STAT 110: Week 2

University of Otago

Outline

- We've started interacting with data
- Data summaries: sample mean and standard deviation
- Summaries are limited
  - To go further we need statistical models
    - Use probability to describe the variation in the data
- Over the next few lectures we will look at probability
  - Start today with foundational knowledge
  - Much of this knowledge remains important even in complex applications

Probability

- Immediate problem:
  - We want to describe the data using probability
  - We need to understand probability

Probability: mathematical language of uncertain events

- What is the probability that:
  - A randomly sampled penguin in the Palmer archipelago is an Adélie?
  - The all black kicker is successful with their next kick?
  - A rat will choose one reward (out of many) when moving through a maze?
  - A person has a certain genotype?
  - A female skink is a breeder?
  - An earthquake of magnitude 5 or larger occurs this year?
  - A cancer patient will die within 12 months?
  - The sliced ham you got at the supermarket is safe to consume?

Probability

- Setup
  - Random process with a number of possible outcomes
    - Roll a die. Possible outcomes: 1, 2, 3, 4, 5, or 6
    - Flip a coin. Possible outcomes: head or tail
    - Observe a penguin at Palmer. Possible outcomes: Adelie, chinstrap, gentoo
    - The set of all possible outcomes is called the sample space
- A probability has to satisfy a number of mathematical principles, including:
  - Between 0 and 1
    - We can't have a probability of -0.4 or 1.2
  - Probabilities sum to 1
    - If we observe the random process, we must see one of the possible outcomes
    - If we flip a coin, we must see either a head, or a tail.

Probability

- From here, things get a little murky
  - There are several definitions (or interpretations) of probability[1]
- We will define probability in terms of relative frequency:
  - The probability of an outcome is the proportion of times the outcome occurs if we were to observe the random process a large (infinite) number of times.
    - Imagine a (bored!) person repeatedly tossing a coin[2]

[1] We may return to this later in the semester.
[2] John Edmund Kerrich.

Mutually exclusive outcomes

- Two outcomes are mutually exclusive (or disjoint) if they cannot both happen
  - e.g. a coin flip cannot land on heads and tails
  - e.g. A penguin we observe cannot be a chinstrap and gentoo
- The probability of mutually exclusive outcomes can be found with addition
  - For two outcomes A and B that are mutually exclusive:

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$$

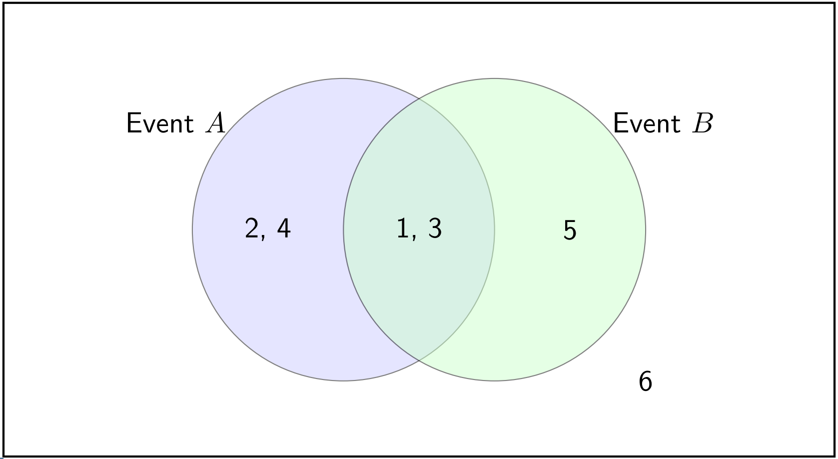
- Die roll: A: roll a 1, B: roll a 6
  - $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) = 1/6 + 1/6 = 1/3$
- Penguins: A: observe a chinstrap, B: observe a gentoo
  - $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$

Events

- We often work with collections of outcomes
  - These are called events
- Examples:
  - Die roll: event A: roll 1, 2, or 4, event B: roll 5 or 6.
  - Penguins: event C: Adelie or chinstrap, event D: gentoo or chinstrap
- Events can be mutually exclusive if they have no outcomes in common
  - Events A and B are mutually exclusive
    - $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) = 3/6 + 2/6 = 5/6$
  - Events C and D are not mutually exclusive
    - What is $\Pr(C \text{ or } D)$?
- An event can comprise a single outcome
  - e.g. the event E: roll a 3

Venn diagram

- Venn diagrams can be used to visualize small sample spaces
- Die roll: event A: roll 4 or less, event B: roll odd number



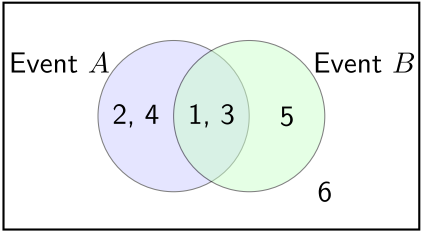
Venn diagram

[2] https://xkcd.com/2090/

Venn diagram and sets

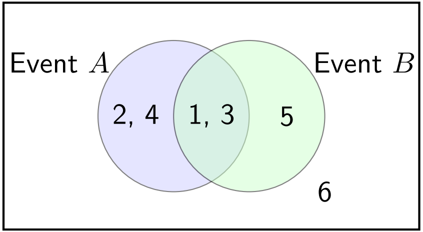
- Venn diagrams are useful when looking at when:
  - Event A or B occurs
    - This is inclusive, i.e. A or B means that event A, B or both A and B occur.
  - Event A and event B occurs

Sets of interest



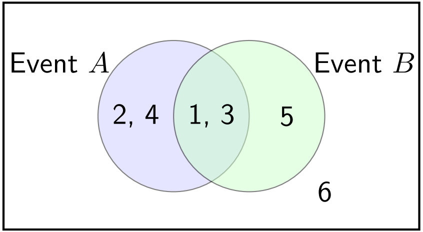
- A or B: 1, 2, 3, 4, 5
- A and B: 1, 3

Probabilities



- What is Pr(A or B)?
  - Same question asked a few slides ago (the events were called C and D then)
  - Events A and B are not mutually exclusive
  - $\mathrm{Pr}(A) + \mathrm{Pr}(B) = 4/6 + 3/6 = 7/6$
    - Clearly incorrect

Probabilities



- What is Pr(A or B)?
  - Probability of observing a 1, 2, 3, 4, or 5: probability of 5/6
- Problem with $\mathrm{Pr}(A) + \mathrm{Pr}(B)$ is that it double counts outcomes 1 and 3
  - Double counting $\mathrm{Pr}(A \text{ and } B)$

General addition rule

- If A and B are any two events, then the probability that at least one of them occurs is

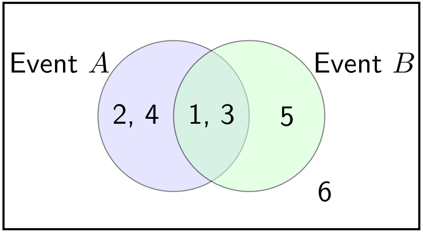
$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$$

- If events A and B are mutually exclusive, then $\Pr(A \text{ and } B) = 0$.
- Example (from previous slide)
  - $\Pr A + \Pr(B) - \Pr(A \text{ and } B)$ = 4/6 + 3/6 - 2/6 = 5/6

Complement

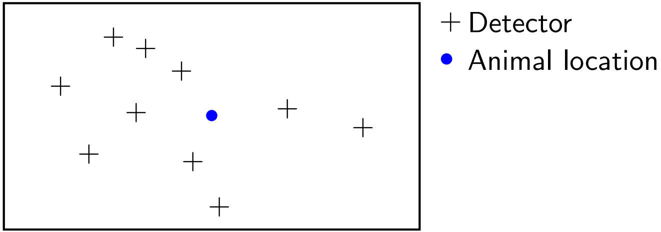
- Compliment: STAT 110 students are amazing!
- Complement of event A: the outcomes in the sample space that are not in A
- Roll a die: sample space is $\{1, 2, 3, 4, 5, 6\}$
  - The event E is rolling even: $\{2, 4, 6\}$
  - Its complement E^{C} is $\{1, 3, 5\}$
- $\Pr(E) + \Pr(E^{\mathsf{C}}) = 1$, or $\Pr(E) = 1 - \Pr(E^{\mathsf{C}})$
  - For the example above: $\Pr(E) = 0.5$, $\Pr(E^{\mathsf{C}}) = 0.5$
- Complements seem obvious and simple
  - I frequently remind 400-level students how useful they can be

Complements



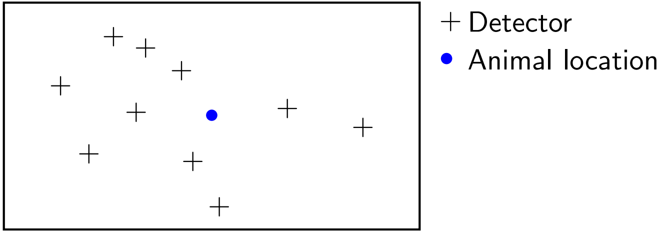
- Complements 'play nice' with Venn diagrams
- What is:
  - $\Pr(A^{\mathsf{C}})$?
  - $\Pr(B^{\mathsf{C}})$?
  - $\Pr((A \text{ or } B)^{\mathsf{C}})$?
  - $\Pr((A \text{ and } B)^{\mathsf{C}})$?

Complement: real example



- The picture represents an array of 'detectors' (e.g. motion activated camera)
  - Assume we know the probability the animal is detected (in some period of time) for each of the 10 detectors, based on its location
    - $p_1, p_2, \ldots, p_{10}$
- What is the probability it is seen by at least one detector?

Complement: real example



- There are over 1000 possible ways an animal could be detected:
  - Seen at one detector: detector 1, detector 2, detector 3, ...
  - Seen at two detectors: detector 1 & 2, detector 1 & 3
  - etc
- There is only one way an animal cannot be seen
  - Complement of being seen by at least one detector

Summary

- We are working toward statistical model for data
  - Use probability to describe the variation in the data
- Foundational knowledge in probability
  - Outcomes and events
  - Sample space, sets, and complements
  - General addition rule
- Relate probability back to examples
- Tomorrow: everyone bring a coin
  - Explore some (interactive) probability results

Outline

- Continue to build our knowledge of probability
- Today we look at two (or more) random processes
  - Independence
  - Conditional probability
  - Contingency tables
- Begin with an interactive exercise

Probability is hard

- It can be easy to trick ourselves that probability is easy
  - What is the probability that a die lands on a 4?

Probability is hard

- It can be easy to trick ourselves that probability is easy
  - What is the probability that a die lands on a 4?
  - It goes from easy to difficult very quickly
- In each of the next two lectures:
  - Start with an exercise that *might* be surprising to you
  - Hopefully broaden understanding of probability
- If I told you that I was flipping coins and saw 7 heads in a row
  - Would you think I am telling the truth?
  - How likely is it that I flip a fair coin and get 7 heads in a row?
    - Without getting calculators (or phones!) out
    - Is it closest to: 1 in one million? 1 in ten thousand? 1 in 100?

Exercise

- Everyone stand up
- Flip your coin (when I tell you to)[3]
  - Head: remain standing
  - Tail: sit down
- Those who are still standing, flip again (when I tell you to)
- Repeat until everyone sits down
  - See how many flips the 'best' person can get
- Is that what you were expecting?
- How would this look if we repeated the experiment at Beaver Stadium?
  - Penn State football stadium: capacity of over 106 000

[3] If you have forgotten a coin, google 'flip a coin' and you can do it online

Examples

- We might want to know the probability that:
  - A try was scored under the posts given the conversion was successful?
  - A participant undertakes a task left handed given they have a certain genotype?
  - A person has a certain genotype given they have brown eyes?
  - A female skink observed without offspring is a breeder?
  - An aftershock of magnitude 5 or larger occurs within six hours of a earthquake of magnitude 6?
  - Chicken is safe to consume given it has been heated to $60\,^{\circ}$C for five minutes?
- Each of these probabilities depends on another variable

Independence

- There are situations where we would expect two random processes to be unrelated
  - Process 1: rolling a die, process 2: flipping a coin
  - Process 1: eye colour of a person, process 2: success of rugby kick (conversion)
- We refer to these as independent
  - Two events A and B are independent if the outcome of one event provides no information about the outcome of the other
    - Knowing that our coin flip landed on heads does not change the probability of rolling a six
- Other processes may not be independent
  - Process 1: stock price of asset A, process 2: stock price of asset B

Independence

- Consider again the two process:
  - Process 1: rolling a die, process 2: flipping a coin
- If A is the event 'roll four or lower', and B is the event 'coin lands head'
  - What is $\Pr(A \text{ and } B)$?

Independence

- Consider again the two process:
  - Process 1: rolling a die, process 2: flipping a coin
- If A is the event 'roll four or lower', and B is the event 'coin lands head'
  - What is $\Pr(A \text{ and } B)$?
- $\Pr(A) = 4/6$ and $\Pr(B) = 1/2$
- Since the two events are independent we can reason that:
  - Event A will occur $4/6$ of the time
  - Event B will subsequently occur $1/2$ of *those* times
  - Event A and B occur together $4/6 \times 1/2 = 2/6$ of the time

Multiplication rule: indpendent processes

- If A and B are independent events, then[4]

$$\Pr(A \text{ and } B) = \Pr(A)\,\Pr(B)$$

- Example: suppose that 10% of the population are left handed, and 50% are female. If handedness and sex are independent, then what is the probability that a randomly selected person is right-handed and female?

$$\begin{aligned}\Pr(\text{right-handed and female}) &= \Pr(\text{right-handed})\,\Pr(\text{female}) \\ &= 0.9 \times 0.5 \\ &= 0.45\end{aligned}$$

[4] This can be extended to more than two events

Diversion: sex and gender

- Often we look at differences to do with sex or gender
- If interest is in biological differences (like the example above)
  - Sex: XX and XY
    - Recognize that intersex individuals exist
    - Usually not accounted for as prevalence is low
- In other applications (e.g. social science) we may be interested in gender
  - Often two genders still used
  - In time, increasingly see wider representation

Conditional probability

- Conditional probability describes the relationship between two events
- The probability of event B given event A has occurred is written $\Pr(B|A)$
  - Let T be the event that a try was scored under the posts
  - Let C be the event that the conversion was successful
    - $\Pr(T|C)$ is the probability that a try was scored under the posts given the conversion was successful?
  - Let B be the event that a female skink is a breeder
  - Let O be the event that a female skink is observed without offspring
    - $\Pr(B|O)$ is the probability that a female skink observed without offspring is a breeder

Conditional probability

- We can find $\Pr(B|A)$ using

$$\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$$

- $\Pr(A \text{ and } B)$: joint probability of events A and B occuring
- $\Pr(A)$: marginal probability of event A
- Two events A and B are independent if
  - $\Pr(B|A) = \Pr(B)$
  - The event A occurring does not change the probability of B occurring.
- Helpful to look at contingency tables

Contingency table: titanic

- Contingency tables allow us to compare two (categorical) variables [5]
- Data from the adult passengers on the titanic. Two variables:
  - Sex: male or female
  - Survived: yes or no
- Two tables: the first gives the counts, the second gives proportions

		survived		
		yes	no	Total
Sex	male	338	1329	1667
	female	316	109	425
	Total	654	1438	2092

		survived		
		yes	no	Total
Sex	male	0.162	0.635	0.797
	female	0.151	0.052	0.203
	Total	0.313	0.687	1.000

[5] They can be extended to more than two variables

Contingency table: titanic

- Two tables: the first gives the counts, the second gives proportions
  - For now, we will treat the proportions as if they are probabilities
  - See better approaches for estimating probabilities from contingency tables later

		survived		
		yes	no	Total
Sex	male	338	1329	1667
	female	316	109	425
	Total	654	1438	2092

		survived		
		yes	no	Total
Sex	male	0.162	0.635	0.797
	female	0.151	0.052	0.203
	Total	0.313	0.687	1.000

- Proportion are found by dividing entries by total, 2092
  - e.g. $316/2092 = 0.151$
  - e.g. $1438/2092 = 0.687$

Contingency table: titanic

- S: randomly selected passenger survived
- M: randomly selected passenger is male
- Marginal probability
  - $\Pr(M) = 0.797$
  - $\Pr(S) = 0.313$
  - Found in margin of contingency table

		survived		
		yes	no	Total
Sex	male	338	1329	1667
	female	316	109	425
	Total	654	1438	2092

		survived		
		yes	no	Total
Sex	male	0.162	0.635	0.797
	female	0.151	0.052	0.203
	Total	0.313	0.687	1.000

Contingency table: titanic

- S: randomly selected passenger survived
- M: randomly selected passenger is male
- Joint probabilities
  - $\Pr(M \text{ and } S) = 0.162$
  - $\Pr(M \text{ and } S^{\complement}) = 0.635$
  - $\Pr(M^{\complement} \text{ and } S) = 0.151$
  - $\Pr(M^{\complement} \text{ and } S^{\complement}) = 0.052$
  - Found in cells of contingency table

		survived		
		yes	no	Total
Sex	male	338	1329	1667
	female	316	109	425
	Total	654	1438	2092

		survived		
		yes	no	Total
Sex	male	0.162	0.635	0.797
	female	0.151	0.052	0.203
	Total	0.313	0.687	1.000

Contingency table: titanic

- S: randomly selected passenger survived
- M: randomly selected passenger is male
- Conditional probabilities
  - $\Pr(S|M) = \frac{\Pr(S \text{ and } M)}{\Pr(M)} = \frac{0.162}{0.797} = \frac{338}{1667} = 0.20$
    - Only consider male row
    - Survival and sex are not independent (why?)
  - $\Pr(S|M^{\complement}) = \frac{0.151}{0.203} = \frac{316}{425} = 0.74$
  - Could also find $\Pr(M|S)$, ...

		survived		
		yes	no	Total
Sex	male	338	1329	1667
	female	316	109	425
	Total	654	1438	2092

		survived		
		yes	no	Total
Sex	male	0.162	0.635	0.797
	female	0.151	0.052	0.203
	Total	0.313	0.687	1.000

Contingency table: smallpox

- Data are 6224 observations from individuals in Boston in 1721 who were exposed to smallpox[6]. There are two variables:
  - Inoculated: yes or no[7]
  - Result: lived or died

		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

		inoculated		
		yes	no	Total
result	lived	0.038	0.825	0.863
	died	0.001	0.136	0.137
	Total	0.039	0.961	1.000

[6] Fenner F. et al. 1988. Smallpox and Its Eradication (History of International Public Health, No. 6). Geneva: World Health Organization. ISBN 92-4-156110-6, p. 257

[7] Exposing a person to the disease in a controlled form

Contingency table: smallpox

- I: individual exposed to smallpox was inoculated
- L: individual exposed to smallpox lived
  - Find marginal probabilities:
    - $\Pr(I)$, $\Pr(L)$, $\Pr(I^C)$, $\Pr(L^C)$
  - Find joint probabilities:
    - $\Pr(I \text{ and } L)$, $\Pr(I^C \text{ and } L)$, $\Pr(I \text{ and } L^C)$, $\Pr(I^C \text{ and } L^C)$
  - Find conditional probabilities:
    - $\Pr(L|I)$, $\Pr(L|I^C)$
    - Are L and I independent?
  - Find $\Pr(I \text{ or } L)$
    - Is this a meaningful quantity in this example?

		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

		inoculated		
		yes	no	Total
result	lived	0.038	0.825	0.863
	died	0.001	0.136	0.137
	Total	0.039	0.961	1.000

Summary

- Looked at two random processes
- Introduced:
  - Independence
  - Multiplication rule (for independent events)
  - Conditional probability
  - Contingency tables
- Still building our knowledge of probability
  - So that we can apply it for statistical modelling
- We will start the next lecture with another probability exercise

Outline

- Look more at (conditional) probability
- Start with another probability exercise

Challenge

- I will select a small group of you
  - Most likely a row or two
- Question: does anyone in this group shares a birthday?
  - Compare our predictions with reality

Recap

- In the last lecture we talked about:
  - Joint probability: $\Pr(A \text{ and } B)$
  - Marginal probability: $\Pr(A)$
  - Conditional probability: $\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$
- Pick up from where we left off

Multiplication rule: general

- Last lecture we saw how to find conditional probability

$$\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$$

- If we rearrange this, we get the general multiplication rule

$$\Pr(A \text{ and } B) = \Pr(A)\,\Pr(B|A)$$

- We can ‘switch’ A and B so that we also have

$$\Pr(A \text{ and } B) = \Pr(B)\,\Pr(A|B)$$

Multiplication rule: smallpox example

- Suppose we were told: "96.1% of the residents were not inoculated, and 85.9% of the residents who were not inoculated ended up surviving."
  - What is the probability that a resident was not inoculated and lived?

Multiplication rule: smallpox example

- Suppose we were told: "96.1% of the residents were not inoculated, and 85.9% of the residents who were not inoculated ended up surviving."
  - What is the probability that a resident was not inoculated and lived?
- $\Pr(I^{\complement}) = 0.961$: 96.1% of the residents were not inoculated
- $\Pr(L|I^{\complement}) = 0.859$: 85.9% of the residents who were not inoculated ended up surviving
- $\Pr(I^{\complement} \text{ and } L)$: probability that a resident was not inoculated and lived

$$\begin{aligned}\Pr(I^{\complement} \text{ and } L) &= \Pr(I^{\complement})\Pr(L|I^{\complement}) \\ &= 0.961 \times 0.859 \\ &= 0.825\end{aligned}$$

Joint and conditional probability (smallpox example)

- Order doesn't matter for the joint probability
  - $\Pr(A \text{ and } B) = \Pr(B \text{ and } A)$
  - Probability both A and B occur
- Order does matter for the conditional probability
  - $\Pr(A|B)$ and $\Pr(B|A)$ are two different quantities
- Smallpox: compare $\Pr(L|I)$ and $\Pr(I|L)$.
  - $\Pr(L|I) = \frac{0.038}{0.039} = \frac{238}{244} = 0.975$
    - Probability of a resident living given inoculation
  - $\Pr(I|L) = \frac{0.038}{0.863} = \frac{238}{5374} = 0.044$
    - Probability of a resident being inoculated given lived

		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

		inoculated		
		yes	no	Total
result	lived	0.038	0.825	0.863
	died	0.001	0.136	0.137
	Total	0.039	0.961	1.000

Marginal probability: law of total probability

- Previously found "intuitively" from contingency table
- To find $\Pr(B)$
  - Sum over possible outcomes that could co-occur with the event B
- If there are two outcomes: A_1 and A_2
  - $\Pr(B) = \Pr(A_1 \text{ and } B) + \Pr(A_2 \text{ and } B)$

  - Smallpox: find $\Pr(L)$ and $\Pr(I)$

		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

		inoculated		
		yes	no	Total
result	lived	0.038	0.825	0.863
	died	0.001	0.136	0.137
	Total	0.039	0.961	1.000

Marginal probability: law of total probability

- Previously found "intuitively" from contingency table
- To find $\Pr(B)$
  - Sum over possible outcomes that could co-occur with the event B
- If there are two outcomes: A_1 and A_2
  - $\Pr(B) = \Pr(A_1 \text{ and } B) + \Pr(A_2 \text{ and } B)$

		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

		inoculated		
		yes	no	Total
result	lived	0.038	0.825	0.863
	died	0.001	0.136	0.137
	Total	0.039	0.961	1.000

- Smallpox: find $\Pr(L)$ and $\Pr(I)$
  - $\Pr(L) = \Pr(I \text{ and } L) + \Pr(I^{\complement} \text{ and } L) = 0.038 + 0.825 = 0.863$

Marginal probability: law of total probability

- Previously found "intuitively" from contingency table
- To find $\Pr(B)$
  - Sum over possible outcomes that could co-occur with the event B
- If there are two outcomes: A_1 and A_2
  - $\Pr(B) = \Pr(A_1 \text{ and } B) + \Pr(A_2 \text{ and } B)$

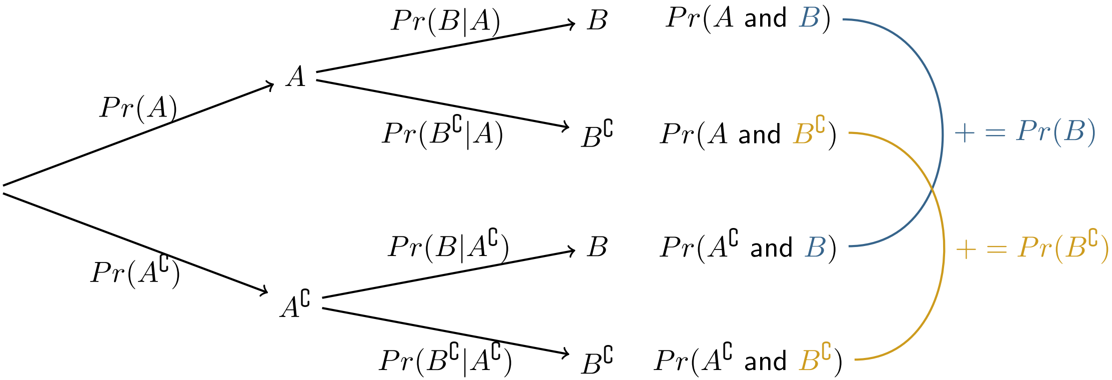
		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

		inoculated		
		yes	no	Total
result	lived	0.038	0.825	0.863
	died	0.001	0.136	0.137
	Total	0.039	0.961	1.000

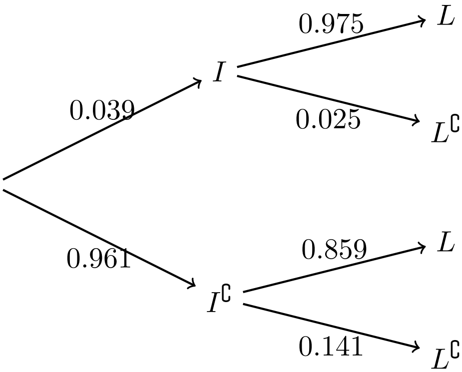
- Smallpox: find $\Pr(L)$ and $\Pr(I)$
  - $\Pr(I) = \Pr(I \text{ and } L) + \Pr(I \text{ and } L^{\complement}) = 0.038 + 0.001 = 0.039$

Tree diagrams

- Tree diagrams are an alternate way to visualize outcomes and probabilities
- General form:



Tree diagrams: smallpox example



$\Pr(I \text{ and } L) = 0.038$

$\Pr(I \text{ and } L^{\complement}) = 0.001$

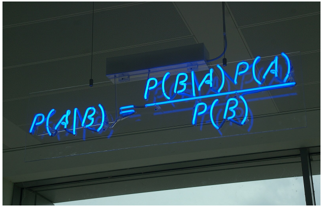
$\Pr(I^{\complement} \text{ and } L) = 0.825$

$\Pr(I^{\complement} \text{ and } L^{\complement}) = 0.136$

$\Pr(L) = 0.863$

$\Pr(L^{\complement}) = 0.137$

Bayes' theorem



- An important result in probability theory
  - Underpins a lot of modern statistics/data science/AI
    - Hopefully return to this at the end of the semester

- Idea: find $\Pr(A|B)$ from $\Pr(B|A)$

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)} = \frac{\Pr(B|A)\Pr(A)}{\Pr(B|A)\Pr(A) + \Pr(B|A^{\complement})\Pr(A^{\complement})}$$

Bayes' theorem

- Example: sleep apnea
- A: patient has sleep apnea
- S: patient snores
- Question: we know our patient snores. What is the prob they sleep apnea?
  - 90% of patients with sleep apnea snore: $\Pr(S|A) = 0.9$
  - 50% of patients without sleep apnea snore: $\Pr(S|A^{\mathsf{C}}) = 0.5$
  - 5% of the population have sleep apnea: $\Pr(A) = 0.05$

Bayes' theorem

- Example: sleep apnea
- A: patient has sleep apnea
- S: patient snores
- Question: we know our patient snores. What is the prob they sleep apnea?
  - 90% of patients with sleep apnea snore: $\Pr(S|A) = 0.9$
  - 50% of patients without sleep apnea snore: $\Pr(S|A^{\complement}) = 0.5$
  - 5% of the population have sleep apnea: $\Pr(A) = 0.05$

$$\begin{aligned}\Pr(A|S) &= \frac{\Pr(S|A)\Pr(A)}{\Pr(S|A)\Pr(A) + \Pr(S|A^{\complement})\Pr(A^{\complement})} \\ &= \frac{0.9 \times 0.05}{0.9 \times 0.05 + 0.5 \times 0.95} = 0.087\end{aligned}$$

Bayes' theorem: example

- We had: $\Pr(S|A) = 0.9$
  - Tempting to think that $\Pr(A|S)$ will also be high
  - $\Pr(A|S) \approx 0.09$ seems surprisingly low
  - Confusing conditional probabilities is a common mistake
- Another perspective:
  - How does the probability of patient having sleep apnea change
- In the general population, we have
  - $\Pr(A) = 0.05$
- After learning that patient snores, the probability increases to
  - $\Pr(A|S) \approx 0.09$

Bayes' theorem: understanding

- It can be difficult to understand why $\Pr(A|S)$ is low when $\Pr(S|A)$ is high
- Construct a hypothetical ('expected') contingency table
  - Pretend there are 100 000 patients
  - Recall: $\Pr(A) = 0.05$

		snores (S)		
		yes	no	Total
sleep apnea (A)	yes			5000
	no			95 000
	Total			100 000

Bayes' theorem: understanding

- It can be difficult to understand why $\Pr(A|S)$ is low when $\Pr(S|A)$ is high
- Construct a hypothetical ('expected') contingency table
  - Pretend there are 100 000 patients
  - Recall: $\Pr(S|A) = 0.9$

		snores (S)		
		yes	no	Total
sleep apnea (A)	yes	4500	500	5000
	no			95 000
	Total			100 000

Bayes' theorem: understanding

- It can be difficult to understand why $\Pr(A|S)$ is low when $\Pr(S|A)$ is high
- Construct a hypothetical ('expected') contingency table
  - Pretend there are 100 000 patients
  - Recall: $\Pr(S|A^{\complement}) = 0.5$

		snores (S)		
		yes	no	Total
sleep apnea (A)	yes	4500	500	5000
	no	47 500	47 500	95 000
	Total			100 000

Bayes' theorem: understanding

- It can be difficult to understand why $\Pr(A|S)$ is low when $\Pr(S|A)$ is high
- Construct a hypothetical ('expected') contingency table
  - Pretend there are 100 000 patients

		snores (S)		
		yes	no	Total
sleep apnea (A)	yes	4500	500	5000
	no	47 500	47 500	95 000
	Total	52 000	48 000	100 000

- Most of those who snore do not have sleep apnea!
- $\Pr(A|S) = \frac{4500}{52\,000} = 0.087$

Summary

- Looked in more detail at conditional probability
- Generalised the multiplication rule
- Tree diagrams
- Bayes' theorem
  - Using formula
  - Constructing an expected contingency table
- Next week: begin exploring how to use probability to model data