# hw1

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## Question 2

### (a)

### (b)

The estimates are given by the following formula:

$$\boxed{\hat{\alpha}_1 = \frac{\sum\limits_{i=1}^{n} \widetilde{x}_i (y_i - \bar{y})}{\sum\limits_{i=1}^{n} \widetilde{x}_i^2}}$$

and

$$\boxed{\hat{\alpha}_0 = \bar{y}}$$

because $\frac{1}{n} \sum_{i=1}^{n} \widetilde{x}_i = 0$

### (c)

We can write:

$$\begin{aligned}
\hat{\alpha}_1 &= \frac{\sum\limits_{i=1}^{n} \widetilde{x}_i (y_i - \bar{y})}{\sum\limits_{i=1}^{n} \widetilde{x}_i^2} \\
&= \frac{(1/s_X) \times \sum\limits_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{(1/s_X)^2 \times \sum\limits_{i=1}^{n} (x_i - \bar{x})^2} \\
&= s_X \times \frac{n \times s_{XY} \times s_Y}{n \times s_X^2 \times s_Y}
\end{aligned}$$

Finally :

$$\boxed{\hat{\alpha}_1 = s_Y \times \rho\widehat{(X, Y)}}$$

### (d)

By noting that $var(Y) = \sigma^2$, the sampling variance of these estimates is given by:

$$var(\hat{\alpha}_1) = s_X^2 \times \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{(\sum_{i=1}^{n}(x_i - \bar{x})^2)^2} \times \sigma^2$$
$$= s_X^2 \times \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Finally :

$$\boxed{var(\hat{\alpha}_1) = \frac{\sigma^2}{n}}$$

Then :

$$var(\hat{\alpha}_0) = var(\bar{y})$$

So then

$$\boxed{var(\hat{\alpha}_0) = \frac{\sigma^2}{n}}$$

Furthermore,

$cov(\hat{\alpha}_0, \hat{\alpha}_1) = \frac{\sum \widetilde{x}_i}{\sum \widetilde{x}_i^2} cov(y_i - \bar{y}, \bar{y})$ with $cov(y_i, \bar{y}) = \frac{1}{n}\sigma^2$ as $cov(y_i, y_k) = 0$ when $i \neq k$ because samples are *iid* and also $cov(\bar{y}, \bar{y}) = \frac{1}{n}\sigma^2$.

So finally:

$$cov(\hat{\alpha}_0, \hat{\alpha}_1) = \sigma^2 \times \frac{\sum \widetilde{x}_i}{\sum \widetilde{x}_i^2} \times (\frac{1}{n}) - \frac{1}{n}) \implies \boxed{cov(\hat{\alpha}_0, \hat{\alpha}_1) = 0}$$

.

### (e)

### (f)