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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

IV SEMESTER B.E DEGREE END SEMESTER EXAMINATION – May, 2008

SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV
(MAT –CSE – 202)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks.

1A. A 2n digit number starts with 2 and all its digits are prime. Find the probability that the sum of every 2 consecutive digits of the number is prime.

1B. Consider families of n children and let A be the event that a family has children of both the sexes and B be the event that there is at most one girl in the family. Find the value of n for which A and B are independent.

1C. It is suspected that a patient has one of the diseases A_1, A_2, A_3 . Suppose that the population percentage suffering from these illness are in the ratio 2:1:1. The patient is given a test which turns out to be positive 25% of the cases of A_1, 50% of the cases of A_2 and 90% of A_3. Given that out of three tests taken by the patient two are positive. Find the probability the patient has illness A_1.

(3 + 3 + 4)

2A. A coin is tossed till first head appears. Let X denote the number of tosses. Find E(X) and V(X).

2B. Suppose that X is uniformly distributed over (– a, +a) where a > 0. Whenever possible determine 'a' such that

(i) $\Pr(X > 1) = \frac{1}{3}$

(ii) $\Pr(X < 1) = \frac{1}{2}$

(iii) $\Pr(|X| < 1) = \Pr(|X| > 1)$

Contd…2

2C. Suppose that joint pdf of the two dimensional random variable (X, Y) is given by

$$f(x,y) = \begin{cases} x^2 + \dfrac{xy}{3}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

compute (i) $\Pr\left(\dfrac{Y}{X} < 1\right)$ (ii) $\Pr\left(X > \frac{1}{2}\right)$

(3 + 3 + 4)

3A. With usual notation show that
$-1 \le \rho \le +1$.

3B. A continuous random variable X has the pdf given by

$$f(x) = \begin{cases} a\ e^{-ax}, & a > 0,\ x \ge 0 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Let $P_j = \Pr(j \le X \le j+1)$, then show that P_j is of the form $(1-b)b^j$ and determine b.

(ii) Show that $\Pr(X > s+t\ / X > s) = \Pr(X > t), t > 0$

3C. An examination is often regarded as being good if the test scores of those taking the examination can be approximated by a normal density function. The instructor often uses the test scores to estimate the normal parameters μ and σ^2 and assign letter grade A to those whose test score is greater than $\mu + \sigma$, B to those whose test score is between μ and $\mu + \sigma$, C to those whose test score is between $\mu - \sigma$ and μ, D to those whose score is between $\mu - 2\sigma$ and $\mu - \sigma$, F to those getting a score below $\mu - 2\sigma$. Find approximate percentage of students in each grade.

(3 + 3 + 4)

4A. Show that for a normal distribution with mean μ and variance σ^2.
$\mu_{2n} = 1.3.5....(2n-1)\sigma^{2n}$.

4B. Let (X_1, X_2) be random sample from a distribution with the pdf $f(x) = e^{-x}, \quad 0 \le x \le \infty$. Show that $Z = X_1/X_2$ has an F distribution.

4C. Show that $\overline{X}$, the sample mean is both an unbiased and consistent estimator for the population mean.

(3 + 3 + 4)

Contd…3

5A. Let (X_1, X_2, …, X_n) denote a random sample from a distribution which is $n(\theta_1, \theta_2)$, $-\infty < \theta_1 < \infty,\ 0 < \theta_2 < \infty$. Find a maximum likelihood estimator for $\theta_1 \& \theta_2$.

5B. Let $\overline{X}$ be the mean of a random sample of size n from distribution which is N(3, 9). Find n such $\Pr\left(\overline{X} - 1 < \mu < \overline{X} + 1\right) = 0.90$, approximately.

5C. Let us assume that the life length of a tyre in miles, say X is normally distributed with mean θ and standard deviation 5000. Past experience indicates that θ = 30,00 the manufacturer claims that the typres made by a new procedure have mean θ > 30,000 and it is very possible that θ = 35, 000. Let us check this claim by testing H_0 : θ < 30,000 against H_1 : θ > 30,000. We shall observe n independent values of X say X_1, X_2, …, X_n and we shall reject H_0 if and only if $\overline{x} \geq c$. Determine n and c so that the power function K(θ) of the test has values K (30,000) = 0.01 and K (35,000) = 0.98.

(3 + 3 + 4)

6A. Let (X_1, X_2, …X_n) be a random sample of size n form a distribution n (θ, 100). Show that $C = \left\{ (x_1, x_2, \ldots, x_n) : c \leq \overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \right\}$ is a best critical region for testing H_0 : θ = 75 against H_1 : θ = 78. Find n and c so that

$\Pr\left[(X_1, X_2, \ldots, X_n) \in C, H_0\right] = 0.05$ and $\Pr\left[(X_1, X_2, \ldots, X_n) \in C, H_1\right] = 0.90$ approximately.

6B. The Mendelian theory states that the probabilities of classification a, b, c, d are respectively $\frac{9}{16}, \frac{3}{16}, \frac{3}{16}, \frac{1}{16}$. From a sample of 160 the actual numbers observed were 86, 35, 26 and 13. Is this data consistent with the theory at 0.01 significance level.

6C. Consider the process $X(t) = A\cos\omega t + B\sin\omega t$
where A and B are uncorrelated random variables with mean 0 and variance 1 and ω is a constant. Show that the process is covariance stationary.

(3 + 3 + 4)

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