Reg.No.										

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THIRD SEMESTER B.E DEGREE END SEMESTER EXAMINATION – DECEMBER, 2012

Sub: MAT 209 - ENGG. MATHEMATICS III (CS/IT)

(REVISED CREDIT SYSTEM – 2011)

Time: 3 Hrs. **Max. Marks: 50**

Note: a). Answer any FIVE full questions b). All questions carry equal marks

1A. Given n = 5 and the five marks 0, 1, 2, 3, 4 what are the 87^{th} and 114^{th} permutations in the following orders (the first permutation is 01234)

a) Reverse Lexicographical order

b) Fike's order.

1B. Let (G, *) be a group and a, b $\in$ G. Suppose that $a^2 = e$ and $a * b * a = b^7$. Prove that $b^{48} = e$.

1C. Let $\langle A, \vee, \wedge, - \rangle$ be a finite Boolean algebra. Let b be any nonzero element in A and $a_1, a_2, \ldots, a_k$ be all the atoms of A such that $a_i \le b$. Prove that $b = a_1 \vee a_2 \vee \ldots \vee a_k$.

(3 +3+4)

2A. Show that a (p, q) graph is a tree if and only if it is connected and p = q + 1

2B. Using the generating functions, show that number of partitions of n is equal to number of partitions of 2n with exactly n parts.

2C. Show that from

(i) $(\exists x)\,(F(x) \wedge S(x)) \rightarrow (y)\,(M(y) \rightarrow W(y))$

(ii) $(\exists y)\,(M(y) \wedge .((y)W \neg$

the conclusion $(x)\,(F(x) \rightarrow$ .swollof $((x)S\neg$ **(3 +3+4)**

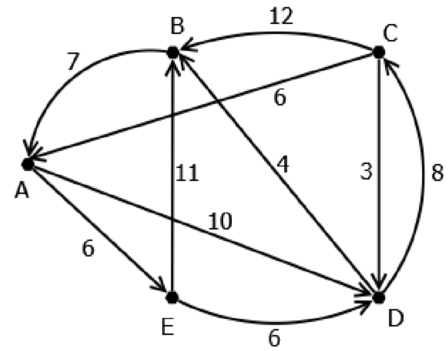
3A. Prove that $(\exists x)\big(P(x) \wedge Q(x)\big) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$. Is the converse true? Why or why not?

3B. Let $E(x_1, x_2, x_3) = \overline{\overline{(x_1 \vee x_2)} \vee \left(\overline{x_1} \wedge x_3\right)}$ be a Boolean expression over the two valued Boolean algebra. Write $E(x_1, x_2, x_3)$ in both disjunctive and conjunctive normal forms.

3C. Prove that the number of permutations of $1, 2, \ldots, n$ in which no number occupies it's proper place is approximately equal to $\frac{n!}{e}$.

(3 +3+4)

4A. Use Dijkstra's algorithm to find the shortest distances from E to the other vertices A, B, C, and D.



4B. (i) In a distributive lattice if an element has a complement, show that it is unique.

(ii) In a Boolean algebra, for any two elements a and b show that

(a) $\overline{a \wedge b} = \bar{a} \vee \bar{b}$ (b) $\overline{a \vee b} = \bar{a} \wedge \bar{b}$

4C. Show that every group of order less than or equal to 5 is abelian.

(3 +3+4)

5A. Construct the truth table for $((p \wedge \sim q) \rightarrow r) \rightarrow (p \rightarrow (q \vee r))$

5B. State and prove Lagrange's theorem.

5C. Let G be connected graph with at least 3 points. Show that G is bipartite if and only if it has no odd cycles. **(3 +3+4)**

6A. Prove that the number of combinations of n distinct objects taken m at a time with unrestricted repetition is ${}^{n+m-1}C_m$. Hence find the number compositions of positive integers n into exactly m parts.

6B. Let $\varphi: G \rightarrow G'$ be a homomorphism. Then prove that
i) $\varphi(e) = e'$, where e' is the identity of G'.
ii) $\varphi(x^{-1}) = \left(\varphi(x)\right)^{-1}, \forall\, x \in G$.

6C. Prove that the meet operation is distributive over join operation in a lattice if and only if the join operation is also distribute over the meet operation.

(3 +3+4)

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