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IV SEMESTER B.E DEGREE END SEMESTER EXAMINATION – May, 2010

SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV
(MAT –CSE – 202)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

Note : a) Answer any FIVE full questions. b) All questions carry equal marks.

1A. Players A and B play a sequences of independent games. Player A throws a die first and wins on a "six". If he fails, B throws and wins on a "five" or "six". If he fails, A throws and wins on a "four", "five", or "six". And so on. Find the probability of A winning the sequence.

1B. Let $(X_1, X_2, \ldots, X_n)$ denote a random sample from a distribution which is $n(\theta_1,\theta_2)$, $-\infty<\theta_1<\infty$, $0<\theta_2<\infty$. Find a maximum likelihood estimator for θ_1 & θ_2

1C. If the random variable X has $N(\mu, \sigma^2)$ distribution, then show that the random variable

$Z=\dfrac{X-\mu}{\sigma}$ has N(0, 1) and that $V=\dfrac{(X-\mu)^2}{\sigma^2}$ has $\chi^2(1)$. (4 + 3 + 3)

2A. Suppose that X has distribution $N(\mu, \sigma^2)$. A sample of size 15 yields $\bar{x} = 3.2$ and $s^2 = 4.24$. Obtain a 90 percent confidence interval for σ^2 and μ.

2B. Consider the process $\{X(t),\ t \in T\}$ whose probability distribution is given by

$$\Pr\{X(t)=n\}=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,\ldots..\\[2ex] \dfrac{at}{1+at}, & n=0\end{cases}$$

Test whether the process is covariance stationary.

2C. Let (X,Y) be a two dimensional continuous random variable with joint pdf

$$f(x,y)=\begin{cases}x^2+\dfrac{xy}{3}, & 0<x<1,\ 0<y<2\\ 0 & \text{elsewhere}\end{cases}$$

Find i) $P(Y<1/2 \mid X<1/2)$ ii) $P(X+Y<1)$ (4 + 3 + 3)

3A. Define the sets $A_1 = \{x: -\infty < x \leq 0\}$, $A_i = \{x : i - 2 < x \leq i - 1\}$, $i = 2, 3.$ and $A_4 = \{x : 2 < x < \infty\}$. A certain hypothesis assigns probabilities p_{i0} to these set A_i in accordance with

$$p_{io} = \int_{A_i} \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{(x-3)^2}{2(4)}\right] dx, \quad i = 1, 2, 3, 4$$

This hypothesis is to be tested, at the 5 percent level of significance, by a chi-square test. The observed frequencies of the sets A_i, $i = 1, 2, 3, 4$ are, respectively, 60, 96, 140, 210. Would H_0 be accepted at the (approximate) 5 percent level of significance?

3B. Find the mgf of the random variable X whose pdf is given by

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}, & x > 0 \\ 0 \qquad\qquad\qquad , & \text{elsewhere} \end{cases}$$

Hence find its mean and variance.

3C. If X and Y are independent random variables show that they are uncorrelated. Give an example to show that the converse is not true.

(4 + 3 + 3)

4A. Let X have a pdf of the form

$$f(x;\theta) = \theta x^{\theta-1}, \quad 0 < x < 1,$$
$$= 0, \quad \text{elsewhere}, \quad \text{where } \theta \in \{\theta : \theta = 1, 2\}.$$

To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_0 : \theta = 2$, use a random sample (X_1, X_2) of size $n = 2$ and define the critical region to be $C = \left\{ (x_1, x_2) : \frac{3}{4} \leq x_1 x_2 \right\}$. Find the power function of the test.

4B. A factory produces 10 glass containers daily. It is assumed that there is a constant probability 0.1 of producing a defective container. Before the containers are stored they are tested and defective ones are set aside. Suppose that there is a constant probability r = 0.1 that a defective container is misclassified. Let X be the number of containers classified as defective. Find

(i) $Pr(X=k)$ (ii) $Pr(X > 3)$

4C. It is suspected that a patient has one of the diseases A_1, A_2, A_3. Suppose that the population percentages suffering from these illnesses are in the ratio 2:1:1. The patient is given a test which turns out to be positive in 25% of the cases of A_1, 50% of A_2 and 90% of A_3. Given that out of three tests taken by a patient two were positive, find the probability that the patient has the disease A_1.

(4 + 3 + 3)

5A. The outside diameter of a shaft, say D, is specified to be 4 inches. Consider D to be a normally distributed random variable with mean 4 inches and variance 0.01 inch^2 . If the actual diameter differs from the specified value by more than 0.05 inch but less than 0.08 inch, the loss to the manufacturer is \$ 0.50. If the actual diameter differs from the specified value by more than 0.08 inch, the loss is \$ 1.00. The loss, L, may be considered as a random variable. Find the probability distribution of L and evaluate E(L).

5B. Let $X_1, X_2, \ldots, X_n$ be mutually independent random variables having, respectively, the normal distributions $N(\mu_1, \sigma_1^2), N(\mu_2, \sigma_2^2) \ldots, N(\mu_n, \sigma_n^2)$. Find the distribution of the random variable $Y = k_1X_1 + k_2X_2 + \ldots + k_nX_n$ where $k_1, k_2 \ldots, k_n$ are constants. Hence deduce the reproductive property of the normal distribution.

5C. Let $X_1, X_2, \ldots, X_{25}$ and $Y_1, Y_2, \ldots, Y_{25}$ are two independent random samples from normal distributions N(3, 16) and N(4, 9) respectively. Evaluate $\Pr\left\{\frac{\overline{X}}{\overline{Y}} > 1\right\}$ (4 + 3 + 3)

6A. A coin is tossed till we get a head for the first time or n times. Let X denote the number of tosses made. Find the probability distribution of X and its expected value.

6B. A continuous random variable X has pdf given by $f(x) = \begin{cases} \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$

Find mean and variance of the distribution.

6C. Let (X_1, X_2) be random sample from a distribution with the pdf $f(x) = e^{-x}, \quad 0 \le x < \infty$. Show that $Z = X_1/X_2$ has F distribution. (4 + 3 + 3)

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