Table of spherical harmonics

This is a **table of orthonormalized spherical harmonics** that employ the Condon-Shortley phase up to degree l = 10. Some of these formulas give the "Cartesian" version. This assumes x, y, z, and r are related to θ and φ through the usual spherical-to-Cartesian coordinate transformation:

$$\left\{egin{array}{ll} x &= r\sin heta\cosarphi \ y &= r\sin heta\sinarphi \ z &= r\cos heta \end{array}
ight.$$

Contents

Spherical harmonics

 $I = 0^{[1]}$

 $I = 1^{[1]}$

 $I = 2^{[1]}$

 $I = 3^{[1]}$

 $I = 4^{[1]}$

 $I = 5^{[1]}$

l = 6

I = 7

I = 8

I = 9

I = 10

Real spherical harmonics

 $I = 0^{[2][3]}$

 $I = 1^{[2][3]}$

 $I = 2^{[2][3]}$

 $|=3^{[2]}$

l = 4

See also

External links

References

Spherical harmonics

$$I = 0^{[1]}$$

$$Y_0^0(heta,arphi)=rac{1}{2}\sqrt{rac{1}{\pi}}$$

$$I = 1^{[1]}$$

$$egin{array}{lll} Y_1^{-1}(heta,arphi) =& & rac{1}{2}\sqrt{rac{3}{2\pi}}\cdot e^{-iarphi}\cdot \sin heta &=& rac{1}{2}\sqrt{rac{3}{2\pi}}\cdot rac{(x-iy)}{r} \ &Y_1^0(heta,arphi) =& & rac{1}{2}\sqrt{rac{3}{\pi}}\cdot \cos heta &=& rac{1}{2}\sqrt{rac{3}{\pi}}\cdot rac{z}{r} \ &Y_1^1(heta,arphi) =& & -rac{1}{2}\sqrt{rac{3}{2\pi}}\cdot e^{iarphi}\cdot \sin heta &=& -rac{1}{2}\sqrt{rac{3}{2\pi}}\cdot rac{(x+iy)}{r} \end{array}$$

$$I = 2^{[1]}$$

$$\begin{array}{lll} Y_{2}^{-2}(\theta,\varphi) = & \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot e^{-2i\varphi} \cdot \sin^{2}\theta & = & \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot \frac{(x-iy)^{2}}{r^{2}} \\ Y_{2}^{-1}(\theta,\varphi) = & \frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot \cos\theta & = & \frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot \frac{(x-iy)z}{r^{2}} \\ Y_{2}^{0}(\theta,\varphi) = & \frac{1}{4}\sqrt{\frac{5}{\pi}} \cdot (3\cos^{2}\theta - 1) & = & \frac{1}{4}\sqrt{\frac{5}{\pi}} \cdot \frac{(2z^{2} - x^{2} - y^{2})}{r^{2}} \\ Y_{2}^{1}(\theta,\varphi) = & -\frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta \cdot \cos\theta & = & -\frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot \frac{(x+iy)z}{r^{2}} \\ Y_{2}^{2}(\theta,\varphi) = & \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^{2}\theta & = & \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot \frac{(x+iy)^{2}}{r^{2}} \end{array}$$

 $I = 3^{[1]}$

$$\begin{split} Y_3^{-3}(\theta,\varphi) &= \frac{1}{8} \sqrt{\frac{35}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^3 \theta &= \frac{1}{8} \sqrt{\frac{35}{\pi}} \cdot \frac{(x-iy)^3}{r^3} \\ Y_3^{-2}(\theta,\varphi) &= \frac{1}{4} \sqrt{\frac{105}{2\pi}} \cdot e^{-2i\varphi} \cdot \sin^2 \theta \cdot \cos \theta &= \frac{1}{4} \sqrt{\frac{105}{2\pi}} \cdot \frac{(x-iy)^2 z}{r^3} \\ Y_3^{-1}(\theta,\varphi) &= \frac{1}{8} \sqrt{\frac{21}{\pi}} \cdot e^{-i\varphi} \cdot \sin \theta \cdot (5\cos^2 \theta - 1) &= \frac{1}{8} \sqrt{\frac{21}{\pi}} \cdot \frac{(x-iy)(4z^2 - x^2 - y^2)}{r^3} \\ Y_3^0(\theta,\varphi) &= \frac{1}{4} \sqrt{\frac{7}{\pi}} \cdot (5\cos^3 \theta - 3\cos \theta) &= \frac{1}{4} \sqrt{\frac{7}{\pi}} \cdot \frac{z(2z^2 - 3x^2 - 3y^2)}{r^3} \\ Y_3^1(\theta,\varphi) &= -\frac{1}{8} \sqrt{\frac{21}{\pi}} \cdot e^{i\varphi} \cdot \sin \theta \cdot (5\cos^2 \theta - 1) &= \frac{-1}{8} \sqrt{\frac{21}{\pi}} \cdot \frac{(x+iy)(4z^2 - x^2 - y^2)}{r^3} \\ Y_3^2(\theta,\varphi) &= \frac{1}{4} \sqrt{\frac{105}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^2 \theta \cdot \cos \theta &= \frac{1}{4} \sqrt{\frac{105}{2\pi}} \cdot \frac{(x+iy)^2 z}{r^3} \\ Y_3^3(\theta,\varphi) &= -\frac{1}{8} \sqrt{\frac{35}{\pi}} \cdot e^{3i\varphi} \cdot \sin^3 \theta &= \frac{-1}{8} \sqrt{\frac{35}{\pi}} \cdot \frac{(x+iy)^3}{r^3} \end{split}$$

 $I = 4^{[1]}$

$$\begin{split} Y_4^{-4}(\theta,\varphi) &= \frac{3}{16} \sqrt{\frac{35}{2\pi}} \cdot e^{-4i\varphi} \cdot \sin^4\theta = \frac{3}{16} \sqrt{\frac{35}{2\pi}} \cdot \frac{(x-iy)^4}{r^4} \\ Y_4^{-3}(\theta,\varphi) &= \frac{3}{8} \sqrt{\frac{35}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^3\theta \cdot \cos\theta = \frac{3}{8} \sqrt{\frac{35}{\pi}} \cdot \frac{(x-iy)^3z}{r^4} \\ Y_4^{-2}(\theta,\varphi) &= \frac{3}{8} \sqrt{\frac{5}{2\pi}} \cdot e^{-2i\varphi} \cdot \sin^2\theta \cdot (7\cos^2\theta - 1) = \frac{3}{8} \sqrt{\frac{5}{2\pi}} \cdot \frac{(x-iy)^2 \cdot (7z^2 - r^2)}{r^4} \\ Y_4^{-1}(\theta,\varphi) &= \frac{3}{8} \sqrt{\frac{5}{\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (7\cos^3\theta - 3\cos\theta) = \frac{3}{8} \sqrt{\frac{5}{\pi}} \cdot \frac{(x-iy) \cdot z \cdot (7z^2 - 3r^2)}{r^4} \\ Y_4^{0}(\theta,\varphi) &= \frac{3}{16} \sqrt{\frac{1}{\pi}} \cdot (35\cos^4\theta - 30\cos^2\theta + 3) = \frac{3}{16} \sqrt{\frac{1}{\pi}} \cdot \frac{(35z^4 - 30z^2r^2 + 3r^4)}{r^4} \\ Y_4^{1}(\theta,\varphi) &= \frac{-3}{8} \sqrt{\frac{5}{\pi}} \cdot e^{i\varphi} \cdot \sin\theta \cdot (7\cos^3\theta - 3\cos\theta) = \frac{-3}{8} \sqrt{\frac{5}{\pi}} \cdot \frac{(x+iy) \cdot z \cdot (7z^2 - 3r^2)}{r^4} \\ Y_4^{2}(\theta,\varphi) &= \frac{3}{8} \sqrt{\frac{5}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^2\theta \cdot (7\cos^2\theta - 1) = \frac{3}{8} \sqrt{\frac{5}{2\pi}} \cdot \frac{(x+iy)^2 \cdot (7z^2 - r^2)}{r^4} \\ Y_4^{3}(\theta,\varphi) &= \frac{-3}{8} \sqrt{\frac{35}{\pi}} \cdot e^{3i\varphi} \cdot \sin^3\theta \cdot \cos\theta = \frac{-3}{8} \sqrt{\frac{35}{\pi}} \cdot \frac{(x+iy)^3z}{r^4} \\ Y_4^{4}(\theta,\varphi) &= \frac{3}{16} \sqrt{\frac{35}{2\pi}} \cdot e^{4i\varphi} \cdot \sin^4\theta = \frac{3}{16} \sqrt{\frac{35}{2\pi}} \cdot \frac{(x+iy)^4}{r^4} \end{split}$$

$$\begin{split} Y_5^{-5}(\theta,\varphi) &= \frac{3}{32} \sqrt{\frac{77}{\pi}} \cdot e^{-5i\varphi} \cdot \sin^5\theta \\ Y_5^{-4}(\theta,\varphi) &= \frac{3}{16} \sqrt{\frac{385}{2\pi}} \cdot e^{-4i\varphi} \cdot \sin^4\theta \cdot \cos\theta \\ Y_5^{-3}(\theta,\varphi) &= \frac{1}{32} \sqrt{\frac{385}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^3\theta \cdot (9\cos^2\theta - 1) \\ Y_5^{-2}(\theta,\varphi) &= \frac{1}{8} \sqrt{\frac{1155}{2\pi}} \cdot e^{-2i\varphi} \cdot \sin^2\theta \cdot (3\cos^3\theta - \cos\theta) \\ Y_5^{-1}(\theta,\varphi) &= \frac{1}{16} \sqrt{\frac{165}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (21\cos^4\theta - 14\cos^2\theta + 1) \\ Y_5^{0}(\theta,\varphi) &= \frac{1}{16} \sqrt{\frac{11}{\pi}} \cdot (63\cos^5\theta - 70\cos^3\theta + 15\cos\theta) \\ Y_5^{1}(\theta,\varphi) &= \frac{-1}{16} \sqrt{\frac{165}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta \cdot (21\cos^4\theta - 14\cos^2\theta + 1) \\ Y_5^{2}(\theta,\varphi) &= \frac{1}{8} \sqrt{\frac{1155}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^2\theta \cdot (3\cos^3\theta - \cos\theta) \\ Y_5^{3}(\theta,\varphi) &= \frac{-1}{32} \sqrt{\frac{385}{\pi}} \cdot e^{3i\varphi} \cdot \sin^3\theta \cdot (9\cos^2\theta - 1) \\ Y_5^{4}(\theta,\varphi) &= \frac{3}{16} \sqrt{\frac{385}{2\pi}} \cdot e^{4i\varphi} \cdot \sin^4\theta \cdot \cos\theta \\ Y_5^{5}(\theta,\varphi) &= \frac{-3}{32} \sqrt{\frac{77}{\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \end{split}$$

$$\begin{split} Y_6^{-6}(\theta,\varphi) &= \frac{1}{64} \sqrt{\frac{3003}{\pi}} \cdot e^{-6i\varphi} \cdot \sin^6\theta \\ Y_6^{-5}(\theta,\varphi) &= \frac{3}{32} \sqrt{\frac{1001}{\pi}} \cdot e^{-5i\varphi} \cdot \sin^5\theta \cdot \cos\theta \\ Y_6^{-4}(\theta,\varphi) &= \frac{3}{32} \sqrt{\frac{91}{2\pi}} \cdot e^{-4i\varphi} \cdot \sin^4\theta \cdot (11\cos^2\theta - 1) \\ Y_6^{-3}(\theta,\varphi) &= \frac{1}{32} \sqrt{\frac{1365}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^3\theta \cdot (11\cos^3\theta - 3\cos\theta) \\ Y_6^{-2}(\theta,\varphi) &= \frac{1}{64} \sqrt{\frac{1365}{\pi}} \cdot e^{-2i\varphi} \cdot \sin^2\theta \cdot (33\cos^4\theta - 18\cos^2\theta + 1) \\ Y_6^{-1}(\theta,\varphi) &= \frac{1}{16} \sqrt{\frac{273}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (33\cos^5\theta - 30\cos^3\theta + 5\cos\theta) \\ Y_6^0(\theta,\varphi) &= \frac{1}{32} \sqrt{\frac{13}{\pi}} \cdot (231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5) \\ Y_6^1(\theta,\varphi) &= -\frac{1}{16} \sqrt{\frac{273}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta \cdot (33\cos^5\theta - 30\cos^3\theta + 5\cos\theta) \\ Y_6^2(\theta,\varphi) &= \frac{1}{64} \sqrt{\frac{1365}{\pi}} \cdot e^{2i\varphi} \cdot \sin\theta \cdot (33\cos^5\theta - 30\cos^3\theta + 5\cos\theta) \\ Y_6^2(\theta,\varphi) &= -\frac{1}{32} \sqrt{\frac{1365}{\pi}} \cdot e^{3i\varphi} \cdot \sin^2\theta \cdot (33\cos^4\theta - 18\cos^2\theta + 1) \\ Y_6^3(\theta,\varphi) &= -\frac{1}{32} \sqrt{\frac{1365}{\pi}} \cdot e^{3i\varphi} \cdot \sin^3\theta \cdot (11\cos^3\theta - 3\cos\theta) \\ Y_6^4(\theta,\varphi) &= \frac{3}{32} \sqrt{\frac{91}{2\pi}} \cdot e^{4i\varphi} \cdot \sin^4\theta \cdot (11\cos^2\theta - 1) \\ Y_6^5(\theta,\varphi) &= -\frac{3}{32} \sqrt{\frac{1001}{\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \cdot \cos\theta \\ Y_6^6(\theta,\varphi) &= \frac{1}{64} \sqrt{\frac{3003}{\pi}} \cdot e^{6i\varphi} \cdot \sin^6\theta \end{split}$$

$$\begin{split} Y_7^{-7}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{715}{2\pi}} \cdot e^{-7i\varphi} \cdot \sin^7 \theta \\ Y_7^{-6}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{5005}{\pi}} \cdot e^{-6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{-5}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{385}{2\pi}} \cdot e^{-5i\varphi} \cdot \sin^5 \theta \cdot (13\cos^2 \theta - 1) \\ Y_7^{-4}(\theta,\varphi) &= \frac{3}{32} \sqrt{\frac{385}{2\pi}} \cdot e^{-4i\varphi} \cdot \sin^4 \theta \cdot (13\cos^3 \theta - 3\cos \theta) \\ Y_7^{-3}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{35}{2\pi}} \cdot e^{-3i\varphi} \cdot \sin^3 \theta \cdot (143\cos^4 \theta - 66\cos^2 \theta + 3) \\ Y_7^{-2}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{35}{\pi}} \cdot e^{-2i\varphi} \cdot \sin^2 \theta \cdot (143\cos^5 \theta - 110\cos^3 \theta + 15\cos \theta) \\ Y_7^{-1}(\theta,\varphi) &= \frac{1}{64} \sqrt{\frac{105}{2\pi}} \cdot e^{-i\varphi} \cdot \sin \theta \cdot (429\cos^6 \theta - 495\cos^4 \theta + 135\cos^2 \theta - 5) \\ Y_7^{0}(\theta,\varphi) &= \frac{1}{32} \sqrt{\frac{15}{\pi}} \cdot (429\cos^7 \theta - 693\cos^5 \theta + 315\cos^3 \theta - 35\cos \theta) \\ Y_7^{1}(\theta,\varphi) &= -\frac{1}{64} \sqrt{\frac{105}{2\pi}} \cdot e^{i\varphi} \cdot \sin \theta \cdot (429\cos^6 \theta - 495\cos^4 \theta + 135\cos^2 \theta - 5) \\ Y_7^{2}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{35}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^2 \theta \cdot (143\cos^5 \theta - 110\cos^3 \theta + 15\cos \theta) \\ Y_7^{3}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{35}{2\pi}} \cdot e^{3i\varphi} \cdot \sin^3 \theta \cdot (143\cos^4 \theta - 66\cos^2 \theta + 3) \\ Y_7^{4}(\theta,\varphi) &= \frac{3}{32} \sqrt{\frac{385}{2\pi}} \cdot e^{3i\varphi} \cdot \sin^4 \theta \cdot (13\cos^3 \theta - 3\cos \theta) \\ Y_7^{5}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{385}{2\pi}} \cdot e^{5i\varphi} \cdot \sin^5 \theta \cdot (13\cos^2 \theta - 1) \\ Y_7^{6}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{5005}{\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^6 \theta \cdot \cos \theta \\ Y_7^{7}(\theta,\varphi) &= -\frac{3}{64} \sqrt{\frac{5005}{2\pi}} \cdot e^{6i\varphi} \cdot \sin^7 \theta \\ \end{pmatrix}$$

$$\begin{split} Y_8^{-8}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{12155}{2\pi}} \cdot e^{-8i\varphi} \cdot \sin^8\theta \\ Y_8^{-7}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{12155}{2\pi}} \cdot e^{-7i\varphi} \cdot \sin^7\theta \cdot \cos\theta \\ Y_8^{-6}(\theta,\varphi) &= \frac{1}{128} \sqrt{\frac{7293}{\pi}} \cdot e^{-6i\varphi} \cdot \sin^6\theta \cdot (15\cos^2\theta - 1) \\ Y_8^{-5}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{17017}{2\pi}} \cdot e^{-5i\varphi} \cdot \sin^5\theta \cdot (5\cos^3\theta - \cos\theta) \\ Y_8^{-4}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{1300}{2\pi}} \cdot e^{-4i\varphi} \cdot \sin^4\theta \cdot (65\cos^4\theta - 26\cos^2\theta + 1) \\ Y_8^{-3}(\theta,\varphi) &= \frac{1}{64} \sqrt{\frac{19635}{2\pi}} \cdot e^{-3i\varphi} \cdot \sin^3\theta \cdot (39\cos^5\theta - 26\cos^3\theta + 3\cos\theta) \\ Y_8^{-2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{595}{\pi}} \cdot e^{-2i\varphi} \cdot \sin^2\theta \cdot (143\cos^6\theta - 143\cos^4\theta + 33\cos^2\theta - 1) \\ Y_8^{-1}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{17}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (715\cos^7\theta - 1001\cos^5\theta + 385\cos^3\theta - 35\cos\theta) \\ Y_8^{0}(\theta,\varphi) &= \frac{1}{256} \sqrt{\frac{17}{\pi}} \cdot (6435\cos^8\theta - 12012\cos^6\theta + 6930\cos^4\theta - 1260\cos^2\theta + 35) \\ Y_8^{1}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{17}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta \cdot (715\cos^7\theta - 1001\cos^5\theta + 385\cos^3\theta - 35\cos\theta) \\ Y_8^{2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{595}{\pi}} \cdot e^{2i\varphi} \cdot \sin\theta \cdot (715\cos^7\theta - 1001\cos^5\theta + 385\cos^3\theta - 35\cos\theta) \\ Y_8^{2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{595}{\pi}} \cdot e^{2i\varphi} \cdot \sin^2\theta \cdot (143\cos^6\theta - 143\cos^4\theta + 33\cos^2\theta - 1) \\ Y_8^{3}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{19635}{2\pi}} \cdot e^{3i\varphi} \cdot \sin^3\theta \cdot (39\cos^5\theta - 26\cos^3\theta + 3\cos\theta) \\ Y_8^{4}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{17017}{2\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \cdot (5\cos^4\theta - 26\cos^2\theta + 1) \\ Y_8^{5}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{17017}{2\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \cdot (5\cos^3\theta - \cos\theta) \\ Y_8^{6}(\theta,\varphi) &= \frac{1}{128} \sqrt{\frac{7293}{2\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \cdot (15\cos^2\theta - 1) \\ Y_8^{7}(\theta,\varphi) &= \frac{3}{64} \sqrt{\frac{12155}{2\pi}} \cdot e^{7i\varphi} \cdot \sin^7\theta \cdot \cos\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \cdot e^{8i\varphi} \cdot \sin^8\theta \\ Y_8^{8}(\theta,\varphi) &= \frac{3}{256$$

$$\begin{split} Y_{9}^{-9}(\theta,\varphi) &= \frac{1}{512} \sqrt{\frac{230945}{\pi}} \cdot e^{-9i\varphi} \cdot \sin^{9}\theta \\ Y_{9}^{-8}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{230945}{2\pi}} \cdot e^{-8i\varphi} \cdot \sin^{8}\theta \cdot \cos\theta \\ Y_{9}^{-7}(\theta,\varphi) &= \frac{3}{512} \sqrt{\frac{13585}{\pi}} \cdot e^{-7i\varphi} \cdot \sin^{7}\theta \cdot (17\cos^{2}\theta - 1) \\ Y_{9}^{-6}(\theta,\varphi) &= \frac{1}{128} \sqrt{\frac{40755}{\pi}} \cdot e^{-6i\varphi} \cdot \sin^{6}\theta \cdot (17\cos^{3}\theta - 3\cos\theta) \\ Y_{9}^{-5}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{2717}{\pi}} \cdot e^{-5i\varphi} \cdot \sin^{5}\theta \cdot (85\cos^{4}\theta - 30\cos^{2}\theta + 1) \\ Y_{9}^{-4}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{95095}{2\pi}} \cdot e^{-4i\varphi} \cdot \sin^{4}\theta \cdot (17\cos^{5}\theta - 10\cos^{3}\theta + \cos\theta) \\ Y_{9}^{-8}(\theta,\varphi) &= \frac{1}{256} \sqrt{\frac{21945}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^{3}\theta \cdot (221\cos^{6}\theta - 195\cos^{4}\theta + 39\cos^{2}\theta - 1) \\ Y_{9}^{-2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{1045}{\pi}} \cdot e^{-2i\varphi} \cdot \sin^{2}\theta \cdot (221\cos^{7}\theta - 273\cos^{5}\theta + 91\cos^{3}\theta - 7\cos\theta) \\ Y_{9}^{-1}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{95}{\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (2431\cos^{8}\theta - 4004\cos^{6}\theta + 2002\cos^{4}\theta - 308\cos^{2}\theta + 7) \\ Y_{9}^{0}(\theta,\varphi) &= \frac{1}{256} \sqrt{\frac{19}{\pi}} \cdot (12155\cos^{9}\theta - 25740\cos^{7}\theta + 18018\cos^{5}\theta - 4620\cos^{3}\theta + 315\cos\theta) \\ Y_{9}^{2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{1045}{\pi}} \cdot e^{2i\varphi} \cdot \sin\theta \cdot (2431\cos^{8}\theta - 4004\cos^{6}\theta + 2002\cos^{4}\theta - 308\cos^{2}\theta + 7) \\ Y_{9}^{2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{1045}{\pi}} \cdot e^{2i\varphi} \cdot \sin^{2}\theta \cdot (221\cos^{7}\theta - 273\cos^{5}\theta + 91\cos^{3}\theta - 7\cos\theta) \\ Y_{9}^{2}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{1045}{\pi}} \cdot e^{2i\varphi} \cdot \sin^{3}\theta \cdot (221\cos^{7}\theta - 273\cos^{5}\theta + 91\cos^{3}\theta - 7\cos\theta) \\ Y_{9}^{4}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{21945}{\pi}} \cdot e^{3i\varphi} \cdot \sin^{3}\theta \cdot (221\cos^{6}\theta - 195\cos^{4}\theta + 39\cos^{2}\theta - 1) \\ Y_{9}^{4}(\theta,\varphi) &= \frac{3}{128} \sqrt{\frac{21717}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{5}\theta \cdot (85\cos^{4}\theta - 30\cos^{2}\theta + 1) \\ Y_{9}^{6}(\theta,\varphi) &= \frac{-3}{128} \sqrt{\frac{40755}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{6}\theta \cdot (17\cos^{3}\theta - 3\cos\theta) \\ Y_{9}^{7}(\theta,\varphi) &= \frac{-3}{512} \sqrt{\frac{13585}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{6}\theta \cdot (17\cos^{3}\theta - 3\cos\theta) \\ Y_{9}^{6}(\theta,\varphi) &= \frac{-3}{512} \sqrt{\frac{230945}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{8}\theta \cdot \cos\theta \\ Y_{9}^{9}(\theta,\varphi) &= \frac{-1}{512} \sqrt{\frac{230945}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{9}\theta \\ \end{pmatrix}$$

$$\begin{split} Y_{10}^{-10}(\theta,\varphi) &= \frac{1}{1024} \sqrt{\frac{969969}{\pi}} \cdot e^{-18i\varphi} \cdot e^{-9i\varphi} \cdot \sin^{10}\theta \\ Y_{10}^{-9}(\theta,\varphi) &= \frac{1}{512} \sqrt{\frac{484945}{\pi}} \cdot e^{-8i\varphi} \cdot \sin^{9}\theta \cdot \cos\theta \\ Y_{10}^{-8}(\theta,\varphi) &= \frac{1}{512} \sqrt{\frac{25525}{\pi}} \cdot e^{-8i\varphi} \cdot \sin^{8}\theta \cdot (19\cos^{2}\theta - 1) \\ Y_{10}^{-7}(\theta,\varphi) &= \frac{3}{512} \sqrt{\frac{5005}{\pi}} \cdot e^{-8i\varphi} \cdot \sin^{7}\theta \cdot (19\cos^{3}\theta - 3\cos\theta) \\ Y_{10}^{-6}(\theta,\varphi) &= \frac{3}{1024} \sqrt{\frac{5005}{\pi}} \cdot e^{-8i\varphi} \cdot \sin^{6}\theta \cdot (323\cos^{4}\theta - 102\cos^{2}\theta + 3) \\ Y_{10}^{-5}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{-8i\varphi} \cdot \sin^{6}\theta \cdot (323\cos^{5}\theta - 170\cos^{3}\theta + 15\cos\theta) \\ Y_{10}^{-4}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{-4i\varphi} \cdot \sin^{4}\theta \cdot (323\cos^{5}\theta - 255\cos^{4}\theta + 45\cos^{2}\theta - 1) \\ Y_{10}^{-3}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^{3}\theta \cdot (323\cos^{5}\theta - 255\cos^{4}\theta + 45\cos^{2}\theta - 1) \\ Y_{10}^{-3}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{-3i\varphi} \cdot \sin^{3}\theta \cdot (323\cos^{7}\theta - 357\cos^{5}\theta + 105\cos^{3}\theta - 7\cos\theta) \\ Y_{10}^{-2}(\theta,\varphi) &= \frac{3}{512} \sqrt{\frac{35}{2\pi}} \cdot e^{-2i\varphi} \cdot \sin^{2}\theta \cdot (4199\cos^{8}\theta - 6188\cos^{6}\theta + 2730\cos^{4}\theta - 364\cos^{2}\theta + 7) \\ Y_{10}^{-1}(\theta,\varphi) &= \frac{1}{512} \sqrt{\frac{1155}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (4199\cos^{8}\theta - 7956\cos^{7}\theta + 4914\cos^{5}\theta - 1092\cos^{3}\theta + 63\cos\theta) \\ Y_{10}^{10}(\theta,\varphi) &= \frac{1}{512} \sqrt{\frac{1155}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot (4199\cos^{8}\theta - 6188\cos^{6}\theta + 2730\cos^{4}\theta - 364\cos^{2}\theta + 7) \\ Y_{10}^{2}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{385}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^{2}\theta \cdot (4199\cos^{8}\theta - 6188\cos^{6}\theta + 2730\cos^{4}\theta - 364\cos^{2}\theta + 7) \\ Y_{10}^{3}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{2\pi}} \cdot e^{3i\varphi} \cdot \sin^{3}\theta \cdot (323\cos^{7}\theta - 357\cos^{5}\theta + 105\cos^{3}\theta - 7\cos\theta) \\ Y_{10}^{4}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{2\pi}} \cdot e^{3i\varphi} \cdot \sin^{4}\theta \cdot (323\cos^{8}\theta - 255\cos^{4}\theta + 45\cos^{2}\theta - 1) \\ Y_{10}^{5}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{2\pi}} \cdot e^{3i\varphi} \cdot \sin^{4}\theta \cdot (323\cos^{8}\theta - 255\cos^{4}\theta + 45\cos^{2}\theta - 1) \\ Y_{10}^{5}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{3i\varphi} \cdot \sin^{4}\theta \cdot (323\cos^{8}\theta - 255\cos^{4}\theta + 45\cos^{2}\theta - 1) \\ Y_{10}^{5}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{6}\theta \cdot (323\cos^{5}\theta - 170\cos^{3}\theta + 15\cos\theta) \\ Y_{10}^{5}(\theta,\varphi) &= \frac{3}{256} \sqrt{\frac{5005}{\pi}} \cdot e^{5i\varphi} \cdot \sin^{6}\theta \cdot (323\cos^{5}\theta - 170\cos^{3}\theta + 15\cos\theta) \\ Y_{10}^{5}(\theta,\varphi) &= \frac{3}{512} \sqrt{\frac{5005}{2\pi}} \cdot e^{5i\varphi} \cdot \sin^{6}\theta \cdot (323\cos^{6}\theta - 255\cos^{6}\theta + 15$$

Real spherical harmonics

For each real spherical harmonic, the corresponding atomic orbital symbol (s, p, d, f, g) is reported as well

 $I = 0^{[2][3]}$

$$Y_{00}=s=Y_0^0=rac{1}{2}\sqrt{rac{1}{\pi}}$$

 $I = 1^{[2][3]}$

$$egin{align} Y_{1,-1} &= p_y = i\sqrt{rac{1}{2}}\left(Y_1^{-1} + Y_1^1
ight) = \sqrt{rac{3}{4\pi}}\cdotrac{y}{r} \ Y_{1,0} &= p_z = Y_1^0 = \sqrt{rac{3}{4\pi}}\cdotrac{z}{r} \ Y_{1,1} &= p_x = \sqrt{rac{1}{2}}\left(Y_1^{-1} - Y_1^1
ight) = \sqrt{rac{3}{4\pi}}\cdotrac{x}{r} \ \end{array}$$

 $I = 2^{[2][3]}$

$$egin{aligned} Y_{2,-2} &= d_{xy} = i \sqrt{rac{1}{2}} \left(Y_2^{-2} - Y_2^2
ight) = rac{1}{2} \sqrt{rac{15}{\pi}} \cdot rac{xy}{r^2} \ Y_{2,-1} &= d_{yz} = i \sqrt{rac{1}{2}} \left(Y_2^{-1} + Y_2^1
ight) = rac{1}{2} \sqrt{rac{15}{\pi}} \cdot rac{yz}{r^2} \ Y_{2,0} &= d_{z^2} = Y_2^0 = rac{1}{4} \sqrt{rac{5}{\pi}} \cdot rac{-x^2 - y^2 + 2z^2}{r^2} \ Y_{2,1} &= d_{xz} = \sqrt{rac{1}{2}} \left(Y_2^{-1} - Y_2^1
ight) = rac{1}{2} \sqrt{rac{15}{\pi}} \cdot rac{zx}{r^2} \ Y_{2,2} &= d_{x^2 - y^2} = \sqrt{rac{1}{2}} \left(Y_2^{-2} + Y_2^2
ight) = rac{1}{4} \sqrt{rac{15}{\pi}} \cdot rac{x^2 - y^2}{r^2} \end{aligned}$$

 $I = 3^{[2]}$

$$egin{aligned} Y_{3,-3} &= f_{y(3x^2-y^2)} = i\sqrt{rac{1}{2}}\left(Y_3^{-3} + Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(3x^2-y^2
ight)y}{r^3} \ Y_{3,-2} &= f_{xyz} = i\sqrt{rac{1}{2}}\left(Y_3^{-2} - Y_3^2
ight) = rac{1}{2}\sqrt{rac{105}{\pi}}\cdotrac{xyz}{r^3} \ Y_{3,-1} &= f_{yz^2} = i\sqrt{rac{1}{2}}\left(Y_3^{-1} + Y_3^1
ight) = rac{1}{4}\sqrt{rac{21}{2\pi}}\cdotrac{y(4z^2-x^2-y^2)}{r^3} \ Y_{3,0} &= f_{z^3} = Y_3^0 = rac{1}{4}\sqrt{rac{7}{\pi}}\cdotrac{z(2z^2-3x^2-3y^2)}{r^3} \ Y_{3,1} &= f_{xz^2} = \sqrt{rac{1}{2}}\left(Y_3^{-1} - Y_3^1
ight) = rac{1}{4}\sqrt{rac{21}{2\pi}}\cdotrac{x(4z^2-x^2-y^2)}{r^3} \ Y_{3,2} &= f_{z(x^2-y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-2} + Y_3^2
ight) = rac{1}{4}\sqrt{rac{105}{\pi}}\cdotrac{\left(x^2-y^2
ight)z}{r^3} \ Y_{3,3} &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_{3,3} &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_{3,3} &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_{3,3} &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_{3,3} &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_3 &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_3 &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_3 &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_3 &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = rac{1}{4}\sqrt{rac{35}{2\pi}}\cdotrac{\left(x^2-3y^2
ight)x}{r^3} \ Y_3 &= f_{x(x^2-3y^2)} = \sqrt{rac{1}{2}}\left(Y_3^{-3} - Y_3^3
ight) = \frac{1}{2}\sqrt{rac{35}{2\pi}} \ Y_3 &= f_{x(x^2-3y^2)} = \sqrt{rac{35}{2}}\left(Y_3^2 - Y_3^3
ight) = \frac{1}{2}\sqrt{rac{35}{2\pi}} \ Y_3 &= f_{x(x^2-3y^2)} + \frac{1}{2}\sqrt{rac{35}{2\pi}} \ Y_3 &$$

$$\begin{split} Y_{4,-4} &= g_{xy(x^2-y^2)} = i\sqrt{\frac{1}{2}} \left(Y_4^{-4} - Y_4^4\right) = \frac{3}{4}\sqrt{\frac{35}{\pi}} \cdot \frac{xy \left(x^2 - y^2\right)}{r^4} \\ Y_{4,-3} &= g_{zy^3} = i\sqrt{\frac{1}{2}} \left(Y_4^{-3} + Y_4^3\right) = \frac{3}{4}\sqrt{\frac{35}{2\pi}} \cdot \frac{(3x^2 - y^2)yz}{r^4} \\ Y_{4,-2} &= g_{z^2xy} = i\sqrt{\frac{1}{2}} \left(Y_4^{-2} - Y_4^2\right) = \frac{3}{4}\sqrt{\frac{5}{\pi}} \cdot \frac{xy \cdot (7z^2 - r^2)}{r^4} \\ Y_{4,-1} &= g_{z^3y} = i\sqrt{\frac{1}{2}} \left(Y_4^{-1} + Y_4^1\right) = \frac{3}{4}\sqrt{\frac{5}{2\pi}} \cdot \frac{yz \cdot (7z^2 - 3r^2)}{r^4} \\ Y_{4,0} &= g_{z^4} = Y_4^0 = \frac{3}{16}\sqrt{\frac{1}{\pi}} \cdot \frac{(35z^4 - 30z^2r^2 + 3r^4)}{r^4} \\ Y_{4,1} &= g_{z^3x} = \sqrt{\frac{1}{2}} \left(Y_4^{-1} - Y_4^1\right) = \frac{3}{4}\sqrt{\frac{5}{2\pi}} \cdot \frac{xz \cdot (7z^2 - 3r^2)}{r^4} \\ Y_{4,2} &= g_{z^2xy} = \sqrt{\frac{1}{2}} \left(Y_4^{-2} + Y_4^2\right) = \frac{3}{8}\sqrt{\frac{5}{\pi}} \cdot \frac{(x^2 - y^2) \cdot (7z^2 - r^2)}{r^4} \\ Y_{4,3} &= g_{zx^3} = \sqrt{\frac{1}{2}} \left(Y_4^{-3} - Y_4^3\right) = \frac{3}{4}\sqrt{\frac{35}{2\pi}} \cdot \frac{(x^2 - 3y^2)xz}{r^4} \\ Y_{4,4} &= g_{x^4 + y^4} = \sqrt{\frac{1}{2}} \left(Y_4^{-4} + Y_4^4\right) = \frac{3}{16}\sqrt{\frac{35}{\pi}} \cdot \frac{x^2 \left(x^2 - 3y^2\right) - y^2 \left(3x^2 - y^2\right)}{r^4} \end{split}$$

See also

Spherical harmonics

External links

• Spherical Harmonic (http://mathworld.wolfram.com/SphericalHarmonic.html) at MathWorld

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General references

- See section 3 in Mathar, R. J. (2009). "Zernike basis to cartesian transformations". Serbian Astronomical Journal. 179 (179): 107–120. arXiv:0809.2368 (https://arxiv.org/abs/0809.2368) Bibcode:2009SerAj.179..107M (http://adsabs.harvard.edu/abs/2009SerAj.179..107M). doi:10.2298/SAJ0979107M (https://doi.org/10.2298%2FSAJ0979107M). (see section 3.3)
- For complex spherical harmonics, see also SphericalHarmonicY[I,m,theta,phi] at Wolfram Alpha (http://www.wolframalpha.com/input/?i=SphericalHarmonicY%5BI,m,theta,phi%5D), especially for specific values of I and m.

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