Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL UN IVERSITY, MANIPAL 576 104

DEPARTMENT OF MATHEMATICS

SIXTH B. E. AND SECOND SEMESTER M. TECH END SEMESTER EXAMINATION, (OPEN ELECTIVE) MAY 2014, (New Credit System -2012).

SUBJECT : APPLIED LINEAR ALGEBRA (MAT-548).

Time: 3Hrs. **Max. Marks: 50**

NOTE: Answer any five full questions. All questions carry equal marks.

1A. State and prove Cauchy-Schwarz inequality and verify the same with an example.

1B. Prove that a linear operator T on V is an isometry if and only if $(u, v) = (T(u), T(v))$ for all u, v in V.

1C. Explain Gram-Schmidt orthogonalization process and use it to find a set of orthonormal vectors from $(1,1,1)$, $(2,-1,\ 2)$, $(1,2,3)$ in E^3.

(3+3+4)

2A. By Lagrange's reduction transform, find the index and signature of $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$.

2B. Prove that any interchange of two elements in a permutation $j_1, j_2, \dots, j_n$ of the set $\{1, 2, \dots, n\}$ changes the index by an odd integer. Give one example.

2C. For $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ show that there is real orthogonal matrix P such that $P^{-1}AP = D$ is diagonal and also find A^6. **(3+3+4)**

3A. State and prove Cayley – Hamilton theorem. Find the inverse of the matrix $\begin{bmatrix} 7 & 1 \\ -4 & 3 \end{bmatrix}$ using the same.

3B. Prove that a mapping f of $V\ X\ V$ into F is an inner product on V iff f is a positive definite hermitian form.

3C. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ using Jacobi's method. Carry out 4 iterations. **(3+3+4)**

4A. Prove that the number of interchanges used to carry a permutation $j_1, j_2, \ldots,$ j_n of $\{1, 2, \ldots, n\}$ into natural ordering is always odd or always even. Give one example.

4B. **i.** Define Null space and column space of a matrix.

ii. Find the spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 0 & -1 & 3 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$ Also find null space and column space of *A*.

4C. Solve the initial value problem $\overrightarrow{x'} = \begin{pmatrix} -5 & 1 \\ 4 & -2 \end{pmatrix} \vec{x},\ \vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

(3+3+4)

5A. Find the SVD of $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$.

5B. Find the matrix of the given bilinear form f on U and V with respect to the given bases $\mathcal{A} = \{(i, 0, 0), (1, i, 0), (0, 0, 2i)\}$, $\mathcal{B} = \{(1 - i, i), (i, -i)\}$, given that $f\big((x_1, x_2, x_3), (y_1, y_2)\big) = 5x_1y_1 + ix_1y_2 - ix_2y_1 + 2x_2y_2 + 2x_3y_1 - x_3y_2$. And also use the matrix to compute the value of $f\big((i, 0, i), (2, 0)\big)$.

(7+3)

6A. Convert the differential equation $2y'' + 5y' - 3y = 0, y(0) = -4,\ y'(0) = 9$ into a system, solve the system and use this solution to get the solution to the original differential equation.

6B. Find the orthogonal transformation which transforms the quadratic forms $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 + 4x_1x_3 - 8x_2x_3$ to canonical form. Determine the index, signature and nature of the quadratic form.

(5+5)

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