Reg.No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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FOURTH SEMESTER B.E. DEGREE END SEMESTER EXAMINATION MAY 2006

SUBJECT : (MAT-CSE-202)
(REVISED CREDIT SYSTEM)

Time: 3 Hours. **Max.Marks: 50**

Instructions to Candidates:

- Answer **ANY FIVE** full questions.
- All questions carry equal marks.
- Statistical tables may be used.

1A] State Kalmogorov's axioms of probability. For any two events A & B,

(i) Prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(i) Find the probability of occurrence of exactly one of the events.

1B] Three balls are randomly chosen from an urn containing 3 white, 3 red and 5 black balls. Suppose that we win \$1 for each white ball selected and lose \$1 for each red selected. Let X denote total winnings from the experiment. Find the distribution of X. Also find the probability of winning money.

1C] The monthly income of 10,000 persons is found to be normally distributed with mean salary of Rs. 7,500 and standard deviation of Rs. 500. What percentage of the persons have income exceeding Rs. 6,680 ? what is the lowest income among the richest 100 ? **(3+4+3)**

2A] A and B throw a pair of fair dice alternatively. A wins if he throws a sum 6 before B throws a sum 7. B wins the game if he throws a sum 7 before A throws a sum 6. If A starts the game, find the probability of A winning the game.

Contd....

2B] A factory produces 10 glass containers daily. It is assumed that there is a constant probability 0.1 of producing a defective container. Before the containers are stored, they are tested and defective ones are set aside. The accuracy of the test is 95%. Let X be the number of containers classified as defective. Find Pr {X=k} and compute Pr {X = 3} & Pr { X >1}.

2C] Find the mgf of Chi-square distribution. Hence find its mean & variance.

(3+4+3)

3A] There are n bags numbered 1, 2, . . . n, each containing m ($m \leq n$) balls. Number of red balls in bag numbered k is k^2, for $1 \leq k \leq n$. Now a bag is selected at random and a ball drawn from it is found to be red. Prove that the probability that it is from the bag numbered k is

$$\frac{6k^2}{n(n+1)(2n+1)}, \quad 1 \leq k \leq n$$

3B] A two dimensional random variable (X,Y) has joint pdf

$$f(x,y) = x^2 + \frac{xy}{3}, \quad 0 \leq x \leq 1, \ 0 \leq y \leq 2$$
$$= 0 \quad , \text{elsewhere}$$

Compute (i) $\Pr\left\{\frac{X}{Y} \geq 1\right\}$ (ii) Pr { X+Y > 1}

3C] X is a random variable with $P(x)=ab^x$, where a and b are positive, a+b = 1 and X takes values 0, 1, 2, . . . Find the mgf of X.

Hence show that $m_2 = m_1(2m_1 + 1)$, m_1 and m_2 being the first two moments.

(3+4+3)

4A] Let (X,Y) be uniformly distributed over a circle of radius 'a' and center at origin. Find the correlation coefficient between X & Y?

Contd....

4B] A pays 1 dollar for each participation in the following game. Three dice are thrown; if one ace appears he gets 1 dollar, if two aces appear he gets 2 dollars and if three aces appear he gets 8 dollars, otherwise he gets nothing.

Is the game fair, that is, is the expected gain of the player is zero ? If not, how much should the player receive when three aces appear to make the game fair.

4C] Show that for the normal distribution with mean μ and variance σ^2,

$$E\left((x-\mu)^{2n}\right) = 1.3.5.\ldots(2n-1)\,\sigma^{2n}$$

(3+4+3)

5A] If the random variable X has $N(\mu, \sigma^2)$, show that $\left(\frac{X-\mu}{\sigma}\right)^2$ has $\chi^2(1)$.

5B] Let (X_1, X_2) be a random sample of size n = 2 from the distribution having pdf

$$f(x;\theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, \quad 0 < x < \infty$$

$$= 0 \text{ elsewhere.}$$

We reject $H_0 : \theta = 2$ and accept $H_1 : \theta = 1$, if the observed values x_1, x_2 are such that $\frac{f(x_1;2)\, f(x_2:2)}{f(x_1;1)\, f(x_2;1)} \leq \frac{1}{2}$

5C] Find the significance level of the test and the power of the test when H_0 is false. let the random variable X have the pdf

$$f(x) = e^{-x}, \; o < x < \infty,$$

$$0, \text{ elsewhere}$$

Compute the probability that the random interval (X, 3X) includes the point X = 3. What is the expected value of length of this random interval?

(3+4+3)

Contd....

6A] A computer, in adding numbers, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (–0.5, 0.5).

(i) If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?

(ii) How many numbers may be added together in order that the magnitude of the total error is less than 10, with probability 0.9?

6B] Let $(X_1, X_2, \ldots\ldots, X_n)$ be a random sample from a distribution that is $n(\theta_1, \theta_2)$, $-\infty < \theta_1 < \infty$, $\theta_2 > 0$. Find MLE for θ_1 and θ_2.

6C] If $\{X_1, X_2, \ldots\ldots X_{25}\}$ and $\{Y_1, Y_2, \ldots\ldots\ldots Y_{25}\}$ are two independent random samples of size 25 from the normal distributions N (0, 9) and N(1, 19) respectively, find $\Pr\left\{\frac{\overline{X}}{\overline{Y}} > 1\right\}$.

(3+4+3)

7A] A die is cast n = 120 independent times and the following resulted.

Spots up	1	2	3	4	5	6
Frequency	B	20	20	20	20	40 – b

If we use chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at 0.025 significance level.

7B] A random sample of size 15 from a population which is N (μ, σ^2) yields $\overline{x} = 3.2$ and $S^2 = 4.24$. Find a 90% confidence interval for μ and σ^2.

7C] X_1 and X_2 are independent random variables having standard normal distribution. Find the pdf of $Y = \frac{X_1}{X_2}$.

(3+4+3)

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