Spin spherical harmonics

In quantum mechanics, **spin spherical harmonics** $Y_{l,\ s,\ j,\ m}$ are spinors eigenstates of the total angular momentum operator squared:

$$\begin{aligned}\mathbf{j}^2 Y_{l,s,j,m} &= j(j+1)Y_{l,s,j,m} \\ \mathbf{j}_z Y_{l,s,j,m} &= mY_{l,s,j,m}\end{aligned}$$

where $\mathbf{j} = \mathbf{l} + \mathbf{s}$. They are the natural spinorial analog of vector spherical harmonics.

For spin-1/2 systems, they are given in matrix form by[1]

$$Y_{j\pm\frac{1}{2},\frac{1}{2},j,m} = \frac{1}{\sqrt{2\left(j\pm\frac{1}{2}\right)+1}}\begin{pmatrix} \mp\sqrt{j\pm\frac{1}{2}\mp m+\frac{1}{2}}Y^{m-\frac{1}{2}}_{j\pm\frac{1}{2}} \\ \sqrt{j\pm\frac{1}{2}\pm m+\frac{1}{2}}Y^{m+\frac{1}{2}}_{j\pm\frac{1}{2}}\end{pmatrix}$$

Notes

1. Biedenharn, L. C.; Louck, J. D. (1981), *Angular momentum in Quantum Physics: Theory and Application*, Encyclopedia of Mathematics, **8**, Reading: Addison-Wesley, p. 283, ISBN 0-201-13507-8

References

- Edmonds, A. R. (1957), *Angular Momentum in Quantum Mechanics*, Princeton University Press, ISBN 978-0-691-07912-7