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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

IV SEMESTER B.E DEGREE END SEMESTER MAKEUP EXAMINATION – July, 2010

SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV
(MAT –CSE – 202)

(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

Note : a) Answer any FIVE full questions. b) All questions carry equal marks.

1A. A bag contains three coins, one of which is two headed and the other two coins are normal and unbiased. One coin is chosen at random and is tossed four times in succession. If each time head comes up, what is the probability that this is a two headed coin?

1B. Consider families of n children and let A be the event that a family has children of both the sexes and B be the event that there is at most one girl in the family. Find the value of n for which A and B are independent.

1C. (i) Two events A and B are such that $P(\overline{A}) = 0.3$, P(B) = 0.4 and $P(A \cap \overline{B}) = 0.5$.

Find $P(B | A \cup \overline{B})$.

(4+4+3)

2A. If the random variable 'K' is uniformly distributed over [0,5], what is the probability that the roots of the equation $4x^2 + 4xK + K + 2 = 0$ are real ?

2B. Suppose that the continuous random variable X has pdf

$f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Find mgf of X and hence find E(X) and V(X).

2C. A computer in adding numbers, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and are uniformly distributed over (– 0.5, 0.5).

a) If 1500 numbers are added what is the probability that magnitude of the total error exceeds 15?
b) How many numbers may be added together in order that the magnitude of the total error is less than 10 with probability 0.90?

(4+4+3)

3A. In a normal distribution 31% of the items are under 45 and 5% are over 64. Find the mean and variance of the distribution.

3B. Let X_1, X_2 and X_3 be uncorrelated random variables having the same standard deviation. Find the correlation coefficient between $X_1 + X_2$ and $X_2 + X_3$.

3C. If $X \sim N\ (0, \sigma^2)$, $Y \sim N\ (0, \sigma^2)$ where X and Y are independent, find the pdf of $R = \sqrt{X^2 + Y^2}$.

(4+4+3)

4A. Let $(X_1, X_2, \ldots\ldots X_n)$ denote a random sample of size n from the distribution with pdf

$$f(x,\theta) = \begin{cases} \dfrac{\theta^x e^{-\theta}}{x!}, & x = 0,1,2,\ldots\ldots, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find MLE for θ.

4B. a) Find the mgf of the random variable which is uniformly distributed in the interval $(-a, a)$. Hence evaluate $E\ (X^{2n})$.

b) Show that for the random variable X having normal distribution with mean μ and variance σ^2, $E\ \ X-\mu\ ^{2n} = 1.3.5.....(2n-1)\sigma^{2n}$.

4C. A random sample of size 15 from the normal distribution $N\ u, \sigma^2$ yields $\bar{x} = 3.2$ and $s^2 = 4.24$. Determine a 90% confidence interval for u and σ^2.

(4 + 4+3)

5A. Let us assume that the life length of a tire in miles, say X is normally distributed with mean θ and standard deviation 5000. Past experience indicates that $\theta = 30{,}000$, the manufacturer claims that the tires made by a new procedure have mean $\theta > 30{,}000$ and it is very possible that $\theta = 35{,}000$. Let us check this claim by testing $H_0 : \theta < 30{,}000$ against $H_1 : \theta > 30{,}000$. We shall observe n independent values of X say $X_1, X_2, \ldots, X_n$ and we shall reject H_0 if and only if $\bar{x} \geq c$. Determine n and c so that the power function $K(\theta)$ of the test has values $K\ (30{,}000) = 0.01$ and $K\ (35{,}000) = 0.98$.

5B. A survey of 320 families with 5 children each, revealed the following distribution. Is the result consistent with the hypothesis that male and female births are equally probable at 0.01 significance level.

No. of Boys	5	4	3	2	1	0
No. of Girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

5C. Let $X \sim \chi^2$ (n). Find M_X (t) and hence evaluate E(X) and V(X).

(4 + 4+ 3)

6A. Let $\hat{\theta}$ be an estimate of θ based on a sample of size n. If $\underset{n\to\infty}{\text{Lt}}\, E\,\hat{\theta} = \theta$ and if $\underset{n\to\infty}{\text{Lt}}\, V\,\hat{\theta} = 0$, then show that $\hat{\theta}$ is a consistent estimate of θ. And hence show that the sample mean is a consistent estimate for population mean.

6B. A coin is tossed till head appears for the first time. Let X denote the number of tosses. Find E(X) and V(X).

6C. Consider the process $X(t) = A\cos\omega t + B\sin\omega t$ where A and B are uncorrelated random variables with mean 0 and variance 1 and ω is a constant. Show that the process is covariance stationary.

(4 + 4+ 3)

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