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**MANIPAL INSTITUTE OF TECHNOLOGY**

**MANIPAL UNIVERSITY, MANIPAL - 576 104**

**VI Semester B.E. / II Semester M.Tech - end Semester Examination – July 2014**

# MAT 540: APPLIED GRAPH THEORY (Elective)

**(New credit System -2012)**

**Time: 3 Hrs.** **Max. Marks: 50**

**Note : a) Answer any FIVE full questions. b) All questions carry equal marks (4+3+3)**

1A. Define Block and cut point in a graph. Show that $\alpha_0 + \beta_0 = p$

1B. Prove that a graph *G* is Eulerian if and only if every vertex is of even degree.

1C. Eleven members of a new club meet each day for lunch at a round table. They decide to sit in such a way that every member has different neighbor at each lunch. How many days can this arrangement last? Give all the possible arrangements.

2.A. Show that the following two nonisomorphic graphs have same chromatic polynomial and find the polynomial. Hence find $\chi(G)$.

$G_1$ $G_2$

2B. Define vertex connectivity and edge connectivity of a graph. State and prove Whitney's theorem.

2C. Write a short note on n-cubes. Obtain an algorithm to construct $Q_n$ from $Q_{n-1}$

3A Show that i) $\beta_1 \le \frac{p}{2} \le \alpha_1$ ii) $\alpha_0 \ge \delta$

3B Show that for a ladder graph $P(G, \lambda) = \lambda(\lambda-1)(\lambda^2 - 3\lambda + 2)^{n-1}$

3C Use Huffman's algorithm to find an optimal prefix code for the message OPERATION SUCCESS Find the corresponding Huffman tree and maximum weight of the tree.

4A Show that i) the chromatic polynomial of a graph G with n vertices is of degree n and leading coefficient is 1. ii) The constant term in chromatic polynomial is zero.

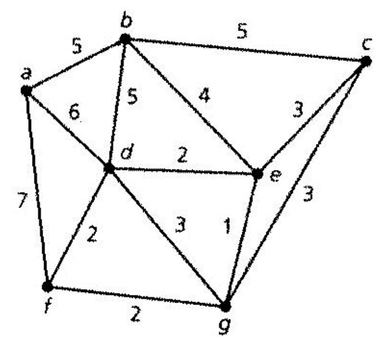
4B Draw the following graphs. Find $\alpha_0$ *and* $\alpha_1$ for each of them. Which of the graphs are Eulerian or Hamiltonian? i) Friendship graph $F_4$ iii) Dodecahederon

4C Show that in a party of six people there always exists three people who know each other or three people who do not know each other.

5A Show that a grapg G is bipartite if and inly if every cycle in G is of even length.

5B Show that every self complimentary graph exists on 4n or 4n+1 vertices. Draw a self complimentary graph on 8 vertices.

5C Find the minimal spanning tree of the following weighted graph using Kruskal's Algorithm




6A. Prove the Nordhaus Gaddum inequality $2\sqrt{p} \le \chi(G) + \bar{\chi}(G) \le p + 1$

6B. Show that $P(C_n, \lambda) = (\lambda - 1)^n + (-1)^n(\lambda - 1)$

6C Let G be graph without triangles with p vertices. The maximum number of edges in G is $\left\lfloor \frac{p^2}{4} \right\rfloor$