Formal Languages
Finite Automata

Chomsky Hierarchy

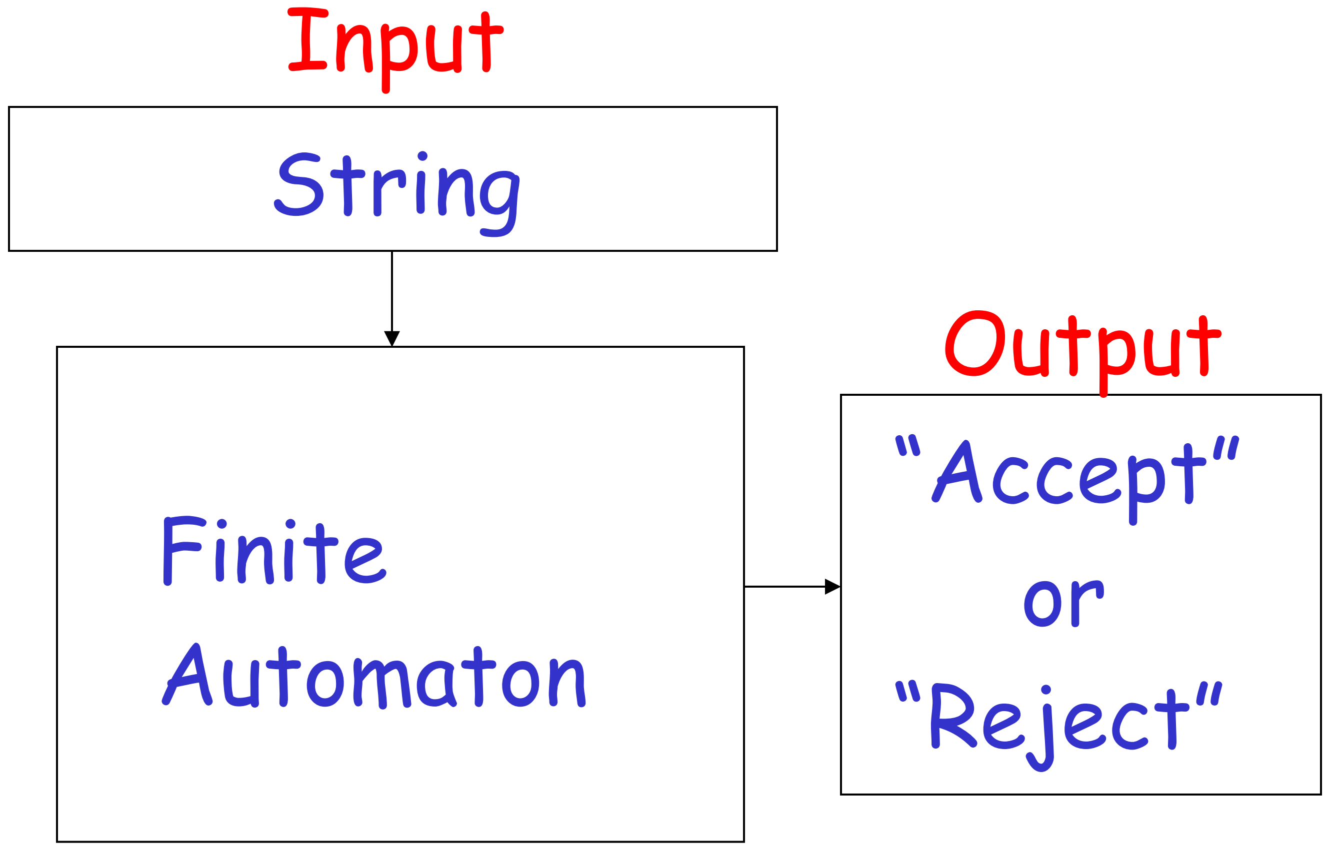
Type-0 : Unrestricted;recursively enumerated; phrase structured; turing m/c

Type-1: CSL;LBA

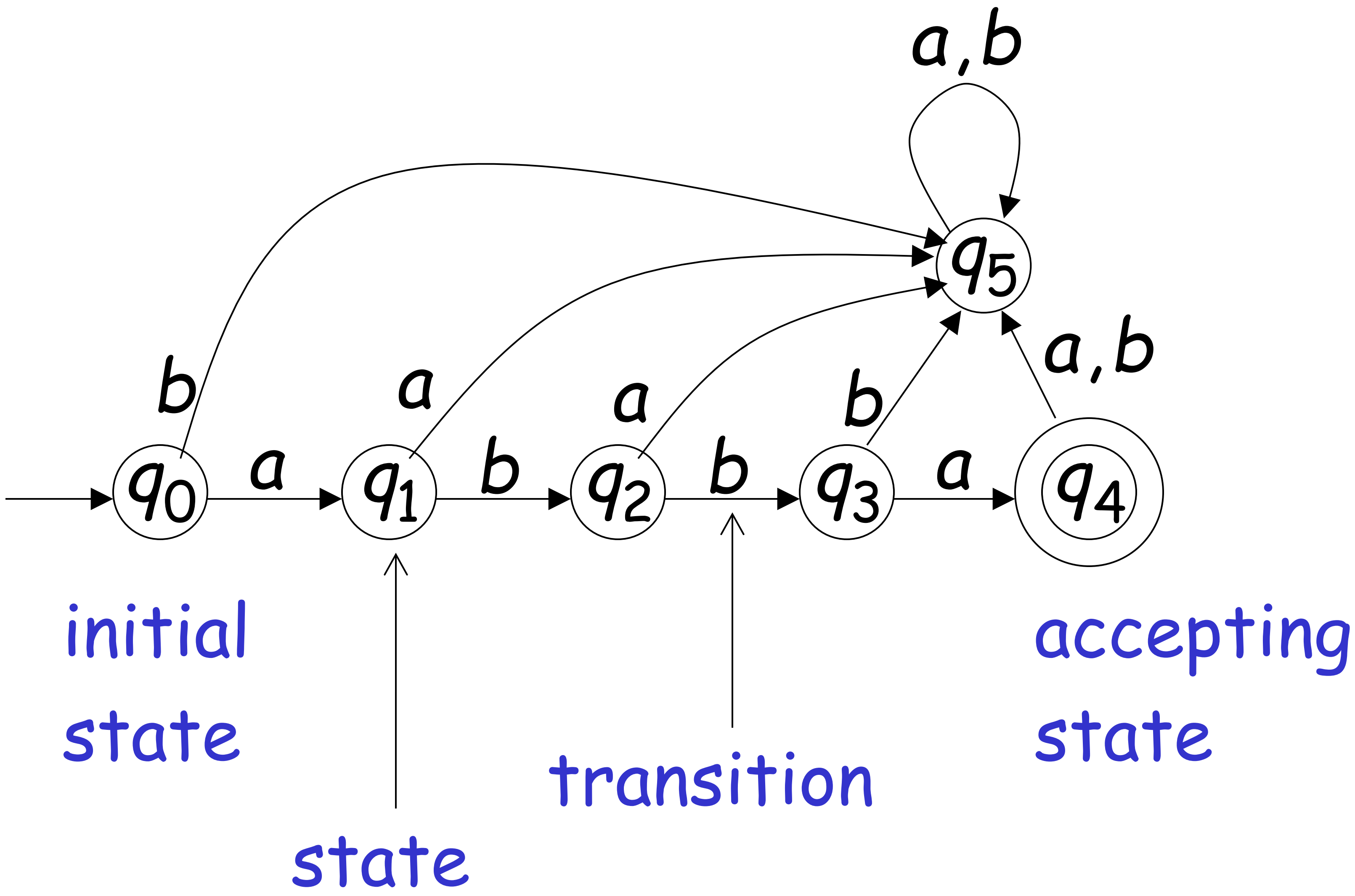
Type-2: CFL;PDA

Type-3:RL:FA

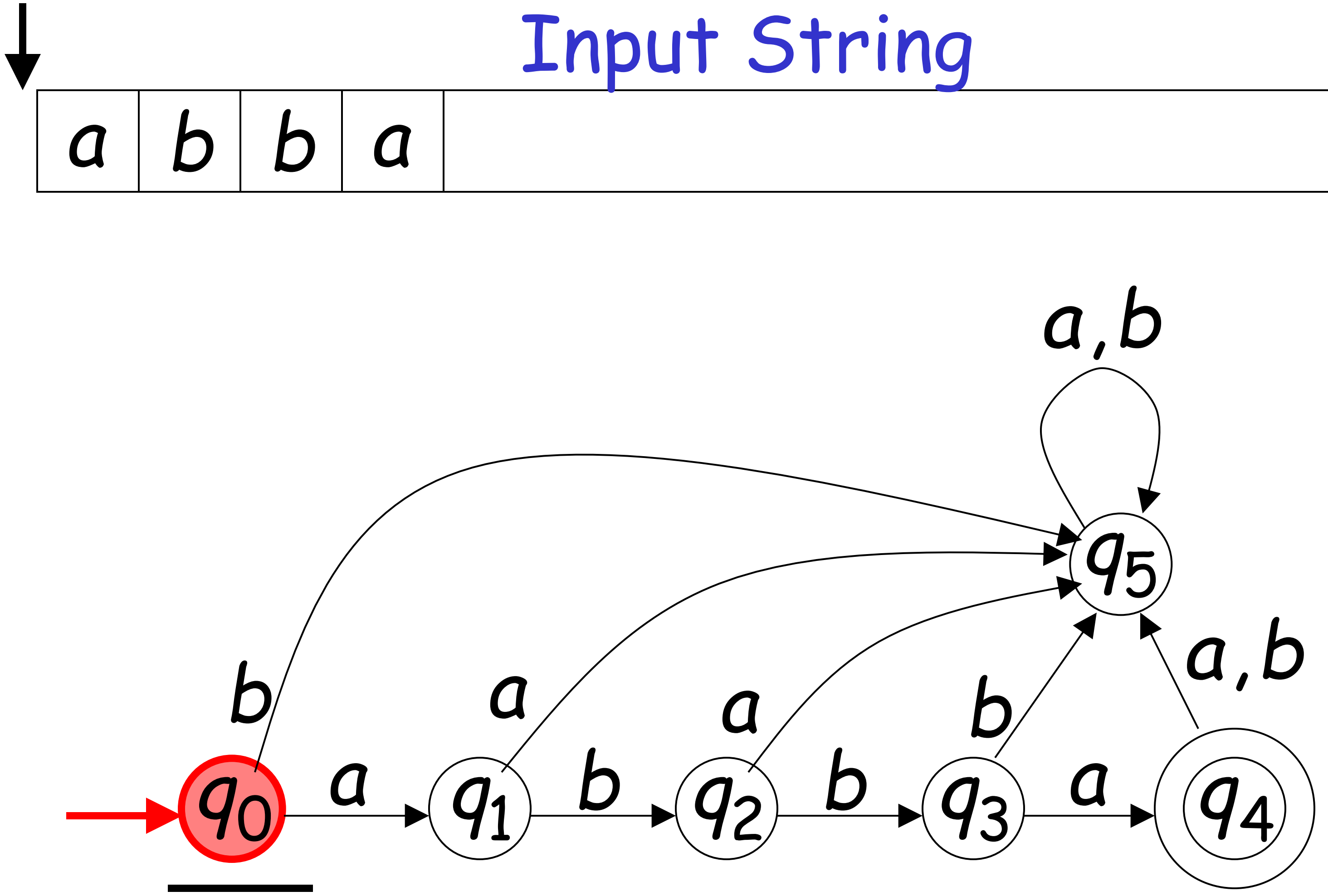
Finite Automaton



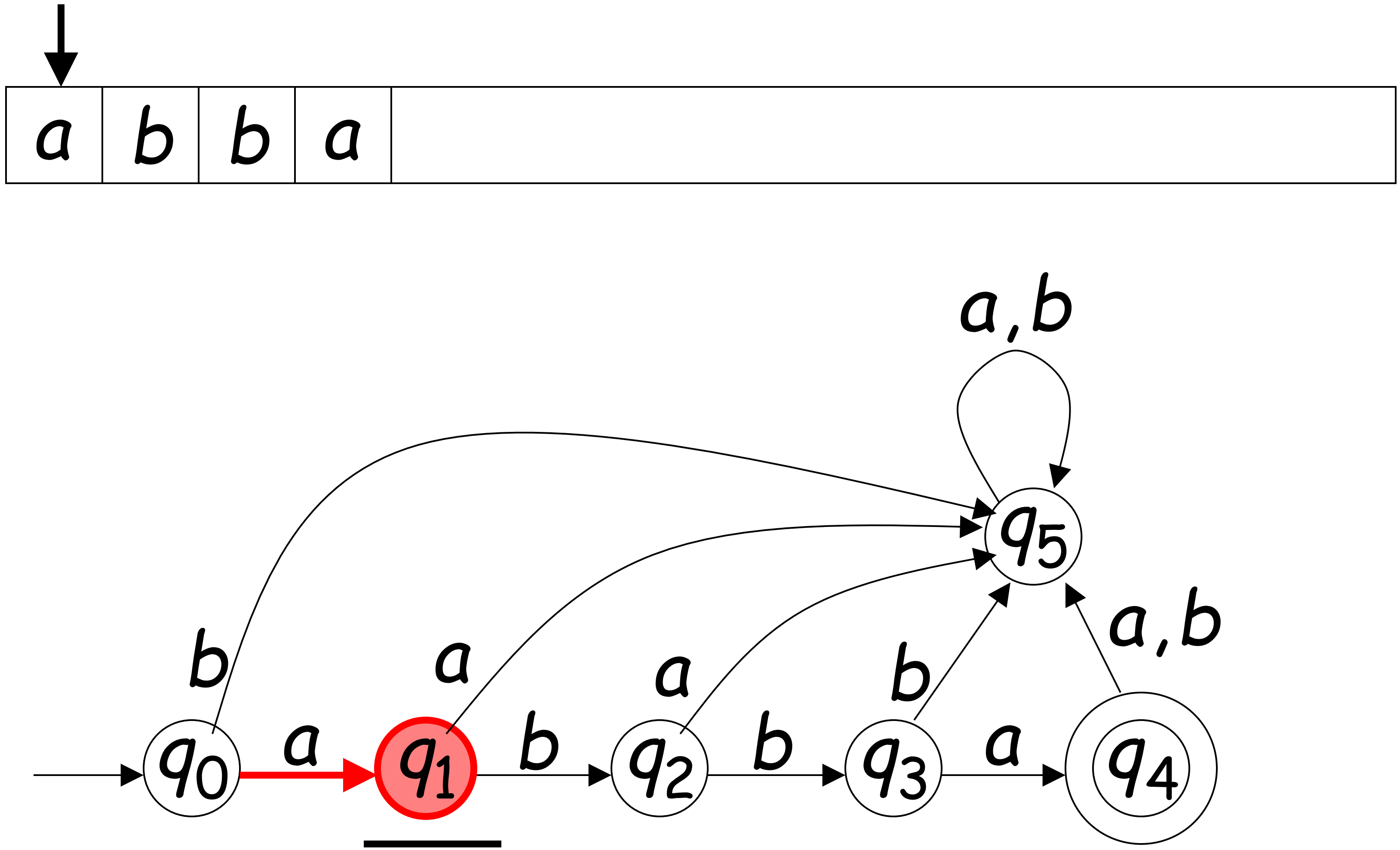
Transition Graph

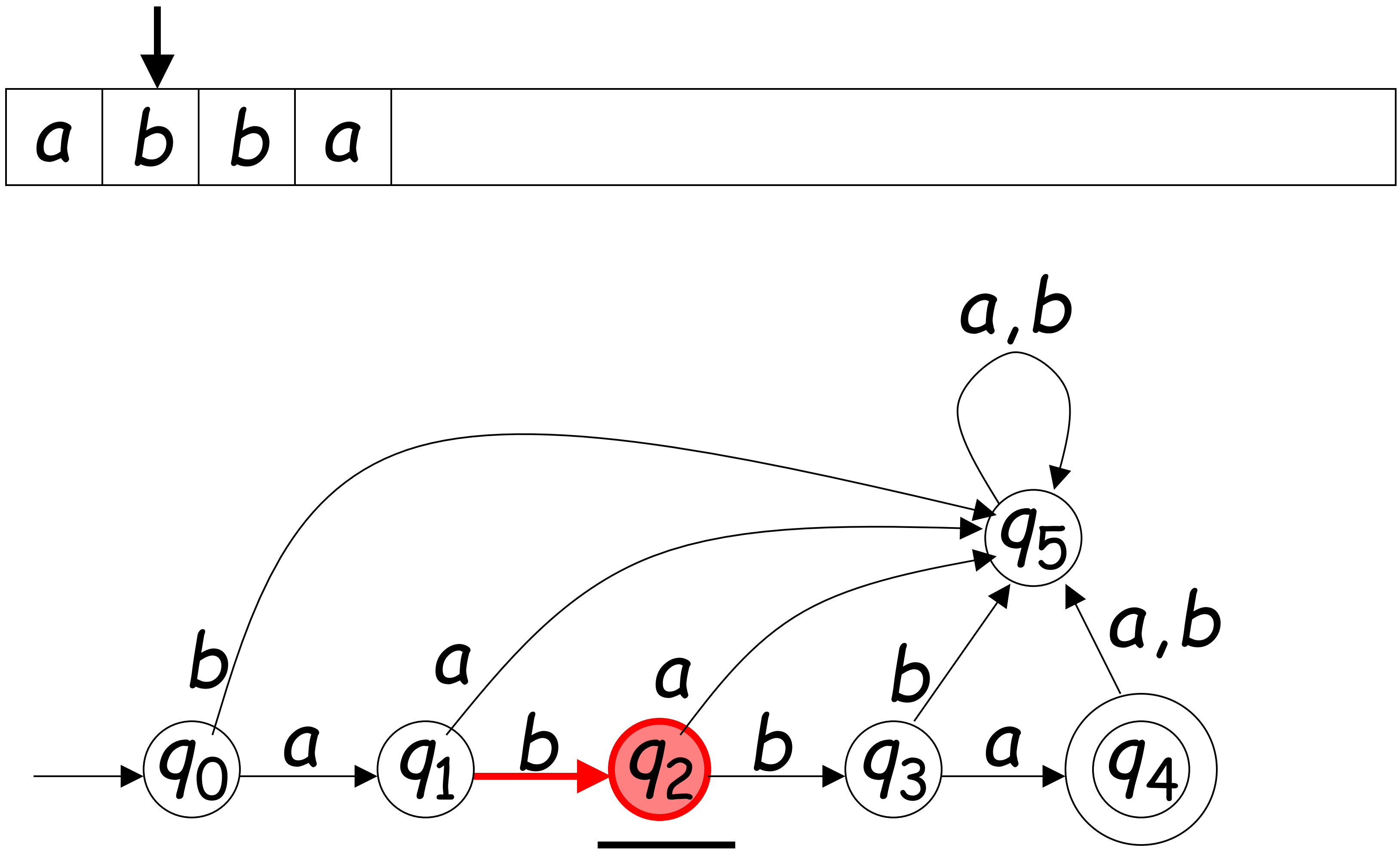


Initial Configuration

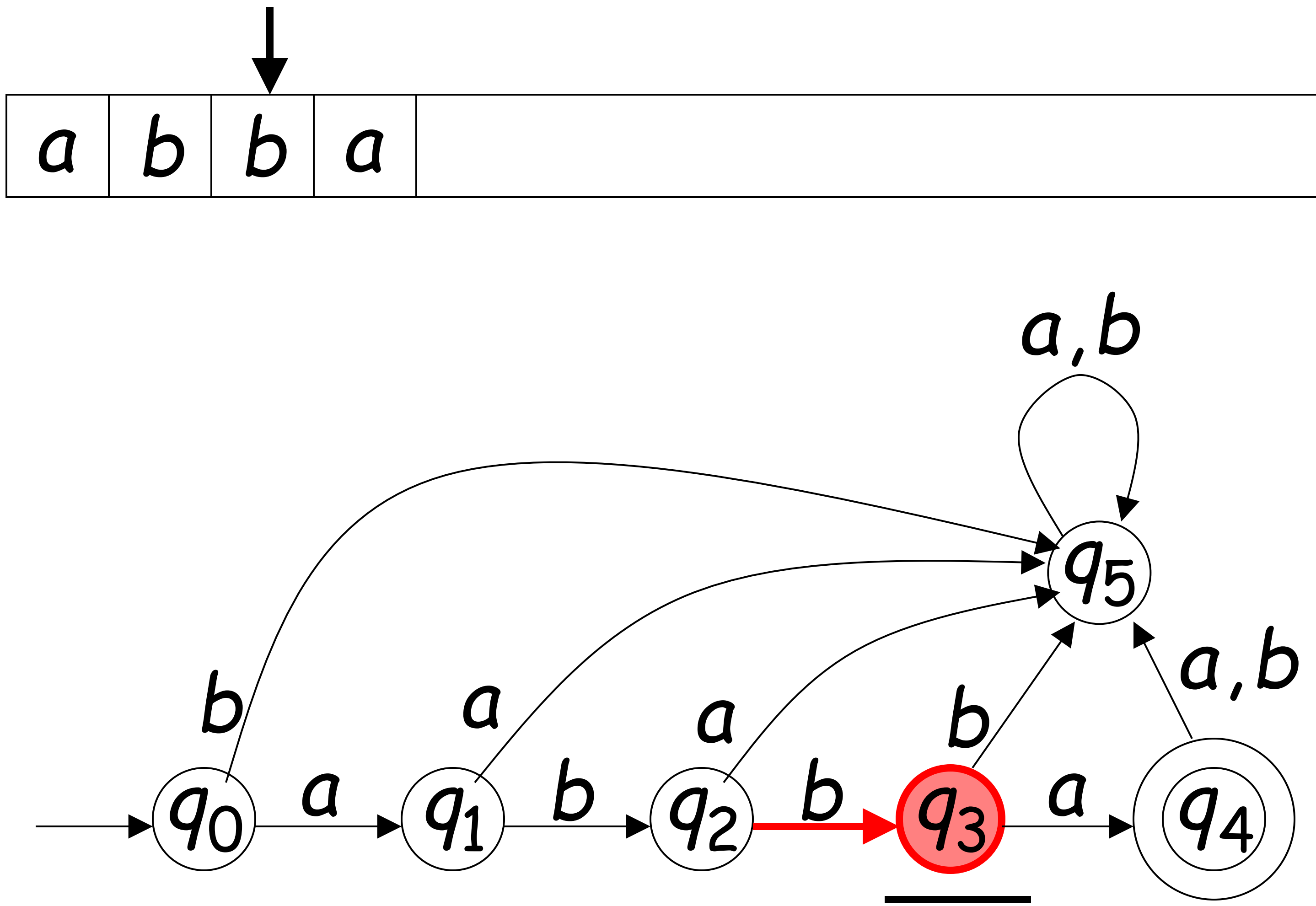


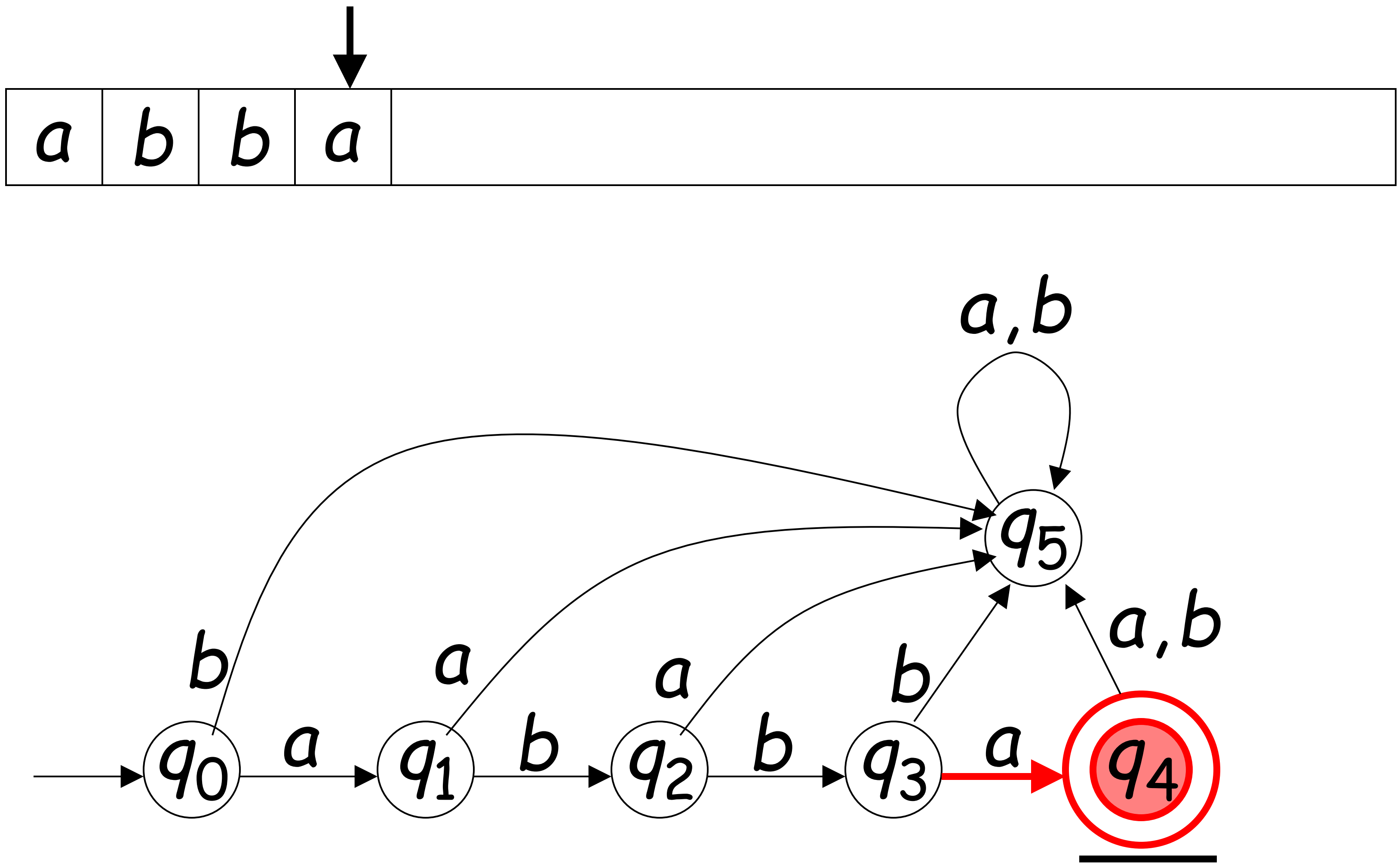
Reading the Input



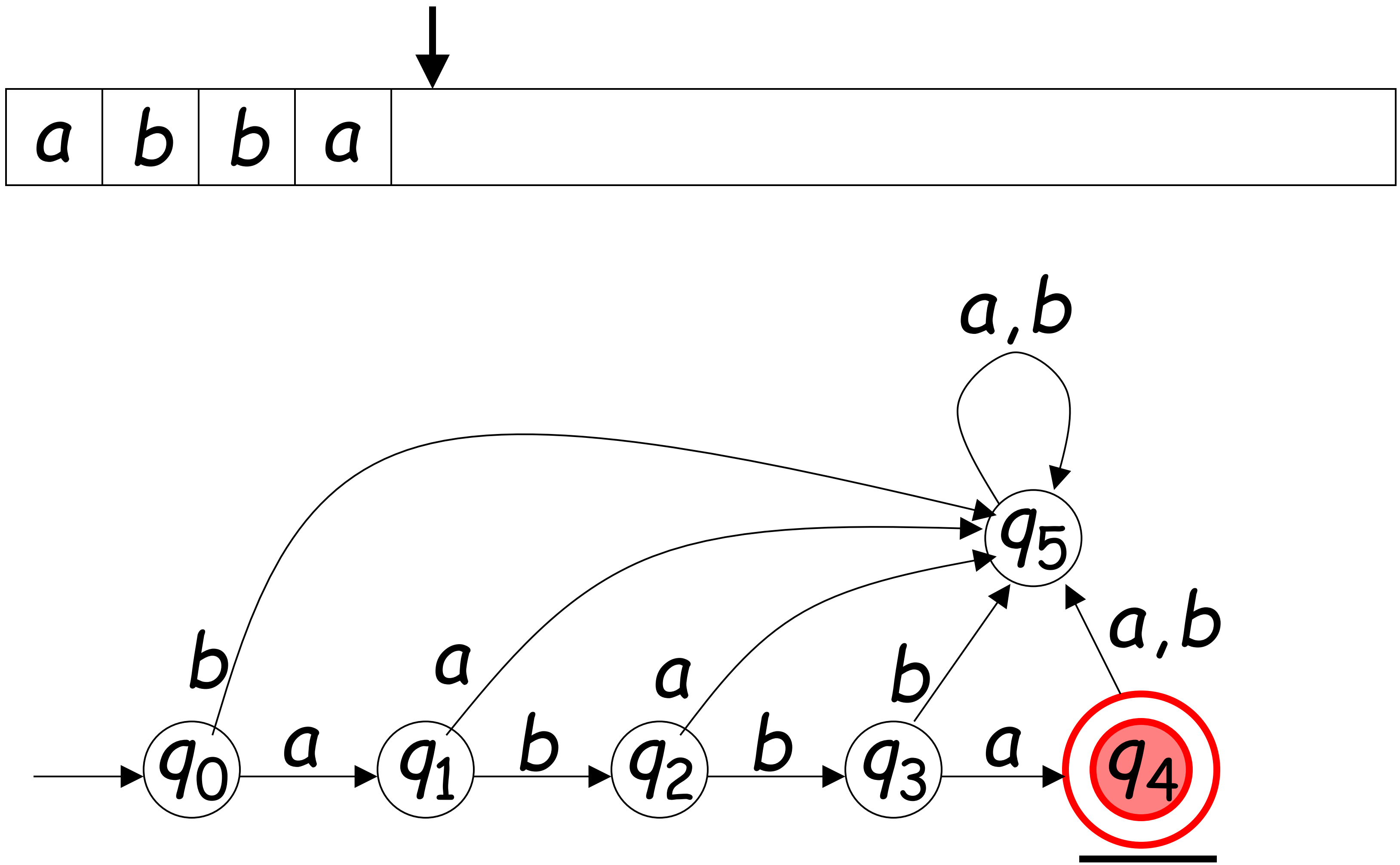


a
b
b
a
a,b
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4
q5


a b b a
a,b
q5
a,b
b
a
a
b
q0
a
q1
b
q2
b
q3
a
q4

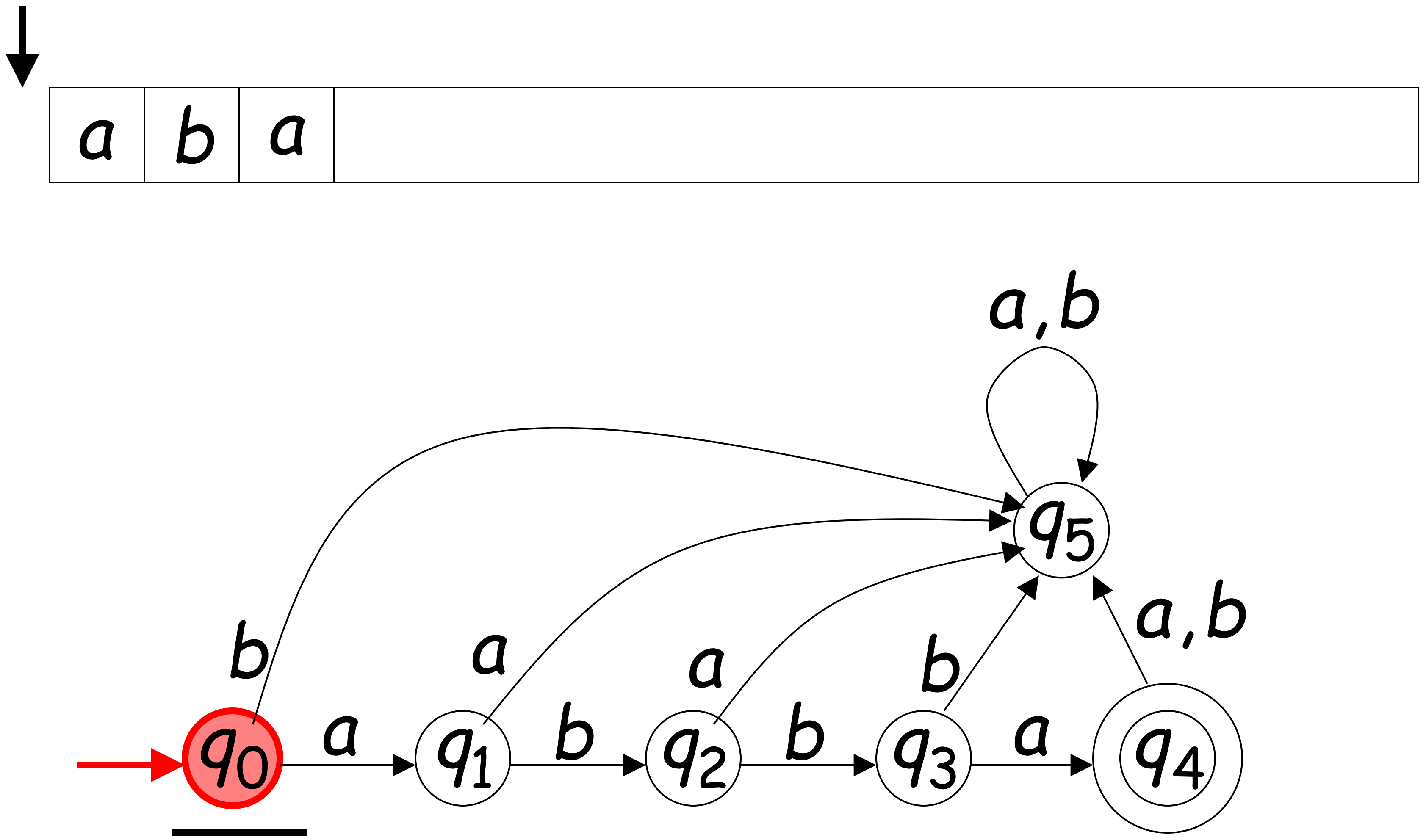

a
b
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

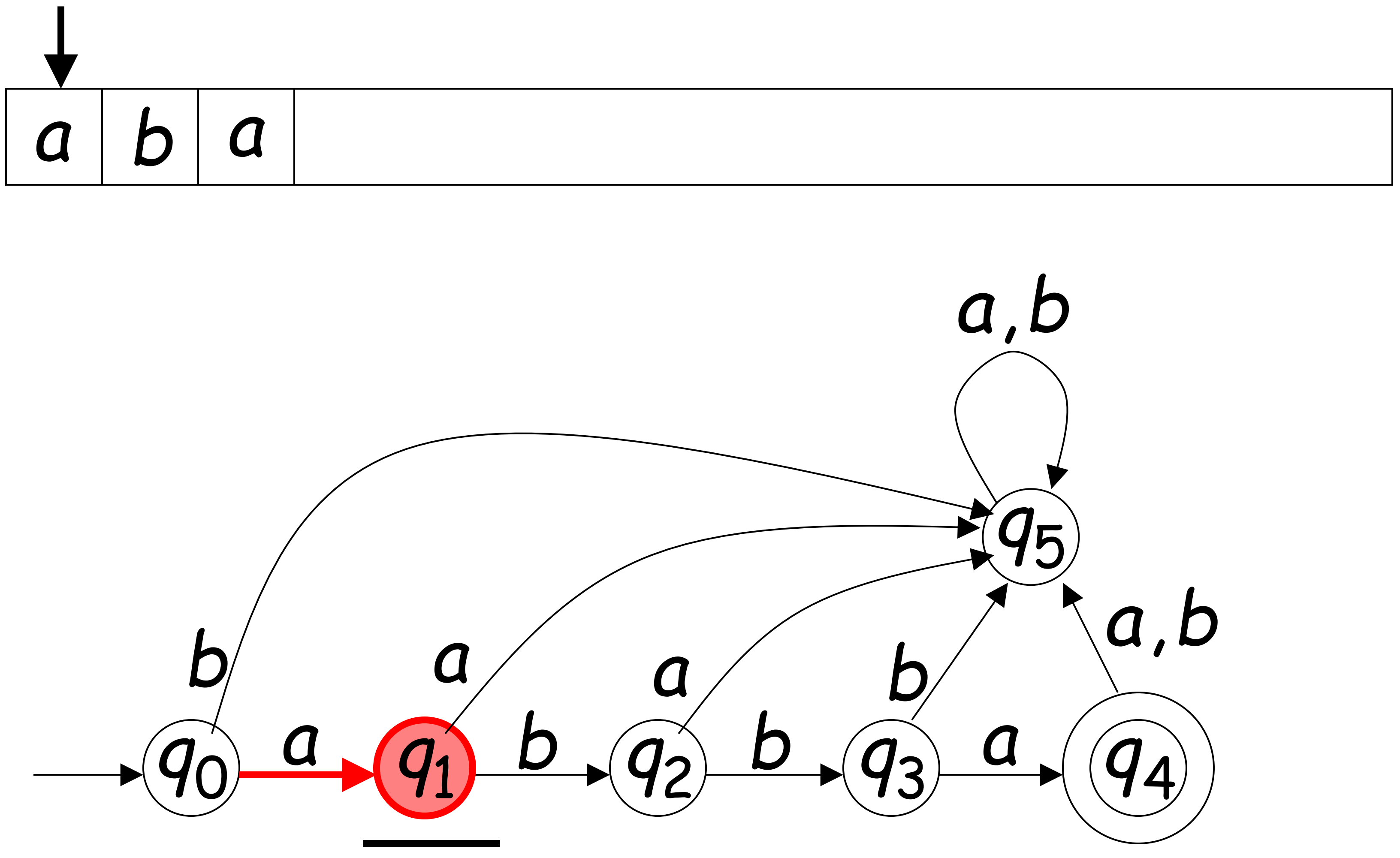
Input finished

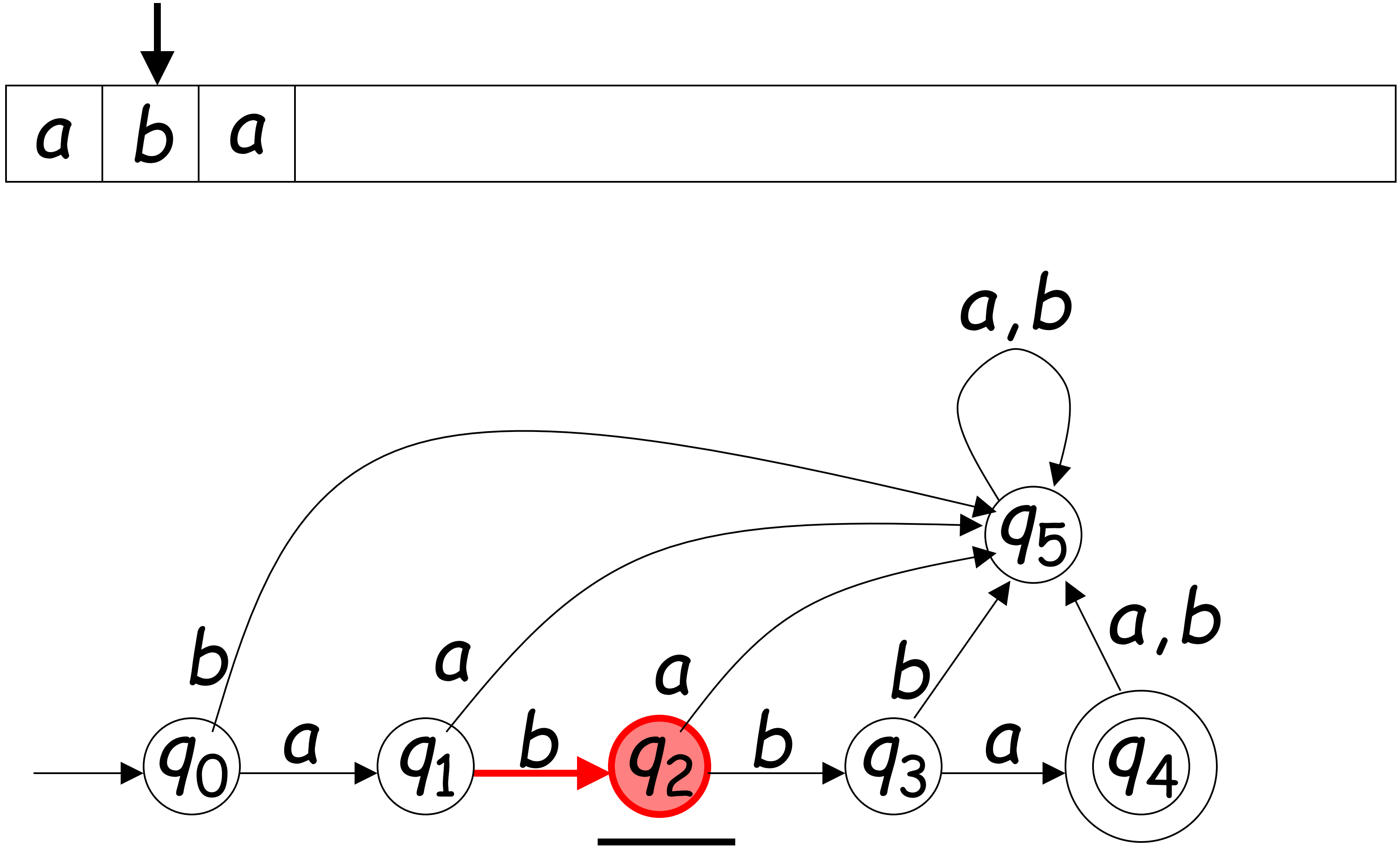


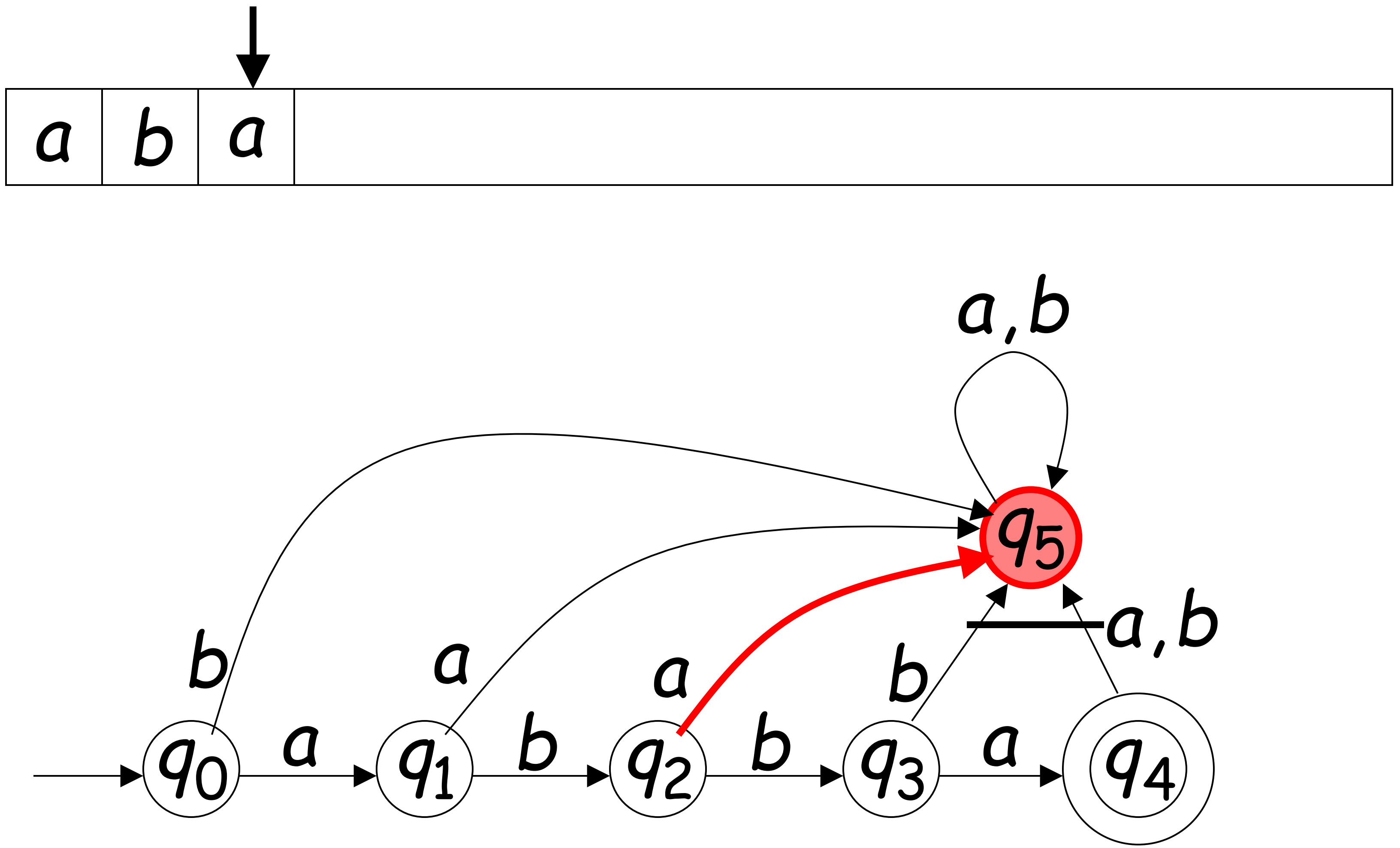
accept

Rejection

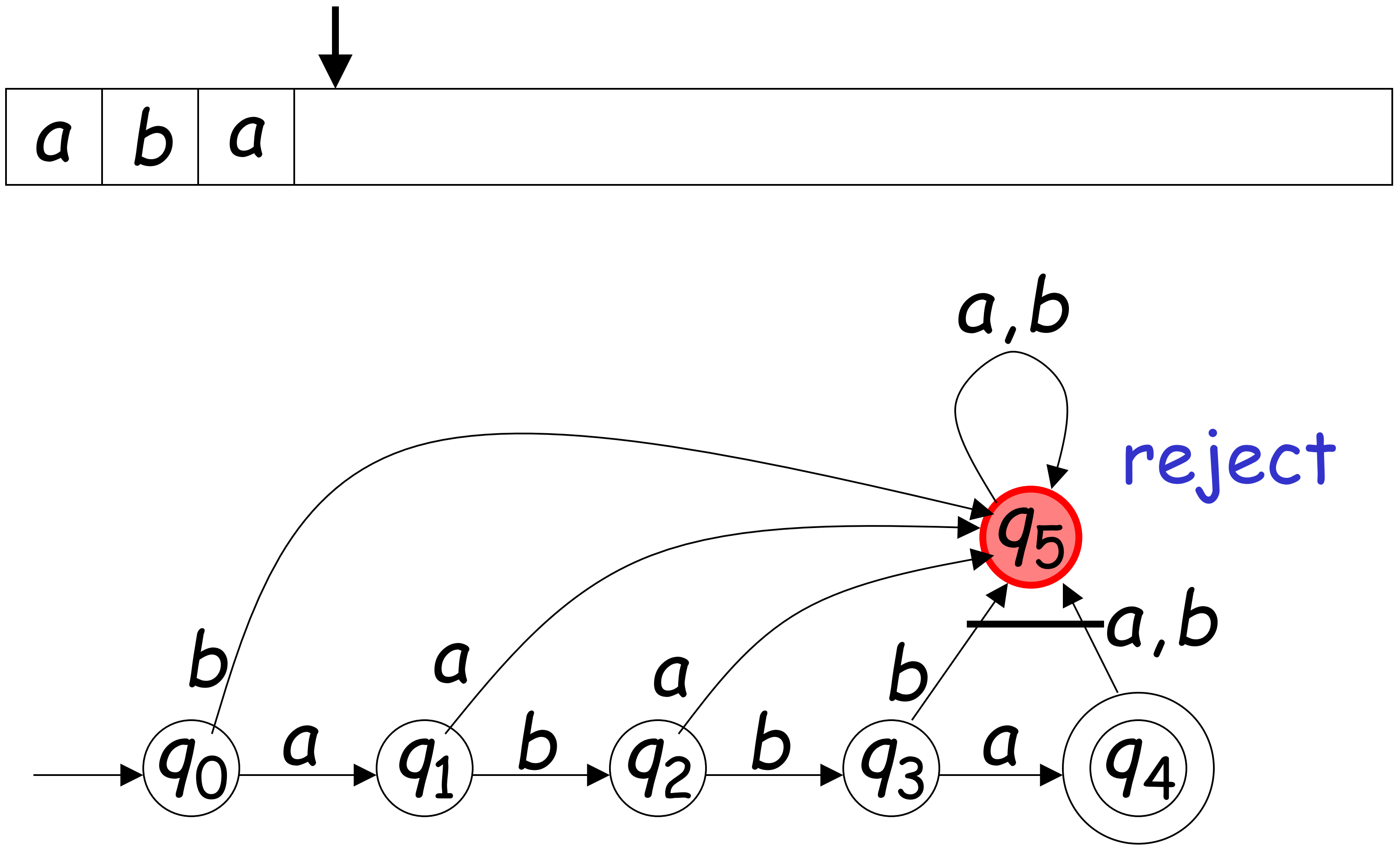



a
b
a
a,b
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4
q5


a
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

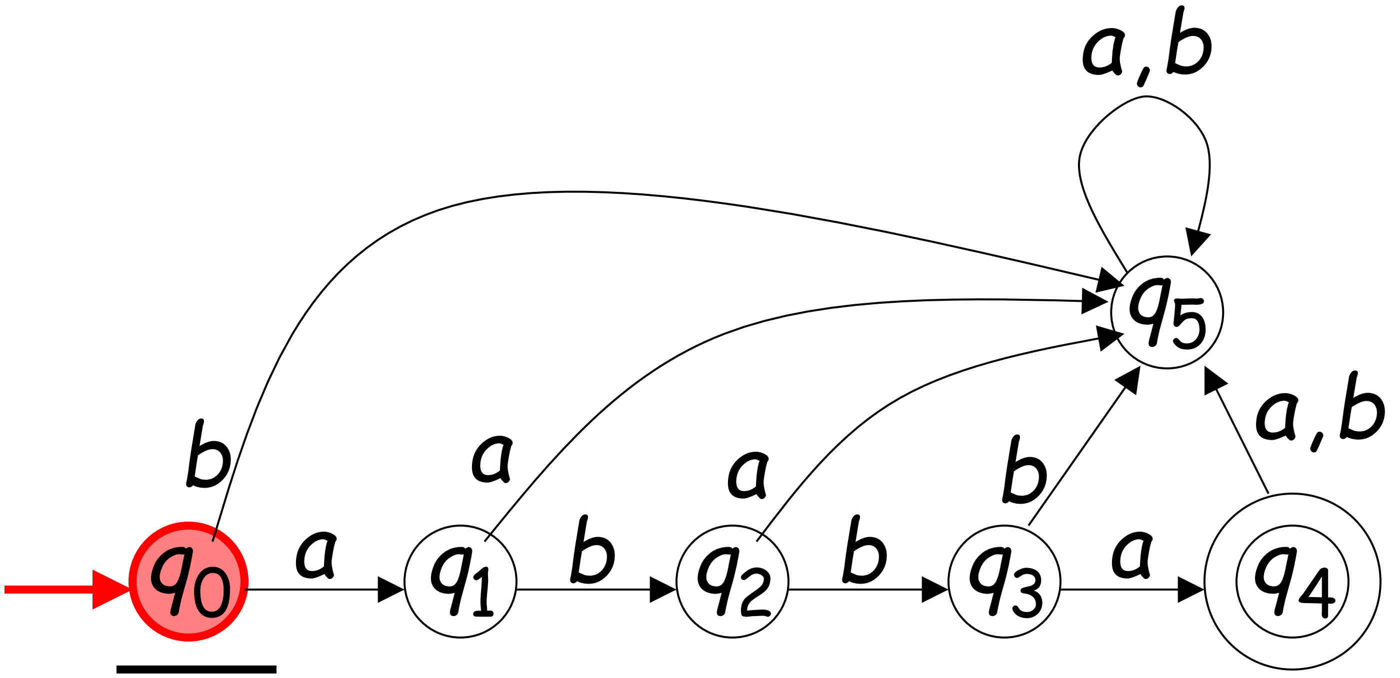

a
b
a
a,b
b
a
a
b
a,b
b
a
b
b
a
q_0
q_1
q_2
q_3
q_4
q_5

Input finished

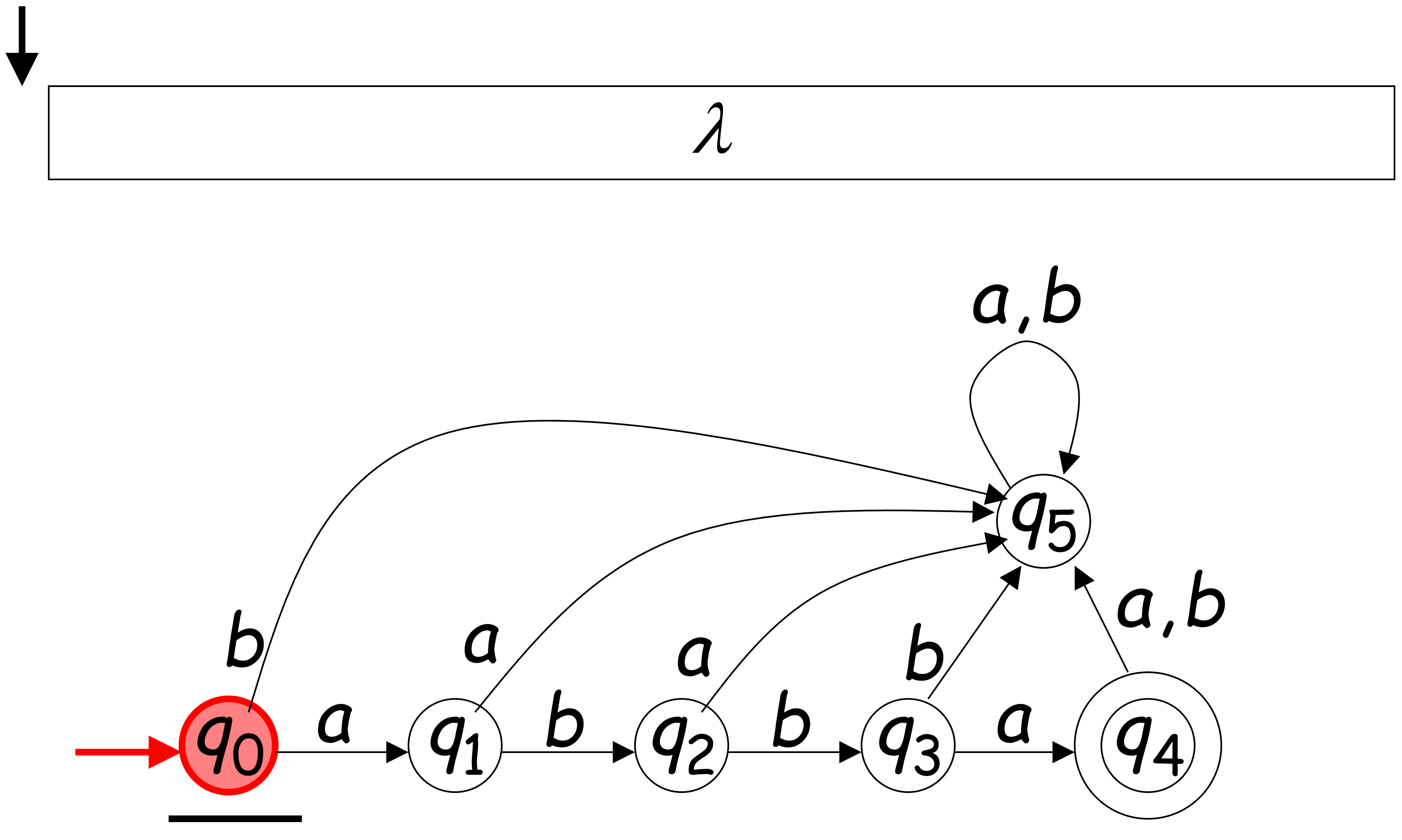


Acceptance or Rejection?

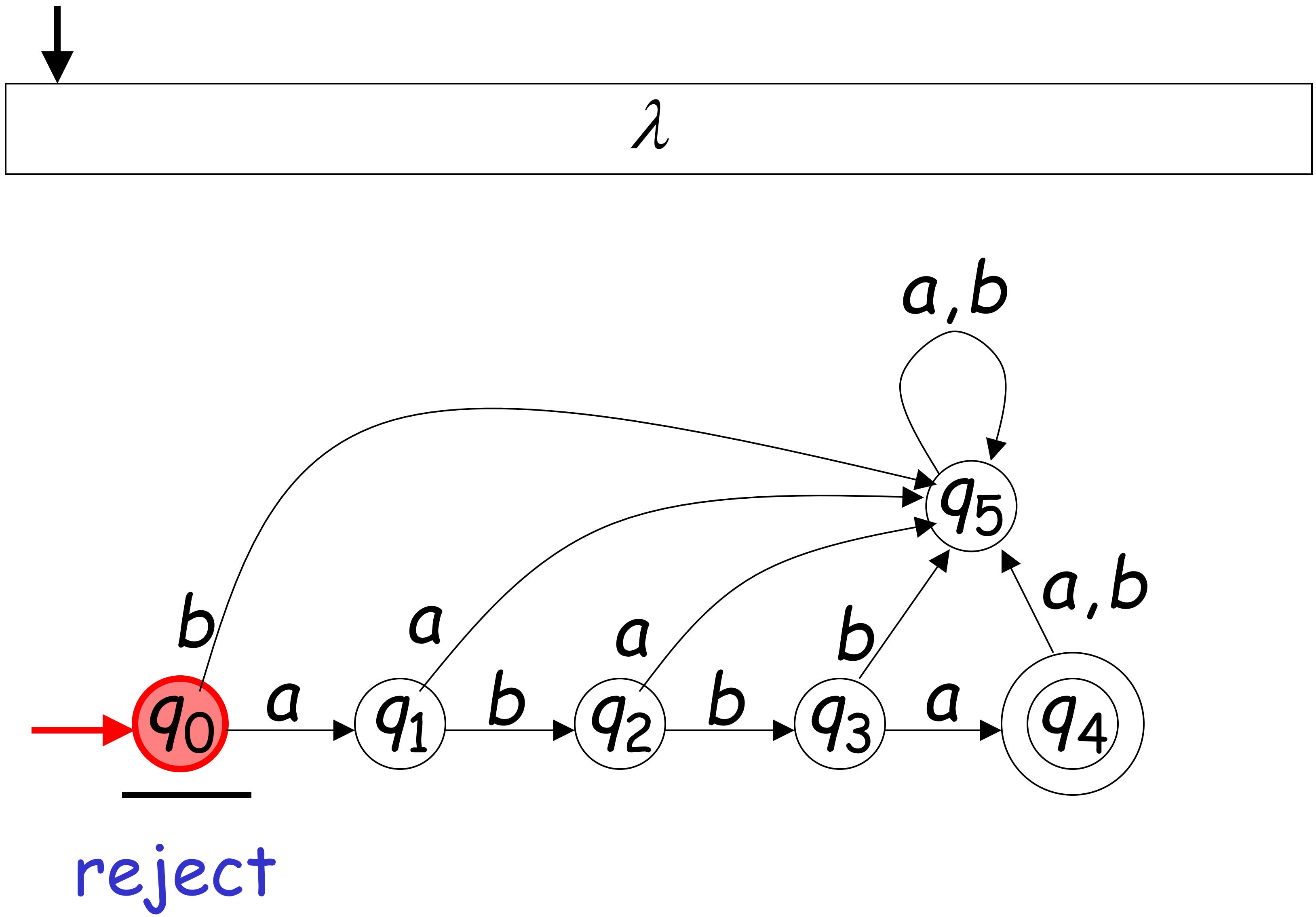
λ



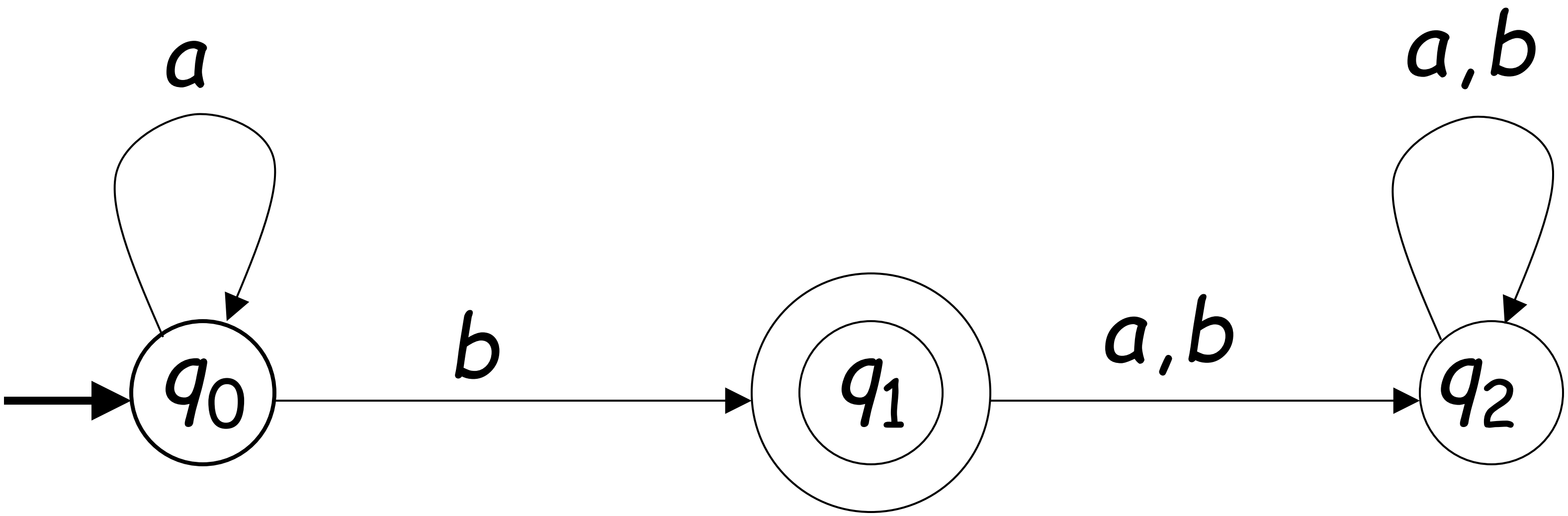
Initial State



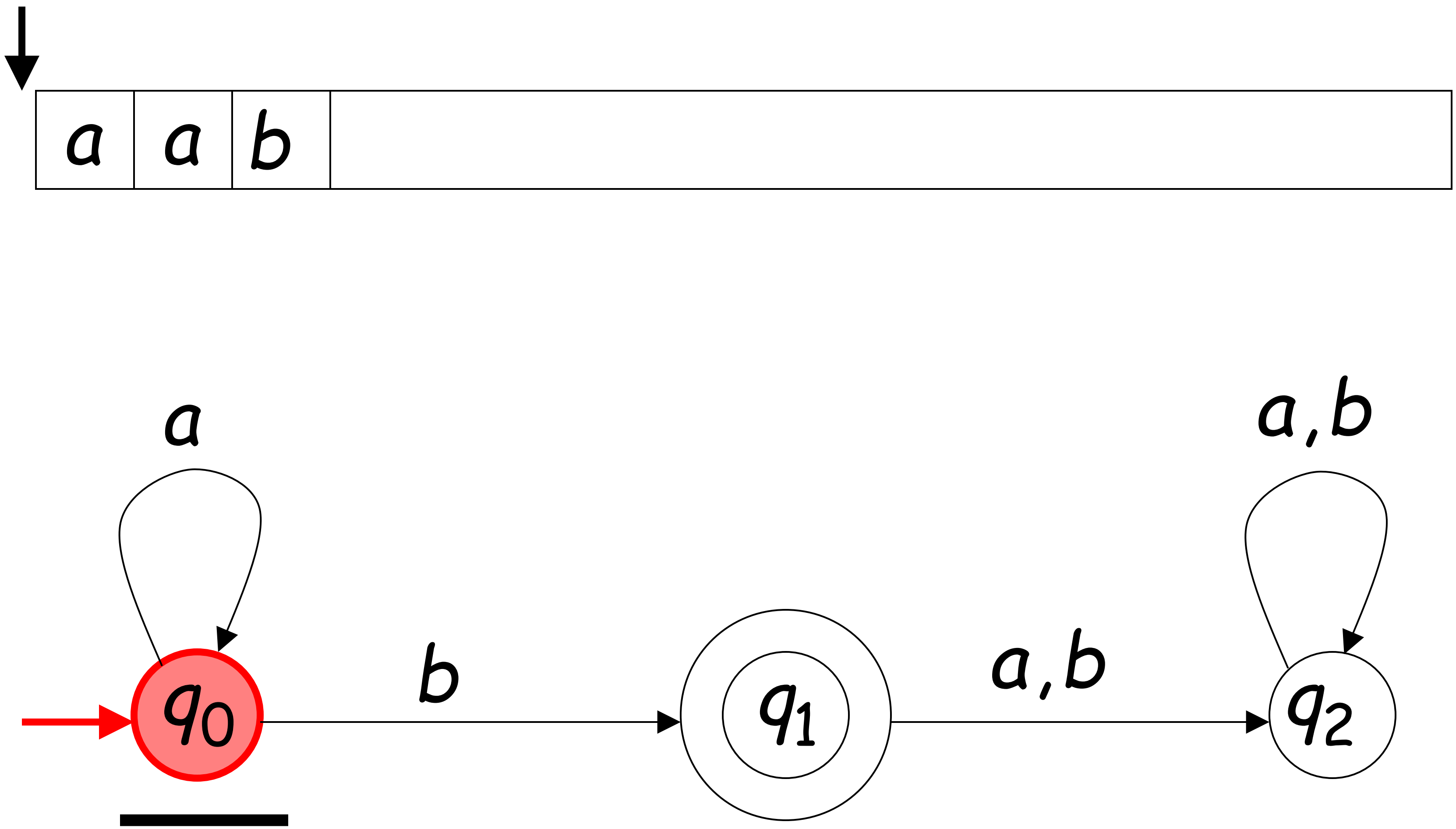
Rejection

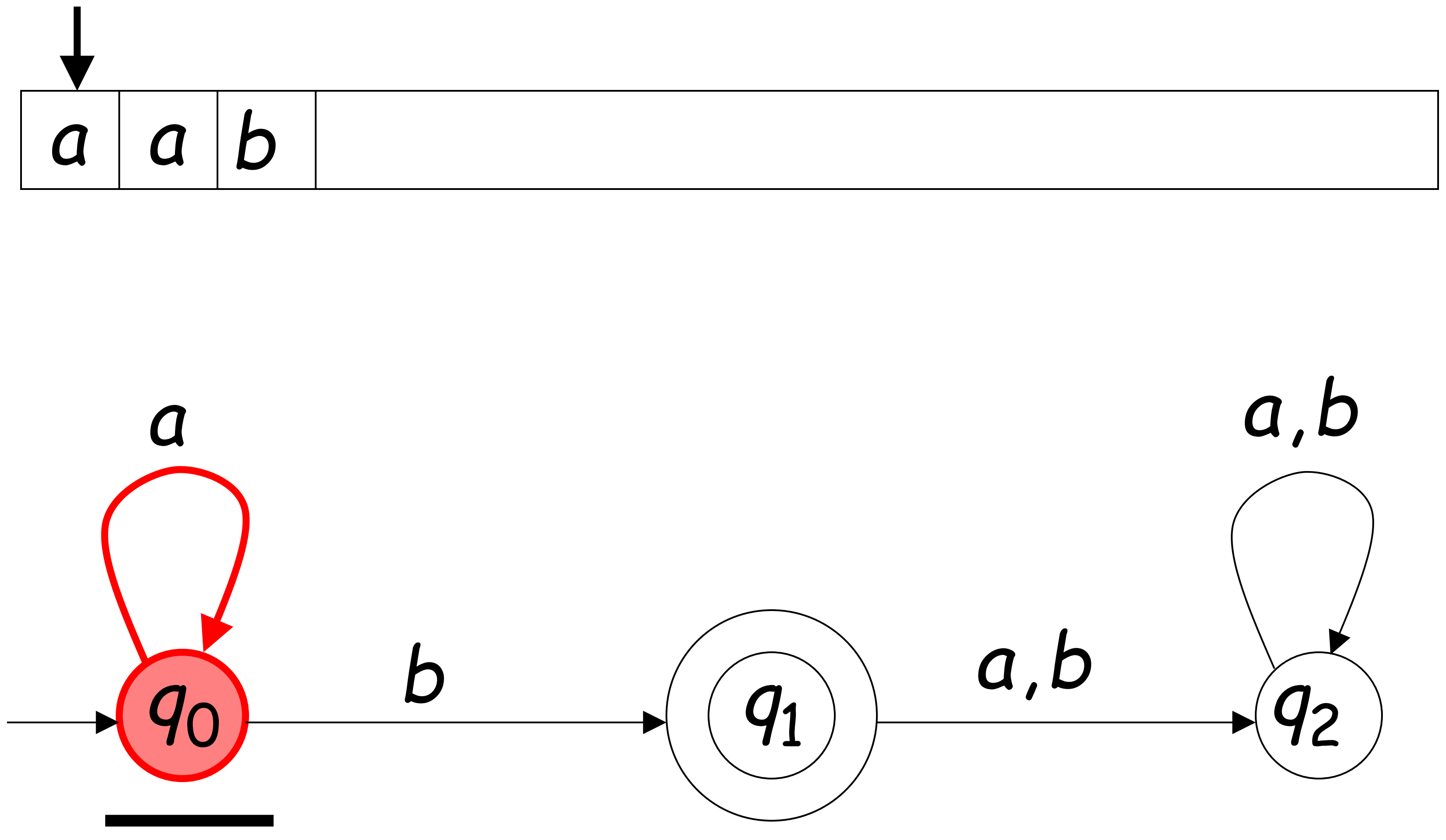


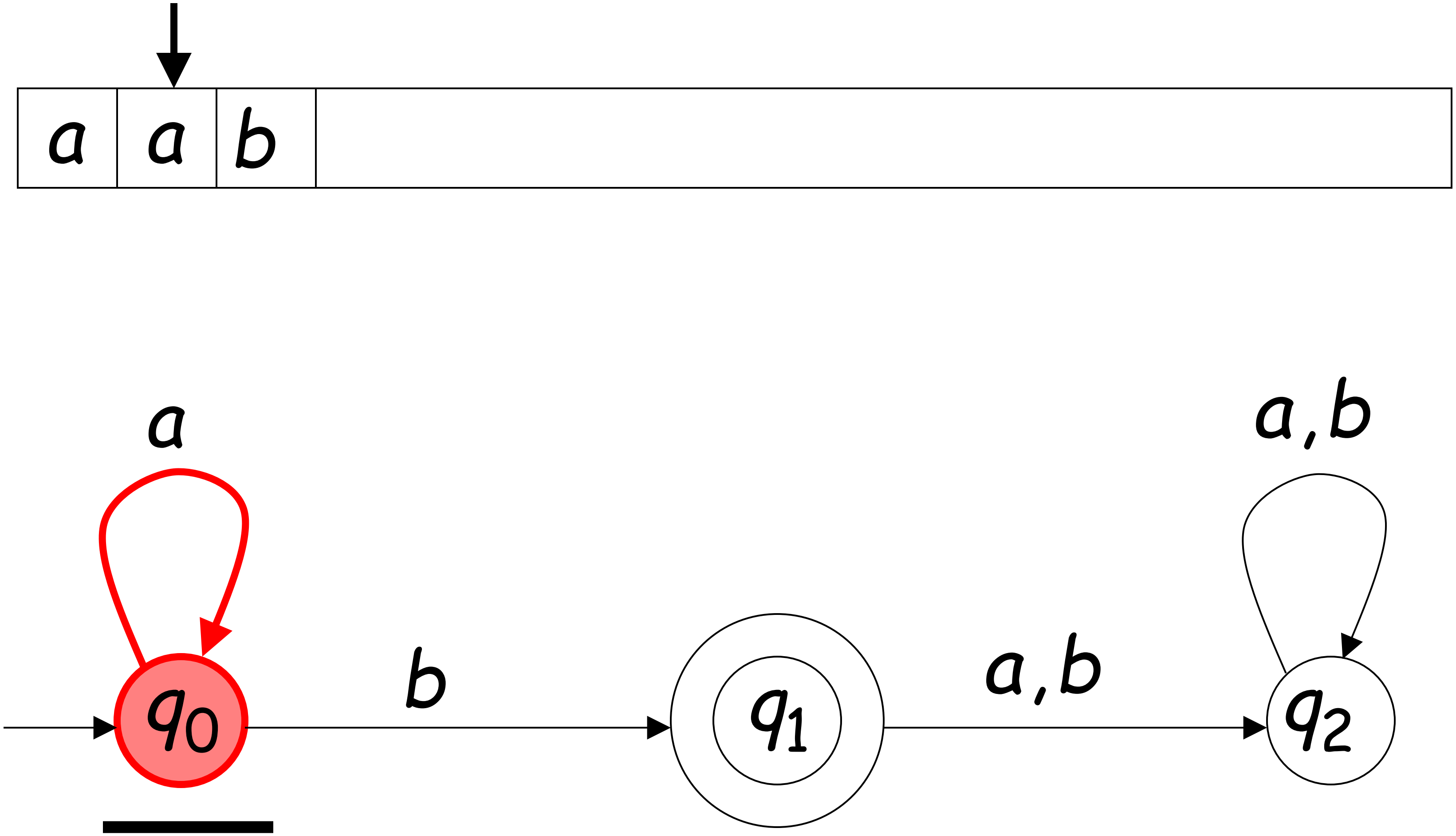
Language?

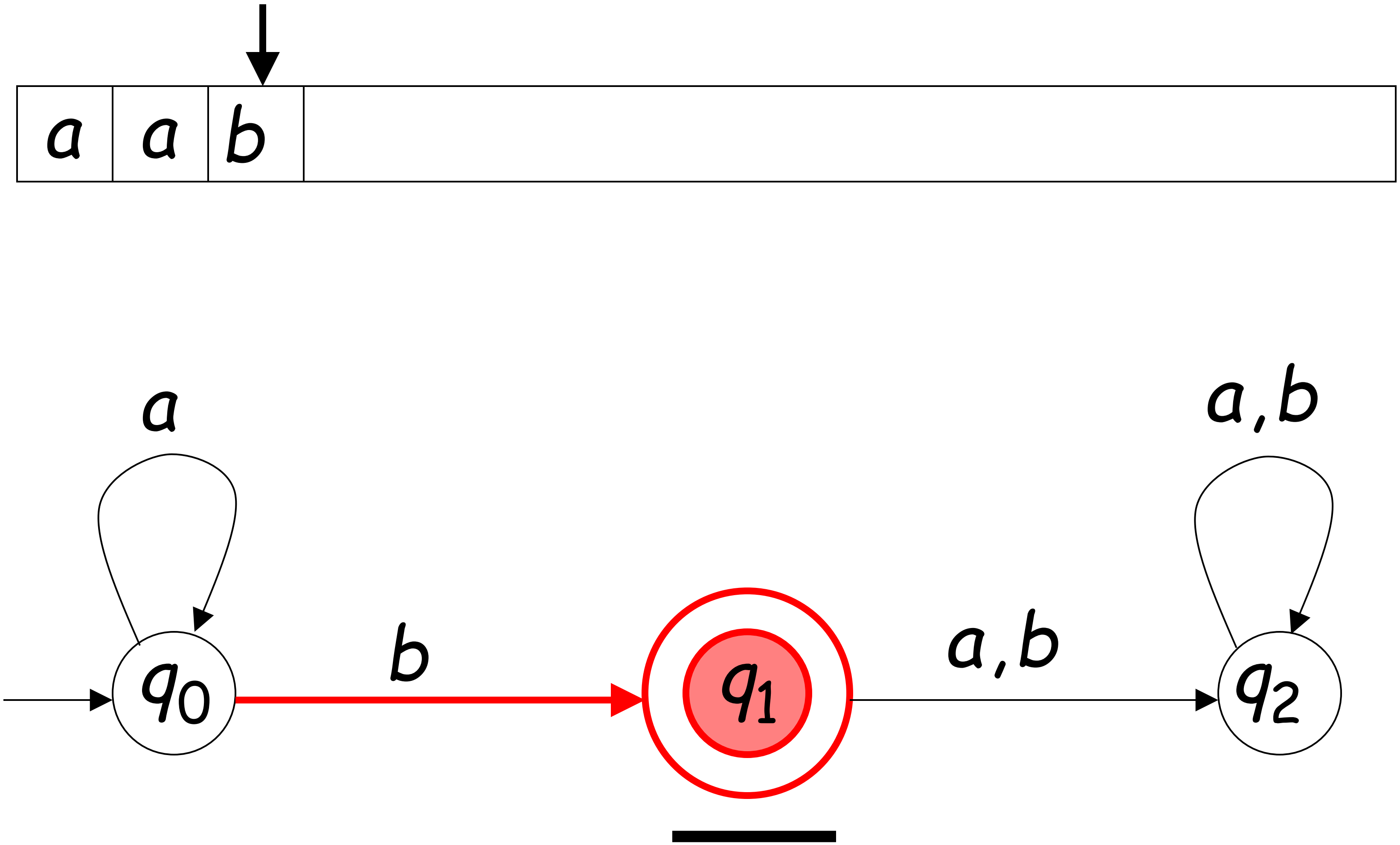


Another Example

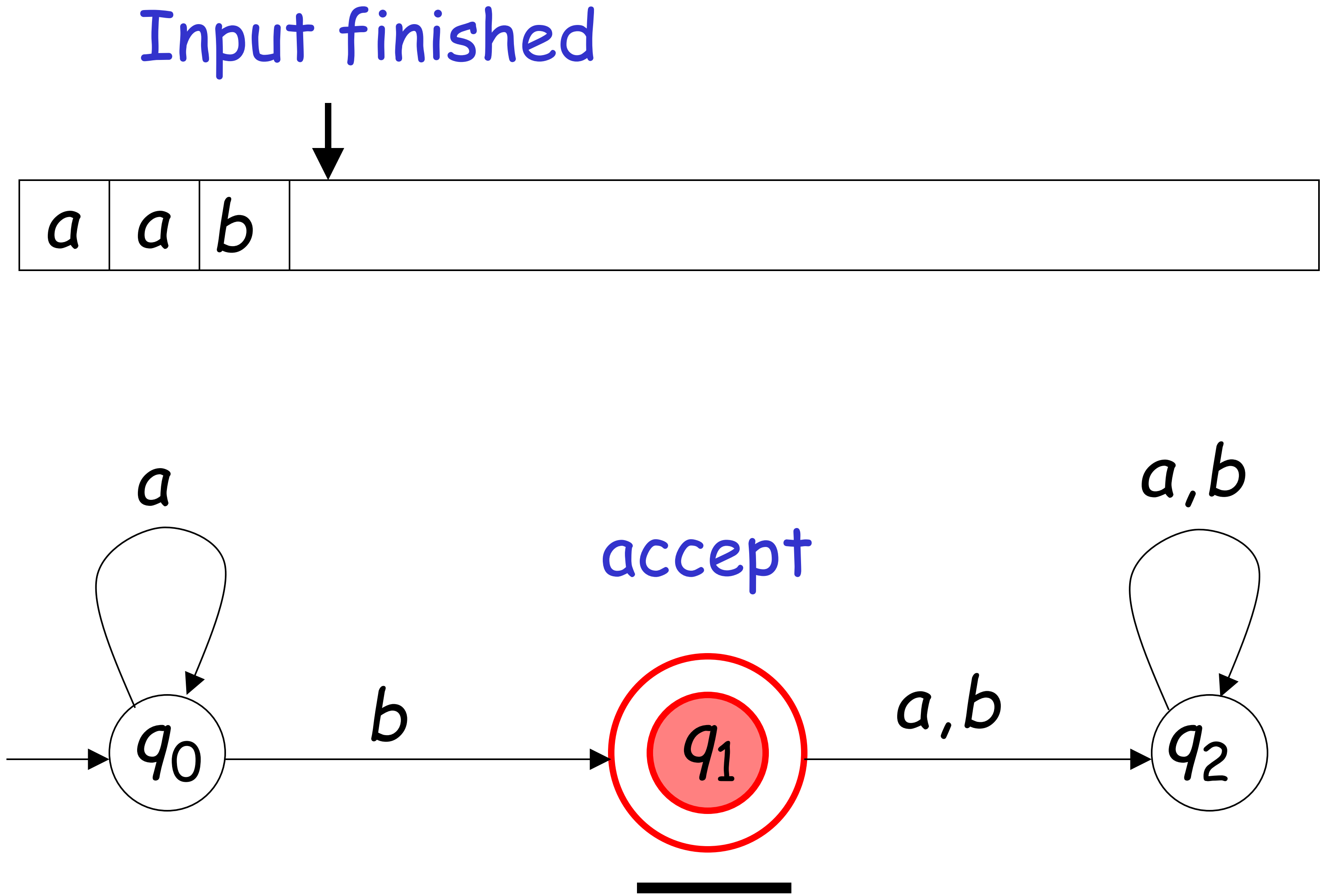



a
a
b
a
a,b
b
q0
q1
a,b
q2


a a b
a
b
a,b
a,b
q_0
q_1
q_2

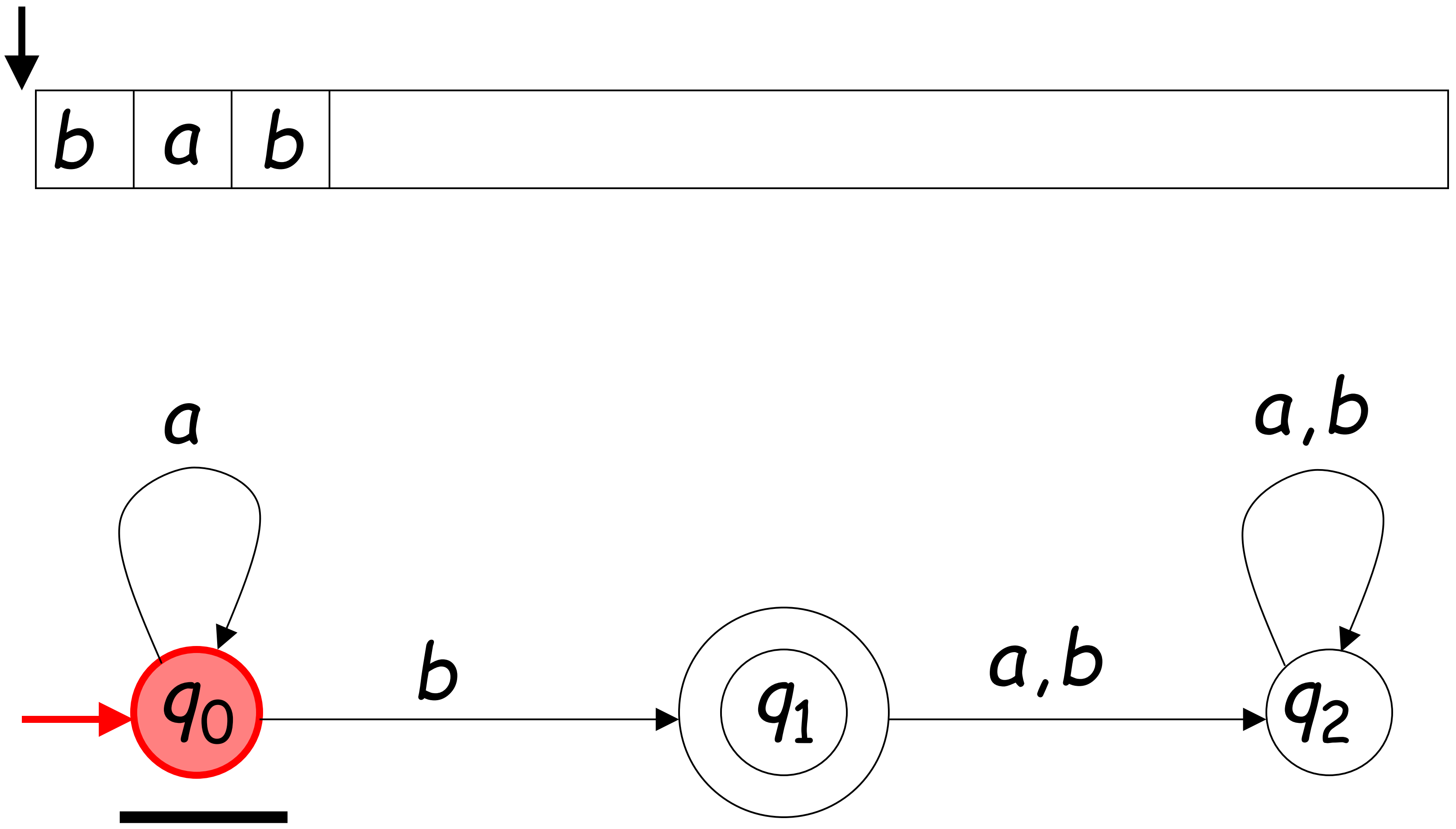


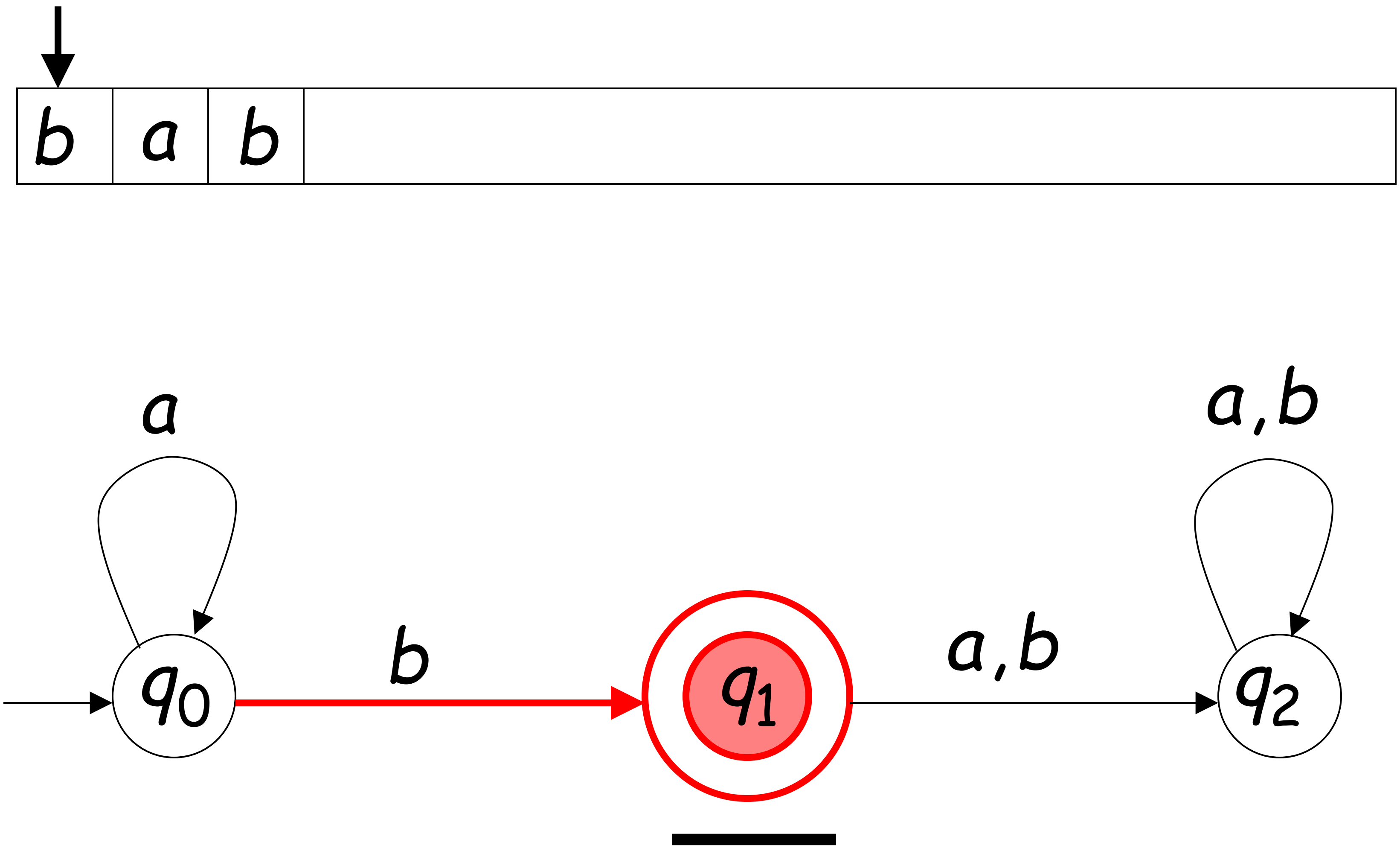
a
a
b
a
a,b
b
q_0
q_1
a,b
q_2



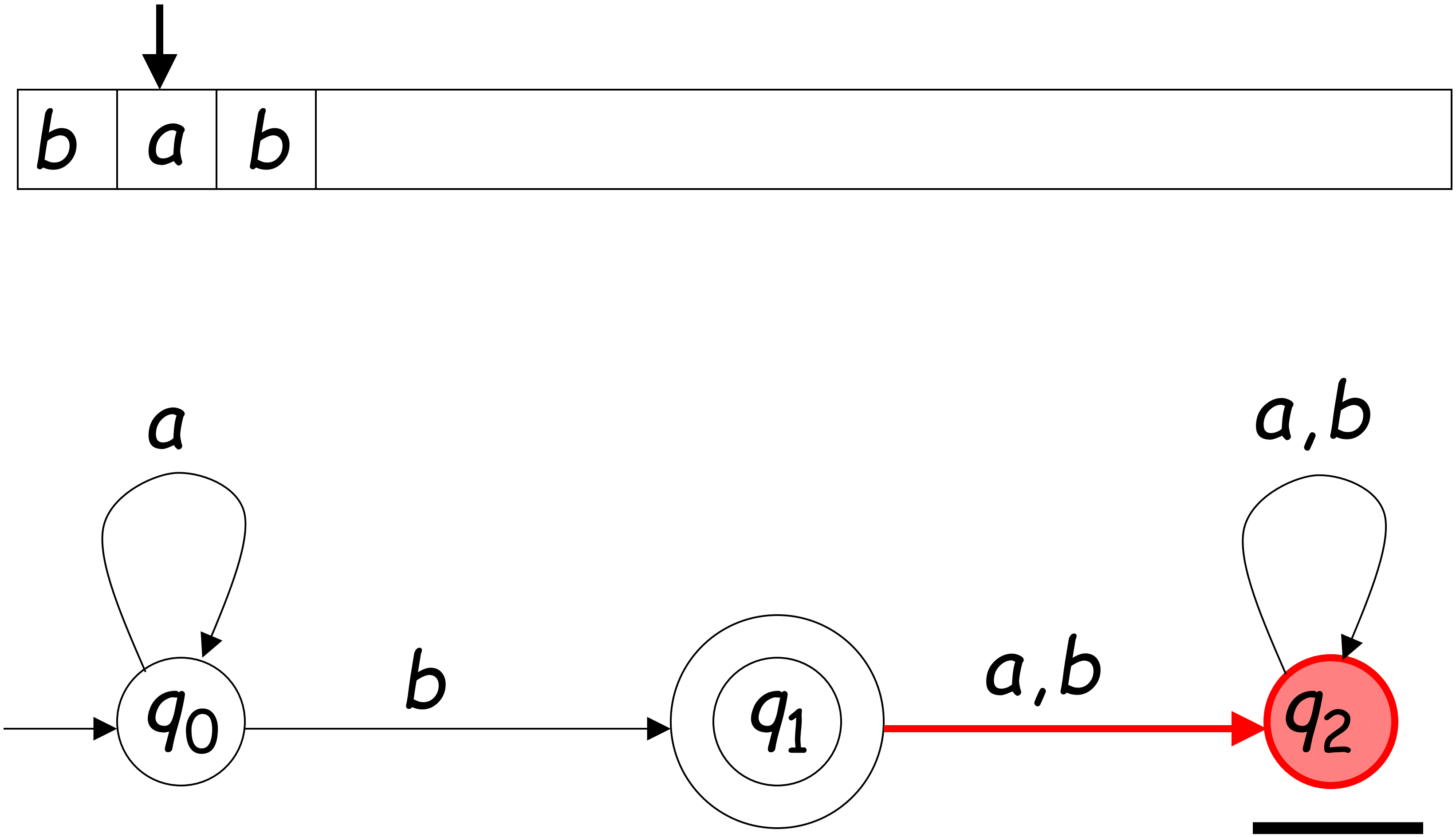
Input finished
a
a
b
a
a,b
accept
b
a,b
q_0
q_1
q_2

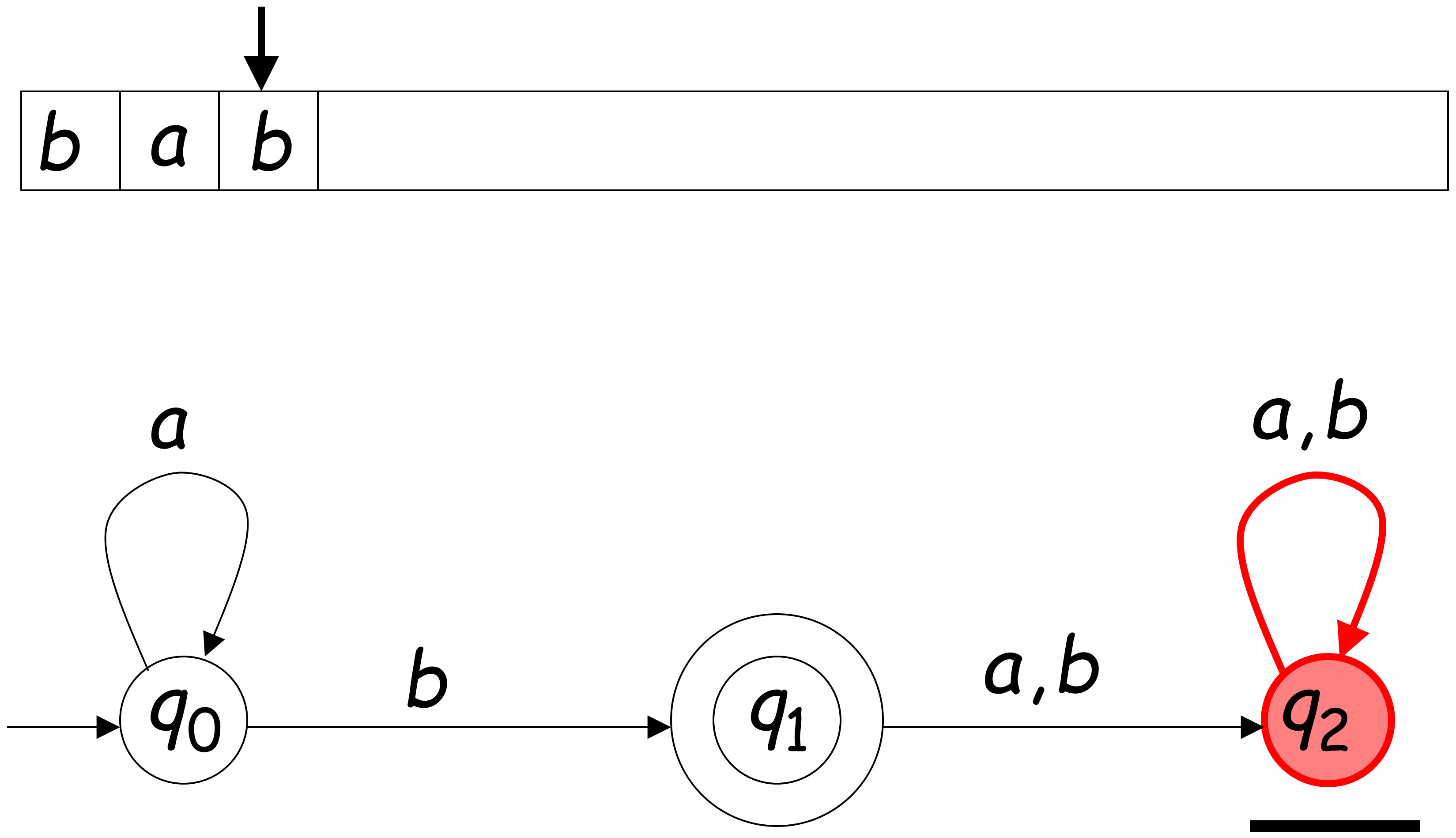
Rejection Example



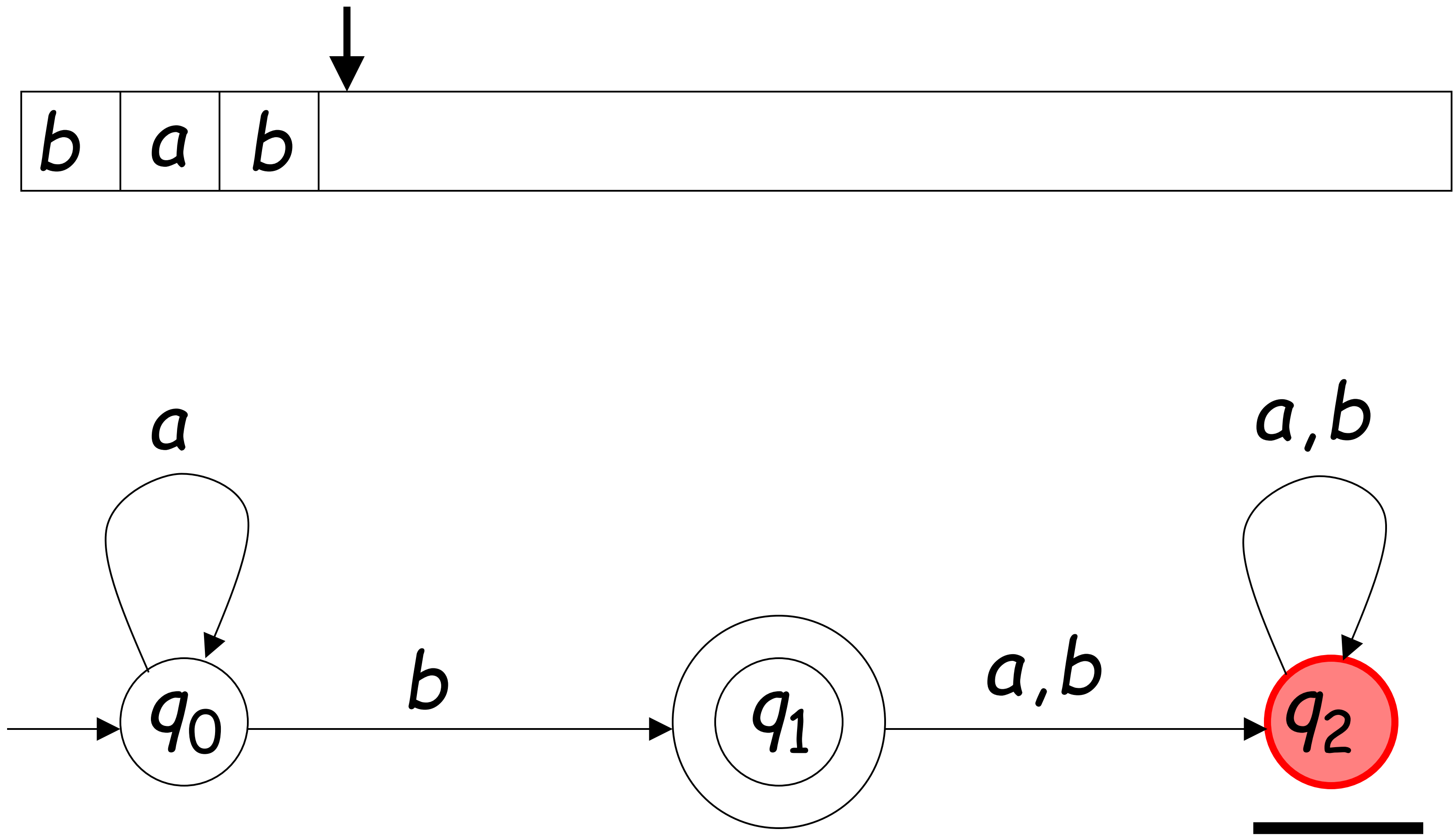


b
a
b
a
a,b
b
a,b
q_0
q_1
q_2


b
a
b
a
a,b
b
q0
q1
a,b
q2


b
a
b
a
a,b
b
a,b
q_0
q_1
q_2

Input finished



reject

Languages Accepted by FAs

FA M

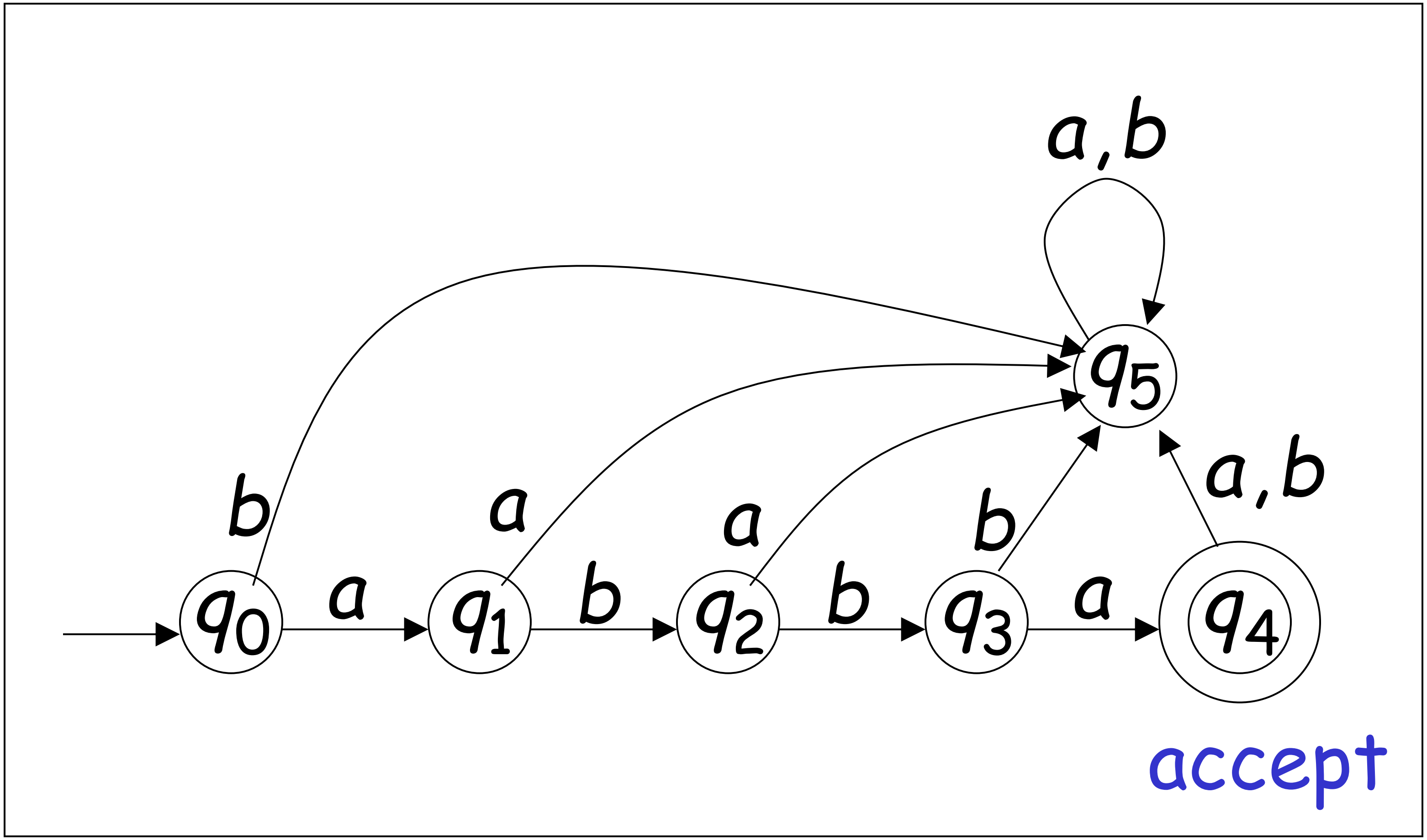
Definition:

The language $L(M)$ contains all input strings accepted by M

$L(M)$ = { strings that bring M to an accepting state}

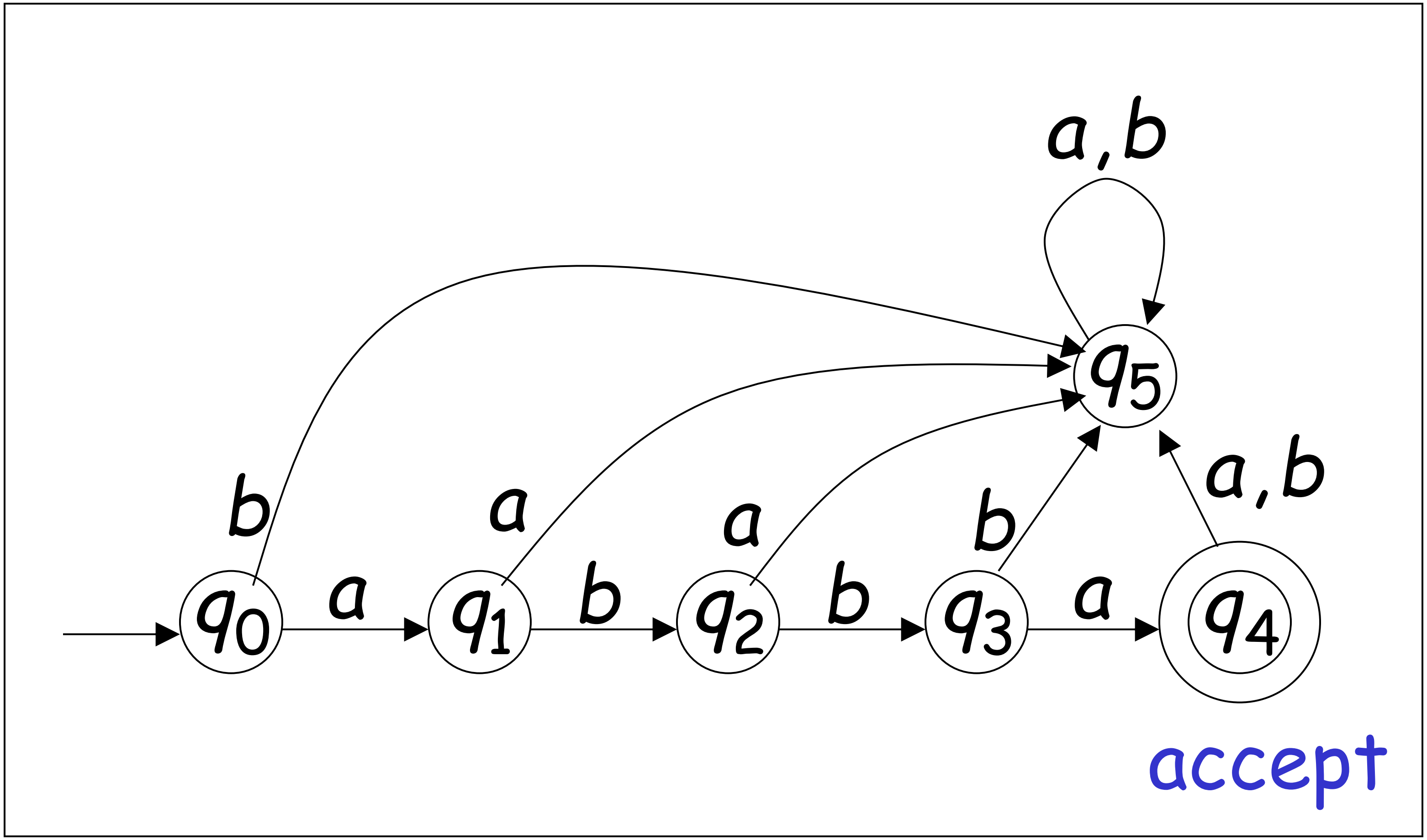
Example: L(M) = ?

M

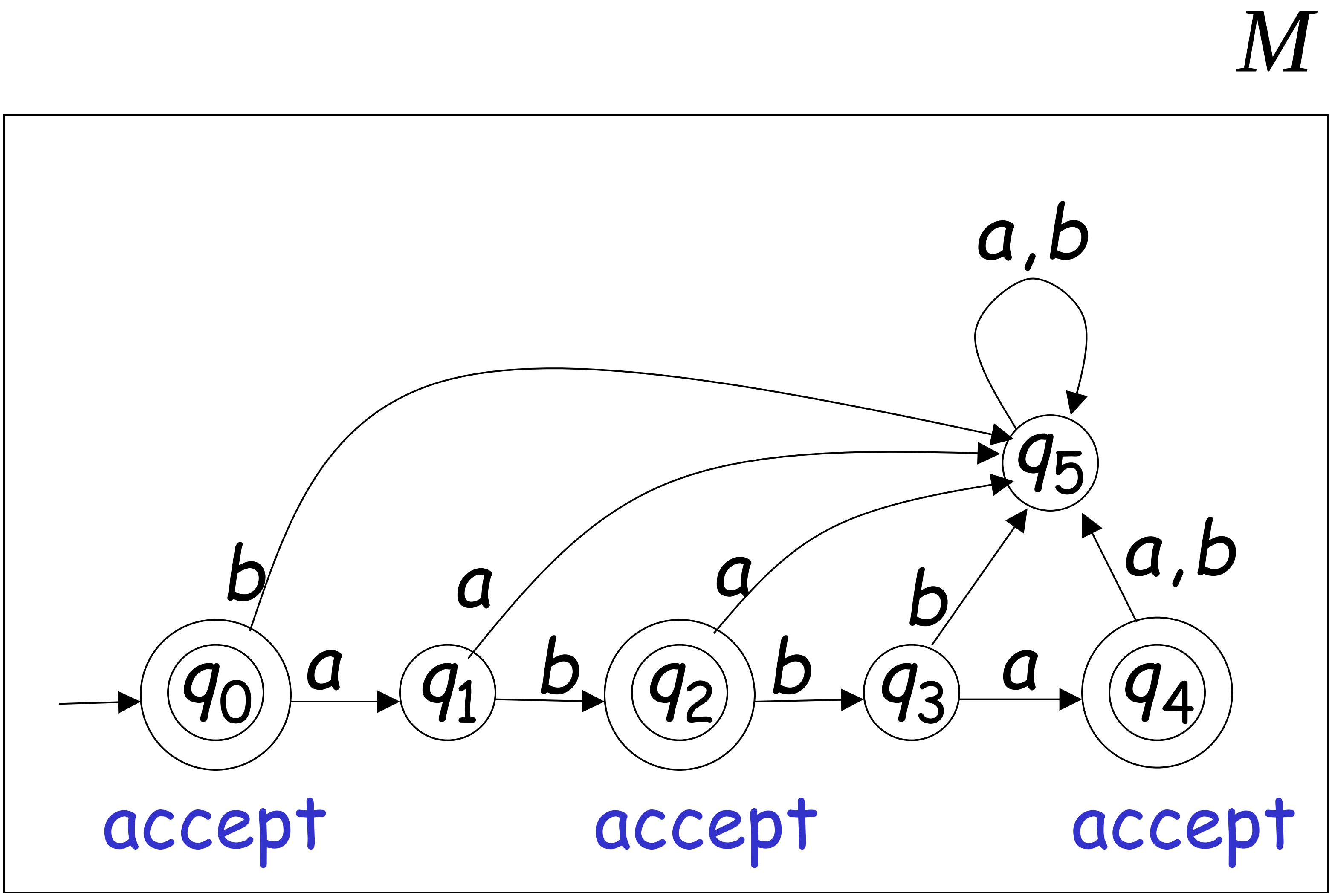


Example

M



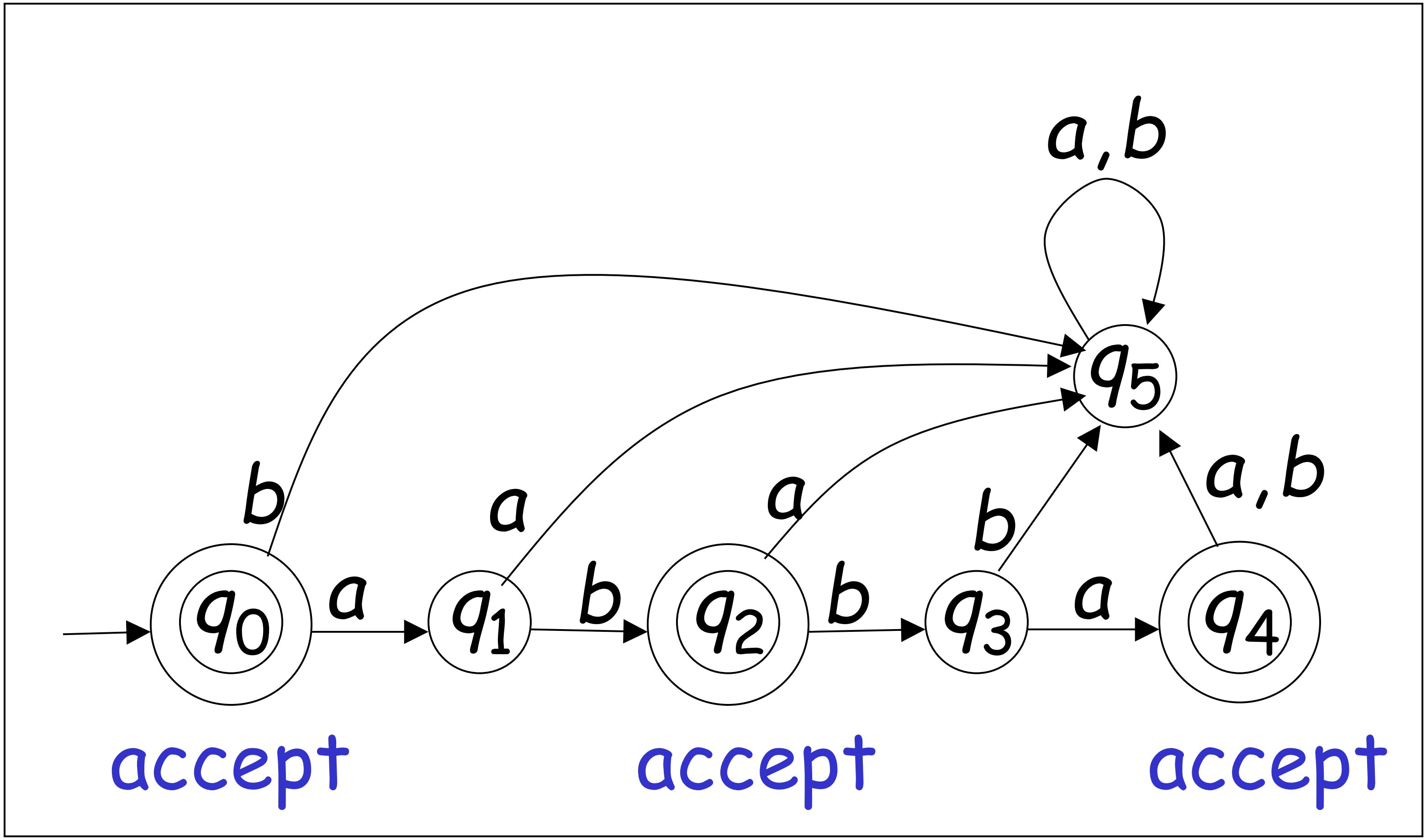
Example: L(M) = ?



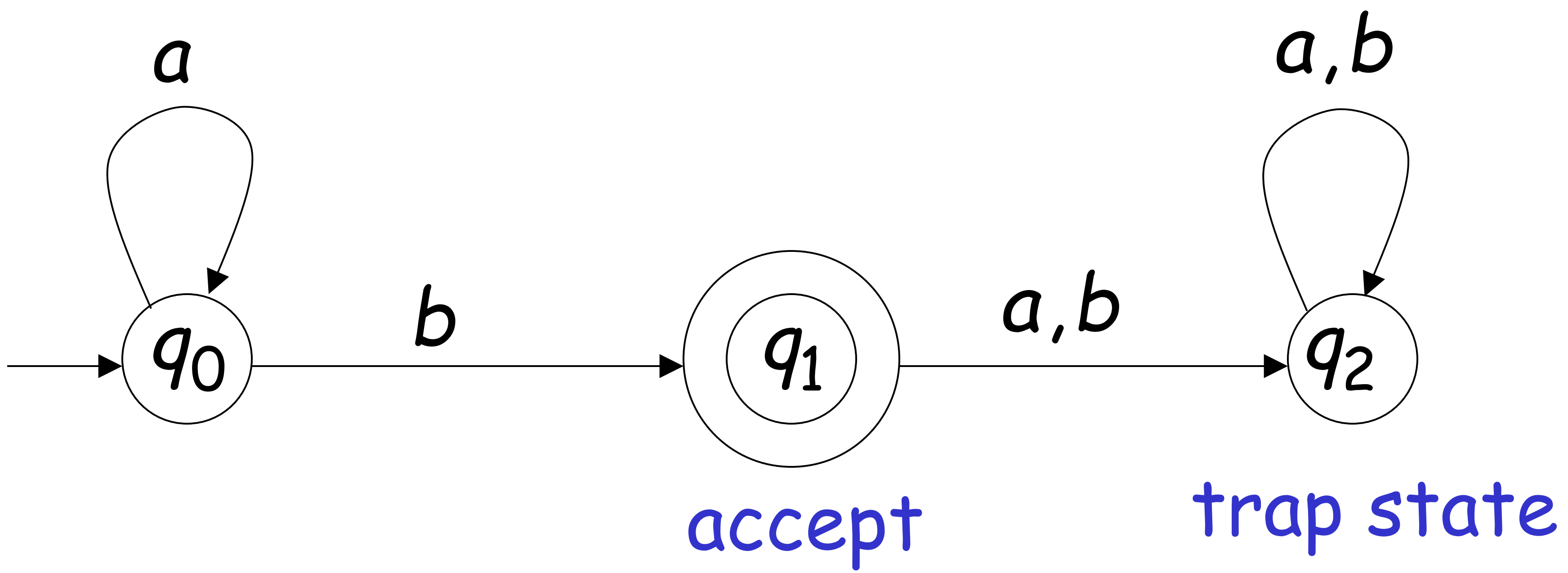
Example

$$L(M) = \{\lambda, ab, abba\}$$

M

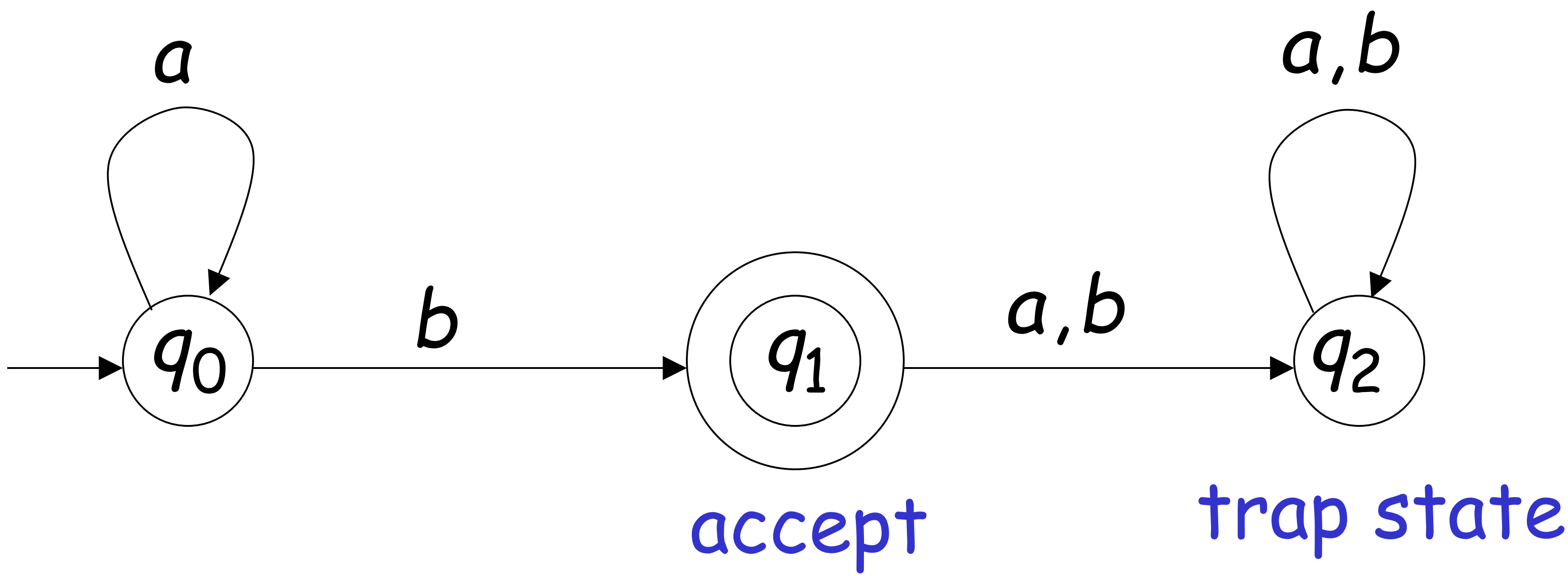


Example: L(M) = ?



Example

$$L(M) = \{a^n b : n \geq 0\}$$



Formal Definition

Finite Automaton (FA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

Σ : input alphabet

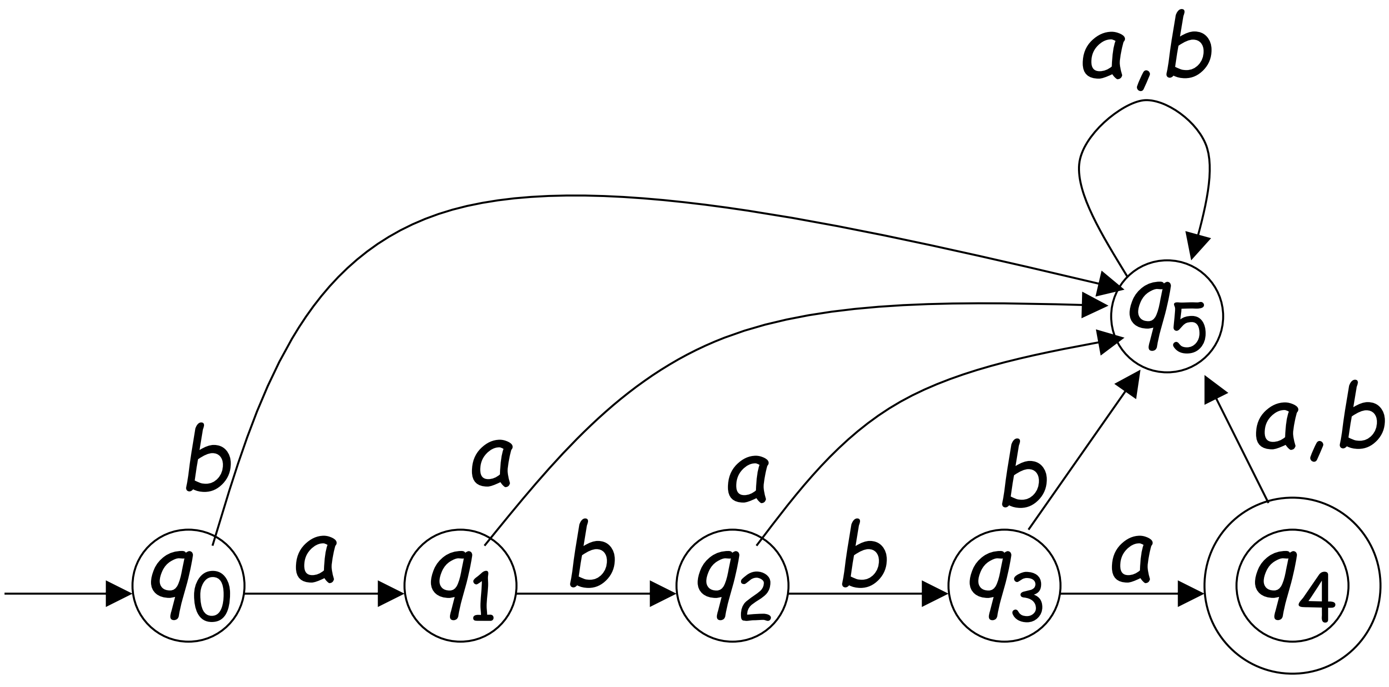
δ : transition function

q_0 : initial state

F : set of accepting states

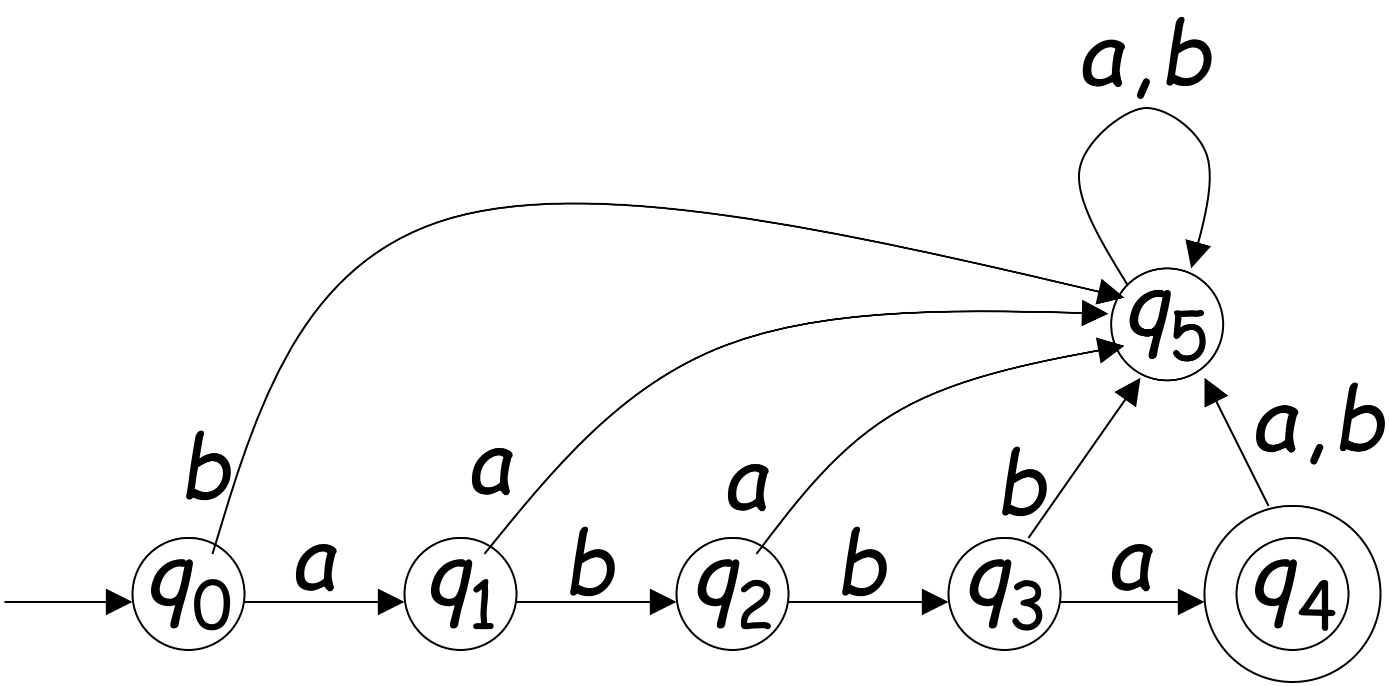
Input Alphabet Σ

$$\Sigma = \{a, b\}$$

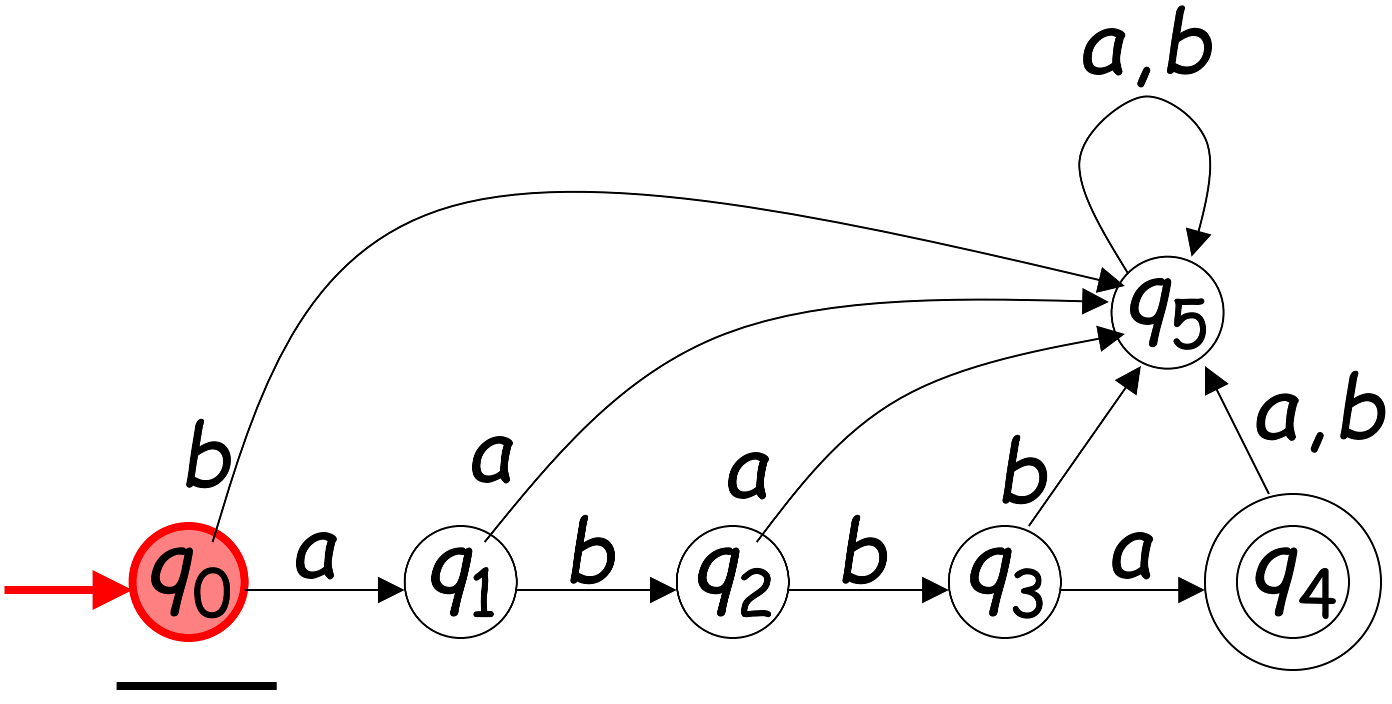


Set of States Q

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

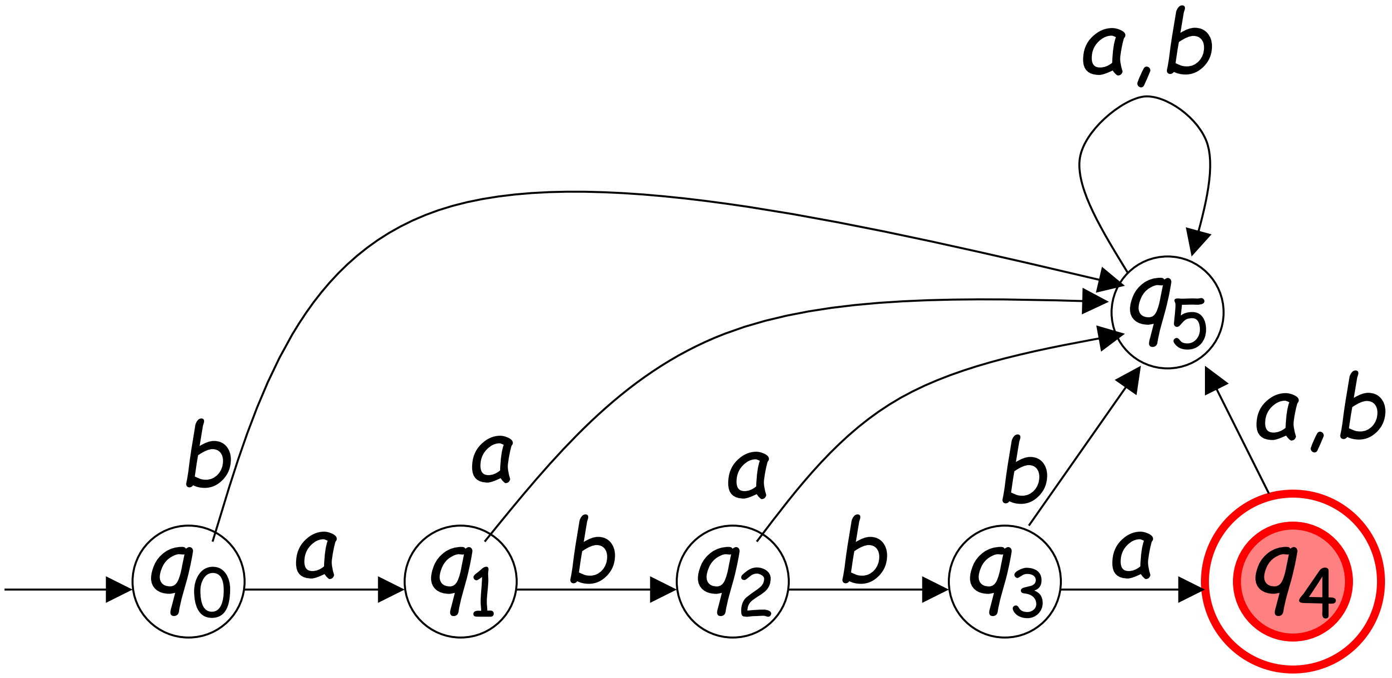


Initial State q_0



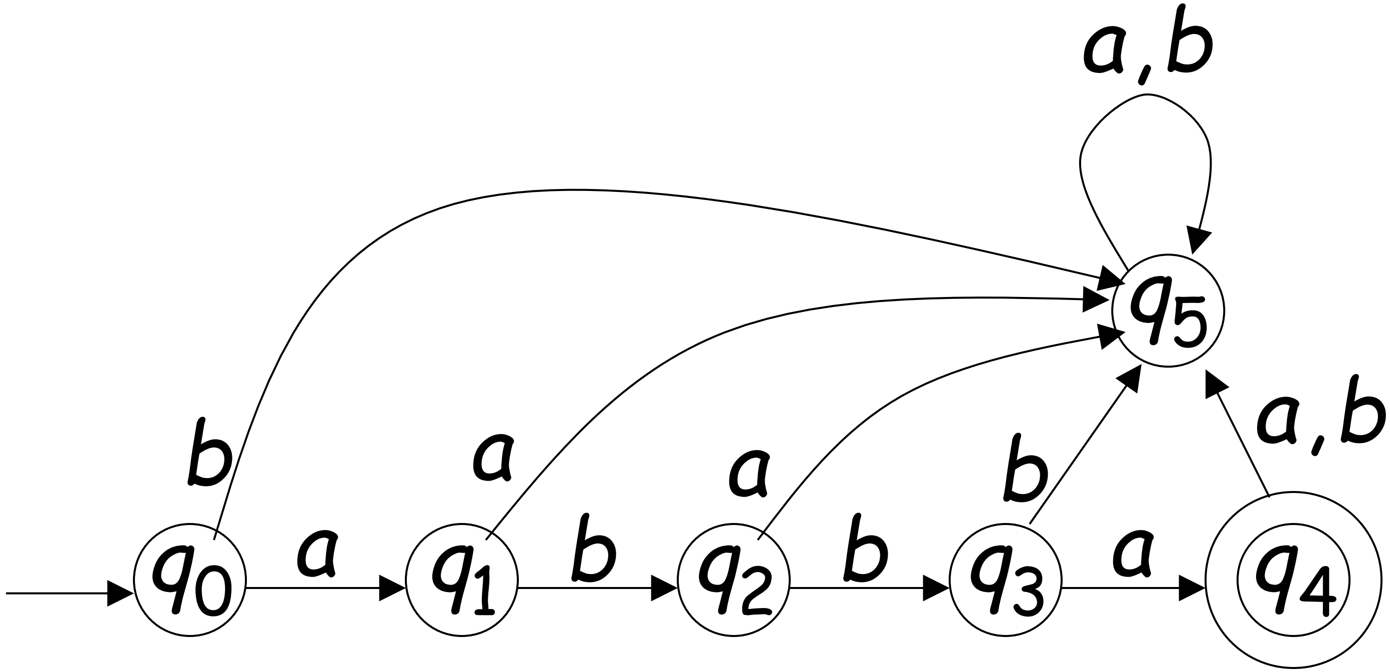
Set of Accepting States F

$$F = \{q_4\}$$

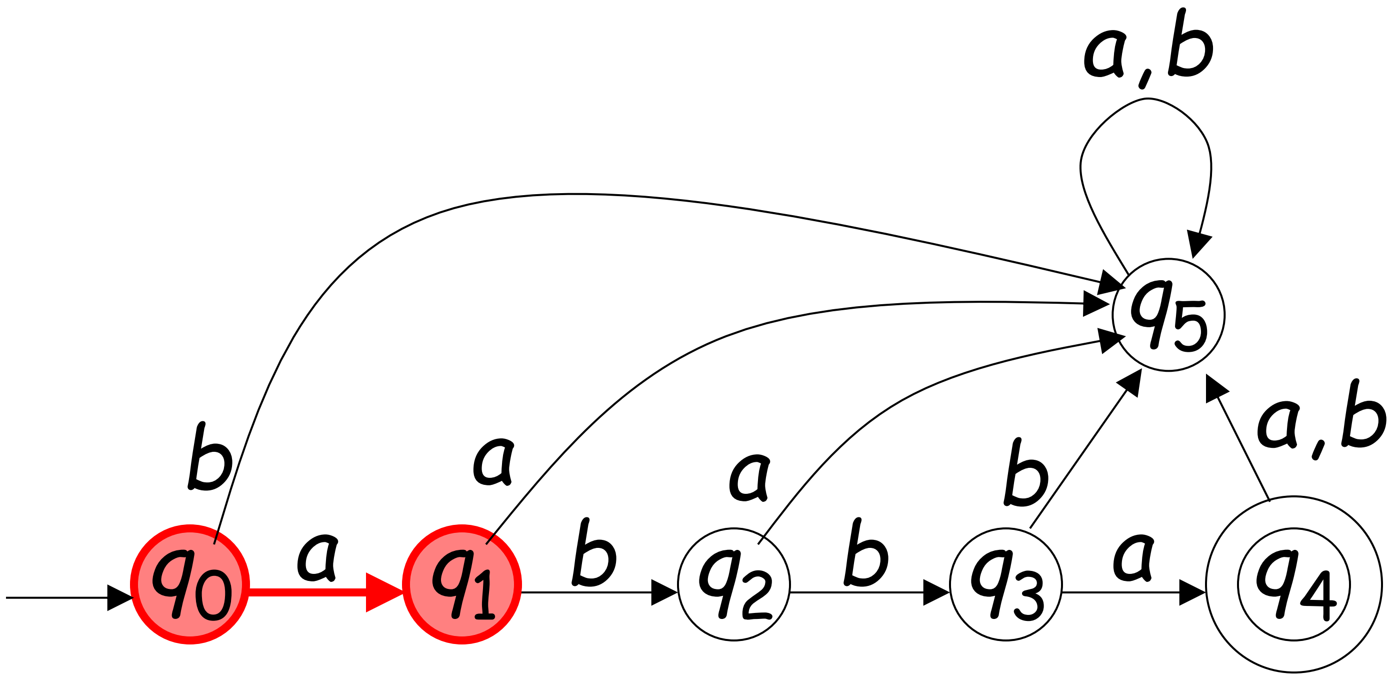


Transition Function δ

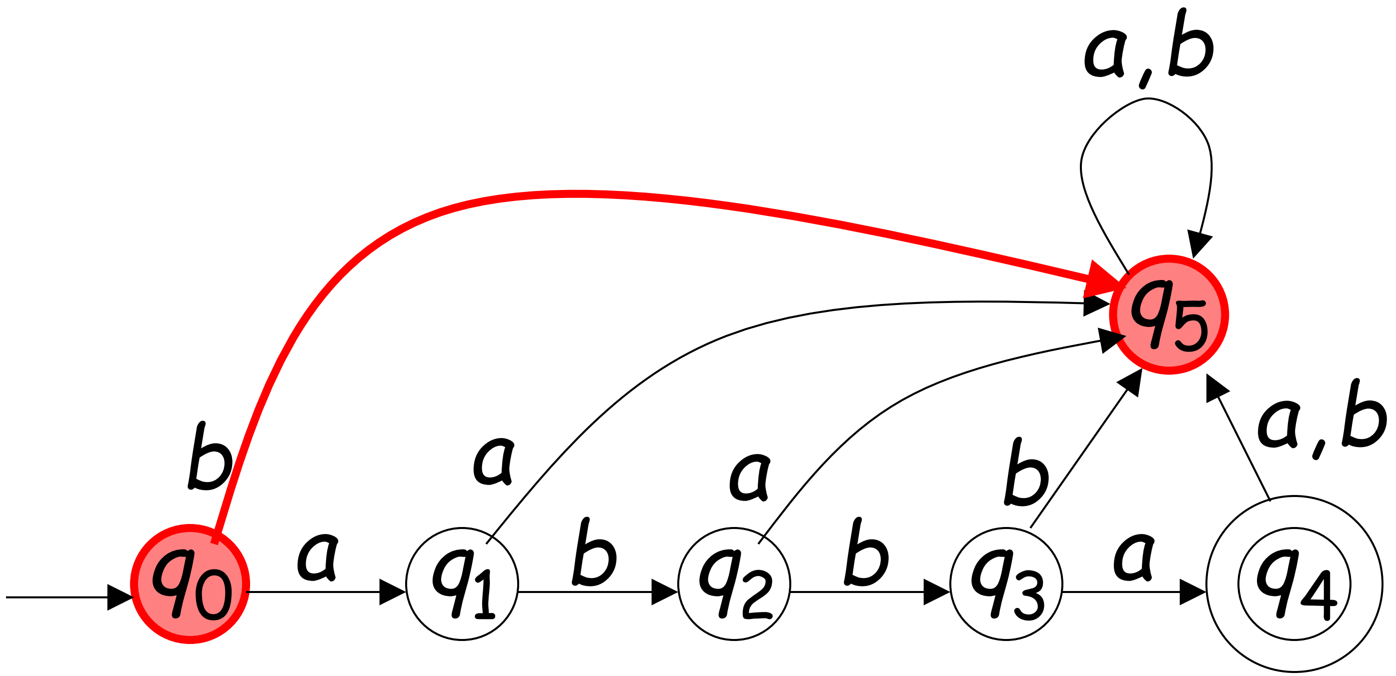
$$\delta : Q \times \Sigma \to Q$$



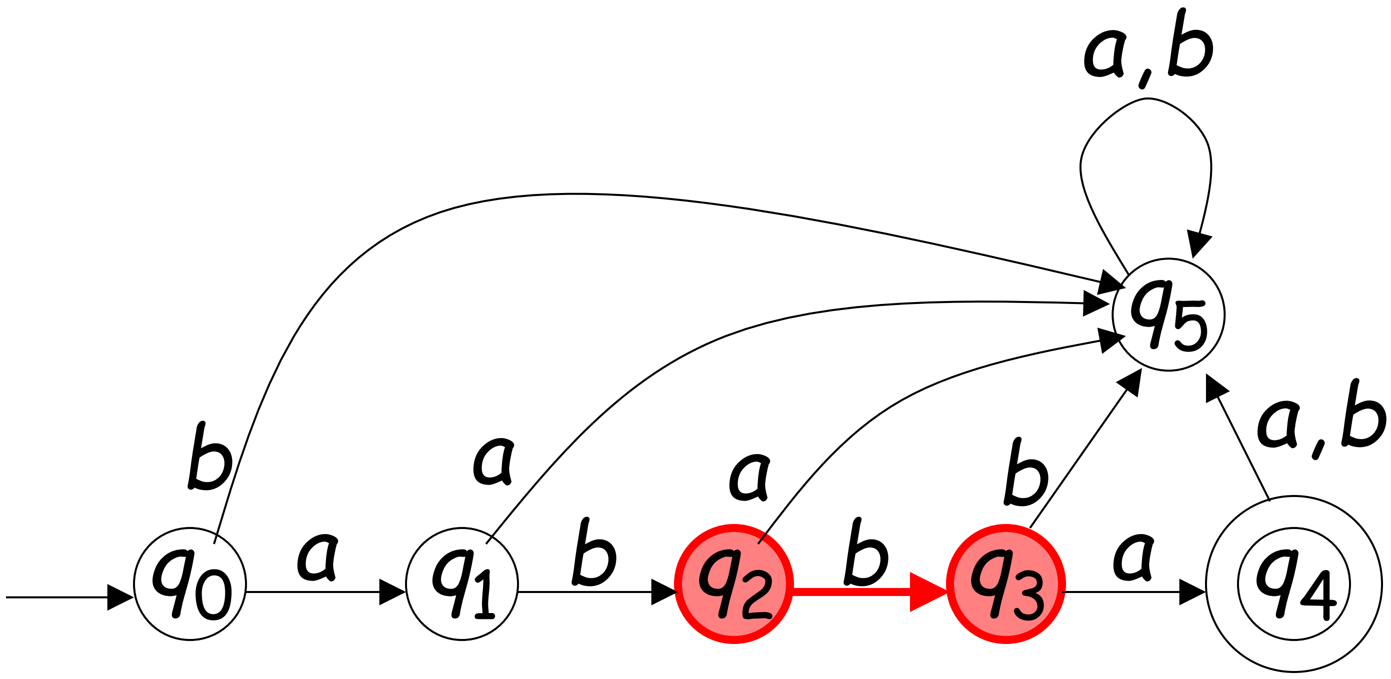
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$

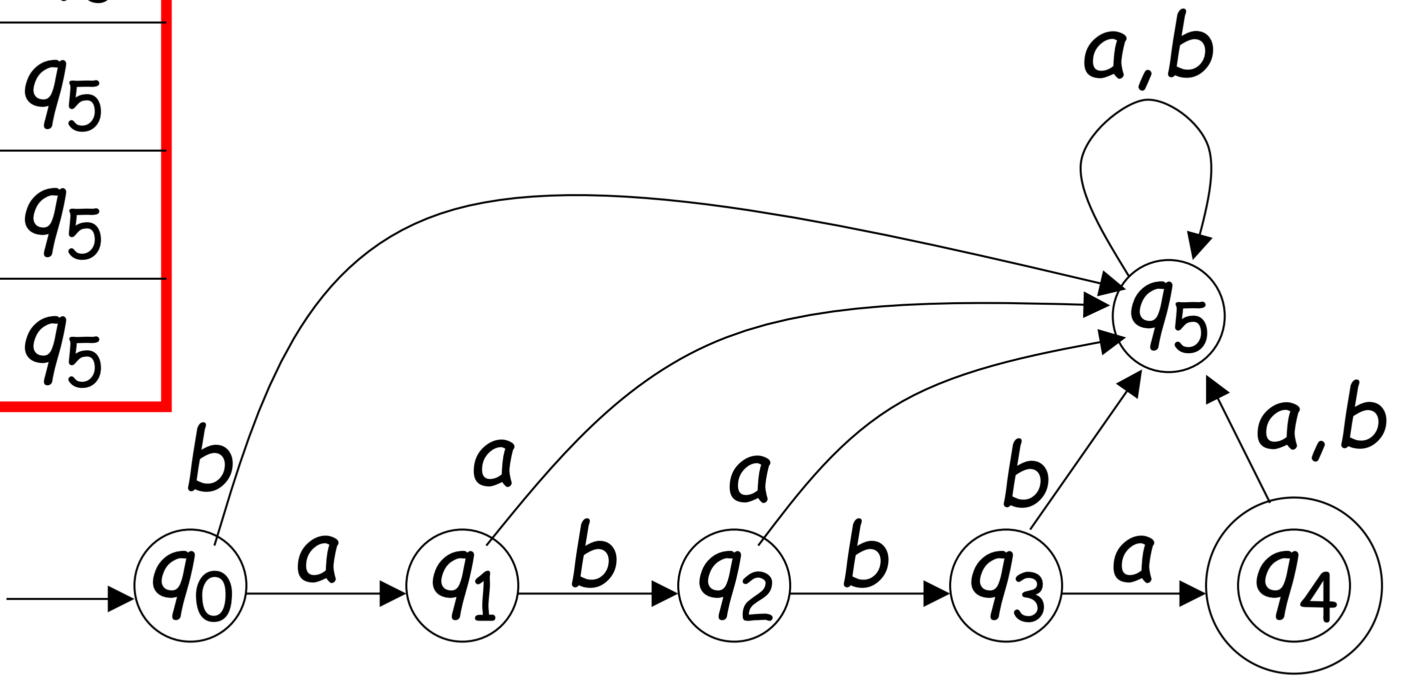


$$\delta(q_2, b) = q_3$$



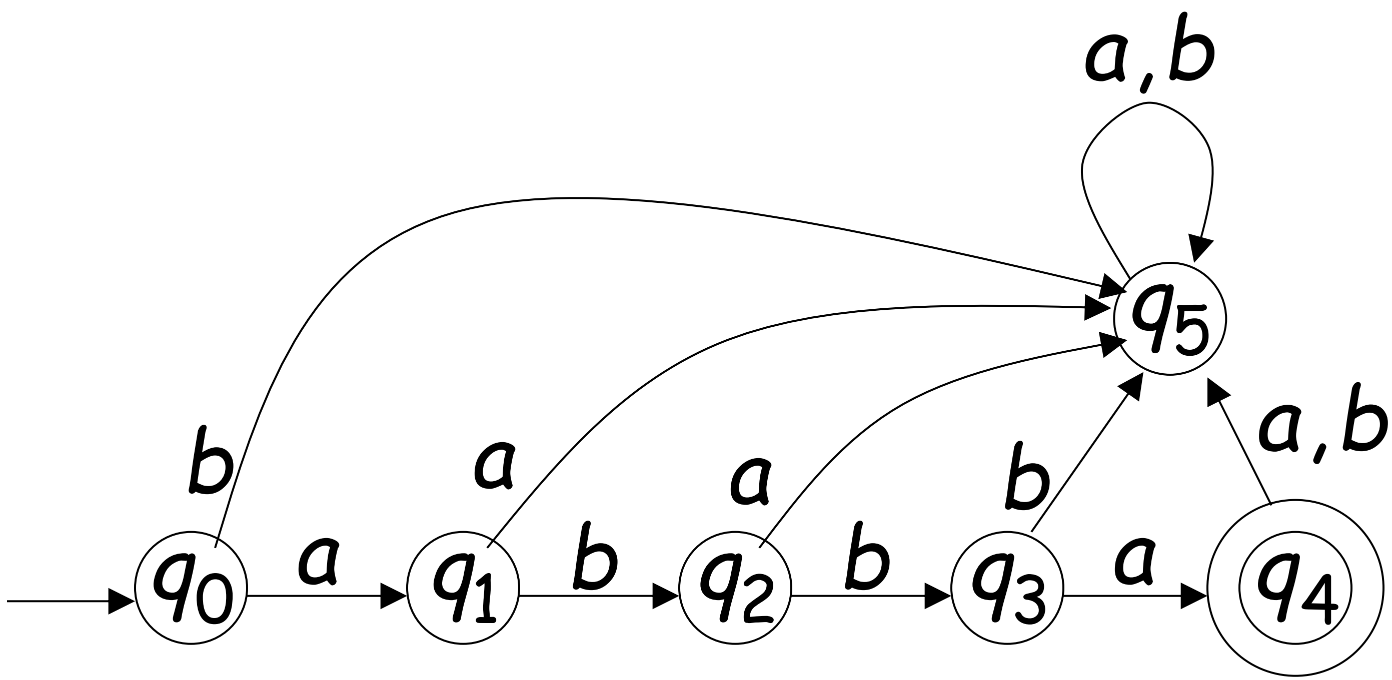
Transition Function δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5

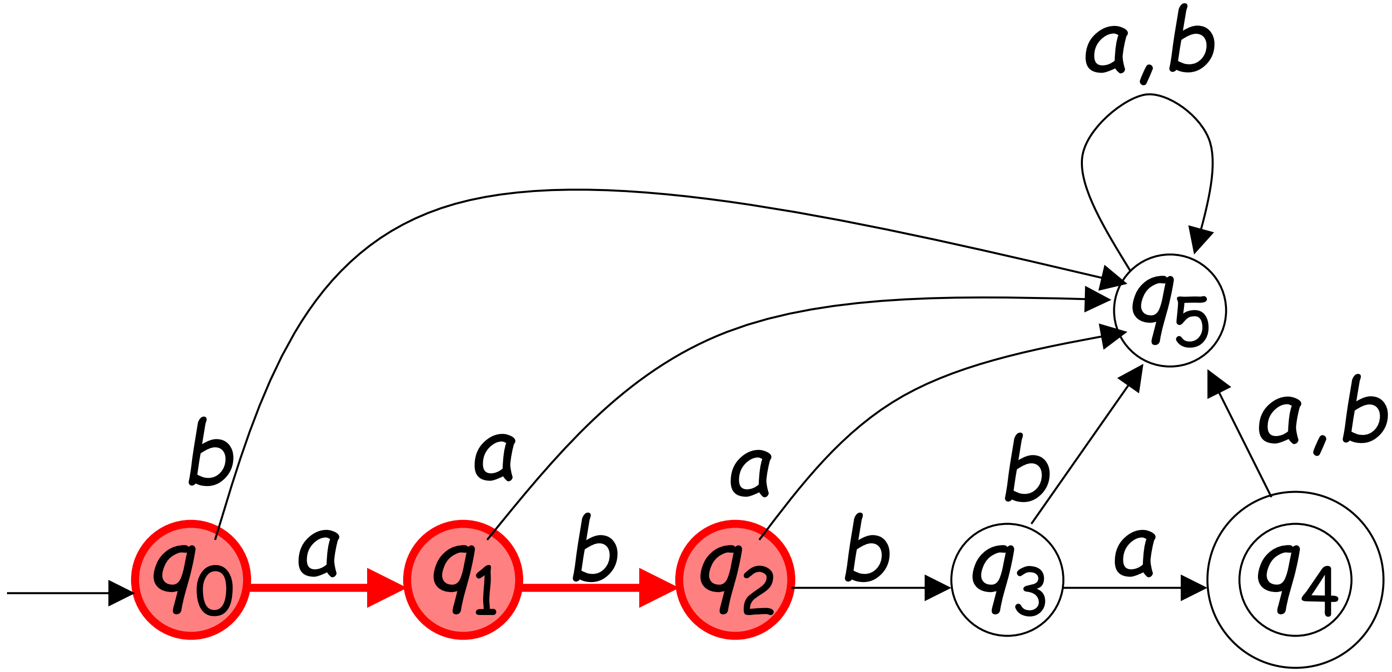


Extended Transition Function δ^*

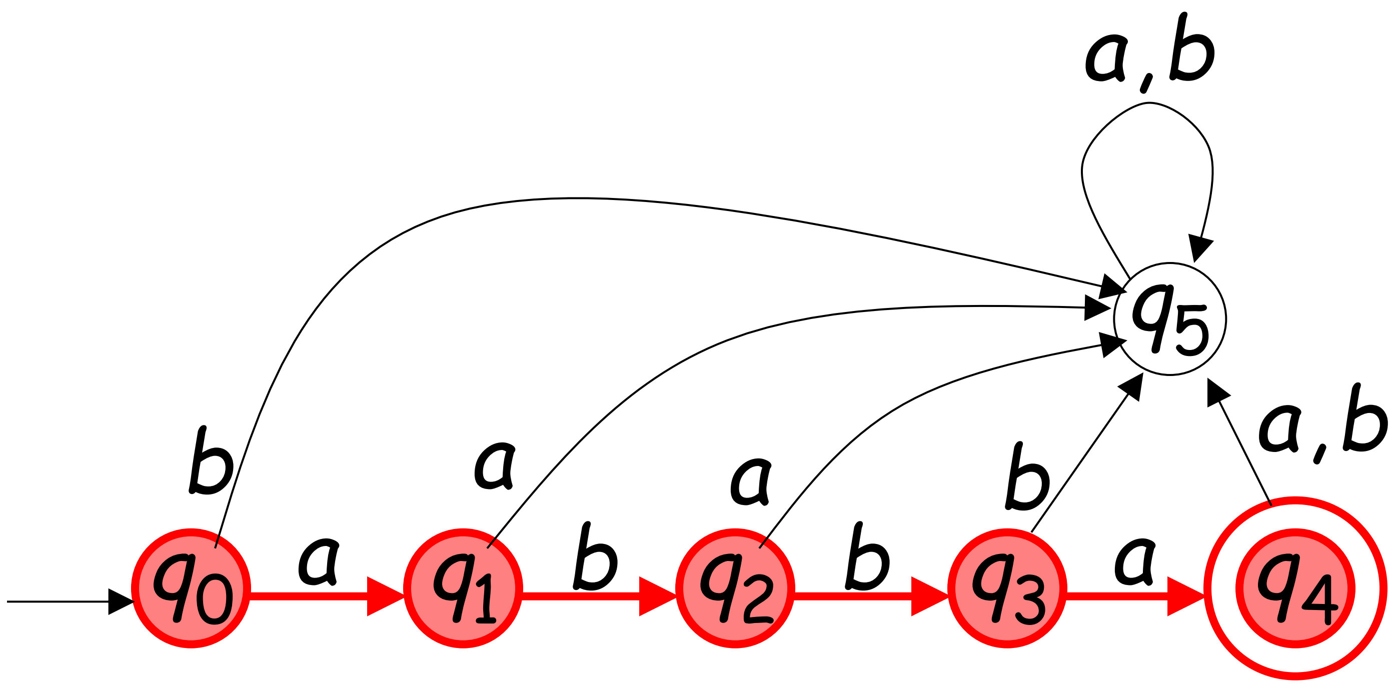
$$\delta^* : Q \times \Sigma^* \to Q$$



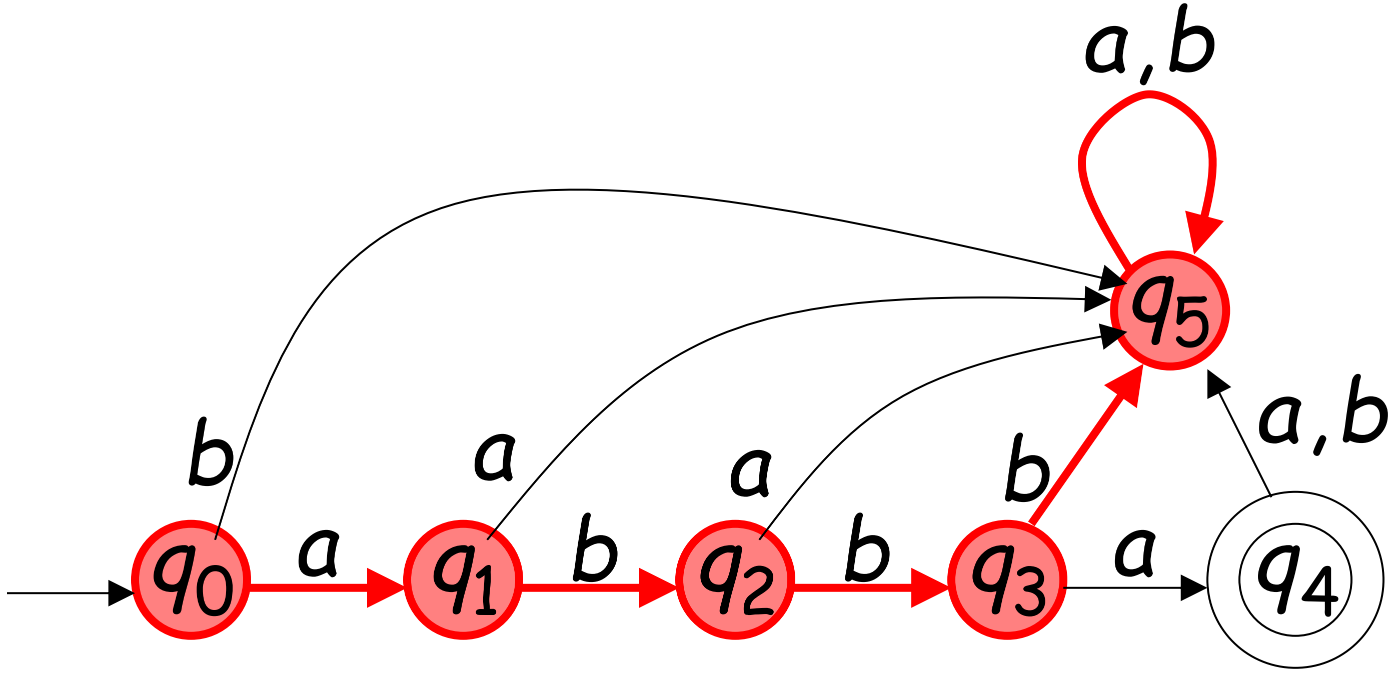
$$\delta^*(q_0, ab) = q_2$$



$$\delta^*(q_0, abba) = q_4$$

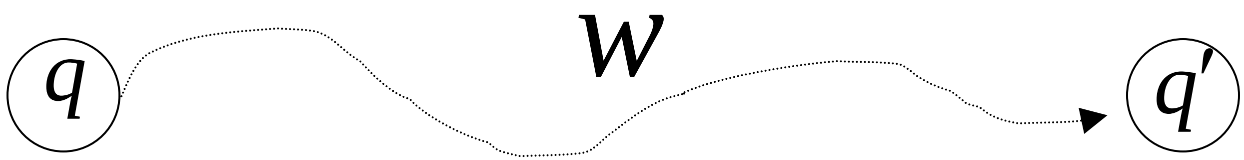


$$\delta^*(q_0, abbbaa) = q_5$$

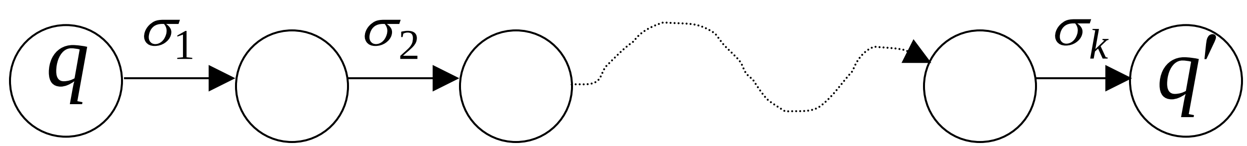


Observation: if there is a walk from q to q' with label w then

$$\delta^*(q,w)=q'$$

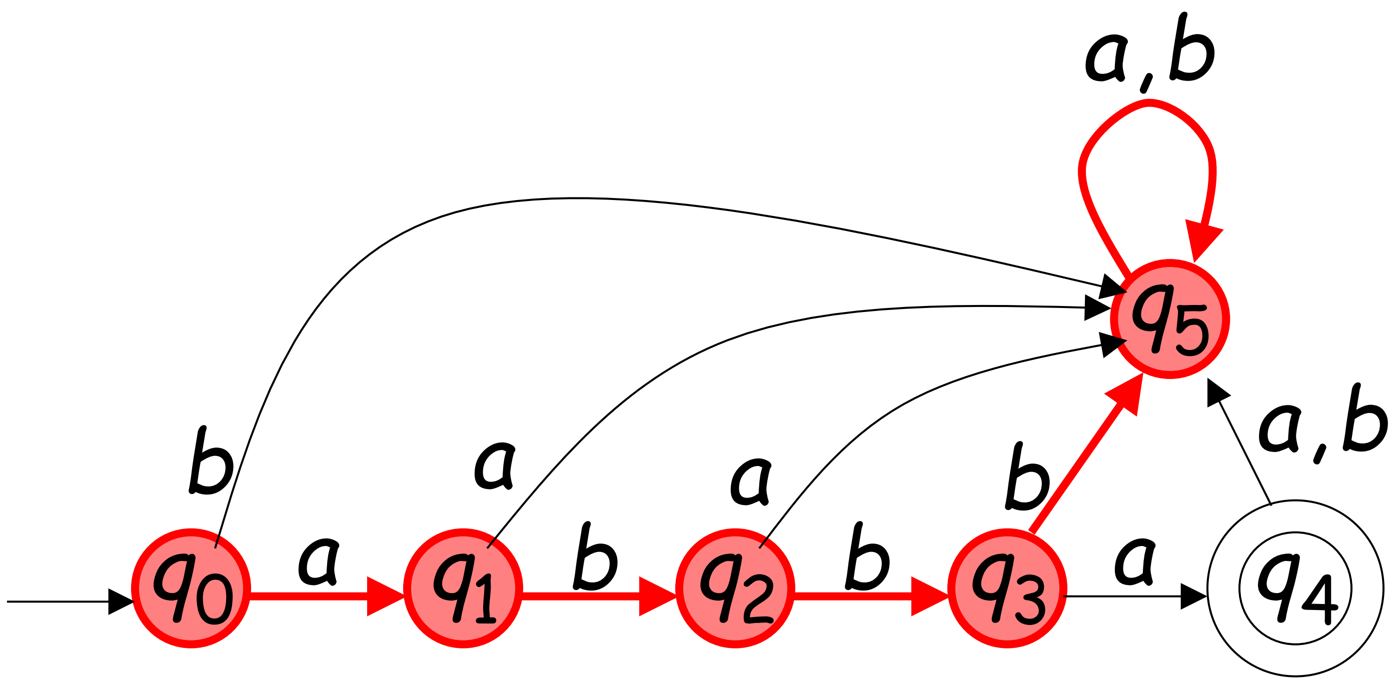


$$w=\sigma_1\sigma_2\cdots\sigma_k$$



Example: There is a walk from q_0 to q_5 with label $abbbaa$

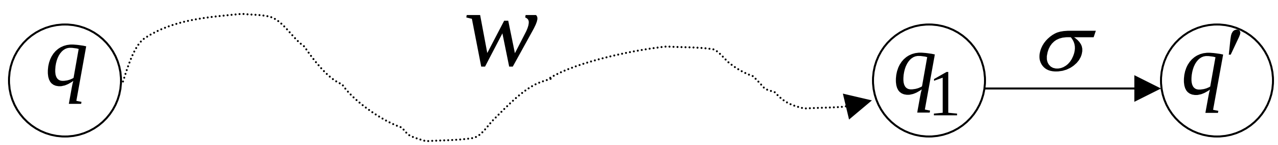
$$\delta^*(q_0, abbbaa) = q_5$$



Recursive Definition

$$\delta^*(q,\lambda)=q$$

$$\delta^*(q,w\sigma)=\delta(\delta^*(q,w),\sigma)$$



$$\left.\begin{array}{l}\delta^*(q,w\sigma)=q' \\ \\ \delta(q_1,\sigma)=q'\end{array}\right\} \Rightarrow \delta^*(q,w\sigma)=\delta(q_1,\sigma)$$

$$\left.\begin{array}{l}\delta^*(q,w\sigma)=\delta(q_1,\sigma) \\ \\ \delta^*(q,w)=q_1\end{array}\right\} \Rightarrow \delta^*(q,w\sigma)=\delta(\delta^*(q,w),\sigma)$$

$$\delta^*(q_0, ab) =$$

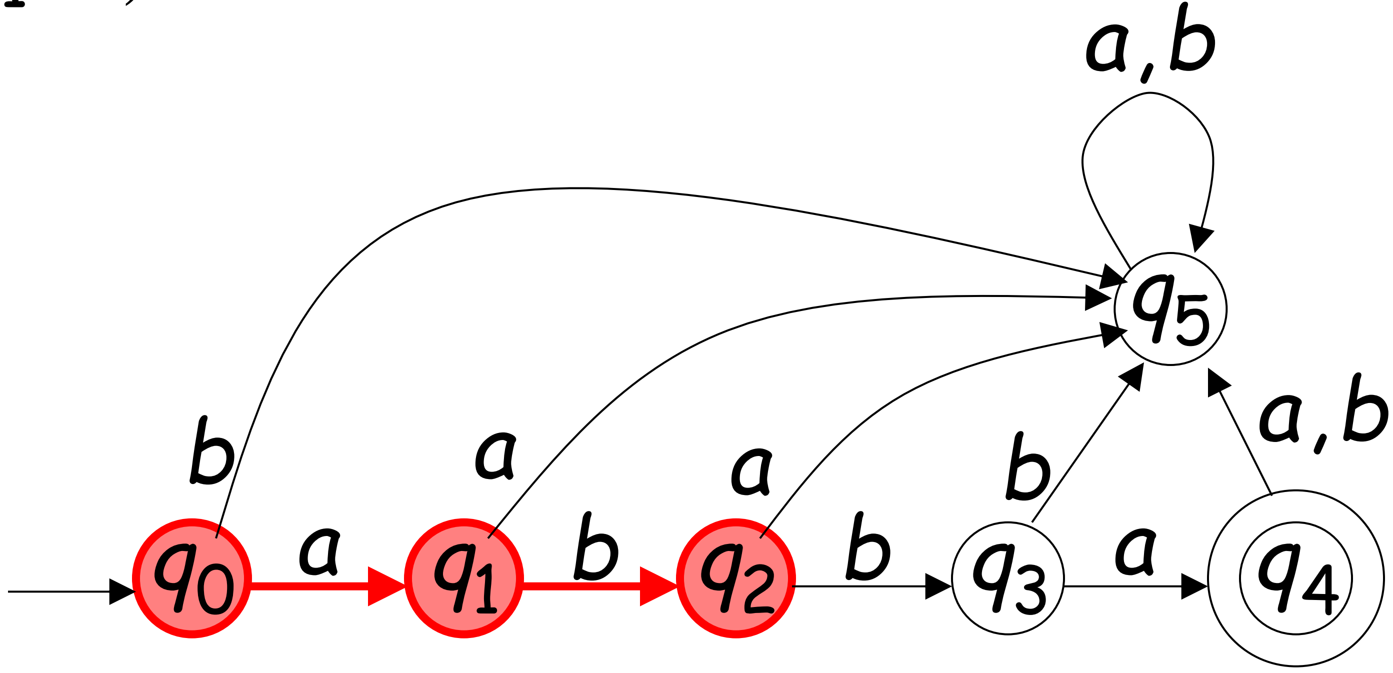
$$\delta(\delta^*(q_0, a), b) =$$

$$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$$

$$\delta(\delta(q_0, a), b) =$$

$$\delta(q_1, b) =$$

$$q_2$$

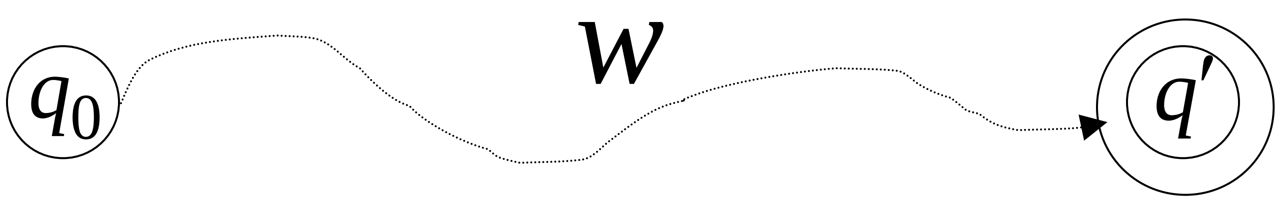


Language Accepted by FAs

For a FA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

$$L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \in F \}$$

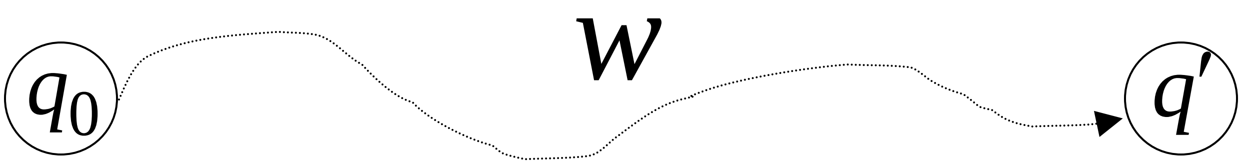


$q' \in F$

Observation

Language rejected by M :

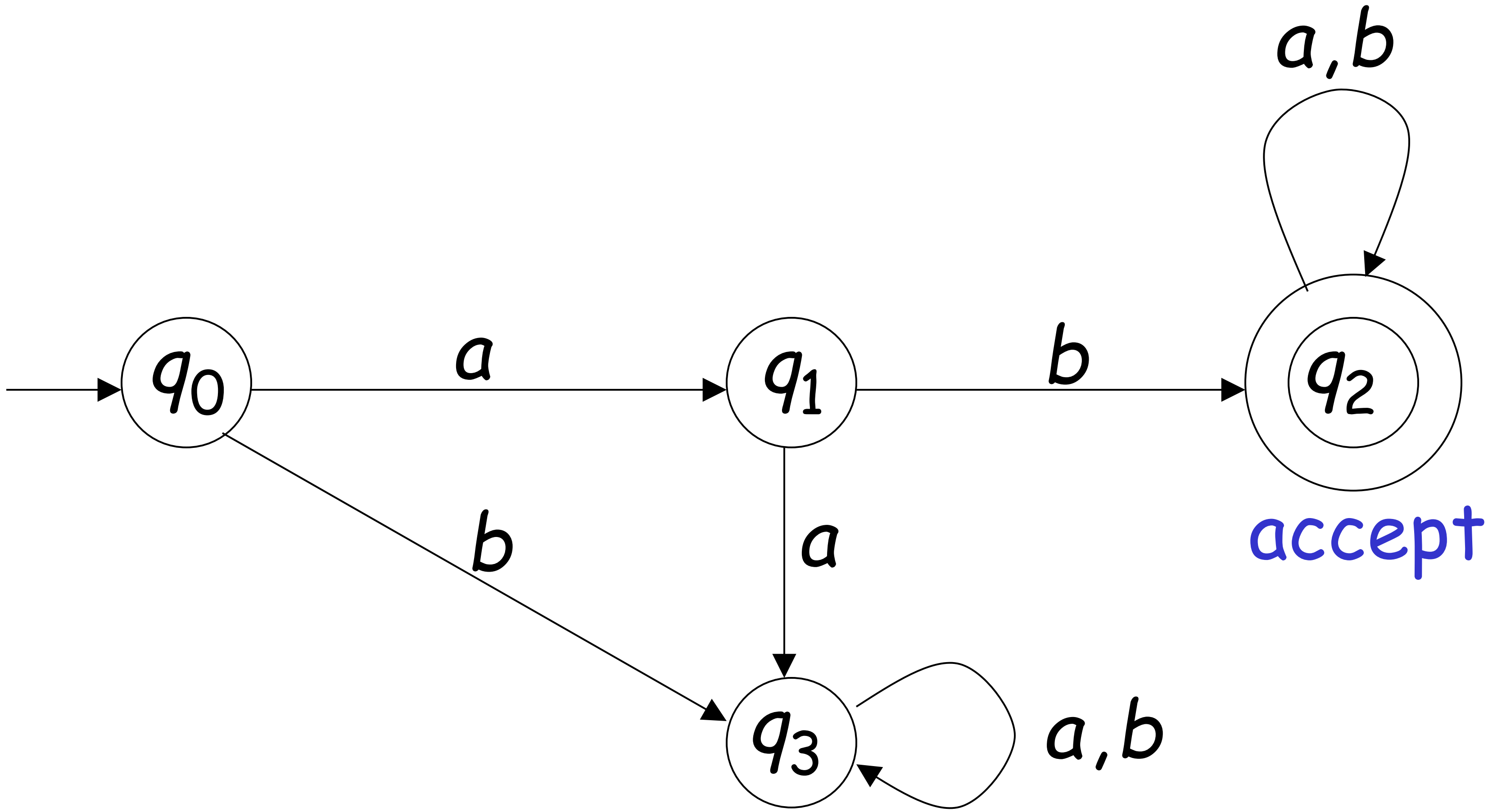
$$\overline{L(M)} = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$$



$q' \notin F$

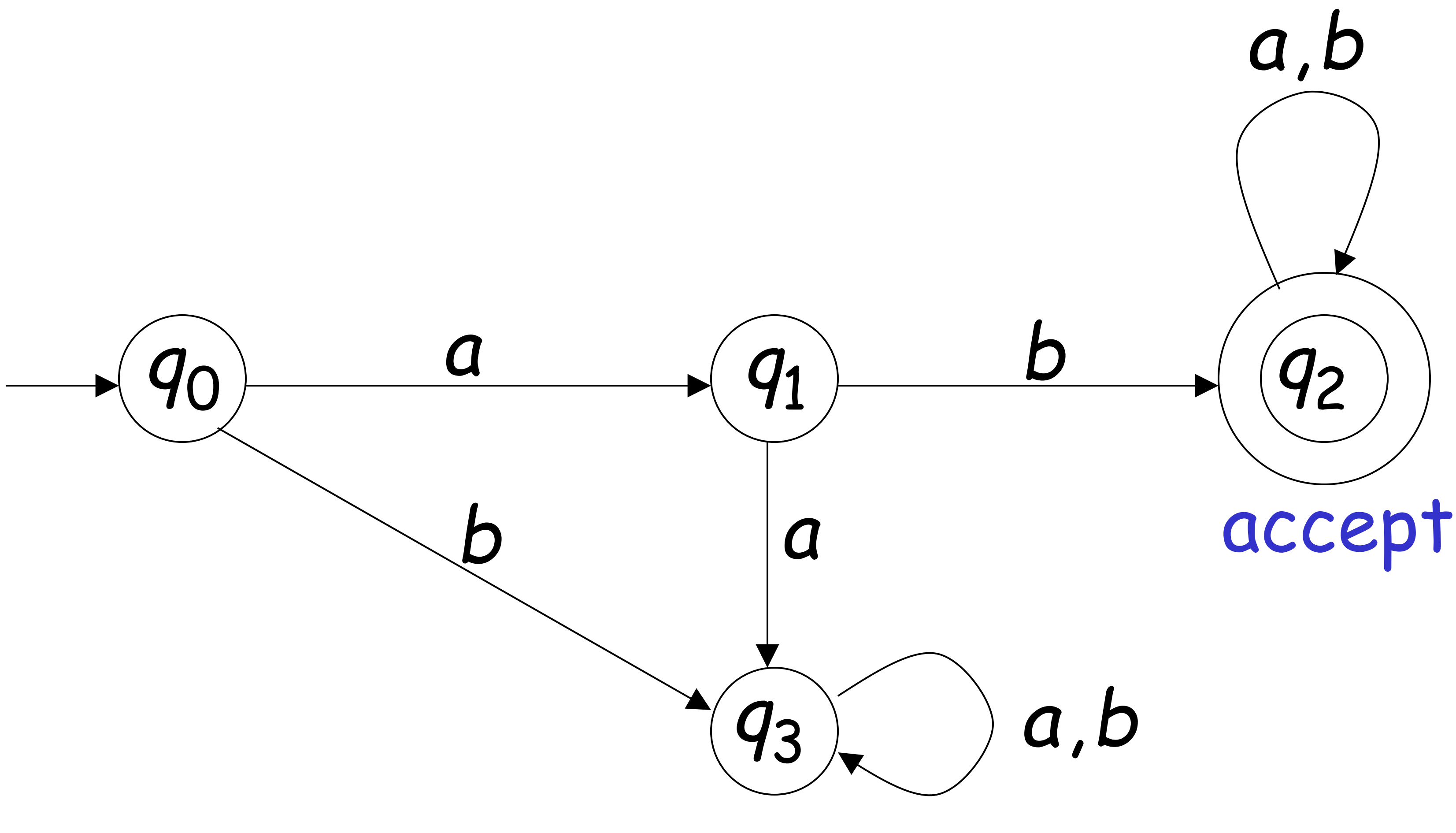
L(M) ?

ab



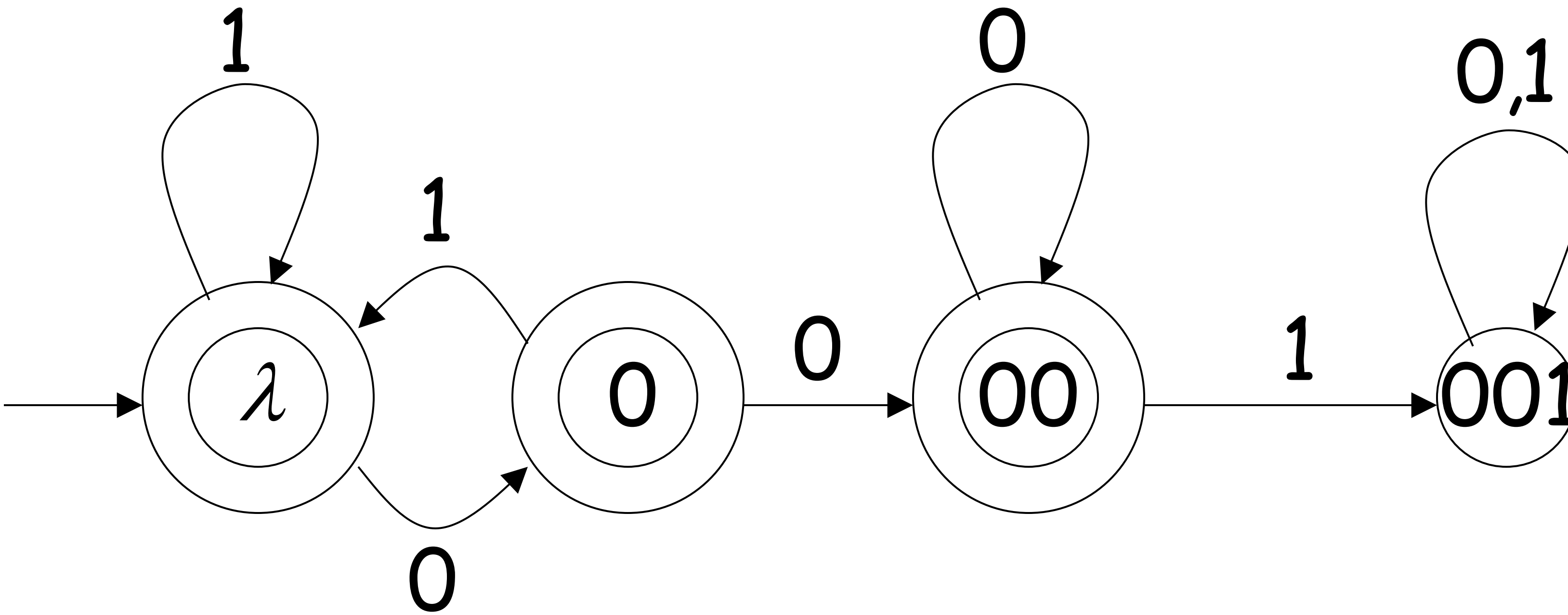
Example

$L(M)$ = { all strings with prefix ab }



Try-Starting with a and ending with b

L(M)?

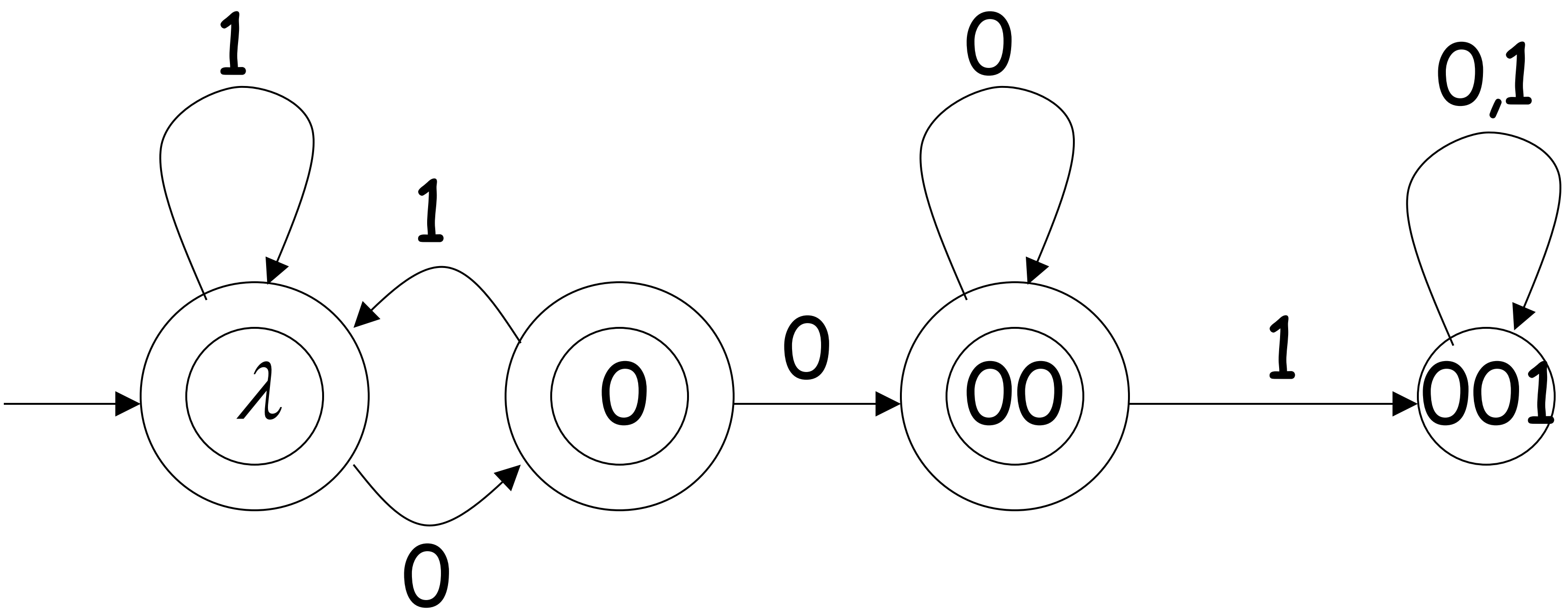


Example

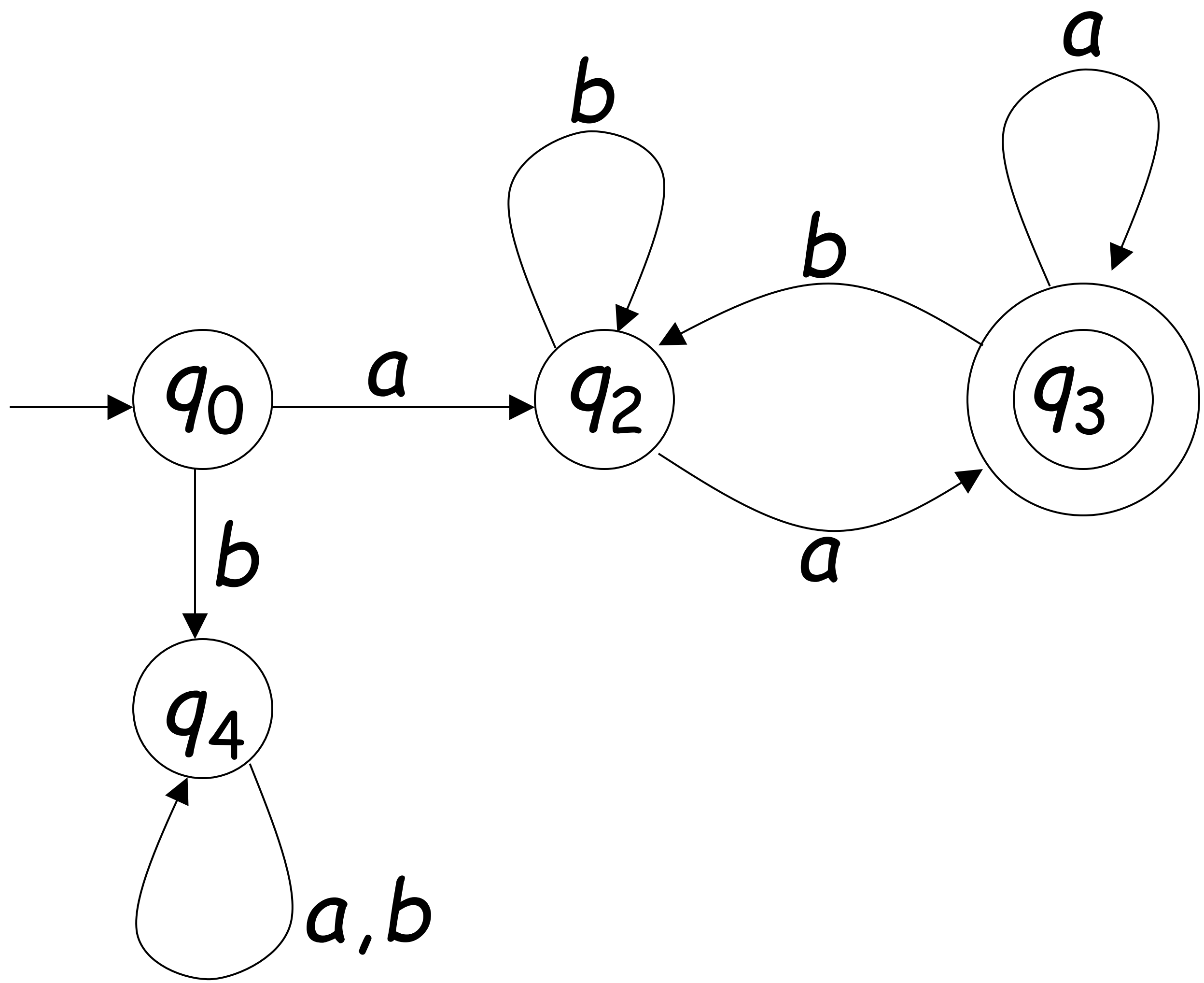
$L(M) =$ { all strings without substring 001 }

Example

$L(M) =$ { all strings without substring 001 }

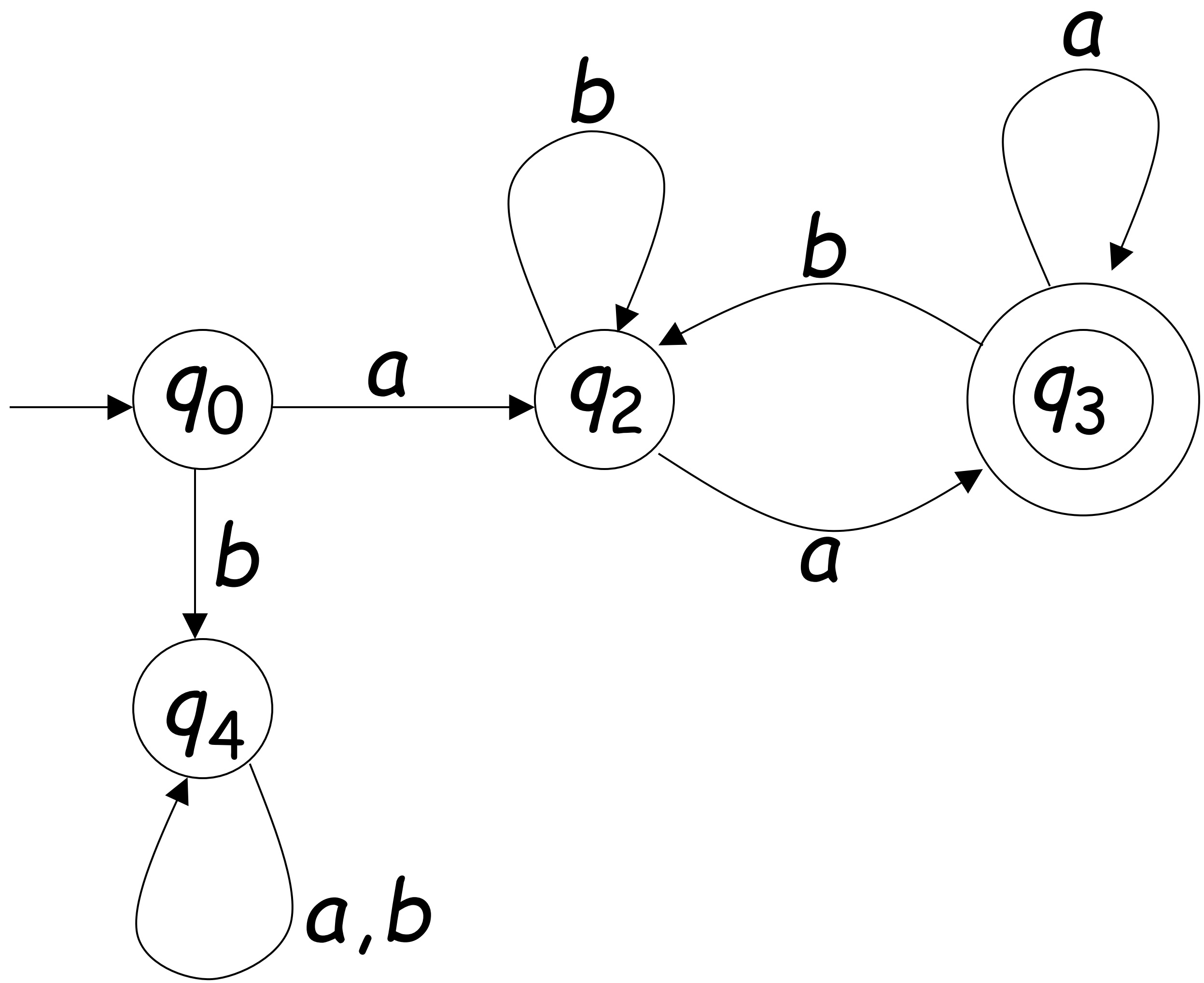


L(M) ?



Example

$$L(M)=\{awa : w\in\{a,b\}^*\}$$



Regular Languages

Definition:

A language L is regular if there is FA M such that $L = L(M)$

Observation:

All languages accepted by FAs form the family of regular languages

Examples of regular languages:

$\{abba\}$ $\{\lambda, ab, abba\}$

$\{awa : w \in \{a,b\}^*\}$ $\{a^n b : n \geq 0\}$

{ all strings with prefix ab }

{ all strings without substring 001 }

There exist automata that accept these Languages (see previous slides).

There exist languages which are not Regular:

Example: $L = \{a^n b^n : n \geq 0\}$

There is no FA that accepts such a language