Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL UNIVERSITY, MANIPAL 576 104

DEPARTMENT OF MATHEMATICS

SIXTH SEMESTER B. E. AND SECOND SEMESTER M. TECH MAKE-UP EXAMINATION, (OPEN ELECTIVE) MAY 2014, (New Credit System -2012).

SUBJECT : APPLIED LINEAR ALGEBRA (MAT-548).

Time: 3Hrs. **Max. Marks: 50**

NOTE: Answer any five full questions. All questions carry equal marks.

1A. Solve the initial value problem $\overrightarrow{x'} = \begin{pmatrix} 3 & -13 \\ 5 & 1 \end{pmatrix} \vec{x},\ \vec{x}(0) = \begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

1B. Solve by relaxation method

$$3x + 9y - 2z = 11;\ \ 4x + 2y + 13z = 24;\ \ 4x - 4y + 3z = -8.$$

1C. The following reaction between potassium permanganate $(KMnO_4)$ and manganese sulphate in water produces manganese dioxide, potassium sulphate and sulphuric acid

$$KMnO_4 + MnSO_4 + H_2O \rightarrow MnO_2 + K_2SO_4 + H_2SO_4.$$

Balance the chemical equation using vector equation approach. **(3+3+4)**

2A. If f is a bilinear form on C^3 and C^2 that is defined by

$$f\left((x_1, x_2, x_3), (y_1, y_2)\right) = x_1y_2 + x_2y_1 + 2x_2y_2 - 2x_3y_1 + 2x_3y_2$$

i. Write the matrix A relative to $\text{Å} = \{(1,0,0),(1,1,0),(1,1,1)\}$, $\mathcal{B} = \{(1,-1),(2,-1)\}$.

ii. Use the matrix A to compute the value of $f\left((i,0,i),(2,0)\right)$.

2B. Prove that a linear operator T on V is an isometry if and only if $\left(T(u), T(v)\right) = (u, v)$ for all u, v in V.

2C. Solve the initial value problem $\overrightarrow{x'} = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} \vec{x},\ \vec{x}(0) = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

(3+3+4)

3A. Orthogonally diagonalize the matrix $\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

3B. Define an inner product space. Prove that a mapping f of $V\ X\ V$ into F is an inner product on V iff f is a positive definite hermitian form.

3C. Apply the Lagrange's reduction method to find the quadratic form
$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ and also find its index and signature. **(3+3+4)**

4A. Use Gram-Schmidt process to find a set of orthonormal vectors from
$(1,1,0),(1,0,-2),(1,1,1)$ in E^3.

4B. Given $A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$ where $A_{11} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$, $A_{12} =$
$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, $A_{22} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ find A^{-1}.

4C. Prove that any interchange of two elements in a permutation $j_1, j_2, \ldots, j_n$ of the set $\{1, 2, \ldots, n\}$ changes the index by an odd integer. Give one example.

(3+3+4)

5A. Find the single value decomposition of $\begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.

5B. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ using Jacobi's method. Carry out 3 iterations.

(6+4)

6A. Find the orthogonal transformation which transforms the quadratic form
$$3x_1^2 - 2x_2^2 - x_3^2 - 4x_1x_2 + 12x_2x_3 + 8x_1x_3$$
to canonical form. Determine its index, signature and nature.

6B. State and prove Cayley – Hamilton theorem.

6C. Define orthogonal and orhonormal sets with an example. **(5+3+2)**

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