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IV SEMESTER B.E DEGREE END SEMESTER EXAMINATION – May, 2009

SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV
(MAT –CSE – 202)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks.

1A. Ten percent of a certain population suffer from a serious disease. A person suspected of the disease is given two independent tests. Each test makes a correct diagnosis 90% of the time. Find the probability that the person really has the illness

(i) given that both tests are positive

(ii) given that only one test is positive.

1B. If X and Y are independent and X has $N(0, \sigma_1^2)$ and Y has $N\left(0, \sigma_2^2\right)$. Find the pdf of $R = \sqrt{X^2 + Y^2}$.

1C. Let (X_1, X_2) be a random sample from a distribution with the pdf $f(x) = e^{-x}, \quad 0 \le x < \infty$. Show that $Z = X_1/X_2$ has an F distribution.

(4 + 3 + 3)

2A. Let $(X_1, X_2, \ldots, X_n)$ denote a random sample from a distribution which is $n\left(\theta_1, \theta_2\right), \ -\infty < \theta_1 < \infty, \ 0 < \theta_2 < \infty$. Find maximum likelihood estimators for $\theta_1 \, \& \, \theta_2$.

2B. A Computer, in adding numbers, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (- 0.5, 0.5)

a) If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?

b) How many numbers may be added together in order that the magnitude of the total error is less than 10 with probability 0.90?

2C. A coin is tossed till a first head appears or n tosses are made. Let X denote the number of tosses. Find E(X).

(4 + 3 + 3)

3A. Two coins C_1 and C_2 have a probability of falling heads P_1 and P_2, respectively. You win a bet if in three tosses you get at least two heads in succession. You toss the coins alternatively starting with either coin. If $P_1 > P_2$, what coin would you select to start the game?

3B. With usual notation show that $-1 \le \rho \le +1$.

3C. Show that $\overline{X}$, the sample mean is both an unbiased and consistent estimator for the population mean.

(4 + 3 + 3)

4A. Find the mean and variance of Gamma distribution. Hence find mean and variance of chi-square distribution.

4B. If X has pdf

$$f(x) = \begin{cases} \frac{1}{4} x e^{-x/2}; & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find mgf of X and hence find μ and σ^2.

4C. Let a random sample of size 17 from a normal distribution $n\ \mu, \sigma^2$ yield $\overline{x} = 4.7$, $S^2 = 5.76$. Determine a 90% confidence interval for μ & σ^2.

(4 + 3 + 3)

5A. Define the sets $A_1 = \{x: -\infty < x \le \infty\}$, $A_i = \{x : i - 2 < x \le i - 1\}$, I =2, 3, …,7, and $A_8 = \{x : 6 < x < \infty\}$. A certain hypothesis assigns probabilities p_{i0} to these set A_i in accordance with

$$p_{io} = \int_{Ai} \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{x-3^{\,2}}{2\;4}\right] dx, \quad i = 1, 2, ..., 7, 8$$

This hypothesis (concerning the multinomial pdf with k = 8) is to be tested, at the 5 percent level of significance, by a chi-square test. If the observed frequencies of the sets A_i, i = 1, 2, …, 8 are, respectively, 60, 96, 140, 210, 172, 160, 88 and 74, would H_0 be accepted at the (approximate) 5 percent level of significance?

5B. Let X be a random variable with pmf $p(x) = ab^x$, x = 0, 1, 2, …where a and b are positive , a + b = 1. Find the mgf of X. Hence show that $m_2 = m_1(2m_1 + 1)$, m_1 and m_2 being the first two moments .

5C. Consider the process $X(t) = A\cos\omega t + B\sin\omega t$

where A and B are uncorrelated random variables with mean 0 and variance 1 and ω is a constant. Show that the process is covariance stationary.

(4 + 3 + 3)

6A. Let X have a pdf of the form

$$f(x;\theta) = \theta x^{\theta-1},\ 0 < x < 1,$$
$$= \text{zero elsewhere, where } \theta \in \{\theta : \theta = 1, 2\}.$$

To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_0 : \theta = 2$, use a random sample X_1, X_2 of size n = 2 and define the critical region to be $C = \left\{ x_1, x_2 : \frac{3}{4} \le x_1 x_2 \right\}$. Find the power function of the test.

6B. A Continuous random variable X has the pdf

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

If 5 independent determinations of X are made, what is the probability that atleast 2 are greater than 1?

6C. Consider families of n children and let A be the event that a family has children of both the sexes and B be the event that there is at most one girl in the family. Find the value of n for which A and B are independent.

(4 + 3 + 3)

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