Reg.No.



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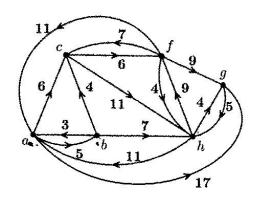
THIRD SEMESTER B.E DEGREE MAKEUP EXAMINATION – DECEMBER, 2012 Sub: MAT 209 - ENGG. MATHEMATICS III (CS/IT)

(REVISED CREDIT SYSTEM – 2011)

Time: 3 Hrs. Max. Marks: 50

Note: a). Answer any FIVE full questions b). All questions carry equal marks

- 1A. A shop sells 6 different flavours of ice-cream. In how many ways, a customer can choose 4 ice-cream cones if
 - (i) They are all of different flavours
 - (ii) They are not necessarily of different flavours
 - (iii) They contain only 2 or 3 flavours.
 - (iv) They contain three flavours.
- 1B. Implement Dijkstraa's algorithm to find shortest path from 'c' to all other vertices, for the following network.



- 1C. If A works hard, then either B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not. If A works hard, then show that D will not enjoy himself, using inference theory of proportional calculus.

 (3+3+4)
- 2A. Show that a subgroup of a cyclic group is cyclic.
- 2B. Let $E(x_1, x_2, x_3) = \overline{(x_1 \vee x_2) \vee (\overline{x_1} \wedge x_3)}$ be a Boolean expression over the two valued Boolean algebra. Write $E(x_1, x_2, x_3)$ in both disjunctive and conjunctive normal forms.

- 2C. Let H and K be subgroups of a group G. Show that $H \cap K$ is also a subgroup of G. (3+3+4)
- 3A. Show that a graph is Eulerian if and only if it is connected and all its vertices have even degree.
- 3B. Define normal subgroup of a group. Show that the kernel of any homomorphism of a group is a normal subgroup.
- 3C. Show that the proportion of permutations of $\{1,2,3,\ldots,n\}$ which contain no consecutive pair (i, i+1), for any i is approximately $\frac{(n+1)}{ne}$. (3+3+4)
- 4A. Let a, b, c be elements of a lattice (A, \leq) . Show that
 - i. $a \lor (b \land c) \le (a \lor b) \land (a \lor c)$
 - ii. $(a \land b) \lor (a \land c) \le a \land (b \lor c)$
- 4B. How many ways are there to distribute 25 identical balls into 7 distinct boxes if the first box can have no more than 10 balls and any amount can roll into each of the other six boxes.
- 4C. Symbolize the predicate
 - (i) "x is the father of the mother of y".
 - (ii) Using inference theory of Predicate calculus prove that $(x)(H(x) \rightarrow M(x)) \wedge H(J) \Rightarrow M(J)$ (3+3+4)
- 5A. Let (G, *) be a group. Let H be a non empty subset of G. Show that H is a subgroup of G if and only if $a * b^{-1} \in H$, for all $a, b \in H$.
- 5B. If G is a graph, show that either G or \overline{G} is connected.
- 5C. Find the 55th and 95th permutations of the five marks 1, 2, 3, 4, 5 in
 (i) Reverse lexicographical order
 (ii) Fike's order
 (3+3+4)
- 6A. i) Symbolize the predicate,

If x is the father of y then y is the child of x.

- ii) Show that $(\exists x) M(x)$ follows logically from the premises, $(x)(H(x) \to M(x))$ and $(\exists x)H(x)$.
- 6B. Let $\langle A, \leq \rangle$ be a distributive lattice. Show that if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a, then x = y.
- 6C. Define self-complementary graph. Show that every self-complementary graph has 4n or 4n + 1 vertices. (3+3+4)
