CME 242 - Lecture 3 : Utility Theory

6 février 2020

Essentials

— Risk Averse $\implies$ Concave Utility Function U since Jensen's inequality yields to : $\mathbf{E}[U(x)] \leq U(\mathbf{E}[x])$.

— **Certainty-Equivalent Value** :

$$x_{CE} = U^{-1}(\mathbf{E}[U(x)]$$

— **Absolute Risk-Premium** :

$$\pi_A = \mathbf{E}[x] - x_{CE}$$

— **Relative Risk-Premium** :

$$\pi_R = 1 - \frac{x_{CE}}{\mathbf{E}[x]}$$

— **Absolute Risk-Aversion** :

$$A(x) = -\frac{U''(x)}{U'(x)}$$

— **Relative Risk-Aversion** :

$$R(x) = -\frac{U''(x)x}{U'(x)}$$

— CARA = Constant Absolute Risk-Aversion. Example if $a \neq 0$:

$$U(x) = \frac{1 - e^{-ax}}{a}$$

If $x \sim \mathcal{N}(\mu, \sigma^2)$, then :

$$\pi_A = \frac{a\sigma^2}{2}$$

— **[Portfolio Application of CARA]** : The Optimal investment fraction in risky asset :

$$\pi_* = \frac{\mu - r}{a\sigma^2}$$

— CRRA = Constant Relative Risk-Aversion. Example if $\gamma \neq 1$:

$$U(x) = \frac{x^{1-\gamma} - 1}{1 - \gamma}$$

If $log(x) \sim \mathcal{N}(\mu, \sigma^2)$, then :

$$\pi_R = 1 - e^{-\frac{\gamma\sigma^2}{2}}$$

— **[Portfolio Application of CARA]** : The Optimal investment fraction in risky asset :

$$\pi_* = \frac{\mu - r}{\gamma\sigma^2}$$