MAT3007 HW#2

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1\. (a) False.

Considering $\min 0$ s.t. $x \geq 0$, the optimal sol. set $[0, +\infty)$ is unbounded.

(b) False.

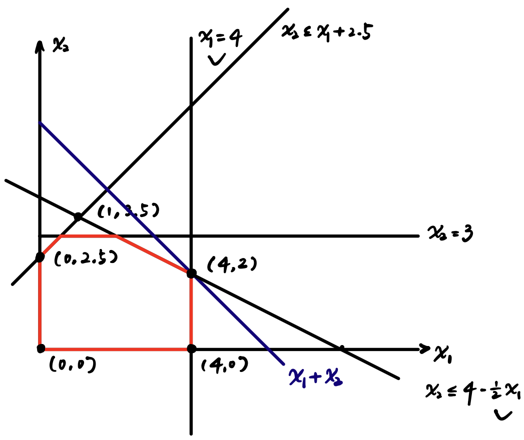
Still considering $\min 0$, any feasible x is optimal regardless of # of positive elements.

(c) True.

It is intuitive that multiple optimal sols. happen when the objective is parallel to an "edge" of feasible region, including a line, a facet, etc. More than 1 sol will lead to infinitely many sols.

If x_1, x_2 are optimal sols, any convex combination of x_1, x_2 will do.

2\.



All the coordinates of vertices are listed in the plotting.

When $x_1 = 4$, $x_2 = 2$, the maximized $x_1 + x_2 = 6$.

$0 \leq x_1 \leq 4$, $x_1 + 2x_2 \leq 8$ are active constraints.

3\. (a)

$$\begin{aligned} \min \quad & -x_1 - 4x_2 - x_3 \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 + s_1 = 4 \\ & -x_1 + x_3 + s_2 = -1 \\ & x_1, x_2, x_3, s_1, s_2 \geq 0 \end{aligned}$$

(b) BS

$(0, 0, 0, 4, -1)$
$(0, 0, 4, 0, -5)$
$(0, 0, -1, 5, 0)$
$(0, 2, 0, 0, -1)$
$(0, \frac{5}{2}, -1, 0, 0)$
$(2, 0, 0, 0, 1)$
$(1, 0, 0, 2, 0)$
$(\frac{5}{3}, 0, \frac{2}{3}, 0, 0)$
$(1, 1, 0, 0, 0)$

BFS

$(2, 0, 0, 0, 3)$
$(1, 0, 0, 2, 0)$
$(\frac{5}{3}, 0, \frac{2}{3}, 0, 0)$
$(1, 1, 0, 0, 0)$

(c) If we are at basis $\{1, 2\}$ with BFS $(1, 1, 0, 0, 0)$

The reduced cost is

$$\bar{c}_3 = -1 - [-1, -4] \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 4$$

$$\bar{c}_4 = 0 - [-1, -4] \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2$$

$$\bar{c}_5 = 0 - [-1, -4] \begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 3$$

Since all reduced costs are non-negative, BFS is optimal.

4. (a) Denote x as the column vector to be # of shares purchased,
i as the row index of the corresponding matrix.

Formulate it as

$$\max_{x} \; \min_{i} \; (A_i^T x - \pi_i^T x)$$

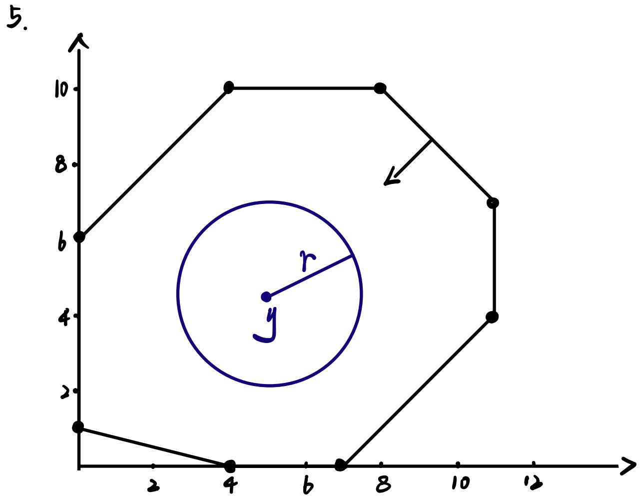
s.t. $\forall i$, $0 \le x_i \le 8_i$ for $i = 1, 2, 3, 4, 5$

(b) Refer to 4b.py for sol.

The optimal selection is 5 shares for Security 3,
5 shares for Security 4,
5 shares for Security 5.

The optimal worst-case value is \$1.

5.



With coding technics, we could represent each edge i as a vector, and figure out the normalized normal vector, guaranteeing the circle has at least the distance of r from each edge.

Formulate it as

$$\max_{y \in \mathbb{R}^2,\, r \in \mathbb{R}} \; r$$

s.t. $\text{normal}_i \cdot y + r \le b_i$ for each edge i

$r \geq 0$

Refer to 5.py.

the maximum radius is 4.596, with the center (5.377, 4.877).