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MANIPAL INSTITUTE OF TECHNOLOGY  
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MANIPAL-576104



FIFTH SEMESTER B.E. (CSE) DEGREE END SEMESTER EXAMINATION

NOV./DEC. 2011

DESIGN AND ANALYSIS OF ALGORITHMS (CSE 301)

DATE: 28-11-2011

TIME: 3 HOURS

MAX.MARKS: 50

**Instructions to Candidates**

- Answer **any five** full questions.

1 A) Mention the two most important properties of any sorting algorithm. --2M

B) Consider the usual Tower of Hanoi problem with three pegs namely source, destination and temp. There are two disks initially on the source peg.

i) Draw the tree of recursive calls denoting each node in the form: Tower (n, src, temp, destn).

ii) What is the total number of calls made by Tower of Hanoi algorithm?

iii) Write all the moves that are required to solve the puzzle. Show each move in the form “move from A to B”.

iv) Set up and solve the recurrence relation for the number of moves  $M(n)$  made by the Tower of Hanoi algorithm --2+1+1+2=6M

C) Arrange the following functions in the increasing order of growth

i.  $\log(n!)$       ii.  $n \log n$       iii.  $\sqrt{\log n}$       iv.  $(\log n)!$  --2M

2 A) Taking  $P$  as the probability of a successful search arrive at the average number of key comparisons,  $C_{avg}(n)$  for the sequential search algorithm. --2M

B) Consider the following instance of Assignment problem.

	Job1	Job2	Job3	Job4
Person1	10	3	8	9
Person2	7	5	4	8
Person3	6	9	2	9
Person4	8	7	10	5

i) How many possible solutions can exist using exhaustive search technique --1M

ii) Show the optimal solution which finds an assignment with the minimum total cost. Denote the solution in the form  $\langle J_1, J_2, \dots, J_n \rangle$ . --1M

C) i) Write a pseudo code for a divide and conquer algorithm for computing the sum of array  $A$  of  $n$  numbers. --2M

ii) Set up and solve a recurrence relation for the number of additions,  $A(n)$  made by the algorithm. --2M

iii) How does this algorithm compare with the brute force technique for this problem? --1M

iv) Solve the recurrence relation by means of Master theorem --1M

3 A) For quick sort, give an example of an array of four elements for which the sentinel is actually needed. What should be its value? Also explain why a single sentinel suffices for any input. --3M

B) Draw the binary tree for the binary search algorithm for a list of 7 elements containing elements {10, 20, 30, 40, 50, 60, 70}. List all the elements of this array that will require the largest number of key comparisons when searched for by binary search. --1+1=2M

C) Consider the recursive algorithm to compute the height of a binary tree with five internal nodes. What is the total number of external nodes in such a tree? How many comparisons are needed to check that the tree is empty? How many comparisons are needed to compute the maximum of two numbers? --1+1+1=3M

D) Consider multiplication of n-digit numbers by applying the classic (pen and paper) method. Set up and solve the recurrence relation  $M(n)$  for the total number of multiplications performed. --2M

4 A) Show that the average number of key comparisons in insertion sort is equal to  $C_{avg}(n) = n^2/4 + n/4 - 1/2$ . --2M

B) For the source removal algorithm to solve the topological sorting problem, how would you find a source in a digraph represented by its adjacency matrix? What is the time efficiency of this operation? --2M

C) Generate all the subsets of a three-element set  $A = \{a_1, a_2, a_3\}$  by bottom-up algorithm. --2M

D) Construct an AVL tree by inserting the elements 100, 200, 300, 250, 270, 70, 40. --2M

E) Construct heap for the list 4, 1, 3, 2, 9, 8, 7 using bottom up approach. --2M

5A) Is the comparison counting algorithm stable? Show by means of an example --2M

B) How many character comparisons will be made in searching for the pattern 01010 in the binary text of 200 zeros using Horspool and Boyer-Moore algorithm? Show all the relevant tables. --1+2=3M

C) For a closed hashing, explain the meaning of "lazy detection" with an example? --2M

D) Compute  $C(6,3)$  by applying the dynamic programming algorithm. Derive an equation for time complexity of the algorithm. --1+2=3M

6 A) Solve the all-pairs shortest path problem for the digraph with the weight matrix --3M

	a	b	c	d	e
a	0	2	$\infty$	1	8
b	6	0	3	2	$\infty$
c	$\infty$	$\infty$	0	4	$\infty$
d	$\infty$	$\infty$	2	0	3
e	3	$\infty$	$\infty$	$\infty$	0

B) Draw a decision tree and find the key comparisons in the worst and average cases for the five element binary search method. --3M

C) What is meant by a minimum spanning tree? Compare greedy solutions with that of exhaustive search techniques in finding minimum spanning trees. --1+1 = 2M

D) Apply backtracking method to solve the following instance of the subset-sum problem:  $S=\{1,2,5,6,8\}$  and  $d=9$  --2M