FIN2020 Homework 4

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Question 1.

If the economic is good, the final wealth is

$$R_G = a(r_G + 1) + (Y_0 - a)(r_f + 1)$$

If it is bad, then

$$R_B = a(r_B + 1) + (Y_0 - a)(r_f + 1)$$

We have the expected utility to be

$$E[u(\tilde{R})] = \sum u_i p_i$$

and specific utility function

$$u(Y) = \ln(Y)$$

Taking in the specific value, we get

$$E[u(\tilde{R})] = \frac{1}{2}\ln(110 + 0.2a) + \frac{1}{2}\ln(110 - 0.05a)$$

Then the allocation problem is converted to the optimization problem as

$$\max_a f(a) = \frac{1}{2}\ln(110 + 0.2a) + \frac{1}{2}\ln(110 - 0.05a)$$

with the constraint

$$0 < a < 100$$

We have the F.O.C to be

$$f'(a) = \frac{0.1}{110 + 0.2a} - \frac{0.025}{110 - 0.05a} = \frac{330 - 0.4a}{40(110 + 0.2a)(110 - 0.05a)} > 0$$

which indicates $f(a)$ increases monotonically within the range.

Thus we find the $a*$ to be 100, so as to optimize $E[u(\tilde{R})]$.

Question 2.

Assume the consumer decides to save $a \leq Y_0$ in stock, thus his consumption at $t = 0$ is

$$c_0 = Y_0 - a$$

In a good economy, his wealth at $t = 1$ is

$$c_{1G} = a(1 + r_G)$$

In a bad one, his wealth at $t = 1$ is

$$c_{1B} = a(1 + r_B)$$

Thus we have the expected utility to be

$$f(a) = \ln(Y_0 - a) + \beta[\ln(a(1 + r_G))\pi + \ln(a(1 + r_B))(1 - \pi)]$$

With $f'(a) = 0$, we have

$$a* = \frac{\beta Y_0}{\beta + 1} < Y_0$$

We find that the optimal saving has nothing to do with π, but this could not be generalized for any case.

Question 3.

(a) We have the mean to be

$$\mu_z = \sum z_i p_i$$

and the variance to be

$$\sigma_z^2 = p_i \sum (z_i - \bar{z})^2$$

With computation, we have

$$\mu_{z1} = 6.75, \sigma_{z1}^2 = 15.1875$$

and

$$\mu_{z2} = 5.36, \sigma_{z2}^2 = 4.2534$$

It shows that

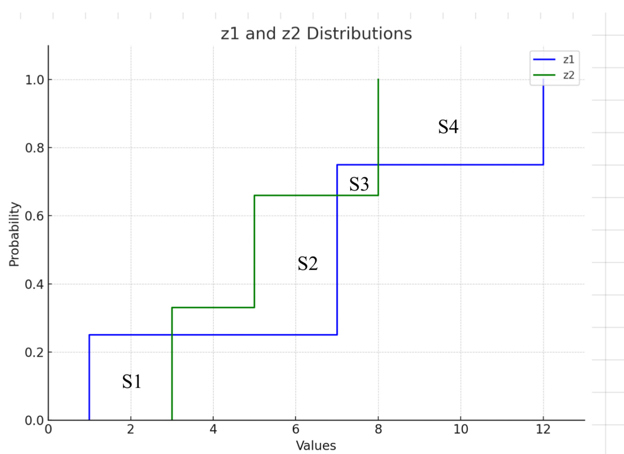
$$\mu_{z1} > \mu_{z2}$$

while

$$\sigma_{z1}^2 > \sigma_{z2}^2$$

No Mean-Variance Dominance.

(b) We draw the graph like:



There is no $F_1 > F_2$ or $F_1 < F_2$ for every z. Thus there is no FOSD.

(c) The following S_i represents $\int_{\Delta v}(F_1 - F_2)dz$, as

$$S_1 = 0.25 \times (3-1) = 0.5$$

$$S_2 = (0.33 - 0.25) \times (5-3) + (0.66 - 0.25) \times (7-5) = -0.98$$

$$S_3 = (0.75 - 0.66) \times (8-7) = 0.09$$

$$S_4 = (1 - 0.75) \times (12-8) = -1$$

Thus we have $G(z) = \int_0^z (F_1 - F_2)dz$ to be

$$G(1) = 0$$

$$G(3) = 0.5$$

$$G(7) = -0.48$$

$$G(8) = -0.39$$

$$G(12) = -1.39$$

There is no $G(z) > 0$ or $G(z) \neq 0$ for every z, no SOSD.

*****This is the end of Homework 4.*****