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MANIPAL INSTITUTE OF TECHNOLOGY
(Manipal University)

Manipal - 576 104

FOURTH SEMESTER B.E DEGREE END SEMESTER EXAMINATION –2010

SUB: PROBABILITY, STATISTICS & STOCHASTIC PROCESS (MAT – CSE- 202)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks.

1A. State Kalmogorov's axioms of probability. For any two events A and B, prove that $P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

1B. Show that $\overline{X}$, the sample mean is both an unbiased and consistent estimator for the population mean.

1C. Let X have a pdf of the form

$$f(x;\theta) = \theta x^{\theta-1},\ 0 < x < 1,$$
$$= \text{zero elsewhere, where } \theta \in \{\theta : \theta = 1, 2\}.$$

To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_0 : \theta = 2$, use a random sample X_1, X_2 of size n = 2 and define the critical region to be $C = \left\{ x_1, x_2 \ : \frac{3}{4} \le x_1 x_2 \right\}$. Find the power function of the test.

(4+3+3)

2A. A bag contains three coins one of which is two headed and the other two are normal and unbiased. A coin is chosen at random from the bag and is tossed four times in succession. If head turns up each time, what is the probability that it a two headed coin?

2B. Two independent r.v. X_1 and X_2 have means 5, 10 and variance 4, 9. Find covariance between $U = 3X_1 + 4X_2$ and $V = 3X_1 - X_2$

2C. Show that for a normal distribution with mean μ and variance σ^2,

$$E(X^{2n}) = 1.3.5....(2n-1)\sigma^{2n}.$$

(4+3+3)

3A. The random variable (X,Y) has joint pdf given by

$$f(x,y) = \begin{cases} x+y, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Compute the correlation coefficient between X & Y.

3B. Find mean and variance of Poisson distribution.

3C. Suppose that a two dimensional continuous random variable has joint pdf

$$f(x,y) = \begin{cases} kx(x-y), & 0 < x < 2,\ -x < y < x \\ 0 & \text{elsewhere} \end{cases}$$

a) Evaluate the constant k

b) Find the marginal pdf of y (4+3+3)

4A. A die is cast n = 120 independent times and the following resulted.

Spots up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40 – b

If we use chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at 0.025 significance level.

4B. The diameter of an electric cable is normally distributed with mean 0.8, variance 0.0004. What is the probability that diameter exceeds 0.81 inches.

4C. Let the observed value of the mean $\overline{X}$ of a random sample of size 20 from the distribution N(μ, 80) be 81.2. Find 95% confidence interval for μ.

(4+3+3)

5A. Suppose that life lengths of two electric devices, say D_1 and D_2 are normally distributed with mean 40 and 45, variance 36 and 9 respectively. If the electric device to be used for a period of 45 hrs? Which device is to be used?

5B. Find the mgf of a random variable which is uniformly distributed over (-a, a). Hence evaluate $E(X^{2n})$.

5C. If X ~ N (0, σ^2) , Y~ N (0, σ^2) where X and Y are independent, find the pdf of $R = \sqrt{X^2 + Y^2}$.

(4+3+3)

6A. Compute an approximate probability that mean of a random sample of size 15 from a distribution having pdf $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ is between $\frac{3}{5}$ & $\frac{4}{5}$.

6B. Let (X_1, X_2, …, X_n) denote a random sample from a distribution which is $n\ \theta_1, \theta_2$, $-\infty < \theta_1 < \infty,\ 0 < \theta_2 < \infty$. Find maximum likelihood estimators for θ_1 & θ_2 .

6C. Let $\overline{X}$ and S^2 be the mean and variance of a random sample of size 25 from a distribution which is n (3, 100). Evaluate Pr{ $0 < \overline{X} < 6$, $55.2 < S^2 < 145.6$}

(4+3+3)

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