mechanics-4

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作业题目：第四次作业
作业说明：4.1，4.4-4.8,4.10-4.12,4.14

4.1
(1)$J_A = m\overrightarrow{d_1} \times \vec{v} = md_1v, \otimes$
$J_B = m\overrightarrow{d_2} \times \vec{v} = md_2v\sin(\frac{\pi}{2}+\theta) = md_1v, \otimes$
$J_C = m\overrightarrow{d_3} \times \vec{v} = 0 \cdots (5)$
(2)$M_A = \overrightarrow{d_1} \times m\vec{g} = md_1g, \otimes$
$M_B = \overrightarrow{d_2} \times m\vec{g} = md_2g\sin(\frac{\pi}{2}+\theta) = md_1g, \otimes$
$M_C = \overrightarrow{d_3} \times m\vec{g} = 0 \cdots (5)$

4.4
这里注意我们取对竖杆的角动量，这个角动量是守恒的，对顶点的角动量不守恒
$J = mrv = ml\sin\theta v$
$$\begin{cases} T\cos\theta = mg \\ T\sin\theta = m\dfrac{v^2}{r} \end{cases}$$
$$v = \left(\frac{gJ\tan\theta}{m}\right)^{\frac{1}{3}} \cdots (5)$$
$$ml\sin\theta\left(\frac{gJ\tan\theta}{m}\right)^{\frac{1}{3}} = J \Rightarrow \sin\theta\tan^{\frac{1}{3}}\theta = \left(\frac{J^2}{gm^2}\right)^{\frac{1}{3}}\frac{1}{l} \propto \frac{1}{l}$$
$\theta \nearrow, v \nearrow, l \searrow$
速度与绳长负相关，顶角与绳长负相关$\cdots (5)$

4.5

$I = \frac{1}{2}mR^2$

$$\begin{cases} J_0 = Iw_0 + \frac{m}{2}R^2 w_0 \\ J = Iw + \frac{m}{2}\left(\frac{R}{2}\right)^2 w \end{cases} \rightarrow J = J_0 \rightarrow w = \frac{8}{5}w_0 \cdots (5)$$

$\Delta E_k = \frac{1}{2}J_0(w - w_0) = \frac{3}{10}J_0 w_0 = \frac{3}{5}E_{k0} \cdots (3)$

动能增大，增大的能量来源于玩具汽车的内部动力所做的功$\cdots (2)$

4.6

$v_{car} = v + Rw \cdots (3)$

$$\begin{cases} J_{car} = \frac{m}{2}Rv_{car} \\ J_{tab} = Iw = \frac{1}{2}mR^2 w \end{cases} \rightarrow J_{car} + J_{tab} = 0 \rightarrow w = -\frac{v}{2R} \cdots (7)$$

负号表示转台旋转方向与车的运动速度方向相反

4.7

$l_{1c} = \frac{m_2 l}{m_1 + m_2}, l_{2c} = \frac{m_1 l}{m_1 + m_2} \cdots (3)$

$$\begin{cases} J_{1c} = m_1 l_1^2 w = \frac{m_1 m_2^2 l^2 w}{(m_1 + m_2)^2} \\ J_{2c} = m_2 l_2^2 w = \frac{m_2 m_1^2 l^2 w}{(m_1 + m_2)^2} \end{cases} \rightarrow \frac{J_{1c}}{J_{2c}} = \frac{m_2}{m_1} < 1 \cdots (7)$$

4.8

$$\begin{cases} J_0 = m_1 l^2 w \\ J = J_c + J_{1c} + J_{2c} = (m_1 + m_2)l_2^2 w + \frac{m_1 m_2^2 l^2 w}{(m_1 + m_2)^2} + \frac{m_2 m_1^2 l^2 w}{(m_1 + m_2)^2} \end{cases}$$

过程中角动量守恒

$J_0 = J \rightarrow w_c = w \cdots (6)$

如果直接得到这个结论也可以给满分，说明同学的物理直觉很强

$\mu = \frac{m_1 m_2}{m_1 + m_2}$

$J_{1c} + J_{2c} = \mu l^2 w \cdots (2)$

$F^e = 0 \rightarrow a_c = 0$

$T = m_1 l_1 w^2 = m_2 l_2 w^2 = \mu l w^2 \cdots (2)$

4.10

过程中角动量守恒

$(1)J = mbv_0 = mlv \to v = \dfrac{b}{l}v_0 \cdots (2)$

$\dfrac{E_k}{E_{k0}} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mv_0^2} = \dfrac{b^2}{l^2} < 1 \cdots (3)$

$(2)L = C \Rightarrow v = C \cdots (5)$

角动量守恒，抛出后的物体做匀速直线运动

4.11

以无穷远点为参考点

$(1)U(r) - 0 = \int_r^\infty f \cdot dr = \int_r^\infty \dfrac{k}{r^2}dr = \dfrac{k}{r} \to U(r) = \dfrac{k}{r} \cdots (5)$

$$(2)\begin{cases} J = mbv_0 = mRv \\ \dfrac{1}{2}mv_0^2 = \dfrac{1}{2}mv^2 + \dfrac{k}{R} \end{cases} \to \begin{cases} v = \dfrac{-k + \sqrt{k^2 + m^2b^2v_0^4}}{mbv_0} \\ R = \dfrac{mb^2v_0^2}{-k + \sqrt{k^2 + m^2b^2v_0^4}} \end{cases} \cdots (5)$$

4.12

$k \to -k \cdots (10)$

其余结果与 4-11 完全相同

4.14

碰撞过程中有动量守恒，角动量守恒，机械能守恒，可下列出方程

$$\begin{cases} 2mv_c + mv = mv_0 \\ Iw + m\dfrac{l\sin\theta}{2}v = m\dfrac{l\sin\theta}{2}v_0, (I = \dfrac{1}{2}ml^2) \\ \dfrac{1}{2}Iw^2 + \dfrac{1}{2}(2m)v_c^2 + \dfrac{1}{2}mv^2 = \dfrac{1}{2}mv_0^2 \end{cases} \cdots (7)$$

$$\begin{cases} v = \dfrac{1 - \sin^2\theta}{3 + \sin^2\theta}v_0 \\ v_c = \dfrac{2}{3 + \sin^2\theta}v_0 \\ w = \dfrac{4\sin\theta}{3 + \sin^2\theta}\dfrac{v_0}{l} \end{cases} \to \theta = \frac{\pi}{4}, \begin{cases} v = -\dfrac{1}{7}v_0 \\ v_c = \dfrac{4}{7}v_0 \\ w = \dfrac{4\sqrt{2}}{7}\dfrac{v_0}{l} \end{cases} \cdots (3)$$

还有另外一个解$v = v_0, v_c = 0, w = 0$,这种情况对应于没有发生碰撞，不符合物理实际