DDA3020 HW4 Written Part

Problem 1.1

(1) M_1

		Prediction +	Prediction −
True	+	3	2
	−	1	4

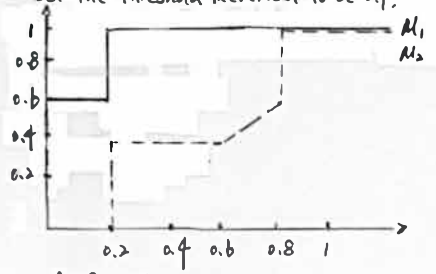
M_2

		Prediction +	Prediction −
True	+	1	4
	−	1	4

M_1: Recall $= \frac{3}{3+2} = 0.6$, Precision $= \frac{3}{3+1} = 0.75$, Acc. $= \frac{3+4}{3+2+1+4} = 0.7$

M_2: Recall $= \frac{1}{1+4} = 0.2$, Precision $= \frac{1}{1+1} = 0.5$, Acc. $= \frac{1+4}{1+4+1+4} = 0.5$

(2) Set the threshold increment to be 0.1.



$AUC_1 = 0.92$

$AUC_2 = 0.46$

$\Rightarrow M_1$ performs better.

Problem 1.2

(1) First iteration:

$C_1 = (0,0)$ $C_2 = (4,-1)$ $C_3 = (5,3)$

A_1: $d_1 = 0$, $d_2 = \sqrt{17}$, $d_3 = \sqrt{34} \Rightarrow C_1$ $A_5: \Rightarrow C_2$

A_2: $d_1 = 1$, $d_2 = \sqrt{20}$, $d_3 = \sqrt{29} \Rightarrow C_1$ $A_6: \Rightarrow C_2$

$A_3: \Rightarrow C_1$ $A_7: \Rightarrow C_2$

$A_4: \Rightarrow C_1$ $A_8: \Rightarrow C_3$

Second iteration:

$C_1 = (\frac{1}{4}, \frac{3}{4})$ $C_2 = (\frac{11}{3}, 0)$ $C_3 = (5,3)$

$A_1: \Rightarrow C_1$ $A_5: \Rightarrow C_2$

$A_2: \Rightarrow C_1$ $A_6: \Rightarrow C_2$

$A_3: \Rightarrow C_1$ $A_7: \Rightarrow C_2$

$A_4: \Rightarrow C_2$ $A_8: \Rightarrow C_3$

From above we see A_4 belongs to C_1, then C_2.

Hence the algorithm still does not converge after the first iteration.

(2) First iteration:

$C_1 = (0,0)$ $C_2 = (4,-1)$ $C_3 = (5,3)$

A_1: $d_1 = 0$, $d_2 = \sqrt{17}$, $d_3 = \sqrt{34} \Rightarrow C_1$ $A_5: \Rightarrow C_2$

$A_2: \Rightarrow C_1$ $A_6: \Rightarrow C_2$ $A_1 \ast A_4$

$A_3: \Rightarrow C_1$ $A_7: \Rightarrow C_2$ $A_7 = A_8$

$A_4: \Rightarrow \cancel{C_1}$ (C_2, cannot link) $A_8: \Rightarrow \cancel{C_3}$ (C_2, must link)

Second iteration:

$C_1 = (-\frac{1}{3}, 1)$ $C_2 = (3, -\frac{1}{3})$ $C_3 = (\frac{9}{2}, 2)$

$A_1: \Rightarrow C_1$ $A_5: \Rightarrow C_2$

$A_2: \Rightarrow C_1$ $A_6: \Rightarrow C_2$

$A_3: \Rightarrow C_1$ $A_7: \Rightarrow C_2$

$A_4: \Rightarrow C_2$ $A_8: \Rightarrow C_2$

Hence the algorithm converges after the first iteration.

Problem 1.3

(1) $\ell = \sum_{i=1}^{3} \log\left(\sum_{i=1}^{2} r_{ik} \pi_k N(x|\mu_k, \sigma_k)\right)$

(2) $\mathcal{L}(\theta, \lambda) = -\ell(\theta) + \lambda(1 - \sum_k \pi_k)$

$\frac{\partial \mathcal{L}}{\partial \pi_k} = 0$, $\pi_k = \frac{\sum_n \gamma_k^{(n)}}{N}$

$\pi_1^{new} = \frac{1.4}{3} = \frac{7}{15}$

$\pi_2^{new} = \frac{1.6}{3} = \frac{8}{15}$

$\Rightarrow \pi_1^{new} = \frac{7}{15}$, $\pi_2^{new} = \frac{8}{15}$

(3) $\frac{\partial \mathcal{L}}{\partial \mu_k} = 0$, $\mu_k = \frac{1}{\sum_n \gamma_k^{(n)}} \gamma_k^{(n)} x^{(n)}$

$\mu_1^{new} = \frac{1+4}{1.4} = \frac{25}{7}$

$\mu_2^{new} = \frac{6+20}{1.6} = \frac{65}{4}$

$\Rightarrow \mu_1^{new} = \frac{25}{7}$, $\mu_2^{new} = \frac{65}{4}$

Problem 1.4

$$\mu = \frac{1}{10}\sum_{i=1}^{10} x^{(i)} = \begin{bmatrix} -0.4 \\ 0.8 \\ 0.2 \\ 0.2 \\ -1.3 \end{bmatrix}$$

$$\Sigma = \frac{1}{10}\sum_{i=1}^{10} (x^{(n)} - \mu)(x^{(n)} - \mu)^T$$

```
[[ 3.04  0.82 -0.02 -0.82 -0.12]
 [ 0.82  3.76 -2.16  1.04  1.04]
 [-0.02 -2.16  3.56  0.76 -0.84]
 [-0.82  1.04  0.76  5.56  1.16]
 [-0.12  1.04 -0.84  1.16  2.21]]
```

As $\Sigma u_k = \lambda_k u_k$,

we have eigenvalues of Σ, with corresponding eigenvectors:

0.82311524
1.53027334
3.07614543
5.93067614
6.76978985

Choose the top 2 as the composition of U

```
[[-0.02625442  0.29809153]
 [ 0.57873744  0.39493632]
 [-0.32862419 -0.58025264]
 [ 0.65356276 -0.64618454]
 [ 0.35949345  0.03031732]]
```

Apply $x' = U^T(x - \mu)$

```
1
[[-3.13194616]
 [ 1.98183693]]
2
[[-0.60746566]
 [ 4.49653708]]
3
[[ 1.97315969]
 [-1.40348923]]
4
[[ 0.4956134 ]
 [-2.87861204]]
5
[[-0.72008587]
 [ 0.48232827]]
6
[[1.87576557]
 [1.74241301]]
7
[[5.38313097]
 [0.53945236]]
8
[[-0.591651  ]
 [-4.45776178]]
9
[[-4.46907149]
 [-1.07184006]]
10
[[-0.20744945]
 [ 0.56913546]]
```