

BIG O NOTATION

TIME and space complexity of algorithms

Big O - in mathematics

Big O - who - what???

$O(n)$

Mathematical notation that describes the limiting behavior of a function when the argument tends towards a particular value or infinity.

Paul Bachmann, Edmund Landau (and others)

Bachmann-Landau notation or asymptotic notation

Big O - in computer science

...that's what i like

$O(n)$

$O(1)$

$O(n^2)$

Used to **classify algorithms** according to how their **run time** or **space** requirements grow as the **input** size grows

Classify algorithm - how long (upper bound = worst case)

Time - CPU (ms, μ s, ns) computation time

Space - RAM (memory) size

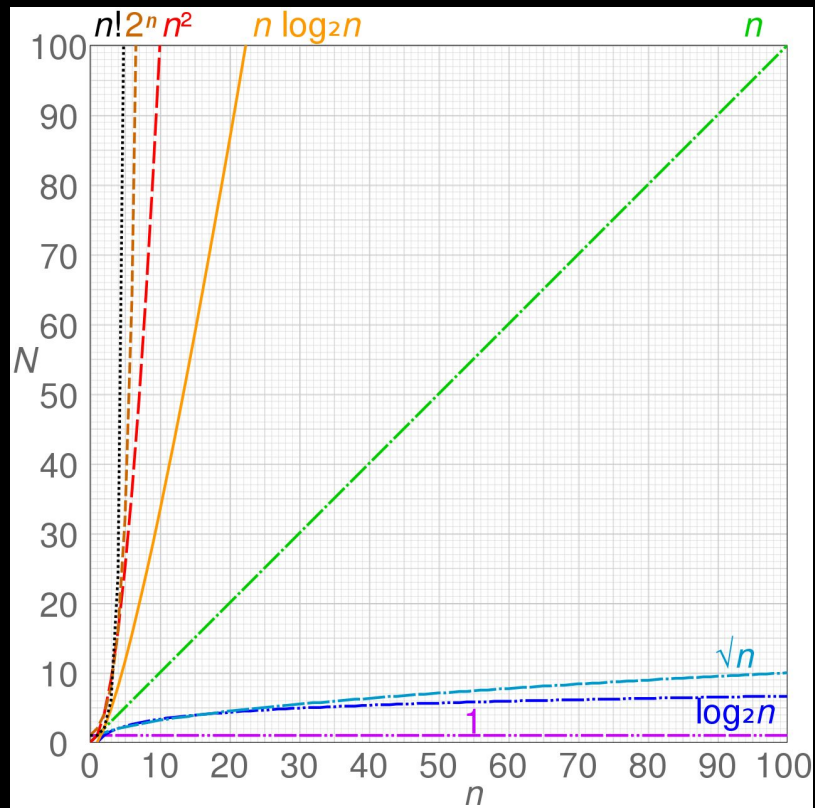
Input (n) - data (variables) to process

O - order of the function

Time consumption

uuuh... a picture :)

Big - O graph



and little insignificant table

| | <i>constant</i> | <i>logarithmic</i> | <i>linear</i> | <i>N-log-N</i> | <i>quadratic</i> | <i>cubic</i> | <i>exponential</i> |
|----------|-----------------|--------------------|---------------|----------------|------------------|--------------|-----------------------|
| <i>n</i> | $O(1)$ | $O(\log n)$ | $O(n)$ | $O(n \log n)$ | $O(n^2)$ | $O(n^3)$ | $O(2^n)$ |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| 2 | 1 | 1 | 2 | 2 | 4 | 8 | 4 |
| 4 | 1 | 2 | 4 | 8 | 16 | 64 | 16 |
| 8 | 1 | 3 | 8 | 24 | 64 | 512 | 256 |
| 16 | 1 | 4 | 16 | 64 | 256 | 4,096 | 65536 |
| 32 | 1 | 5 | 32 | 160 | 1,024 | 32,768 | 4,294,967,296 |
| 64 | 1 | 6 | 64 | 384 | 4,096 | 262,144 | 1.84×10^{19} |

Constant

$O(1)$

some code, ...finally!

```
void constant(Variable n) {  
    statement;  
}
```

```
int firstNumber(int[] numbers) {  
    return numbers[0];  
}
```

Linear

$O(n)$

straight ahead sailor!

```
void linear(Variable[] n) {  
    for (int i = 0; i < n.length; i++) {  
        statement;  
    }  
}
```

Quadratic

$O(n^2)$

...some curves at last

```
void quadratic(Variable[] n) {  
    for (int i = 0; i < n.length; i++) {  
        for (int j = 0; j < n.length; j++) {  
            statement;  
        }  
    }  
}
```


Worst case

n -> infinity

...and beyond...

$$f(n) = a*n^2 + b*n + c$$

$$f(n) = n/a$$

Other bounds

Ω , Θ , O

“I have no strong feelings one way or the other.”

Ω

lower bound - BEST CASE

Θ

average

O

upper bound - WORST CASE

Data structures operations

operative operations

| Data Structure | Time Complexity | | | | | | | | Space Complexity |
|--------------------|-----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|------------------|
| | Average | | | | Worst | | | | Worst |
| | Access | Search | Insertion | Deletion | Access | Search | Insertion | Deletion | |
| Array | $O(1)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(1)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| Stack | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ |
| Queue | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ |
| Singly-Linked List | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ |
| Doubly-Linked List | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ | $O(n)$ |
| Skip List | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n \log(n))$ |
| Hash Table | N/A | $O(1)$ | $O(1)$ | $O(1)$ | N/A | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| Binary Search Tree | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| Cartesian Tree | N/A | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | N/A | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |
| B-Tree | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ |
| Red-Black Tree | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ |
| Splay Tree | N/A | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | N/A | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ |
| AVL Tree | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ |
| KD Tree | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(\log(n))$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ | $O(n)$ |

Array sorting algorithms

what about sorting your time for LT Martin?

| Algorithm | Time Complexity | | | Space Complexity |
|-----------------------|---------------------|------------------------|-------------------|------------------|
| | Best | Average | Worst | Worst |
| <u>Quicksort</u> | $\Omega(n \log(n))$ | $\Theta(n \log(n))$ | $O(n^2)$ | $O(\log(n))$ |
| <u>Mergesort</u> | $\Omega(n \log(n))$ | $\Theta(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |
| <u>Timsort</u> | $\Omega(n)$ | $\Theta(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |
| <u>Heapsort</u> | $\Omega(n \log(n))$ | $\Theta(n \log(n))$ | $O(n \log(n))$ | $O(1)$ |
| <u>Bubble Sort</u> | $\Omega(n)$ | $\Theta(n^2)$ | $O(n^2)$ | $O(1)$ |
| <u>Insertion Sort</u> | $\Omega(n)$ | $\Theta(n^2)$ | $O(n^2)$ | $O(1)$ |
| <u>Selection Sort</u> | $\Omega(n^2)$ | $\Theta(n^2)$ | $O(n^2)$ | $O(1)$ |
| <u>Tree Sort</u> | $\Omega(n \log(n))$ | $\Theta(n \log(n))$ | $O(n^2)$ | $O(n)$ |
| <u>Shell Sort</u> | $\Omega(n \log(n))$ | $\Theta(n(\log(n))^2)$ | $O(n(\log(n))^2)$ | $O(1)$ |
| <u>Bucket Sort</u> | $\Omega(n+k)$ | $\Theta(n+k)$ | $O(n^2)$ | $O(n)$ |
| <u>Radix Sort</u> | $\Omega(nk)$ | $\Theta(nk)$ | $O(nk)$ | $O(n+k)$ |
| <u>Counting Sort</u> | $\Omega(n+k)$ | $\Theta(n+k)$ | $O(n+k)$ | $O(k)$ |
| <u>Cubesort</u> | $\Omega(n)$ | $\Theta(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |

THE END

...these aren't the answers you are looking for...