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MANIPAL INSTITUTE OF TECHNOLOGY

(Constituent Institute of MAHE – Deemed University)
Manipal - 576 104

FOURTH SEMESTER B.E DEGREE END SEMESTER EXAMINATION –2006

SUB: PROBABILITY, STATISTICS & STOCHASTIC PROCESS (MAT – CSE- 202)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

Note : a) Answer any FIVE full questions.
b) All questions carry equal marks.

1A. (i) Two events A and B are such that $P(\overline{A}) = 0.3$, $P(B) = 0.4$ and $P(A \cap \overline{B}) = 0.5$. Find $P(B | A \cup \overline{B})$.

1B. A continuous random variable X has pdf given by

$$f(x) = \begin{cases} \dfrac{x}{a^2} e^{\frac{-x^2}{2a^2}} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Find mean, variance and mode of the distribution.

1C. A bag contains some white and some black balls, all combinations of balls being equally likely. The total number of balls in the bag is ten. If three balls are drawn at random without replacement and all of them are found to be black, find the probability that the bag contains one white and nine black balls.

2A. Box one contains four black and five green balls. Box two contains five black and four green balls. Three balls are selected at random from box one and put into box 2. And then a ball is drawn randomly from box two and is found to be green. What is the probability that two green and one black are transferred from box one ?

2B. If the random variable 'K' is uniformly distributed over [0,5], what is the probability that the roots of the equation $4x^2 + 4xK + K + 2 = 0$ are real ?

Contd....2

2C. Obtain Poisson distribution as a limiting case of Binomial distribution.

3A. The random variable (X,Y) has joint pdf given by

$$f(x,y)=\begin{cases} x+y, & 0\le x\le 1,\ 0\le y\le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Compute the correlation coefficient between X & Y.

3B. Find mean and variance of Gamma distribution.

3C. Suppose that a two dimensional continuous random variable has joint pdf

$$f(x,y)=\begin{cases} kx(x-y), & 0<x<2,\ -x<y<x \\ 0 & \text{elsewhere} \end{cases}$$

a) Evaluate the constant k

b) Find the marginal pdf of y

4A. In a normal distribution 31% of the items are under 45 and 5% are over 64. Find the mean and variance of the distribution.

4B. Two dice are thrown until we get either a sum of 5 or a sum of 7. Find the probability that a sum of 5 appears before a sum of 7 ?

4C. Let X_1, X_2 and X_3 be uncorrelated random variables having the same standard deviation. Find the correlation coefficient between $X_1 + X_2$ and $X_2 + X_3$.

5A. Suppose that the continuous random variable X has pdf

$f(x)=\frac{1}{2}e^{-|x|}, -\infty<x<\infty$. Find mgf of X and hence find E(X) and V(X).

5B. Let X have uniform distribution over the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Find the pdf of Y where Y = tanX.

5C. If $X \sim N(0, \sigma^2)$, $Y \sim N(0, \sigma^2)$ where X and Y are independent, find the pdf of $R=\sqrt{X^2+Y^2}$.

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Contd...3

6A. Compute an approximate probability that mean of a random sample of size 15 from a distribution having pdf

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is between $\frac{3}{5}$ & $\frac{4}{5}$.

6B. Let $(X_1, X_2, \ldots\ldots X_n)$ denote a random sample of size n from the distribution with pdf

$$f(x,\theta) = \begin{cases} \dfrac{\theta^x e^{-\theta}}{x!}, & x = 0,1,2,\ldots\ldots,\theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find MLE for θ.

6C. Let a random sample of size 17 from a normal distribution $n(\mu,\sigma^2)$ yield $\bar{x}=4.7$, $S^2 = 5.76$. Determine a 90% confidence interval for μ & σ^2.

7A. Define the following

(i) Type I error

(ii) Type II error

(iii) Critical error

7B. Let X have binomial distribution with parameters n = 10 and p. The simple hypothesis $H_0 : p = \frac{1}{2}$ is rejected and the alternative simple hypothesis $H_1 : p = \frac{1}{4}$ is accepted, if the observed value of X_1, a random sample of size 1, is less than that or equal to 3. Find the power function of the test.

7C. A die is cast n = 120 independent times and the following resulted.

Spots up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40 – b

If we use chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at 0.025 significance level.