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**MANIPAL INSTITUTE OF TECHNOLOGY  
MANIPAL UNIVERSITY, MANIPAL - 576 104**



**IV SEMESTER B.E DEGREE END SEMESTER EXAMINATION – 2008**

**SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV**  
**(MAT –CSE – 202)**  
**(REVISED CREDIT SYSTEM)**

**Time : 3 Hrs.**

**Max.Marks : 50**

 **Note :** a) Answer any FIVE full questions.  
b) All questions carry equal mark

- 1A. (i) Two events A and B are such that  $P(\bar{A}) = 0.3$ ,  $P(B) = 0.4$  and  $P(A \cap \bar{B}) = 0.5$ . Find  $P(B|(A \cup \bar{B}))$ .
- 1B. A continuous random variable X has pdf given by
- $$f(x) = \begin{cases} \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$
- Find mean, variance and mode of the distribution.
- 1C. If the random variable 'K' is uniformly distributed over [0,5], what is the probability that the roots of the equation  $4x^2 + 4xK + K + 2 = 0$  are real ?
- (4 + 3 + 3 )
- 2A. Ten percent of a certain population suffer from a serious disease. A person suspected of the disease is given two independent tests. Each test makes a correct diagnosis 90% of the time. Find the probability that the person really has the illness given that both tests are positive

- 2B. A two dimensional random variable (X, Y) is uniformly distributed over a rectangle with vertices  $(-1, 0)$ ,  $(1, 0)$ ,  $(0, -1)$  and  $(0, 1)$ . Find  $E(x)$  and  $E(Y)$ .

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- 2C. Suppose that a two dimensional continuous random variable has joint pdf

$$f(x,y) = \begin{cases} kx(x-y), & 0 < x < 2, -x < y < x \\ 0 & \text{elsewhere} \end{cases}$$

- a) Evaluate the constant k  
b) Find the marginal pdf of y

(4 +3 +3 )

- 3A. (i) A random variable X assumes 4 values with probabilities  $\frac{1+3x}{4}$ ,  $\frac{1-x}{4}$ ,  $\frac{1+2x}{4}$  and  $\frac{1-4x}{4}$ . For what values X of this is a probability distribution?

- (ii) Suppose that the random variable X has possible values 1, 2, 3 ,....., and  $P(X = j) = \frac{1}{2^j}$ ,  $j = 1, 2, \dots$ . Compute  $P(X \text{ is even})$  and  $P(X \geq 6)$ .

- 3B. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation.

- 3C. Find mean and variance of Poisson distribution.

(4 +3 +3 )

- 4A. Let X have uniform distribution over the interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ . Find the pdf of Y where  $Y = \tan X$ .

- 4B. Compute an approximate probability that mean of a random sample of size 15 from a distribution having pdf

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is between  $\frac{3}{5}$  &  $\frac{4}{5}$ .

- 4C. Suppose that the continuous random variable X has pdf

$$f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty. \text{ Find mgf of X and hence find } E(X) \text{ and } V(X).$$

(4 +3 +3 )

- 5A. Show that the sample mean  $\bar{X}$  is both unbiased and consistent estimator for the population mean.

Contd..3

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- 5B. Let  $(X_1, X_2, \dots, X_n)$  denote a random sample of size n from the distribution with pdf

$$f(x, \theta) = \begin{cases} \frac{\theta^x e^{-\theta}}{x!}, & x = 0, 1, 2, \dots, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find MLE for  $\theta$ .

- 5C. Let a random sample of size 17 from a normal distribution  $n(\mu, \sigma^2)$  yield  $\bar{x} = 4.7$ ,  $S^2 = 5.76$ . Determine a 90% confidence interval for  $\mu$  &  $\sigma^2$ .

(4 +3 +3 )

- 6A. Let X have binomial distribution with parameters  $n = 10$  and  $p$ . The simple hypothesis  $H_0 : p = \frac{1}{2}$  is rejected and the alternative simple hypothesis  $H_1 : p = \frac{1}{4}$  is accepted, if the observed value of  $X_1$ , a random sample of size 1, is less than that or equal to 3. Find the power function of the test.

- 6B. A die is cast  $n = 120$  independent times and the following resulted.

Spots up	1	2	3	4	5	6
Frequency	b	20	20	20	20	$40 - b$

If we use chi-square test, for what values of  $b$  would the hypothesis that the die is unbiased be rejected at 0.025 significance level.

- 6C. Define a Stochastic Process. With example explain the classification.

(4 +3 +3 )

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