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Laboratory Assignment # 7

**1a) Using the Karnaugh map method, find a boolean expression (in the form of a sum of products) for the following function. Show the Karnaugh map and indicate the relationship between the terms in your answer and the circled "1"s in the Karnaugh map:**

		CD			
		00	01	11	10
AB	00	0	0	1	0
	01	0	0	1	1
	11	1	0	1	1
	10	0	0	0	0

The Boolean Expression will be:

$$A\bar{C} + BC + \bar{A}CD$$

The answer of 1b is the screenshot of Simcir.

**2a) Using De Morgan's laws, find the negation of the following expressions, expressing the negation as a product of sums:**

$$\begin{aligned} & \bullet A * \bar{B} * \bar{C} + \bar{A} * \bar{B} + \bar{B} * C \\ &= (A * \bar{B} * \bar{C}) + (\bar{A} * \bar{B}) + (\bar{B} * C) \\ &= (\overline{A + B + \bar{C}}) * (\overline{\bar{A} + B}) * (\overline{\bar{B} + C}) \\ &= (\bar{A} + \bar{B} + \bar{\bar{C}}) * (\bar{\bar{A}} + \bar{B}) * \bar{\bar{B} + C} \\ &= (\bar{A} + \bar{B} + C) * (A + \bar{B}) * (B + \bar{C}) \end{aligned}$$

$$\begin{aligned} & \bullet \bar{A} * C + B * C + \bar{A} * \bar{B} * \bar{C} \\ &= (\bar{A} * C) + (B * C) + (\bar{A} * \bar{B} * \bar{C}) \\ &= (\overline{\bar{A} + C}) * (\overline{B + C}) * (\overline{\bar{A} + \bar{B} + \bar{C}}) \\ &= (\bar{\bar{A}} + \bar{\bar{C}}) * (\bar{B} + \bar{\bar{C}}) * (\bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}}) \\ &= (A + \bar{C}) * (\bar{B} + \bar{C}) * (A + B + C) \end{aligned}$$

2b) Using De Morgan's laws, find the negation of the following expressions, expressing the negation as a sum of products.

$$\begin{aligned}
 & \bullet (\bar{A} + B + C) * (\bar{B} + \bar{C}) * (A + \bar{C}) \\
 &= \overline{(\bar{A} * B * C) + (\bar{B} * \bar{C}) + (A * \bar{C})} \\
 &= (\bar{\bar{A}} * \bar{B} * \bar{C}) + (\bar{\bar{B}} * \bar{\bar{C}} + (\bar{A} * \bar{\bar{C}}) \\
 &= (A * \bar{B} * \bar{C}) + (B * C) + (\bar{A} * C)
 \end{aligned}$$

$$\begin{aligned}
 & \bullet (A + B + \bar{C}) * (\bar{A} + \bar{B} + C) * (A + B) \\
 &= \overline{(A * B * \bar{C}) + (\bar{A} * \bar{B} * C) + (A * B)} \\
 &= (\bar{A} * \bar{B} * \bar{C}) + (\bar{\bar{A}} * \bar{\bar{B}} * \bar{C}) + (\bar{A} * \bar{B}) \\
 &= (\bar{A} * \bar{B} * C) + (A * B * \bar{C}) + (\bar{A} + \bar{B})
 \end{aligned}$$

3) Find the CNF representation to the function  $f$  described in problem 1. Use the Karnaugh map method to find the DNF representation of the complement, then use the DeMorgan's laws to convert that into a CNF formula for  $f$

		CD			
		00	01	11	10
AB	00	0	0	1	0
	01	0	0	1	1
	11	1	0	1	1
	10	0	0	0	0

$$A\bar{C} + BC + \bar{A}CD$$

$$\begin{aligned}
 & \overline{(A + \bar{C}) * (B + C) * (\bar{A} + C + D)} \\
 &= (\bar{A} + \bar{\bar{C}}) * (\bar{B} + \bar{C}) * (\bar{\bar{A}} + \bar{C} + \bar{D}) \\
 &= (\bar{A} + C) * (\bar{B} + \bar{C}) * (A + \bar{C} + \bar{D})
 \end{aligned}$$

4) For each of the following expressions, transform it from its current form (DNF or CNF) into the other using the above laws of boolean algebra. Show several intermediate steps; don't just show the final answers. Your final expression should be as simple as you can make it subject to the restriction that it is in CNF or DNF form). Thus,  $AB + ABC$  should be simplified to just  $AB$  since  $AB + ABC = AB(1 + C) = AB1 = AB$ , etc.

a)  $AB + \bar{B}\bar{C} + \bar{A}C$  (transform into CNF)

$$\begin{aligned}
 &= (AB + \bar{B})(AB + \bar{C} + \bar{A}C) \\
 &= (AB + \bar{B})(AB + \bar{C}) + \bar{A}(AB + \bar{B})(AB + \bar{C}) + C \\
 &= (AB + \bar{B})(AB + \bar{C}) + \bar{A}(\bar{B} + \bar{C}) + C \\
 &= (AB + \bar{B}\bar{C} + \bar{B}C + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}C) + C \\
 &= (AB + \bar{B}\bar{C} + \bar{B}C + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}C) + C \\
 &= (AB + \bar{B}\bar{C} + \bar{A}\bar{B})(AB + \bar{B}\bar{C} + C)
 \end{aligned}$$

$$\begin{aligned}
& b) (\bar{A} + B + C) * (A + \bar{B}) \text{ (transform into DNF)} \\
& = \bar{A}A + \bar{A}\bar{B} + BA + B\bar{B} + CA + C\bar{B} \\
& = 0 + \bar{A}\bar{B} + AB + 0 + CA + C\bar{B} \\
& = \bar{A}\bar{B} + AB + CA + C\bar{B} \\
& = \bar{A}\bar{B} + AB + C(A + \bar{B}) \\
& = 1 + C(A + \bar{B})
\end{aligned}$$