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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

IV SEMESTER B.E DEGREE END SEMESTER MAKEUP EXAMINATION - 2009

SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV
(MAT –CSE – 202)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks.

1A. At an art exhibition there are 12 paintings of which 10 are original. A visitor selects a painting at random and before he decides to buy, he asks the opinion of an expert about the authenticity of the painting. The expert is right in 9 out of 10 cases on average.
a) Given that the expert decides that the painting is authentic, what is the probability that this is really the case?
b) If the expert decides that the painting is a copy, then the visitor returns it and chooses another one ; what is the probability that his second choice is an original.

1B. Suppose that a continuous random variable X has the pdf
$f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Find mgf of X and hence find its mean and variance.

1C. Find the coefficient of correlation between X & Y, if (X, Y) has joint pdf
$$f(x,y) = \begin{cases} e^{-|y|}/2, & y > |x| \\ 0, & \text{elsewhere} \end{cases}$$

(4 + 3 + 3)

2A. Let X and Y be two independent random variables having the pdf
$f(x) = e^{-x}$, $g(y) = 2e^{-2y}$, $0 \le x, y < \infty$. Find the pdf of Z = X + Y.

2B. The diameter of an electric cable is normally distributed with mean 0.8, variance 0.0004. What is the probability that diameter exceeds 0.81 inches. Suppose that the cable is considered to be defective if the diameter differs from its mean by more than 0.025, what is the probability of obtaining a defective cable.

2C. A die is cast n = 120 independent times and the following resulted.

Spots up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40 – b

If we use chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at 0.025 significance level.

(4 + 3 + 3)

3A. Urn A contains w_1 white balls and b_1 black balls. Urn B contains w_2 white balls and b_2 black balls. A ball is drawn from A and placed into B, and then a ball is transferred from B to A. Finally a ball is chosen from A. What is the probability that the ball will be white?

3B. A pays 1 dollar for each participation in the following game: three dice are thrown; if one ace appears he gets 1 dollar, if two aces appear he gets 2 dollars and if three aces appear he gets 8 dollars; otherwise he gets nothing. Is the game fair, i.e., is the expected gain of the player zero? If not, how much should the player receive when three aces appear to make the game fair?

3C. State and prove central limit theorem.

(4 + 3 + 3)

4A. Suppose that X is uniformly distributed over (– a, a) where a > 0. Whenever possible determine 'a' such that

(i) $\Pr\ X > 1\ = 1/3$ (ii) $\Pr\ X < 1\ = 1/2$

(iii) $\Pr\ |X| < 1\ = \Pr\ |X| > 1$ (iv) $\Pr\ X<1\ = 0.7$

4B. Two friends decide to meet at a place anywhere between 5 pm to 6 pm. Each one can come to the place and wait for the other for 10 minutes. What is the probability that they meet.

4C. Let $\overline{X}$ and S^2 be the mean and variance of a random sample of size 25 from a distribution which is n (3, 100). Evaluate $\Pr\{ 0 < \overline{X} < 6,\ 55.2 < S^2 < 145.6\}$

(4 + 3 + 3)

5A. Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size n from a distribution having pdf

$$f(x,\theta) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1,\ 0<\theta<\infty \\ 0, & \text{elsewhere} \end{cases}$$

Find M. L. E. for θ.

5B. Suppose that a two dimensional random variable (X, Y) has joint pdf

$$f(x,y) = \begin{cases} kx(x-y), & 0 < x < 2,\ -x < y < x \\ 0 & \text{elsewhere} \end{cases}$$

(i) Evaluate the constant k (ii) Find the marginal pdf of Y

5C. Let (X_1, X_2) be a sample of size 2 from the distribution having the pdf

$$f(x;\ \theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}},\quad 0 < x < \infty,\ \theta > 0.$$

$$= 0, \quad \text{elsewhere}$$

We reject $H_0 : \theta = 2$ and accept $H_1 : \theta = 1$, if the observed values (x_1, x_2) are such that

$$\frac{f\ x_1;2\ f\ x_2;2}{f\ x_1;1\ f\ x_2;1} \le \frac{1}{2}.$$

Find significance level of the test and the power of the test when H_0 is false.

(4 + 3 + 3)

6A. A random sample of size 15 from a normal distribution $n\ \mu, \sigma^2$ yield $\overline{X} = 4.26$, $S^2 = 5.64$. Determine a 95% confidence interval for $\mu\ \&\ \sigma^2$.

6B. Let $X_1, X_2, \ldots, X_n$ be mutually independent random variables having, respectively, the normal distribution $n\ \mu_1, \sigma_1^2\ ,\ n\ \mu_2, \sigma_2^2\ \ldots, n\ \mu_n, \sigma_n^2$. Find the mgf of the random variable $Y = k_1X_1 + k_2X_2 + \ldots + k_nX_n$ where $k_1, k_2 \ldots, k_n$ are constants. Deduce the reproductive property of the normal variate.

6C. Compute an approximate probability that the mean of a random sample of size 15 from a distribution having pdf

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is between $\frac{3}{5} \& \frac{4}{5}$.

(4 + 3 + 3)

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