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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

VI SEMESTER B.E DEGREE END SEMESTER EXAMINATION –May 2011
SUB: GRAPH THEORY (MAT –301) ELECTIVE – I
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions. b) All questions carry equal marks.**

1A. (i) Let G be a (p,q) – graph all of whose vertices have degree k or k+1. If G has $p_k>0$ vertices of degree k and p_{k+1} vertices of degree k+1, show that $p_k = (k+1)p - 2q$.

(ii) Construct a cubic graph with 2n vertices ($n\geq3$) having no triangles.

1B. Define a bipartite graph.
Show that a graph is bipartite, if and only if, all its cycles (if any) are even.

1C. If G is a graph (connected or disconnected) having exactly two vertices u_0 and v_0 of odd degrees, then show that u_0 and v_0 are connected.

(03 + 04+03)

2A. (i) Illustrate with an example that every graph is an intersection graph.
(ii) Let G be a graph of order 13 with 3 components. Show that atleast one component of G has atleast 5 vertices.

2B. Define a self – complementary graph.
Show that every self – complementary graph has 4n or 4n+1 vertices. Draw a self – complementary graph with p = 5.

2C. If G is not connected, show that $\overline{G}$ is connected. Give an example of a graph G in which both G and $\overline{G}$ are connected. (04 + 03+03)

3A. Define the product $G_1\times G_2$ of two graphs. If G_1 is (p_1, q_1) – graph and G_2 is (p_2, q_2)- graph, show that the graph $G_1\times G_2$ has $p_1 p_2$ vertices and $p_1q_2 + p_2q_1$ edges. Draw the graph $G_1 \times G_2$ if $G_1 = P_2$ and $G_2 = P_3$.

3B. If G_1 and G_2 are regular (or bipartite) prove or disprove $G_1+ G_2$ is regular (or bipartite).

3C. (i) Define : block graph B (G) and cut point graph C(G) of a graph G.

(ii) Illustrate with an example that if G is connected with atleast one cutpoint, then B(B(G)) = C(G) (04 +03 +03)

4A. State Havel – Hakimi theorem on graphical sequence. Given an algorithm for constructing a graph with a given degree sequence π = (d_1, d_2 , …….., d_p). Illustrate the algorithm for the sequence π = (5,5,3,3,2,2,2).

4B. If G is a tree, then prove that every two vertices of G are joined by a unique path and that p = q+1.

4C. Define a cut vertex. Show that every non trivial connected graph has atleast two vertices which are not cutvertices. (04 +03 + 03)

5A. State and prove Whitney's theorem.

5B. (i) Define : a outer planar graph and a maximal outer planar graph. Give one example for each. Draw the forbidden graphs for outer planarity.

(ii) Define the topological invariant thickness $\theta(G)$ of a graph. Show that for any (p,q) – graph $\theta(G) \geq \frac{q}{3p-6}$. Hence, find $\theta(K_5)$.

5C. For any non trivial graph G, prove that $\alpha_0 + \beta_0 = p$ (04 +03+03)

6A. (i) Define the total graph T(G) of a graph G. Draw $T(K_3)$

(ii) If G is a (p,q) – graph whose vertices have degrees d_i, show that T(G) has $p_T = p+q$ vertices and $q_T = 2q + \frac{1}{2}\sum_i d_i^2$ edges.

6B. Show that the number of labeled spanning trees of the complete graph K_p is p^{p-2}.

6C. Define : Coloring, an n – coloring and the chromatic number $\chi(G)$ of a graph G. Determine $\chi(G)$ for following graph :

K_p, $K_p - x$, C_{2n}, C_{2n+1} and wheel W_n.

(04 + 03+03)

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