



MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL UNIVERSITY, MANIPAL 576 104

DEPARTMENT OF MATHEMATICS

SIXTH SEMESTER B. E. AND SECOND SEMESTER M. TECH MAKE-UP

EXAMINATION, (OPEN ELECTIVE) MAY 2014, (New Credit System -2012).

SUBJECT : APPLIED LINEAR ALGEBRA (MAT-548).



Time: 3Hrs.

Max. Marks: 50

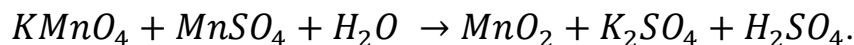
NOTE: Answer any five full questions. All questions carry equal marks.

1A. Solve the initial value problem $\vec{x}' = \begin{pmatrix} 3 & -13 \\ 5 & 1 \end{pmatrix} \vec{x}$, $\vec{x}(0) = \begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

1B. Solve by relaxation method

$$3x + 9y - 2z = 11; \quad 4x + 2y + 13z = 24; \quad 4x - 4y + 3z = -8.$$

1C. The following reaction between potassium permanganate ($KMnO_4$) and manganese sulphate in water produces manganese dioxide, potassium sulphate and sulphuric acid



Balance the chemical equation using vector equation approach. (3+3+4)

2A. If f is a bilinear form on C^3 and C^2 that is defined by

$$f((x_1, x_2, x_3), (y_1, y_2)) = x_1y_2 + x_2y_1 + 2x_2y_2 - 2x_3y_1 + 2x_3y_2$$

i. Write the matrix A relative to $\mathcal{A} = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$,
 $\mathcal{B} = \{(1, -1), (2, -1)\}$.

ii. Use the matrix A to compute the value of $f((i, 0, i), (2, 0))$.

2B. Prove that a linear operator T on V is an isometry if and only if

$$(T(u), T(v)) = (u, v) \text{ for all } u, v \text{ in } V.$$

2C. Solve the initial value problem $\vec{x}' = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} \vec{x}$, $\vec{x}(0) = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

(3+3+4)

3A. Orthogonally diagonalize the matrix $\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

3B. Define an inner product space. Prove that a mapping f of $V \times V$ into F is an inner product on V iff f is a positive definite hermitian form.

- 3C. Apply the Lagrange's reduction method to find the quadratic form

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
 and also find its index and signature. **(3+3+4)**
- 4A. Use Gram-Schmidt process to find a set of orthonormal vectors from
 $(1, 1, 0), (1, 0, -2), (1, 1, 1)$ in E^3 .
- 4B. Given $A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$ where $A_{11} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$, $A_{12} =$
 $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, $A_{22} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ find A^{-1} .
- 4C. Prove that any interchange of two elements in a permutation j_1, j_2, \dots, j_n of the set $\{1, 2, \dots, n\}$ changes the index by an odd integer. Give one example.
(3+3+4)
- 5A. Find the single value decomposition of $\begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.
- 5B. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ using Jacobi's method. Carry out 3 iterations.
(6+4)
- 6A. Find the orthogonal transformation which transforms the quadratic form
 $3x_1^2 - 2x_2^2 - x_3^2 - 4x_1x_2 + 12x_2x_3 + 8x_1x_3$
to canonical form. Determine its index, signature and nature.
- 6B. State and prove Cayley – Hamilton theorem.
- 6C. Define orthogonal and orthonormal sets with an example. **(5+3+2)**
