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Multiple Mixing-Cell Method for MMP Calculations

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Abstract

The minimum miscibility pressure (MMP) is a key parameter governing the displacement efficiency of gas floods. There are several methods to determine the MMP, but the most accurate methods are slim-tube experiments, analytical methods, and numerical simulation/cell-to-cell methods. Slim-tube experiments are important to perform because they use actual crude oil, but they are costly and time consuming. Analytical methods that use the method of characteristics (MOC) are very fast and help to understand the structure of gas floods. MOC, however, relies on finding the unique and correct set of key tie lines in the displacements, which can be difficult. Slim-tube simulation methods and their simplified cell-to-cell derivatives can be slow and their MMP estimates clouded by dispersion.

This paper presents a simple and accurate multiple mixing cell method for MMP calculations that corrects for dispersion, and is substantially faster than simulation methods. Unlike previous "mixing cell" methods, our cell-to-cell mixing model uses a variable number of cells, and is independent of gas-oil ratio, volume of the cells, excess oil volumes, and the amount of gas injected. The new method relies on robust P-T flash calculations using any cubic equation-of-state (EOS). The calculations begin with only two cells and perform additional cell-to-cell contacts between resulting equilibrium phase compositions based on equilibrium gas moving ahead of the equilibrium liquid phase. We show for a variety of oil and gas compositions that all key tie lines can be found to the desired accuracy, and that they are nearly identical to those found using analytical MOC methods. Our approach, however, is more accurate and robust than those from MOC because we do not make approximations regarding shocks along nontie-line paths, and the unique set of key tie lines converge automatically.

The MMP using our mixing cell method is estimated from four or five pressure calculations based on a power-law extrapolation of the smallest tie line to zero length. Our multiple mixing cell method can calculate either the MMP or the minimum miscibility for enrichment (MME) independent of the number of components in the gas or oil. Our approach further supports the notion that the MMP is independent of fractional flow since we obtain the same key tie lines independent of how much fluid is moved from one cell to another.

Introduction

Slim-tube experiments are traditionally used to estimate MMPs because they capture the interaction of flow in porous media with the phase behavior of the crude (Yellig and Metcalfe 1980). Slim-tube experiments, however, are slow and expensive, and their MMP estimates may not be accurate because of dispersion, and the lack of data points (Johns *et al.* 2002). Besides slim-tube experiments, multicontact mixing-cell experiments can provide accurate MMP estimates for vaporizing or condensing gas floods. Most gas floods, however, have features of both condensing and vaporizing drives (CV), which means MMPs estimated by mixing-cell experiments are typically not accurate (Zick 1986, Johns *et al*. 1993). Other experimental MMP methods, such as vanishing-interfacial-tension tests (Rao 1997) and rising bubble experiments (Christianson and Haines 1987), are suspect because they may not duplicate the complex interaction of phase behavior and flow for CV drives (Jessen and Orr 2007).

Because of the drawbacks of experimental methods, computational methods for MMP estimation have been developed over the years to estimate the MMP from cubic equations-of-state (EOS). There are primarily three computational methods: analytical calculations with MOC, slim-tube compositional simulation, and multicontact mixing cell models. Analytical methods for MMP estimation are based on the analytical solution of dispersion free 1D flow equations (Orr 2007). Monroe *et al.* (1990) first examined the analytical theory for quaternary systems and showed that there exists a third key tie line in the displacement path, called the crossover tie line. Orr *et al.* (1993) and Johns *et al.* (1993) confirmed the existence of the crossover tie line and presented a simple geometric construction to find the key tie lines (gas, oil, and crossover). Their geometric construction assumed that consecutive tie lines are connected by shocks along nontie-line paths. They

demonstrated that the MMP occurs at the pressure when any one of the three key tie lines first intersects a critical point (becomes zero length). Johns *et al.* (1993) further showed that the crossover tie line controls the development of miscibility in CV drives.

Johns and Orr (1996) gave a procedure to calculate the MMP for more than four components, and extended their geometric construction to construct the first multi-component displacement of (ten-component) oil by CO_2. They provided a general methodology for composition path construction for any number of oil and gas components and showed that N_C-*1* key tie lines exist that the composition path must follow. They showed that the composition path consists of either a rarefaction wave (continuous variation) or shock along the nontie-line paths from each key tie line to the next. They further stated that miscibility develops when any one of these key tie lines becomes zero length (intersects a critical point). The MMP calculation, therefore, was reduced to finding a series of key tie lines from the oil to injection gas composition whose tie-line extensions intersect sequentially. Wang and Orr (1997) demonstrated their multi-component approach by calculating the MMP for injection gases with more than one component. They found the intersection points of the key tie-line extensions by applying a Newton-Raphson scheme in compositional space. In their iteration method, they also assumed that only shock jumps occur from one key tie line to the next, which Johns (1992) and Johns and Orr (1996) speculated to be a close approximation. Jessen *et al.* (1998) improved the speed of Wang and Orr's method by the inclusion of fugacity equations in the Newton-Raphson scheme. Yuan and Johns (2005) recently simplified the size of the Newton-Raphson problem, and showed that it is possible to converge to the wrong set of key tie lines, which is a potential drawback of these analytical MOC methods.

Another approach to estimating the MMP is by 1D slim-tube simulation. Determination of MMP by 1D compositional simulation mimics the flow in porous media that occurs in slim-tube experiments (Yellig and Metcalfe 1980). Fine-grid compositional simulations, however, can suffer from numerical dispersion effects causing the MMP to be in error (Stalkup 1987, Johns *et al.* 2002). Simulation is also time-consuming to perform. A typical procedure uses a one-dimensional compositional simulator to predict the recoveries of oil displacements by gas at different pressures. Like slim-tube experiments, the MMP is determined from an arbitrary bend in the recovery curves, typically called the "knee" (Jarrell *et al.* 2002). Better accuracy of the MMP is obtained by repeating the simulations for different levels of dispersion, and extrapolating the estimated MMP to zero dispersion. Even this extrapolation, however, can be in error from the true MMP (Johns *et al.* 2002).

There is also a variety of published mixing-cell methods to estimate the MMP or MME (Hutchinson and Braun 1961, Cook *et al*. 1969, Metcalf *et al*. 1973, Pedersen *et al*. 1986, Clancy *et al*. 1986, Lake 1989, Jensen and Michelsen 1990, Neau *et al*. 1996). The basic idea in these single and multiple mixing cell methods is to mix (analytically) gas and oil in repeated contacts, resulting in new equilibrium compositions. In the case of a vaporizing, drive (lean gas injection), the intermediate component in the oil is vaporized into the more mobile gas phase, and miscibility is developed when the equilibrium gas is repeatedly mixed with fresh oil, causing the equilibrium gas composition to move towards the oil tie line. Thus, in vaporizing drives, the tie line that extends through the oil controls the development of miscibility. Miscibility in vaporizing drives is developed at the leading edge of the displacement. For a condensing gas drive (enriched gas injection), however, the intermediate component in the gas is condensed into the oil, and the gas tie line controls miscibility. Miscibility for condensing drives, therefore, is developed at the trailing edge of the displacement. These mixing-cell methods can give reliable MMPs for either condensing or vaporizing drives. Most field displacements, however, are combined CV drives, and thus MMPs from these earlier mixing cell methods are unreliable.

Jaubert *et al*. (1998a,1998b) developed mixing-cell algorithms that are similar to that of Metcalf *et al*. (1973) in that they transfer specified amounts of gas and excess oil from cell-to-cell without solving the flow equations. These methods attempt to account for all displacement mechanisms including CV, but like 1-D simulation, they can be affected by dispersion. To reduce the effect of dispersion, recovery factors at 1.2 PVI are determined for a fixed pressure and number of cells. The number of cells is changed, and the calculation of recovery factor at the same pressure is repeated. A plot of the recovery factor versus $1/\sqrt{N}$, where N is the number of cells, gives the estimated recovery factor at zero dispersion for that pressure. A graph of the estimated zero-dispersion recovery factor is then made for several pressures, an extrapolation of which to near 100% recovery gives the pressure corresponding to the MMP. Jaubert *et al*. give examples of MMP calculations, but like simulation methods, these are affected by dispersion. Zhao *et al*. (2008a, 2008b) present a more complex multiple mixing-cell method, where they include fractional flow to determine transfer amounts. They find key tie lines by graphing the tie-line lengths along profiles of their fixed cell model. Like Jaubert *et al*., their approach is susceptible to dispersion, although this is difficult to evaluate because they do not give example MMP calculations.

In this paper, we present a simple, practical, and robust multiple mixing-cell method to determine the MMP for systems with any number of components. Our mixing cell method differs from the previous ones in that we start with two cells, and increase them until the desired accuracy in the key tie lines are obtained. We then determine the MMP based on the pressure at which the first key tie-line length becomes zero. In the first section, we describe our mixing cell algorithm in detail, and show that we can capture all key tie lines of the displacement to any accuracy desired. Subsequent sections show that in practice, the MMP is determined not by finding all of the key tie lines, but by tracking only the shortest one. Several example MMP calculations are given and compared to analytical MOC calculations showing good agreement.

Multiple Mixing-Cell Model

In this section, we describe the basic steps in our mixing-cell method, which is remarkably simple. Our method does not calculate recovery factors nor transfer specific amounts of fluid based on cell volumes. Further, unlike other methods, there is no extrapolation required to correct for dispersion as the mixing cell approach automatically reduces dispersion with repeated contacts. Our method only relies on performing PT flash calculations using any EOS, and on moving the injected and equilibrium gas ahead of the equilibrium liquid in each cell.

We begin with two cells at fixed temperature and pressure, where the injection gas is located in the upstream cell and the reservoir fluid in the downstream cell (see **Fig. 1**). The reservoir oil ($\boldsymbol{x}^{\boldsymbol{O}}$) and injection gas ($\boldsymbol{y}^{\boldsymbol{G}}$) are then mixed, in any mole fraction desired, although we generally use the mass balance equation $\boldsymbol{z}=\boldsymbol{x}^{\boldsymbol{O}}+\alpha(\boldsymbol{y}^{\boldsymbol{G}}-\boldsymbol{x}^{\boldsymbol{O}})$ with α equal to 0.5. As long as the pressure is below the MMP, the resulting overall composition $\boldsymbol{z}$ will be either in the two-phase region, or in the region of tie-line extensions. Thus, a flash or negative flash with a cubic EOS can be performed at this overall composition, resulting in two equilibrium compositions, one for liquid ($\boldsymbol{x}$), and one for vapor ($\boldsymbol{y}$). The equilibrium vapor moves ahead of the equilibrium liquid since gas is injected. This is the first contact.

The second series of contacts contains both an upstream and downstream contact (see Fig. 1). The downstream contact mixes the equilibrium vapor ($\boldsymbol{y}$) with fresh oil, and the upstream contact mixes the equilibrium liquid ($\boldsymbol{x}$) with fresh injection gas. When mixing vapor and liquid, we always use the same material balance equation, $\boldsymbol{z}=\boldsymbol{x}+\alpha(\boldsymbol{y}-\boldsymbol{x})$. Two new sets of equilibrium liquid and vapor phases result from these flash calculations, so that we now have six cells, including the reservoir oil, and injection gas. This completes the second contact.

We then make additional contacts until all key tie lines develop and converge to within a specified tolerance. Thus, after N contacts, we have a total of $2N+2$ cells.

The following steps summarize a possible procedure for the estimation of MMP with our mixing-cell model:

1- Specify the reservoir temperature, and an initial pressure that is substantially below the MMP. We typically use an initial pressure of 500 psia.

2- Start with two cells filled with injection gas, and reservoir oil. Mix the gas and oil, and flash the resulting overall composition (using any cubic EOS) to get two new equilibrium compositions, liquid $\boldsymbol{x}$ and vapor $\boldsymbol{y}$.

3- Mix the resulting equilibrium liquid(s) ($\boldsymbol{x}$) with equilibrium vapor(s) ($\boldsymbol{y}$) assuming gas moves ahead of the oil phase. Each of the contacts results in new compositions for the next set of contacts.

4- Continue with additional contacts by mixing neighboring cells, as shown in Fig. 1, until all N_C-1 key tie lines develop and converge to a specified tolerance. We automatically identify a converged tie line by checking the tie-line lengths between neighboring cells. That is, we calculate for each cell the slope of the tie-line length as a function of the cell number. A key tie line is developed when three successive cells have a slope of zero, to within a specified tolerance. In this paper, we use a very strict tolerance of 1E-8 because we want to demonstrate the accuracy of the method. Once all key tie lines are found, we proceed to the next step.

5- Calculate the tie-line length of each key tie line found in step 4), and store the minimum tie-line length (TL).

6- Increase the pressure and repeat steps 2) through 5). There are varieties of ways to determine the next pressure to use. For the second pressure, we simply increase the pressure by a small amount, say 200 psia. To determine the third pressure, we use a linear extrapolation of the tie-line lengths (from the previous two pressures) to zero pressure to give the first MMP estimate. The increment in pressure, ΔP, is determined by dividing the difference in the estimated MMP and the current pressure by three or by how many additional pressure points you want. Smaller increases are possible, but at the expense of time and little improved accuracy in the final MMP estimate. For subsequent pressures, ΔP is based on a power-law extrapolation of the minimum tie-line lengths found at the previous three pressures. That is, we do a multiple parameter regression of the minimum tie-line lengths to determine the exponent n in $TL^n = mP+C$, where m is the slope and C is a constant. The three parameters are determined when the correlation coefficient exceeds 0.999. We then determine a first estimate of the MMP as the pressure at which the power-law extrapolation gives zero length. Typical values for n are between 1.5 and 10, although the extrapolated MMP near the actual MMP changes little over this range.

7- Repeat step 6) for two or three additional pressures, fitting the last three pressure points, until the MMP is estimated to within the desired accuracy, say 20 psia. If the accuracy is not satisfactory, then use smaller pressure increments in step 5). This approach not only yields the final MMP, but also gives an estimate of the uncertainty in the MMP based on the MMP estimate from the previous regression. We take the absolute value of the difference in the previous MMP estimate and the final MMP estimate as the uncertainty of the MMP. It is likely that this MMP range is larger than the true error in the final MMP.

The proposed method has several advantages over other MMP estimation methods. First, compared to the analytical MOC methods it is very robust since all flash calculations are in positive composition space, and there are no Newton-Raphson iterations to solve. The mixing cell model always results in a single unique solution compared to MOC, which requires careful checking to ensure convergence to the correct set of key tie lines. Further, there are no assumptions regarding whether shocks connect key tie lines or continuous variations. Thus, the mixing cell approach given here will locate all key tie lines, whether composition paths have shocks or continuous variations that connect key tie lines. In theory, these key tie lines could include ones that are transitional in nature, although we did not locate them in this research

(Seto *et al*. 2006). As is discussed later, the proposed mixing model should also be able to handle displacements where three or more phases are present, something that is currently very difficult with MOC methods (LaForce and Johns 2005).

Our new mixing cell method, however, is substantially faster than slim-tube simulation methods, and does not require the input of petrophysical parameters or the solution of the flow equations. Our approach also corrects for dispersion by continuing cell-to-cell contacts until all key tie lines are converged, without the need for determining recoveries or making plots of recovery as a function of the number of cells. In addition, our approach is automatic, and gives an estimate of the MMP and its uncertainty.

Example MMP Calculations

In this section, we give examples of MMP calculations for gas floods published in the literature. We also show the effect of the assumed relative mobility of the phases on the MMP calculation. We use the Peng-Robinson EOS (1978) for all calculations in this research.

Case 1: Four-Component Condensing and Vaporizing (CV) Displacement. Consider first a simple example of a CV displacement of three-component oil (CH_4, C_4, and C_{10}) by a two-component injection gas (CH_4, CO_2) at a temperature of 160°F. The MMP and the key tie lines for this displacement using the MOC are reported by Orr *et al.* (1993) and Wang and Orr (1997). The composition of oil and gas considered and the EOS properties are shown in **Table 1**.

For such a displacement, there are three key tie lines: the oil tie line; the gas tie line; and the crossover tie line. **Figure 2** presents four profiles of the key tie-line lengths for the displacement at 2000 psia (about 300 psia below MMP) as a function of the cell number and the number of contacts made. As is shown, the three key tie lines develop as the number of contacts proceeds. The key tie lines are nearly fully developed after 50 contacts, although after 250 contacts, all three key tie lines converge to within the slope tolerance of 1E-8 described in the previous section. As shown, the crossover tie line for this displacement controls miscibility as its tie-line length is shorter than either the gas or oil tie line. **Figure 3** illustrates the CV behavior further in that the K-values for the crossover tie line are nearest to 1.0. The condensing portion of the displacement occurs at the leading edge (downstream), while the vaporizing portion occurs at the trailing edge (upstream). Miscibility is developed in the middle of the displacement for CV drives. Figure 1 illustrates that for CV drives miscibility occurs in a cell somewhere between the oil and gas tie lines, whereas for vaporizing drives miscibility occurs at the oil tie line (most downstream tie line), and for condensing drives the gas tie line (most upstream tie line).

Figure 4 shows the key tie-line lengths for this displacement as a function of pressure. The dashed lines are the key tie lines from Wang and Orr, while the symbols are the converged key tie lines from our mixing cell model. Agreement between the two is identical, within the tolerances specified. The MMP is estimated by the power-law extrapolation using $n = 1.8$ for the last three pressure points (see **Fig. 5**). For this system, the calculated MMP from our mixing-cell method is 2303±4 psia. This compares well to the MMP reported in Wang and Orr (1997) of 2297 psi.

The K-values for each component in each cell at a pressure (2305 psia) just slightly greatly than our estimated MMP is given in **Fig. 6.** The profile of the K-values clearly shows the development of miscibility in the middle of the displacement characteristic of a CV drive. At this pressure, the crossover tie line, however, does not exist for MOC methods, but for our mixing-cell model, a slight amount of dispersion causes some two-phase flow so that the crossover tie-line is retained. Continued contacts would decrease its length further, although zero length can not be reached. This is not a problem for our mixing-cell method as the MMP can be estimated accurately, as it is solely based on tie-line lengths determined for pressures below the estimated MMP. In the event that a pressure greater than the MMP is selected, a key tie line like this one can be assumed converged by setting a cutoff for the tie-line length. That is, when a cell tie-line length becomes smaller than say 0.05, that tie line is assumed to be converged, although additional contacts can still take place to find other key tie lines that may not yet be fully developed.

Case 2: CO_2 Displacement of Ten-Component Oil. Our second example is the displacement of ten-component oil at 120°F by pure CO2, a case that was previously examined by both Johns and Orr (1996) and Wang and Orr (1997) using MOC. Like case 1, the displacement is CV because a crossover tie line controls miscibility. **Table 2** gives the composition of the oil, and EOS properties of the components.

Johns and Orr predicted a MMP of 1350 psia for the system based on an extrapolation of crossover tie-line 1 becoming zero length. Wang and Orr calculated tie-line lengths, however, well above 1300 psia and obtained a MMP estimate of 1466 psia for the same system. The difference between the two models was attributed to the fact that Johns and Orr did not have a robust cubic EOS for tie lines above 1300 psia. However, our MMP prediction using an in-house MOC code named UTPVT gives nearly perfect agreement with that of Johns and Orr, up to 1300 psia. In addition, the key tie line lengths calculated from UTPVT and our new mixing cell method agrees to within four-digit accuracy (see **Fig. 7**). As is shown, crossover tie-line 2 becomes the minimum length tie line at about 1283 psia, nearly the same pressure at which the cubic EOS for Johns and Orr could no longer converge. **Figure 8** shows a close-up view of Fig. 7, which shows that crossover tie line 2 (using our MOC code) drops suddenly to the MMP (estimated to be 1298 +/- 2 psia). This is confirmed by our mixing cell method, which also finds that crossover tie-line 2 becomes the smallest tie line at 1283 psia. Our new mixing cell method also agrees well with the MMP predicted from our MOC code, in that it indicates a MMP of about 1298 +/- 5 psia using power-law extrapolation (see **Fig. 9**).

The agreement between our new mixing cell method, our MOC code (UTPVT), and the results by Johns and Orr (1996) reinforces that we have the correct MMP and key tie lines for this displacement. Further, the small differences in the key tie-line lengths observed with our mixing cell model and that of MOC show a trend in the errors. That is, the magnitude of the errors from the gas to oil tie lines are 0.0 (gas tie line), 10^{-4} (crossover tie-line 1), 10^{-5}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-7}, 10^{-8}, 10^{-9} (crossover tie-line 8) and 0.0 (oil tie line). The trend is explained by the fact that unlike our mixing-cell method, MOC assumes that all key tie lines are connected by shocks along their nontie-line paths, and there is no non-tie line rarefaction wave present. Johns and Orr (1996), however, showed that the actual composition path takes a rarefaction wave (continuous variation) between the oil tie line and crossover tie-line 1, which is exactly where the differences between the two methods are greatest.

Our results explain why Johns and Orr (1996) could not converge to key tie lines above 1300 psia. We do not know why Wang and Orr (1997) failed to locate these key tie lines. It is likely that either they mistakenly used different EOS parameters than they reported, or they converged to the wrong set of key tie lines above 1300 psia. This displacement is a very difficult one for MOC methods to calculate, primarily because of the steep drop in crossover tie line 2 over just a few psia. Our in-house MOC code, UTPVT, has been developed to ensure that the correct set of key tie lines are found as is evidenced here by good agreement with our mixing cell model.

Case 3: Lean Gas Displacement of Eight-Component Oil. Consider next a displacement of eight-component oil by a five-component gas, as is described by Hearn and Whitson (1995) and Wang (1998). **Table 3** lists the composition of the oil and lean gas, and the EOS properties. The temperature for this system is 212°F.

The minimum tie-line length from the mixing-cell method as a function of pressure is shown in **Fig. 10**. The MMP from the mixing-cell method is 3179 +/- 10 psia (see **Fig. 11**). This agrees favorably with Hearn and Whitson, who reported a MMP of 3190 psia from compositional simulation. Wang also reported a similar, but larger MMP of 3259 psia from MOC. (Wang actually reported 3279 psia in his text, but his figure clearly shows a MMP of 3259 psia). This displacement is also combined condensing and vaporizing (CV), because a crossover tie line controls miscibility.

Calculation Speed Using Our Mixing-Cell Method. The new mixing-cell method is faster than slim-tube simulation, but it is still substantially slower than MOC. The simple four-component example of case 1 only took a few minutes on a 2.0 GHz, Intel Xeon PC to develop all of the key tie lines even for pressures close to the MMP. For the eleven-component displacement of case 2, however, it takes much longer (sometimes hours) to develop the key tie lines, depending on the pressure selected. The mixing-cell method is much faster for pressures far below the MMP compared to pressures that are near the MMP.

We have developed three ways to increase the speed of the mixing-cell method. First, we only calculate the key tie lines for four or five pressure points that are not too close to the MMP. The power-law extrapolation to estimate the MMP is very good when those points are properly regressed.

Second, we eliminate cells, with what we call a "freezing" technique, from future contacts. Once a cell is converged based on having the same tie-line slopes (see procedure in the first section) with its neighbor, that cell is marked and frozen; it is no longer involved in additional contacts, although its neighboring cells may be. This freezing technique can significantly reduce computational time as is shown in **Fig. 12**.

Last, we can significantly decrease the computational time if we are only interested in estimating the MMP, instead of also finding all key tie lines. In this approach, we focus only on the development of the key tie line(s) with the smallest length. After about 200 contacts, we identify the region where the tie-line length is the shortest, as this region develops miscibility first. We then identify the nearest tie lines that are fully developed upstream and downstream of this region. All other cells outside of this region are frozen, and are no longer involved in contacts.

Discussion

Many have claimed (including this author) that the MMP is independent of fractional flow (relative permeability and viscosities), and thus mobility of the phases (Zhao *et al*. (2008a; 2008b), Pires and Bedrikovetsky (2005)). This hypothesis is reinforced using our mixing cell model in that we obtained the exact same key tie lines for any value of α, where α determines how much of the equilibrium oil ($\boldsymbol{x}^{\boldsymbol{O}}$) and gas ($\boldsymbol{y}^{\boldsymbol{G}}$) are mixed from cell to cell using $\boldsymbol{z}=\boldsymbol{x}^{\boldsymbol{O}}+\alpha(\boldsymbol{y}^{\boldsymbol{G}}-\boldsymbol{x}^{\boldsymbol{O}})$. Thus, no matter the amount of fluid that moves ahead, the key tie lines remain unchanged.

The literature is also replete with statements that equilibrium gas moves out ahead of the equilibrium liquid during mixing cell contacts because the gas phase is more mobile than the oil. While gas is more mobile than liquid, the reason that equilibrium gas moves ahead of liquid is the result of the simple fact that gas must move ahead since it is the injected phase. Simple 1D simulations can show that even if the gas phase mobility is decreased below that of oil (a favorable mobility ratio), the same key tie lines are obtained as those for an adverse mobility ratio.

Conclusions

We have developed a true mixing-cell model to estimate MMP (or MME) and the key tie lines in the composition path. The method is simple, robust, and faster than conventional 1D slim-tube simulation. The main conclusions from this research are

1- The mixing-cell method can accurately find all key tie lines for a displacement, regardless of the number of components in the injection gas and reservoir fluid. Unlike other mixing-cell methods, our method automatically corrects for dispersion by performing additional contacts until a small tolerance in the tie-line slopes are met. Further, our method can be accurately applied whether the reservoir fluid is a single phase oil, or a two-phase mixture.
2- The results of the mixing-cell method agree well with the MMP and key tie-lines from our analytical MOC software. The tie lines determined from our mixing cell method are slightly more accurate than those from MOC because MOC methods generally assume shocks along nontie-line paths from one key tie line to the next.
3- Our mixing-cell method is accurate for all displacement types, including combined condensing/vaporizing drives.
4- The relative value of the phase mobilities (fractional flow) do not affect the MMP. In all cases studies here, we converge to the same key tie lines independent of the fraction at which equilibrium gas and oil are mixed.
5- The speed of the mixing-cell method can be substantially increased by omitting (or freezing) cells from future calculations that have already converged to a key tie line. Further, the speed can be increased by omitting contacts between cells away from where miscibility is being developed.

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Nomenclature

Roman symbols

N = number of cells
N_c = number of components
P = pressure, m/Lt^2, psi
T = temperature, T, Fahrenheit
TL = tie-line length
x = liquid composition
y = vapor composition
z = overall composition

Greek symbols

α = dilution factor
ΔP = increment in pressure, m/Lt^2, psi

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Table 1 - Composition of oil and gas and properties of the components for case 1 (from Orr *et al.* 1993 and Wang and Orr 1997).

	Composition		Properties			Binary Interaction Parameters			
	Oil	**Gas**	**T_c (^{o}F)**	**P_c (psia)**	**ω**	**C_1**	**C_4**	**C_{10}**	**CO_2**
C_1	0.2	0.2	-116.63	667.8	0.0104	-	-	-	-
C_4	0.15	-	305.65	550.7	0.201	0.027	-	-	-
C_{10}	0.65	-	652.1	305.7	0.49	0.042	0.008	-	-
CO_2	-	0.8	87.9	1071	0.225	0.1	0.1257	0.0942	-

Table 2- Composition of oil and key EOS parameters for case 2 (from Metcalfe and Yarborough 1979 and Johns and Orr 1996).

	Composition		Properties			Binary Interaction Parameters				
	Oil	**Gas**	**T_c (^{o}F)**	**P_c (psia)**	**ω**	**CO_2**	**C_1**	**C_2**	**C_3**	**C_4**
CO_2	-	1	87.6	1071	0.225	-	-	-	-	-
C_1	0.35	-	-116.6	667.8	0.01	0.1	-	-	-	-
C_2	0.03	-	89.7	708.4	0.099	0.13	0	-	-	-
C_3	0.04	-	205.8	617.4	0.152	0.135	0	0	-	-
C_4	0.06	-	294.6	543.3	0.187	0.13	0	0	0	-
C_5	0.04	-	366.5	475.3	0.252	0.125	0	0	0	0
C_6	0.03	-	453.9	431	0.296	0.12	0.02	0.03	0.03	0.03
C_7	0.05	-	512.9	397	0.351	0.12	0.03	0.03	0.03	0.03
C_8	0.05	-	564.4	364	0.394	0.12	0.035	0.03	0.03	0.03
C_{10}	0.3	-	647.6	320	0.491	0.12	0.04	0.03	0.03	0.03
C_{14}	0.05	-	790	230	0.755	0.12	0.06	0.03	0.03	0.03

Table 3 – Composition of oil and gas, and EOS properties for case 3 (from Hearn and Whitson 1995).

	Composition		Properties			EOS CF*		BIP	
	Oil	**Gas**	**T_c (^{o}F)**	**P_c (psia)**	**ω**	**Ω_a**	**Ω_b**	**C_1+N_2**	**C_2+CO_2**
C_1+N_2	0.341	0.646	-120.87	661.5	0.0127	1.00351	1.0855	-	-
C_2+CO_2	0.056	0.124	89.73	760.9	0.1113	1.0175	1.03587	0.01511	-
C_3	0.055	0.103	206.03	616.3	0.1454	1	1	0.00326	0.01828
C_4's	0.059	0.089	294.93	543.1	0.1868	1.00229	0.99748	0.00344	0.01708
C_5's-C_6	0.092	0.038	417.53	462.2	0.2693	0.99558	0.9928	0.00391	0.01682
C_{7+} (1-2)	0.195	-	632.43	376.8	0.3663	0.98309	0.99562	0.02678	0.01682
C_{7+} (3-4)	0.155	-	932.63	237.8	0.6797	0.96443	0.99911	0.0434	0.01682
C_{7+} (5)	0.047	-	1186.03	158.2	1.0468	1.00216	1.00958	0.05834	0.01682

*** EOS Constant Correction Factors**

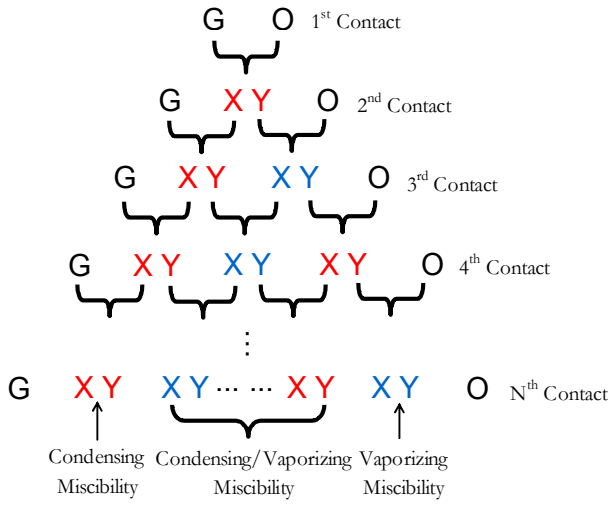

Figure 1 – Illustration of repeated contacts in the multiple mixing cell method. G: injecting gas composition; O: oil composition; Y: equilibrium gas composition; X: equilibrium liquid composition.

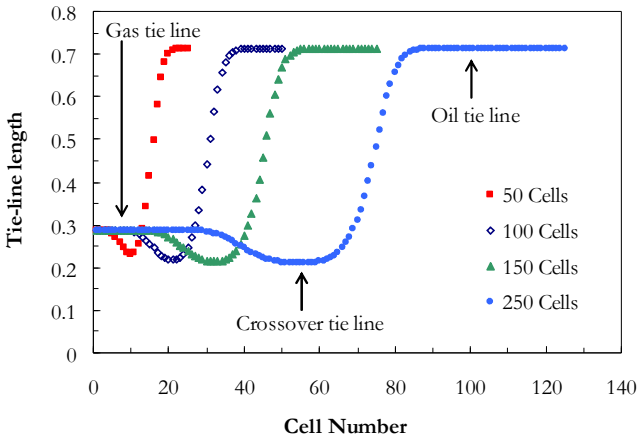

Figure 2 – Development of key tie lines using our mixing cell model for the four-component displacement (case 1) at 2000 psi and 160°F

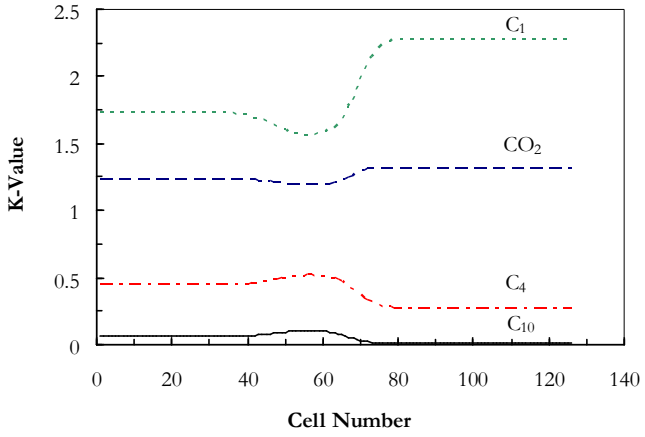

Figure 3 – K-values for the four-component displacement (case 1) at 2000 psi and 160 °F after all key tie lines are developed after 250 contacts.

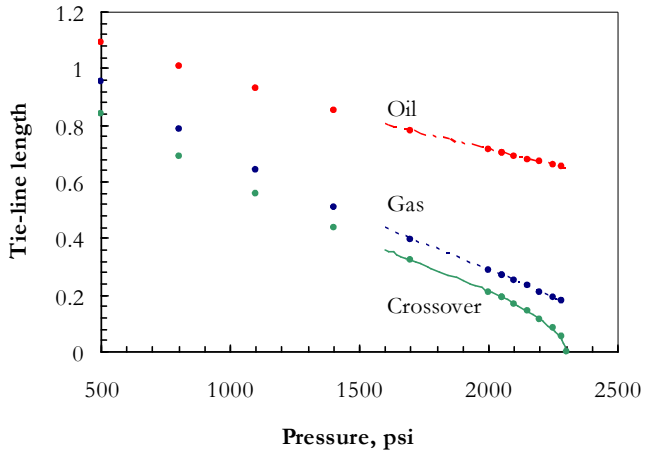

Figure 4 – Key tie-line lengths as a function of pressure for the four-component displacement of case 1. The MMP occurs where the crossover tie line has zero length at about 2303 psia.

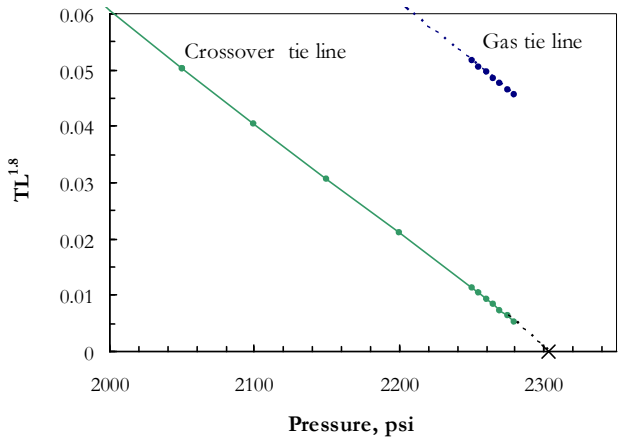

Figure 5 – Extrapolation of the tie-line lengths for case 1 (see Fig. 4).

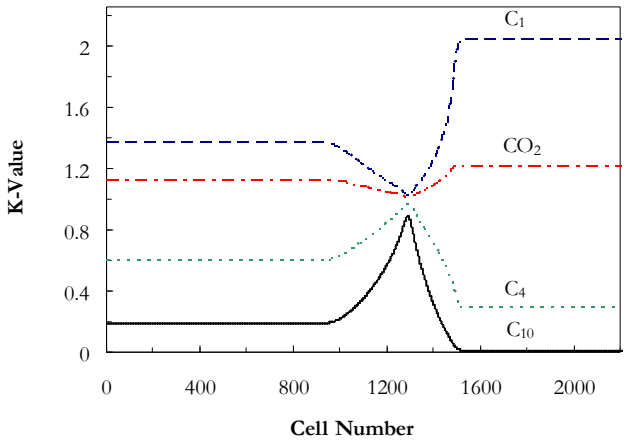

Figure 6 – K-values for the four-component displacement (case 1) at 2305 psi and 160°F. The crossover tie line is approaching zero length as the pressure is slightly above the estimated MMP.

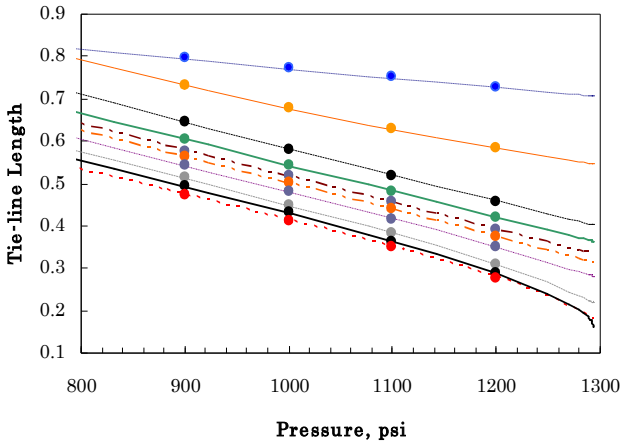

Figure 7 – Comparison of key tie lines for case 2 found from mixing cell method (solid dots) and MOC using UTPVT package (lines).

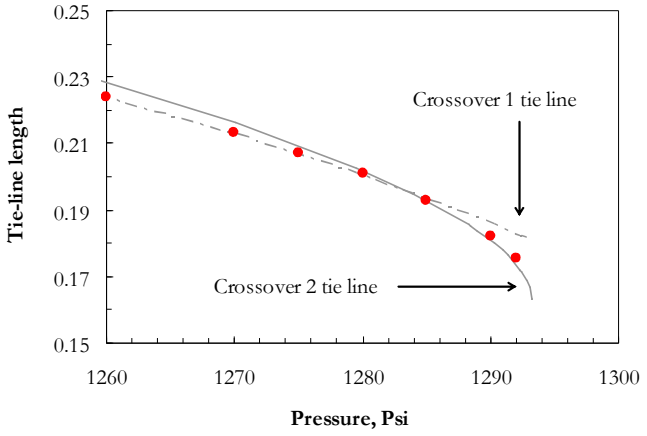

Figure 8 – Close-up view of Fig. 7 showing key tie lines for case 2 from MOC (UTPVT), and those from the new mixing-cell method (solid dots). Only the minimum tie-line length is tracked by the mixing cell method.

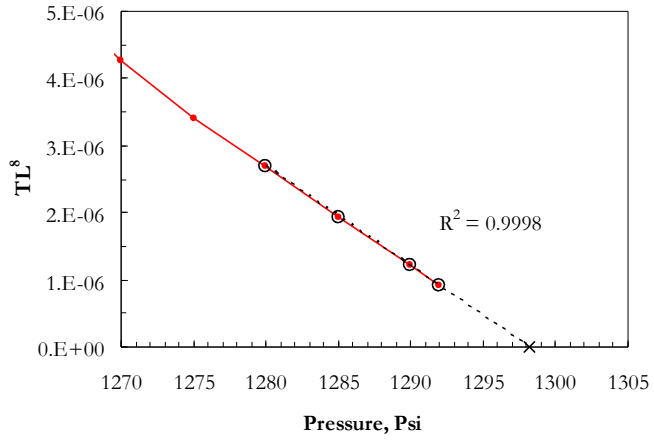

Figure 9 – Extrapolation of the minimum tie-line lengths from the mixing cell method for case 2. The MMP is estimated to be 1298 psia based on the last four data points.

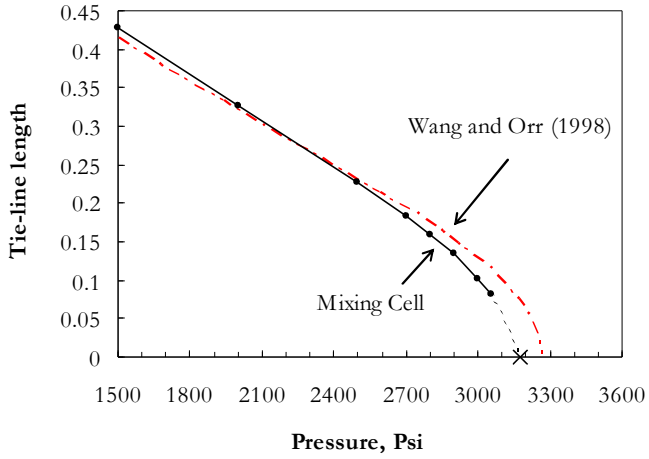

Figure 10 – Minimum tie-line length for case 3 from the mixing cell method, as compared to that of Wang and Orr (1998). The MMP is controlled by crossover tie line 4, and is approximately 3179 +/- 10 psia from our mixing cell method.

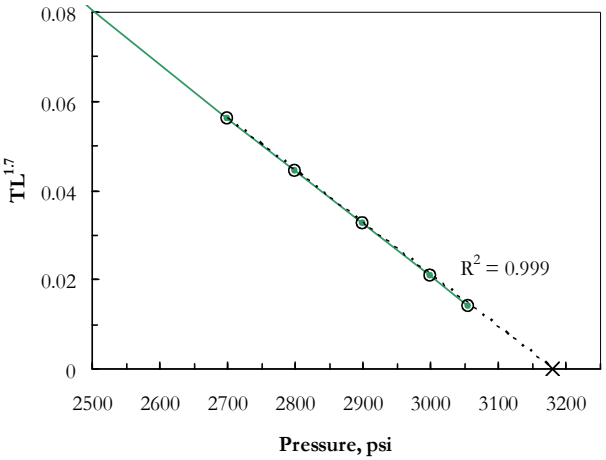

Figure 11 – Power-law extrapolation to the MMP for case 3, using our mixing-cell model. The MMP is 3179 +/- 10 psia.

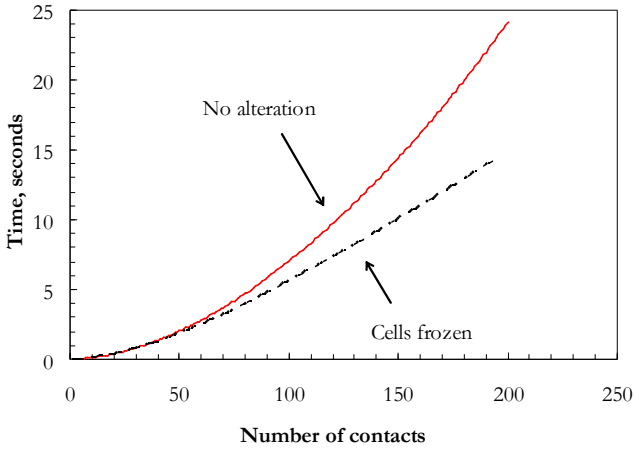

Figure 12 – Example of how the freezing of cells can save time for case 1.