bar{y}_{i.}$$

- **Sample mean for the  $i$ th group:**

$$\bar{y}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij}$$

- **Formula for residual sum of squares in ANOVA** (no need to memorise):

$$RSS = \sum_{i=1}^K \sum_{j=1}^{n_i} (y_{ij} - \hat{\mu}_i)^2 = \sum_{i=1}^K \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2$$

- **Total sum of squares in ANOVA** (no need to memorise):

$$TSS = \sum_{i=1}^K \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2$$

- $\bar{y}_{..}$  is the sample mean overall the data.

- **Formula for GSS in ANOVA** (no need to memorise):

$$GSS = TSS - RSS$$

$$GSS = \sum_{i=1}^k n_i (\bar{y}_{i.} - \bar{y}_{..})^2$$

GSS can be interpreted as a measure of the variation that is explained by differences between groups.

- **Setting up the hypotheses to test ANOVA:**

As usual, the null hypothesis will be the 'no difference' hypothesis:

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_K$$

The alternative is simply an expression that the null is incorrect:

$$H_A : \mu_1, \mu_2, \dots, \mu_K \text{ not all equal.}$$

- **Equation for F statistic:**

$$F = \frac{GSS/(K-1)}{RSS/(n-K)}$$

- **GMS:**  $GSS/(K-1)$  is the group mean square.
- **RMS:**  $RSS/(n-K)$  is the residual mean square.
- **What situations would let  $H_0$  fail:** Large differences between group means. Relatively large value of GSS. A large value of F.
- **ANOVA table:**

Source	SS	DF	MS
Groups	GSS	$K-1$	$\frac{GSS}{K-1}$
Residuals	RSS	$n-K$	$\frac{RSS}{n-K}$
Total	TSS	$n-1$	

- **P-value is right censored.**
- **Blocking variable:** A second treatment variable that when included in ANOVA analysis will have the effect of reducing the SSE term (noise).