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The Distributed Ontology, Modeling, and Specification Language (DOL)

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Preface

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`Courier - 10 pt. Bold:` Programming language elements.

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0 Submission-Specific Material

0.1 Submission Preface

Fraunhofer FOKUS, MITRE, and Thematix Partners LLC are pleased to submit this joint proposal in response to the Ontology, Modeling and Specification Integration and Interoperability (OntoIOp) RFP (OMG document ad/2013-12-02). The joint proposal is supported by Athan Services and the Otto-von-Guericke University Magdeburg. The contacts for this submission are:

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0.2 Mandatory Requirements

ID	RFP requirement	How this proposal addresses requirement
6.5.1(a)	Proposals shall provide a specification of a metalanguage for relationships between the components of logically heterogeneous OMS, particularly, given a language translation from a language L1 to another language L2, the application of the language translation to an OMS that is written in the language L1.	DOL provides the required translation construct using syntax `O with translation t`, see **??** and **??**. Moreover, DOL provides heterogeneous interpretations between OMS, see **??** and **??**.
6.5.1(b)	Proposals shall provide a specification of a metalanguage for the union of OMS written in different languages, which implicitly involves the application of suitable default translations in order to reach a common target language.	The syntax for unions is `O1 and O2`, see **??** and **??**. Default translations are discussed in **??**, and DOL's notion of heterogeneous logical environment explicitly specifies default translations, see **??**.
6.5.1(c)	Proposals shall provide a specification of a metalanguage for importation in modular OMS.	DOL allows the import of OMS by their IRI, see **??** and **??**.
6.5.1(d)	Proposals shall provide a specification of a metalanguage for relationships between OMS and their extracted modules e.g. the whole theory is a conservative extension of the module.	DOL provides such a construct with syntax `module m : o1 of o2 for sig`, see **??** and **??**.
6.5.1(e)	Proposals shall provide a specification of a metalanguage for relationships between OMS and their approximation in less expressive languages such that the approximation is logically implied by the original theory, where the approximation generally has to be maximal in some suitable sense.	DOL provides such a construct with syntax `o keep logic`, see **??** and **??**.
6.5.1(f)	Proposals shall provide a specification of a metalanguage for links such as imports, interpretations, refinements, and alignments between OMS/modules.	DOL covers several metalogical relationships, namely entailments, interpretations, equivalences, refinements, alignments and module relations, see **??** and **??**.
6.5.1(g)	Proposals shall provide a specification of a metalanguage for combination of OMS along links.	DOL provides such a construct with syntax `combine n`, where `n` is a network of OMS and mappings (links), see **??** and **??**.
6.5.2(a)	The constructs of the metalanguage shall be applicable to different logics.	The semantics of DOL is based on a heterogeneous logical environment, which can contain arbitrary logics, see **??**.

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Table 0.1 – *Continued from previous page*

ID	RFP requirement	How this proposal addresses requirement
6.5.2(b)	The metalanguage shall neither be restricted to OMS in a specific domain, nor to OMS represented in a specific logical language.	The semantics of DOL is based on a heterogeneous logical environment, which can contain arbitrary logics, see ??.
6.5.2(c)	The metalanguage shall not replace the object language constructs of the conforming logical languages.	The syntax of a `NativeDocument` is left unspecified in this standard. Rather, here this standard relies on other standards and language definitions. See ?? and ??.
6.5.2(d)	The metalanguage shall provide syntactic constructs for (i) structuring OMS regardless of the logic in which their sentences are formalized and (ii) basic and structured OMS and facilities to identify them in a globally unique way.	The structuring constructs for OMS in ?? and ?? can be used for any logic, see the semantics in ??. DOL uses IRIs for referencing both basic and structured OMS, see ??.
6.5.3(a)	An abstract syntax specified as an SMOF compliant meta model.	The abstract syntax is specified using SMOF, see clause ??. An EBNF variant is given in annex ??.
6.5.3(b)	A human-readable lexical concrete syntax in EBNF and serialization in XML, for the latter XMI shall be used.	The concrete syntax (in EBNF) is specified in clause ??. The XMI representation is automatically derived from the SMOF meta model.
6.5.3(c)	Complete round-trip mappings from the human-readable concrete syntax to the abstract syntax and vice versa.	The metaclasses of the MOF abstract syntax are used as non-terminals of the EBNF concrete syntax (clause ??); this makes a round-trip mapping between both straight-forward. Moreover, the round-trip mapping has been implemented in form of a parser and a printer as part of the heterogeneous tool set (see appendix ?? and `http://hets.eu`).
6.5.3(d)	A formal semantics for the abstract syntax.	The formal semantics is given in clause ??.
6.5.4(a)	Existing OMS in existing serializations shall validate as OMS in the metalanguage with a minimum amount of syntactic adaptation.	Any document providing an OMS in a serialization of a DOL conforming language can be used as-is in DOL, by reference to its IRI. See ??.
6.5.4(b)	It shall be possible to refer to existing files/documents from an OMS implemented in the metalanguage without the need for modifying these files/-documents.	Documents can be referenced by IRIs, see ??.
6.5.4(c)	Translations between logical languages shall preserve (possibly to different degrees) the semantics of the logical languages. Between a given pair of logical languages, several translations are possible.	The semantics of DOL is based on a heterogeneous logical environment, which contains institution comorphisms as translations, see ??. Institution comorphisms preserve semantics in a weak form through their satisfaction condition. The DOL Ontology specifies properties of translations (comorphisms) preserving more and more of the semantics, see annex ??.
6.5.5(a)	Informative annexes shall establish the conformance of a number of relevant logical languages. An initial set of language translations may be part of an informative annex.	For conformance of logical languages, see 6.5.5(b) below. Conformance of some translations is established in annex ??.
6.5.5(b)	Conformance of the following subset of logical languages shall be established: OWL2 (with profiles EL, RL, QL), CLIF, RDF, UML class diagrams.	Conformance of the following languages is established: OWL 2 (annex ??), CLIF (annex ??), RDF and RDF Schema (annex ??), UML class diagrams (annex ??).
6.5.5(c)	Conformance of a suitable set of translations among the languages mentioned in the previous bullet point shall be established.	Conformance of some translations is established in annex ??.
6.5.6	Existing standards and best practices for allocating globally unique identifiers shall be reused. The same standards and best practices shall also be applied to associate different representations of the same content to one unique identifier.	DOL uses IRIs to reference documents (both DOL documents, as well as documents written in some conforming language). See ??.

0.3 Optional Requirements

ID	RFP requirement	How this proposal addresses requirement
6.6.1	Submissions may include additional languages without a standardized model theory.	This has been left for forthcoming versions.
6.6.2	Proposals may provide constructs for non-monotonic logics.	Currently, only monotonic logics are supported. However, DOL provides a circumscription-like non-monotonic structuring construct with syntax `o1 then %minimize o2`, see ?? and ??.
6.6.3	A characterization of the trade-offs among different translations.	This is left for future work.

0.4 Issues to be Discussed

ID	Discussion item	Resolution
6.7.(a)	Do existing language standards need to be extended or adapted in order to make them OntoIOp conforming.	The goal of DOL is to support existing languages without any adaptations, see also 6.5.4(a). However, in order to meet requirement 6.5.6, DOL-conforming languages should support the use of IRIs. If they do not, there is a mechanism for assigning IRIs to (fragments of) language documents even if the language itself does not support this, see 2.2. Moreover, there is a mechanism for injecting IRIs in existing language serializations, see **??** and **??**.
6.7.(b)	Proposals should discuss whether the semantics of the metalanguage shall be included into the standard	The semantics of the DOL metalanguage is included in this specification. The reasons are discussed in the introduction of clause **??**.
6.7.(c)	Proposals should discuss the chosen list of logics and translations.	The chosen list of logics and translations is discussed in the introduction of annex **??**.
6.7.(d)	Proposals should discuss a meta-ontology of logical languages and theories.	The DOL Ontology is discussed in annex **??**.
6.7.(e)	Proposals should discuss the use of QVT for expressing logic translations.	This is discussed in annex **??**.
6.7.(f)	Proposals should discuss the role of APIs.	The role of APIs is discussed in section **??**.
6.7.(g)	Proposals should discuss availability and use of tools.	Tools for DOL are discussed in annex **??**.
6.7.(h)	Proposals should discuss a registry of logical languages.	A registry is discussed in annex **??**.

0.5 Evaluation Criteria

ID	Criterion	Comment
6.8(a)	Proposals covering a broader range of features and of use cases will be favored. As a minimum, proposals shall define conformance criteria for logical languages and translations, and their proposed metalanguage shall cover some metalogical relationships and shall be applicable to multiple logics.	Based on the notion of institution, conformance criteria for logical languages are defined in 2.1 and those for translations in 2.1.1. DOL covers several metalogical relationships, namely entailments, interpretations, equivalences, refinements, alignments and module relations, see ?? and ??. DOL is applicable to multiple logics (see also 6.8(c) and ?? below).
6.8(b)	Proposals covering existing language standards without (or with fewer) modifications will be favored.	Any document providing an OMS in a serialization of a DOL conforming language can be used as-is in DOL, by reference to its IRI. See ??.
6.8(c)	Proposals establishing actually (or making this at least possible in theory) OntoIOp conformance of more logical languages and translations will be favored.	The conformance of OWL 2 (annex ??), Common Logic (annex ??), RDF and RDF Schema (annex ??), UML class diagrams (annex ??) and CASL (annex ??) is established.

0.6 Proof of Concept

Prototypical open source tools for DOL are already available, see annex ??. It is expected that they will reach industrial strength within two or three years.

0.7 Changes to Adopted OMG Specifications

This specification proposes no changes to adopted OMG specifications.

1 Scope

This OMG Specification specifies the Distributed Ontology, Modeling and Specification Language (DOL). DOL is designed to achieve integration and interoperability of ontologies, specifications and MDE models (OMS for short). DOL is a language for distributed knowledge representation, system specification and model-driven development across multiple OMS, particularly OMS that have been formalized in different OMS languages. This OMG Specification responds to the OntoIOp Request for Proposals [?].

1.1 Background Information

Logical languages are used in several fields of computing for the development of formal, machine-processable texts that carry a formal semantics. Among those fields are 1) **O**ntologies formalizing domain knowledge, 2) (formal) **M**odels of systems, and 3) the formal **S**pecification of systems. Ontologies, MDE models and specifications will (for the purpose of this document) henceforth be abbreviated as **OMS**.

An OMS provides formal descriptions, which range in scope from domain knowledge and activities (ontologies, MDE models) to properties and behaviors of hardware and software systems (MDE models, specifications). These formal descriptions can be used for the analysis and verification of domain models, system models and systems themselves, using rigorous and effective reasoning tools. As systems increase in complexity, it becomes concomitantly less practical to provide a monolithic logical cover for all. Instead various MDE models are developed to represent different viewpoints or perspectives on a domain or system. Hence, interoperability becomes a crucial issue, in particular, formal interoperability, i.e. interoperability that is based on the formal semantics of the different viewpoints. Interoperability is both about the ability to interface different domains and systems and the ability to use several OMS in a common application scenario. Further, interoperability is about coherence and consistency, ensuring at an early stage of the development that a coherent system can be reached.

In complex applications, which involve multiple OMS with overlapping concept spaces, it is often necessary to identify correspondences between concepts in the different OMS; this is called OMS alignment. While OMS alignment is most commonly studied for OMS formalized in the same OMS language, the different OMS used by complex applications may also be written in different OMS languages, which may even vary in their expressiveness. This OMG Specification faces this diversity not by proposing yet another OMS language that would subsume all the others. Instead, it accepts the diverse reality and formulates means (on a sound and formal semantic basis) to compare and integrate OMS that are written in different formalisms. It specifies DOL, a formal language for expressing not only OMS but also mappings between OMS formalized in different OMS languages.

Thus, DOL gives interoperability a formal grounding and makes heterogeneous OMS and services based on them amenable to checking of coherence (e.g. consistency, conservativity, intended consequences, and compliance).

1.2 Features Within Scope

The following are within the scope of this OMG Specification:

1. homogeneous OMS as well as heterogeneous OMS (OMS that consist of parts written in different languages);
2. mappings between OMS (which map OMS symbols to OMS symbols);
3. OMS networks (involving several OMS and mappings between them);
4. translations between different OMS languages conforming with DOL (translating a whole OMS to another language);
5. structuring constructs for modeling non-monotonic behavior;
6. annotation and documentation of OMS, mappings between OMS, symbols, and sentences;
7. recommendations of vocabularies for annotating and documenting OMS;
8. a syntax for embedding the constructs mentioned under (1)–(6) as annotations into existing OMS;
9. a syntax for expressing (1)–(5) as standoff markup that points into existing OMS;
10. a formal semantics of (1)–(5);
11. criteria for existing or future OMS languages to conform with DOL.

The following are outside the scope of this OMG Specification:

1. the (re)definition of elementary OMS languages, i.e. languages that allow the declaration of OMS symbols (non-logical symbols) and stating sentences about them;

2. algorithms for obtaining mappings between OMS;
3. concrete OMS and their conceptualization and application;
4. mappings between services and devices, and definitions of service and device interoperability;
5. non-monotonic logics[1].

This OMG Specification describes the syntax and the semantics of the Distributed Ontology, Modeling and Specification Language (DOL) by defining an abstract syntax and an associated model-theoretic semantics for DOL.

[1]Only monotonic logics are within scope of this specification. Conformance criteria for non-monotonic logics are still under development. However, closure (i.e. employing a closed-world assumption) provides non-monotonic reasoning in DOL. It is also possible to include non-monotonic logics by construing entailments between formulas as sentences of the logic (formalized as an institution).

2 Conformance

This clause defines conformance criteria for languages and logics that can be used with DOL, as well as conformance criteria for serializations, translations and applications. The conformance of a number of OMS languages (namely OWL 2, Common Logic, RDF and RDF Schema, UML Class Diagrams, TPTP, CASL) as well as translations among these is discussed in informative annexes of this OMG Specification.

2.1 Conformance of an OMS Language/a Logic with DOL

Rationale: for an OMS language to conform with DOL,

- its logical language aspect either needs to satisfy certain criteria related to its own abstract syntax and formal semantics, or there must be a translation (again satisfying certain criteria) to a language that already is DOL-conforming.
- its structuring language aspect (if present) must be compatible with DOL's own structuring mechanisms
- its annotation language aspect must be compatible with DOL's meta-language constructs.

Several conformance levels are defined. They differ with respect to the usage of IRIs as identifiers for all kinds of entities that the OMS language supports.

An OMS language is conforming with DOL if it satisfies the following conditions:

1. its abstract syntax is conformant. This means that a) it is specified as an SMOF compliant meta model or as an EBNF grammar. Moreover, b) an SMOF metaclass or an EBNF non-terminal has to be declared to be a subclass of `NativeDocument`, and optionally another metaclass or non-terminal may be declared to be a subclass of `BasicOMS` (see clause ??);
2. it has at least one serialization in the sense of section 2.2;
3. either there exists a translation of it into a conforming language[1], or:
   a) the logical language aspect (for expressing basic OMS) is conforming, and in particular has a semantics (see below),
   b) the structuring language aspect (for expressing structured OMS and relations between those) is conforming (see below), and
   c) the annotation language aspect (for expressing comments and annotations) is conforming (see below).

The *logical language aspect* of an OMS language is conforming with DOL if each logic corresponding to a profile (including the logic corresponding to the whole logical language aspect) is presented as an institution in the sense of Definition **??** in clause **??** , and there is a mapping from the abstract syntax of the OMS language to signatures and sentences of the institution. Note that one OMS language can have several sublanguages or profiles corresponding to several logics (for example, OWL 2 has profiles EL, RL and QL, apart from the whole OWL 2 itself).

The *structuring language aspect* of an OMS language is conforming with DOL if it can be mapped to DOL's structuring language in a semantics-preserving way. The structuring language aspect **may** be empty.

The *annotation language aspect* of an OMS language is conforming with DOL if its constructs have no impact on the semantics. The annotation language aspect **shall** be non-empty; it **shall** provide the facility to express comments.

Concerning item 1. in the definition of DOL conformance of OMS languages above, the following levels of conformance of the abstract syntax of an OMS language with DOL are defined, listed from highest to lowest:

Full IRI conformance The abstract syntax specifies that IRIs be used for identifying all symbols and entities.

No mandatory use of IRIs The abstract syntax does not require IRIs to be used to identify entities. Note that this includes the case of optionally supporting IRIs without enforcing their use (such as in Common Logic).

Any conforming language and logic shall have a machine-processable description as detailed in clause 2.3.

[1] For example, consider the translation of OBO1.4 to OWL, giving a formal semantics to OBO1.4.

2.1.1 Conformance of language/logic translations with DOL

Rationale: a translation between logics must satisfy certain criteria in order to conform with DOL. Also, a translation between OMS languages based on such logics must be consistent with the translation between these logics. Translations should break neither structuring language aspects nor comments/annotations.

A logic translation is conforming with DOL if it is presented either as an institution morphism or as an institution comorphism.

A language translation **shall** provide a mapping between the abstract syntaxes (it **may** also provide mappings between concrete syntaxes). A language translation from language L_1 (based on institution I_1) to language L_2 (based on institution I_2) is conforming with DOL if it is based on a logic translation such that the following diagram commutes (i.e. following both possible paths from L_1 to I_2 leads to the same result):

$$\begin{tikzcd}
L_1 \arrow[r, "\text{mapping between abstract syntaxes}"] \arrow[d, "\text{abstract syntax to institution}"'] & L_2 \arrow[d, "\text{abstract syntax to institution}"] \\
I_1 \arrow[r, "\text{institution (co)morphism}"] & I_2
\end{tikzcd}$$

Language translations **may** also translate the structuring language aspect, in this case, they **shall** preserve the semantics of the structuring language aspect. Furthermore, language translations **should** preserve comments and annotations. All comments attached to a sentence (or symbol) in the source **should** be attached to its translation in the target (if there are more than one sentences (respectively symbols) expressing the translation, to at least one of them).

2.2 Conformance of a Serialization of an OMS Language With DOL

Rationale: The main reason for the following specifications is identifier injection. DOL is capable of assigning identifiers to entities (symbols, axioms, modules, etc.) inside fragments of OMS languages that occur in a DOL document, even if that OMS language does not support such identifiers by its own means. Such identifiers will be visible to a DOL tool, but not to a tool that only supports the OMS language. To achieve this without breaking the formal semantics of that OMS language, DOL utilizes the annotation or commenting features that the OMS language supports, in order to place such identifiers inside annotations/ comments. Depending on the nature of a given concrete serialization of the OMS language (be it plain text, some serialization of RDF, XML, or some other structured text format), one can be more specific about what the annotation/commenting facilities of that serialization must look like in order to support this identifier injection. Well-behaved XML and RDF schemas support identifier injection in a 'nice' way (rather than using text-level comments). In the worst case it is not possible to inject something into an OMS language fragment, because the OMS language serialization does not enable the addition of suitable comments. In this case the solution is to point into the OMS language fragment from the enclosing context by using standoff markup.

Further conformance criteria in this section are introduced to facilitate the convenient reuse of verbatim fragments of OMS language inside a DOL document.

Independently from these criteria, several levels of conformance of a serialization are distinguished. They differ with respect to their means of conveniently abbreviating long IRI identifiers.

There are seven levels of conformance of a serialization of an OMS language with DOL.

XMI conformance An XMI serialization for OMS written in the OMS language has been automatically derived from the SMOF specification of the abstract syntax, using the canonical MOF 2 XMI Mapping.

XML conformance The given serialization has to be specified as an XML schema that satisfies all of the following conditions:

1. The elements of the schema belong to one or more non-empty XML namespaces.
2. The serialization shall use XML *elements* to represent all structural elements of an OMS.
3. XML elements that represent structural elements of an OMS shall support identifier injection in at least one of the following two ways:
   a) Such elements shall be able to carry annotations that comprise at least an object (the value of the annotation) and a IRI-valued predicate (the type of annotation), where the structural element is the subject. The value of the predicate shall either be full IRI according to IETF/RFC 3987:2005, or the serialization shall specify a way of interpreting the value of the predicate as a full IRI – for example if it is a relative URI or if an

abbreviating notation is used. Analogously, the serialization shall permit the object to be a full IRI or anything that can be interpreted as a full IRI.

b) The schema shall not forbid both attributes and child elements from foreign namespaces (here: the DOL namespace `http://www.omg.org/spec/DOL/1.0/xml`) on ~~any elements. (This is~~ such elements.

This requirement is necessary because either an annotation or an attribute or a child element is used to inject identifiers into elements of the XML serialization; cf. clause ??. ~~)~~

RDF conformance The given serialization has to be specified as an RDF vocabulary that satisfies all of the following conditions:

1. The elements of the vocabulary belong to one or more RDF namespaces identified by absolute URIs.
2. The serialization shall specify ways of giving IRIs or URIs to all structural elements of an OMS. (The rationale is that RDF syntax supports the identification of any kinds of items, so an RDF-based serialization of an OMS language should not forbid making use of such RDF constructs that do allow for identifying arbitrary items.)
3. There shall be no additional rules (stated in writing in the specification of the serialization, or formalized in its implementation in, e.g., OWL) that forbid properties from foreign vocabulary namespaces to be stated about arbitrary subjects for the purpose of annotation.

See annex ?? for an example.

Text conformance The given serialization has to satisfy all of the following conditions:

- The serialization conforms with the requirements for the *text/plain* media type specified in IETF/RFC 2046, section 4.1.3.
- The serialization shall provide a designated comment construct that can be placed sufficiently flexibly as to be uniquely associated with any non-comment construct of the language. That means, for example, one of the following:
  - The serialization provides a construct that indicates the start and end of a comment and may be placed before/after each token that represents a structural element of an OMS.
  - The serialization provides line-based comments (ranging from an indicated position to the end of a line) but at the same time allows the flexible placement of line breaks before/after each token that represents a structural element of an OMS.

Standoff markup conformance ~~An OMS language is standoff markup conforming with if one of its serializations~~ The given serialization has to satisfy at least one of the following conditions:

1. The serialization conforms with the requirements for the *text/plain* media type specified in IETF/RFC 2046, section 4.1.3. Note that conformance with *text/plain* is a prerequisite for using, for example, fragment URIs in the style of IETF/RFC 5147 for identifying text ranges.
2. The serialization conforms with XML W3C/TR REC-xml:2008, which is a prerequisite for using XPointer fragment URIs for addressing subresources of an XML resource (cf. W3C/TR REC-xptr-framework:2003).

Independently from the conformance levels given above, there is the following hierarchy of conformance w.r.t. CURIEs (compact URIs) as a means of abbreviating IRIs (grammar specified in clause ??), listed from highest to lowest:

Prefixed CURIE conformance The given serialization allows non-logical symbol identifiers to have the syntactic form of a CURIE, or any subset of the CURIE grammar that allows named prefixes (`prefix:reference`, where a declaration of DOL-conformance of a serialization **may** redefine the separator character to a character different from `:`). A serialization that conforms w.r.t. a prefixed CURIE is **not required** to support CURIEs with no prefix: its declaration of DOL-conformance **may** forbid the use of prefixed CURIEs.
Informative comments:

- In the case that CURIEs are used, a prefix map with multiple prefixes **may** be used to map the non-logical symbol identifiers of a native OMS to IRIs in multiple namespaces (cf. clause ??)
- The reason for allowing redefinitions of the prefix/reference separator character is that certain serializations of OMS languages may not allow the colon (`:`) in identifiers.

Non-prefixed names only The given serialization only supports CURIEs with no prefix, or any subset of the grammar of the `REFERENCE` nonterminal in the CURIE grammar.
Informative comment: In this case, a binding for the empty prefix **must** be declared, as this is the only possibility of mapping the identifiers of the native OMS to IRIs that are located in one flat namespace.

Any conforming serialization of an OMS language shall have a machine-processable description as detailed in clause 2.3.

2.3 Machine-Processable Description of Conforming Languages, Logics, and Serializations

Rationale: When a parser processes a DOL OMS found somewhere that refers to modules in OMS languages, or includes them verbatim, the parser needs to know what language to expect; further DOL-supporting software needs to know, e.g., what other DOL-conforming languages the module in the given OMS language can be translated to. Therefore, all languages/logics/serializations that conform with DOL are required to describe themselves in a machine-processable way.

For any conforming OMS language, logic, and serialization of an OMS language, it is required that it be assigned an HTTP IRI, by which it can be identified. It is also required that a machine-processable description of this language/-logic/serialization is retrievable by dereferencing this IRI; this requirement follows the linked data principles W3C/TR REC-ldp-20150226:2015. As a minimal requirement, there must be a RDF description conforming to the vocabulary specified in annex **??**. That description must be made available in the RDF/XML serialization when a client requests content of the MIME type *application/rdf+xml*. Descriptions of the language/logic/serialization in further representations, having different content types, may be provided.

2.4 Conformance of a Document With DOL

Rationale: for exchanging DOL documents with other users/tools, nothing that has a formal semantics must be left implicit. One DOL tool may assume that by default any OMS fragments inside a DOL document are in some fixed OMS language unless specified otherwise, but another DOL tool can't be assumed to understand such DOL documents. Defaults are, however, practically convenient, which is the reason for having the following section about the conformance of an *application*.

A document conforms with DOL if it contains a DOL text that is well-formed according to the grammar. That means, in particular, that any information related to logics must be made explicit (as foreseen by the DOL abstract syntax specified in clause **??**), such as:

- the logic of each OMS that is part of the DOL document,
- any translation that is employed between two logics (unless it is one of the default translations specified in annex **??**)

However, details about aspects of an OMS that do not have a formal, logic-based semantics, may be left implicit. For example, a conforming document may omit explicit references to matching algorithms that have been employed in obtaining an alignment.

2.5 Conformance of an Application With DOL

In the ~~sequel~~following, “DOL abstract syntax” means an XMI document that conforms to the DOL metamodel. Optionally, further representations (e.g. as JSON) can be supported.

- A *parser* is DOL-conformant if it can parse the DOL textual syntax and produce the corresponding DOL abstract syntax.
- A *printer* is DOL-conformant if it can read DOL abstract syntax and produce DOL textual syntax.
- DOL-conformant software that is used to *edit, format or manage* DOL libraries **must** be capable of reading and writing DOL abstract syntax. Moreover, it **must** meet the requirements for a DOL-conformant parser if it is able to read in DOL textual input. It **must** meet the requirements of a DOL-conformant printer if it is able to generate DOL textual output. However, it is also possible that a software for DOL management will work on the abstract syntax only, delegating the reading and generation of DOL text to external parsers and/or printers.
- a *static analyzer* is DOL-conformant if it can compute the logic and the signature of an OMS according to the semantics defined in section **??**. In more detail, a static analyzer can have the following capabilities:
  - *simple analysis*: static analysis of DOL excluding networks and alignments;
  - *full analysis*: static analysis of full DOL.
- a *transformation tool* is DOL-conformant if it implements one (or more) language translations, logic translations, language projections and/or logic projections.
- Software that implements machine *reasoning* about OMS (e.g., theorem proving, approximation) complies with this specification if and only if it interprets DOL documents according to the semantics defined in section **??**. In more detail, a reasoning tool can have the following capabilities:
  - *simple logical consequence*, i.e. checking whether all sentences that are marked as `%implied` within basic OMS and extensions are logical consequences of the enclosing OMS;
  - *structured logical consequence*, i.e. checking whether all sentences that are marked as `%implied` are logical consequences of the enclosing OMS and whether all entailments in a DOL document have a defined semantics;
  - *interpretation*, i.e. checking whether all interpretations in a DOL document have a defined semantics;
  - *simple refinement*, i.e. checking whether all refinements of OMS in a DOL document have a defined semantics;

- *full refinement*, i.e. checking whether all refinements (both of OMS and networks) in a DOL document have a defined semantics;
- *simple conservativity*, i.e. checking whether all conservativity statements in a DOL document have a defined semantics;
- *full conservativity*, i.e. checking whether all statements about conservative, monomorphic, definitional and weakly definitional extensions in a DOL document have a defined semantics;
- *module extraction*, i.e. the ability to compute modules (typically, a given tool will provide this only for some logics);
- *approximation*, i.e. the ability to compute approximations (typically, a given tool will provide this only for some logics and logic projections);
- *full* DOL *reasoning*, i.e. checking whether an DOL document has a defined semantics.

In practice, DOL-aware *applications* may also deal with documents that are not conforming with DOL according to the criteria established in clause 2.4. However, an application only *conforms* with DOL if it is capable of producing DOL-conforming documents as its output when requested.

DOL-aware applications **shall** support a fixed (possibly extensible) set of OMS languages conforming with DOL.

It is, for example, possible that a DOL-aware application only supports OWL and Common Logic. In that case, the application may process DOL documents that mix OWL and Common Logic ontologies, as well as native OWL and Common Logic documents.

DOL-aware applications also **shall** be able to strip DOL annotations from embedded fragments in other OMS languages. Moreover, they **shall** be able to expand CURIEs into IRIs when requested.

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4 Terms and Definitions

For the purposes of this document, the following terms and definitions apply.

4.1 Distributed Ontology, Modeling and Specification Language

Distributed Ontology, Modeling and Specification Language; DOL unified metalanguage for the structured and heterogeneous expression of ontologies, specifications, and MDE models, using DOL libraries of OMS, OMS mappings and OMS networks, whose syntax and semantics are specified in this OMG Specification.

DOL library collection of named OMS and OMS networks, possibly written in different OMS languages, linked by named OMS mappings.

4.2 Native OMS, OMS, and OMS Languages

native OMS collection of expressions (like non-logical symbols, sentences and structuring elements) from a given OMS language.
Example A UML class diagram, an ontology written in OWL 2 EL, and a specification written in CASL are three different native OMS.
Note An OMS can be written in different OMS language serializations.

native document document containing a native OMS.

DOL document document containing a DOL library.

OMS language language equipped with a formal, declarative, logic-based semantics, plus non-logical annotations.
Example OMS languages include OWL 2 DL, Common Logic, F-logic, UML class diagrams, RDF Schema, and OBO.
Note An OMS language is used for the formal specification of native OMS.
Note An OMS language has a logical language aspect, a structuring language aspect, and an annotation language aspect.

DOL structured OMS syntactically valid DOL expression denoting an OMS that is built from smaller OMS as building blocks.

Note DOL structured OMS, typically, use basic OMS as building blocks for defining other structured OMS, OMS mappings or OMS networks.
Note All DOL structured OMS are structured OMS.

ontology logical theory that is used as a shard conceptualization

MDE model logical theory that is used as an abstract representation of a domain or of a system, in the sense of model-driven engineering (MDE)
Note Not to be confused with the term model in the sense of logic (model theory).

specification logical theory that is used to express formal constraint in mathematical structures, software systems and/or hardware systems

OMS (ontology, specification or MDE model) basic OMS or structured OMS.
NOTE
An OMS is either a basic OMS (which is always a native OMS, and can occur as a text fragment in a DOL document) or a structured OMS (which can be either a native structured OMS contained in some native document, or a DOL structured OMS contained in a DOL document).

NOTE An OMS has a single signature and model class over that signature as its model-theoretic semantics.

basic OMS; flat OMS native OMS that does not utilize any elements from the structuring language aspect of its language.
NOTE Basic OMS are self-contained in the sense that their semantics does not depend on some other OMS. In particular, a basic OMS does not involve any imports.
NOTE Since a basic OMS has no structuring elements, it consists of (or at least denotes) a signature equipped with a set of sentences and annotations.
NOTE In signature-free logics like Common Logic or TPTP, a basic OMS only consists of sentences. A signature can be obtained a posteriori by collecting all non-logical symbols occuring in the sentences.

non-logical symbol; OMS symbol atomic expression or syntactic constituent of an OMS that requires an interpretation through a model.
NOTE This differs from the notion of "atomic sentence": such sentences may involve several non-logical symbols.

EXAMPLE Non-logical symbols in OWL ~~W3C/TR REC-owl2-syntax:2009~~ W3C/TR REC-owl2-syntax:2012 (there called "entities") comprise

- individuals (denoting objects from the domain of discourse),
- classes (denoting sets of objects; also called concepts), and
- properties (denoting binary relations over objects; also called roles).

These non-logical symbols are distinguished from logical symbols in OWL, e.g., those for intersection and union of classes.

EXAMPLE Non-logical symbols in Common Logic ISO/IEC 24707:2007 comprise

- names (denoting objects from the domain of discourse),
- sequence markers (denoting sequences of objects).

These non-logical symbols are distinguished from logical symbols in Common Logic, e.g. logical connectives and quantifiers.

signature; vocabulary set (or otherwise structured collection) of non-logical symbols of an OMS.
NOTE The signature of a term is the set of all non-logical symbols occurring in the term. The notion of signature depends on the OMS language or logic.
NOTE The signature of an OMS is usually unequivocally determinable.

model semantic interpretation of all non-logical symbols of a signature.
NOTE A model of an OMS is a model of the signature of the OMS that also satisfies all the additional constraints expressed by the OMS. In case of flattenable OMS, these constraints are expressed by the axioms of the OMS.
NOTE This term refers to *model* in the sense of model theory (a branch of logic). It is not to be confused with MDE model in the sense of modeling (i.e., the "M" in OMS).
NOTE The notion of model depends on the OMS language or logic.

expression a finite combination of symbols that are well-formed according to applicable rules (depending on the language)

term syntactic expression either consisting of a single non-logical symbol or recursively composed of other terms (a.k.a. its subterms).
NOTE A term belongs to the logical language aspect of an OMS language.

sentence term that is either true or false in a given model, i.e. which is assigned a truth value in this model.
NOTE In a model, on the one hand, a sentence is always true or false. In an OMS, on the other hand, a sentence can have several logical statuses. For example, a sentence can be: an axiom, if postulated to be true; a theorem, if proven from other axioms and theorems; or a conjecture, if expecting to be proven from other axioms and theorems.
NOTE A sentence can conform to one or more signatures (namely those signatures containing all non-logical symbols used in the sentence).
NOTE It is quite common that sentences are required to be closed (i.e. have no free variables). However, this depends on the OMS language at hand.

NOTE A sentence belongs to the logical language aspect of an OMS language.
NOTE The notion of sentence depends on the OMS language or logic.

satisfaction relation relation between models and sentences indicating which sentences hold true in the model.
NOTE The satisfaction relation depends on the OMS language or logic.

logical theory signature equipped with a set of sentences over the signature.

NOTE Each logical theory can also be written a basic OMS, and conversely each basic OMS has as its semantics a logical theory.

entailment; logical consequence; specialization relation between two OMS (or an OMS and a sentence, or two OMS networks, or an OMS network and an OMS) expressing that the second item (the conclusion) is logically implied by the first one (the premise).
NOTE Entailment expresses that each model satisfying the premise also satisfies the conclusion.

NOTE The converse is generalization.

axiom sentence that is postulated to be valid (i.e. true in every model).

theorem sentence that has been proven from other axioms and theorems and therefore has been demonstrated to be a logical consequence of the axioms.

tool software for processing DOL libraries and OMS.

theorem proving process of demonstraing that a sentence (or OMS) is the logical consequence of some OMS.

theorem prover tool implementing theorem proving.

4.3 Structured OMS

structured OMS OMS that results from other (basic and structured) OMS by import, union, combination, OMS translation, OMS reduction or other structuring operations.
NOTE Structured OMS are either DOL structured OMS or native OMS that utilize elements of the structuring language aspect of their OMS language.

flattenable OMS OMS that can be seen, by purely syntactical means, to be logically equivalent to a flat OMS.
NOTE More precisely, an OMS is flattenable if and only if it is either a basic OMS or it is an extension, union, translation, module, approximation, filtering, or reference of named OMS involving only flattenable OMS.

elusive OMS OMS that is not flattenable.

subOMS OMS whose associated sets of non-logical symbols and sentences are subsets of those present in a given larger OMS.

import reference to an OMS behaving as if it were verbatim included; also import of DOL libraries.
NOTE Semantically, an import of O_2 into O_1 is equivalent to the verbatim inclusion of O_2 in place of the import declaration.
NOTE The purpose of O_2 importing O_1 is to make non-logical symbols and sentences of O_1 available in O_2.
NOTE Importing O_1 into O_2 turns O_2 into an extension of O_1.
NOTE An owl:import in OWL is an import.
NOTE The import of a whole DOL library into another DOL library is also called import.

union DOL structured OMS expressing the aggregation of several OMS to a new OMS, without any renaming.

OMS translation DOL structured OMS expressing the assignment of new names to some non-logical symbols of an OMS, or translation of an OMS along a language translation.
NOTE An OMS translation results in an OMS mapping between the original and the renamed OMS.
NOTE Typically, the resulting OMS mapping of a translation is surjective: the symbols of the original OMS can be identified by the renaming, but no new symbols are added.

OMS reduction DOL structured OMS expressing the restriction of an OMS to a smaller signature.

local environment context for an OMS, being the signature built from all previously-declared symbols and axioms.

extension structured OMS extending a given OMS with new symbols and sentences.
NOTE The new symbols and sentences are interpreted relative to the local envorinment, which is the signature of the "given OMS".

extension mapping inclusion OMS mapping between two OMS where the sets of non-logical symbols and sentences of the second OMS are supersets of those present in the first OMS.
NOTE The second OMS is said to extend the first, and is an extension of the first OMS.

conservative extension extension that does not add new logical properties with respect to the signature of the extended OMS.
NOTE An extension is a consequence-theoretic or model-theoretic conservative extension. If used without qualification, the ~~consequence-theoretic~~ model-theoretic version is meant.

consequence-theoretic conservative extension extension that does not add new theorems (in terms of the unextended signature).
NOTE An extension O_2 of an OMS O_1 is a consequence-theoretic conservative extension, if all properties formulated in the signature of O_1 hold for O_1 whenever they hold for O_2.

model-theoretic conservative extension extension that does not lead to a restriction of class of models of an OMS.
NOTE An extension O_2 of an OMS O_1 is a model-theoretic conservative extension, if each model of O_1 can be expanded to a model of O_2.
NOTE Each model-theoretic conservative extension is also a consequence-theoretic one, but not vice versa.

monomorphic extension extension whose newly introduced non-logical symbols are interpreted in a way unique up to isomorphism.
NOTE An extension O_2 of an OMS O_1 is a monomorphic extension, if each model of O_1 can be expanded to a model of O_2 that is unique up to isomorphism.
NOTE Each monomorphic extension is also a model-theoretic conservative extension but not vice versa.

definitional extension extension whose newly introduced non-logical symbols are interpreted in a unique way.
NOTE An extension O_2 of an OMS O_1 is a definitional extension, if each model of O_1 can be uniquely expanded to a model of O_2.
NOTE O_2 being a definitional extension of O_1 implies a bijective correspondence between the classes of models of O_2 and O_1.
NOTE Each definitional extension is also a monomorphic extension but not vice versa.

weak definitional extension extension whose newly introduced non-logical symbols can be interpreted in at most one way.
NOTE An extension O_2 of an OMS O_1 is a weak definitional extension, if each model of O_1 can be expanded to at most one model of O_2.
NOTE An extension is definitional if and only if it is both weakly definitional and model-theoretically conservative.

implied extension model-theoretic conservative extension that does not introduce new non-logical symbols.
NOTE A conservative extension O_2 of an OMS O_1 is an implied extension, if and only if the signature of O_2 is the signature of O_1. O_2 is an implied extension of O_1 if and only if the model class of O_2 is the model class of O_1.
NOTE Each implied extension is also a definitional extension but not vice versa.

consistency property of an OMS expressing that it has a non-trivial set of logical consequences in the sense that not every sentence follows from the OMS.
NOTE The opposite is inconsistency.
NOTE In many (but not all) logics, consistency of an OMS equivalently can be defined as *false* not being a logical consequence of the OMS. However, this does not work for logics that e.g. do not feature a *false*. See [?] for a more detailed discussion.

satisfiability property of an OMS expressing that it is satisfied by least one model.
NOTE The opposite is unsatisfiability.
NOTE Any satisfiable OMS is consistent, but there are some logics where the converse does not hold.

model finding process that finds models of an OMS and thus proves it to be satisfiable.

model finder tool that implements model finding.

module structured OMS expressing a subOMS that conservatively extends to the whole OMS.
NOTE The conservative extension can be either model-theoretic or consequence-theoretic; without qualification, the ~~consequence-theoretic~~ model-theoretic version is used.

module extraction activity of obtaining from an OMS concrete modules to be used for a particular purpose (e.g. to contain a particular sub-signature of the original OMS).
NOTE Cited and slightly adapted from [?].
NOTE The goal of module extraction is "decomposing an OMS into smaller, more manageable modules with appropriate dependencies" [?].
EXAMPLE Assume one extracts a module about white wines from an OWL DL ontology about wines of any kind. That module would contain the declaration of the non-logical symbol "white wine", all declarations of non-logical symbols related to "white wine", and all sentences about all of these non-logical symbols.

approximant logically implied theory (possibly after suitable translation) of an OMS in a smaller signature or a sublanguage.

maximum approximant best possible approximant of an OMS in a smaller signature or a sublanguage.
NOTE Technically, a maximum approximant is a uniform interpolant, see [?].

approximation structured OMS that expresses a maximum approximant.

filtering structured OMS expressing the verbatim removal of symbols or axioms from an OMS.
NOTE If a symbol is removed, all axioms containing that symbol are removed, too.

closed world assumption assumption that facts whose status is unknown are true.

closure; circumscription structured OMS expressing a variant of the closed world assumption by restricting the models to those that are minimal, maximal, free or cofree (with respect to the local environment).
NOTE Symbols from the local environment are assumed to have a fixed interpretation. Only the symbols newly declared in the closure are forced to have minimal or maximal interpretation.
NOTE DOL supports four different forms of closure: minimization and maximization as well as freeness and cofreeness (explained below).

NOTE See [?], [?].

minimization form of closure that restricts the models to those that are minimal (with respect to the local environment).

maximization form of closure that restricts the models to those that are maximal (with respect to the local environment).

freeness special type of closure, restriction of models to those that are free (with respect to the local environment).
NOTE In first-order logic (and similar logics), freeness means minimal interpretation of predicates and minimal equality among data values. Freeness can be used for the specification of inductive datatypes like numbers, lists, trees, bags etc. In order to specify e.g. lists over some elements, the specification of the elements should be in the local environment.

cofreeness special type of closure, restriction of models to those that are cofree (with respect to the local environment).
NOTE In first-order logic (and similar logics), cofreeness means maximal interpretation of predicates and equality being observable equivalence. Cofreeness can be used for the specification of coinductive datatypes like infinite lists and streams.

combination structured OMS expressing the aggregation of all the OMS in an OMS network, where non-logical symbols are shared according to the OMS mappings in the OMS network.
EXAMPLE Consider an ontology involving a concept `Person`, and another one involving `Human being`, and an alignment that relates these two concepts. In the combination of the ontologies along the alignment, there is only one concept, representing both `Person` and `Human being`.

sharing property of OMS symbols being mapped to the same symbol when computing a combination of an OMS network.
NOTE Sharing is always relative to a given OMS network that relates different OMS. That is, two given OMS symbols can share with respect to one OMS network, and not share with respect to some other OMS network.

4.4 Mappings Between OMS

OMS mapping; link relationship between two OMS.

symbol map item pair of symbols of two OMS, indicating how a symbol from the first OMS is mapped by a signature morphism to a symbol of the second OMS
NOTE A symbol map item is given as $s_1 \mapsto s_2$, where s_1 is a symbol from the *source* OMS and s_2 is a symbol from the *target* of the OMS mapping.
NOTE Similar to correspondence.

signature morphism mapping between two signatures, preserving the structure of the source signature within the target signature
NOTE Each signature morphism has an underlying list of symbol map items. Conversely, a list of symbol map items may induce a signature morphism (but generally, it does not so in all cases).

interpretation; view; refinement OMS mapping that postulates a specialization relation between two OMS along a morphism between their signatures.
NOTE An interpretation typically leads to proof obligations, i.e. one has to prove that translations of axioms of the source OMS along the morphism accompanying the interpretation are theorems in the target OMS.

equivalence OMS mapping ensuring that two OMS share the same definable concepts.
NOTE Two OMS are equivalent if they have a common definitional extension. The OMS may be written in different OMS languages.

interface signature signature mediating between an OMS and a module of that OMS in the sense that it contains those non-logical symbols that the sentences of the module and the sentences of the OMS have in common.
NOTE Adapted from [?].

alignment an OMS mapping expressing a collection of semantic relations between entities of the two OMS.
NOTE Alignments consist of correspondences, each of which may have a confidence value. If all confidence values are 1, the alignment can be given a formal, logic-based semantics.

correspondence relationship between an non-logical symbol e_1 from an OMS O_1 and an non-logical symbol e_2 from an OMS O_2, or between an non-logical symbol e_1 from O_1 and a term t_2 formed from non-logical symbols from O_2, with a confidence level.
NOTE A correspondence is given as a quadruple $(e_1, R, \left\{ \begin{array}{c} e_2 \\ t_2 \end{array} \right\}, c)$, where R denotes the type of relationship that is asserted to hold between the two non-logical symbols/terms, and $0 \leq c \leq 1$ is a confidence value. R and c may be omitted: When R is omitted, it defaults to the equivalence relation, unless another default relation has been explicitly specified; when c is omitted, it defaults to 1.
NOTE A confidence value of 1 does not imply logical equivalence (cf. [?] for a worked-out example).
NOTE Not all OMS languages implement logical equivalence. For example, OWL does not implement logical equivalence in general, but separately implements equivalence relations restricted to individuals (*owl:sameAs*), classes (*owl:equivalentClass*) and properties (*owl:equivalentProperty*).
NOTE A default correspondence can be used for stating that all symbols with the same name in the two ontologies are equivalent. A correspondence block can be used for specifying the relation and/or the confidence value of several single correspondences in the same time: if the relation or the confidence value of a single correspondence in a block are missing, they will be replaced with those specified as parameters of the block.

matching algorithmic procedure that generates an alignment for two given OMS.
NOTE For both matching and alignment, see [?, ?].

matcher tool that implements matching.

OMS network; distributed OMS; hyperontology graph with OMS as nodes and OMS mappings as edges, showing how the OMS are interlinked.
NOTE In [?], a distinction between focused and distributed heterogeneous specifications is made. In the terminology of this standard, this is the distinction between OMS and OMS networks.
NOTE An OMS network is a diagram of OMS in the sense of category theory, but different from a diagram in the sense of model-driven engineering.
NOTE The links between the nodes of a network can be given using interpretations or alignments. Imports between the nodes of a network are automatically included in the network. By including an interpretation or an alignment in a distributed OMS, the involved nodes are automatically included.
EXAMPLE Consider two ontologies and an interpretation between them. In the network of the interpretation there are two nodes, one for each ontology, and one edge from the source ontology to the target ontology of the interpretation.

category a collection of objects with suitable morphisms between them.
NOTE In this standard, objects of a category are usually signatures or OMS, and morphisms are signature morphisms, or OMS mappings. In principle, no assumption about the exact nature of objects and morphisms is made.
NOTE The morphisms determine which part of the structure of the objects is relevant, i.e. preserved by morphisms. Hence, objects can be seen as "sets with structure", and morphisms as "structure-preserving maps". However note that not all categories can be obtained in this way.

4.5 Features of OMS Languages

mapping; function relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.
NOTE In some cases is a morphism, as in category theory.

language mapping mapping between languages
NOTE This is a general term, subsuming OMS language translation, logic translation and logic reduction below.

OMS language translation mapping from constructs in the source OMS language to their equivalents in the target OMS language.
NOTE An OMS language translation shall satisfy the property that the result of a translation is a well-formed text in the target language.

graph set of objects (nodes) that are connected by links (edges).

OMS language graph graph of OMS languages and OMS language translations, typically used in a heterogeneous environment.
NOTE In an OMS language graph, some of the OMS language translations can be marked to be default translations.

default translation specially marked OMS language translation or logic translation that will be used whenever a translation is needed and no explicit translation is given.

heterogeneous environment environment for the expression of homogeneous and heterogeneous OMS, comprising a logic graph, an OMS language graph and supports relations.
NOTE The support relations specify which language supports which logics and which serializations, and which language translation supports which logic translation or reduction. Moreover, each language has a default logic and a default serialization.
NOTE Although in principle, there can be many heterogeneous environments, for ensuring interoperability, there will be a global heterogeneous environment (maintained in some registry), with subenvironments for specific purposes.

sublanguage syntactically specified subset of a given language, consisting of a subset of its meta classes (abstract syntax) and terminal and nonterminal symbols and grammar rules (concrete syntax).

language aspect a set of language constructs of a given language, not necessarily forming a sublanguage.

logical language aspect the (unique) language aspect of an OMS language that enables the expression of non-logical symbols and sentences in a logic.

structuring language aspect the (unique) language aspect of an OMS language that covers structured OMS as well as the relations of basic OMS and structured OMS to each other, including, but not limited to imports, OMS mappings, conservative extensions, and the handling of prefixes for CURIEs.

annotation language aspect the (unique) language aspect of an OMS language that enables the expression of comments and annotations.

profile (syntactic) sublanguage of an OMS language interpreted according to a particular logic that targets specific applications or reasoning methods.
EXAMPLE Profiles of OWL 2 include OWL 2 EL, OWL 2 QL, OWL 2 RL, OWL 2 DL, and OWL 2 Full.
NOTE Profiles typically correspond to sublogics.
NOTE Profiles can have different logics, even with completely different semantics, e.g. OWL 2 DL versus OWL 2 Full.
NOTE The logic needs to support the language.

4.6 Logic

logic specification of valid reasoning that comprises signatures (user defined vocabularies), models (interpretations of these), sentences (constraints on models), and a satisfaction relation between models and sentences.
NOTE Most OMS languages have an underlying logic.
EXAMPLE $\mathcal{SROIQ}(D)$ is the logic underlying OWL 2 DL.
NOTE See annex ?? for the organization of the relation between OMS languages and their logics and serializations.

supports relation relation between OMS languages and logics expressing the logical language aspect of the former, namely that the constructs of the former lead to a logical theory in the latter.
NOTE There is also a supports relation between OMS languages and serializations, and one betwwen language translations and logic translations/reductions.

exact logical expressivity strengthening of the supports relation between languages and logics, stating that the language has exactly the expressivity of the logic.

institution metaframework mathematically formalizing the notion of a logic in terms of notions of signature, model, sentence and satisfaction.
NOTE In order to support a broad range of OMS languages and enable interoperability between them, the DOL semantics has to abstract from the differences of the logic language aspects of OMS languages. Institutions provide a formal framework that enables this abstraction.
NOTE The notion of institution uses category theory for providing formal interfaces for the notions of signature, model, sentence and satisfaction.

NOTE See Definition ?? in clause ?? for a formal definition.

plain mapping logic mapping that maps signatures to signatures and therefore does not use infrastructure axioms.

translation mapping between languages or logics representing all structure, in contrast to reduction.

reduction mapping between languages or logics forgetting parts of the structure, projection to a smaller language or logic.

logic translation translation of a source logic into a target logic (mapping signatures, sentences and models) that keeps or encodes the logical content of OMS.

logic reduction reduction of a source logic onto a (usually less expressive) target logic (mapping signatures, sentences and models) that forgets those parts of the logical structure not fitting the target logic.
~~translation that maps signatures of the source logic to theories (i.e. signatures and sets of sentences) of the target logic .~~

simple theoroidal logic translation translation that maps signatures of the source logic to theories (i.e. signatures and sets of sentences, playing the role of infrastructure axioms) of the target logic.
EXAMPLE The translation from OWL to multi-sorted first-order logic translates each OWL built-in type to its first-order axiomatization as a datatype.

infrastructure axiom axiom that is used in the target of a logic translation in order to encode a signature of the source logic
EXAMPLE The translation from OWL to multi-sorted first-order logic translates each OWL built-in type to its first-order axiomatization as a datatype. These first order axioms are infrastructure axioms.

sublogic a logic that is a syntactic restriction of another logic, inheriting its semantics.

logic graph graph of logics, logic translations and logic reductions, typically used in a heterogeneous environment.
NOTE In a logic graph, some of the logic translations and reductions can be marked to be default translations.

homogeneous OMS OMS whose parts are all formulated in one and the same logic.
NOTE The opposite of heterogeneous OMS.

heterogeneous OMS OMS whose parts are formulated in different logics.
NOTE The opposite of homogeneous OMS.
EXAMPLE See section **??**.

faithful mapping logic mapping that preserves and reflects logical consequence.

model-expansive mapping logic mapping that has a surjective model translation (ensuring faithfulness of the mapping).

model-bijective mapping logic mapping that has a bijective mapping of models.

exact mapping logic mapping that is compatible with certain DOL structuring constructs, e.g. union, OMS translation and OMS reduction.

weakly exact mapping logic mapping that is weakly compatible with certain DOL structuring constructs, e.g. union, OMS translation and OMS reduction.

embedding logic mapping that embeds the source into the target logic, using components that are embeddings and (in the case of model translations) isomorphism.

sublogic logic embedding that is "syntactic" in the sense that signature and sentence translations are inclusions.

adjointness relation between a logic translation and a logic reduction, expressing that they share their sentence and model translations, while the signature translations are adjoint to each other (in the sense of category theory).

4.7 Interoperability

logically interoperable property of structured OMS, which may be written in different OMS languages supporting different logics, of being usable jointly in a coherent way (via suitable OMS language translations), such that the notions of their overall consistency and logical entailment have a precise logical semantics.
NOTE Within ISO 19763 and ISO 20943, metamodel interoperability is equivalent to the existence of mapping, which are statements that the domains represented by two MDE models intersect and there is a need to register details of the correspondence between the structures in the MDE models that semantically represent this overlap. Within these standards, an MDE model is a representation of some aspect of a domain of interest using a normative modeling facility and modeling constructs.

The notion of logical interoperability is distinct from the notion of interoperability used in ISO/IEC 2381-1 Information Technology Vocabulary – Part 1: Fundamental Terms, which is restricted to the capability to communicate, execute programs, or transfer data among various hardware or software entities in a manner that requires the user to have little or no knowledge of the unique characteristics of those entities.

OMS interoperability relation among OMS (via OMS alignments) which are logically interoperable.

4.8 Abstract and Concrete Syntax

concrete syntax ; serialization specific syntactic encoding of a given OMS language or of DOL.
NOTE Serializations serve as standard formats for exchanging DOL documents and OMS between human beings and tools.
EXAMPLE OWL uses the term "serialization"; the following are standard OWL serializations: OWL functional-style syntax, OWL/XML, OWL Manchester syntax, plus any standard serialization of RDF (e.g. RDF/XML, Turtle, ...). However, W3C specifications only require an RDF/XML implementation for OWL2 tools.
EXAMPLE Common Logic uses the term "dialect"; the following are standard Common Logic dialects: Common Logic Interchange Format (CLIF), Conceptual Graph Interchange Format (CGIF), eXtended Common Logic Markup Language (XCL).

document result of serializing an OMS or DOL library using a given serialization.

standoff markup way of providing annotations to subjects in external resources, without embedding them into the original resource (here: OMS).

abstract syntax; parse tree term language for representing documents in a machine-processable way
NOTE An abstract syntax can be specified as a MOF metamodel. Then abstract abstract syntax documents can be represented as XMI documents.

4.9 Semantics

formalization precise mathematical entity capturing an informal or semi-formal entity.

formal semantics assignment of a mathematical meaning to a language by mapping the abstract syntax to suitable semantic domains.
NOTE A formal semantics is a formalization of the meaning of a language.

semantic domain mathematically-defined set of values that can represent the intended meanings of language constructs.

semantic rule specification of a mapping from expressions for some meta class in the abstract syntax to a semantic domain.

global environment mapping from identifiers (IRIs) to values in semantics domains representing the global knowledge about OMS.

4.10 Semantic Web

resource something that can be globally identified.
NOTE IETF/RFC 3986:2005, Section 1.1 deliberately defines a resource as "in a general sense [...] whatever might be identified by [an IRI]". The original source refers to URIs, but DOL uses the compatible IRI standard IETF/RFC 3987:2005 for identification.
EXAMPLE Familiar examples include an electronic document, an image, a source of information with a consistent purpose (e.g., "today's weather report for Los Angeles"), a service (e.g., an HTTP-to-SMS gateway), and a collection of other resources. A resource is not necessarily accessible via the Internet; e.g., human beings, corporations, and bound books in a library can also be resources. Likewise, abstract concepts can be resources, such as the operators and operands of a mathematical equation, the types of a relationship (e.g., "parent" or "employee"), or numeric values (e.g., zero, one, and infinity). See IETF/RFC 3986:2005, Section 1.1

element (of an OMS) any resource in an OMS (e.g. a non-logical symbol, a sentence, a correspondence, the OMS itself, ...) or a named set of such resources.

linked data structured data that is published on the Web in a machine-processable way, according to principles specified in W3C/TR REC-ldp-20150226:2015[1].

NOTE The linked data principles (adapted from W3C/TR REC-ldp-20150226:2015 and its paraphrase at [?]) are the following:

1. Use IRIs as names for things.
2. Use HTTP IRIs so that these things can be referred to and looked up ("dereferenced") by people and user agents. (I.e., the IRI is treated as a URL (uniform resource locator).)
3. Provide useful machine-processable (plus optionally human-readable) information about the thing when its IRI is dereferenced, using standard formats.
4. Include links to other, related IRIs in the exposed data to improve discovery of other related information on the Web.

NOTE RDF, serialized as RDF/XML [?], is the most common format for publishing linked data. However, its usage is not mandatory.

NOTE Using HTTP content negotiation [?] it is possible to serve representations in different formats from the same URL.

4.11 OMS Annotation and Documentation

annotation additional information without a logical semantics that is attached to an element of an OMS.

NOTE Formally, an annotation is given as a (subject, predicate, object) triple as defined by SOURCE: W3C/TR REC-rdf11-concepts:2014, Section 3.1. The subject of an annotation is an element of an OMS. The predicate is an RDF property defined in an external OMS and describes in what way the annotation object is related to the annotation subject.

NOTE According to ~~note 4.11~~ the preceding note, it is possible to interpret annotations under an RDF semantics. "Without a logical semantics" in this definition means that annotations to an OMS are not considered sentences of that OMS.

OMS documentation set of all annotations to an OMS, plus any other documents and explanatory comments generated during or after development or deployment of the OMS.

NOTE Adapted from [?].

[1] The original source is widely accepted but not formally a standard [?].

5 Symbols

As listed below, these symbols and abbreviations are generally for the main clauses of the OMG Specification. Some annexes may introduce their own symbols and abbreviations which will be grouped together within that annex.

CASL	Common Algebraic Specification Language, specified by the Common Framework Initiative
CGIF	Conceptual Graph Interchange Format
CL	Common Logic
CLIF	Common Logic Interchange Format
CURIE	Compact URI expression
DDL	Distributed description logic [?]
DOL	Distributed Ontology, Modeling and Specification Language
DTV	Date-Time Vocabulary
EBNF	Extended Backus-Naur Form
E-connections	a modular ontology language (closely related to DDL) [?]
F-logic	frame logic, an object-oriented ontology language
IRI	Internationalized Resource Identifier
MOF	Meta-Object Facility
OCL	Object Constraint Language
OWL 2	Web Ontology Language (W3C), version 2: family of knowledge representation languages for authoring ontologies
OWL 2 DL	description logic profile of OWL 2
OWL 2 EL	a sub-Boolean profile of OWL 2 (used often e.g. in medical ontologies)
OWL 2 Full	the language that is determined by RDF graphs being interpreted using the OWL 2 RDF-Based Semantics [?]
OWL 2 QL	profile of OWL 2 designed to support fast query answering over large amounts of data
OWL 2 RL	fragment of OWL 2 designed to support rule-based reasoning
OWL/XML	XML-based serialization of the OWL 2 language
P-DL	Package-based description logic
RDF	Resource Description Framework, a graph data model
RDFS	RDF Schema
RDFa	a set of XML attributes for embedding RDF graphs into XML documents
RDF/XML	an XML serialization of the RDF data model
RIF	Rule Interchange Format
SBVR	Semantics of Business Vocabulary and Business Rules
SMOF	MOF Support for Semantic Structures
UML	Unified Modeling Language
URI	Uniform Resource Identifier
URL	Uniform Resource Locator
W3C	World Wide Web Consortium
XMI	XML Metadata Interchange
XML	eXtensible Markup Language

6 Additional Information

(Informative)

6.1 Changes to Adopted OMG Specifications

This specification does not require or request any change to any other OMG specification.

6.2 How to Read This Specification

The initial five clauses of this specification describe the scope of the specification, determine conformance criteria, provide normative references, define terms and definitions, and introduce symbols that are used in the specification. The next three clauses are *informative*. This clause provides some background information, the next two provide a high-level summary of usage scenarios and goals (clause 7) and an overview over the design of DOL (clause ??).

Clause ?? defines the abstract syntax of DOL (*normative*) as an SMOF compliant meta model. Further, the same clause also provides a human friendly text serialization of the abstract syntax of DOL (*normative*).

Annex ?? contains the abstract syntax specified using Extended Backus–Naur Form (EBNF) (*informative*).

Clause ?? defines the model-theoretic semantics of DOL on the abstract syntax, and also makes the notion of heterogeneous logical environment (providing languages, logics and translations) precise (*normative*).

Annex ?? specifies an RDF vocabulary for the terms in clause 4, and for OMS languages and translation that conform with DOL (*normative*).

Various languages are shown to conform to DOL in informative annexes: OWL2 (annex ??), Common Logic (annex ??), RDF and RDF Schema (annex ??), UML class diagrams (annex ??), TPTP (annex ??), and CASL (annex ??).

Annex ?? provides a core graph of logics and translations, covering those OMS languages whose conformance with DOL is established in the preceding annexes (*informative*). Annex ?? extends the graph presented in Annex ?? by a list of OMS language whose conformance with DOL will be established by a registry (*informative*).

Annex ?? discusses an extension of DOL by queries. This extension is needed to support query languages (e.g., SQL or SPARQL) in DOL and to enable query related constructs for OMS in other DOL conformant languages (*informative*).

Annex ?? provides of DOL texts, which provide examples for all DOL constructs, which are specified in the abstract syntax (*informative*). ~~Annex ?? sketches scenarios that outline how is intended to be applied (*informative*). For each scenario, a brief description is provided, and the utilized features as well as the status of its implementation are listed.~~

Annex ?? gives an overview of available software tools for DOL. Annex ?? discusses the implementation of a linked-data compliant IRI scheme used in one of these tools (*informative*).

The bibliography contains ?? references to the literature that is cited in this document (*informative*).

6.3 Acknowledgments

6.3.1 Submitting and supporting organizations

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- Fraunhofer FOKUS
- MITRE
- Thematix Partners LLC

The following organizations are supporting this specification:

- Otto-von-Guericke University Magdeburg
- Athan Services

6.3.2 Participants

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7 Goals and Usage Scenarios

(Informative)

Often, engineering tasks require the use of several different OMS, which represent knowledge about a given domain or specify a given system from different perspectives or for different purposes. (E.g., a software engineer will typically use different OMS to model different aspects of a software system, including its behavior, its components, and its interactions with other systems.) Further, the OMS are often represented in different OMS languages (e.g., UML class diagrams, OWL, or Common Logic), which may differ in style, expressivity, and different computational properties.

The use of different OMS within the same context leads to several challenges in the design and deployment of OMS, which have been addressed by current research in ontological engineering, formal software specification and formal modeling:

- How is it possible to support shareability and reusability of OMS within the same domain?
- How is it possible to merge OMS in different domains, particularly in the cases in which the OMS are axiomatized in different logical languages?
- What notions of modularity play a role when only part of an OMS is being shared or reused?
- What are the relationships between versions of an OMS axiomatized in different logical languages?

To illustrate these challenges, this clause presents a set of usage scenarios that involve the use of more than one OMS. These scenarios address the areas of ontology design, formal specification, and model-driven development. In spite of their many differences, they all highlight one common theme: the use of multiple OMS leads to interoperability challenges.

The purpose of DOL is to provide a standardized representation language, which can be used to represent structured OMS and the relations between OMS as part of OMS networks in a semantically well-defined way. Thus, tools that implement DOL are able to integrate different OMS into a coherent whole, thereby enabling users of DOL to overcome the different kind of interoperability issues that are illustrated by the usage scenarios in this clause.

Most of the following subsections are illustrated with sample DOL libraries. These are always written in DOL, see the DOL Text Serialization in clause ??. Naturally, they also contain parts written in different OMS languages (e.g. OWL), the syntax of which is not described in this standard, but in other standard documents.

7.1 Use Case Onto-1: Interoperability Between OWL and FOL Ontologies

In order to achieve interoperability during ontology development it is often necessary to describe concepts in a language more expressive than OWL. Therefore, it is common practice to informally annotate OWL ontologies with FOL axioms (e.g., Keet's mereotopological ontology ~~Part-Whole~~[?], Dolce Lite ~~Dolce-lite~~[?], BFO-OWL). OWL is used because of better tool support, FOL because of greater expressiveness. However, relegating FOL axioms to informal annotations means that these are not available for machine processing. Another example of this problem is the following: For formally representing concept schemes (including taxonomies, thesauri and classification schemes) and provenance information there are the two W3C standards SKOS (Simple Knowledge Organization System; W3C/TR REC-skos-reference:2009) and PROV, as well as [1]ISO and other domain-specific standards for metadata representation. The semantics for the SKOS and PROV languages Note(1) are largely specified as OWL ontologies; however, as OWL cannot capture the full semantics, the rest is specified using some informal first-order rules. In other words, valid instance models that use SKOS or PROV may be required to satisfy both OWL and FOL axioms. When solving reasoning tasks over either SKOS or PROV ontologies, OWL reasoners are not able to consider the FOL axioms. Hence, the information contained in these axioms is lost.

DOL allows the user to replace such informal annotations by formal axioms in a suitable ontology language. The relation between the OWL ontology and the FOL axioms is that of a heterogeneous import. In the result, both the OWL and the FOL axioms are amenable to, e.g., automated consistency checking and theorem proving. Hence, all available information can be used in the reasoning process. For example, the ontology below extends the OWL definition of `isProperPartOf` as an asymmetric relation with a first-order axiom (in Common Logic) asserting that the relation is also transitive. *@ @* *@

[1] Note: CL: Did we mean something like "ISO 12345" here, i.e. some specific ISO standard that we reference by number? Terry: (re: footnote 29) Probably the family headed by ISO 11179 Information Technology – Metadata registries (MDR) (http://metadata-stds.org/) The 11179 standard is a multipart standard that includes the following parts:· ' Part 1: Framework · Part 2: Classification · Part 3: Registry metamodel and basic attributes · Part 4: Formulation of data definitions · Part 5: Naming and identification principles · Part 6: Registration Other standards in the series are ISO 19583 Concepts and usage of metadata ISO 19763-* Metamodel interoperability ISO 19773 Metadata modules ISO 24706 Metadata for technical stds ISO 24707 Common logic

```
\DIFaddend * trans:  <http://purl.net/DOL/translations/> )language lang:CommonLogicontology Parthood
=ObjectProperty:  isProperPartOfCharacteristics:  AsymmetricSubPropertyOf:  isPartOfwith translation
trans:SROIQtoCLthen(if (and (isProperPartOf x y) (isProperPartOf y z))(isProperPartOf x z))
```

OWL can express transitivity, but not together with asymmetry.

7.2 Use Case Onto-2: Ontology Integration by Means of a Foundational Ontology

One major use case for ontologies in industry is to achieve interoperability and data integration. However if ontologies are developed independently and used within the same domain, the differences between the ontologies may actually impede interoperability. One strategy to avoid this problem is the use of a shared foundational ontology (e.g., DOLCE or BFO), which can be used to harmonize different domain ontologies. One challenge for this approach is that foundational ontologies typically rely on expressive ontology languages (e.g., Common Logic), while domain ontologies may be represented in languages that are optimized for performance (e.g., OWL EL). For this reason, currently the role of the foundational ontology is mainly to provide a conceptual framework that may be reused by the domain ontologies; further, watered-down versions of the foundational ontologies in OWL (like DOLCE-lite or the OWL version of BFO) are used as basis for the development of domain ontologies, be this as is, in an even less expressive version (e.g., a DOLCE-lite in OWL 2 EL), or only a relevant subset thereof (e.g., only the branch of endurants). A sample ~~orchestration of interactions between the~~ interplay between foundational and domain ontologies in various languages is depicted in Figure ?? below.

DOL provides the framework for integrating different domain ontologies, aligning these to foundational ontologies ~~Alignment1-2~~[?, ?] a combining the aligned ontologies into a coherent integrated ontology – even across different ontology languages. Thus, DOL enables ontology developers to utilize the complete, and most expressive, foundational ontologies for ontology integration and validation purposes.

The foundational ontology (FO) repository Repository of Ontologies for MULtiple USes (ROMULUS)[1] contains alignments between a number of foundational ontologies, expressing semantic relations between the aligned entities. For this use-case three such ontologies are considered, containing spatial and temporal concepts: DOLCE[2], GFO[3] and BFO[4], and present alignments between them using DOL syntax:

@ @ *@

```
\DIFaddend * gfo:  <http://www.onto-med.de/ontologies/>dolce:  <http://www.loa-cnr.it/ontologies/>bfo:  <http://w
<http://purl.net/DOL/languages/>)language lang:OWLalignment DolceLite2BFO :dolce:DOLCE-Lite.owltobfo:1.1
=endurant = IndependentContinuant,physical-endurant = MaterialEntity,physical-object = Object, perdurant
= Occurrent,process = Process, quality = Quality,spatio-temporal-region = SpatiotemporalRegion,temporal-region
= TemporalRegion, space-region = SpatialRegionalignment DolceLite2GFO :dolce:DOLCE-Lite.owl to gfo:gfo.owl
=particular = Individual, endurant = Presential,physical-object = Material
```
$_object, amount-of-matter = Amount_of_substrat$ $Occurrent, quality = Property, time-interval = Chronoid, generic-dependent < necessary_for, part < abstract_has_part, part-of < abstract_part_of, proper-part < has_proper_part, proper-part-of < proper_part_of, generic-location < occupies, generic-location-of < occupied_by alignment BFO2GFO : bfo : 1.1 to gfo : gfo.owl = Entity = Entity, Object = Material_object, ObjectBoundary = Material_boundary, Role < Role, Occurrent = Occurrent, Process = Process, Quality = Property SpatialRegion = Spatial_region, TemporalReg$ $Temporal_region$

DOL can be used to combine ontologies, while taking into account the semantic dependencies given by the alignments. In the following example the ontology `Space` is defined as a combination of three different ontologies (BFO, GFO, DolceLite) along three alignments.

@ @ *@

```
\DIFaddend * ontology Space =combine BFO2GFO, DolceLite2GFO, DolceLite2BFO
```

7.3 Use Case Onto-3: Module Extraction From Large Ontologies

Especially in the biomedical domain, ontologies tend to become very large (e.g., SNOMED CT, FMA) with over 100000 concepts and relationships. Yet, none of these ontologies covers all aspects of a domain, and frequently provide coverage at various levels of specificity, with excessive detail in some areas that may not be required for all usage scenarios. Often, for a given knowledge representation problem in industry, only relevant knowledge from two such large reference ontologies needs to be integrated, so a comprehensive integration would be both unfeasible and unwieldy. Hence, parts (modules) of these ontologies are obtained by selecting the concepts and relationships (roles) relevant for the intended application. An integrated version will then be based on these excerpts from the original ontologies (i.e., modules). For example, the Juvenile Rheumatoid Arthritis ontology JRAO has been created using modules from the NCI thesaurus and GALEN medical ontology. (See [2]Figure 7.1) DOL supports the description of such subsets (modules) of ontologies, as well as their alignment Note(2)

[1] See `http://www.thezfiles.co.za/ROMULUS/home.html`
[2] See `http://www.loa.istc.cnr.it/DOLCE.html`
[3] See `http://www.onto-med.de/ontologies/gfo/`
[4] See `http://www.ifomis.org/bfo/`
[2] NOTE: CL: can we please have a vector graphic here?

and integration.

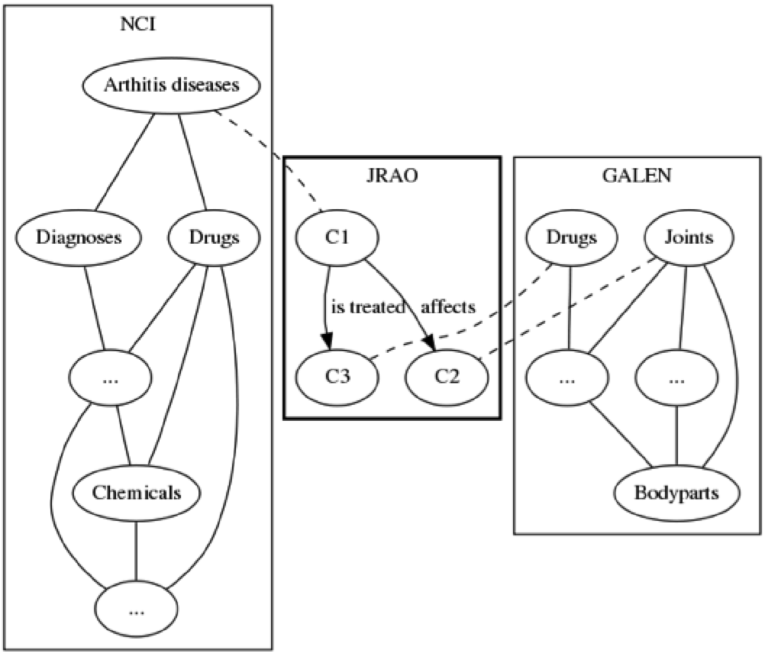


Figure 7.1: JRAO – Example for Module Extraction

```
  *@  @* *@
\DIFaddend ⋆ library GalenModulelanguage lang:OWLontology myGalen =http://purl.bioontology.org/ontolo
Drugs, Joints, Bodypartsend⋆ \DIFdelbegin \DIFdel{module myGalenIsAModule : myGalen of }\DIFdelend ⋆
⋆ \DIFaddbegin \DIFadd{cons-ext myGalenIsAModule : }\DIFaddend ⋆ http://purl.bioontology.org/ontology
⋆ \DIFaddbegin \DIFadd{of myGalen@⋆ for Drugs, Joints, Bodypartsend
```

7.4 Use Case Onto-4: Interoperability Between Closed-World Data and Open-World Metadata

Data collection has become easier and much more widespread over the years. This data has to be assigned a meaning somehow, which occurs traditionally in the form of metadata annotations. For instance, consider geographical datasets derived from satellite data and raw sensor readings. Current implementations in, e.g., ecological economics [?] require manual annotation of datasets with the information relevant for their processes. While there have been attempts to standardize such information [?], metadata for datasets of simulation results are more difficult to standardize. Moreover, it is resource-consuming to link the data to the metadata, to ensure the metadata itself is of good quality and consistent, and to actually exploit the metadata when querying the data for data analysis.

The data is usually represented in a database or RDF triple store, which work with a closed world assumption on the dataset, and are not expressive enough to incorporate the metadata ‘background knowledge’, such as the conditions for validity of the physical laws in the MDE model of the object of observation. These metadata require a more expressive language, such as OWL or Common Logic, which operate under an open-world semantics. However, it is unfeasible to translate the whole large dataset into OWL or first-order logic. To ‘meet in the middle’, it is possible to declare bridge rules (i.e., a mapping layer) that can link the metadata to the data. This approach can be used for intelligent data analysis that combines the data and metadata through querying the system. It enables the analysis of the data on the conceptual layer, instead of users having to learn the SQL/SPARQL query languages and how the data is stored. There are various tools and theories to realize this, which is collectively called Ontology-Based Data Access/Management, see also ~~OBDA~~[?] .

The languages for representing the metadata or ontology, for representing the bridge rules or mapping assertions, and for representing the data are different yet they need to be orchestrated and handled smoothly in the system, be this for data analytics for large enterprises, for formulating policies, or in silico biology in the sciences.

DOL provides the framework for expressing such bridge rules in a systematic way, maintaining these, and building tools for them.

7.5 Use Case Onto-5: Verification of Rules Translating Dublin Core Into PROV

The Dublin Core Metadata terms, which have been formalized as an RDF Schema vocabulary, developed initially by the digital library community, are less comprehensive but more widely used than PROV (cf.~~Use Case Onto-1~~ subclause 7.1). The rules for translating Dublin Core to the OWL subset of PROV (and, with restrictions, vice versa) are not known to yield valid instances of the PROV data model, i.e. they are not known to yield OWL ontologies consistent with respect to the OWL axioms that capture part of the PROV data model. This may disrupt systems that would like to reason about the provenance of an entity, and thus the assessment of the entity's quality, reliability or trustworthiness. The Dublin Core to PROV ontology translation[5] is expressed partly by a symbol mapping and partly by FOL rules. These FOL rules are implemented by CONSTRUCT patterns in the SPARQL RDF query language.[6] SPARQL has a formal specification of the evaluation semantics of its algebraic expressions, which is different from the model-theoretic semantics of the OWL and RDF Schema languages; nevertheless SPARQL CONSTRUCT is a popular and immediately executable syntax for expressing translation rules between ontologies in RDF-based languages in a subset of FOL. DOL not only supports the reuse of the existing Dublin Core RDF Schema and PROV OWL ontologies as modules of a distributed ontology (= OMS network), but it is also able to support the description of the FOL translation rules in a sufficiently expressive ontology language, e.g. Common Logic, and thus enable formal verification of the translation from Dublin Core to PROV.

7.6 Use Case Onto-6: Maintaining Different Versions of an Ontology in Languages with Different Expressivity

Often is useful to maintain different versions of an ontology within languages, which differ in their expressivity.

For example, DOLCE is a foundational ontology that has primarily been formalized in the first-order logic ontology language KIF (a predecessor of Common Logic), but also in OWL ("DOLCE Lite") [?] . This "OWLized" version was targeting use in semantic web services and domain ontology interoperability, and to provide the generic categories and relationships to aid domain ontology development. DOLCE has been used also for semantic middleware, and in OWL-formalized ontologies of different domains, including neuroimaging, computing, and ecology. Given the differences in expressivity between KIF and OWL, DOLCE Lite had to simplify certain notions. For example, the DOLCE Lite formalization of "temporary parthood" (something is part of something else at a certain point or interval in time) omits any information about the time, as OWL only supports binary predicates (a.k.a. "properties"). That leaves ambiguities for modeling a view from DOLCE Lite to the first-order DOLCE, as such a view would have to reintroduce the third (temporal) component of such predicates:

- Should a relation asserted in terms of DOLCE Lite be assumed to hold for *all* possible points/intervals in time, i.e.should it be universally quantified?
- Or should such a relation be assumed to hold for *some* points/intervals in time, i.e.should it be existentially quantified?
- Or should a concrete value for the temporal component be assumed, e.g."0" or "now"?

DOLsupports the formalization of all of these views. Given suitable consistency checking tools, DOL enables the analysis of whether any such view satisfies all further axioms that the first-order DOLCE states about temporal parthood.

7.7 Use Case Onto-7: Metadata within OMS Repositories

DOL provides a language for the metadata within OMS Repositories. For example, the Common Logic Repository (COLORE) [7] is an open repository of more than 150 ontologies as of December 2011, all formalized in Common Logic. COLORE stores metadata about its ontologies, which are represented using a custom XML schema that covers the following aspects[8], without specifying a formal semantics for them:

module provenance author, date, version, description, keyword, parent ontology[9]

axiom source provenance name, author, year[10]

direct relations maps (signature morphisms), definitional extension, conservative extension, inconsistency between ontologies, imports, relative interpretation, faithful interpretation, definable equivalence

DOLprovides built-in support for a subset of the "direct relations" and specifies a formal semantics for them. In addition, it supports the implementation of the remainder of the COLORE metadata vocabulary as an ontology, reusing suitable existing metadata vocabularies such as OMV, and it supports the implementation of one or multiple Common Logic ontologies plus their annotations as one coherent DOLlibrary.

[5] http://www.w3.org/TR/2013/NOTE-prov-dc-20130430/
[6] E.g., http://www.w3.org/TR/2013/NOTE-prov-dc-20130430/#dct-creator
[7] http://stl.mie.utoronto.ca/colore/
[8] http://stl.mie.utoronto.ca/colore/metadata.html
[9] Note that this use of the term "module" in COLORE corresponds to the term structured OMS in this OMG Specification.
[10] Note that this may cover any sentences in the sense of this OMG Specification.

7.8 Use Case Spec-1: Modularity of Specifications

Often specifications become so large that it is necessary to structure them in a modular way, for human readability and maintainability, and for more efficient tool support. The lack of a standard for such modular structuring hinders interoperability among different development efforts and the reuse of specifications. DOL provides a notion of structured modular specification that is equally applicable to all DOL-conforming logical languages.

Structuring pays off even for small specifications. For example, it makes structuring a simple specification of sorting lists in the following way enhances both readability and potential for re-use of specifications:
@ @ *@

```
\DIFaddend ⋆ library Sorting⋆ \DIFdelbegin %DIFDELCMD <

%DIFDELCMD < %%%
\DIFdelend ⋆ language lang:CASLspec TotalOrder =sort Elempred
```
$<=_{:Elem*Elemforallx,y,z:Elem.x<=x.x<=zifx<=y/\ y<=z.x=yifx<=y/}$

In the last step, the structuring operation of hiding is used to restrict the specification to an export interface: predicates `is_ordered` and `permutation` are hidden, because they are only auxiliary and need not be implemented.

7.9 Use Case Spec-2: Specification Refinements

Formal software and hardware development methods are often used to ensure the correct function of systems which have safety-critical requirements or which may not be easily accessible for repair or replacement. Examples of such requirements can be found in safety-critical areas such as medical systems, or in the automotive, avionics and aerospace industries, as well as in components used by those industries such as in microprocessor design.

Typically, a requirement specification is refined into a design specification and then an implementation, often involving several intermediate steps (see, e.g. the V-model ~~V-model~~[?] , although this does not require formal specification). There are numerous specification formalisms in use, including the OMG's SysML language; moreover, often during development, the formalism needs to be changed (e.g. from a specification to a programming language, or from a temporal logic to a state machine). For each of these formalisms, notions of refinement have been defined and implemented. However, the lack of a standardized, logically sound language and methodology for such refinement hinders interoperability among different development efforts and the reuse of refinements. DOL provides the capability to represent refinement that is equally applicable to all DOL-conforming logical languages, and that covers at least the most relevant of the industrial use cases of specification refinement.

A simple example is the refinement of the (purely declarative) sorting specification from use case in section **??** into a specification of a particular sorting algorithm (for simplicity, insert sort is used for demonstration):
@ @ *@

`\DIFaddend ⋆ spec InsertSort =TotalOrder and Listthenops insert : Elem⋆List -> List;insert`$_sort : List- > Listvarsx,y : Elem; L : List.insert(x,[]) = x :: [].insert(x,y :: L) = x :: insert(y,L)whenx <= yelsey :: insert(x,L).insert_sort([]) = [].insert_sort(x :: L) = insert(x,insert_sort(L))hideinsertendrefinementInsertSortCorrectness = Sortingrefinedviasorter|- > insert_sorttoInsertSortend$

Note that hiding is essential here to make the signatures of both specifications compatible. If the predicates `is_ordered` and `permutation` had not been hidden in the `Sorting` specification, a refinement would not have been possible, since `InsertSort` does not implement these predicates (and it would be rather artificial to add an implementation for them).

Refinements can be composed. A simple example below illustrates this by expressing that natural numbers with addition form a monoid, and that natural numbers can be efficiently represented for implementation as lists of binary digits, together with several equivalent ways of composing these refinements.
@ @ *@

`\DIFaddend ⋆ spec Monoid =sort Elemops 0 : Elem;`$_{+:Elem*Elem->Elem,assoc,unit0endspecNatWithSuc=freetypeNat::=0|suc(Nat)op_{+:Nat*N}}$

7.10 Use Case Model-1: Consistency Among UML Diagrams of Different Types

A typical UML model involves diagrams of different types. Such UML models may have intrinsic errors because diagrams of different types may specify conflicting requirements. Typical questions that arise in this context are ask for semantic consistency, e.g.,

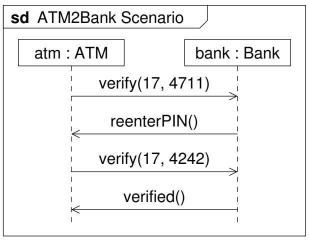
- whether the multiplicities in a class diagram are semantically consistent with each other; ~~whether the attributes and operations in a state machine are available in a class diagram;~~
- whether the sequential composition of actions in an interaction diagram is justified by an accompanying OCL specification;

- whether cooperating state machines comply with pre-/post-conditions and invariants;
- whether the behavior prescribed in an interaction diagram is realizable by several state machines cooperating according to a composite structure diagram.

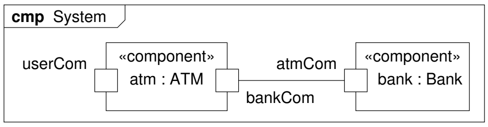
Such questions are currently hard to answer in a systematic manner. One method to answer these questions and find such errors is a check for semantic consistency. Under some restrictions, the proof of semantic consistency can be (at least partially) performed using model-checking tools like Hugo/RT [?]. Once a formal semantics for the different diagram types has been chosen (see, e.g. [?]), it is possible to use DOL to specify in which sense the diagrams need to be consistent, and check this by suitable tools.

7.10.1 The ATM Example

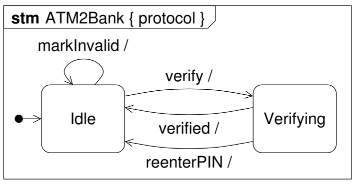
The ATM example, which illustrates model-driven development using UML, is taken from [?]. The example involves the design of a traditional automatic teller machine (ATM) connected to a bank. For simplicity, the example focuses on the ATM's processing of card and PIN entry actions. After entering the card, one has three trials for entering the correct PIN (which is checked by the bank). After three unsuccessful trials the card is kept.



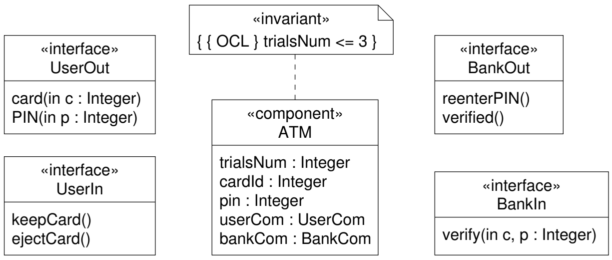
(a) Interaction



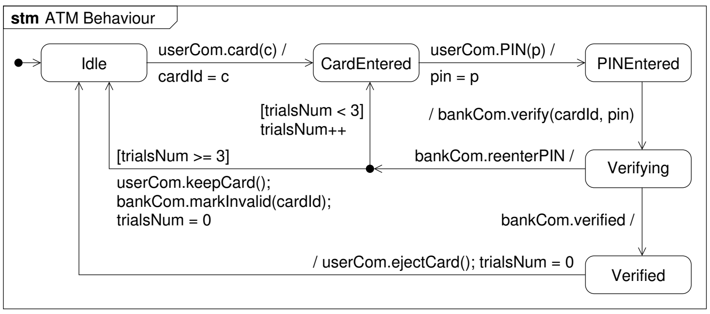
(b) Composite structure



(c) Protocol state machine



(d) Interfaces and components



(e) State machine

Figure 7.2: ATM example

Figure ?? shows a possible *interaction* between an atm and a bank object, which consists of four messages: the atm requests the bank to verify if a card and PIN number combination is valid, in the first case the bank requests to reenter the PIN, in the second case the verification is successful. This interaction presumes that the system has an atm and a bank as objects. This can, e.g., be ensured by a *composite structure diagram*, see Fig. ??, which – among other things – specifies the objects in the initial system state. Furthermore, it specifies that the communication between atm and bank goes through the two ports bankCom and atmCom linked by a connector. The communication protocol on this connector is captured with a *protocol state machine*, see Fig. ??. The protocol state machine fixes in which order the messages verify, verified, reenterPIN, and markInvalid between atm and bank may occur. Figure ?? provides structural information in form of ~~an interface~~ interfaces specifying what is provided and required at the userCom port and the bankCom port of the atm instance. An interface is a set of operations that other MDE model elements have to implement. In our case, the ~~interface is~~ interfaces are described in a *class diagram*. ~~Here, the operation~~ In this class diagram also the component type ATM is enriched with the OCL constraint trialsNum <= 3, which refines its semantics ~~: can only be invoked if the OCL constraints holds~~requiring that trialsNum must not exceed three.

Finally, the dynamic behavior of the atm object is specified by the *behavioral state machine* shown in Fig. ??. The machine consists of five states including Idle, CardEntered, etc. Beginning in the initial Idle state, the user can *trigger* a state change by entering the card. This has the *effect* that the parameter c from the card event is assigned to the cardId in the atm object (parameter names are not shown on triggers). Entering a PIN triggers another transition to PINEntered. Then the ATM requests verification from the bank using its bankCom port. The transition to Verifying uses a *completion event*: No explicit trigger is declared and the machine autonomously creates such an event whenever a state is completed, i.e., all internal activities of the state are finished (in our example there are no such activities). If the interaction with the bank results in reenterPIN, and the *guard* trialsNum < 3 is true, the user can again enter a PIN.

The ATM example in Fig. ?? consists of five different UML models, which naturally form a network. Coherence of this network is expressed as its consistency. It is assumed that XMI representations of the relevant UML models have been stored at `http://www.example.org/uml/`, that is under URL `http://www.example.org/uml/xxx.xmi`, where `xxx` is determined as follows:

Figure	xxx	diagram type
Fig. ??	sd	sequence diagram
Fig. ??	cmp	composite structure diagram
Fig. ??	psm	protocol state machine
Fig. ??	cd	class diagram
Fig. ??	stm	state machine

@ @ *@

```
\DIFaddend * uml:  &lt;http://www.uml.org/spec/UML/&gt;log:  &lt;http://purl.net/DOL/logics/&gt; )library
ATMview cd2stm = cd to  atm hide along stm2cd endview cd2psm = cd to  psm hide along psm2cd endnetwo
```

ATM$_network = cd, stm, psm, cmp, cd2stm, cd2psm, abstract_to_concrete_atmentailmentatminATM_networkentailssdnetworkSome_refine$

$...refinementr = ATM_networkrefinedtoSome_refined_ATM_networkentailmente = Some_refined_ATM_networkentailsATM_network$

Here, `abstract_to_concrete_atm` is defined in the next section, and `stm2cd` and `psm2cd` are suitable logic projections extracting the classes, attributes and operations from a (protocol) state machine, delivering a class diagram.

7.11 Use Case Model-2: Refinements Between UML Models of Different Types, and Their Reuse

A problem is a lack of reusability of refinements: Consider a controller for an elevator, which is specified with a UML protocol state machine, enriched with UML sequence diagrams and OCL constraints. Assume further that this UML model is not directly implemented, but first refined to a UML behavior state machine (which then can be automatically or semi-automatically transformed into some implementation using standard UML tools). However, there is no standardized language to express, document and maintain the refinement relation itself (UML only allows very simple refinements, namely between state machines). This hinders both the reuse of such refinements in different contexts, as well as the interoperability of tools proving such refinements to be correct. DOL addresses these problems by providing a standardized notation with formal semantics for such refinements. Refinements expressed in this language could, e.g., be parameterized and reused in different contexts.

This can be illustrated based on the state machine of the atm, shown in Fig. ??, which is a refinement of the protocol state machine in Fig. ??. This can be stated as follows in DOL. [11]
@ @ *@

`\DIFaddend * refinement abstract`$_to_concrete_atm = psmrefinedviatranslationpsm2atmtoatmwithIdle|->Idle, CardEntere$

The refinement uses an abstraction of the atm, expressed by the translation via symbol map `Idle |-> Idle, CardEntered |-> Idle, PINEntered |-> Idle, Verified |-> Idle, Verifying |-> Verifying`, resulting in a two-state machine. Moreover, some detail of the atm is hidden using `hide`. Then, the protocol state machine can be refined to the thus abstracted atm.

7.12 Use Case Model-3: Coherent Semantics for Multi-Language Models

Often a single problem area within a given domain must be represented using several formalisms, e.g., because of user community requirements, expressiveness or tool support and usage. Typically the different representations are written by different people using formalisms that are based on different logics. Thus, it is a challenge to maintain consistency across the

[11] It is assumed that XMI representations of the relevant UML models have been stored at `http://www.example.org/uml/`, e.g. `http://www.example.org/uml/atm.xmi`

different representations. The need for the use of multiple OMS languages, even within the OMG community, is also reflected by the OMG Ontology Definition Metamodel (ODM), which provides a number of syntactic transformations between such languages. One example is the OMG Date-Time Vocabulary (DTV). DTV has been formulated in different languages, each of which addresses different audiences:

- SBVR: business users
- UML (class diagrams and OCL): software implementors
- OWL: ontology developers and users
- Common Logic: (foundational) ontology developers and users

With DOL, one can, e.g.,

- formally relate the different formalizations used for DTV, relate the different formalizations using translations,
- check consistency across the different formalizations (using suitable tools),
- extract sub-modules covering specific aspects, and
- specify the OWL version to be an approximation of the Common Logic version (using a heterogeneous interpretation of OMS).

Note that the last point does not specify what information is lost in the approximation. Indeed, DOL provides the means to specify requirements on the approximation, e.g., that it maximally preserves the information.

Coming to a DOL example, a UML model like the ATM model developed in section **??** typically is part of an application context that also contains some common terminology. This terminology often is specified by an ontology, and then it is desirable to relate the model to the ontology. Consider the following financial ontology fragment:

@ @ *@

```
\DIFaddend * ontology myTaxonomy =ObjectProperty: ownsCharacteristics: Irreflexive, AsymmetricClass
FinancialIntermediarySubClassOf: CorporatePersonClass: CorporatePersonSubClassOf: ImmaterialEntity
ImmaterialEntityDisjointWith: MaterialEntitySubClassOf: has_partonlyImmaterialEntityClass : LivestockSubCla
MaterialEntity...end
```

To relate this ontology with the ATM model, various aspects need to be taken care of:

- Translating into shared language (in this case, Common Logic)
- Unifying terminology (Bank vs. FinancialIntermediary)
- Connecting related concepts (bank.owns.ATM vs. owns)
- Removing irrelevant parts (livestock)

@ @ *@

```
\DIFaddend * model xmiStateModel = <https://ontohub.org/ATM/state.xmi>model clStateModel = xmiStateMo
withtranslation UMLState2CLmodel xmiClassModel = <https://ontohub.org/ATM/class.xmi>model clClassMode
= xmiClassModel withtranslation UMLClass2CLBank |-> FinancialIntermediaryontology BigTaxonomy
= <https://ontohub.org/ATM/mytaxonmy.owl>ontology NoLivestockTaxonomy = BigTaxonomy rejectClass:
Livestockendontology ExtendedTaxonomy = NoLivestockTaxonomy thenObjectProperty FinancialIntermediary.
ownsDomain: FinancialIntermediaryRange: ATMendontology clTaxonomy = ExtendedTaxonomy withtranslatio
OWL22CommonLogicoms JointModel = clStateModel andclClassModel andclTaxonomyend
```

7.13 Conclusion

In this section, several use cases have been introduced. They illustrate many aspects of DOL and its usefulness in many situations in which different OMS artifacts might be leveraged and augmented to produce broader or more tractable MDE models, ontologies, and specifications.

DOL has been designed to support of a wide range of formalisms and provides the ability to specify the basis for formal interoperability even among heterogeneous OMS and OMS networks. DOL enables the solutions of the problems described in the use cases above. It also enables the development of DOL documents, tools and workflows that allow a better exchange and reuse of OMS. Eventually, this will also lead to better, easier developed and maintained systems based on these OMS.

The next sections present the metalanguage DOL; in particular, the syntax and the model-theoretic semantics. Further, various features of DOL will be discussed, which are based on best practices of modularity across the three areas of ontology design, formal specification, and model-driven development.

8 Design Overview

(Informative)

The purpose of this clause is to briefly describe the overall guiding principles and constraints of DOL's syntax and semantics. It provides an overview of the most important and innovative language constructs of DOL. Details can be found in clause ??.

8.1 DOL in a Nutshell

As the usage scenarios in clause 7 illustrate, the use of multiple OMS may lead to lack of interoperability. The goal of DOL is to enable users to overcome these interoperability issues by providing a language for representing structured OMS and the relations between OMS as part of an OMS network in a semantically well-defined way. One particular challenge that needs to be addressed is that OMS are written in a wide variety of OMS languages, which differ in style, expressivity and logical properties. To address this diversity this specification does **not** propose a "universal" language that is intended to subsume all the others. Quite the opposite, the authors of this specification embrace the pluralism of OMS languages, and the purpose of DOL is to provide means (on a sound and formal semantic basis) to compare and integrate OMS written in different formalisms. Thus, DOL is not 'yet-another-modeling language', but a meta-language that is used on top of existing OMS languages.

The major functions of DOL are the following:

- DOL allows the use of OMS in other OMS languages (e.g., UML ~~class diagrams~~models, CASL, OWL, Common Logic) without requiring any changes. These are called *native OMS*. A native OMS is serialized in a *native document*.
- DOL provides for defining new, *structured OMS* based on existing OMS.[1] DOL provides a number of operations for this purpose; e.g., it is possible to define a structured OMS C as the union of an OWL ontology A and a Common Logic ontology B.
- DOL provides for defining connections between two OMS by using *OMS mappings*. DOL provides a variety of mappings; e.g., one can align terminology between different OMS or specify that some OMS is an extension of another. A set of OMS and OMS mappings may form together an *OMS network*.
- Native OMS inherit their semantics from the underlying OMS languages. The DOL operations for defining structured OMS, OMS mappings, and OMS networks have a declarative model-theoretic semantics, which is defined in clause ??.

Each of these functions corresponds to a syntactic category in DOL: native OMS, structured OMS, OMS mappings, and OMS networks. They (together with imports) form the items in a DOL *library*, and are, in this sense, the most important metaclasses of DOL.

8.2 Features of DOL

DOL is a language enabling OMS interoperability. DOL is

free DOL is freely available for unrestricted use (as any OMG specification is).

generally applicable DOL is neither restricted to OMS in a specific domain, nor to foundational OMS, nor to OMS represented in a specific OMS language, nor to OMS stored in any specific repositories.

open DOL supports mapping, integrating, and annotating OMS across arbitrary internet locations. It makes use of existing open standards wherever suitable. The criteria for extending DOL (see next item) are transparent and explicit.

extensible DOL provides a framework into which any existing, and, desirably, any future OMS language can be plugged.

DOL is applicable to any OMS language that has a formal, logic-based semantics or a semantics defined by translation to another OMS language with such a formal semantics. The annotation framework of DOL is additionally applicable to the non-logical constructs of such languages. This OMG Specification specifies formal criteria for establishing the conformance of an OMS language with DOL. The annex establishes the conformance of a number of relevant OMS languages with DOL; a registry shall offer the possibility to add further (including non-standardized) languages.

[1] Native OMS can also use the structuring constructs from their OMS language. However, these structuring constructs are often quite limited, and moreover, they differ from OMS language to OMS language.

DOL provides syntactic constructs for structuring OMS regardless of the logic their sentences are formalized in. Since DOL is a meta-language, it *inherits* the logical language aspects of conforming OMS languages. It is possible to literally include sentences expressed in such OMS languages in a DOL OMS.

DOL provides an initial vocabulary for expressing relations in correspondences (as part of alignments between OMS). Additionally, it provides a means of reusing relation types defined externally of this OMG Specification. DOL does not provide an annotation vocabulary, i.e. it neither provides annotation properties nor datatypes to be used with literal annotation objects.

8.3 OMS Languages

OMS languages are declarative languages for making ontological distinctions formally precise, for modeling a domain in an unambiguous way, or for expressing algebraic specifications of software. OMS languages are distinguished by the following features:

Logic Most commonly, OMS languages are based on a description logic or some other subset of first-order logic, but in some cases, higher-order, modal, paraconsistent and other logics are used.

Modularity A means of structuring an OMS into reusable parts, reusing parts of other OMS, mapping imported symbols to those in the importing OMS, and asserting additional properties about imported symbols.

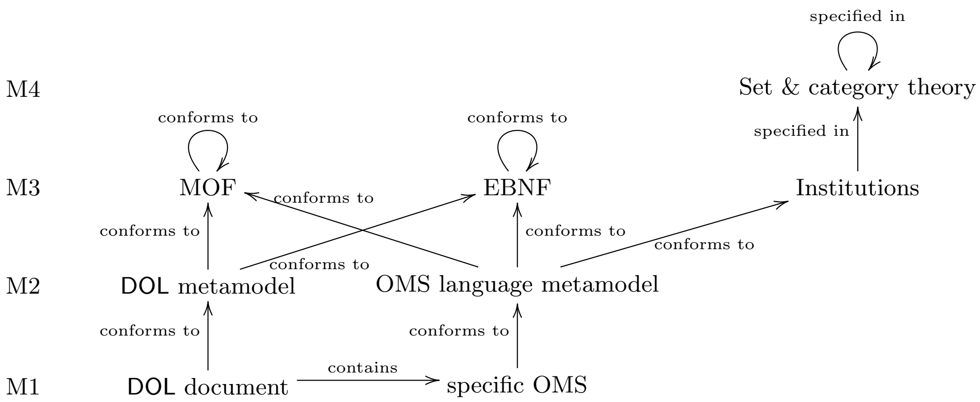
Annotation A means of enabling the attachment of human-readable descriptions to OMS symbols, addressing knowledge engineers and service developers, but also end users of OMS-based services.

Whereas the first feature determines the expressivity of the language and the possibilities for automated reasoning (decidability, tractability, etc.), the latter two facilitate OMS engineering as well as the engineering of OMS-based software.

Acknowledging the wide tool support that conforming established languages such as OWL, RDF, Common Logic, UML, MOF, or CASL enjoy, existing OMS in these (and any other) conforming languages remain as they are within the DOL framework. DOL enhances their modularity and annotation facilities to a superset of the modularity and annotation facilities they provide themselves. Using DOL's modularity constructs to make statements about modules of existing OMS works by making relevant parts of these OMS, e.g., sets of axioms, identifiable, and then referring to these identifiers from DOL statements. DOL's modularity constructs are semantically well-founded within a library of formal relationships between the logics underlying the different supported OMS languages. General annotation of OMS and their parts works in a similar way. Here, DOL does not provide its own annotation constructs, but once again DOL's general mechanism of making things of interest identifiable can be employed. Once these things have been identified, the actual annotations can be added using external mechanisms such as RDF. [3] Note(3)

8.4 DOL in the Metamodeling Hierarchy

DOL uses the metamodeling hierarchy known from model-driven engineering:



The syntax of a DOL conformant language can be written in MOF or EBNF, which are self-describing. The semantics of a DOL conformant language is its presentation as an institution. Institutions themselves are specified in the language of set theory and category theory.

In the future, it may be possible to specify the semantics of a DOL conformant language using a semantics-based logical framework such as LF or MMT. Since LF can be specified in LF itself, this would close the loop already at M3 also for the semantics.

[3] NOTE: Till, is this standoff markup remark up to date? TM: we promise standoff markup at various places, but we never are specific how this can be written in DOL.
CL: I have now emphasized that DOL itself doesn't do annotation but only identification, whereas annotation is left to RDF.

8.5 Semantic Foundations of DOL

A large variety of OMS languages in use can be captured at an abstract level using the concept of *institutions* [?]. This allows the development of DOL independently of the particularities of a logical system and to use the notions of institution and logical language interchangeably. The main idea is to collect the non-logical symbols of the language in signatures and to assign to each signature the set of sentences that can be formed with its symbols. For each signature, DOL provides means for extracting the symbols it consists of, together with their kind. Institutions also provide a model theory, which introduces semantics for the language and gives a satisfaction relation between the models and the sentences of a signature.

It is also possible to complement an institution with a proof theory, introducing a derivability relation between sentences, formalized as an *entailment system* [?]. In particular, this can be done for all logics that have so far been in use in DOL.

Since institutions allow the differences between OMS languages to be elided to common abstractions, the semantics of basic OMS is presented in a uniform way. The semantics of structured OMS, OMS mappings, OMS networks, and other DOL expressions is defined using model-theoretic constructions on top of institutions.

8.6 DOL Enables Expression of Logically Heterogeneous OMS and Literal Reuse of Existing OMS

DOL is a mechanism for expressing logically heterogeneous OMS. It can be used to combine sentences and structured OMS expressed in different conforming OMS languages and logics into single documents or modules. With DOL, sentences or structured OMS of previously existing OMS in conforming languages can be reused by literally including them into a DOL OMS. A minimum of wrapping constructs and other annotations (e.g., for identifying the language of a sentence) are provided. See the MOF metaclass `OMS` in clause ??.

A heterogeneous OMS can import several OMS expressed in different conforming logics, for which suitable translations have been defined in the logic graph provided in annex ?? or in an extension to it that has been provided when establishing the conformance of some other logic with DOL. Determining the semantics of the heterogeneous OMS requires a translation into a common target language to be applied (cf. clause ??). This translation is determined via a lookup in the transitive closure of the logic graph. Depending on the reasoners available in the given application setting, it can, however, be necessary to employ a different translation. Authors can express which one to employ. However, DOL provides default translations, which are applied unless the user specifies a translation that deviates from the default. Both default and non-default translations may be combined to multi-step translations.

8.7 DOL Includes Provisions for Expressing Mappings Between OMS

DOL provides a syntax for expressing mappings between OMS. One use case illustrating both is sketched in Figure ??. OMS mappings supported by DOL include:

- imports (particularly including imports that lead to conservative extensions), see the MOF metaclasses `OMSRef` and `ExtensionOMS` in clause ??.
- interpretations (both between OMS and OMS networks), see the MOF metaclass `InterpretationDefinition` in clause ??.
- alignments between OMS, see the MOF metaclass `AlignmentDefinition` in clause ??.
- conservative extensions, e.g. mappings between OMS and their modules, see the MOF metaclass `ConservativeExtensionDefi` in clause ??.

DOL uses symbol maps to express signature translations in such OMS mappings; see the MOF metaclass `SymbolMap` in clause ??.

DOL need not be able to fully represent logical translations but is capable of referring to them.

DOL can also be used to combine or merge OMS along such OMS mappings, see the rule for `combination` for the MOF metaclass `OMS` in clause ??.

8.8 DOL Provides a Mechanism for Rich Annotation and Documentation of OMS

DOL provides a mechanism for identifying anything of relevance in OMS by assigning an IRI to it. With RDF there is a standard mechanism for annotating things identified by IRIs. Thus, DOL supports annotations in the full generality specified in clause 4.11.

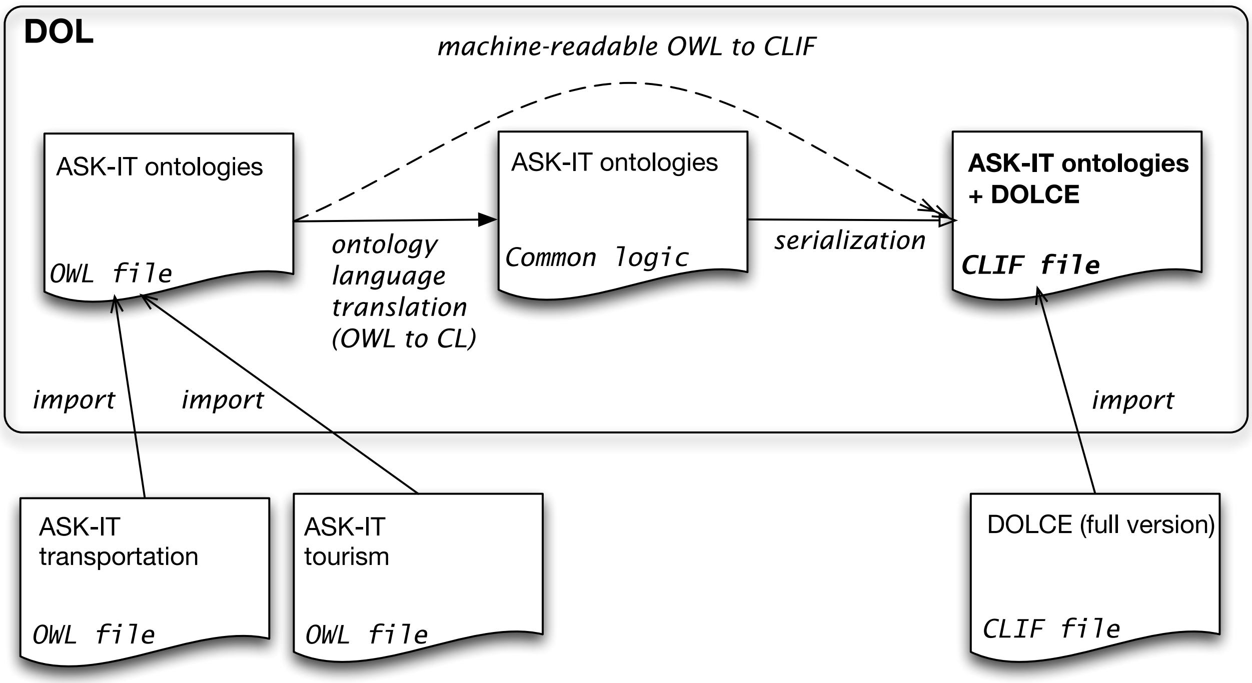


Figure 8.1: Mapping between two OMS formulated in different OMS languages

9 DOL Syntax

This clause specifies the DOL abstract syntax as a MOF metamodel. In annex ??, the same abstract syntax is specified using EBNF. We further include the DOL concrete syntax, which uses the metaclasses of the abstract syntax as non-terminals of an EBNF grammar.

At several places, the concrete syntax uses the non-terminal `'end'` to mark the end of a definition or declaration. Tools may make this `'end'` optional. However, in this standard, the `'end'` is not marked as optional, because it may be needed to effectively disambiguate heterogeneous texts.

The DOL document types are as follows

MIME type *application/dol+text*

Filename extension .dol

9.1 MOF Metaclasses

DOL provides MOF metaclasses for (among others):

- OMS (which can be native OMS in some OMS language, or unions, translations, closures, combinations, approximations of OMS, among others)
- OMS mappings
- OMS networks
- DOL libraries (items in these are: definitions of OMS, OMS mappings, and OMS networks, as well as qualifications choosing (1) the logic, (2) the OMS language and/or (3) the serialization)
- identifiers
- annotations

~~Additionally, the MOF~~ The DOL metaclasses `NativeDocument` and `BasicOMS` are abstract metaclasses without any instances within the normative DOLmetamodel. In order to use DOLwith some specific conforming OMS language, the top-level MOF metaclass of the abstract ~~syntaxes of any conforming OMS languages~~ syntax of this language (cf. clause 2.1) ~~are subclasses~~ has to be a subclass (in the sense of SMOF multiple classification) of the DOL metaclass `NativeDocument`~~.~~ ~~If a~~, see Fig. ??. Likewise, if the conforming OMS language has a metaclass for basic OMS, this ~~is~~ has to be a subclass of the metaclass `BasicOMS`, see Fig. ??. See the informative annexes ?? to ?? for details.

9.2 Documents

9.2.1 Abstract Syntax

The DOL metamodel for documents and libraries is shown in Fig. ??. A document (`Document`) can be a

- a DOL library, or
- a `NativeDocument`, which is the verbatim inclusion of an OMS written in an OMS language that conforms with DOL; cf. 2.1).

A DOL library

consists of a collection of (named) OMS, OMS networks, and mappings between these. More specifically, a DOL library consists of a name, followed by a list of `LibraryItems`. A `LibraryItem` is either a `Definition`, an import of another DOL library (`LibraryImport`), or a `Qualification` selecting a specific OMS language, logic and/or syntax that is used

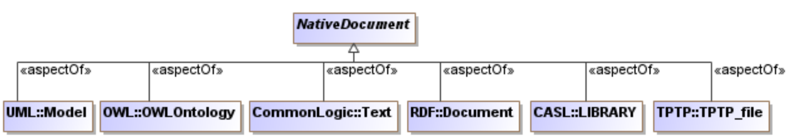


Figure 9.1: Informative diagram showing subclasses of `NativeDocument`

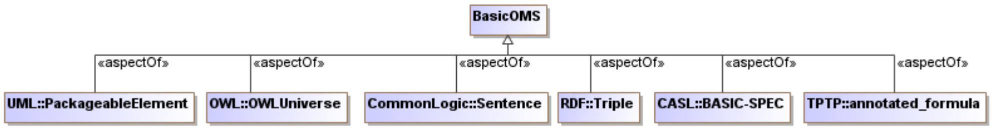


Figure 9.2: Informative diagram showing subclasses of `BasicOMS`

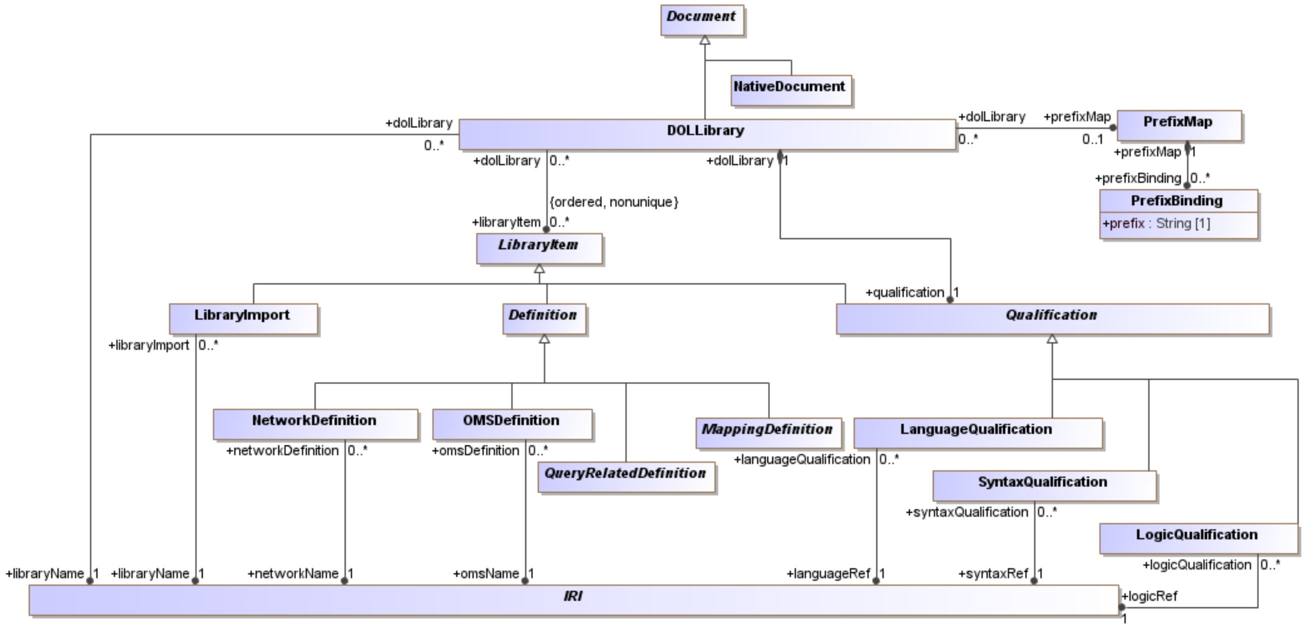


Figure 9.3: DOL metamodel: Documents and libraries

to interpret the subsequent `LibraryItems`. A `LibraryImport` leads to the inclusion of all `LibraryItems` of the imported DOL library into the importing one. A `Definition` assigns an IRI to an OMS (`OMSDefinition`), to a mapping between OMS (`MappingDefinition`), or an OMS network (`NetworkDefinition`). Moreover, annex ?? informatively introduces `QueryRelatedDefinition`.

At the beginning of a DOL library, one can declare a `PrefixMap` for abbreviating long IRIs using CURIEs; see clause ?? for further details. Examples of the use of DOLlibrary can be found in Appendix ?? and Section 7.

9.2.2 Concrete Syntax

9.2.2.1 Documents

```
Document             ::= DOLLibrary | NativeDocument
DOLLibrary           ::= [PrefixMap] 'library' LibraryName
                             Qualification LibraryItem*
NativeDocument       ::= <language and serialization specific >
LibraryItem          ::= LibraryImport | Definition | Qualification
Definition           ::= OMSDefinition
                       | NetworkDefinition
                       | MappingDefinition
LibraryImport        ::= 'import' LibraryName
Qualification        ::= LanguageQualification
                       | LogicQualification
                       | SyntaxQualification
LanguageQualification ::= 'language' LanguageRef
LogicQualification ::= 'logic' LogicRef
SyntaxQualification ::= 'serialization' SyntaxRef
LibraryName          ::= IRI
```

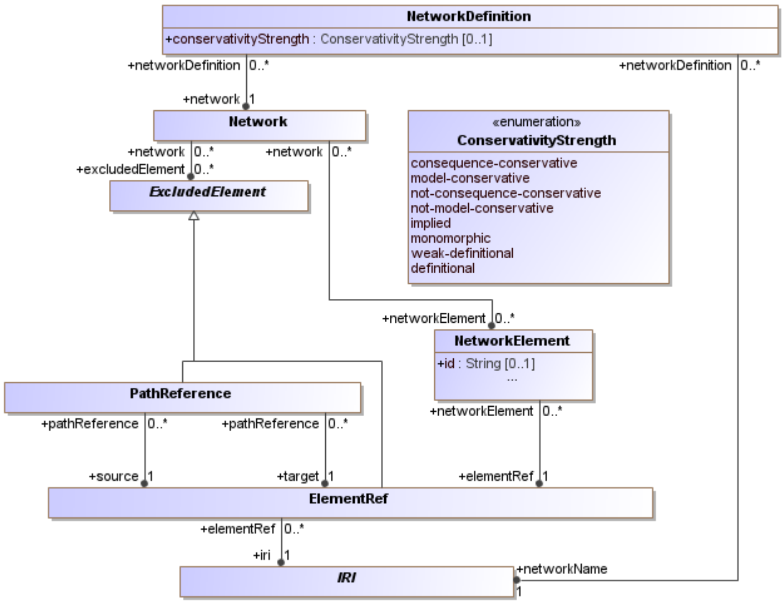


Figure 9.4:

```
LanguageRef          ::= IRI
LogicRef             ::= IRI
SyntaxRef            ::= IRI
```

```
PrefixMap        ::= '%prefix(' PrefixBinding* ')%'
PrefixBinding    ::= BoundPrefix IRIBoundToPrefix [Separators]
BoundPrefix      ::= ':' | Prefix <see definition in clause ??>[4]
IRIBoundToPrefix ::= '<' FullIRI '>'
Separators       ::= 'separators' SeparatorString SeparatorString
SeparatorString  ::= SeparatorChar SeparatorChar*
SeparatorChar    ::= ipchar | gen-delims - '#'<as defined in IETF/RFC 3987>
```

Note(4)

Note that the empty prefix (called “no prefix” in ~~W3C/TR REC-rdfa-core:2013, Section 6~~W3C/TR REC-rdfa-core:2015, Section 6) is denoted by a colon inside the prefix map, but it is omitted in CURIEs. This is the style of the OWL Manchester syntax [?] but differs from the RDFa Core 1.1 syntax.

9.3 OMS Networks

9.3.1 Abstract Syntax

The DOL metamodel for documents and libraries is shown in Fig. ??. Inside a DOL library, with a `NetworkDefinition`, one can define OMS networks (also called distributed OMS). OMS networks are typically used for complex viewpoint specifications; they also can be used in `combinations` (see clause ?? below). A `NetworkDefinition` names an OMS network consisting of `NetworkElements`. These can be `ElementRefs`, i.e. IRIs that name OMS, OMS mappings, or previously-defined OMS networks. `ElementRefs` that are OMS can be prefixed with an `Id`; this is then used for ~~disambugation~~ disambiguation in a `combination`. An optional `ConservativityStrength` specifies e.g. consistency of the network (analogously to `OMSDefinitions`, see clause ?? below for details).

An OMS network by default also includes all inclusions (between the extended and the extending OMS of an `ExtensionOMS`) between the involved OMS—unless these are explicitly excluded. The latter can be achieved using `ExcludingElements`. They consist of `ElementRefs` naming OMS or OMS mappings, and of `PathReferences`. A `PathReference` refers to an unnamed OMS mapping (e.g. one generated by an `Extension`) by specifying its source and target OMS. See Appendix ?? for an example of the use of `combination`.

[4] Note: Q-AUT: I think that, in contrast to OWL Manchester, we can allow prefix names that match keywords of the DOL syntax, as we are enclosing the whole prefix map into an annotation construct – right?

9.3.2 Concrete Syntax

```
NetworkDefinition ::= 'network' NetworkName '='
                      [ConservativityStrength] Network
NetworkName      ::= IRI
Network          ::= NetworkElements [ExcludedElements]
NetworkElements ::= NetworkElement  ',' NetworkElement *
NetworkElement  ::= [Id ':'] ElementRef
ExcludedElements ::= 'excluding' ExcludedElement  ',' ExcludedElement *
ExcludedElement ::= PathReference | ElementRef
PathReference   ::= IRI '->' IRI
ElementRef      ::= IRI
Id              ::= Letter LetterOrDigit*
```

9.4 OMS

9.4.1 Abstract Syntax

The DOL metamodel for OMS is shown in Fig. ??. DOL provides a rich structuring language for OMS, providing extension, translation, unions of OMS and many more. For each of these alternatives, a subclass is introduced. An OMS can be

- a `TranslationOMS` involving both an OMS (to be translated), and a specification of the translation, which is covered by the class `OMSTranslation`; (See Appendix ??, ??, for examples.)
- a `UnionsOMS`, uniting two given OMS; (See Appendix ?? for an example.)
- a `ClosureOMS`, applying a closure operator (given by a `Closure`) to an OMS;
- an `ExtensionOMS`, extending a given OMS with another OMS (given by the `Extension`). The major difference between a union and extension is that the members of the unions need to be self-contained OMS, while the extensions may reuse the signature of the extended OMS; (See Appendix ??, ??, ?? for examples.)
- an `ExtendingOMS`, which is a very simple form of OMS, namely a basic OMS or an OMS reference (see below);
- a `FilteringOMS`, applying a filtering operator (given by a `Filtering`) to an OMS; (See Appendix ?? for an example.)
- an `ApproximationOMS`, applying an approximation operator (given by an `Approximation`) to an OMS;
- a `CombinationOMS`, giving a combination of (the OMS contained in) an OMS network (technically, this is a colimit, see [?]); (See Appendix ?? for an example of the use of `combination`.)
- a `ReductionOMS`, applying a reduction (given by an `Reduction`) to an OMS;
- a `ExtractionOMS`, applying a module extraction operator (given by an `Extraction`) to an OMS; (See use case 7.3 for an example.)
- a `QualifiedOMS`, which is an OMS qualified with the OMS language that is used to express it.

Moreover, annex ?? informatively introduces `Applications`, which apply a ~~substition~~ substitution to an OMS.

A `ConservativityStrength` specifies additional relations that may hold between an OMS and its extension (or union with other OMS), like conservative or definitional extension. The rationale is that the extension should not have impact on the original OMS that is being extended.

An OMS definition `OMSDefinition` names an OMS. It can be optionally marked as inconsistent, consistent, monomorphic or having a unique model using `ConservativityStrength`. More precisely, `'consequence-conservative'` here requires the OMS to have only tautologies as signature-free logical consequences, while `'notconsequence-conservative'` expresses that this is not the case. `'model-conservative'` requires satisfiability of the OMS, `'not-model-conservative'` its unsatisfiability. `'definitional'` expresses that the OMS has a unique model (see Appendix ?? for an example); this may be interesting for characterizing OMS (e.g. returned by model finders) that are used to describe single models.

The DOL metamodel for extension OMS is shown in Fig. ??. ExtendingOMS is a subclass of OMS, containing those OMS that may be used to extend a given OMS within an ExtensionOMS. An ExtendingOMS can be one of the following:

- a basic OMS BasicOMS written inline, in a conforming serialization of a conforming OMS language (which is defined outside this standard)[1];
- a reference (through an IRI) to an OMS ~~existing on the Web~~ (OMSReference); or
- a RelativeClosureOMS, applying a closure operator to a basic OMS or OMS reference (these two are hence joined into ClosableOMS). A closure forces the subsequently declared non-logical symbols to be interpreted in a minimal or maximal way, while the non-logical symbols declared in the local environment are fixed. Variants of closure are minimization, maximization, freeness (minimizing also data sets and equalities on these, which enables the inductive definition of relations and datatypes), and cofreeness (enabling the coinductive definition of relations and datatypes).

Recall that the local environment is the OMS built from all previously-declared symbols and axioms.

Using ExtendingOMS, ~~further OMS can be built:~~ extensions of an OMS with an ~~, which is an~~ ExtendingOMS ~~optionally~~ can be built. The latter can optionally be named and/or marked as conservative, monomorphic, definitional, weakly definitional or implied (using a ConservativityStrength, see clause 4.3 for details)~~,~~.

Furthermore, OMS can be constructed using

- closures of an OMS with a Closure. This is similar to a RelativeClosureOMS, but the non-logical symbols to be minimized/maximized and to be varied are explicitly declared here (while a RelativeClosureOMS takes the local environment to be fixed, i.e. not varied)~~. Recall that the local environment is the OMS built from all previously-declared symbols and axioms.~~

  ~~Furthermore, OMS can be constructed using~~ ;
- a translation OMSTranslation of an OMS into a different signature or OMS language. The former is done using a SymbolMap, specifying a map of symbols to symbols. The latter is done using an OMS language translation OMSLanguageTranslation can be either specified by its name, or be inferred as the default translation to a given target (the source will be inferred as the OMS language of the current OMS);
- a Reduction of an OMS to a smaller signature and/or less expressive logic (that is, some non-logical symbols and/or some parts of the model structure are hidden, but the semantic effect of sentences involving these is kept). The former is done using a SymbolList, which is a list of non-logical symbols that are to be hidden. The latter uses an OMSLanguageTranslation denoting a logic projection that is used as logic reduction to a less expressive OMS language.
- an Approximation of an OMS, in a subsignature (InterfaceSignature) or sublogic, with the effect that sentences not expressible in the subsignature respectively sublogic are replaced with a suitable approximation,
- a Filtering of an OMS, with the effect that some signature symbols and axioms (specified by a BasicOMS) are removed from the OMS,
- a module Extraction of an OMS, using a restriction signature (InterfaceSignature).

In all of these cases except for translation, a RemovalKind specifies whether the listed symbols are removed from the OMS, or whether they are kept (and the other ones are removed).

The DOL metamodel for closure OMS is shown in Fig. ??, that for translation and reduction OMS in Fig. ??.

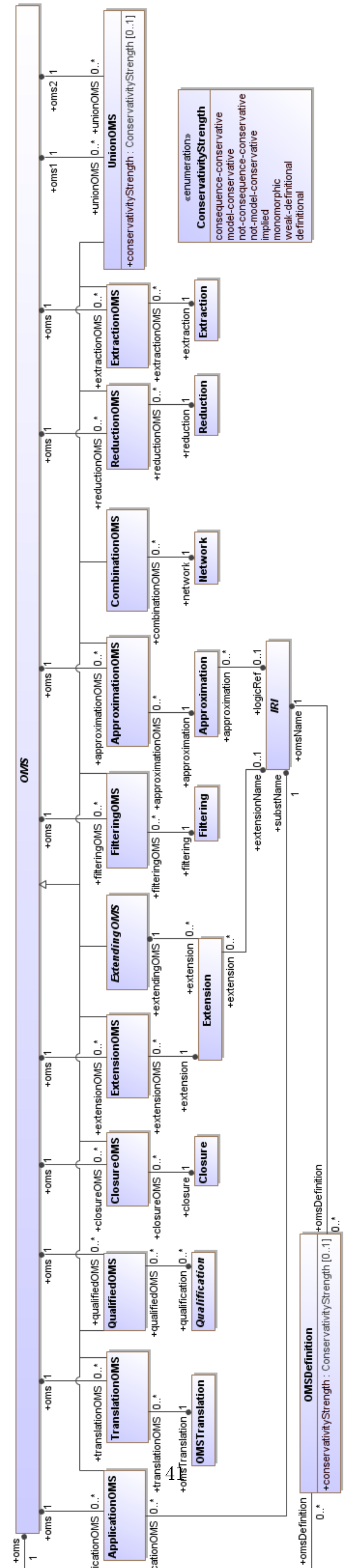
9.4.2 Concrete Syntax

While in most cases the translation from concrete to abstract syntax is obvious (the structure is largely the same),

- both %satisfiable, %cons and %mcons are translated to model-conservative,
- both %consistent and %ccons are translated to consequence-conservative,,
- both %unsatisfiable and %notmcons are translated to not-model-conservative,
- both %inconsistent and %notccons are translated to not-consequence-conservative,,
- Moreover, both closed-world and minimize are translated to minimize.

```
BasicOMS            ::= <language and serialization specific>
ClosableOMS         ::= BasicOMS | OMSRef [ImportName]
ExtendingOMS        ::= ClosableOMS | RelativeClosureOMS
RelativeClosureOMS  ::= ClosureType '{' ClosableOMS '}'
```

[1]In this place, any OMS in a conforming serialization of a conforming OMS language is permitted. However, DOL's module sublanguage should be ~~given preference over~~ used instead of the module sublanguage of the respective conforming OMS language; e.g. DOL's OMS reference and extension construct should be preferred over OWL's import construct.


OMS
UnionOMS
+conservativityStrength : ConservativityStrength [0..1]
«enumeration»
ConservativityStrength
consequence-conservative
model-conservative
not-consequence-conservative
not-model-conservative
implied
monomorphic
weak-definitional
definitional
ExtractionOMS
Extraction
ReductionOMS
Reduction
CombinationOMS
Network
ApproximationOMS
Approximation
IRI
FilteringOMS
Filtering
ExtendingOMS
Extension
ExtensionOMS
ClosureOMS
Closure
QualifiedOMS
Qualification
TranslationOMS
OMSTranslation
ApplicationOMS
OMSDefinition
+conservativityStrength : ConservativityStrength [0..1]

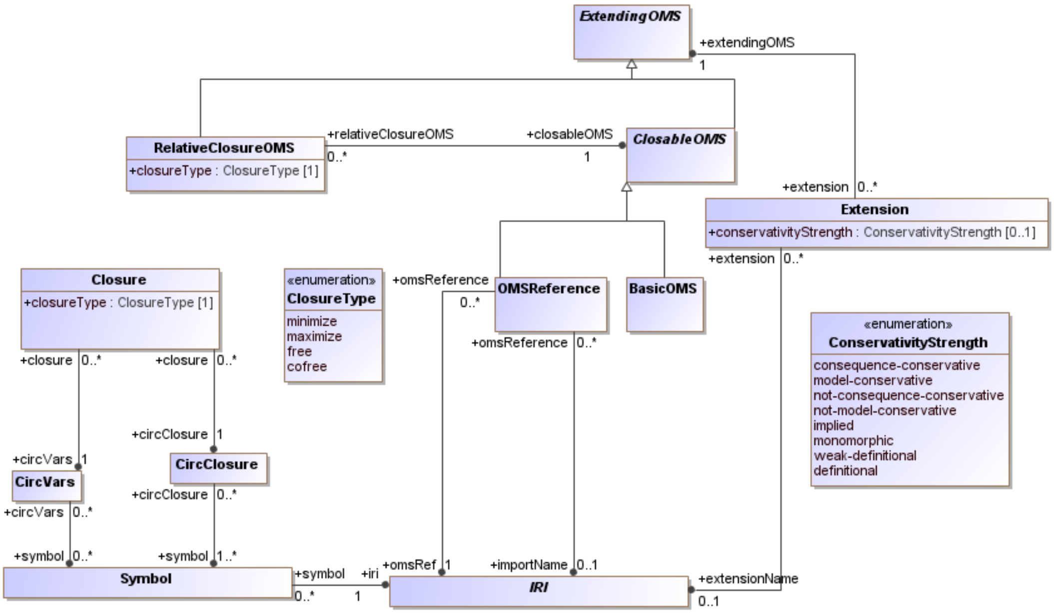


Figure 9.6: DOL metamodel: Extension and closure OMS

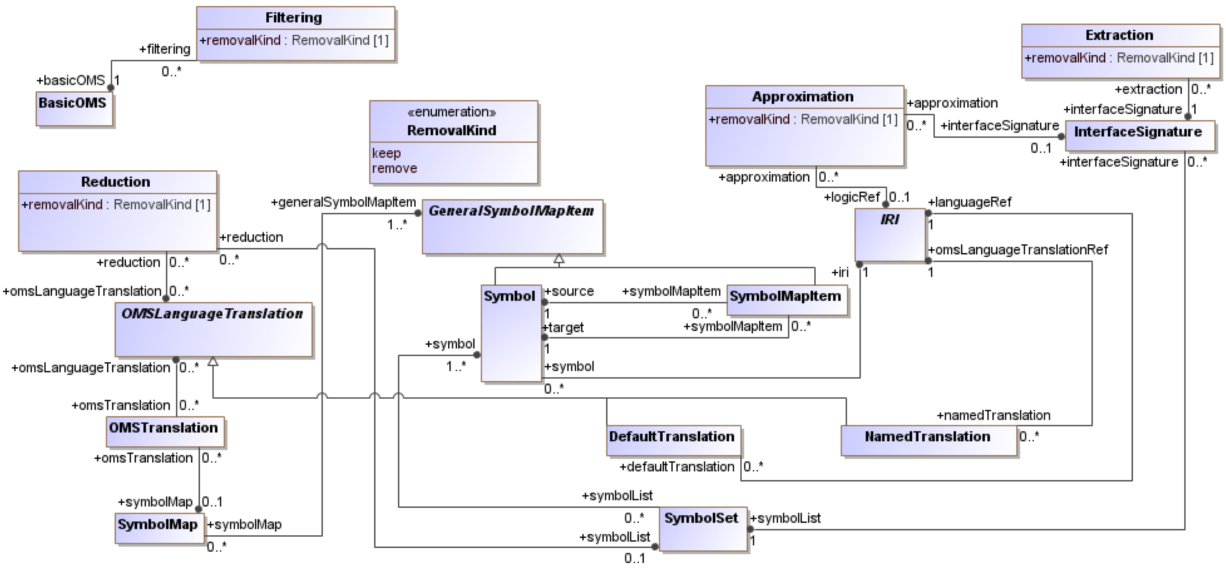


Figure 9.7: DOL metamodel: translation and reduction OMS

```
OMS                   ::= ExtendingOMS
                        | OMS Closure
                        | OMS OMSTranslation
                        | OMS Reduction
                        | OMS   Extraction| OMS   Approximation
                        | OMS Filtering
                        | OMS 'and' [ConservativityStrength] OMS
                        | OMS 'then' ExtensionOMS
                        | Qualification* ':' GroupOMS
                        | 'combine' NetworkElements [ExcludeExtensions]
                        | GroupOMS
Closure               ::= ClosureType CircMin [CircVars]
ClosureType           ::= 'minimize'
                        | 'closed-world'
                        | 'maximize'
                        | 'free'
                        | 'cofree'
CircMin               ::= Symbol Symbol*
CircVars              ::= 'vars' Symbol Symbol*
GroupOMS              ::= '{' OMS '}' | OMSRef
OMSTranslation        ::= 'with' LanguageTranslation* SymbolMap
                        | 'with' LanguageTranslation  ~~LanguageTranslation*~~   +  LanguageTranslation ::=
Reduction             ::= 'hide' LogicReduction* SymbolList
                        | 'hide' LogicReduction  ~~LogicReduction*~~   +  | 'reveal' SymbolList
LogicReduction        ::= 'along' OMSLanguageTranslation
SymbolList            ::= Symbol  ',' Symbol *
SymbolMap             ::= GeneralSymbolMapItem  ',' GeneralSymbolMapItem *
Extraction            ::= 'extract' InterfaceSignature
                        | 'remove' InterfaceSignature
Approximation         ::= 'forget' InterfaceSignature ['keep' LogicRef]
                        | 'keep' InterfaceSignature ['keep' LogicRef]
                        | 'keep' LogicRef
Filtering             ::= 'select' SymbolList
                        | 'select'   ~~BasicOMS~~   '' BasicOMS ''  | 'reject' SymbolList
                        | 'reject'   ~~BasicOMS~~   '' BasicOMS ''  ExtensionOMS       ::= [ExtConservativi
                          [ExtensionName]
                          ExtendingOMS
ConservativityStrength ::= Conservative | '%mono' | '%wdef' | '%def'
ExtConservativityStrength ::= ConservativityStrength | '%implied'
Conservative          ::= '%DIF␣>␣cons'
                        | '  %ccons'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%mcons'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%notccons'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%notmcons'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%consistent'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%inconsistent'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%satifsiable'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'%unsatisfiable'
InterfaceSignature␣::=␣SymbolList
ImportName␣␣␣␣␣␣␣␣␣::=␣'%' IRI '%'
ExtensionName␣␣␣␣␣␣::=␣'%' IRI '%'
OMSkeyword␣␣␣␣␣␣␣␣␣::=␣'ontology'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'onto'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'specification'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'spec'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'model'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣|␣'oms'
OMSDefinition␣␣␣␣␣␣::=␣OMSkeyword␣OMSName␣'='
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣[ConservativityStrength]␣OMS␣'end'
Symbol␣␣␣␣␣␣␣␣␣␣␣␣␣::=␣IRI
SymbolMapItem␣␣␣␣␣␣::=␣Symbol␣'|->'␣Symbol
```

```
GeneralSymbolMapItem␣::=␣Symbol␣|␣SymbolMapItem
Sentence␣␣␣␣␣␣␣␣␣␣::=␣<an expression specific to an OMS language>
OMSName              ::= IRI
OMSRef               ::= IRI
LoLaRef              ::= LanguageRef | LogicRef


OMSLanguageTranslation ::= OMSLanguageTranslationRef | '->' LoLaRef
OMSLanguageTranslationRef ::= IRI
```

The above grammar allows for some grouping ambiguity when using operators in OMS definitions. These ambiguities are resolved according to the following list, listing operators in decreasing order of precedence:

- `minimize`, `maximize`, `free`, and `cofree`.
- `extract`, `forget`, `hide`, `keep`, `reject`, `remove`, `reveal`, `select`, and `with`.
- `and`.
- `then`.

Multiple occurrences of the same operator are grouped in a left associative manner. In all other cases operators on the same precedence level are not implicitly grouped and have to be grouped explicitly. Omitting such an explicit grouping results in a parse error.

9.5 OMS Mappings

9.5.1 Abstract Syntax

An OMS mapping provides a connection between two OMS. An OMS mapping definition is the definition of either a named interpretation (`InterpretationDefinition`, entailment (`EntailmentDefinition`), refinement (`RefinementDefinition`) or equivalence (`EquivalenceDefinition`), a named declaration of the relation between a module of an OMS and the whole OMS (`ConservativeExtensionDefinition`), or a named alignment (`AlignmentDefinition`).

~~Both interpretation~~ The DOL metamodel for interpretations and refinements is shown in Fig. ??. Both interpretations and refinements specify a logical entailment or specialization relation between OMS.
An `InterpretationDefinition` specifies source and target OMS (forming the `InterpretationType`), as well as a `SymbolMap` and/or an `OMSLanguageTranslation`. The `SymbolMap` in an interpretation always must lead to a signature morphism. A proof obligation expressing that the source OMS, when translated along the signature morphism and/or the `OMSLanguageTranslation`, logically follows from the target OMS.

A symbol map in an interpretation is **required** to cover all non-logical symbols of the source OMS; the semantics specification in clause **??** makes this assumption. (Mapping a non-logical symbol twice is an error. Mapping two source non-logical symbols to the same target non-logical symbol is legal, this is a non-injective OMS mapping.)

`Refinements` subsume interpretations (via `SimpleRefinements`), but allow the specification of much more complex relation between OMS (and OMS networks). The style differs from interpretation in that even a single OMS is a refinement (via `RefinementOMS`); this corresponds to the source of an interpretation. Using `SimpleOMSRefinements`, a refinement can be further specialized to a (target) OMS via an `OMSRefinementMap`. The latter involves a symbol map and/or OMS language translation, analogously to interpretations. With this style of notation, simple refinements can be easily chained up (which cannot be done using interpretations). Refinements themselves can also be ~~composed (), provided that the target of the first refinement matches the source of the second one~~refined, also by other refinements—this amounts to the possibility of composing refinements. Furthermore, refinements can also be specified between networks (`SimpleNetworkRefinement`). A refinement between OMS networks ~~has to specify both a mapping () between the nodes of the OMSnetwork, as well as,~~ consists of a list of ordinary refinements (between OMS), one for each node ~~, a symbol map from the OMS of that node to the target OMS to which it is mapped (this is a).~~ in the source network (the OMS refinement is then required to refine the node in the source network to some node in the target network). The list may also include network refinements, much in the same way as network definitions also may include other networks.

The DOL metamodel for entailments and equivalences is shown in Fig. ??. An entailment is a variant of an interpretation where all symbols are mapped identically, while an equivalence states that the model classes of two OMS are in bijective correspondence. As for refinements, entailments and equivalences are also possible between networks (`NetworkNetworkEntailment` and `NetworkEquivalence`). An entailment between a network as premise and an OMS as conclusion (`NetworkOMSEntailment`) specifies that all models of the network, when restricted to a given node (given by an IRI), are models of the OMS.

The DOL metamodel for alignments is shown in Fig. ??. Signature morphisms used in interpretations and refinements use a functional style of mapping symbols of OMS. In contrast to this style, an alignment provides a relational connection between two OMS, using a set of `Correspondences`. Each correspondence may relate some OMS non-logical symbol to

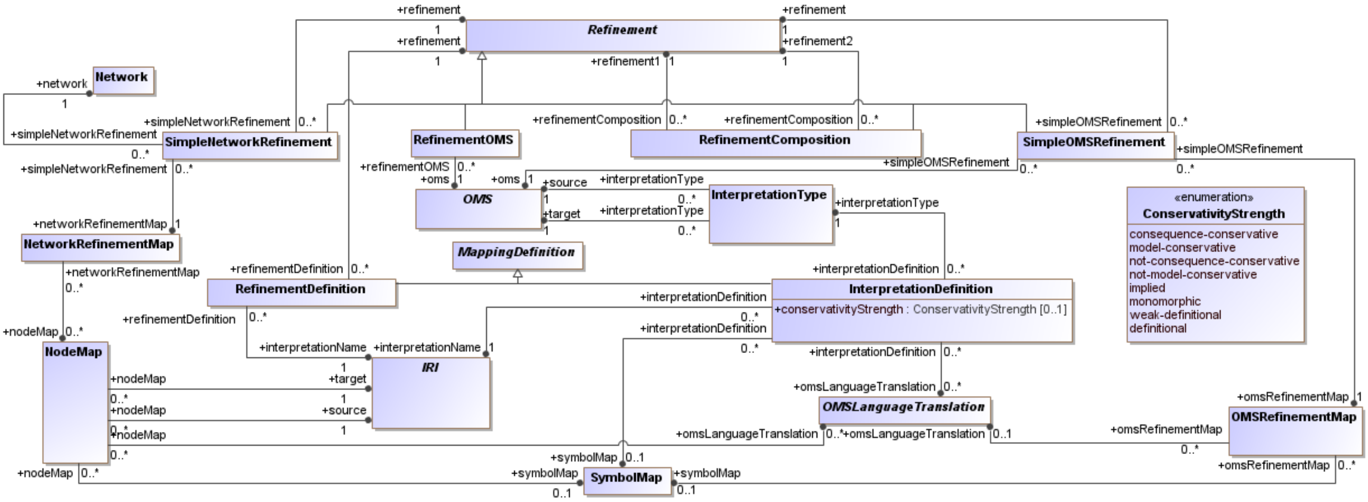


Figure 9.8: DOL metamodel: Interpretations and refinements

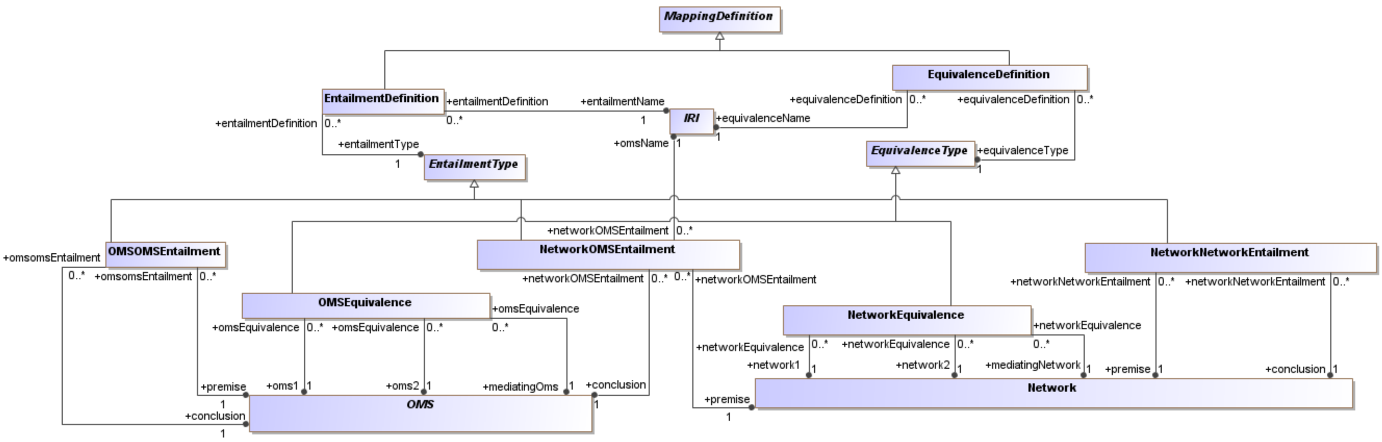


Figure 9.9: DOL metamodel: Entailments and equivalences

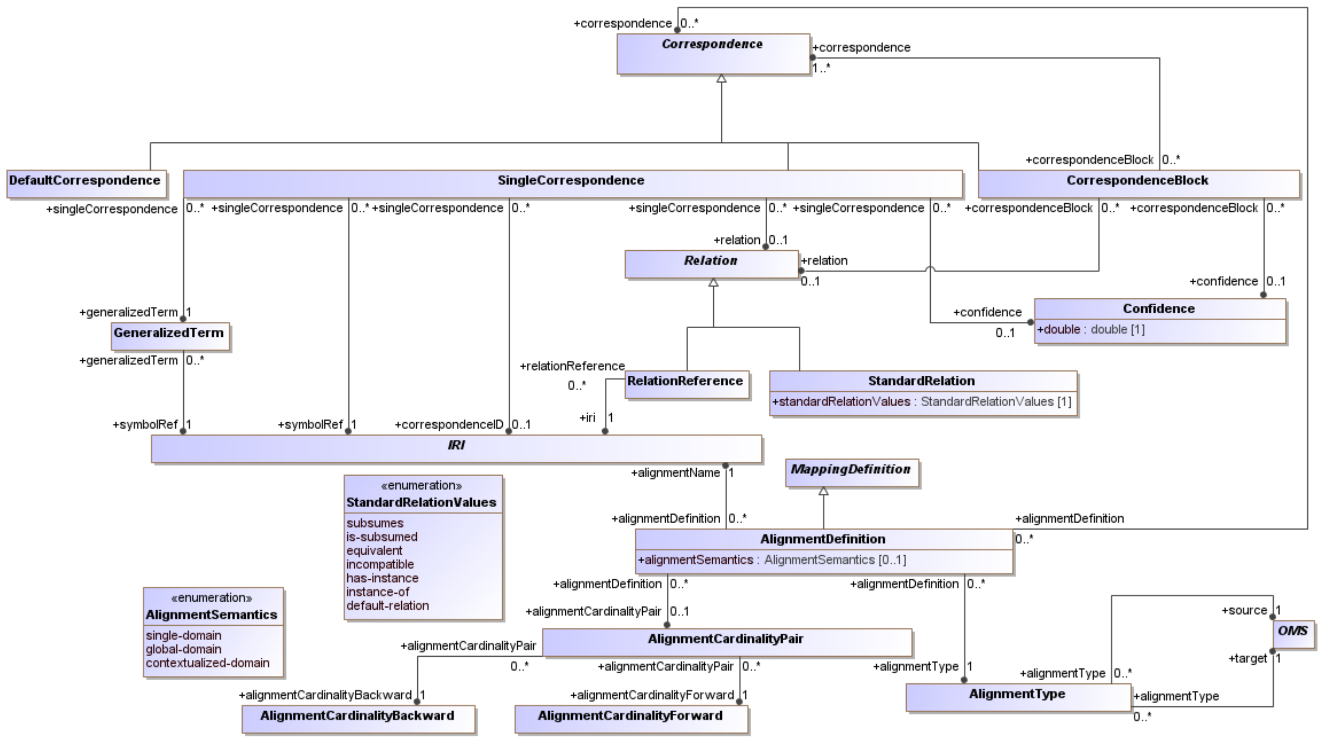


Figure 9.10: DOL metamodel: Alignments

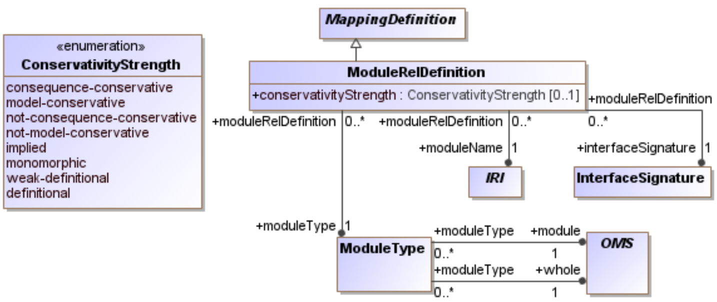


Figure 9.11: DOL metamodel: conservative extension definitions

another one (possibly given by a term) with an optional confidence value. Moreover, the relation between the two non-logical symbols can be explicitly specified (like being equal, or only being subsumed) in a similar way to the Alignment API [?]. The relations that can be used in a correspondence are equivalence, disjointness, subsumption, membership (the last two with a variant for each direction) or a user-defined relation that is stored in a registry and must be prefixed with `http://www.omg.org/spec/DOL/correspondences/`. A default correspondence can be used; it is applied to all pairs of non-logical symbols with the same local names. The default relation in a correspondence is equivalence, unless a different relation is specified in a surrounding 'CorrespondenceBlock'. Using an `AlignmentCardinality`, left and right injectivity and totality of the alignment can be specified (the default is left-injective, right-injective, left-total and right-total). With `AlignmentSemantics`, different styles of networks of aligned ontologies (to be interpreted in a logic-specific way) of alignments can be specified: whether a single domain is assumed, all domains are embedded into a global domain, or whether several local domains are linked ("contextualized") by relations.

~~A~~ The DOL metamodel for conservative extension definitions is shown in Fig. ??. A `ConservativeExtensionDefinition` declares that a certain ("~~module"~~whole) OMS actually is a ~~module of~~ conservative extension some other ("~~whole~~module") OMS with respect to the `InterfaceSignature`.

9.5.2 Concrete Syntax

```
MappingDefinition   ::= InterpretationDefinition
```

| EntailmentDefinition
| EquivalenceDefinition
| ~~ModuleRelDefinition~~ ConservativeExtensionDefinition | AlignmentDefinitio
InterpretationDefinition ::= InterpretationKeyword
InterpretationName
[Conservative] ':'
InterpretationType 'end'
| InterpretationKeyword
InterpretationName
[Conservative] ':'
InterpretationType '='
LanguageTranslation*
[SymbolMap] 'end'
| InterpretationKeyword
InterpretationName '='
Refinement 'end'
InterpretationKeyword ::= 'interpretation' | 'view' | 'refinement'
InterpretationName ::= IRI
InterpretationType ::= GroupOMS 'to' GroupOMS
Refinement ::= GroupOMS
| NetworkName
| ~~Refinement 'then' Refinement| GroupOMS 'refined' RefMap'to' Refinement|~~
~~NetworkName~~ Refinment 'refined' [RefMap] 'to' Refinement
RefMap ::= 'via' LanguageTranslation [SymbolMap]
| 'via' [LanguageTranslation] SymbolMap
| 'via' ~~NodeMap~~ Refinement (',' ~~NodeMap~~ Refinement)*
~~NodeMap ::= OMSName '|->' OMSName'using' LanguageTranslation* SymbolMap~~ EntailmentDefinition ::= '
EntailmentType 'end'
EntailmentName ::= IRI
EntailmentType ::= GroupOMS 'entails' GroupOMS
| OMSName 'in' Network 'entails' GroupOMS
| Network 'entails' Network
EquivalenceDefinition ::= 'equivalence' EquivalenceName ':'
EquivalenceType 'end'
EquivalenceName ::= IRI
EquivalenceType ::= GroupOMS '<->' GroupOMS ['=' OMS] | Network '<->' Network [
'=' Network ~~ModuleRelDefinition~~]ConservativeExtensionDefinition ::= '␣ ~~module' ModuleName~~
cons-ext' ConservativeExtensionName [Conservative] '

:'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣ ~~ModuleType~~ ConservativeExtensionType 'for'␣InterfaceSignature
␣ ~~ModuleName~~ ConservativeExtensionName ::= IRI
~~ModuleType~~ ConservativeExtensionType ::= GroupOMS 'of'␣GroupOMS
AlignmentDefinition␣::=␣'alignment'␣AlignmentName
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣[␣ ~~AlignmentCardinalityPair~~ AlignmentCardinality AlignmentCardinality
] ':'
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣AlignmentType
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣['='␣Correspondence␣(␣','␣Correspondence␣)*]
␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣␣['assuming'␣AlignmentSemantics]␣'end'
AlignmentName␣␣␣␣␣␣::=␣IRI
␣ ~~AlignmentCardinalityPair ::= AlignmentCardinalityForwardAlignmentCardinalityBackwardAlignmentCardi~~
~~::= AlignmentCardinalityAlignmentCardinalityBackward ::= AlignmentCardinality~~ AlignmentCardinality :

1'␣|␣'?' | '+' | '*'
AlignmentType ::= GroupOMS 'to' GroupOMS
AlignmentSemantics ::= 'SingleDomain'
| 'GlobalDomain'
| 'ContextualizedDomain'
Correspondence ::= CorrespondenceBlock | SingleCorrespondence | ''*

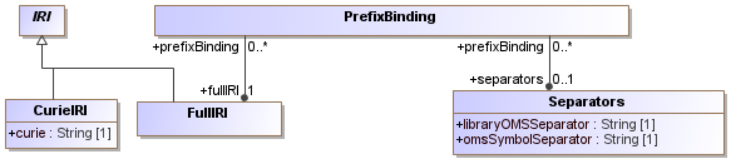


Figure 9.12: DOL metamodel: Prefixes

CorrespondenceBlock ::= 'relation' [Relation] [Confidence] '{'
Correspondence (',' Correspondence) '}'*
SingleCorrespondence ::= ~~SymbolRef~~ Symbol [Relation] [Confidence]
GeneralizedTerm [CorrespondenceId]
CorrespondenceId ::= '%(' IRI ')%'
~~SymbolRef~~ Symbol ::= IRI
GeneralizedTerm ::= ~~SymbolRef~~ Symbol Relation ::= '>' | '<' | '=' | '%' | 'ni' | '
Confidence ::= Double

`Double ::=` < `a number` $\in [0,1]$ >

9.6 Identifiers

This section specifies the abstract syntax of identifiers of DOL OMS and their elements. Further, it introduces the concrete syntax that is used in the DOL serialization.

9.6.1 IRIs

In accordance with best practices for publishing OMS on the Web, identifiers of OMS and their elements **should** not just serve as *names*, but also as *locators*, which, when dereferenced, give access to a concrete representation of an OMS or one of its elements. (For the specific case of RDF Schema and OWL OMS, these best practices are documented in [?]. The latter is a specialization of the linked data principles, which apply to any machine-processable data published on the Web [?].) It is recommended that publicly accessible DOL OMS be published as linked data.

Therefore, in order to impose fewer conformance requirements on applications, DOL requires the use of IRIs for identification per IETF/RFC 3987:2005. It is **recommended** that DOL libraries use IRIs that translate to URLs when applying the algorithm for mapping IRIs to URIs specified in IETF/RFC 3987:2005, Section 3.1. DOL descriptions of any element of a DOL library that is identified by a certain IRI **should** be *located* at the corresponding URL, so that agents can locate them. As IRIs are specified with a concrete syntax only in IETF/RFC 3987:2005, DOL adopts the latter into its abstract syntax as well as all of its concrete syntaxes (serializations). The DOL metamodel for IRIs and prefixes is shown in Fig. ??.

In accordance with semantic web best practices such as the OWL Manchester Syntax [?], this OMG Specification does not allow relative IRIs, and does not offer a mechanism for defining a base IRI, against which relative IRIs could be resolved.

Concerning these languages, note that they allow arbitrary IRIs in principle, but in practice they strongly recommend using IRIs consisting of two components [?]:

namespace an IRI that identifies an OMS, usually ending with # or /. (See annex **??** for a specific linked-data compliant URL scheme for DOL.)

local name a name that identifies a non-logical symbol within an OMS

9.6.2 Abbreviating IRIs using CURIEs

As IRIs tend to be long, and as syntactic mechanisms for abbreviating them have been standardized, it is **recommended** that applications employ such mechanisms and support expanding abbreviatory notations into full IRIs. For specifying the *semantics* of DOL, this OMG Specification assumes full IRIs everywhere, but the DOL abstract *syntax* adopts CURIEs (compact URI expressions) as an abbreviation mechanism, as it is the most flexible one that has been standardized to date.

The CURIE abbreviation mechanism works by binding prefixes to IRIs. A CURIE consists of a *prefix*, which may be empty, and a *reference*. If there is an in-scope binding for the prefix, the CURIE is valid and expands into a full IRI, which is created by concatenating the IRI bound to the prefix and the reference. In the following example that uses DOL prefix map mechanism, one the prefix `lang` is bound to `http://purl.net/DOL/languages/`, which means that the CURIE `lang:OWL2` will be expanded to the IRI `http://purl.net/DOL/languages/OWL2`.

@ @ *@

```
\DIFaddend * owl:  <http://www.w3.org/2002/07/owl>lang:  <http://purl.net/DOL/languages/>ser:
<http://purl.net/DOL/serializations/>log:  <http://purl.net/DOL/logics/>trans:  <http://purl.net/DOL/
)library Mereologylanguage lang:OWL2 logic log:SROIQ syntax ser:OWL2/Manchester[...]
```

DOL adopts the CURIE specification of RDFa Core 1.1 ~~W3C/TR REC-rdfa-core:2013, Section 6~~ W3C/TR REC-rdfa-core:2015, Secti with the following changes:

- DOL does not support the declaration of a "default prefix" mapping (covering CURIEs such as `:name`).
- DOL does support the declaration of a "no prefix" mapping (covering CURIEs such as `name`). If there is no explicit declaration for the "no prefix", it defaults to a context-sensitive expansion mechanism, which always prepends the DOL library IRI (in the context of a structured OMS where named OMS are referenced) respectively the current OMS IRI (in the context of a basic OMS) to a symbol name. Both the separator between the DOL library and the OMS name and that between the OMS name and the symbol name can be declared (using the keyword `separators`), and both default to "//".
- DOL does not make use of the `safe_curie` production.
- DOL does not allow binding a relative IRI to a prefix.
- Concrete syntaxes of DOL are encouraged but **not required** to support CURIEs.

CURIES are not required as a concession to having an RDF-based concrete syntax among the normative concrete syntaxes. RDFa is the only standardized RDF serialization to support CURIEs so far. Other serializations, such as RDF/XML or Turtle, support a subset of the CURIE syntax, whereas some machine-oriented serializations, including N-Triples, only support full IRIs.

CURIEs can occur in any place where IRIs are allowed, as stated in clause ??. ~~Informatively, the CURIE grammar supported by can be restated as follows:~~)

(~~CURIE ::= MaybeEmptyCURIE - MaybeEmptyCURIE ::= PrefixRefWithoutComma RefWithoutComma ::= Reference - StringWithComma StringWithComma ::= UChar* ',' UChar* UChar ::= (< any Unicode ISO/IEC 10646 character >) Prefix ::= NCName ':'(< see "NCName" in W3C/TR REC-xml-names:2009, Section 3 >) Reference ::= Path QueryFragmentPath ::= ipath-absolute | ipath-rootless | ipath-empty(< as defined in IETF/RFC 3987 >) Query ::= '?' iquery(< as defined in IETF/RFC 3987 >) Fragment ::= '' ifragment(< as defined in IETF/RFC 3987 >)~~ The CURIE grammar of DOLis summarized in clause ??.

Note that outside the context of a basic OMS the prefix/reference separator of a CURIE is always the colon (`:`); only for serializations of OMS languages other than DOL it may be redefined as stated in clause 2.2.

Prefix mappings can be defined at the beginning of a DOL library (specified in clause ??; these apply to all parts of the DOL library, including basic OMS as clarified in clause ??).

Bindings in a prefix map are evaluated from left to right. Authors **should not** bind the same prefix twice, but if they do, the later binding takes precedence.

9.6.3 Mapping identifiers in basic OMS to IRIs

While DOL uses IRIs as identifiers throughout, OMS languages do not necessarily do; for example:

- OWL ~~W3C/TR REC-owl2-syntax:2009, Section 5.5~~ W3C/TR REC-owl2-syntax:2012, Section 5.5 does use IRIs.
- Common Logic ISO/IEC 24707:2007 supports them but does not enforce their use.
- F-logic [?] does not use them at all.

However, DOL OMS mappings as well as certain operations on OMS require making unambiguous references to non-logical symbols of basic OMS (`Symbol`). Therefore, DOL provides a function that maps global identifiers used within basic OMS to IRIs. This mapping affects all non-logical symbol identifiers (such as class names in an OWL ontology), but not locally-scoped identifiers such as bound variables in Common Logic ontologies. DOL reuses the CURIE mechanism for abbreviating IRIs for this purpose (cf. clause ??).

The IRI of a non-logical symbol identifier in a basic OMS O is determined by the following function:

Require: D is a DOL library
Require: O is a basic OMS in serialization S
Require: id is the identifier in question, identifying a symbol in O according to the specification of S
Ensure: i is an IRI
  if id represents a full IRI according to the specification of S **then**
    $i \leftarrow id$
  else
    {first construct a pattern cp for CURIEs in S, then match id against that pattern}
    if the declaration of DOL-conformance of S redefines the prefix/reference separator character cs (cf. clause 2.2) **then**
      $sep \leftarrow cs$
    else if S forbids prefixed CURIEs **then**
      $sep \leftarrow$ undefined

else
$sep \leftarrow$: {the standard CURIE separator character}
end if
{The following statements construct a modified EBNF grammar of CURIEs; see ISO/IEC 14977:1996 for EBNF, and clause **??** for the original grammar of CURIEs.}
if sep is defined **then**
$cp \leftarrow [NCName, sep], Reference$
else
$cp \leftarrow Reference$
end if
if id matches the pattern cp, where ref matches $Reference$ **then**
if the match succeeded with a non-empty $NCName$ pn **then**
$p \leftarrow concat(pn, :)$
else
$p \leftarrow$ no prefix
end if
if O binds p to an IRI pi according to the specification of S **then**
$nsi \leftarrow pi$
else
$P \leftarrow$ the innermost prefix map in D, starting from the place of O inside D, and going up the abstract syntax tree towards the root of D
while P is defined **do**
if P binds p to an IRI pi **then**
$nsi \leftarrow pi$
break out of the **while** loop
end if
$P \leftarrow$ the next prefix map in D, starting from the place of the current P inside D, and going up the abstract syntax tree towards the root of D
end while
return an error
end if
$i \leftarrow concat(nsi, ref)$
else
return an error
end if
end if
return i

This mechanism applies to basic OMS given inline in a DOL library (`BasicOMS`), not to OMS in external documents (`NativeDocument`); the latter **shall** be self-contained.

While CURIEs used for identifying parts of a DOL library (cf. clause **??**) are merely syntactic sugar, the prefix map for a basic OMS is essential to determining the semantics of the basic OMS within the DOL library.

9.6.4 Concrete Syntax

```
IRI             ::= '<' FullIRI '>' | CURIE
FullIRI         ::= < an IRI as defined in IETF/RFC 3987:2005 >
CURIE           ::= MaybeEmptyCURIE -
MaybeEmptyCURIE ::= [Prefix] RefWithoutComma
RefWithoutComma ::= Reference - StringWithComma
StringWithComma ::= UChar* ',' UChar*
UChar           ::= < any Unicode ISO/IEC 10646 character >
Prefix          ::= NCName ':'< see "NCName" in W3C/TR REC-xml-names:2009, Section 3 >
Reference       ::= Path [Query] [Fragment]
Path            ::= ipath-absolute | ipath-rootless | ipath-empty< as defined in IETF/RFC 3987 >
Query           ::= '?' iquery< as defined in IETF/RFC 3987 >
Fragment        ::= '#' ifragment< as defined in IETF/RFC 3987 >
```

In a CURIE without a prefix, the `reference` part is **not allowed** to match any of the keywords of the DOL syntax (cf. clause ??).

9.7 Lexical Symbols

The character set for the DOL text serialization is the UTF-8 encoding of Unicode ISO/IEC 10646. However, OMS can always be input in the Basic Latin subset, also known as US-ASCII.[2] For enhanced readability of OMS, the DOL text serialization particularly supports the native Unicode glyphs that represent common mathematical symbols (e.g. Greek letters) or operators (e.g. ∂ for partial derivatives).

9.7.1 Key words and signs

The lexical symbols of the DOL text serialization include various key words and signs that occur as terminal symbols in the context-free grammar in annex ??. Key words and signs that represent mathematical signs are displayed as such, when possible, and those signs that are available in the Unicode character set may also be used for input.

9.7.1.1 Key words

Key words are always written lowercase. The following key words are reserved, and are not available for use as variables or as CURIEs with no prefix[3], although they can be used as parts of tokens.

```
alignment
along
assuming
and
closed-world
cofree
combine
  cons-ext  end
entails
entailment
equivalence
excluding
extract
free
hide
import
in
for
forget
interpretation
keep
language
library
logic
maximize
model
  module  minimize
network
ni
of
oms
onto
ontology
refined
refinement
reject
relation
remove
```

[2]In this case, IRIs will have to be mapped to URIs following section 3.1 of IETF/RFC 3987:2005.

[3]In such a case, one can still rename affected variables, or declare a prefix binding for affected CURIEs, or use absolute IRIs instead. These rewritings do not change the semantics.

Table 9.1: Key Signs

Sign	Unicode Code Point	Basic Latin substitute
{	U+007B LEFT CURLY BRACKET	
}	U+007D RIGHT CURLY BRACKET	
:	U+003A COLON	
=	U+003D EQUALS SIGN	
,	U+002C COMMA	
↦	U+21A6 RIGHTWARDS ARROW FROM BAR	|->
→	U+2192 RIGHTWARDS ARROW	->

```
result
reveal
select
separators
serialization
spec
specification
substitution
  syntax  then
to
translation
using
vars
via
view
where
with
%DIF > cons
%ccons
%complete
%consistent
%def
%implied
%inconsistent
%mcons
%mono
%notccons
%notmcons
%prefix
%wdef
```

9.7.1.2 Key signs

Table ?? following key signs are reserved, and are not available for use as complete identifiers. Key signs that are outside of the Basic Latin subset of Unicode may alternatively be encoded as a sequence of Basic Latin characters.

9.8 Integration of Serializations of Conforming Languages

Any document providing an OMS in a serialization of a DOL conforming language can be used as-is in DOL, by reference to its IRI.

The following cases apply for injecting identifiers into fragments of OMS languages, depending on the conformance level of the respective serialization of the OMS language used in terms of section 2.2:

XML conformance ~~Identifiers are added to XML elements by using the IRI-valued~~

1. If the serialization supports annotation of the root element of the fragment of interest, as specified in XML conformance requirement 3a, an identifier is assigned by way of an annotation whose predicate is `http://www.w3.org/2002/07/owl#sameAs` from the OWL language W3C/TR REC-owl2-primer:2012 and whose object is expected to be the desired IRI identifier.

2. If the *dol:id* XML attribute from the `http://www.omg.org/spec/DOL/1.0/xml` namespace ~~, or, if the serialization does not support this attribute, by adding a *dol:id* XML element~~ is supported on the element, as specified in XML conformance requirement 3b, its value is expected to be the IRI identifier.
3. If the `dol:id` XML element is supported as the first child ~~, containing~~ of the element, as specified in XML conformance requirement 3b, it is expected to contain exactly one text node ~~with the IRI~~ whose value is the IRI identifier.

It is a DOL syntax error if 1. an `owl:sameAs` annotation or a `dol:id` attribute or child element is present and its value is not, or cannot be interpreted, as a full IRI, or if 2. more than one of these three alternative fields (annotation, attribute or child element) is present on an element.

RDF conformance The RDF data model itself enables the assignment of IRI identifiers to all resources.

Text conformance Identifiers are added by inserting a special comment immediately[4] after the structural OMS element to be annotated, or, if this is not allowed and no ambiguity arises from inserting the comment *before* the structural element, by doing the latter. The complete comment **shall** read `%(I)%` if the language uses the `%` character to introduce comments, where `I` is the identifier IRI. If the language uses a different comment syntax, the *content* of the comment **shall** start with `%(I)%`, possibly preceded by whitespace.

Standoff markup conformance ~~Standard mechanisms like XPointer (W3C/TR REC-xptr-framework:2003) or~~ If the given OMS serialization conforms with the *text/plain* media type as per standoff markup conformance requirement 1 but not with XML, IETF/RFC 5147 shall be used as means of non-destructively assigning a URI to pieces of ~~XML or~~ text in the given OMS serialization. If the serialization conforms with XML as per requirement 2, one of IETF/RFC 5147 or XPointer (W3C/TR REC-xptr-framework:2003) shall be used. (As ~~a use case for XPointer~~an example, consider the identification of ~~axioms~~ imports in the OWL~~2~~/XML serialization [?], which does not provide a native way for assigning identifiers to ~~axioms. If, for~~ imports unless modified as suggested in annex ??. For example, in an OMS file *~~birds~~cars.owx*, the ~~axiom~~ import <~~SubClassOf~~Import>~~<Class IRI="Penguin"~~http:/~~><Class IRI="FlightlessBird"~~/~~>~~example.org/engines</~~SubClassOf~~Import> ~~is the first one (in document order) to declare a superclass for *Penguin*, it~~ can be referred to by the IRI `cars.owx#xpointer(/owl:Ontology/owl:Import[text()='http://example.org/engines'])` assuming the right binding for the namespace prefix *owl* in scope~~, whereas unique reference by the axiom's structure rather than by position would require a more complex expression~~. The same ~~axiom~~ import in the text-based OWL Manchester syntax [?] could be referred to as `cars.omn#line=27` according to IETF/RFC 5147 if it is on line 27 of the document.)

[5] Note(5)

Where the given OMS language does not provide a way of assigning IRIs to a desired subject of an annotation (e.g. if one wants to annotate an import in OWL), a document may employ RDF annotations that use XPointer or IETF/RFC 5147 as means of non-destructively referencing pieces of XML or text by ~~URI.~~ IRI, as specified above. (The extensibility of the XPointer framework may be utilized by developing additional XPointer schemes, e.g. for pointing to subterms of ~~Common Logic sentences.)~~ sentences in the XCL serialization of Common Logic.)

[4] The serialization **may** allow whitespace between the keyword and the comment.

[5] NOTE: TODO: injection of identifiers addressed, but we also had %implies etc. The latter will probably be handled in a similar way, but I don't know how exactly.

10 DOL Semantics

DOL is a logical language with a precise formal semantics. The semantics gives DOL a rock-solid foundation, and provides increased trustworthiness in applications based on OMS written in DOL. The semantics of DOL is moreover the basis for formal interoperability, as well as for the meaningful use of logic-based tools for DOL, such as theorem provers, model-checkers, satisfiability modulo theories (SMT) solvers etc. Last but not least, the semantics has provided valuable feedback on the language design, and has led to some corrections on the abstract syntax. These reasons have lead to inclusion of the semantics in the standard document proper, even though the semantics is quite technical and therefore has a more limited readership than the other clauses of this standard.

The semantics starts with the theoretical foundations. Since DOL is a language that can be applied to a variety of logics and logic translations, it is based on a heterogeneous logical environment. Hence, the most important need is to capture precisely what a heterogeneous logical environment is.

The DOL semantics itself gives a formal meaning to DOL libraries, OMS networks, OMS, and OMS mappings. For each syntactic construct in the abstract syntax, a *semantic domain* is given. It specifies the range of possible values for the semantics. Additionally, *semantic rules* are presented, mapping abstract syntax trees to some suitable semantic domain.

10.1 Theoretical Foundations of the DOL Semantics

In the following the theoretical foundations of the semantics of DOL are specified. The notions of *institution* and institution *comorphism* and *morphism* are introduced, which provide formalizations of the terms logic, ~~respectively~~ logic translation ~~,~~ ~~respectively~~ and logic reduction, respectively.

Since DOL covers OMS written in one or several logical systems, the DOL semantics needs to clarify the notion of logical system. Traditionally, logicians have studied abstract logical systems as sets of sentences equipped with an entailment relation $\vdash$. Such an entailment relation can be generated in two ways: either via a proof system, or as the logical consequence relation for some model theory. This specification follows the model-theoretic approach, since this is needed for many of the DOL constructs, and moreover, ontology, modeling and specification languages like OWL, Common Logic, or CASL come with a model-theoretic semantics, or (like UML class diagrams) can be equipped with one.

An abstract notion of logical system is given by the notion of satisfaction system [?], called 'rooms' in the terminology of [?]. They capture the Tarskian notion of satisfaction of a sentence in a model in an abstract way.

Definition 1 *A triple $\mathcal{R} = (Sen, \mathcal{M}, \models)$ is called a* ***satisfaction system****, or* ***room****, if $\mathcal{R}$ consists of*

- *a set Sen of* ***sentences****,*
- *a class $\mathcal{M}$ of* ***models****, and*
- *a binary relation $\models\ \subseteq \mathcal{M} \times Sen$, called the* ***satisfaction relation****.*

□

While this signature-free treatment enjoys simplicity and is wide-spread in the literature, many concepts and definitions found in logics, e.g. the notion of a conservative extension, involve the *vocabulary* or *signature* Σ used in sentences. Signatures can be extended with new non-logical symbols, or some of these symbols can be renamed; abstractly, this is captured using signature morphisms. Moreover, morphisms between models are also needed in order to give a semantics to `minimize`, `maximize`, `free` and `cofree`—these constructs use model morphisms to select certain models, e.g. the minimal ones. This leads to the notion of *institution*. An institution is nothing more than a family of satisfaction systems, indexed by signatures, and linked coherently by signature morphisms.

Definition 2 *Let $\mathbb{S}et$ be the category having all small sets as objects and functions as arrows, and let $\mathbb{C}at$ be the category of categories and functors.*[1] *An* ***institution*** *[?] is a quadruple $I = (\mathsf{Sig}, \mathbf{Sen}, \mathbf{Mod}, \models)$ consisting of the following:*

- *a category*[2] Sig *of* signatures *and* signature morphisms*,*
- *a functor* $\mathbf{Sen}\colon \mathsf{Sig} \longrightarrow \mathbb{S}et$ *giving, for each signature Σ, the set of* sentences $\mathbf{Sen}(\Sigma)$*, and for each signature morphism $\sigma : \Sigma \to \Sigma'$, the* sentence translation map $\mathbf{Sen}(\sigma) : \mathbf{Sen}(\Sigma) \to \mathbf{Sen}(\Sigma')$*, where often $\mathbf{Sen}(\sigma)(\varphi)$ is written as $\sigma(\varphi)$,*

[1] Strictly speaking, $\mathbb{C}at$ is not a category but only a so-called quasicategory, which is a category that lives in a higher set-theoretic universe.

[2] See [?, ?] for an introduction into category theory.

- *a functor* $\mathbf{Mod}: \mathsf{Sig}^{op} \to \mathbb{C}at$ *giving, for each signature* Σ, *the category of* models $\mathbf{Mod}(\Sigma)$, *and for each signature morphism* $\sigma\colon \Sigma \longrightarrow \Sigma'$, *the* reduct functor $\mathbf{Mod}(\sigma): \mathbf{Mod}(\Sigma') \to \mathbf{Mod}(\Sigma)$, *where often* $\mathbf{Mod}(\sigma)(M')$ *is written as* $M'|_\sigma$, *and* $M'|_\sigma$ *is called the* σ-reduct *of* M', *while* M' *is called a* σ-expansion *of* $M'|_\sigma$,
- *a satisfaction relation* $\models_\Sigma \subseteq |\mathbf{Mod}(\Sigma)| \times \mathbf{Sen}(\Sigma)$ *for each* $\Sigma \in |\mathsf{Sig}|$,

such that for each $\sigma\colon \Sigma \longrightarrow \Sigma'$ *in* Sig *the following **satisfaction condition** holds:*

$$(\star) \qquad M' \models_{\Sigma'} \sigma(\varphi) \text{ iff } M'|_\sigma \models_\Sigma \varphi$$

for each $M' \in |\mathbf{Mod}(\Sigma')|$ *and* $\varphi \in \mathbf{Sen}(\Sigma)$, *expressing that truth is invariant under change of notation and context.* □

Definition 3 (Propositional Logic) *The signatures of propositional logic are sets* Σ *of propositional symbols, and signature morphisms are just functions* $\sigma: \Sigma_1 \to \Sigma_2$ *between these sets. A* Σ*-model is a function* $M: \Sigma \to \{True, False\}$, *and the reduct of a* Σ_2*-model* M_2 *along a signature morphism* $\sigma: \Sigma_1 \to \Sigma_2$ *is the* Σ_1*-model given by the composition of* σ *with* M_2. Σ*-sentences are built from the propositional symbols with the usual connectives, and sentence translation is replacing the propositional symbols in* Σ *along the morphism. Finally, the satisfaction relation is defined by the standard truth-tables semantics. It is straightforward to see that the satisfaction condition holds.* □

Definition 4 (Common Logic — CL) *A common logic signature* Σ *(called vocabulary in Common Logic terminology) consists of a set of names, with a subset called the set of discourse names, and a set of sequence markers. A* Σ*-model consists of a set UR, the universe of reference, with a non-empty subset* $UD \subseteq UR$, *the universe of discourse, and four mappings:*

- *rel from UR to subsets of* $UD^* = \{\langle x_1, \ldots, x_n\rangle \mid x_1, \ldots, x_n \in UD\}$ *(i.e., the set of finite sequences of elements of UD);*
- *fun from UR to total functions from* UD^* *into UD;*
- *int from names in* Σ *to UR, such that* $int(v)$ *is in UD if and only if* v *is a discourse name;*
- *seq from sequence markers in* Σ *to* UD^*.

A Σ*-sentence is a first-order sentence, where predications and function applications are written in a higher-order like syntax:* $t(s)$. *Here,* t *is an arbitrary term, and* s *is a sequence term, which can be a sequence of terms* $t_1 \ldots t_n$, *or a sequence marker. A predication* $t(s)$ *is interpreted by evaluating the term* t, *mapping it to a relation using rel, and then asking whether the sequence given by the interpretation* s *is in this relation. Similarly, a function application* $t(s)$ *is interpreted using fun. Otherwise, interpretation of terms and formulae is as in first-order logic. A difference to first-order logic is the presence of sequence terms (namely sequence markers and juxtapositions of terms), which denote sequences in* UD^*, *with term juxtaposition interpreted by sequence concatenation. Note that sequences are essentially a non-first-order feature that can be expressed in second-order logic. For details, see [?].*

A CL *signature morphism consists of two maps between the sets of names and of sequence markers, such that the property of being a discourse name is preserved and reflected.*[3] *Model reducts leave UR, UD, rel and fun untouched, while int and seq are composed with the appropriate signature morphism component.* □

Further examples of institutions are: $\mathcal{SROIQ}(D)$, unsorted first-order logic, many-sorted first-order logic, and many others. Note that ~~reduct~~ the reduct of a model is generally given by forgetting ~~parts of the model.~~ some of its parts.

For the rest of the section, an arbitrary institution is considered.

Definition 5 (Theory) *A **theory** is a pair* (Σ, Δ) *where* Σ *is a signature and* Δ *is a set of* Σ*-sentences.*

Given a theory $T = (\Sigma, \Delta)$, the class of T-models is the class of all Σ-models M such that $M \models \delta$, for each sentence $\delta \in \Delta$. A theory (Σ, Δ) is **consistent** if there exists a Σ-model M such that $M \models \varphi$ for each $\varphi \in \Delta$. **Semantic entailment** is defined as usual: for a theory $\Delta \subseteq \mathbf{Sen}(\Sigma)$ and $\varphi \in \mathbf{Sen}(\Sigma)$, Δ entails φ, $\Delta \models \varphi$, if all models satisfying all sentences in Δ also satisfy φ. For a theory (Σ, Δ), we write $\Delta^\bullet$ for the set of all Σ-sentences φ such that $\Delta \models \varphi$.

Definition 6 (Theory morphism) *A **theory morphism*** $\phi: (\Sigma, \Delta) \to (\Sigma', \Delta')$ *is a signature morphism* $\phi: \Sigma \to \Sigma'$ *such that* $\Delta' \models \phi(\Delta)$.

Institution comorphisms capture the intuition of encoding or embedding a logic into a more expressive one.

Definition 7 (Institution Comorphism) *An **institution comorphism** from an institution* $I = (\mathsf{Sig}^I, \mathsf{Mod}^I, \mathsf{Sen}^I, \models^I)$ *to an institution* $J = (\mathsf{Sig}^J, \mathsf{Mod}^J, \mathsf{Sen}^J, \models^J)$ *consists of a functor* $\Phi: \mathsf{Sig}^I \longrightarrow \mathsf{Sig}^J$, *and two natural transformations* ~~$\beta: \mathsf{Mod}^J \circ \Phi \Longrightarrow \mathsf{Mod}^I$~~ $\beta: \mathsf{Mod}^J \circ \Phi^{op} \Longrightarrow \mathsf{Mod}^I$ *and* $\alpha: \mathsf{Sen}^I \Longrightarrow \mathsf{Sen}^J \circ \Phi$, *such that for each* I*-signature* Σ, *each sentence* $\varphi \in \mathsf{Sen}^I(\Sigma)$ *and each model* ~~$M' \in \mathsf{Mod}^J(\Phi(\Sigma))$~~ $M' \in |\mathsf{Mod}^J(\Phi(\Sigma))|$

$$M' \models^J_{\Phi(\Sigma)} \alpha_\Sigma(\varphi) \Leftrightarrow \beta_\Sigma(M') \models^I_\Sigma \varphi.$$

*holds, called the **satisfaction condition**.* □

[3] That is, a name is a discourse name if and only if its image under the signature morphism is.

Here, $\Phi(\Sigma)$ is the translation of the signature Σ from institution I to institution J, $\alpha_\Sigma(\varphi)$ is the translation of the Σ-sentence φ to a $\Phi(\Sigma)$-sentence, and $\beta_\Sigma(M')$ is the translation (or perhaps better: reduction) of the $\Phi(\Sigma)$-model M' to a Σ-model. Naturality of α and β means that for each signature morphism $\sigma : \Sigma_1 \to \Sigma_2$ in I the following squares commute:

$$\begin{tikzcd}
\mathsf{Sen}^I(\Sigma_1) \arrow[r, "\alpha_{\Sigma_1}"] \arrow[d, "\mathsf{Sen}^I(\sigma)"'] & \mathsf{Sen}^J(\Phi(\Sigma_1)) \arrow[d, "\mathsf{Sen}^J(\Phi(\sigma))"] \\
\mathsf{Sen}^I(\Sigma_2) \arrow[r, "\alpha_{\Sigma_2}"'] & \mathsf{Sen}^J(\Phi(\Sigma_2))
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathsf{Mod}^J(\Phi(\Sigma_2)) \arrow[r, "\beta_{\Sigma_2}"] \arrow[d, "\mathsf{Mod}^J(\Phi(\sigma))"] & \mathsf{Mod}^I(\Sigma_2) \arrow[d, "\mathsf{Mod}^I(\sigma)"] \\
\mathsf{Mod}^J(\Phi(\Sigma_1)) \arrow[r, "\beta_{\Sigma_1}"'] & \mathsf{Mod}^I(\Sigma_1)
\end{tikzcd}$$

A comorphism is:

- *faithful* if logical consequence is preserved and reflected along the comorphism:

$$\Gamma \models^I \varphi \text{ iff } \alpha(\Gamma) \models^J \alpha(\varphi)$$

- *model-expansive* if each β_Σ is surjective;
- *(weakly) exact* if for each signature morphism $\sigma \colon \Sigma_1 \longrightarrow \Sigma_2$, the naturality diagram

$$\begin{tikzcd}
\mathsf{Mod}^J(\Phi(\Sigma_2)) \arrow[r, "\beta_{\Sigma_2}"] \arrow[d, "\mathsf{Mod}^J(\Phi(\sigma))"] & \mathsf{Mod}^I(\Sigma_2) \arrow[d, "\mathsf{Mod}^I(\sigma)"] \\
\mathsf{Mod}^J(\Phi(\Sigma_1)) \arrow[r, "\beta_{\Sigma_1}"'] & \mathsf{Mod}^I(\Sigma_1)
\end{tikzcd}$$

  admits (weak) amalgamation, i.e. any for any two models ~~$M_2 \in \mathsf{Mod}^I(\Sigma_2)$ and $M_1' \in \mathsf{Mod}^J(\Phi(\Sigma_1))$~~ $M_2 \in |\mathsf{Mod}^I(\Sigma_2)|$ and $M_1' \in |\mathsf{Mod}^J(\Phi(\Sigma_1))|$ with $M_2|_\sigma = \beta_{\Sigma_1}(M_1')$, there is a unique (not necessarily unique) ~~$M_2' \in \mathsf{Mod}^J(\Phi(\Sigma_2))$~~ $M_2' \in |\mathsf{Mod}^J(\Phi(\Sigma_2))|$ with $\beta_{\Sigma_2}(M_2') = M_2$ and $M_2'|_{\Phi(\sigma)} = M_1'$;
- a *subinstitution comorphism* if Φ is an embedding, each α_Σ is injective and each β_Σ is bijective[4];
- an *inclusion comorphism* if Φ and each α_Σ are inclusions, and each β_Σ is the identity.

It is known that each subinstitution comorphism is model-expansive and each model-expansive comorphism is also faithful. Faithfulness means that a proof goal $\Gamma \models^I \varphi$ in I can be solved by a theorem prover for J by just feeding the theorem prover with $\alpha(\Gamma) \models^J \alpha(\varphi)$. Subinstitution comorphism preserve the semantics of more advanced DOL structuring constructs such as OMS translation and OMS reduction.

Definition 8 *Given an institution $I = (\mathsf{Sig}^I, \mathsf{Mod}^I, \mathsf{Sen}^I, \models^I)$, the institution of its theories~~can be defined~~, denoted I^{th} ~~,~~ can be defined as follows. The category of signatures of I^{th} is the category of I-theories and I-theory morphisms, that is denoted Th^I. For each theory (Σ, Δ), its sentences are just Σ-sentences in I, and its models are just Σ-models in I that satisfy the sentences in Δ, while the (Σ, Δ)-satisfaction is the Σ-satisfaction of sentences in models of I.* □

Using this notion, logic translations can be defined that include axiomatization of parts of the syntax of the source logic into the target logic.

Definition 9 *Let $I = (\mathsf{Sig}^I, \mathsf{Mod}^I, \mathsf{Sen}^I, \models^I)$ and $J = (\mathsf{Sig}^J, \mathsf{Mod}^J, \mathsf{Sen}^J, \models^J)$ be two institutions. ~~An~~ A* ***theoroidal institution comorphism*** *from I to J is a institution comorphism from I to J^{th}.* □

Institution morphisms capture the intuition of projecting from a more expressive logic to a less expressive one.

Definition 10 (Institution Morphism) *An* ***institution morphism*** *from an institution $I = (\mathsf{Sig}^I, \mathsf{Mod}^I, \mathsf{Sen}^I, \models^I)$ to an institution $J = (\mathsf{Sig}^J, \mathsf{Mod}^J, \mathsf{Sen}^J, \models^J)$ consists of a functor $\Phi : \mathsf{Sig}^I \longrightarrow \mathsf{Sig}^J$, and two natural transformations ~~$\beta : \mathsf{Mod}^I \Longrightarrow \mathsf{Mod}^J \circ \Phi$~~ $\beta : \mathsf{Mod}^I \Longrightarrow \mathsf{Mod}^J \circ \Phi^{op}$ and $\alpha : \mathsf{Sen}^J \circ \Phi \Longrightarrow \mathsf{Sen}^I$, such that for each I-signature Σ, each sentence $\varphi \in \mathsf{Sen}^J(\Phi(\Sigma))$ and each model $M \in \mathsf{Mod}^I(\Sigma)$*

$$M \models^I_\Sigma \alpha_\Sigma(\varphi) \Leftrightarrow \beta_\Sigma(M) \models^J_{\Phi(\Sigma)} \varphi.$$

holds, called the ***satisfaction condition****.* □

[4] An isomorphism if model morphisms are taken into account.

Colimits are a categorical concept providing means of combining objects interconnected by morphisms, where the colimit glues together objects along the morphisms. They can be employed for constructing larger theories from already available smaller ones, see [?].

~~A *network*[5] in a category C is a functor $D : G \to C$, where G is a small category[5], and can be thought of as the shape of the graph of interconnections between the objects of C selected by the functor D. A *cocone* of a network $D : G \to C$ consists of an object c of C and a family of morphisms $\alpha_i : D(i) \longrightarrow c$, for each object i of G, such that for each edge of the network, $e: i \longrightarrow i'$ it holds that $D(e); \alpha_{i'} = \alpha_i$. A *colimiting cocone* (or colimit) $(c, \{\alpha_i\}_{i \in |G|})$ can be intuitively understood as a minimal cocone, i.e.has the property that for any cocone $(d, \{\beta_i\}_{i \in |G|})$ there exists a unique morphism $\gamma : c \longrightarrow d$ such that $\alpha_i; \gamma = \beta_i$. By dropping the uniqueness condition and requiring only that a morphism γ should exist, a *weak* colimit is obtained.~~

~~When G is the category $\bullet \longleftarrow \bullet \longrightarrow \bullet$ with 3 objects and 2 non-identity arrows, G-colimits are called *pushouts*~~For a formal mathematical definition, see ??.

A major property of colimits of specifications is *amalgamation* (also related to 'exactness' [?]). It can be intuitively explained as stating that models of given specifications can be combined to yield a uniquely determined model of a colimit specification, provided that the original models coincide on common components. Amalgamation is a common technical assumption in the study of specification semantics [?].

In the ~~sequel~~following, fix an arbitrary institution $I = (\mathsf{Sig}, \mathsf{Sen}, \mathsf{Mod}, \models)$.

Definition 11 *Given a network* $D\colon J \longrightarrow \mathsf{Sig}^I$*, a family of models* $\mathcal{M} = \{M_p\}_{j \in |J|}$ *is* consistent with D *(or sometimes* compatible with D*) if for each node p of D, $M_p \in Mod(D(p))$ and for each edge $e : p \to q$, $M_p = M_q|_{D(e)}$. A* cocone $(\Sigma, (\mu_j)_{j \in |J|})$ *over the network* $D\colon J \longrightarrow \mathsf{Sig}^I$ *is called* weakly amalgamable *if it is mapped to a weak limit by* Mod*. For models, this means that for each D-compatible family of models $(M_j)_{j \in |J|}$, there is a Σ-model M, called an amalgamation of $(M_j)_{j \in |J|}$, with $M|_{\mu_j} = M_j$ $(j \in |J|)$, and similarly for model morphisms. If this model is unique, the cocone is called* amalgamable. I *(or* Mod*) admits* (finite) (weak) amalgamation *if (finite) colimit cocones are (weakly) amalgamable. Finally,* I *is called* (weakly) semi-amalgamable *if it has pushouts and admits (weak) amalgamation for these.* □

[?] studies conditions for existence of weakly amalgamable cocones in a heterogeneous setting, where the network consists of signatures (or theories) in different logics. Since a network may admit more than one weakly amalgamable cocone, a selection operation is required both for the weakly amalgamable cocone of a network and for the (potentially non-unique) amalgamation of a family of models compatible with the network. This allows us to define a function *colimit* taking as argument a network of heterogeneous signatures and returning the selected weakly amalgamable cocone for the network and a function $\oplus$ taking as argument a family of models compatible with a network and returning its selected amalgamation.

~~To be able to talk about the symbols of a signature in a formal way, it is required that the category of signatures of an institution is an inclusive category with symbols, as defined below: An *inclusive category with symbols* is an inclusive category $\mathbb{C}$ equipped with a faithful functor $|\_| : \mathbb{C} \to \mathbb{S}et$[5] that preserves inclusions.~~

10.2 Semantics of DOL Language Constructs

The semantics of DOL is based on a fixed (but in principle arbitrary) heterogeneous logical environment. The semantic domains are based on this heterogeneous logical environment. A specific heterogeneous logical environment is given in the annexes.

A heterogeneous logical environment is given by a collection of OMS languages and OMS language translations[5], a collection of institutions, institution morphisms and institution comorphisms (serving as logics, logic reductions and logic translations), and a collection of serializations. Moreover, some of the institution comorphisms are marked as default translations (but only at most one between a given source and target institution), and there is a binary ~~supports relation~~ relation *supports* between OMS languages and institutions, and a binary ~~supports relation~~ relation *supports* between OMS languages and serializations. Each language is required to have a default logic and serialization. Moreover, we assume that institutions, institution morphisms and institution comorphisms are uniquely identified by names, and we use the notation $\Gamma(n)$ for the institution, institution morphism and institution comorphism identified by the name n int the heterogeneous logical environment Γ.

~~It is required that for each institution in the heterogeneous logical environment there is a trivial signature $\emptyset$ with model class $\mathcal{M}_\emptyset$ and such that there exists a unique signature morphism from $\emptyset$ to any signature of the institution . Moreover, the existence of a partial union operation on logics is required; denoted $\bigcup$: $L_1 \bigcup L_2 = (L, \rho_1 : L_1 \to L, \rho_2 : L_2 \to L)$, when defined. Finally, some of the comorphisms are marked as default translations and some of the morphisms as default projection, with the condition that between two institutions at most one comorphism and at most one morphism is marked as default.~~

[5] ~~A network is called a diagram in category theory texts. This terminology is introduced to disambiguate OMS networks from UML diagrams.~~

[5] ~~That is, it has a set of objects and sets of morphisms between them instead of classes.~~

[5] ~~That is, $(\mathbb{C}, |\_|)$ is a concrete category.~~

[5] The terms *OMS language* and *serialization* are not defined formally. For this semantics, it suffices to know that there is a language-specific semantics of basic OMS as defined below.

~~For each logic in the heterogeneous logical environment, it is further required that there is: a function giving the semantics of basic OMS. It has format~~

$$(\Sigma', \Delta') = semBasic_{(lang,logic,ser)}(\Sigma, \texttt{BasicOMS}),$$

~~where Σ is a signature giving the local environment, $\Sigma' \supseteq \Sigma$ is the resulting signature, and Δ' the resulting set of sentences, a function that turns a symbol map into a signature morphism , a relativization procedure taking as argument a theory and giving as result a theory, and three procedures for translating correspondences of alignments into sentences in the logic, as needed in Section ??.~~

~~Further, for each institution, it is required that there exist (possibly partial)~~ We are going to require existence of union and difference operations on ~~signatures~~the signatures of an institution in the heterogeneous logical environment. These concepts could be captured in a categorical setting using *inclusion systems* [?]. However, inclusion systems are too strong for the purposes of this specification. Therefore, weaker assumptions will be used.

Definition 12 *An* inclusive category *[?] is a category having a broad subcategory*[6] *which is a partially ordered class with a least element (denoted $\emptyset$), finite products and coproducts, called intersection (denoted $\cap$) and union (denoted $\cup$) such that for each pair of objects A, B, $A \cup B$ is a pushout of $A \cap B$ in the category.* □

A category *has pushouts which preserve inclusions* iff there exists a pushout

$$\begin{tikzcd} A \arrow[r, hook] \arrow[d] & A' \arrow[d] \\ B \arrow[r, hook] & B' \end{tikzcd}$$

for each span where one arrow is an inclusion.

A functor between two inclusive categories is inclusive if it takes inclusions in the source category to inclusions in the target category.

Definition 13 *An institution is* weakly inclusive *if*

- Sig *is inclusive and has pushouts which preserve inclusions,*
- Sen *is inclusive, and*
- *each model category ~~have~~ has a broad subcategory of inclusions.*

□

Let I be a weakly inclusive institution. I *has differences*, if there is a binary operation $\setminus$ on signatures, such that for each pair of signatures Σ_1, Σ_2, the greatest signature Σ such that

1. $\Sigma \subseteq \Sigma_1$
2. $\Sigma \cap \Sigma_2 = \emptyset$

exists and is equal to $\Sigma_1 \setminus \Sigma_2$.

~~This concludes the definition of~~ We will write $\iota_{A \subseteq B}$ for the inclusion of A in B in an inclusive category, when such an inclusion exists. If $\mathcal{I}$ is an inclusive institution and $\Sigma \subseteq \Sigma'$ is an inclusion of signatures, we write $M'|_{\Sigma}$ for the reduct of a Σ'-model M' along the inclusion $\iota_{\Sigma \subseteq \Sigma'}$.

To be able to talk about the symbols of a signature in a formal way, it is required that the category of signatures of an institution is an inclusive category with symbols, as defined below:

Definition 14 An inclusive category with symbols *is an inclusive category $\mathbb{C}$ equipped with a faithful functor $|\_| : \mathbb{C} \to \mathbb{S}et$*[7] *that preserves inclusions.*

After these preliminaries, we can now list the assumptions made about the institutions in a heterogeneous logical environment. It is required that for each institution in the heterogeneous logical environment there is a trivial signature $\emptyset$ with model class $\mathcal{M}_\emptyset$ and such that there exists a unique signature morphism from $\emptyset$ to any signature of the institution. Moreover, the existence of a partial union operation on institutions is required, denoted $\bigcup$: $L_1 \bigcup L_2 = (L, \rho_1 : L_1 \to L, \rho_2 : L_2 \to L)$, when defined, where L is an institution and ρ_1 and ~~the assumptions made about it~~ ρ_2 are institution comorphisms, giving the embedding of L_1 and respectively L_2 in L. Finally, some of the comorphisms are marked as default translations and some of the morphisms as default projections, with the condition that between any two institutions at most one comorphism and at most one morphism is marked as default.

For each institution $\mathcal{I}$ in the heterogeneous logical environment, it is further required that there is:

[6] That is, with the same objects as the original category.
[7] That is, $(\mathbb{C}, |\_|)$ is a concrete category.

- a function giving the semantics of a basic OMS. It has the format

$$semBasic_{(lang,logic,ser)}(\Sigma, O) = (\Sigma', \Delta')$$

where O is a `BasicOMS`, Σ gives the context of previous declarations, Σ' is the resulting signature and Δ' is the resulting set of sentences. It is required then that $\Sigma \subseteq \Sigma'$.

- a function $makeMorphism_{\mathcal{I}}$ that turns symbol maps into signature morphisms,
- a function $matchSymbols_{\mathcal{I}}$ that takes as arguments two signatures Σ_1 and Σ_2 of $\mathcal{I}$ and returns as result the list of all pairs of symbols (s_i^1, s_i^2) such that $s_i^1 \in |\Sigma_1|$ and $s_i^2 \in |\Sigma_2|$ and the symbols have the same name [6]. Note(6)
- a relativization function $relativize_{\mathcal{I}}$ taking as argument a theory and giving as result a theory, and a function $theoryOfCorrespondences$ for translating correspondences of alignments into sentences in the logic according to a given assumption about the semantics of the alignment, both needed in Section ??.

Further, for each institution, it is required that there exist (possibly partial) union [7] and difference operations on signatures. Note(7)

DOL follows a model-theoretic approach on semantics: the semantics of OMS will be defined as a class of models over some signature of an institution. This is called *model-level* semantics. In some cases, but not in all, one can also define a *theory-level* semantics of an OMS as a set of sentences over some signature of an institution. The two semantics are related by the fact that, when both the model-level and the theory-level semantics of an OMS are defined, they are compatible in the sense that the class of models given by the model-level semantics is exactly the model class of the theory given by the theory-level semantics.

The following unifying notation is used for the two semantics of an OMS O:

- the institution of O is denoted $\mathbf{Inst}(O)$,
- the signature of O is denoted $\mathsf{Sig}(O)$ (which is a signature in $\mathbf{Inst}(O)$),
- the class of models of O is denoted $\mathsf{Mod}(O)$ (which is a class of models over $\mathsf{Sig}(O)$),
- the set of axioms of O is denoted $\mathsf{Th}(O)$ (which is a set of sentences over $\mathsf{Sig}(O)$).

Moreover, the semantics of O is the tuple $sem(O) = (I, \Sigma, \mathcal{M}, \Delta)$ where $\mathbf{Inst}(O) = I$, $\mathsf{Sig}(O) = \Sigma$, $\mathsf{Mod}(O) = \mathcal{M}$ and $\mathsf{Th}(O) = \Delta$. In the following, we will freely mix these two equivalent descriptions of the semantics. That is, whenever $sem(O)$ is determined in some the context, then also its components $\mathbf{Inst}(O)$, $\mathsf{Sig}(O)$, $\mathsf{Mod}(O)$ and $\mathsf{Th}(O)$ are determined. Vice versa, if the four components are determined, then so is $sem(O)$.

The theory-level semantics of O can be undefined, and then so is $\mathsf{Th}(O)$. When $\mathsf{Th}(O)$ is defined, $\mathsf{Mod}(O)$ can be obtained as $\mathsf{Mod}(O) = \{M \in \mathsf{Mod}(\mathsf{Sig}(O)) \mid M \models \mathsf{Th}(O)\}$.

Intuitively, OMS mappings denote various types of links between two or more OMS. The semantics of OMS mappings can be captured uniformly as a graph whose nodes N are labeled with

- $\mathbf{Name}(N)$, the name of the node
- $\mathbf{Inst}(N)$, the institution of the node
- $\mathsf{Sig}(N)$, the signature of the node
- $\mathsf{Mod}(N)$, the class of $\mathsf{Sig}(N)$-models of the node
- $\mathsf{Th}(N)$, the set of $\mathsf{Sig}(N)$-sentences of the node

and which has two kinds of edges:

- import links (written using single arrows, $S \to T$)
- theorem links (written using double arrows, $S \Rightarrow T$)

both labeled with heterogeneous signature morphisms between the signatures of the source and target nodes. The theory of a node may be undefined, as in the case of OMS, and when it is defined, the class of models of that node is the class of models of $\mathsf{Th}(N)$. For brevity, the label of a node may be written as a tuple. Further, it is required that any OMS can be assigned a unique name.

The semantics of a network of OMS is a graph whose nodes are labeled like in the semantics of OMS mappings and edges are labeled with heterogeneous signature morphisms (i.e. an edge from the node S to the node T is labeled with a pair (ρ, σ) where $\rho = (\Phi, \alpha, \beta) : \mathbf{Inst}(S) \to \mathbf{Inst}(T)$ is an institution comorphism and $\sigma : \Phi(\mathsf{Sig}(S)) \to \mathsf{Sig}(T)$ is a signature morphism in $\mathbf{Inst}(T)$). The intuition is that network provide means of putting together graphs of OMS and OMS mappings and of removing sub-graphs of existing networks.

The semantics of OMS generally depends on a global environment Γ containing:

[6] NOTE: @Till: this is not formal... TM: yes, it is formal. Maybe rename it to $sameName$ in order to stress that sameness of names is formalized here in some institution-specific way? It would make sense to require that this is an equivalence relation.

[7] NOTE: If the institutions are inclusive, should the union not always be defined? TM: yes, it should.

- a graph of imports between OMS, as in the semantics of OMS mappings but only with import links between nodes, denoted $\Gamma.imports$
- a mapping from `IRI`s to semantics of OMS, OMS mappings, and OMS networks, that is also denoted by Γ, providing access to previous definitions,
- a prefix map, denoted $\Gamma.prefix$, that stores the declared prefixes,
- a triple $\Gamma.current$ that stores the current language, logic and serialization.

If Γ is such a global environment, $\Gamma[\mathtt{IRI} \mapsto \mathcal{S}]$ extends the domain of Γ with `IRI` and the newly added value of ~~Γ in~~ `IRI` in Γ is the semantic entity $\mathcal{S}$. $\Gamma_\emptyset$ is the empty global environment, i.e. the domain of $\Gamma_\emptyset$ is the empty set, its import graph $\Gamma.imports$ is empty, the prefix map is empty and the current triple contains the error logic together with its language and serialization. The union of two global environments Γ_1 and Γ_2, denoted $\Gamma_1 \cup \Gamma_2$, is defined only if the domains of Γ_1 and Γ_2, and of $\Gamma_1.prefix$ and $\Gamma_2.prefix$ are disjoint, and then $\Gamma_1 \cup \Gamma_2(\mathtt{IRI}) = \begin{cases} \Gamma_1(\mathtt{IRI}) & \text{if } \mathtt{IRI} \in dom(\Gamma_1) \\ \Gamma_2(\mathtt{IRI}) & \text{if } \mathtt{IRI} \in dom(\Gamma_2) \end{cases}$, $\Gamma_1 \cup \Gamma_2.imports = \Gamma_1.imports \cup \Gamma_2.imports$, $\Gamma_1 \cup \Gamma_2.current = \Gamma_1.current$ and $\Gamma_1 \cup \Gamma_2.prefix = \Gamma_1.prefix \cup \Gamma_2.prefix$. $\Gamma.\{prefix = PMap\}$ represents the global environment that sets the prefix map of Γ to $PMap$ and $\Gamma.\{current = (lang, logic, ser)\}$ is used for updating the current triple of Γ to $(lang, logic, ser)$.

DOL assumes a *language-specific semantics* of native structured OMS, inherited from the OMS language. For a native ~~structured OMS O~~ document D in a language L, logic L' and serialization S, $\cancel{sem_{(L,L',S)}(O)}$ $semNative_{(L,L',S)}(D)$ denotes the language-specific semantics of ~~O. Further, assumes similar language-specific semantics of a basic OMS fragment O in the context of previous declarations, which is denoted $sem^{(\mathcal{I},\Sigma,\mathcal{M},\Delta)}_{(L,L',S)}(O)$.~~ D.

10.2.1 Semantics of Documents

In this section the semantics of DOL constructs regarding documents and DOL libraries is defined.

$$\boxed{\begin{array}{ll} sem(\mathtt{Document}) & = \Gamma \\ & : LogicalEnvironment \end{array}}$$

A document is either a DOL library, or a native document written in one of the languages supported by the heterogeneous logical environment.

For a `NativeDocument` $nativeDocument$,

$$sem(nativeDocument) = \Gamma''$$

where $\cancel{\Gamma' = \Gamma_\emptyset.\{current = L\}}$, ~~with~~ $\cancel{L}$ $\Gamma' = \Gamma_\emptyset.\{current = (lang, logic, ser)\}$, with $lang, logic, ser$ determined from the extension of the file containing the native document~~and~~ $\cancel{\Gamma'' = \Gamma[\mathtt{IRI} \mapsto sem_{(\Gamma'.lang,\Gamma'.logic,\Gamma'.ser)}(nativeDocument)]}$.

$postfixLogicIRI(o, l)$ is the string $o?logic = l$,
$l_1, \ldots, l_n$ are the logics supported by $lang$ for some natural number n,
$\Gamma_1 = \Gamma'[postfixLogicIRI(IRI, l_1) \mapsto semNative_{(lang, l_1, ser)}(nativeDocument)]$,
$\Gamma_2 = \Gamma_1[postfixLogicIRI(IRI, l_2) \mapsto semNative_{(lang, l_2, ser)}(nativeDocument)], \ldots$
$\Gamma'' = \Gamma_{n-1}[postfixLogicIRI(IRI, l_n) \mapsto semNative_{(lang, l_n, ser)}(nativeDocument)]$.
[8] Note that if the OMS in the ~~library~~ native document does not conform with the logic determined by the extension of the ~~library~~file where the document is stored, $sem(nativeDocument)$ will be undefined. Note(8)

The rule for `DOLLibrary` is given below.

10.2.1.1 Semantics of libraries

$$\boxed{\begin{array}{ll} sem(\mathtt{DOLLibrary}) & = \Gamma \\ & : LogicalEnvironment \end{array}}$$

A DOL library is list of definitions of OMS, OMS mappings and OMS networks, starting with an optional prefix map and a qualification.

For a `DOLLibrary` $dolLibrary$,

$$sem(dolLibrary) = \Gamma'$$

where
$sem(dolLibrary.\mathtt{prefixMap}) = PMap$,
$\Gamma_1 = \Gamma_\emptyset.\{prefix = PMap\}$,

[8] Note: TM: aha, here we need to consider all logics l supported by $lang$ and store the respective semantics under $postfixLogicIRI(IRI, l)$. MC: done

$sem(\Gamma_1, dolLibrary.\mathtt{qualification}) = \Gamma_2$,
$sem(\Gamma_2, dolLibrary.\mathtt{libraryItem}) = \Gamma'$.

Note that $dolLibrary.$`libraryName` is just discarded here. However, this name should be the IRI of the document containing the `Document`. This is known as "linked data compliance". Tools can issue a warning (not an error), if a `Document` does not follow this practice.

10.2.1.2 Semantics of lists of library items

$$\boxed{sem(\Gamma, Sequence(\mathtt{LibraryItem})) = \Gamma' : LogicalEnvironment}$$

If $libItem_1, \ldots, libItem_n$ are all `LibraryItems`,

$$sem(\Gamma, Sequence\{libItem_1, \ldots, libItem_n\}) = \Gamma'$$

where
$sem(\Gamma, libItem_1) = \Gamma_1$,
$sem(\Gamma_1, libItem_2) = \Gamma_2$, ...
$sem(\Gamma_{n-1}, libItem_n) = \Gamma'$.

10.2.1.3 Semantics of library items

$$\boxed{sem(\Gamma, \mathtt{LibraryItem}) = \Gamma' : LogicalEnvironment}$$

For a `LibraryImport` ~~*libImport*,~~ *libImport*,

$$sem(\Gamma, \cancel{libImport}libImport) = \Gamma \cup \Gamma'$$

where $sem(\Gamma, libImport.\mathtt{libraryName}) = anIRI$ and $sem(anIRI) = \Gamma'$.

A `LibraryItem` can also be an `OMSDefinition`, `NetworkDefinition` or `MappingDefinition`, and equations for these are given in the next sections. (Annex ?? also introduces `QueryRelatedDefinition`.)

10.2.1.4 Semantics of a list of qualifications

$$\boxed{sem(\Gamma, Sequence(\mathtt{Qualification})) = \Gamma' : LogicalEnvironment}$$

If $q_1, \ldots, q_n$ are all `Qualifications`,

$$sem(\Gamma, Sequence(q_1, \ldots, q_n)) = \Gamma'$$

where $sem(\Gamma, q_1) = \Gamma_1$, $sem(\Gamma_1, q_2) = \Gamma_2$, ..., $sem(\Gamma_{n-1}, q_n) = \Gamma'$.

10.2.1.5 Semantics of qualifications

$$\boxed{sem(\Gamma, \mathtt{Qualification}) = \Gamma' : LogicalEnvironment}$$

For a `LanguageQualification` q,

$$sem(\Gamma, q) = \Gamma'$$

where $\Gamma' = \Gamma.\{current = (q.\mathtt{languageRef}, logic', ser')\}$ and

$$logic' = \begin{cases} logic(\Gamma.current), & \text{if } q.\mathtt{languageRef} \text{ supports } logic(\Gamma.current) \\ \text{default logic for } q.\mathtt{languageRef}, & \text{otherwise} \end{cases}$$

$$ser' = \begin{cases} ser(\Gamma.current), & \text{if } q.\mathtt{languageRef} \text{ supports } ser(\Gamma.current) \\ \text{default serialization for } q.\mathtt{languageRef}, & \text{otherwise} \end{cases}$$

For a `LogicQualification` q,

$$sem(\Gamma, q) = \Gamma'$$

where $\Gamma' = \Gamma.\{current = (lang', q.\mathtt{logicRef}, ser')\}$
$lang = lang(\Gamma.current)$, $ser = ser(\Gamma.current)$

$$lang' = \begin{cases} lang, & \text{if } lang \text{ supports } q.\texttt{logicRef} \\ \text{the unique language supporting } q.\texttt{logicRef}, & \text{otherwise} \end{cases}$$

$$ser' = \begin{cases} ser, & \text{if } lang' \text{ supports } ser \\ \text{the default serialization for } lang', & \text{otherwise} \end{cases}$$

Note that "the unique language supporting q.`logicRef`" may be undefined; in this case, the semantics of q construct is undefined.

For a `SyntaxQualification` q,

$$sem(\Gamma, q) = \Gamma'$$

where $lang = lang(\Gamma.current), logic = logic(\Gamma.current)$ and
$\Gamma' = \Gamma.\{current = (lang, logic, q.\texttt{syntaxRef})\}$. The semantics is defined only if $lang$ supports q.`syntaxRef`.

10.2.2 Semantics of Networks

The semantics of networks of OMS is given with the help of a directed graph. Its nodes and edges are specified by the `NetworkElements`, which can be OMS, OMS mappings, or OMS networks. Intuitively, the graph of a network consists of the union of all graphs of the network elements it contains, where an OMS yields a graph with one isolated node. By convention, all imports in the graph $\Gamma.imports$ of the current context between nodes that are specified in the list of `NetworkElements` are also included in the graph of the network. The nodes and edges given in the `ExcludeExtensions` list are then removed from the graph of the network.

An additional `Id` can be specified for each node, with the purpose of letting the user specify a prefix in the colimit of a network for the symbols with the origin in that node that must be disambiguated.

The following auxiliary functions are used:

- $insert(G, \Gamma, iri, id)$, where G is a graph, Γ is a global environment, iri is an `IRI` and id is an `Id`, defined as follows:
  - if iri denotes an OMS in Γ, then a new node named iri and labeled with $\Gamma(iri)$ and with id is added to G, unless a node named iri already exists in G, and in this case G is left unchanged,
  - if iri denotes an OMS mapping or a network in Γ, the result is the union of G with the graph of $\Gamma(iri)$.
- $removeElement(\Gamma, G, anIRI)$, where G is a graph, Γ is a global environment and $anIRI$ is an `IRI`, defined as follows:
  - if $anIRI$ denotes an OMS in Γ, then the node labeled with $anIRI$ and all its incoming and outgoing edges are removed from G,
  - if $anIRI$ denotes an OMS mapping in Γ, then $\Gamma(anIRI)$ gives a graph G' and two nodes N_1 and N_2. Then all nodes of G' other than N_1 and N_2 and all the edges of G' are removed from G.
  - if $anIRI$ is a network in Γ, then all the nodes of its graph and all their incoming and outgoing edges are removed from G.
- $removePaths(\Gamma, G, iri_1, iri_2)$, where G is a graph, Γ is a global environment and iri_1, iri_2 are `IRI`s, whose result is that all paths of imports in G between the nodes labeled with iri_1 and iri_2 are removed from G.

Finally, the operation $addImports(\Gamma, G, [iri_1, \ldots, iri_n])$ adds to G all import edges in $\Gamma.imports$ between nodes which appear in the subgraph determined by $\Gamma(iri_1), \ldots, \Gamma(iri_n)$.

10.2.2.1 Semantics of network definitions

$$\boxed{\begin{array}{ll} sem(\Gamma, \texttt{NetworkDefinition}) & = \Gamma' \\ & : LogicalEnvironment \end{array}}$$

If n is a `NetworkDefinition`,

$$sem(\Gamma, n) = \Gamma'$$

where $\Gamma' = \Gamma[n.\texttt{networkName} \mapsto sem(\Gamma, n.\texttt{network})]$.

If n.`ConservativityStrength` is `model-conservative`, the semantics is only defined if ~~$sem(\Gamma, n.\texttt{network}) \neq \emptyset$~~the class of families of models compatible with the graph $sem(\Gamma, n.\texttt{network})$ is not empty.

If n.`ConservativityStrength` is `consequence-conservative`, the semantics is defined only if all signature-free sentences that follow from the network, see entailment of OMS by networks, are tautologies.

If n.`ConservativityStrength` is `monomorphic`, the semantics is only defined if the class of families of models compatible with the graph $sem(\Gamma, n.\texttt{network})$ consist of exactly one isomorphism class of families of models.

If n.`ConservativityStrength` is `weak-definitional`, the semantics is only defined if the class of families of models compatible with the graph $sem(\Gamma, n.\texttt{network})$ is at most a singleton.

If n.`ConservativityStrength` is `definitional`, the semantics is only defined if the class of families of models compatible with the graph $sem(\Gamma, n.\texttt{network})$ is a singleton.

If n.`ConservativityStrength` is `not-model-conservative`, the semantics is only defined if ~~$sem(\Gamma, n.\texttt{network}) = \emptyset$~~the class of families of models compatible with the graph $sem(\Gamma, n.\texttt{network})$ is the empty set.

If n.`ConservativityStrength` is `not-consequence-conservative`, the semantics is defined only if not all signature-free sentences that follow from the network, see entailment of OMS by networks, are tautologies.

10.2.2.2 Semantics of networks

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{Network}) \quad &= G \\ &: OMSGraph \end{aligned}}$$

If n is a network,

$$sem(\Gamma, n) = G'$$

where $sem(\Gamma, n.\texttt{networkElement}) = G$ and $sem(\Gamma, G, n.\texttt{excludedElement}) = G'$.

10.2.2.3 Semantics of sets of network elements

$$\boxed{\begin{aligned} sem(\Gamma, Set(\texttt{NetworkElement})) \quad &= G \\ &: OMSGraph \end{aligned}}$$

If $elem_1, \ldots, elem_n$ are all `NetworkElements`,

$$sem(\Gamma, Set\{elem_1, \ldots, elem_n\}) = G'$$

where
$G_1 = sem(\Gamma, G_\emptyset, elem_1)$, where $G_\emptyset$ is the empty graph,
$G_2 = sem(\Gamma, G_1, elem_2)$
...
$G_n = sem(\Gamma, G_{n-1}, elem_n)$,
~~$G' = addImports(\Gamma, G_n, [elem_1, \ldots, elem_n])$~~$G = addImports(\Gamma, G_n, [elem_1, \ldots, elem_n])$.

10.2.2.4 Semantics of network elements

$$\boxed{\begin{aligned} sem(\Gamma, G, \texttt{NetworkElement}) \quad &= G' \\ &: OMSGraph \end{aligned}}$$

If $networkElement$ is a `NetworkElement`,

$$sem(\Gamma, G, networkElement) = insert(G, \Gamma, networkElement.\texttt{elementRef.iri}, networkElement.\texttt{id})$$

10.2.2.5 Semantics of sets of excluded elements

$$\boxed{\begin{aligned} sem(\Gamma, G, Set(\texttt{ExcludedElement})) \quad &= G' \\ &: OMSGraph \end{aligned}}$$

If $elem_1, \ldots, elem_n$ are all `ExcludedElements`,

$$sem(\Gamma, G, Set\{elem_1, \ldots, elem_n\}) = G'$$

where
$G_1 = sem(\Gamma, G, elem_1)$
$G_2 = sem(\Gamma, G_1, elem_2)$
...
$G' = sem(\Gamma, G_{n-1}, elem_n)$

10.2.2.6 Semantics of excluded elements

$$\boxed{\begin{aligned} sem(\Gamma, G, \texttt{ExcludedElement}) \quad &= G' \\ &: OMSGraph \end{aligned}}$$

If $excludedElem$ is a `ElementRef`,

$$sem(\Gamma, G, excludedElem) = removeElement(\Gamma, G, excludedElem.\texttt{iri})$$

.

If $excludedElem$ is a `PathReference`,

$$sem(\Gamma, G, excludedElem) = removePaths(\Gamma, G, iri_1, iri_2)$$

where $iri_1 = excludedElem.\texttt{elementRef.iri}$ and $iri_2 = excludedElem.\texttt{elementRef2.iri}$).

10.2.3 Semantics of OMS

In the rest of this section, given a global environment Γ and an OMS O, the notation $Env(\Gamma, O)$ is used for the global environment Γ' such that $sem(\Gamma, O) = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$.

10.2.3.1 Semantics of basic OMS

$$\boxed{\begin{array}{ll} sem(\Gamma, \texttt{BasicOMS}) & = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) \\ & : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences)) \end{array}}$$

For a BasicOMS ~~$basicOMS$~~ Q in a global environment Γ, the semantics is defined as follows:

$$sem(\Gamma, \cancel{basicOMS}Q) = (\Gamma', (\Gamma.logic, \Sigma', \mathcal{M}', \Delta'))$$

where

- $\cancel{(\Sigma', \Delta') = semBasic_{(\Gamma.lang, \Gamma.logic, \Gamma.ser)}(\emptyset, basicOMS)}$ $(\Sigma', \Delta') = semBasic_{(\Gamma.lang, \Gamma.logic, \Gamma.ser)}(O)$,
- $\mathcal{M}' = \{M \in \mathsf{Mod}(\Sigma') \mid M \models \Delta'\}$
- Γ' is obtained from Γ by adding to $\Gamma.imports$ a new node labeled with the name of ~~$basicOMS$~~Q, $\Gamma.logic$,Σ', $\mathcal{M}'$ and Δ'.

10.2.3.2 Semantics of basic OMS in a local environment

$$\boxed{\begin{array}{ll} sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), \texttt{BasicOMS}) & = (\Gamma', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta')) \\ & : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences)) \end{array}}$$

For a BasicOMS ~~$basicOMS$~~ Q in a global environment Γ and local environment $(\mathcal{I}, \Sigma, \mathcal{M}, \Delta)$, its semantics is defined only if $\Gamma.logic = \mathcal{I}$ as follows:

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), \cancel{basicOMS}Q) = (\Gamma', (\Gamma.logic, \Sigma', \mathcal{M}', \Delta'))$$

where

- $\cancel{(\Sigma', \Delta') = semBasic_{(\Gamma.lang, \Gamma.logic, \Gamma.ser)}(\Sigma, basicOMS)}$ $(\Sigma', \Delta') = semBasic_{(\Gamma.lang, \Gamma.logic, \Gamma.ser)}(\Sigma, O)$
- $\mathcal{M}' = \{M \in \mathcal{M} \mid M \models \Delta'\}$
- Γ' is obtained from Γ by adding to $\Gamma.imports$ a new node labeled with the name of ~~$basicOMS$ and the other components as given by $sem_{(\Gamma.lang, \Gamma.logic, \Gamma.ser)}(basicOMS)$~~$O$, $\Gamma.logic$,Σ', $\mathcal{M}'$ and Δ'.

10.2.3.3 Semantics of closable OMS

~~In the rest of this section, given a global environment Γ and an oms, the notation $Env(\Gamma, oms)$ is used for the global environment Γ' such that $sem(\Gamma, oms) = \Gamma'$.~~

$$\boxed{\begin{array}{ll} sem(\Gamma, \texttt{ClosableOMS}) & = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) \\ & : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences)) \end{array}}$$

The semantics of a BasicOMS has been defined above.
The semantics of an OMSReference ~~o~~ O is given by

$$sem(\Gamma, O) = (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)$$

where $locId = postfixLogicIRI(O.\texttt{omsRef}, name(\Gamma.logic))$ and

- ~~$\mathbf{Inst}(o) = \mathbf{Inst}(\Gamma(o.\texttt{omsRef}))$~~ $postfixLogicIRI(o, l)$ is the string $o?logic = l$
- ~~$\mathsf{Sig}(o) = \mathsf{Sig}(\Gamma(o.\texttt{omsRef}))$~~ $\mathcal{I} = \mathbf{Inst}(\Gamma(locId))$,
- ~~$\mathsf{Mod}(o) = \mathsf{Mod}(\Gamma(o.\texttt{omsRef}))$~~ $\Sigma = \mathsf{Sig}(\Gamma(locId))$
- ~~$\mathsf{Th}(o) = \mathsf{Th}(\Gamma(o.\texttt{omsRef}))$~~ $\mathcal{M} = \mathsf{Mod}(\Gamma(locId))$
- ~~$Env(\Gamma, o)$~~ $\Delta = \mathsf{Th}(\Gamma(locId))$
- $Env(\Gamma, O)$ extends the graph of imports $\Gamma.imports$ with a new node for ~~o labeled~~ O whose name is either $O.importName$, or, if $O.importName$ is missing, $locId$, and whose other components of the label are as defined in the items above, and with a new edge from the node labeled with ~~$o.\texttt{omsRef}$ to o, named $o.\texttt{importName}$~~ $locId$ to O, named $O.\texttt{importName}$ and labeled with the identity on ~~$\mathsf{Sig}(\Gamma(o.\texttt{omsRef}))$~~$\mathsf{Sig}(\Gamma(locId))$.

10.2.3.4 Semantics of closable OMS in a local environment

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), \texttt{ClosableOMS}) = (\Gamma', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta')) : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences))$$

The semantics of a `BasicOMS` has been defined above.

The semantics of an `OMSReference` ~~o~~ O is defined only if ~~$\mathbf{Inst}(\Gamma(o.\texttt{omsRef})) = \mathcal{I}$~~ $\mathbf{Inst}(\Gamma(locId)) = \mathcal{I}$, where $locId = postfixLogicIR$ as follows:

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), O) = (\Gamma', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta'))$$

where

- ~~$\mathbf{Inst}(o) = \mathbf{Inst}(\Gamma(o.\texttt{omsRef}))$~~ $\mathcal{I}' = \mathbf{Inst}(\Gamma(locId))$
- ~~$\mathsf{Sig}(o) = \mathsf{Sig}(\Gamma(o.\texttt{omsRef})) \cup \Sigma$~~ $\Sigma' = \mathsf{Sig}(\Gamma(locId)) \cup \Sigma$
- ~~$\mathsf{Mod}(o) = \{M \in \mathsf{Mod}(\mathsf{Sig}(o)) \mid M|_\Sigma \in \mathcal{M} \text{ and } M|_{\mathsf{Sig}}(\Gamma(o.\texttt{omsRef})) \in \mathsf{Mod}(\Gamma(o.\texttt{omsRef}))\}$~~ $\mathcal{M}' = \{M \in \mathsf{Mod}(\Sigma') \mid M|_\Sigma \in \mathcal{M}$ and
- ~~$\mathsf{Th}(o) = \iota_{\mathsf{Sig}(\Gamma(o.\texttt{omsRef})) \subseteq \mathsf{Sig}(o)}(\mathsf{Th}(\Gamma(o.\texttt{omsRef}))) \cup \iota_{\Sigma \subseteq \mathsf{Sig}(o)}(\Delta)$~~ $\Delta' = \iota_{\mathsf{Sig}(\Gamma(locId)) \subseteq \Sigma'}(\mathsf{Th}(\Gamma(locId))) \cup \iota_{\Sigma \subseteq \Sigma'}(\Delta)$
- ~~$Env(\Gamma, o)$~~ $Env(\Gamma, O)$ extends the graph of imports $\Gamma.imports$ with a new node for ~~o~~ O labeled as defined in the items above and with a new edge from the node labeled with ~~o.`omsRef` to o, named o.`importName`~~ $locId$ to O, named O.`importName` and labeled with the inclusion of Σ in ~~$\mathsf{Sig}(\Gamma(o.\texttt{omsRef}))$~~ Σ'.

10.2.3.5 Semantics of ExtendingOMS

$$sem(\Gamma, \texttt{ExtendingOMS}) = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences))$$

The semantics for `ClosableOMS` has been defined above.

If O is a `RelativeClosureOMS`, O.`closureType` = `minimize` and $O' = O$.`closableOMS`, then

$$sem(\Gamma, O) = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$$

where

- $\mathcal{I} = \mathbf{Inst}(O')$
- $\Sigma = \mathsf{Sig}(O')$
- $\mathcal{M} = \{M \in \mathsf{Mod}(O') \mid M \text{ is minimal in } \mathsf{Mod}(O')\}$ and "minimal" is interpreted in the pre-order defined by $M_1 \leq M_2$ if there is a model homomorphism $\mathcal{M}_1 \to \mathcal{M}_2$.
- $\Delta = \perp$
- Γ' is obtained from $\Gamma'' = Env(\Gamma, O')$ by adding to $\Gamma''.imports$ a new node labeled with $(Name(O), \mathbf{Inst}(O), \mathsf{Sig}(O), \mathsf{Mod}(O), \mathsf{Th}(O$ and an edge from the node of O' to the node of O labeled with the identity morphism on $\mathsf{Sig}(O')$.

The semantics of O is defined similarly for the other three alternatives of O.`closureType`, only the model class differs:

- if O.`closureType` = `maximize`, $\mathcal{M} = \{M \in \mathsf{Mod}(O') \mid M \text{ is maximal in } \mathsf{Mod}(O')\}$
- if O.`closureType` = `free`, $\mathcal{M} = \{M \in \mathsf{Mod}(O') \mid M \text{ is initial in } \mathsf{Mod}(O')\}$
- if O.`closureType` = `cofree`, $\mathcal{M} = \{M \in \mathsf{Mod}(O') \mid M \text{ is terminal in } \mathsf{Mod}(O')\}$

Here, initial and terminal models are defined as in category theory, see **??**.

10.2.3.6 Semantics of ExtendingOMS in a local environment

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), \texttt{ExtendingOMS})) = (\Gamma', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta')) : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences))$$

The semantics for `ClosableOMS` has been defined above.

The semantics for minimization selects the models that are minimal in the class of all models with the same interpretation for the local environment (= fixed non-logical symbols, in the terminology of circumscription).

Formally, if ~~O'~~ O is a `RelativeClosureOMS`, ~~O'.`closureType` = `minimize` and $O = O'$.`closableOMS`, then~~ O.`closureType` = and $O' = O$.`closableOMS`, and $sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), O') = (\Gamma', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta'))$ then

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), O)) = (\Gamma'', (\mathcal{I}'', \Sigma'', \mathcal{M}'', \Delta''))$$

where

- ~~$\mathsf{Inst}(O') = \mathsf{Inst}(O)$ $\mathsf{Sig}(O') = \mathsf{Sig}(O)$~~ $\mathcal{I}'' = \mathcal{I}'$
- ~~$\mathsf{Mod}(O') = \{M \in \mathsf{Mod}(O) \mid M$ is minimal in $\{M' \in \mathsf{Mod}(O) \mid M'|_\Sigma = M|_\Sigma\}\}$~~ $\Sigma'' = \Sigma'$
- ~~$\mathsf{Th}(O) = \bot$ where the semantics of the O is given relative to the environment Γ and the context $(\mathcal{I}, \Sigma, \mathcal{M}, \Delta)$,~~ $\mathcal{M}'' = \{M \in \mathcal{M} \mid M$ is minimal in $\{M' \in \mathcal{M} \mid M'|_\Sigma = M|_\Sigma\}\}$ and "minimal" is interpreted in the pre-order defined by ~~$\Sigma_1 \leq \Sigma_2$~~ $M_1 \leq M_2$ if there is a ~~signature morphism $\Sigma_1 \to \Sigma_2$.~~
  ~~The theory-level semantics for O' cannot be defined.~~
  ~~$Env(\Gamma, O')$~~ model homomorphism $\mathcal{M}_1 \to \mathcal{M}_2$
- $\Delta'' = \bot$
- Γ'' is obtained from ~~Γ~~ Γ' by adding to ~~$\Gamma.imports$~~ $\Gamma'.imports$ a new node labeled with ~~$(Name(O'), \mathsf{Inst}(O'), \mathsf{Sig}(O'), \mathsf{Mod}(O'), \mathsf{Th}(O$~~ $(Name(O), \mathsf{Inst}(O), \mathsf{Sig}(O), \mathsf{Mod}(O), \mathsf{Th}(O))$ and an edge from the node of ~~O~~ O' to the node of ~~O'~~ O labeled with the identity morphism on ~~$\mathsf{Sig}(O')$~~ Σ''.

The theory-level semantics for O cannot be defined.

The semantics of ~~O'~~ O is defined similarly for the other three alternatives of ~~O'.closureType~~ O.closureType, only the model class differs:

- if ~~O'.closureType = maximize, $\mathsf{Mod}(O') = \{M \in \mathsf{Mod}(O) \mid M$ is maximal in $\{M' \in \mathsf{Mod}(O) \mid M'|_\Sigma = M|_\Sigma\}\}$~~ O.closureTyp
  $\mathcal{M}'' = \{M \in \mathcal{M} \mid M$ is maximal in $\{M' \in \mathcal{M} \mid M'|_\Sigma = M|_\Sigma\}\}$
- if ~~O'.closureType = free, $\mathsf{Mod}(O') = \{M \in \mathsf{Mod}(O) \mid M$ is initial in $\{M' \in \mathsf{Mod}(O) \mid M'|_\Sigma = M|_\Sigma\}\}$~~ O.closureType = fre
  $\mathcal{M}'' = \{M \in \mathcal{M} \mid M$ is initial in $\{M' \in \mathcal{M} \mid M'|_\Sigma = M|_\Sigma\}\}$
- if ~~O'.closureType = cofree, $\mathsf{Mod}(O') = \{M \in \mathsf{Mod}(O) \mid M$ is terminal in $\{M' \in \mathsf{Mod}(O) \mid M'|_\Sigma = M|_\Sigma\}\}$~~ O.closureType =
  $\mathcal{M}'' = \{M \in \mathcal{M} \mid M$ is terminal in $\{M' \in \mathcal{M} \mid M'|_\Sigma = M|_\Sigma\}\}$

~~Here, initial and terminal models are defined as in category theory: M is initial (terminal) in $\mathcal{M}$ if for each $N \in \mathcal{M}$, there is exactly one morphism $h : M \to N$ ($h : N \to M$).~~

10.2.3.7 Semantics of OMS

~~An is interpreted in a context similar to that~~

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{OMS}) &= (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) \\ &: (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences)) \end{aligned}}$$

The semantics for a `ClosableOMS` ~~, the difference being that there is no local environment~~ has been defined above.
The semantics for an `ExtendingOMS` has been defined above.
If o is a `ClosureOMS`,

$$sem(\Gamma, \cancel{o}O) = (\Gamma'', (I, \Sigma, \mathcal{M}', \bot))$$

where

$$(\Gamma', (I, \Sigma, \mathcal{M}, \Delta)) = sem(\Gamma, \cancel{o}O.\texttt{oms}), \qquad \Sigma_{\cancel{min}\,closure} = sem(\cancel{o}\Gamma', \Sigma, O.\texttt{closure.}\cancel{\Sigma}\texttt{circClosure}),$$

$$\Sigma_{var} = sem(\cancel{o}\Gamma', \Sigma, O.\texttt{closure.}\cancel{\Sigma}\texttt{circVars}), \qquad \Sigma_{fixed} = \Sigma \setminus (\Sigma_{\cancel{min}\,closure} \cup \Sigma_{var})$$

and

- if ~~o.closure.closureType = minimize~~ O.closure.closureType = minimize, then
  ~~$\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{min} \cup \Sigma_{fixed}}$ is minimal in $\{M' \in \mathcal{M}|_{\Sigma_{min} \cup \Sigma_{fixed}} \mid M'|_{\Sigma_{fixed}} = M|_{\Sigma_{fixed}}\}\}$~~ $\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{\cancel{closure}} \cup \Sigma_{fixed}}$ is mini
- if ~~o.closure.closureType = maximize~~ O.closure.closureType = maximize, then
  ~~$\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{min} \cup \Sigma_{fixed}}$ is maximal in $\{M' \in \mathcal{M}|_{\Sigma_{min} \cup \Sigma_{fixed}} \mid M'|_{\Sigma_{fixed}} = M|_{\Sigma_{fixed}}\}\}$~~ $\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{\cancel{closure}} \cup \Sigma_{fixed}}$ is max
- if ~~o.closure.closureType = free~~ O.closure.closureType = free, then
  ~~$\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{min} \cup \Sigma_{fixed}}$ is initial in $\{M' \in \mathcal{M}|_{\Sigma_{min} \cup \Sigma_{fixed}} \mid M'|_{\Sigma_{fixed}} = M|_{\Sigma_{fixed}}\}\}$~~ $\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{\cancel{closure}} \cup \Sigma_{fixed}}$ is initial
- if ~~o.closure.closureType = cofree~~ O.closure.closureType = cofree, then
  ~~$\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{min} \cup \Sigma_{fixed}}$ is terminal in $\{M' \in \mathcal{M}|_{\Sigma_{min} \cup \Sigma_{fixed}} \mid M'|_{\Sigma_{fixed}} = M|_{\Sigma_{fixed}}\}\}$~~ $\mathcal{M}' = \{M \in \mathcal{M} \mid M|_{\Sigma_{\cancel{closure}} \cup \Sigma_{fixed}}$ is tern

Γ'' is obtained from $\Gamma' = Env(\Gamma, O.\mathtt{oms})$ by extending $\Gamma'.imports$ with a new node for O labeled as in the items above and with a new edge from the node of $O.\mathtt{oms}$ to the node of O labeled with the identity morphism on Σ.

The semantics of a `TranslationOMS` ~~O'~~ O is given by

$$sem(\Gamma, O) = (\Sigma'', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$$

where

- ~~$\mathbf{Inst}(O') = J$, when $\mathbf{Inst}_{\mathsf{Sig}(O'.\mathtt{oms})}(O'.\mathtt{omsTranslation}) = (\Phi, \alpha, \beta) : \mathbf{Inst}(O'.\mathtt{oms}) \to J$~~ $\mathcal{I} = J$,
- ~~$\mathsf{Sig}(O') = \Sigma'$, when $\mathsf{Mor}_{\mathsf{Sig}(O'.\mathtt{oms})}(O'.\mathtt{omsTranslation}) = \sigma : \Phi(\mathsf{Sig}(O'.\mathtt{oms})) \to \Sigma'$~~ $\Sigma = \Sigma'$, when $sem(\Gamma, \mathsf{Sig}(O.\mathtt{oms}), O.\mathtt{omsTra}$
- ~~$\mathsf{Mod}(O') = \{M \in \mathsf{Mod}(\Sigma') \mid \beta_\Sigma(M|_\sigma) \in \mathsf{Mod}(O'.\mathtt{oms})\}$~~ $\mathcal{M} = \{M \in \mathsf{Mod}(\Sigma') \mid \beta_{\mathsf{Sig}(O.\mathtt{oms})}(M|_\sigma) \in \mathsf{Mod}(O.\mathtt{oms})\}$
- ~~$\mathsf{Th}(O') = \{Sen^J(\sigma)(\alpha_\Sigma(\delta)) \mid \delta \in \mathsf{Th}(O'.\mathtt{oms})\}$~~ $\Delta = \{Sen^J(\sigma)(\alpha_{\mathsf{Sig}(O.\mathtt{oms})}(\delta)) \mid \delta \in \mathsf{Th}(O.\mathtt{oms})\}$. It is defined only if ~~$O'.\mathtt{oms}$~~ $O.\mathtt{oms}$ is flattenable.
- ~~$Env(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = Env(\Gamma, O'.\mathtt{oms})$ by extending $\Gamma''.imports$~~ $\Gamma' = Env(\Gamma, O.\mathtt{oms})$ by extending $\Gamma'.imports$ with a new node for ~~O'~~ O labeled as in the items above and with a new edge from the node of ~~$O'.\mathtt{oms}$~~ $O.\mathtt{oms}$ to the node of ~~O'~~ O labeled with $((\Phi, \alpha, \beta), \sigma)$.

The semantics of a `ReductionOMS` ~~O' is~~ O is

$$sem(\Gamma, O) = (\Gamma'', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$$

where

- ~~$\mathbf{Inst}(O') = J$, when $\mathbf{Inst}_{\mathsf{Sig}(O'.\mathtt{oms})}(O'.\mathtt{reduction}) = (\Phi, \alpha, \beta) : \mathbf{Inst}(O'.\mathtt{oms}) \to J$~~ $\mathcal{I} = J$,
- ~~$\mathsf{Sig}(O') = \Sigma'$, when $\mathsf{Mor}_{\mathsf{Sig}(O'.\mathtt{oms})}(O'.\mathtt{reduction}) = \sigma : \Sigma' \to \Phi(\mathsf{Sig}(O'.\mathtt{oms}))$~~ $\Sigma = \Sigma'$, when $sem(\Gamma, \mathsf{Sig}(O.\mathtt{oms}), O.\mathtt{reduction}) =$
- ~~$\mathsf{Mod}(O') = \{\beta_\Sigma(M)|_\sigma \mid M \in \mathsf{Mod}(O'.\mathtt{oms})\}$~~ $\mathcal{M} = \{\beta_\Sigma(M)|_\sigma \mid M \in \mathsf{Mod}(O.\mathtt{oms})\}$
- ~~$\mathsf{Th}(O') = \bot$~~ $\Delta = \bot$
- ~~$Env(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = Env(\Gamma, O'.\mathtt{oms})$ by extending $\Gamma''.imports$~~ $\Gamma' = Env(\Gamma, O.\mathtt{oms})$ by extending $\Gamma'.imports$ with a new node for ~~O'~~ O labeled as in the items above and with a new edge from the node of ~~O'~~ O to the node of ~~$O'.\mathtt{oms}$~~ $O.\mathtt{oms}$ labeled with $((\Phi, \alpha, \beta), \sigma)$.

The semantics of an `ExtractionOMS` ~~O' is~~ O is

$$sem(\Gamma, O) = (\Gamma'', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$$

where

- ~~$\mathbf{Inst}(O') = \mathbf{Inst}(O'.\mathtt{oms})$~~ $\mathcal{I} = \mathbf{Inst}(O.\mathtt{oms})$
- ~~$\mathsf{Sig}(O') = \Sigma'$~~ $\Sigma = \Sigma'$,
- ~~$\mathsf{Th}(O') = \Delta'$~~ $\Delta = \Delta'$
- ~~$\mathsf{Mod}(O')$~~ $\mathcal{M}$ is the class of ~~$Th(O)$~~ Δ-models
- ~~$Env(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = Env(\Gamma, O'.\mathtt{oms})$ by extending $\Gamma''.imports$~~ $\Gamma' = Env(\Gamma, O.\mathtt{oms})$ by extending $\Gamma'.imports$ with a new node for ~~O'~~ O labeled as in the items above and with a new edge from the node of ~~O'~~ O to the node of ~~$O'.\mathtt{oms}$~~ $O.\mathtt{oms}$ labeled with the inclusion of Σ' in ~~$\mathsf{Sig}(O'.\mathtt{oms})$~~ $\mathsf{Sig}(O.\mathtt{oms})$

where ~~$sem(\Gamma, (\mathsf{Sig}(O'.\mathtt{oms}), \mathsf{Th}(O'.\mathtt{oms})), O'.\mathtt{extraction}) = (\Sigma', \Delta')$~~ $sem(\Gamma, (\mathsf{Sig}(O.\mathtt{oms}), \mathsf{Th}(O.\mathtt{oms})), O.\mathtt{extraction}) = (\Sigma', \Delta')$.

The semantics of an `ApproximationOMS` ~~O' is~~ O is

$$sem(\Gamma, O) = (\Gamma'', (\mathcal{I}'', \Sigma', \mathcal{M}, \Delta))$$

where

- ~~$\mathbf{Inst}(O) = \mathcal{I}$ when $(\Phi, \alpha, \beta) : \mathbf{Inst}(O'.\mathtt{oms}) \to \mathcal{I})$ is the default projection (in case $O'.\mathtt{approximation.logicRef}$ is missing, it is the identity on $\mathbf{Inst}(O'.\mathtt{oms})$)~~ $\mathcal{I}'' = \mathcal{I}'$,
- ~~$\mathsf{Sig}(O) = \Phi(\Sigma)$~~ $\Sigma' = \Phi(\Sigma)$
- ~~$\mathsf{Th}(O) = \alpha^{-1}_{\mathsf{Sig}(O'.\mathtt{oms})}(\mathsf{Th}(O'.\mathtt{oms})^\bullet) \cap \mathsf{Sen}^I(\mathsf{Sig}(O'.\mathtt{oms}))$[8]~~ $\Delta = \alpha^{-1}_{\mathsf{Sig}(O.\mathtt{oms})}(\mathsf{Th}(O.\mathtt{oms})^\bullet) \cap \mathsf{Sen}^{\mathcal{I}'}(\mathsf{Sig}(O.\mathtt{oms}))$. , i.e. that part of ~~$\mathsf{Th}(O'.\mathtt{oms})$~~ $\mathsf{Th}(O.\mathtt{oms})$ that can be expressed in the smaller signature and logic . In practice, one looks for a finite subset that still is logically equivalent to this set.

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[8] ~~In practice, one looks for a finite subset that still is logically equivalent to this set. Note that $\Delta^\bullet$ is the set of logical consequences of Δ, i.e. $\Delta^\bullet = \mathbf{Th}(\Delta)$.~~

- ~~$\mathsf{Mod}(O)$~~ $\mathcal{M}$ is the class of ~~$\mathsf{Th}(O)$~~Δ-models
- ~~$\mathit{Env}(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = \mathit{Env}(\Gamma, O'.\texttt{oms})$ by extending $\Gamma''.\mathit{imports}$~~ $\Gamma' = \mathit{Env}(\Gamma, O.\texttt{oms})$ by extending $\Gamma'.\mathit{imports}$ with a new node for ~~O'~~ O labeled as in the items above and with a new edge from the node of ~~$O'.\texttt{oms}$~~ $O.\texttt{oms}$ to the node of ~~O' labeled with $((\Phi, \alpha, \beta), \iota : \Phi(\Sigma) \to \mathsf{Sig}(O'.\texttt{oms}))$~~ O labeled with $(\rho, \iota : \Phi(\Sigma) \to \mathsf{Sig}(O.\texttt{oms}))$

where ~~$(\mathcal{I}, \Sigma) = \mathit{sem}(\Gamma, (\mathbf{Inst}(O'.\texttt{oms}), \mathsf{Sig}(O'.\texttt{oms})), O'.\texttt{approximation})$~~$(\rho = (\Phi, \alpha, \beta) : \mathcal{I} \to \mathcal{I}', \Sigma) = \mathit{sem}(\Gamma, (\mathbf{Inst}(O.\texttt{oms}), \mathsf{Sig}(O.\texttt{oms}$

The semantics of a `FilteringOMS` O ~~, where $O' = O.\texttt{filtering.basicOMS}$, is defined only if $\mathsf{Sig}(O') \subseteq \mathsf{Sig}(O.\texttt{oms})$~~ is defined by case distinction. Let $(\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) = \mathit{sem}(\Gamma, O.\texttt{oms})$ and ~~$\mathsf{Th}(O') \subseteq \mathsf{Th}(O.\texttt{oms})$. Two cases are distinguished based on the value of $O.\texttt{filtering.removalKind}$. If $O.\texttt{filtering.removalKind} = \mathit{keep}$~~$(c, \mathcal{I}', \Sigma', \Delta') = \mathit{sem}(\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), O.\texttt{fi}$

If $c = \mathit{keep}$, the semantics of O is given by

$$\mathit{sem}(\Gamma, O) = (\Gamma'', (\mathcal{I}'', \Sigma'', \mathcal{M}'', \Delta''))$$

where

- ~~$\mathbf{Inst}(O) = \mathbf{Inst}(O')$~~ $\mathcal{I}'' = \mathcal{I}'$
- ~~$\mathsf{Sig}(O) = \Sigma'$ where Σ'~~ Σ'' is the smallest signature with ~~$\mathsf{Sig}(O') \subseteq \Sigma'$ and $\mathsf{Th}(O') \subseteq \mathsf{Sen}(\Sigma')$~~$\Sigma' \subseteq \Sigma''$ and $\Delta' \subseteq \mathsf{Sen}(\Sigma'')$. (If this smallest signature does not exist, the semantics is undefined.)
- ~~$\mathsf{Th}(O) = (\mathsf{Th}(O.\texttt{oms}) \cap \mathsf{Sen}(\mathsf{Sig}(O))) \cup \mathsf{Th}(O')$~~ $\Delta'' = (\Delta \cap \mathsf{Sen}(\Sigma'')) \cup \Delta'$
- $\mathsf{Mod}(O)$ is the class of all ~~$\mathsf{Th}(O)$~~Δ-models.
- ~~$\mathit{Env}(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = \mathit{Env}(\Gamma, O.\texttt{oms})$ by extending $\Gamma''.\mathit{imports}$~~ Γ' by extending $\Gamma'.\mathit{imports}$ with a new node for O labeled as in the items above and with a new edge from the node of O to the node of $O.\texttt{oms}$ labeled with the inclusion of ~~Σ' in $\mathsf{Sig}(O.\texttt{oms})$~~$\Sigma''$ in Σ.

If ~~$O.\texttt{filtering.removalKind} = \mathit{remove}$~~$c = \mathit{remove}$, the semantics of O is

$$\mathit{sem}(\Gamma, O) = (\Gamma'', (\mathcal{I}'', \Sigma'', \mathcal{M}'', \Delta''))$$

where

- ~~$\mathbf{Inst}(O) = \mathbf{Inst}(O')$~~ $c\mathcal{I}'' = \mathcal{I}'$
- ~~$\mathsf{Sig}(O) = \mathsf{Sig}(O.\texttt{oms}) \setminus \mathsf{Sig}(O')$~~ $\Sigma'' = \Sigma \setminus \Sigma'$
- ~~$\mathsf{Th}(O) = \mathsf{Th}(O.\texttt{oms}) \cap \mathsf{Sen}(\mathsf{Sig}(O)) \setminus \mathsf{Th}(O')$~~ $\Delta'' = \Delta \cap \mathsf{Sen}(\Sigma'') \setminus \Delta'$
- ~~$\mathsf{Mod}(O)$~~ $\mathcal{M}''$ is the class of all $\mathsf{Th}(O)$-models.
- ~~$\mathit{Env}(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = \mathit{Env}(\Gamma, O.\texttt{oms})$ by extending $\Gamma''.\mathit{imports}$~~ Γ' by extending $\Gamma'.\mathit{imports}$ with a new node for O labeled as in the items above and with a new edge from the node of O to the node of $O.\texttt{oms}$ labeled with the inclusion of ~~Σ' in $\mathsf{Sig}(O.\texttt{oms})$~~$\Sigma''$ in Σ.

The semantics of an `UnionOMS` O is

$$\mathit{sem}(\Gamma, O) = (\Gamma'', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta'))$$

where

- ~~$\mathbf{Inst}(O) = I$ where $\mathbf{Inst}(O_1) \bigsqcup \mathbf{Inst}(O_2) = (I, (\Phi_1, \alpha_1, \beta_1) : \mathbf{Inst}(O_1) \to I, (\Phi_2, \alpha_2, \beta_2) : \mathbf{Inst}(O_2) \to I)$~~ $c\mathcal{I}' = \mathcal{I}$ where $\mathbf{Inst}(O_1) \bigsqcup \mathbf{Inst}(O_2) = (\mathcal{I}, (\Phi_1, \alpha_1, \beta_1) : \mathbf{Inst}(O_1) \to \mathcal{I}, (\Phi_2, \alpha_2, \beta_2) : \mathbf{Inst}(O_2) \to \mathcal{I})$
- ~~$\mathsf{Sig}(O) = \Phi_1(\mathsf{Sig}(O_1)) \cup \Phi_2(\mathsf{Sig}(O_2))$~~ $\Sigma' = \Phi_1(\mathsf{Sig}(O_1)) \cup \Phi_2(\mathsf{Sig}(O_2))$
- ~~$\mathsf{Mod}(O) = \{M \in \mathsf{Mod}(\mathsf{Sig}(O)) \mid \beta_{\Sigma_i}(M|_{\Phi_i(\mathsf{Sig}(O_i))}) \in \mathsf{Mod}(O_i), \text{ for } i = 1, 2\}$~~ $\mathcal{M}' = \{M \in \mathsf{Mod}^{\mathcal{I}}(\Sigma') \mid \beta_{\Sigma_i}(M|_{\Phi_i(\mathsf{Sig}(O_i))}) \in \mathsf{Mod}(O_i$
- ~~$\mathsf{Th}(O) = \alpha_1(\mathsf{Th}(O_1)) \cup \alpha_2(\mathsf{Th}(O_2))$~~$\Delta' = \alpha_1(\mathsf{Th}(O_1)) \cup \alpha_2(\mathsf{Th}(O_2))$.
- ~~$\mathit{Env}(\Gamma, O')$~~ Γ'' is obtained from ~~$\Gamma'' = \mathit{Env}(\mathit{Env}(\Gamma, O_1), O_2)$ by extending $\Gamma''.\mathit{imports}$~~ $\Gamma' = \mathit{Env}(\mathit{Env}(\Gamma, O_1), O_2)$ by extending $\Gamma'.\mathit{imports}$ with a new node for O labeled as in the items above and with edges from the nodes of O_1 and O_2, respectively, to the node of O, labeled for each $i = 1, 2$ with ~~$(\Phi_i, \alpha_i, \beta_i, \iota_i : \Phi_i(O_i) \to \mathsf{Sig}(O))$~~$(\Phi_i, \alpha_i, \beta_i, \iota_i : \Phi_i(O_i) \to \Sigma')$.

where $O_1 = O.\texttt{oms}$ and $O_2 = O.\texttt{oms2}$.

If $O.\texttt{conservativityStrength}$ is present, then O must be a conservative extension of the appropriate strength of O_1.

The semantics of an `ExtensionOMS` O is

$$\mathit{sem}(\Gamma, O) = (\Gamma'', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta'))$$

where

- ~~$\mathbf{Inst}(O) = \mathbf{Inst}(O.\texttt{oms}) = \mathbf{Inst}(O.\texttt{extension})$~~ $\mathcal{I}' = \mathbf{Inst}(O.\texttt{oms}) = \mathbf{Inst}(O.\texttt{extension})$ (which means that the institutions of $O.\texttt{oms}$ and $O.\texttt{extension}$ must be the same)
- ~~$\mathsf{Sig}(O) = \mathsf{Sig}(O.\texttt{oms}) \cup \mathsf{Sig}((\mathbf{Inst}(O.\texttt{oms}), \mathsf{Sig}(O.\texttt{oms}), \mathsf{Mod}(O.\texttt{oms}), \mathsf{Th}(O.\texttt{oms})), O.\texttt{extension})$~~ $\Sigma' = \mathsf{Sig}(O.\texttt{oms}) \cup \mathsf{Sig}(sem(\Gamma, ($
- ~~$\mathsf{Mod}(O) = \{M \in \mathsf{Mod}(\mathsf{Sig}(O)) \mid M|_{\mathsf{Sig}(O.\texttt{oms})} \in \mathsf{Mod}(O.\texttt{oms})$ and $M|_{\mathsf{Sig}(O.\texttt{extension})} \in \mathsf{Mod}(O.\texttt{extension})\}$~~ $\mathcal{M}' = \{M \in \mathsf{Mod}(\Sigma')$
- ~~$\mathsf{Th}(O) = \mathsf{Th}(O.\texttt{oms}) \cup \mathsf{Th}(O.\texttt{extension})$~~ $\Delta' = \mathsf{Th}(O.\texttt{oms}) \cup \mathsf{Th}(O.\texttt{extension})$
- ~~$Env(\Gamma, O')$ is obtained from $\Gamma'' = Env(\Gamma, O.\texttt{oms})$ by extending $\Gamma''.imports$ with a new node for O labeled as in the items above and with a new edge from the node of $O.\texttt{oms}$ to the node of O labeled with the inclusion of $\mathsf{Sig}(O.\texttt{oms})$ in $\mathsf{Sig}(O)$~~ Γ'' is $Env(Env(\Gamma, O.\texttt{oms}), O.\texttt{extension})$.

The semantics of a `QualifiedOMS` O in the context Γ is the same as the semantics of $O.\texttt{oms}$ in the context Γ' given by the semantics of $O.\texttt{qualification}$ in the context Γ. The change of context is local to $O.\texttt{oms}$, which means that if the qualification appears as a term in a larger expression, after its analysis the context will be Γ and not Γ'. Formally,

$$sem(\Gamma, O) = (\Gamma'', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$$

where $(\Gamma'', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) = sem(sem(\Gamma, O.\texttt{qualification}), O.\texttt{oms})$.

The semantics of a `CombinationOMS` O is

$$sem(\Gamma, O) = (\Gamma'', (\mathcal{I}', \Sigma', \mathcal{M}', \Delta'))$$

where

- ~~$\mathbf{Inst}(O) = I$~~ $\mathcal{I}' = \mathcal{I}$,
- ~~$\mathsf{Sig}(O) = \Sigma$, where $(I, \Sigma, \{\mu_i\}_{i\in|G|})$~~ $\Sigma' = \Sigma$, where $(\mathcal{I}, \Sigma, \{\mu_i\}_{i\in|G|})$ is the colimit of the graph ~~G given by the semantics of $O.\texttt{network}$~~ $G = sem(\Gamma, O.\texttt{network})$,
- ~~$\mathsf{Th}(O) = \cup_{i\in|G|}\mu_i(\mathsf{Th}(O_i))$~~ $\Delta' = \cup_{i\in|G|}\mu_i(\mathsf{Th}(O_i))$, where O_i is the OMS label of the node i in G
- ~~$\mathsf{Mod}(O) = \{M \in \mathsf{Mod}(\Sigma) \mid M|_{\mu_i} \in \mathsf{Mod}(O_i), i \in |G|\}$~~ $\mathcal{M}' = \{M \in \mathsf{Mod}(\Sigma) \mid M|_{\mu_i} \in \mathsf{Mod}(O_i), i \in |G|\}$, where O_i is the OMS label of the node i in G.
- ~~$Env(\Gamma, O)$~~ Γ'' is obtained from Γ by adding to $\Gamma.imports$ a new node for O labeled as in the items above and with edges from each node in G to this new node labeled with the morphisms μ_i for each $i \in |G|$.

10.2.3.8 Semantics of `CircClosure`

$$\boxed{\begin{aligned} sem(\Gamma, \Sigma, \texttt{CircClosure}) &= \Sigma' \\ &: Signature \end{aligned}}$$

If c is a `CircClosure`,

$$sem(\Gamma, \Sigma, c) = sem(\Gamma, \Sigma, c.symbol)$$

10.2.3.9 Semantics of `CircVar`

$$\boxed{\begin{aligned} sem(\Gamma, \Sigma, \texttt{CircVar}) &= \Sigma' \\ &: Signature \end{aligned}}$$

If c is a `CircVar`,

$$sem(\Gamma, \Sigma, c) = sem(\Gamma, \Sigma, c.symbol)$$

10.2.3.10 Semantics of OMS translations

$$\boxed{\begin{aligned} sem(\Gamma, \Sigma, \texttt{OMSTranslation}) &= (\rho, \sigma) \\ &: (Comorphism, SignatureMorphism) \end{aligned}}$$

The semantics of a `OMSTranslation` $O =$ is given by

- ~~$\mathbf{Inst}(O) = sem(O.\texttt{omsLanguageTranslation}) : \Gamma.logic \to logic'$~~ $\rho = sem(O.\texttt{omsLanguageTranslation}) : \Gamma.logic \to logic'$
- ~~$\mathsf{Mor}(O) = sem(\Gamma.\{current = (lang', logic', ser')\}, \Phi(\Sigma), O.\texttt{symbolMap})$~~ $\sigma = sem(\Gamma.\{current = (lang', logic', ser')\}, \Phi(\Sigma), O.\texttt{sym}$

where $lang'$ and ser' are the default language and serialization for logic $logic'$. If $O.\texttt{omsLanguageTranslation}$ is missing, it defaults to the identity comorphism of the current logic.

10.2.3.11 Semantics of OMS language translations

~~The semantics of a O is $sem(\Gamma, O.\texttt{iri}) = (\Phi, \alpha, \beta)$.~~

$$\boxed{sem(\Gamma, \texttt{OMSLanguageTranslation}) \quad = \rho \quad : Translation}$$

If t is a NamedLanguageTranslation,

$$sem(\Gamma, t) = \Gamma(O.\texttt{omsLanguageTranslationRef})$$

where ~~(Φ, α, β) is the institution comorphism named by $O.\texttt{iri}$ in~~ $\Gamma(O.\texttt{omsLanguageTranslationRef})$ is an institution comorphism. This is defined only if the domain of ρ is the current logic of Γ.

If t is a DefaultTranslation,

$$sem(\Gamma, t) = \rho$$

where ρ is the unique default institution comorphism of the heterogeneous logical environment running from $\Gamma.logic$ to $t.\texttt{languageRef}$ (if this is a logic) or to some logic supported by $t.\texttt{languageRef}$ (if this is a language). If there is no or no unique such comorphism, the semantics is undefined.

$$\boxed{sem(\Gamma, Sequence(\texttt{OMSLanguageTranslation})) \quad = \rho \quad : Translation}$$

If $t_1, \ldots, t_n$ are all OMSLanguageTranslations, ~~$sem(\Gamma, Sequence\{t_1, \ldots, t_n\}) = (\Phi, \alpha, \beta)$, where $sem(\Gamma, t_i) = (\Phi_i, \alpha_i, \beta_i)$~~ $sem(\Gamma, Sequence\{t_1, \ldots, t_n\}) = \rho$, where $sem(\Gamma, t_i) = \rho_i$ for $i = 1, \ldots, n$ and ~~$(\Phi, \alpha, \beta) = (\Phi_1, \alpha_1, \beta_1); \ldots; (\Phi_n, \alpha_n, \beta_n)$~~ $(\Phi, \alpha, \beta) = \rho_1; \rho_2; \ldots$

10.2.3.12 Semantics of reductions

$$\boxed{sem(\Gamma, \Sigma, \texttt{Reduction}) \quad = (\rho, \sigma) \quad : (Morphism, SignatureMorphism)}$$

The semantics of a Reduction $O =$ with $O.\texttt{reduction.removalKind} = remove$ is given by

- ~~**Inst**(O) $= sem(O.\texttt{reduction.omsLanguageTranslation}) : \Gamma.logic \to logic'$~~ $\rho = sem(O.\texttt{reduction.omsLanguageTransla}$

- ~~**Mor**(O) $= \iota : \Sigma' \to \Phi(\Sigma)$~~ $\sigma = \iota : \Sigma' \to \Phi(\Sigma)$, where $\Sigma' = sem(\Gamma.\{current = (lang', logic', ser')\}, \Phi(\Sigma), O.\texttt{reduction.symbolLis}$ $lang'$ and ser' are the default language and serialization for logic $logic'$ and ι is the inclusion morphism.

If $O.\texttt{reduction.omsLanguageTranslation}$ is missing, it defaults to the identity morphism of the current logic of Γ.
The semantics of a reduction $O =$ with $O.\texttt{reduction.removalKind} = keep$ is

- ~~**Inst**(O)~~ ρ is the identity morphism on the current logic of Γ
- ~~**Mor**(O)~~ σ is the inclusion of $sem(\Gamma, \Sigma, O.\texttt{reduction.symbolList})$ in Σ.

10.2.3.13 Semantics of sets of symbols

$$\boxed{sem(\Gamma, \Sigma, Set(\texttt{Symbol})) \quad = \Sigma' \quad : Signature}$$

If $s_1, \ldots, s_n$ are all Symbols,

$$sem(\Gamma, \Sigma, Set\{s_1, \ldots, s_n\}) = \Sigma'$$

where Σ' is the smallest sub-signature of Σ containing $sem(\Gamma, \Sigma, s_1), \ldots, sem(\Gamma, \Sigma, s_n)$, if such a sub-signature exists and is otherwise undefined.

10.2.3.14 Semantics of symbol maps

$$\boxed{sem(\Gamma, \Sigma, \texttt{SymbolMap}) \quad = \sigma : \Sigma \to \Sigma' \quad : SignatureMorphism}$$

[9] Note(9)

[9] NOTE: This is needed for translations, same as induced_from vs. induced_from_to in CASL. How to deal with this? TM: couldn't we use the same notions as in the CASL reference manual p. 198/199? I have packed the sources into sem.tgz

$$\boxed{sem(\Gamma, \Sigma, \Sigma', \texttt{SymbolMap}) \quad = \sigma : \Sigma \to \Sigma' \quad : SignatureMorphism}$$

If m is a `SymbolMap`,

$$sem(\Gamma, \Sigma, \Sigma', m) = \sigma$$

where $\sigma = makeMorphism_{logic(\Gamma.current)}((s_1, t_1), \ldots, (s_n, t_n))$ ~~and $(s, t) = sem(\Gamma, \Sigma_1, \Sigma_2, m.\texttt{generalSymbolMapItem})$,~~and

~~$Set\{(s_1, t_1), \ldots, (s_n, t_n)\} = sem(\Gamma, \Sigma_1, \Sigma_2, m.\texttt{generalSymbolMapItem2})$~~ $Set\{(s_1, t_1), \ldots, (s_n, t_n)\} = sem(\Gamma, \Sigma_1, \Sigma_2, m.\texttt{generalSymb}$

Applications **shall** implicitly map those non-logical symbols of the source OMS, for which an explicit mapping is not given, to non-logical symbols of the same (local) name in the target OMS, wherever this is uniquely defined – in detail:

Require: O_s, O_t are OMS
Require: $M \subseteq |\mathsf{Sig}(O_s)| \times |\mathsf{Sig}(O_t)|$ maps non-logical symbols (i.e. elements of the signature) of O_s to non-logical symbols of O_t
  for all $e_s \in |\mathsf{Sig}(O_s)|$ not covered by M **do**
    $n_s \leftarrow \text{localname}(e_s)$
    ~~$N_t \leftarrow \{\text{localname}(e) \mid e \in |\Sigma(O_t)|\}$~~ $N_t \leftarrow \{\mathrm{e} \in |\mathsf{Sig}(\mathrm{O_t})| \mid \text{localname}(e) = n_s\}$
    if $N_t = \{e_t\}$ **then** {i.e. if there is a unique target}
      $M \leftarrow M \cup \{(e_s, e_t)\}$
    end if
  end for
Ensure: M completely covers $|\Sigma(O_s)|$

10 Note(10

The local name of a non-logical symbol e is determined as follows[8]:

Require: e is a non-logical symbol (identified by an IRI; cf. clause **??**)
  if e has a fragment f **then** {production `ifragment` in IETF/RFC 3987:2005}
    return f
  else
    $n \leftarrow$ the longest suffix of e that matches the `Nmtoken` production of XML W3C/TR REC-xml:2008
    return n
  end if

10.2.3.15 Semantics of extractions

$$\boxed{sem(\Gamma, (\Sigma, \Delta), \texttt{Extraction}) \quad = (\Sigma', \Delta') \quad : (Signature, Sentences)}$$

If e is an `Extraction`,

$$sem(\Gamma, (\Sigma, \Delta), e) = (\Sigma', \Delta')$$

where $sem(\Gamma, \Sigma, e.\texttt{removalKind}, e.\texttt{interfaceSignature}) = \Sigma''$, $\langle \Sigma', \Delta' \rangle$ is the smallest depleting Σ''-module~~(see [?] for the definition in a description logic context and [?] for a generalization to an arbitrary institution)~~, i.e. the smallest sub-theory $\langle \Sigma', \Delta' \rangle$ of (Σ, Δ) such that the following model-theoretic inseparability holds

$$\Delta \setminus \Delta' \equiv_{\Sigma' \cup \Sigma''} \emptyset.$$

~~(In [?], it is shown that the smallest depleting Σ''-module exists in description logics, and in [?] this is generalized to arbitrary institutions.)~~

This means intuitively that $\Delta \setminus \Delta'$ cannot be distinguished from $\emptyset$ (~~what~~ as far as $\Sigma' \cup \Sigma''$ ~~concerns~~is concerned) and formally that

$$\begin{aligned} & \{M|_{\Sigma' \cup \Sigma''} \mid M \in \mathsf{Mod}(\Sigma), M \models \Delta \setminus \Delta'\} \\ = \; & \{M|_{\Sigma' \cup \Sigma''} \mid M \in \mathsf{Mod}(\Sigma)\}. \end{aligned}$$

[?] defines the concept of smallest depleting Σ-module in a description logic context and shows that the smallest depleting Σ''-module exists in description logics. [?] generalizes both the definition of smallest depleting Σ''-module and the mentioned result to arbitrary institutions.

[10] NOTE: We need institutions with a symbol functor, otherwise signatures do not have symbols. The “unique target” method does not cover cases where the target is unique because the target has a unique symbol of the required kind and type. It does not cover the case where just the symbol with the same name is used. But maybe all this should be made institution-specific? TM: I think my answer to the previous ednote should solve this.

[8] In practice, this can often have the effect of undoing an IRI abbreviation mechanism that was used when writing the respective OMS (cf. clause **??**). In general, however, functions that turn abbreviations into IRIs are not invertible. For this reason, the implicit mapping of non-logical symbols is specified independently from IRI abbreviation mechanisms possibly employed in the OMS.

10.2.3.16 Semantics of approximations

$$sem(\Gamma, (\mathcal{I}, \Sigma), \texttt{Approximation}) = (\rho : \mathcal{I} \to \mathcal{I}', \Sigma') : (Morphism, Signature)$$

If a is an Approximation,

$$sem(\Gamma, (\mathcal{I},\Sigma), a) = (\rho, \Sigma')$$

where $\Sigma' = sem(\Gamma, \Sigma, a.\texttt{removalKind}, a.\texttt{interfaceSignature})$ ~~and $sem(a.\texttt{logicRef}) = \mathcal{I}$~~, and ρ is the default projection (institution morphism) from $\mathcal{I}$ to $sem(\Gamma, a.\texttt{logicRef}) = \mathcal{I}'$, when $a.\texttt{logicRef}$ is present and the identity institution morphism on $\mathcal{I}$, when $a.\texttt{logicRef}$ is missing.

10.2.3.17 Semantics of filtering

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), \texttt{Filtering}) = (c, \mathcal{I}', \Sigma', \Delta') : ('keep'|'remove', Institution, Signature, Sentences)$$

If f is a Filtering such that $f.\texttt{removalKind} = keep$,

$$sem(\Gamma, (\mathcal{I},\Sigma, \mathcal{M},\Delta), f) = (keep, \mathcal{I}', \Sigma', \Delta')$$

where ~~$sem(\Gamma, (\Sigma, \Delta), f.\texttt{basicOMS}) = (\mathcal{I}, \Sigma', \Delta')$~~ $sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), f.\texttt{basicOMS}) = (\mathcal{I}', \Sigma', \Delta')$.

If f is a Filtering such that $f.\texttt{removalKind} = remove$,

$$sem(\Gamma, (\mathcal{I},\Sigma, \mathcal{M},\Delta), f) = (remove, \mathcal{I}', \Sigma', \Delta')$$

where ~~$sem(\Gamma, (\Sigma, \Delta), f.\texttt{basicOMS}) = (\mathcal{I}, \Sigma', \Delta')$~~ $sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), f.\texttt{basicOMS}) = (\mathcal{I}', \Sigma', \Delta')$.

10.2.3.18 Semantics of extension

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), \texttt{Extension}) = (\Gamma', (\mathcal{I}, \Sigma', \mathcal{M}', \Delta')) : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences))$$

If e is an Extension,

$$sem(\Gamma, (\mathcal{I}, \Sigma, \mathcal{M}, \Delta), e) = (\Gamma', (\mathcal{I}, \Sigma', \mathcal{M}', \Delta'))$$

where ~~$(\mathcal{I}, \Sigma', \mathcal{M}', \Delta') = sem(\Gamma, (\Sigma, \mathcal{M}), e.\texttt{extendingOMS})$~~ $(\Gamma', (\mathcal{I}, \Sigma', \mathcal{M}', \Delta')) = sem(\Gamma, (\Sigma, \mathcal{M}), e.\texttt{extendingOMS})$.

If $e.\texttt{conservativityStrength}$ is model-conservative or implied, the semantics is only defined if each model in $\mathcal{M}$ is the Σ-reduct of some model in $\mathcal{M}'$. In case that $e.\texttt{conservativityStrength}$ is implied, it is furthermore required that $\Sigma = \Sigma'$. If $e.\texttt{conservativityStrength}$ is consequenceconservative, the semantics is only defined if for each Σ-sentence φ, $\mathcal{M}' \models \varphi$ implies $\mathcal{M} \models \varphi$. If $e.\texttt{conservativityStrength}$ is definitional, the semantics is only defined if each model in $\mathcal{M}$ is the Σ-reduct of a unique model in $\mathcal{M}'$.

If $e.\texttt{extensionName}$ is present, the inclusion link is labeled with this name.

10.2.3.19 Semantics of interface signatures

$$sem(\Gamma, \Sigma, \texttt{RemovalKind}, \texttt{InterfaceSignature}) = \Sigma' : Signature$$

If r is a RemovalKind and s is an InterfaceSignature, ~~$sem(\Gamma, \Sigma, \texttt{Qual Symbol+}) = \Sigma'$,~~

$$sem(\Gamma, \Sigma, r, s) = \Sigma'$$

where

$$\Sigma' = \begin{cases} \Sigma \cap sem(\Gamma, \Sigma, s.\texttt{symbolList}) & \text{if} \quad r = keep \\ \Sigma \setminus sem(\Gamma, \Sigma, s.\texttt{symbolList}) & \text{if} \quad r = remove \end{cases}$$

10.2.3.20 Semantics of OMS definitions

$$\boxed{\begin{array}{ll} sem(\Gamma, \texttt{OMSDefinition}) & = \Gamma' \\ & : LogicalEnvironment \end{array}}$$

An OMSDefinition O extends the global environment~~:~~

$$\underline{sem(\Gamma, O) = \Gamma[O.\texttt{omsName} \mapsto sem(\Gamma, O.\texttt{oms})]}$$

:

$$sem(\Gamma, O) = \Gamma''$$

where $\Gamma.logic$ and ~~**Inst**$(O.\texttt{oms})$~~ $\mathcal{I}$[11] must be the same ~~.~~ and for each of the institutions $\mathcal{I}_1, \ldots, \mathcal{I}_n$ supported by $\Gamma.lang$, we have Note(11)
$\Gamma_1 = \Gamma_1'[postfixLogicIRI(O.\texttt{omsName}, \mathcal{I}_1) \mapsto (\mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1)]$ where $sem(\Gamma.logic = \mathcal{I}_1, O.\texttt{oms}) = (\Gamma_1', (\mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1))$,
$\Gamma_2 = \Gamma_2'[postfixLogicIRI(O.\texttt{omsName}, \mathcal{I}_2) \mapsto (\mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2)]$ where $sem(\Gamma_1'.logic = \mathcal{I}_2, O.\texttt{oms}) = (\Gamma_2', (\mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2))$,
...
$\Gamma'' = \Gamma_n'[postfixLogicIRI(O.\texttt{omsName}, \mathcal{I}_n) \mapsto (\mathcal{I}_n, \Sigma_n, \mathcal{M}_n, \Delta_n)]$ where $sem(\Gamma_{n-1}'.logic = \mathcal{I}_n, O.\texttt{oms}) = (\Gamma_n', (\mathcal{I}_n, \Sigma_n, \mathcal{M}_n, \Delta_n))$

[12] If O.conservativityStrength is model-conservative, the semantics is only defined if ~~$sem(\Gamma, O.\texttt{oms}) \neq \emptyset$.~~ $\mathcal{M} \neq \emptyset$. Note(12)
If O.conservativityStrength is consequence-conservative, the semantics is only defined if ~~$sem(\Gamma, O.\texttt{oms})$~~ Δ has only tautologies[9] as signature-free[10] logical consequences.
If O.conservativityStrength is monomorphic, the semantics is only defined if ~~$sem(\Gamma, O.\texttt{oms})$~~ $\mathcal{M}$ consist of exactly one isomorphism class of models.
If O.conservativityStrength is weak-definitional, the semantics is only defined if ~~$sem(\Gamma, O.\texttt{oms})$~~ $\mathcal{M}$ is empty or a singleton.
If O.conservativityStrength is definitional, the semantics is only defined if ~~$sem(\Gamma, O.\texttt{oms})$~~ $\mathcal{M}$ is a singleton.

10.2.3.21 Semantics of OMS references

$$\boxed{\begin{array}{ll} sem(\Gamma, \texttt{OMSReference}) & = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)) \\ & : (LogicalEnvironment, (Institution, Signature, ModelClass, Sentences)) \end{array}}$$

~~If O is an~~
The rule for OMSReference~~,~~

$$\underline{sem(\Gamma, O) = \Gamma(O.\texttt{omsRef})}$$

s has been given above, as OMSReferences are a particular case of ClosableOMS.

10.2.3.22 Semantics of symbols

$$\boxed{\begin{array}{ll} sem(\Gamma, \Sigma, \texttt{Symbol}) & = s \\ & : LogicalSymbol \end{array}}$$

If sym is a Symbol

$$sem(\Gamma, \Sigma, sym) = s$$

where s is a logic-specific symbol with the name sym.iri from $|\Sigma|$.[13] If such symbol does not exist, the semantics is undefined. Note(13)

10.2.3.23 Semantics of symbol map items

$$\boxed{\begin{array}{ll} sem(\Gamma, \Sigma_1, \Sigma_2, \texttt{SymbolMapItem}) & = (s_1, s_2) \\ & : (LogicalSymbol, LogicalSymbol) \end{array}}$$

If smi is a SymbolMapItem,

$$sem(\Gamma, \Sigma_1, \Sigma_2, smi) = (s_1, s_2)$$

where ~~$sem(\Gamma, \Sigma_1, smi.\texttt{symbol1}) = s_1$ and $sem(\Gamma, \Sigma_2, smi.\texttt{symbol2}) = s_2$~~ $sem(\Gamma, \Sigma_1, smi.\texttt{source}) = s_1$ and $sem(\Gamma, \Sigma_2, smi.\texttt{target}) =$

[11] Note: @Mihai: what is $\mathcal{I}$?
[12] Note: @Till: to which of the possible n institutions do these annotations refer to? TM: to $\Gamma.logic$.
[9] A tautology is a sentence holding in every model.
[10] A signature-free sentence is one over the empty signature.
[13] Note: Is there some conversion going on here? Then this should be added to the heterogeneous logical env. TM: no, I think no conversion is needed.

10.2.3.24 Semantics of general symbol map items

$$\boxed{\begin{aligned} sem(\Gamma, \Sigma_1, \Sigma_2, \texttt{GeneralSymbolMapItem}) &= (s, t) \\ &: (LogicalSymbol, LogicalSymbol) \end{aligned}}$$

If $gsmi$ is a `SymbolMapItem`, then its semantics has been given in the previous rule.
If $gsmi$ is a `Symbol`, $sem(\Gamma, \Sigma_1, \Sigma_2, gsmi) = (s, s)$ where $sem(\Gamma, \Sigma_1, gsmi) = s$ ~~.~~
~~If sen is a ,~~

$$sem(\Gamma, \Sigma, sen) = \varphi$$

~~where $\varphi \in Sen(\Sigma)$ and the analysis is done in a logic-specific way.~~

10.2.3.25 Semantics of references

$$\boxed{\begin{aligned} sem(\texttt{LolaRef}) &= L \\ &: Language|Institution \end{aligned}}$$

L is the language or the institution from the heterogeneous logical environment named by `LolaRef`.

$$\boxed{\begin{aligned} sem(\texttt{LanguageRef}) &= L \\ &: Language \end{aligned}}$$

L is the language from the heterogeneous logical environment named by `LanguageRef`.

$$\boxed{\begin{aligned} sem(\texttt{SyntaxRef}) &= S \\ &: Serialization \end{aligned}}$$

S is the serialization from the heterogeneous logical environment named by `SyntaxRef`.

$$\boxed{\begin{aligned} sem(\texttt{LogicRef}) &= L \\ &: Institution \end{aligned}}$$

L is the institution from the heterogeneous logical environment named by `LogicRef`.
~~If t is a , $sem(\Gamma, t) = \rho$ where ρ is the institution comorphism from the heterogeneous logical environment named by t.`omsLanguageTranslationRef`. This is defined only if the domain of ρ is the current logic of Γ.~~
~~If t is a , $sem(\Gamma, t) = \rho$ where ρ is the unique default institution comorphism from the heterogeneous logical environment running from $\Gamma.logic$ to t.`languageRef` (if this is a logic) or to some logic supported by t.`languageRef` (if this is a language). If there is no or no unique such comorphism, the semantics is undefined.~~

10.2.4 Semantics of OMS Mappings

10.2.4.1 Semantics of mapping definitions

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{MappingDefinition}) &= \Gamma' \\ &: LogicalEnvironment \end{aligned}}$$

See equations for `InterpretationDefinition`, `EntailmentDefinition`, `EquivalenceDefinition`, `ConservativeExtensionDefinition` and `AlignmentDefinition`.

10.2.4.2 Semantics of interpretation definitions

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{InterpretationDefinition}) &= \Gamma' \\ &: LogicalEnvironment \end{aligned}}$$

If d is an `InterpretationDefinition`,

$$sem(\Gamma, d) = \Gamma'$$

where $\Gamma' = \Gamma[d.\texttt{interpretationName} \to (G, (\rho, \sigma), L_1, L_2)]$
and G is the graph $L_1 \xrightarrow{(\rho,\sigma)} L_2$ where

- $(L_1, L_2) = sem(\Gamma, d.\texttt{interpretationType})$

- $\rho = (\Phi, \alpha, \beta) : \mathbf{Inst}(L_1) \to \mathbf{Inst}(L_2)$ is the comorphism given by $sem(\Gamma, d.$`omsLanguageTranslation`$)$. If $d.$`OMSLanguageTranslation` is missing, the default translations between the logics is selected.
- ~~$sem(\Gamma.\{current = (lang, logic', ser)\}, \Phi(\mathsf{Sig}(L_1)), \mathsf{Sig}(L_2), d.$`symbolMap`$) = \sigma$,~~ $\sigma = sem(\Gamma.\{current = (lang, logic', ser)\}, \Phi(\mathsf{Sig}(L$ where $\Gamma.current = (lang, logic, ser)$ and $logic'$ is the target logic of ρ, or, if $d.$`symbolMap` is missing, σ is the identity signature morphism on $\Phi(\mathsf{Sig}(L_1))$ which must be equal with $\mathsf{Sig}(L_2)$.

The semantics is only defined if $\beta_{\mathsf{Sig}(L_1)}(M_2|_\sigma) \in \mathsf{Mod}(L_1)$ for each $M_2 \in \mathsf{Mod}(L_2)$.
If the optional argument $d.$`conservativityStrength` is

- `model-conservative`, for each model $M_1 \in \mathsf{Mod}(L_1)$ there must exist a model $M_2 \in \mathsf{Mod}(L_2)$ such that $\beta_{\mathsf{Sig}(L_1)}(M_2|_\sigma) = M_1$.
- `consequence-conservative`, for each $\mathsf{Sig}(L_1)$-sentence φ, if $\mathcal{M}_2 \models \sigma(\alpha_{\mathsf{Sig}(L_1)}(\varphi))$ then $\mathcal{M}_1 \models \varphi$.
- `not-model-conservative`, there must exist a model $M_1 \in \mathsf{Mod}(L_1)$ such that there is no model $M_2 \in \mathsf{Mod}(L_2)$ such that $\beta_{\mathsf{Sig}(L_1)}(M_2|_\sigma) = M_1$.
- `not-consequence-conservative`, there is a $\mathsf{Sig}(L_1)$-sentence φ, such that $\mathcal{M}_2 \models \sigma(\alpha_{\mathsf{Sig}(L_1)}(\varphi))$ and $\mathcal{M}_1 \not\models \varphi$.

10.2.4.3 Semantics of refinement definitions

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{RefinementDefinition}) &= \Gamma' \\ &: LogicalEnvironment \end{aligned}}$$

If d is a `RefinementDefinition`,

$$sem(\Gamma, d) = \Gamma'$$

where ~~$\Gamma' = \Gamma[d.$`interpretationName` $\mapsto (G, \sigma, N_1, N_2)]$ and $sem(\Gamma, d.$`refinement`$) = (G, \sigma, N_1, N_2)$~~$\Gamma' = \Gamma[d.$`interpretationNam`

10.2.4.4 Semantics of interpretation types

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{InterpretationType}) &= ((N_1, \mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1), (N_2, \mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2)) \\ &: (NodeLabel, NodeLabel) \end{aligned}}$$

If t is an `InterpretationType`,

$$sem(\Gamma, t) = (L_1, L_2)$$

where

- ~~$\mathbf{Name}(L_1) = \mathbf{Name}(t.$`oms`$)$ and $\mathbf{Name}(L_2) = \mathbf{Name}(t.$`oms2`$)$~~$\mathbf{Name}(L_1) = \mathbf{Name}(t.$`source`$)$ and $\mathbf{Name}(L_2) = \mathbf{Name}(t.$`targ`
- ~~$(\mathbf{Inst}(L_1), \mathsf{Sig}(L_1), \mathsf{Mod}(L_1), \mathsf{Th}(L_1)) = sem(\Gamma, t.$`oms`$)$~~$(\mathbf{Inst}(L_1), \mathsf{Sig}(L_1), \mathsf{Mod}(L_1), \mathsf{Th}(L_1)) = sem(\Gamma, t.$`source`$)$,
- ~~$(\mathbf{Inst}(L_2), \mathsf{Sig}(L_2), \mathsf{Mod}(L_2), \mathsf{Th}(L_2)) = sem(\Gamma, t.$`oms2`$)$~~$(\mathbf{Inst}(L_2), \mathsf{Sig}(L_2), \mathsf{Mod}(L_2), \mathsf{Th}(L_2)) = sem(\Gamma, t.$`target`$)$,

10.2.4.5 Semantics of refinements

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{Refinement}) &= (((G_1, G_2), \sigma, \mathcal{M}) \\ &: (OMSGraph, OMSGraph, GraphMorphism, ModelClass) \end{aligned}}$$

The signature of a refinement is a pair consisting of the graph of the OMS or network of OMS being refined and the graph of the OMS or network of OMS after refinement. Together with this pair the mapping is stored along which the refinement is done. Given two networks G_1 and G_2, a *network morphism* $\sigma : G_1 \to G_2$ is

1. a ~~functor~~ graph homomorphism $\sigma^G : Shape(G_1) \to Shape(G_2)$, ~~together with~~ where given a network G, its shape $Shape(G)$ is a graph with same nodes and edges as G but with no labels of nodes,
2. a natural transformation $\sigma^M : G_1 \to \sigma^G; G_2$

such that

1. for each node N_1 in G_1 labeled with $(\mathcal{I}_1, \Sigma_1, \mathcal{M}_1)$ such that $\sigma^G(N_1)$ is a node N_2 labeled with $(\mathcal{I}_2, \Sigma_2, \mathcal{M}_2)$ in G_2, there is a signature morphism $(\rho^M_{N_1}, \sigma^M_{N_1}) : (\mathcal{I}_1, \Sigma_1) \to (\mathcal{I}_2, \Sigma_2)$, where
2. $\rho^M_{N_1} = (\Phi, \alpha, \beta) : \mathcal{I}_1 \to \mathcal{I}_2$ is an institution comorphism between the logics of the two nodes and $\sigma^M_{N_1} : \Phi(\Sigma_1) \to \Sigma_2$ is a signature morphism, such that $\beta_{\Sigma_1}(M_2|_{\sigma^M_{N_1}}) \in \mathcal{M}_1$ for each $M_2 \in \mathcal{M}_2$.

A refinement model is a class $\mathcal{M}$ of pairs of families of models compatible with the two networks. Given a network morphism $\sigma : G_1 \to G_2$ and a G_2 model F, $F|_\sigma$ is defined as the family of models $\{M_i\}_{i \in Nodes(G_1)}$ such that $M_i = F_{\sigma^G(i)}|_{\sigma^M_i}$ for each $i \in Nodes(G_1)$.

Thus, the semantics of a `Refinement` consists of

- a refinement signature (G_1, G_2),
- a network morphism σ and
- a refinement model $\mathcal{M}$.

If r is `RefinementOMS`,

$$sem(\Gamma, \mathtt{r}) = ((G, G), \sigma, \mathcal{M})$$

where

- G is a graph with just one isolated node N such that $\mathbf{Name}(N) = \mathbf{Name}(r.\mathtt{oms})$ and the other elements of the tuple labeling ~~L~~ N are given by $sem(\Gamma, r.\mathtt{oms})$,
- σ is the identity morphism on $\mathsf{Sig}(r.\mathtt{oms})$,
- $\mathcal{M} = \{((M), (M)) \mid M \in \mathsf{Mod}(r.\mathtt{oms})\}$, where (M) is the singleton family consisting of M.

If r is `RefinementNetwork`,

$$sem(\Gamma, r) = ((G, G), \sigma, \mathcal{M})$$

where $sem(\Gamma, r.\mathtt{network}) = G$, σ is the identity network morphism on G and $\mathcal{M} = \{(F, F) \mid F \in \mathsf{Mod}(G)\}$.

~~If r is ,~~

$$\cancel{sem(\Gamma, r) = ((G_1, G_2'), \sigma, \mathcal{M})}$$

~~where $sem(\Gamma, r.\mathtt{refinement}) = ((G_1, G_1'), \sigma_1, \mathcal{M}_1)$, $sem(\Gamma, r.\mathtt{refinement2}) = ((G_2, G_2'), \sigma_2, \mathcal{M}_2)$ such that $G_1' = G_2$, $\sigma = \sigma_1; \sigma_2$ is a network morphism from G_1 to G_2', and $\mathcal{M} = \{(F_1, F_3) \mid \exists F_2$ such that $(F_1, F_2) \in \mathcal{M}_1$ and $(F_2, F_3) \in \mathcal{M}_2\}$~~

If r is ~~,~~ `SimpleRefinement`,[14] Note(14

$$sem(\Gamma, r) = ((G_1, G'_2), \sigma', \mathcal{M}')$$

where

~~$sem^M(\Gamma, r.\mathtt{oms}) = (\mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1)$, $sem(\Gamma, r.\mathtt{refinement}) = ((G_1, G_2), (\rho_2, \sigma_2), \mathcal{M}')$ such that G_1 consists of an isolated node labeled with $(\mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2)$, $sem(\Gamma, (\mathcal{I}_1, \Sigma_1), (\mathcal{I}_2, \Sigma_2), r.\mathtt{omsRefinementMap}) = (\rho_1 = (\Phi, \alpha, \beta) : \mathcal{I}_1 \to \mathcal{I}_2, \sigma_1 : \Phi(\Sigma_1) \to \Sigma_2)$, for each $(M_1, M_2) \in \mathcal{M}'$, $\beta_{\Sigma_1}(M_1|_{\sigma_1}) \in \mathcal{M}_1$, G consists of an isolated node labeled with $sem^M(\Gamma, r.\mathtt{oms})$, $\sigma = (\rho_1, \sigma_1); (\rho_2, \sigma_2)$ and $\mathcal{M} = \{(\beta_{\Sigma_1}(M_1|_{\sigma_1}), M_2) \mid (M_1, M_2) \in \mathcal{M}'\}$.~~

~~If r is ,~~

$$\cancel{sem(\Gamma, r) = ((G_1, G_2), \sigma, \mathcal{M})}$$

~~where~~ $sem(\Gamma, r.\mathtt{refinementSource}) = ((G_1, G_1'), \sigma_1, \mathcal{M}_1)$,
~~$sem^M(\Gamma, r.\mathtt{network}) = G_1$~~ $sem(\Gamma, r.\mathtt{refinementTarget}) = ((G_2, G_2'), \sigma_2, \mathcal{M}_2)$,
~~$sem(\Gamma, r.\mathtt{refinement}) = ((G_1', G_2), \sigma_2, \mathcal{M}')$~~ $sem(\Gamma, G_1', G_2, r.\mathtt{refMap}) = \sigma : G_1' \to G_2$,
$sem(\Gamma, G_1, G_2, r.\mathtt{networkRefinementMap}) = \sigma_1 : G_1 \to G_1'$,
~~$\sigma = \sigma_1; \sigma_2$ is a network morphism and $\mathcal{M} = \{(F_2|_\sigma, F_2) \mid (F_1, F_2) \in \mathcal{M}'\}$.~~

~~If m is an ,~~

$$\cancel{sem(\Gamma, (I_1, \Sigma_1), (I_2, \Sigma_2), m) = ((\Phi, \alpha, \beta), \sigma)}$$

~~where $sem(\Gamma, m.\mathtt{omsLanguageTranslation}) = (\Phi, \alpha, \beta) : \mathcal{I}_1' \to \mathcal{I}_2'$ such that $\mathcal{I}_1' = \mathcal{I}_1$ and $\mathcal{I}_2' = \mathcal{I}_2$~~ $\sigma' = \sigma_1; \sigma; \sigma_2$
and ~~$sem(\Gamma.current = (lang', logic', ser'), \Phi(\Sigma_1), \Sigma_2, m.\mathtt{symbolMap}) = \sigma : \Phi(\Sigma_1) \to \Sigma_2$ where $\Gamma.current = (lang, logic, ser)$, $logic'$ is the target logic of (Φ, α, β), and $lang'$ and ser' are the default language and serializations for $logic'$~~ $\mathcal{M} = \{(F_1, F_3) \mid \exists F_2$ such tha

~~If m is a ,~~

$$\cancel{sem(\Gamma, G_1, G_2, m) = sem(\Gamma, G_1, G_2, m.nodeMap)}$$

10.2.4.6 Semantics of a set of refinements

$$\boxed{\begin{array}{ll} sem(\Gamma, G_1, G_2, Set(\mathtt{Refinement})) & = \sigma \\ & : GraphMorphism \end{array}}$$

If ~~$m_1, \ldots, m_n$ are all s,~~ $r_1, \ldots, r_n$ are all `Refinements`,

$$sem(\Gamma, G_1, G_2, Set\{m(r_1, \ldots, mr_n\})) = \sigma:$$

where

~~$sem(\Gamma, G_1, G_2, m_1) = (name_1^1, name_2^1, \rho_1, \sigma_1), \ldots$~~ $sem(\Gamma, r_1) = ((G_1^1, G_2^1), \sigma_1, \mathcal{M}_1), \ldots,$

~~$sem(\Gamma, G_1, G_2, m_n) = (name_1^n, name_2^n, \rho_n, \sigma_n)$ and~~$sem(\Gamma, r_n) = ((G_1^n, G_2^n), \sigma_n, \mathcal{M}_n)$ ~~$\sigma^G(name_1^i) = name_2^i$ and $\sigma^M_{name_1^i} = (\rho_i, \sigma_i)$ for each $i = 1, \ldots, n$. The map is required to be~~ such that [15] $G_1 = \bigcup_{i=1,\ldots,n} G_1^i$, Note(15 $G_2 = \bigcup_{i=1,\ldots,n} G_2^i$ and $\sigma : G_1 \to G_2 = \bigcup_{i=1,\ldots,n} \sigma_i$ is total on the nodes of G_1.

10.2.4.7 Semantics of refinement maps

$$\boxed{\begin{aligned} sem(\Gamma, G_1, G_2, \texttt{RefinementMap}) &= \sigma \\ &: GraphMorphism \end{aligned}}$$

If m is ~~a~~ , an `OMSRefinementMap`,

$$sem(\Gamma, G_1, G_2, nm) = (m.name_1, m.name_2, \rho, \sigma)$$

where
~~$(\mathcal{I}_1, \Sigma_1, \mathcal{M}_1)$ is the label of $m.\texttt{omsName}$ in~~ G_1 ~~,~~must be a graph with just one isolated node labeled $(name_1, \mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1)$

~~$(\mathcal{I}_2, \Sigma_2, \mathcal{M}_2)$ is the label of $m.\texttt{omsName2}$ in~~ G_2 must be a graph with just one isolated node labeled $(name_2, \mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2)$,
~~$sem(\Gamma, m.\texttt{omsLanguageTranslation}) = \rho = (\Phi, \alpha, \beta) : \mathcal{I}_1 \to \mathcal{I}_2$,~~ $sem(\Gamma, m.\texttt{translation}) = \rho = (\Phi, \alpha, \beta) : \mathcal{I}_1 \to \mathcal{I}_2$, or if $m.\texttt{translation}$ is missing, the default comorphism between $\mathcal{I}_1$ and $\mathcal{I}_2$,
~~$sem(\Gamma.current = (lang', logic', ser'), \Phi(\Sigma_1), \Sigma_2, m.\texttt{symbolMap}) = \sigma : \Phi(\Sigma_1) \to \Sigma_2$,~~$sem(\Gamma', \Phi(\Sigma_1), \Sigma_2, m.\texttt{symbolMap}) = \sigma : \Phi(\Sigma_1) \to \Sigma$
where $\Gamma.current = (lang, logic, ser)$, $logic'$ is the target logic of (Φ, α, β)~~and~~ , $lang'$ and ser' are the default language and serialization for $logic'$ and $\Gamma' = \Gamma.current = (lang', logic', ser')$, or, when $m.\texttt{symbolMap}$ is missing, $\Phi(\Sigma_1)$ and Σ_2 must be the same and σ is the identity signature morphism on Σ_2.

If m is a `NetworkRefinementMap`,

$$sem(\Gamma, G_1, G_2, m) = \sigma$$

where $sem(\Gamma, G_1, G_2, m.refinements) = \sigma$.

10.2.4.8 Semantics of entailment definitions

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{EntailmentDefinition}) &= \Gamma' \\ &: LogicalEnvironment \end{aligned}}$$

If e is an `EntailmentDefinition`,

$$sem(\Gamma, e) = \Gamma'$$

where $\Gamma' = \Gamma[e.\texttt{entailmentName} \mapsto sem(\Gamma, e.\texttt{entailmentType})]$.

10.2.4.9 Semantics of entailment types

$$\boxed{\begin{aligned} sem(\Gamma, \texttt{EntailmentType}) &= G \\ &: OMSGraph \end{aligned}}$$

If t is an `OMSOMSEntailment`,

$$sem(\Gamma, t) = L_2 \stackrel{\text{id}}{\to} L_1$$

where ~~$\mathbf{Name}(L_1) = \mathbf{Name}(t.\texttt{oms})$, $\mathbf{Name}(L_2) = \mathbf{Name}(t.\texttt{oms2})$~~$\mathbf{Name}(L_1) = \mathbf{Name}(t.\texttt{premise})$, $\mathbf{Name}(L_2) = \mathbf{Name}(t.\texttt{conclusi}$ ~~$(\mathbf{Inst}(L_1), \mathsf{Sig}(L_1), \mathsf{Mod}(L_1), \mathsf{Th}(L_1)) = sem(\Gamma, t.\texttt{oms})$, $(\mathbf{Inst}(L_2), \mathsf{Sig}(L_2), \mathsf{Mod}(L_2), \mathsf{Th}(L_2)) = sem(\Gamma, t.\texttt{oms2})$~~ $(\mathbf{Inst}(L_1), \mathsf{Sig}(L_1), \mathsf{Mod}$ $(\mathbf{Inst}(L_2), \mathsf{Sig}(L_2), \mathsf{Mod}(L_2), \mathsf{Th}(L_2)) = sem(\Gamma, t.\texttt{conclusion})$ such that $\mathsf{Sig}(L_1) = \mathsf{Sig}(L_2)$ and $\mathsf{Mod}(L_1) \subseteq \mathsf{Mod}(L_2)$ and id is the identity morphism on $\mathsf{Sig}(L_1)$.

If t is a `NetworkOMSEntailment`, $sem(\Gamma, t) = G$
where $sem(\Gamma, t.\texttt{network}) = G'$ such that G' contains a node n labeled with ~~$\mathbf{Name}(t.\texttt{omsName})$~~$\mathbf{Name}(t.\texttt{premise})$, $sem(\Gamma, t.\texttt{oms}) = (\mathcal{I}, \Sigma, \mathcal{M}_2, \Delta_2)$ and $\{\mathcal{M}_n \mid \mathcal{M}$ is compatible with $G'\} \subseteq \mathcal{M}_2$. Then G extends G' with a new node whose label has the name $\mathbf{Name}(t.\texttt{oms})$ and the other components given by $sem(\Gamma, t.\texttt{oms})$ and with a new theorem link from this new node to the node $\mathbf{Name}(t.\texttt{omsName})$, labeled with the identity morphism on Σ.

If t is a `NetworkNetworkEntailment`,

$$sem(\Gamma, t) = G$$

[14] NOTE: @MC: check!
[15] NOTE: the union operations should be defined. Note that it would be possible to include even network morphisms in a refinement map. TM: cool! I did not think of this possibility, but it clearly makes sense. It is analogous to the possibility to include networks into the definition of other networks. Note hwever that the semantics of a set of refinements should be a network morphism, not a graph morphism.

where $\cancel{sem(\Gamma, t.\mathtt{network}) = G_1, sem(\Gamma, t.\mathtt{network2}) = G_2}$ $sem(\Gamma, t.\mathtt{premise}) = G_1$, $sem(\Gamma, t.\mathtt{conclusion}) = G_2$, such that $Shape(G_1) = Shape(G_2)$ and, for each node $i \in |Shape(G_1)|$, its names in the networks G_1 and G_2 are the same, its signatures are the same and the class of models obtained by projecting each family of models compatible with G_1 to the component i is included in the class of models obtained by projecting each family of models compatible with G_2 to the component i. Then G extends the union of G_1 and G_2 for each pair of nodes (i_1, i_2), where i_1 and i_2 identify the occurrences of the same node i in G_1 and G_2 respectively, with a theorem link from i_1 to i_2 labeled with the identity on $\mathsf{Sig}(i_1)$.

10.2.4.10 Semantics of equivalence definitions

$$\boxed{\begin{aligned} sem(\Gamma, \mathtt{EquivalenceDefinition}) \quad &= \Gamma' \\ &: LogicalEnvironment \end{aligned}}$$

If d is an `EquivalenceDefinition`,

$$sem(\Gamma, d) = \Gamma'$$

where $\Gamma' = \Gamma[d.\mathtt{equivalenceName} \mapsto sem(\Gamma, d.\mathtt{equivalenceType})]$.

10.2.4.11 Semantics of OMS equivalences

$$\boxed{\begin{aligned} sem(\Gamma, \mathtt{OMSEquivalence}) \quad &= (G, N_1, N_2) \\ &: (OMSGraph, Node, Node) \end{aligned}}$$

If t is an `OMSEquivalence`,

$$sem(\Gamma, t) = (G, N_1, N_2)$$

where $O_1 = t.\mathtt{oms}$, $O_2 = t.\mathtt{oms2}$, $\cancel{O_3 = t.\mathtt{oms3}, sem^{(\mathsf{Sig}(O_1)\cup\mathsf{Sig}(O_2),\emptyset)}_{\Gamma.lang,\Gamma.logic,\Gamma.ser}(O_3) = (\mathcal{I}, \Sigma, \mathcal{M}, \Delta)}$ $O_3 = t.\mathtt{mediatingOMS}$, $sem(\Gamma, (\mathcal{I}, \mathsf{Sig}(O_1) \cup \mathsf{Sig}(O_2), \mathsf{Mod}^{\mathcal{I}}(\mathsf{Sig}(O_1) \cup \mathsf{Sig}(O_2)), \emptyset), O_3) = (\Gamma', (\mathcal{I}, \Sigma, \mathcal{M}, \Delta))$
G is the graph $N_1 \xrightarrow{\iota_1} N_3 \xleftarrow{\iota_2} N_3$ where

1. N_1 is labeled with $(\mathbf{Name}(O_1), \mathbf{Inst}(O_1), \mathsf{Sig}(O_1), \mathsf{Mod}(O_1), \mathsf{Th}(O_1))$,
2. N_2 is labeled with $(\mathbf{Name}(O_2), \mathbf{Inst}(O_2), \mathsf{Sig}(O_2), \mathsf{Mod}(O_2), \mathsf{Th}(O_2))$ and
3. N_3 is labeled with $(\mathbf{Name}(O_3), \mathcal{I}, \Sigma, \mathcal{M}, \Delta)$

such that

1. $\iota_i : \mathsf{Sig}(O_i) \to \Sigma$ are signature inclusions,
2. $\cancel{\mathbf{Inst}(O_1) = \mathbf{Inst}(O_2) = \mathbf{Inst}(O_3)}$ $\mathcal{I} = \mathbf{Inst}(O_1) = \mathbf{Inst}(O_2)$ and
3. for each $i = 1, 2$ and each model $M_i \in \mathsf{Mod}(O_i)$ there exists a unique model $M \in \mathcal{M}$ such that $M|_{\mathsf{Sig}(O_i)} = M_i$.

10.2.4.12 Semantics of network equivalences

$$\boxed{\begin{aligned} sem(\Gamma, \mathtt{NetworkEquivalence}) \quad &= (G_1, G_2, G_3) \\ &: (OMSGraph, OMSGraph, OMSGraph) \end{aligned}}$$

If t is a `NetworkEquivalence`,

$$sem(\Gamma, t) = (G_1, G_2, G_3)$$

where $n_1 = t.\mathtt{network}$, $n_2 = t.\mathtt{network2}$, $\cancel{n_3 = t.\mathtt{network3}}$ $n_3 = t.\mathtt{mediatingNetwork}$, $sem(\Gamma, n_1) = G_1$, $sem(\Gamma, n_2) = G_2$, $sem(\Gamma, n_3) = G_3$ such that G_1 and G_2 are subgraphs of G_3 and for each $i = 1, 2$ and each family of models $\mathcal{M}_i$ compatible with G_i there is a unique family of models $\mathcal{M}$ compatible with G_3 such that the projection of $\mathcal{M}$ to the nodes in G_i is $\mathcal{M}_i$.

10.2.4.13 Semantics of conservative extension definitions

$$\boxed{\begin{aligned} sem(\Gamma, \mathtt{ConservativeExtensionDefinition}) \quad &= \Gamma' \\ &: LogicalEnvironment \end{aligned}}$$

[16] Note(16

If d is a `ConservativeExtensionDefinition`,

$$sem(\Gamma, d) = \Gamma'$$

[16] NOTE: in the manifesto this is called a conservative extension. TM: we have decided to change this into conservative extension! “moduleType” needs to be changed once this has been changed in the metamodel.

where ~~$O_1 = d.\mathtt{oms}, O_2 = d.\mathtt{oms2}$~~$O_1 = d.\mathtt{moduleType.module}, O_2 = d.\mathtt{moduleType.whole}$, $c = d.\mathtt{conservativityType}$, $\Sigma = sem(\Gamma, d.\mathtt{interfaceSignature})$, $\Gamma' = \Gamma[d.\mathtt{moduleName} \mapsto (G, \iota, N_2, N_1)]$ and G is the graph $N_1 \xrightarrow{\iota} N_2$ where N_1 is labeled with $(O_1, \mathbf{Inst}(O_1), \mathsf{Sig}(O_1), \mathsf{Mod}(O_1), \mathsf{Th}(O_1))$, N_2 with $(O_2, \mathbf{Inst}(O_2), \mathsf{Sig}(O_2), \mathsf{Mod}(O_2), \mathsf{Th}(O_2))$, and ι is an inclusion, when $\Sigma \subseteq \mathsf{Sig}(O_2) \subseteq \mathsf{Sig}(O_1)$ and if ~~$c = \mathtt{mcons}$~~ $c = \mathtt{model\text{-}conservative}$ and for each $M \in \mathsf{Mod}(O_2)$ there is a model $M' \in \mathsf{Mod}(O_1)$ such that $M'|_\Sigma = M|_\Sigma$, or if ~~$c = \mathtt{ccons}$~~ $c = \mathtt{consequence\text{-}conservative}$ and for each $\varphi \in \mathsf{Sen}(\Sigma)$, $O_1 \models \varphi$ implies $O_2 \models \varphi$.

10.2.4.14 Semantics of alignment definitions

$$\boxed{\begin{aligned} sem(\Gamma, \mathtt{AlignmentDefinition}) &= \Gamma' \\ &: \mathit{LogicalEnvironment} \end{aligned}}$$

If d is an $\mathtt{AlignmentDefinition}$,

$$sem(\Gamma, d) = \Gamma'$$

where ~~$sem(\Gamma, d.\mathtt{alignmentType}) = (L_1, L_2)$ and $\Gamma' = \Gamma[d.\mathtt{AlignmentName} \mapsto (G, id, L_1, L_2)]$~~$sem(\Gamma, d.\mathtt{alignmentType}) = (\Gamma_0, L_1, L$
and $\Gamma' = \Gamma_0[d.\mathtt{AlignmentName} \mapsto (G, L_1', L_2')]$,
where $(L_1', L_2') = sem(\Gamma, L_1, L_2, d.\mathtt{alignmentSemantics})$~~and~~,
~~$G = sem(\Gamma, L_1', L_2', d.\mathtt{alignmentCardinalityPair}, d.\mathtt{alignmentSemantics}, d.\mathtt{correspondence})$~~$card = d.\mathtt{alignmentCardinal}$
or, when this is missing, $card = ('1', '1')$,
$aSem = d.\mathtt{alignmentSemantics}$ or, when this is missing, $aSem = \mathtt{single\text{-}domain}$,
and $G = sem(\Gamma_0, L_1', L_2', card, aSem, d.\mathtt{correspondence})$.

10.2.4.15 Semantics of alignment types

$$\boxed{\begin{aligned} sem(\Gamma, \mathtt{AlignmentType}) &= (\Gamma', L_1, L_2) \\ &: (\mathit{LogicalEnvironment}, \mathit{NodeLabel}, \mathit{NodeLabel}) \end{aligned}}$$

If t is an $\mathtt{AlignmentType}$

$$sem(\Gamma, t) = (\Gamma'', L_1, L_2)$$

where $sem(\Gamma, t.\mathtt{source}) = (\Gamma', (\mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1))$, $sem(\Gamma', t.\mathtt{target}) = (\Gamma'', (\mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2))$, L_1 ~~is a node label whose name is $\mathbf{Name}(t.\mathtt{oms})$ and whose other components are given by $sem(\Gamma, t.\mathtt{oms})$ and similarly,~~ and L_2 ~~is a node label whose name is $\mathbf{Name}(t.\mathtt{oms2})$ and whose other components are given by $sem(\Gamma, t.\mathtt{oms2})$~~are the labels of the nodes of $t.\mathtt{source}$ and $t.\mathtt{target}$ in $\Gamma'.imports$.

10.2.4.16 Semantics of alignments

$$\boxed{\begin{aligned} sem(\Gamma, L_1, L_2, (\mathtt{AlignmentCardinality}, \mathtt{AlignmentCardinality}), \mathtt{AlignmentSemantics}, Set(\mathtt{Correspondence})) &= G \\ &: OMSC \end{aligned}}$$

If ~~$card$ is a set of s, sem~~ $card_1, card_2$ are $\mathtt{AlignmentCardinality}$, $aSem$ is an $\mathtt{AlignmentSemantics}$ and $C = Set\{c_1, \ldots, c_n\}$ a set of $\mathtt{Correspondence}$s,

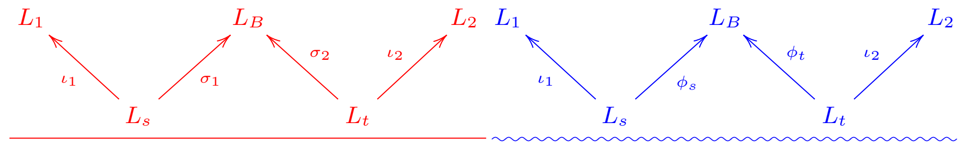
$$sem(\Gamma, L_1, L_2, (card_1, card_2), \cancel{sem}\,aSem, C) = G$$

where $sem(\Gamma, \mathsf{Sig}(L_1), \mathsf{Sig}(L_2), aSem, C) = (\Sigma_s, \Sigma_t, (\Sigma, \Delta), \phi_s : \Sigma_s \to \Sigma, \phi_t : \Sigma_t \to \Sigma, smap, cvalues)$,
~~if~~ where the semantics of the alignment is not defined in the following cases:

- if $cvalues = True$ and then at least one of the correspondences ~~$c_1, \ldots, c_n$~~ in C has a confidence value different than 1 or
- if the alignment does not have the specified cardinality, i.e.
  - if $card_1 = '?'$, then $smap$ must be injective,
  - if $card_1 = '+'$, then $smap$ must be total on the symbols of $\mathsf{Sig}(L_1)$,
  - if $card_1 = '1'$, then $smap$ must be injective and total,
  - if $card_1 = '*'$, then no cardinality restriction on $smap$ is made,
  - if $card_2 = '?'$, then $smap^{-1}$ must be injective,
  - if $card_2 = '+'$, then $smap^{-1}$ must be total on the symbols of $\mathsf{Sig}(L_2)$, ~~then the semantics of the alignment is not defined, and the alignment is ill-formed if the alignment mapping does not have the arities given by $card$, otherwise~~

- if $card_2 =' 1'$, then $smap^{-1}$ must be injective and total,
- if $card_2 =' *'$, then no cardinality restriction on $smap^{-1}$ is made,

and when the above conditions are met, G is a W-shaped graph as below



where ~~L_B, L_s and L_t are built in a logic-specific way from the correspondences $C_1, \ldots, C_n$ taking into account sem. [?] illustrates how this construction works in the case of OWL, in a way that can be generalized to other logics~~$L_s = (alignName + +"\_so$
$L_t = (alignName + +"\_target", \mathbf{Inst}(L_2), \Sigma_t, \mathsf{Mod}(\Sigma_t), \emptyset)$ and $L_B = (alignName + +"\_bridge", \mathcal{I}, \Sigma, \mathsf{Mod}((\Sigma, \Delta)), \Delta)$.

10.2.4.17 Semantics of sets of correspondences

$$sem(\Gamma, \Sigma_1, \Sigma_2, \texttt{AlignmentSemantics}, Set(\texttt{Correspondence})) = (\Sigma_s, \Sigma_t, (\mathcal{I}, \Sigma, \Delta), \phi_s : \Sigma_s \to \Sigma, \phi_t : \Sigma_t \to \Sigma, smap, cvalues)$$
$$: (Signature, Signature, (Institution, Signature, Sentences), Sig$$

If $c_1, \ldots, c_n$ are all `Correspondence`s and $aSem$ is an `AlignmentSemantics`,

$$sem(\Gamma, \Sigma_1, \Sigma_2, aSem, Set(c_1, \ldots, c_n)) = (\Sigma_s, \Sigma_t, (\Sigma, \Delta), \phi_s, \phi_t, smap, cvalues)$$

where $sem(\Gamma, \Sigma_1, \Sigma_2, (1, \texttt{equivalent}), c_i) = (clist_i, cvalues_i)$ for $i = 1, \ldots, n$,
$cvalues = \bigvee_{i=1,\ldots,n} cvalues_i$,
$smap = \{s_i^1 \mapsto s_i^2\}$,
$(\mathcal{I}, \Sigma, \Delta, \phi_s : \Sigma_s \to \Sigma, \phi_t : \Sigma_t \to \Sigma) = theoryOfCorrespondences_{\Gamma.logic}(aSem, \Sigma_1, \Sigma_2, clist_1 + + \ldots + +clist_n)$.

10.2.4.18 Semantics of correspondences

$$sem(\Gamma, \Sigma_1, \Sigma_2, (defaultConf, defaultRel), \texttt{Correspondence}) = (clist, cvalues)$$
$$: (Sequence((Relation, Symbol, Symbol)), Bool)$$

If c is a `DefaultCorrespondence`,

$$sem(\Gamma, \Sigma_1, \Sigma_2, (defaultConf, defaultRel), c) = (clist, cvalues)$$

where $cvalues = True$ if $defaultConf$ is different than 1 and $False$ otherwise,
$Sequence((sym_1^1, sym_1^2), \ldots, (sym_k^1, sym_k^2)) = matchSymbols_{\Gamma.logic}(\Sigma_1, \Sigma_2)$,
$clist = Sequence((defaultRel, sym_i^1, sym_i^2)\}_{i=1,\ldots,k})$.

If c is a `SingleCorrespondence`,

$$sem(\Gamma, \Sigma_1, \Sigma_2, (defaultConf, defaultRel), c) = (clist, cvalues)$$

where $conf = \begin{cases} defaultConf & c.\texttt{confidence} \text{ is missing,} \\ c.\texttt{confidence} & \text{otherwise} \end{cases}$

$rel = \begin{cases} defaultRel & c.\texttt{relation} \text{ is missing,} \\ c.\texttt{relation} & \text{otherwise} \end{cases}$

$cvalues = \begin{cases} False & conf = 1 \\ True & \text{otherwise} \end{cases}$

$sem(\Gamma, \Sigma_1, c.generalizedTerm) = sym1$, [17] $sem(\Gamma, \Sigma_2, c.symbolRef) = sym2$, $clist = Sequence((rel, sym1, sym2))$. Note(17

If c is a `CorrespondenceBlock`,

$$sem(\Gamma, \Sigma_1, \Sigma_2, (defaultConf, defaultRel), c) = (clist, cvalues)$$

where if $c.\texttt{relation}$ is missing, $rel = defaultRel$, else $rel = c.\texttt{relation}$
if $c.\texttt{confidence}$ is missing, $conf = defaultConf$, else $conf = c.\texttt{confidence}$
for all correspondences $c_1, \ldots, c_n$ in $c.\texttt{correspondence}$, $(clist_i, cvalues_i) = sem(\Gamma, \Sigma_1, \Sigma_2, (conf, rel), c_i)$,
$clist = clist_1 + + \ldots + +clist_n$ and $cvalues = \bigvee_{i=1,\ldots,n} cvalues_i$.

[17] NOTE: How do we get a symbol from an IRI? TM: we would need an institution-specific function for this. But I think it is easier to assume that symbols *are* IRIs.

$$\boxed{\begin{array}{ll} sem(\Gamma, L_1, L_2, \texttt{AlignmentSemantics}) & = ((Name_1, \mathcal{I}'_1, \Sigma'_1, \mathcal{M}'_1, \Delta'_1), (Name_2, cI'_2, \Sigma'_2, \mathcal{M}'_2, \Delta'_2)) \\ & : (NodeLabel, NodeLabel) \end{array}}$$

If s is an `AlignmentSemantics`, $L_1 = (aName, \mathcal{I}_1, \Sigma_1, \mathcal{M}_1, \Delta_1)$ and $L_2 = (aName', \mathcal{I}_2, \Sigma_2, \mathcal{M}_2, \Delta_2)$

$$sem(\Gamma, L_1, L_2, s) = (rel(L_1), rel(L_2))$$

where

$$rel(L) = \begin{cases} (L_1, L_2) & \text{if } s = \texttt{single-domain} \\ relativize_{logic(\Gamma.current)}(Lname_1, \mathcal{I}_1, \Sigma'_1, \mathcal{M}'_1, \Delta'_1), (name_2, \mathcal{I}_2, \Sigma'_2, \mathcal{M}'_2, \Delta'_2) & \text{otherwise} \end{cases}$$

~~where the relativization procedure is logic-specific. An example of this is done for OWL can be found in [?]~~ $relativize_{\mathcal{I}_1}(\Sigma_1, \Delta_1) = (\Sigma'_1, \Delta$
$\mathcal{M}'_1$ is the class of Δ'_1-models, $name_1 =' relativized' + +aName$,
$relativize_{\mathcal{I}_2}(\Sigma_2, \Delta_2) = (\Sigma'_2, \Delta'_2)$, $\mathcal{M}'_2$ is the class of Δ'_2-models and $name_2 =' relativized' + +aName'$.

Annex

A Annex: DOL Ontology

(Informative)

This annex specifies the DOL Ontology, an RDF vocabulary that implements the terms and definitions from clause 4. Part of the background and design considerations of the DOL Ontology can be found in [?].

A.1 Normative State and Normative References

The canonical namespace IRI for the DOL Ontology is `http://www.omg.org/spec/DOL/dol-language/`. Normative snapshots of the implementation are published there. The IRI for the version of the DOL Ontology that corresponds to this version of the OMG standard is `http://www.omg.org/spec/DOL/20150801/dol-language/`.

The DOL Ontology is currently implemented in OWL 2 (~~W3C/TR REC-owl2-syntax:2009~~W3C/TR REC-owl2-syntax:2012). The normative snapshots are encoded in RDF/XML using the OWL 2 mapping to RDF graphs (W3C/TR REC-owl2-mapping-to-rdf:2012).

The ontology makes use of the following standard ontologies and vocabularies:

- DCMI Metadata Terms (DCMI Metadata Terms:2012)
- OMG Specification Metadata (SM) Vocabulary (OMG Specification Metadata:2014)
- SKOS (W3C/TR REC-skos-reference:2009)

~~The sources of the ontology are being maintained in OWL Manchester syntax [?] at .~~

It is intended to implement future versions of the DOL ontology as a DOL document in DOL.

A.2 Intended Applications of the DOL Ontology

Applications of the DOL Ontology include modeling statements about OMS in RDF, e.g., when annotating OMS, or when describing new conforming logics, OMS languages, serializations, translations, etc., in the registry of DOL-conforming languages and translations detailed below in clause ??.

A.3 Classes and Object Properties of the DOL Ontology

The classes in the DOL Ontology (and their annotations) correspond to the terms (and their definitions) in clause 4. Classes that are reifications of relations also have been introduced as object properties. All classes and object properties are assumed to be in the DOL Ontology namespace unless stated otherwise. The DOL Ontology additionally contains some top-level abstract classes as ~~follows:~~ shown in Fig. ??.

This reflects central issues in the structure of DOL: while DOL, as a language, is a linguistic entity, it is related to mathematical entities like logics, signatures and models through its semantics. That is, semantic entities provide the bridge bewteen linguistic and mathematical entities. Moreover, processes (like theorem proving) provide algorithmic procedcures for manipulating DOL librares, and tools implement these in software.

Below the top-level classes, the class structure is as shown in Figs. ??–??.

The top level object properties are structured in a ~~similar way :~~ way similar to that of the top-level classes, see Fig. ??.

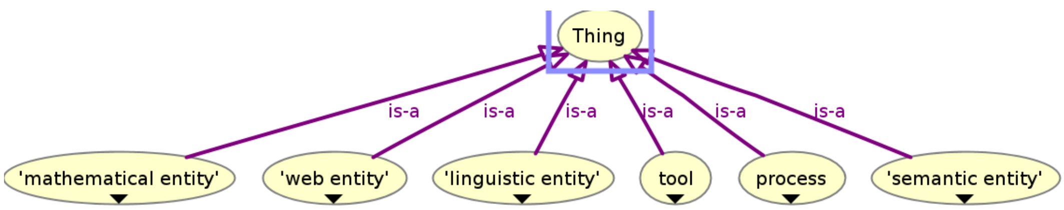


Figure A.1: DOL term ontology: top-level classes

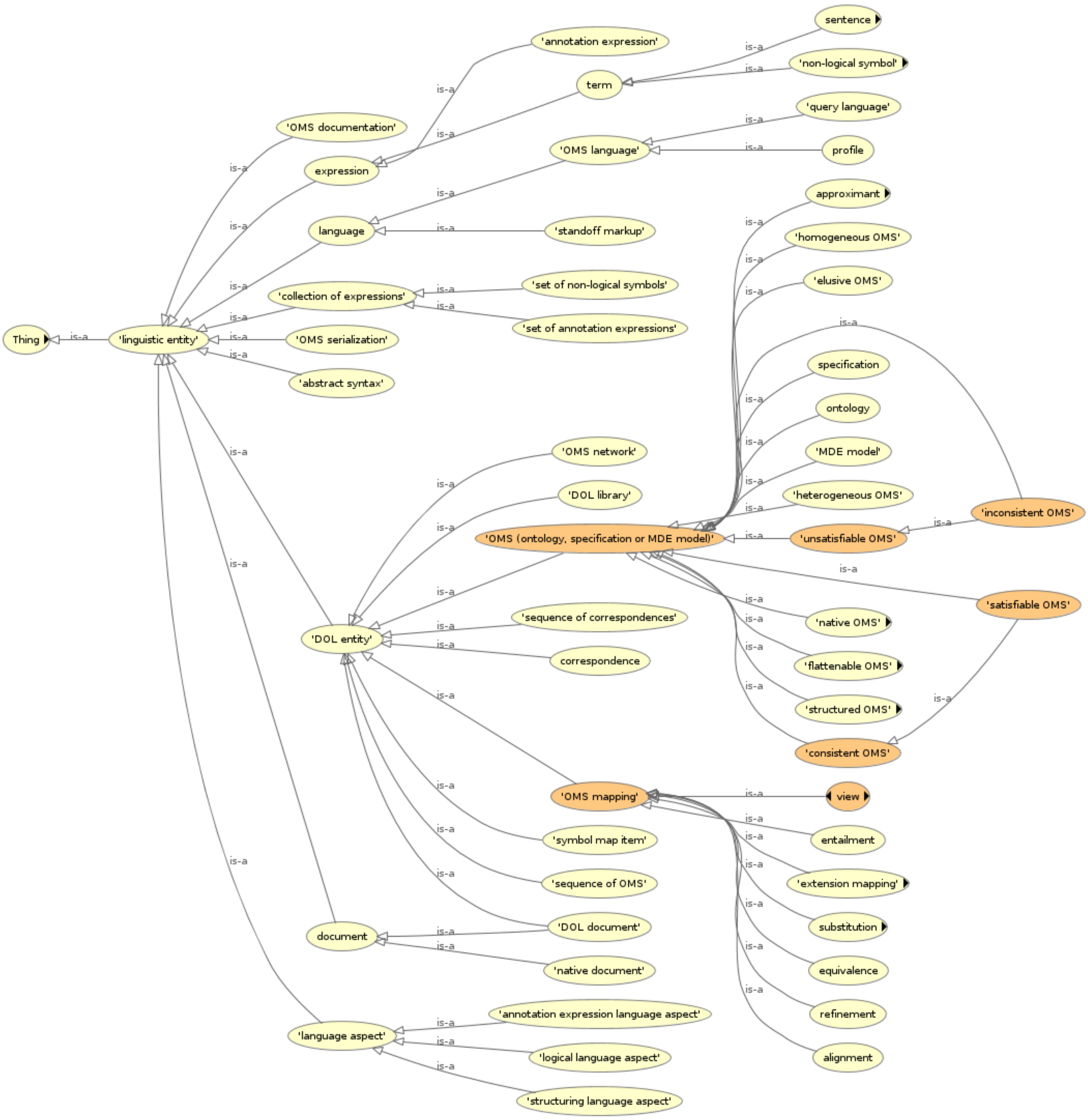


Figure A.2: DOL term ontology: linguistic entities

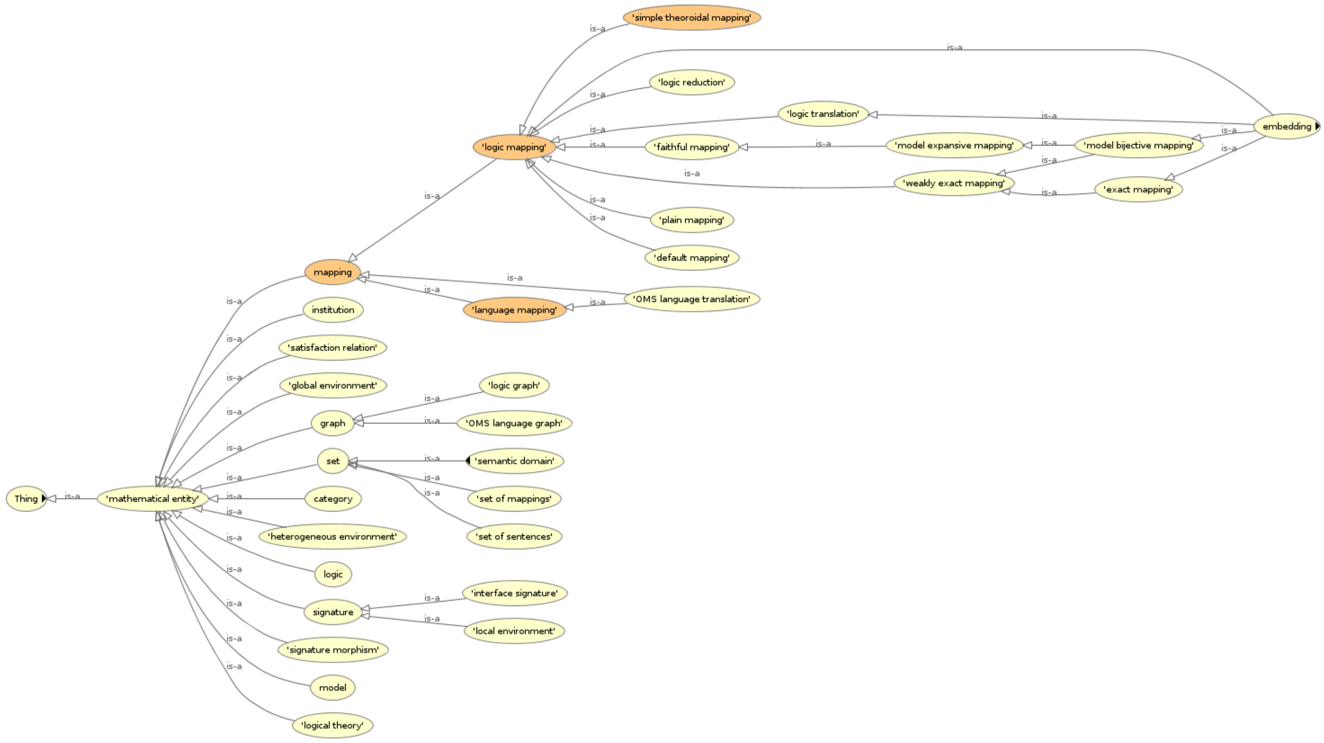


Figure A.3: DOL term ontology: mathematical entities

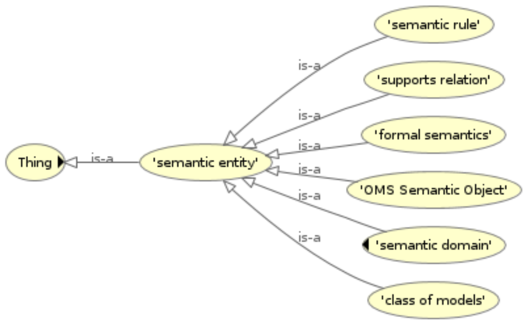


Figure A.4: DOL term ontology: semantic entities

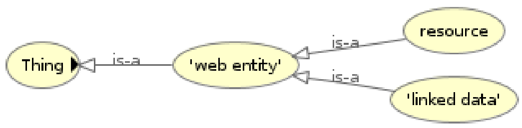


Figure A.5: DOL term ontology: web entities

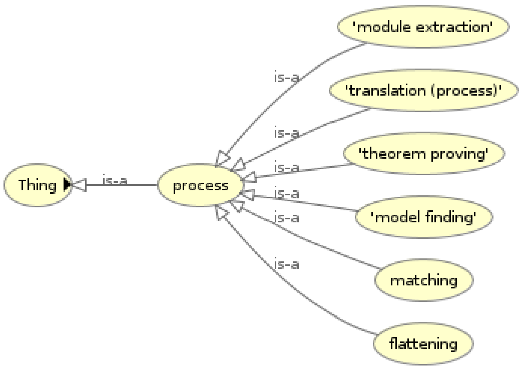


Figure A.6: DOL term ontology: processes

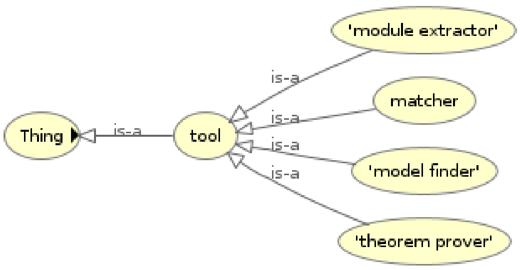


Figure A.7: DOL term ontology: tools

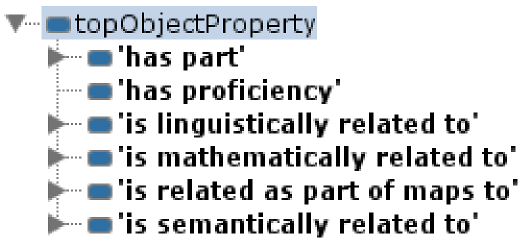


Figure A.8: DOL term ontology: properties

A.4 DOL Registry

~~It is expected that will be used for other languages than the set of -conforming languages~~ OMG hosts a registry for DOL-conforming languages and translations. This registry will enable the use of other DOL-conforming languages than the ones that are discussed in this ~~. There is a~~ ~~**for -conforming languages and translations**~~ ~~hosted at~~ OMG Specification. The registry also includes descriptions of ~~-conforming~~ DOL-conforming languages and translations (as well as other information needed by implementors and users) in both human-readable and machine-processable form.

~~There will be Maintenance Authority (MA) or, depending on advisability, a Registration Authorityestablished to maintain~~ OMG maintains the registry as an informative resource governed by the standard. The registry contents itself ~~will not be normative; however, it is expected to become the basis for normative activities~~is informative.

B Annex: Conformance of OWL 2 DL With DOL

(Informative)

[18] [19] The semantic conformance of OWL 2 DL (as specified in ~~W3C/TR REC-owl2-syntax:2009~~W3C/TR REC-owl2-syntax:2012) with DOL is established in [?]. Note(18) Note(19)

B.1 Abstract Syntax Conformance of OWL 2 With DOL

The metaclass `OWL Ontology` is a subclass (in the sense of SMOF multiple classification) of `NativeDocument`. The metaclass `OWL Universe` is a subclass (in the sense of SMOF multiple classification) of `BasicOMS`.

B.2 Conformance of the OWL Serializations With DOL

B.2.1 Text Conformance of the OWL 2 Manchester Syntax With DOL

The OWL 2 Manchester syntax satisfies the criteria for text conformance established in clause 2.2 in a straightforward way thanks to its line-based comment syntax (comments starting with #) and its flexible handling of line breaks.

B.2.2 Conformance of the XML and RDF Serializations of OWL With DOL

B.2.2.1 General Issues

With minor modifications detailed below, the OWL/XML serialization [?] satisfies the criteria for XML conformance and the serialization of OWL in RDF (W3C/TR REC-owl2-mapping-to-rdf:2012) satisfies RDF the criteria for RDF conformance. Both modifications define a super-language of the respective OWL serialization. Any OWL ontology serialization S' in one of these two super-languages can be translated into an OWL ontology serialization S that fully conforms to the original specification OWL/XML or "OWL serialized in RDF" and is semantically equivalent to the extended serialization S' with regard to the semantics of OWL. Without these modifications, neither OWL/XML nor "OWL serialized in RDF" satisfies the XML or RDF conformance requirements, respectively. The reason is that with imports there is a structural element supported by OWL that cannot have identifiers nor carry annotations, and that these two OWL serializations do not permit the use of XML or RDF constructs that would enable assigning identifiers to imports.

B.2.2.2 XML Conformance of a Modified OWL/XML With DOL

In the OWL/XML serialization, the *Import* element does not have annotations and is only allowed to carry the attributes *xml:base*, *xml:lang* and *xml:space*, but no further attributes or child elements from foreign namespaces (requirement (3b)), and therefore in particularly not a *dol:id* attribute or child elements, as would be required for adding identifiers (cf. clause ??).

An extended specification of OWL/XML that does allow the *dol:id* attribute on *Import* satisfies the XML conformance criteria. From an ontology serialized in this super-language of OWL/XML, one can obtain a semantically equivalent ontology (with regard to the semantics of OWL) by stripping all *dol:id* attributes.

B.2.2.3 RDF Conformance of a Modified Serialization of OWL in RDF With DOL

The serialization of OWL in RDF (regardless of the concrete *RDF* serialization employed to serialize the RDF graph that represents the OWL ontology) does not satisfy requirement (2) for RDF conformance because there is an `owl:imports` property but no class representing imports. Therefore, it is not possible to represent a concrete import, of an ontology O_1 importing an ontology O_2, as an RDF resource. However, only resources can have identifiers in RDF. RDF reification would allow for turning the statement O_1 `owl:imports` O_2 into a resource and thus giving it an identifier. However, the RDF triples required for expressing this reification, including, e.g., the triple `:import_id rdf:predicate owl:imports`, would not match the head of any rule in the mapping from RDF graphs to the OWL structural specification[1]. They

[18] NOTE: Is it correct that "With" and "of" use different capitalizations?

[19] NOTE: Why refer to this external paper when $\mathcal{SROIQ}$ is actually formalized as an institution below? Maybe cut this sentence and replace it by a general introduction? Or at least rephrase "is established" into "has originally been established".

[1] W3C/TR REC-owl2-mapping-to-rdf:2012, section 3

would thus remain left over in the RDF graph that is attempted to be parsed into an OWL ontology, and thus violate the requirement that at the end of this parsing process, the RDF graph must be empty[2].

After extending the specification of the serialization of OWL in RDF in the following way, it satisfies the RDF conformance criteria: if the input RDF graph G considered in section 3 of W3C/TR REC-owl2-mapping-to-rdf:2012 contains the pattern

i `rdf:subject` s .
i `rdf:predicate owl:imports` .
i `rdf:object` o .

and thus introduces a resource i to represent that the ontology s imports the ontology o, these three triples are removed from G. From an ontology serialized in this super-language of the serialization of OWL in RDF, one can obtain semantically equivalent ontologies (with regard to the semantics of OWL) by stripping all triples whose predicate is *rdf:subject*, *rdf:predicate* or *rdf:object*, or by adding triples that declare these three properties to be *annotation properties*.

B.3 Conformance of the $\mathcal{SROIQ}$ Logic With DOL

The logic $\mathcal{SROIQ}$ underlying OWL can be formalized as an institution as follows:

Definition 15 ***OWL 2 DL.*** *OWL 2 DL is the description logic (DL) based fragment of the web ontology language OWL. First, the simple description logic $\mathcal{ALC}$ is discussed, afterward the approach is generalized to the more complex description logic $\mathcal{SROIQ}$, which is underlying OWL 2 DL. Signatures of the description logic $\mathcal{ALC}$ consist of a set $\mathcal{A}$ of atomic concepts, a set $\mathcal{R}$ of roles and a set $\mathcal{I}$ of individual constants. Signature morphisms are tuples of functions, one for each signature component. Models are first-order structures $I = (\Delta^I, \cdot^I)$ with universe Δ^I that interpret concepts as unary and roles as binary predicates (using $\cdot^I$). $I_1 \leq I_2$ if $\Delta^{I_1} = \Delta^{I_2}$ and all concepts and roles of I_1 are subconcepts and subroles of those in I_2. Sentences are subsumption relations $C_1 \sqsubseteq C_2$ between concepts, where concepts follow the grammar*

$$C ::= \mathcal{A} \mid \top \mid \bot \mid C_1 \sqcup C_2 \mid C_1 \sqcap C_2 \mid \neg C \mid \forall R.C \mid \exists R.C$$

These kind of sentences are also called TBox sentences. Sentences can also be ABox sentences, which are membership assertions of individuals in concepts (written $a : C$ for $a \in \mathcal{I}$) or pairs of individuals in roles (written $R(a, b)$ for $a, b \in \mathcal{I}, R \in \mathcal{R}$). Satisfaction is the standard satisfaction of description logics.

The logic $\mathcal{SROIQ}$ [?], which is the logical core of the Web Ontology Language OWL 2 DL[3], extends $\mathcal{ALC}$ with the following constructs: (i) complex role inclusions such as $R \circ S \sqsubseteq S$ as well as simple role hierarchies such as $R \sqsubseteq S$, assertions for symmetric, transitive, reflexive, asymmetric and disjoint roles (called RBox sentences, denoted by $\mathcal{SR}$), as well as the construct $\exists R.\mathsf{Self}$ (collecting the set of 'R-reflexive points'); (ii) nominals, i.e. concepts of the form $\{a\}$, where $a \in \mathcal{I}$ (denoted by $\mathcal{O}$); (iii) inverse roles (denoted by $\mathcal{I}$); qualified and unqualified number restrictions ($\mathcal{Q}$). For details on the rather complex grammatical restrictions for $\mathcal{SROIQ}$ (e.g. regular role inclusions, simple roles) compare [?].

OWL profiles *are syntactic restrictions of OWL 2 DL that support specific modeling and reasoning tasks, and which are accordingly based on DLs with appropriate computational properties. Specifically, OWL 2 EL is designed for ontologies containing large numbers of concepts or relations, OWL 2 QL to support query answering over large amounts of data, and OWL 2 RL to support scalable reasoning using rule languages (EL, QL, and RL for short) .*

The logic $\mathcal{EL}$ is underlying the EL profile. (To be exact, EL adds various 'harmless' expressive means and syntactic sugar to $\mathcal{EL}$ resulting in the DL $\mathcal{EL}$ ++.) $\mathcal{EL}$ is a syntactic restriction of $\mathcal{ALC}$ to existential restriction, concept intersection, and the top concept:

$$C ::= \mathcal{A} \mid \top \mid C_1 \sqcap C_2 \mid \exists R.C$$

Note that $\mathcal{EL}$ does not have disjunction or negation, and is therefore a sub-Boolean logic. □

OWL itself is more complicated than $\mathcal{SROIQ}$ due to the presence of datatypes. Following the direct model-theoretic semantics of OWL [?]:

Definition 16 *A datatype map, formalizing datatype maps from the OWL 2 Specification [?], is a 6-tuple*

$$D = (N_{DT}, N_{LS}, N_{FS}, \cdot^{DT}, \cdot^{LS}, \cdot^{FS})$$

with the following components:

- *N_{DT} is a set of datatypes (more precisely, names of datatypes) that does not contain the datatype rdfs:Literal.*
- *N_{LS} is a function that assigns to each datatype $DT \in N_{DT}$ a set $N_{LS}(DT)$ of strings called lexical forms. The set $N_{LS}(DT)$ is called the lexical space of DT.*
- *N_{FS} is a function that assigns to each datatype $DT \in N_{DT}$ a set $N_{FS}(DT)$ of pairs (F, v), where F is a constraining facet and v is an arbitrary data value called the constraining value. The set $N_{FS}(DT)$ is called the facet space of DT.*

[2] See the last sentence of section 3.2.5 of W3C/TR REC-owl2-mapping-to-rdf:2012

[3] See also `http://www.w3.org/TR/owl2-overview/`

- *For each datatype $DT \in N_{DT}$, the interpretation function $\cdot^{DT}$ assigns to DT a set $(DT)^{DT}$ called the value space of DT.*
- *For each datatype $DT \in N_{DT}$ and each lexical form $LV \in N_{LS}(DT)$, the interpretation function $\cdot^{LS}$ assigns to the pair (LV, DT) a data value $(LV, DT)^{LS} \in (DT)^{DT}$.*
- *For each datatype $DT \in N_{DT}$ and each pair $(F, v) \in N_{FS}(DT)$, the interpretation function $\cdot^{FS}$ assigns to (F, v) the set $(F, v)^{FS} \subseteq (DT)^{DT}$.*

The set of datatypes N_{DT} of a datatype map D is not required to contain all datatypes from the OWL 2 datatype map; this allows one to talk about subsets of the OWL 2 datatype map, which may be necessary for the various profiles of OWL 2. If, however, D contains a datatype DT from the OWL 2 datatype map, then $N_{LS}(DT)$, $N_{FS}(DT)$, $(DT)^{DT}$, $(LV, DT)^{LS}$ for each $LV \in N_{LS}(DT)$, and $(F, v)^{FS}$ for each $(F, v) \in N_{FS}(DT)$ are required to coincide with the definitions for DT in the OWL 2 datatype map. □

Given two datatype maps $D = (N_{DT}, N_{LS}, N_{FS}, \cdot^{DT}, \cdot^{LS}, \cdot^{FS})$ and $D' = (N'_{DT}, N'_{LS}, N'_{FS}, \cdot^{DT'}, \cdot^{LS'}, \cdot^{FS'})$, we write $D \subseteq D'$ if $N_{DT} \subseteq N'_{DT}$, and the other components of D are restrictions (as functions) of those of D'.

Definition 17 *A vocabulary $V = (V_C, V_{OP}, V_{DP}, V_I, V_{DT}, V_{LT}, V_{FA})$ over a datatype map D is a 7-tuple consisting of the following elements:*

- *V_C is a set of classes as defined in the OWL 2 Specification [?], containing at least the classes owl:Thing and owl:Nothing.*
- *V_{OP} is a set of object properties as defined in the OWL 2 Specification [?], containing at least the object properties owl:topObjectProperty and owl:bottomObjectProperty.*
- *V_{DP} is a set of data properties as defined in the OWL 2 Specification [?], containing at least the data properties owl:topDataProperty and owl:bottomDataProperty.*
- *V_I is a set of individuals (named and anonymous) as defined in the OWL 2 Specification [?].*
- *V_{DT} is a set containing all datatypes of D, the datatype rdfs:Literal, and possibly other datatypes; that is, $N_{DT} \cup \{rdfs{:}Literal\} \subseteq V_{DT}$.*
- *V_{LT} is a set of literals LV^{DT} for each datatype $DT \in N_{DT}$ and each lexical form $LV \in N_{LS}(DT)$.*
- *V_{FA} is the set of pairs (F, lt) for each constraining facet F, datatype $DT \in N_{DT}$, and literal $lt \in V_{LT}$ such that $(F, (LV, DT_1)^{LS}) \in N_{FS}(DT)$, where LV is the lexical form of lt and DT_1 is the datatype of lt.*

□

Definition 18 *Given a datatype map D and a vocabulary V over D, an interpretation*

$$I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA}, NAMED)$$

for D and V is a 10-tuple with the following structure:

- *Δ_I is a nonempty set called the object domain.*
- *Δ_D is a nonempty set disjoint with Δ_I called the data domain such that $(DT)^{DT} \subseteq \Delta_D$ for each datatype $DT \in V_{DT}$.*
- *$\cdot^C$ is the class interpretation function that assigns to each class $C \in V_C$ a subset $(C)^C \subseteq \Delta_I$ such that*
  - *$(owl{:}Thing)^C = \Delta_I$ and*
  - *$(owl{:}Nothing)^C = \emptyset$.*
- *$\cdot^{OP}$ is the object property interpretation function that assigns to each object property $OP \in V_{OP}$ a subset $(OP)^{OP} \subseteq \Delta_I \times \Delta_I$ such that*
  - *$(owl{:}topObjectProperty)^{OP} = \Delta_I \times \Delta_I$ and*
  - *$(owl{:}bottomObjectProperty)^{OP} = \emptyset$.*
- *$\cdot^{DP}$ is the data property interpretation function that assigns to each data property $DP \in V_{DP}$ a subset $(DP)^{DP} \subseteq \Delta_I \times \Delta_D$ such that*
  - *$(owl{:}topDataProperty)^{DP} = \Delta_I \times \Delta_D$ and*
  - *$(owl{:}bottomDataProperty)^{DP} = \emptyset$.*
- *$\cdot^I$ is the individual interpretation function that assigns to each individual $a \in V_I$ an element $(a)^I \in \Delta_I$.*
- *$\cdot^{DT}$ is the datatype interpretation function that assigns to each datatype $DT \in V_{DT}$ a subset $(DT)^{DT} \subseteq \Delta_D$ such that*
  - *$\cdot^{DT}$ is the same as in D for each datatype $DT \in N_{DT}$, and*
  - *$(rdfs{:}Literal)^{DT} = \Delta_D$.*
- *$\cdot^{LT}$ is the literal interpretation function that is defined as $(lt)^{LT} = (LV, DT)^{LS}$ for each $lt \in V_{LT}$, where LV is the lexical form of lt and DT is the datatype of lt.*

- $\cdot^{FA}$ *is the facet interpretation function that is defined as* $(F, lt)^{FA} = (F, (lt)^{LT})^{FS}$ *for each* $(F, lt) \in V_{FA}$.
- *NAMED is a subset of* Δ_I *such that* $(a)^I \in NAMED$ *for each named individual* $a \in V_I$.

□

The institution $\mathcal{SROIQ}(D)$ underlying OWL is now defined as follows:

Definition 19
- *An* $\mathcal{SROIQ}(D)$ *signature is a pair* (D, V), *where* D *is a datatype map and* V *a vocabulary over* D.
- *Given* $\mathcal{SROIQ}(D)$ *signatures* (D, V) *and* (D', V'), *a* $\mathcal{SROIQ}(D)$ *signature morphism* $\sigma\colon (D, V) \to (D', V')$ *only exists if* $D \subseteq D'$. *In this case, such a signature morphism consists of*
  - *a map* $\sigma_C\colon V_C \to V'_C$,
  - *a map* $\sigma_{OP}\colon V_{OP} \to V'_{OP}$,
  - *a map* $\sigma_{DP}\colon V_{DP} \to V'_{DP}$,
  - *a map* $\sigma_I\colon V_I \to V'_I$,
  - *a map* $\sigma_{DT}\colon V_{DT} \to V'_{DT}$ *that is the identity on* $N_{DT} \cup \{rdfs{:}Literal\}$,
  - *a map* $\sigma_{LT}\colon V_{LT} \to V'_{LT}$
- *The sentences for a signature are definded as in the direct model-theoretic semantics of OWL [?]. Sentence translation is substitution of symbols.*
- (D, V)*-models are interpretations for* D *and* V. (D, V)*-model morphisms are maps between the domains* Δ_I *preserving membership in classes and properties, where* Δ_D *is mapped identically. Model reducts are built by first translating along the signature morphism and then looking up the interpretation in the model to be reduced.*
- *The satisfaction relation is defined as in direct model-theoretic semantics of OWL [?].*

□

Remark: strictly speaking, the institution defined above is *OWL 2 DL without restrictions* in the sense of [?]. The reason is that in an institution, the sentences can be used for arbitrary formation of theories. This is related to the presence of DOL's union operator on OMS. OWL 2 DL's specific restrictions on theory formation can be modeled *inside* this institution, as a constraint on OMS. This constraint is generally not preserved under unions or extensions. DOL's multi-logic capability allows the clean distinction between ordinary OWL 2 DL and OWL 2 DL without restrictions.

B.3.1 Relativization in OWL

Definition 20 *Given an* OWL *theory* $T = ((C, R, I), \Delta)$, *the relativization of* T, *denoted* $\tilde{T}$, *is the theory* $((C', R, I), \Delta')$ *where*

- $C' = C \cup \{\top_T\}$
- Δ' *contains axioms stating that:*
  - *each concept in* C *is subsumed by* $\top_T$,
  - *each individual in* I *is an instance of* $\top_T$,
  - *each role* r *has its domain and range intersected with* $\top_T$, *if they are present in* Δ, *otherwise they are* $\top_T$,

  and the sentences in Δ *where the following replacement of concepts are made:*
  - *each occurence of* $\top$ *is replaced with* $\top_T$,
  - *each occurence of* $\neg C$ *is replaced with* $\top_T \sqcap \neg C$,
  - *each occurence of* $\forall\ r \bullet C$ *is replaced with* $\top_T \sqcap \forall\ r \bullet C$.

Definition 21 *Given an* OWL *theory* $T = ((C, R, I), \Delta)$, *we define* $\beta : Mod^{\mathsf{OWL}}(\tilde{T}) \to Mod^{\mathsf{OWL}}(T)$ *as follows: if* $M' \in Mod^{\mathsf{OWL}}(\tilde{T})$, *then* $M = \beta(M')$ *has as universe* Δ^M *the set* $(\top_T)^{M'}$ *and each concept, role and individual are interpreted in* M *in the same way as in* M'. *Since* M' *is a* Δ'*-model, we get that* M *is indeed a* (C, R, I)*-model and moreover* $M \models \Delta$.

B.3.2 Translating correspondences to a bridge theory in OWL

We define the function $theoryOfCorrespondences_{\mathsf{OWL}}$ that takes as arguments the assumption made on the semantics of the alignment where the correspondences come from, the signatures of the two ontologies being aligned and a list of processed correspondences, in the sense that the default correspondence and any correspondence block, if present, are replaced with the lists of single correspondences they induce, represented as triples of the form $(relation, sourceSymbol, targetSymbol)$.

The result of the function is a co-span of theories: $(\Sigma_s, \emptyset) \xrightarrow{\varphi_s} (\Sigma, \Delta) \xleftarrow{\varphi_t} (\Sigma_t, \emptyset)$. Intuitively, Σ_s and Σ_t gather the symbols of the aligned ontologies that appear in the list of correspondences passed as an argument, while (Σ, Δ) contains an OWL sentence representing each correspondence.

We distinguish three cases.

1. Single domain:
   - no other symbols occur in the signatures Σ_s and Σ_t than the ones that appear in correspondences: $\Sigma_s = (C_s, R_s, I_s)$ and $\Sigma_t = (C_t, R_t, I_t)$, where C_s, R_s and I_s are the sets of all concept names, roles and individuals that appear in the list of correspondences as source symbols and C_t, R_t and I_t are the sets of all concept names, roles and individuals that appear in the list of correspondences as target symbols.
   - $\Sigma = \Sigma_s \uplus \Sigma_t$, where we prefix the symbols of Σ coming from Σ_s with $1:$ and those coming from Σ_t with $2:$
   - φ_s maps each symbol s in Σ_s to $1:s$ and φ_t maps each symbol s in Σ_t to $2:s$.
   - Δ contains the translation of correspondences to Σ-sentences using the following rules:

$('equivalent', c1, c2)$	`Class: 1:c1 EquivalentTo: 2:c2`
$('equivalent', r1, r2)$	`ObjectProperty: 1:r1 EquivalentTo: 2:r2`
$('equivalent', i1, i2)$	`Individual: 1:i1 SameAs: 2:i2`
$('incompatible', c1, c2)$	`Class: 1:c1 DisjointWith: 2:c2`
$('incompatible', r1, r2)$	`ObjectProperty: 1:r1 DisjointWith: 2:r2`
$('incompatible', i1, i2)$	`Individual: 1:i1 DifferentFrom: 2:i2`
$('subsumes', c1, c2)$	`Class: 2:c2 SubClassOf: 1:c1`
$('subsumes', r1, r2)$	`ObjectProperty: 2:r2 SubPropertyOf: 1:r1`
$('is\text{-}subsumed', c1, c2)$	`Class: 1:c1 SubClassOf: 2:c2`
$('is\text{-}subsumed', r1, r2)$	`ObjectProperty: 1:r1`$SubPropertyOf: 2: r2$
$('has\text{-}instance', c1, i2)$	`Individual: 2:i2 Types: 1:c1`
$('instance\text{-}of', i1, c2)$	`Individual: 1:i1 Types: 2:c2`

2. Global domain: [20] Note(20)
3. Contextualized domain:
   - Σ_s and Σ_t are constructed as before, but now they also include the relativized top concepts $\top_S$ and $\top_T$ respectively.
   - Σ extends the disjoint union $\Sigma_s \uplus \Sigma_t$ with new roles r_{st} and r_{ts}.
   - Δ contains the following axioms:

     `ObjectProperty:` r_{st} `Domain:`$\top_S$ `Range:`$\top_T$

     and

     `ObjectProperty:` r_{ts} `Domain:`$\top_T$ `Range:`$\top_S$

     together with the property that r_{st} is the converse of r_{ts}:

     `ObjectProperty:` r_{st} `InverseOf:` r_{ts}

     and translation of correspondences to Σ-sentences using the following rules:[21] Note(21)

$('equivalent', c1, c2)$	`Class: 1:c1 EquivalentTo:` r_{st} `some 2:c2`
$('equivalent', r1, r2)$	`ObjectProperty: 1:r1 EquivalentTo: 2:r2`
$('equivalent', i1, i2)$	`Individual: 1:i1 Facts:` r_{st} `2:i2`
$('incompatible', c1, c2)$	`EquivalentClasses:` $1: c1$ `and` r_{st} `some 2:c2, Nothing`
$('incompatible', r1, r2)$	`ObjectProperty: 1:r1 DisjointWith: 2:r2`
$('incompatible', i1, i2)$	`Individual: 1:i1 DifferentFrom: 2:i2`
$('subsumes', c1, c2)$	`Class: 2:c2 SubClassOf:` r_{sz} `some 1:c1`
$('subsumes', r1, r2)$	`ObjectProperty: 2:r2 SubPropertyOf: 1:r1`
$('is\text{-}subsumed', c1, c2)$	`Class: 1:c1 SubClassOf:` r_{ts} `some 2:c2`
$('is\text{-}subsumed', r1, r2)$	`ObjectProperty: 1:r1 SubPropertyOf: 2:r2`
$('has\text{-}instance', c1, i2)$	`Individual: 2:i2 Types:` r_{ts} `some 1:c1`
$('instance\text{-}of', i1, c2)$	`Individual: 1:i1 Types:` r_{st} `some 2:c2`

[20] NOTE: in the absence of terms, there is no difference to first case

[21] NOTE: correct incompatible, what to do with 2, 5,6, 8,10? TM: I do not understand the question.

C Annex: Conformance of Common Logic with DOL

(Informative)

C.1 Abstract Syntax Conformance of Common Logic With DOL

The metaclass `Text` is a subclass (in the sense of SMOF multiple classification) of `NativeDocument`. The metaclass `Sentence` is a subclass (in the sense of SMOF multiple classification) of `BasicOMS`.

C.2 Text and Semantic Conformance of Common Logic With DOL

The semantic conformance of Common Logic (as specified in ISO/IEC 24707:2007) with DOL is established in [?].

The XCF dialect of Common Logic has a serialization that satisfies the criteria for XML conformance. The CLIF dialect of Common Logic has a serialization that satisfies the criteria for text conformance.

Common Logic can be defined as an institution as follows:

Definition 22 ***Common Logic.*** *A common logic signature Σ (called vocabulary in Common Logic terminology) consists of a set of names, with a subset called the set of discourse names, and a set of sequence markers. An signature morphism maps names and sequence markers separately, subject to the requirement that a name is a discourse name in the smaller signature if and only if it is one in the larger signature. A Σ-model $I = (UR, UD, rel, fun, int, seq)$ consists of a set UR, the universe of reference, with a non-empty subset $UD \subseteq UR$, the universe of discourse, and four mappings:*

- *rel from UR to subsets of $UD^* = \{< x_1, \ldots, x_n >| x_1, \ldots, x_n \in UD\}$ (i.e., the set of finite sequences of elements of UD);*
- *fun from UR to total functions from UD^* into UD;*
- *int from names in Σ to UR, such that $int(v)$ is in UD if and only if v is a discourse name;*
- *seq from sequence markers in Σ to UD^*.*

A Σ-sentence is a first-order sentence, where predications and function applications are written in a higher-order like syntax: $t(s)$. Here, t is an arbitrary term, and s is a sequence term, which can be a sequence of terms $t_1 \ldots t_n$, or a sequence marker. A predication $t(s)$ is interpreted by evaluating the term t, mapping it to a relation using rel, and then asking whether the sequence given by the interpretation s is in this relation. Similarly, a function application $t(s)$ is interpreted using fun. Otherwise, interpretation of terms and formulae is as in first-order logic. A further difference to first-order logic is the presence of sequence terms (namely sequence markers and juxtapositions of terms), which denote sequences in UD^*, with term juxtaposition interpreted by sequence concatenation. Note that sequences are essentially a non-first-order feature that can be expressed in second-order logic.

Model reducts are defined in the following way: Given a signature morphism $\sigma : \Sigma_1 \to \Sigma_2$ and a Σ_2-model $I_2 = (UR, UD, rel, fun, int, seq)$, $I|_\sigma = (UR, UD, rel, fun, int \circ \sigma, seq \circ \sigma)$.

Given two CL models $I_1 = (UR_1, UD_1, rel_1, fun_1, int_1, seq_1)$ and $I_2 = (UR_2, UD_2, rel_2, fun_2, int_2, seq_2)$, a homomorphism $h : I_1 \to I_2$ is a function $h : UR_1 \to UR_2$ such that

- *h restricts to $k : UD_1 \to UD_2$,*
- *for each $x \in UR_1$ and $s \in UD_1^*$, if $s \in rel_1(x)$, then $k^*(s) \in rel_2(h(x))$[1],*
- *for each $x \in UR_1$, $k \circ fun_1(x) = fun_2(h(x)) \circ k^*$,*
- *for each name n in Σ, $int_2(n) = h(int_1(n))$,*
- *for each sequence marker n in Σ, $seq_2(n) = k^*(seq_1(n))$.*

CL $^-$ *is the restriction of* CL *to sentence without sequence markers.* □

Note that Common Logic also includes sentence formation constructs like `cl:imports` that in DOL terms belong to the structuring language. They have been omitted from the institution, because they must not occur in basic OMS. They can occur in structured native OMS, however, and need to be flattened out in order to obtain a theory in the CL institution.

[1] k^* is the extension of h to sequences.

D Annex: Conformance of RDF and RDF Schema with DOL

(Informative)

D.1 Abstract Syntax Conformance of RDF and RDF Schema With DOL

The metaclass `Document` is a subclass (in the sense of SMOF multiple classification) of `NativeDocument`. The metaclass `Triple` is a subclass (in the sense of SMOF multiple classification) of `BasicOMS`.

D.2 Semantic Conformance of RDF and RDF Schema With DOL

The semantic conformance of RDF Schema (as specified in W3C/TR REC-rdf-schema:2014) with DOL is established in [?].

The way of representing RDF Schema ontologies as RDF graphs satisfies the criteria for RDF conformance.

Definition 23 (RDF **and** RDF **Schema**) *The institutions for the Resource Description Framework (*RDF*) and* RDF *Schema (also known as* RDFS*), respectively, are defined following [?]. Both* RDF *and* RDFS *are based on a logic called* bare RDF *(*SimpleRDF*), which consists of triples only (without any predefined resources).*

A signature $\mathbf{R_s}$ *in* SimpleRDF *is a set of resource references. For* $sub, pred, obj \in \mathbf{R_s}$*, a triple of the form* $(sub, pred, obj)$ *is a sentence in* SimpleRDF*, where* sub*,* $pred$*,* obj *represent subject name, predicate name, object name, respectively. An* $\mathbf{R_s}$*-model* $M = \langle R_m, P_m, S_m, EXT_m\rangle$ *consists of a set* R_m *of resources, a set* $P_m \subseteq R_m$ *of predicates, a mapping function* $S_m : \mathbf{R_s} \to R_m$*, and an extension function* $EXT_m : P_m \to \mathcal{P}(R_m \times R_m)$ *mapping every predicate to a set of pairs of resources. Satisfaction is defined as follows:*

$$\mathfrak{M} \models_{\mathbf{R_s}} (sub, pred, obj) \Leftrightarrow (S_m(sub), (S_m(obj)) \in EXT_m(S_m(pred)).$$

Both RDF *and* RDFS *are built on top of* SimpleRDF *by fixing a certain standard vocabulary both as part of each signature and in the models.*[22] Note(22

Actually, the standard vocabulary is given by a certain theory. In case of RDF*, it contains e.g. resources* `rdf:type` *and* `rdf:Property` *and* `rdf:subject`*, and sentences like, e.g.*

$$(\texttt{rdf:type}, \texttt{rdf:type}, \texttt{rdf:Property}),$$

and

$$(\texttt{rdf:subject}, \texttt{rdf:type}, \texttt{rdf:Property}).$$

In the models, the standard vocabulary is interpreted with a fixed model. Moreover, for each RDF*-model* $M = \langle R_m, P_m, S_m, EXT_m\rangle$*, if* $p \in P_m$*, then it must hold* $(p, S_m(\texttt{rdf:Property})) \in EXT_m(\texttt{rdf:type})$*. For* RDFS*, similar conditions are formulated (here, for example also the subclass relation is fixed).*

In the case of RDFS*, the standard vocabulary contains more elements, like* `rdfs:domain`*,* `rdfs:range`*,* `rdfs:Resource`*,* `rdfs:Literal`*,* `rdfs:Datatype`*,* `rdfs:Class`*,* `rdfs:subClassOf`*,* `rdfs:subPropertyOf`*,* `rdfs:member`*,* `rdfs:Container`*,* `rdfs:ContainerMembershipProperty`*.*

There is also OWL *Full, an extension of* RDFS *with resources such as* `owl:Thing` *and* `owl:oneOf`*, tailored towards the representation of* OWL *[?].* □

[22] Note: Refer to the RDF standard here.

E Annex: Conformance of UML class and object diagrams with DOL

(Informative)

This informative annex demonstrates conformance of a subset of UML class and object diagrams with DOL by defining an institution for both. The subset is restricted to the static aspects of class diagrams; that is, change of state is ignored. This means that all operations are query operations.

The institution of UML class and object diagrams is defined using a translation of UML class diagrams to Common Logic, following the fUML specification and [?].

E.1 Abstract Syntax Conformance of UML With DOL

The metaclass `OWL Model` is a subclass (in the sense of SMOF multiple classification) of `NativeDocument`. The metaclass `OWL PackageableElement` is a subclass (in the sense of SMOF multiple classification) of `BasicOMS`.

E.2 Semantic Conformance: Preliminaries

The axioms for primitive types are imported from the fUML specification, section 10.3.1: Booleans, numbers, sequences and strings. These axiomatize (among others) predicates corresponding to primitive types, e.g. `buml:Boolean`, `form:Number`, `form:NaturalNumber`, `buml:Integer`, `form:Sequence`, `form:Character`, and `buml:String`.

The following infrastructure, consisting off a number of predicates axiomatized in Common Logic, provides a foundation for an institution for UML class diagrams described in the later sections of this Annex.

```
logic CLIF

oms pairs =
  (forall (x y) (= (form:first (form:pair x y)) x))
  (forall (x y) (= (form:second (form:pair x y)) y))
  (forall (x y) (form:Pair (form:pair x y)))
  (forall (p) (if (form:Pair p)
                  (= (form:pair (form:first p) (form:second p)) p)))
end

oms sequences =
fuml:sequences.clif and pairs
then
  // fuml:sequence - membership of an element in a sequence
  (forall (x s)
      (if (form:sequence-member x s)
          (form:Sequence s)))

  (forall (x s)
      (iff (form:sequence-member x s)
           (exists (pt)
                (and (form:in-sequence s pt)
                     (form:in-position pt x)) )))

  // selection of elements
  (forall (o) (= (form:select1 o form:empty-sequence) form:empty-sequence))
  (forall (o y s)
          (= (form:select1 o (form:sequence-insert (form:pair o y) s))
             (form:sequence-insert y (form:select1 o s))))
```

```
  (forall (o x y s)
          (if (not (= x o))
              (= (form:select1 o (form:sequence-insert (form:pair x y) s))
                 (form:select1 o s))))
  (forall (o) (= (form:select2 o form:empty-sequence) form:empty-sequence))
  (forall (o x s)
          (= (form:select2 o (form:sequence-insert (form:pair x o) s))
             (form:sequence-insert x (form:select2 o s))))
  (forall (o x y s)
          (if (not (= y o))
              (= (form:select2 o (form:sequence-insert (form:pair x y) s))
                 (form:select2 o s))))

  (forall (i s)
          (= (form:n-select form:empty-sequence i s)
             form:empty-sequence))
  (forall (a i s t x)
          (if (= (insert-i i x t) s)
              (= (form:n-select (form:sequence-insert s a) i t)
                 (form:sequence-insert s (form:n-select a i t)))))
  (forall (a i s t)
          (if (not (exists (x) (= (insert-i i x t) s)))
              (= (form:n-select (form:sequence-insert s a) i t)
                 (form:n-select a i t))))

  // insert element at i-th position
  (forall (x s)
          (= (insert-i form:0 x s) (form:sequence-insert x s)))
  (forall (i j x y s)
          (if (form:add-one i j)
              (= (insert-i j x (form:sequence-insert y s))
                 (form:sequence-insert y (insert-i i x s)))))
end

oms sequences-insert =
sequences then
  // insertion of elements
  (forall (x s1 s2)
    // inserting an element means...
    (if (= (form:sequence-insert x s1) s2)
        (and (form:Sequence s1)
             (form:Sequence s2)
             // the new element is at the first position
             (form:in-position-count s2 form:1 x)
             // and all other elements are shifted by one
             (forall (n1 n2 y)
               (if (form:add-one n1 n2)
                   (iff (form:in-position-count s1 n1 y)
                        (form:in-position-count s2 n2 y)))))))
  // synonym
 (forall (s) (= (form:sequence-length s) (form:sequence-size s)))
end

oms ordered-sets =
sequences with
  form:Sequence |-> form:Ordered-Set,
  form:empty-sequence |-> form:empty-ordered-set,
  form:sequence-length |-> form:ordered-set-size,
  form:same-sequence |-> form:same-ordered-set,
  form:sequence-member |-> form:ordered-set-member,
  form:in-sequence |-> form:in-ordered-set,
  form:before-in-sequence |-> form:before-in-ordered-set,
```

```
  form:position-count |-> form:ordered-set-position-count,
  form:in-position-count |-> form:in-ordered-set-position-count
then
  //Different positions contain different elements
  (forall (s x1 x2 n1 n2)
            (if (and (form:in-ordered-set-position-count s n1 x1)
                     (form:in-ordered-set-position-count s n2 x2)
                     (= x1 x2))
                (= n1 n2)))
  // insertion of elements
  (forall (x s1 s2)
    (if (= (form:ordered-set-insert x s1) s2)
        (and (form:Ordererd-Set s1)
             (form:Ordererd-Set s2)
  // no element can be inserted twice
  (forall (x s)
    (if (from:ordered-set-member x s)
        (= (form:ordered-set-insert x s) s)))
  // inserting a new element
  (forall (x s)
    (if (not (from:ordered-set-member x s1))
        (exists (s2)
          (and (= (form:ordered-set-insert x s1) s2)
               // the new element is at the first position
               (form:in-ordered-set-position-count s2 form:1 x)
               // and all other elements are shifted by one
               (forall (n1 n2 y)
                 (if (form:add-one n1 n2)
                     (iff (form:in-ordered-set-position-count s1 n1 y)
                          (form:in-ordered-set-position-count s2 n2 y)))))))
end

oms sets =
//An empty set has no members.
(forall (s)
        (if (form:empty-set s)
            (form:Set s)))
(forall (s)
        (if (form:Set s)
            (iff (form:empty-set s)
                 (not (exists (x)
                              (form:set-member x s))))))
//Size of sets
(forall (s n)
        (if (form:set-size s n)
            (and (form:Set s)
                 (buml:UnlimitedNatural n))))
(= (form:set-size form:empty-set) form:0)
(forall (x s)
        (if (not (form:set-member x s))
            (exists (n)
              (and (form:add-one (form:set-size s) n)
                   (= (form:set-size (form:set-insert x s))
                      n)))))

//The same-set relation is true for sets that have the same members.
  // but:  why not replace same-set with = ?  (forall (s1 s2)
        (if (form:same-set s1 s2)
            (and (form:Set s1)
                 (form:Set s2))))
(forall (s1 s2)
        (iff (form:same-set s1 s2)
```

```
             (forall (x)
                     (iff (form:set-member x s1)
                          (form:set-member x s2)))))
//Insertion of elements into sets and set membership
(forall (x s)
        (if (form:Set s)
            (form:Set (form:set-insert x s))))
(forall (x y s)
        (iff (form:set-member x (form:set-insert y s))
             (or (= x y)
                 (form:set-member x s))))
end

oms bags =
//An empty bag has no members.
(forall (s)
        (if (form:empty-bag s)
            (form:Bag s)))
(forall (s)
        (if (form:Bag s)
            (iff (form:empty-bag s)
                 (not (exists (x)
                              (form:bag-member x s))))))
//Size of bags
(forall (s n)
        (if (form:bag-size s n)
            (and (form:Bag s)
                 (buml:UnlimitedNatural n))))
(= (form:bag-size form:empty-bag) form:0)
(forall (x s)
        (exists (n)
            (and (form:add-one (form:bag-size s) n)
                 (= (form:bag-size (form:bag-insert x s))
                    n))))

//The same-bag relation is true for bags that have the same members.
(forall (s1 s2)
        (if (form:same-bag s1 s2)
            (and (form:Bag s1)
                 (form:Bag s2))))
(forall (s1 s2)
        (iff (form:same-bag s1 s2)
             (forall (x)
                     (iff (form:bag-member-count x s1)
                          (form:bag-member-count x s2)))))
//Insertion of elements into bags and bag membership
(forall (x s)
        (if (form:Bag s)
            (form:Bag (form:bag-insert x s))))
(forall (x y s)
        (iff (form:bag-member x (form:bag-insert y s))
             (or (= x y)
                 (form:bag-member x s))))
//Member count
(forall (x s)
        (if (form:Bag s)
            (buml:UnlimitedNatural (form:bag-member-count x s))))
(= (form:bag-member-count form:empty-bag) form:0)
(forall (x s)
        (exists (n)
           (and (form:add-one (form:bag-member-count x s) n)
                (= (form:bag-member-count x (form:bag-insert x s))
```

```
                    n))))
(forall (x y s)
        (if (not (= x y))
            (= (form:bag-member-count x (form:bag-insert y s))
               (form:bag-member-count x s))))
end

oms collection-types =
  sequences-insert and ordered-sets and sets and bags
then
//bag to set
(forall (b)
        (if (form:Bag s)
            (form:Set (form:bag2set b))))
(= (form:bag2set form:empty-bag) form:empty-set)
(forall (x b)
        (if (form:Bag b)
            (= (form:bag2set (form:set-insert x b))
               (form:bag-insert x (form:bag2set b)))))

//sequence to ordered set
(forall (s)
        (if (form:Sequence s)
            (form:Ordered-Set (form:seq2ordset s))))
(= (form:seq2ordset form:empty-sequence) form:empty-ordered-set)
(forall (x s)
        (if (form:Sequence s)
            (= (form:seq2ordset (form:sequence-insert x s))
               (form:ordered-set-insert x (form:seq2ordset s)))))

//sequence to bag
(forall (s)
        (if (form:Sequence s)
            (form:Bag (form:seq2bag s))))
(= (form:seq2bag form:empty-sequence) form:empty-bag)
(forall (x s)
        (if (form:Sequence s)
            (= (form:seq2bag (form:sequence-insert x s))
               (form:bag-insert x (form:seq2bag s)))))

//ordered-set to set
(forall (b)
        (if (form:Ordered-Set s)
            (form:Set (form:ordset2set b))))
(= (form:ordset2set form:empty-ordered-set) form:empty-set)
(forall (x b)
        (if (form:Ordered-Set b)
            (= (form:ordset2set (form:set-insert x b))
               (form:ordered-set-insert x (form:ordset2set b)))))

//sequence to set
(forall (s)
        (if (form:Sequence s)
            (form:Set (form:seq2set s))))
(forall (s) (= (form:seq2set s) (form:ordset2set (form:seq2ordset s))))

// leq
(forall (x y)
   (iff (buml:leq x y)
        (or (= x y)
            (buml:less-than x y))))
end
```

```
oms uml-cd-preliminaries =
  collection-types and pairs
end
```

E.3 Signatures

Class/data type hierarchies. A *class/data type hierarchy* $(C, \leq_C)$ is given by a partial order where the set C contains the *class/data type names*, which are closed w.r.t. the *built-in data types* Boolean, UnlimitedNatural, Integer, Real, and String, i.e., $\{\mathsf{Boolean}, \mathsf{UnlimitedNatural}, \mathsf{Integer}, \mathsf{Real}, \mathsf{String}\} \subseteq C$; and the partial ordering relation $\leq_C$ represents a *generalization relation* on C, where c_1 is a *sub-class/data type* of c_2 if $c_1 \leq_C c_2$.

A *class/data type hierarchy map* $\gamma : (C, \leq_C) \to (D, \leq_D)$ is given by a monotone map from $(C, \leq_C)$ to $(D, \leq_D)$, i.e., $\gamma(c) \leq_D \gamma(c')$ if $c \leq_C c'$, such that $\gamma(c) = c$ for all $c \in \{\mathsf{Boolean}, \mathsf{UnlimitedNatural}, \mathsf{Integer}, \mathsf{Real}, \mathsf{String}\}$.

The *collection type constructors* OrderedSet, Set, Sequence, and Bag are used for representing the meta-attributes "ordered" and "unique" of MultiplicityElement according to the following table:[1]

	ordered	not ordered
unique	OrderedSet	Set
not unique	Sequence	Bag

The default is "not ordered" and "unique".[2]

For a class/data type $c \in C$ of a class/data type-hierarchy $(C, \leq_C)$ and a collection type constructor

$$\tau \in \{\mathsf{OrderedSet}, \mathsf{Set}, \mathsf{Sequence}, \mathsf{Bag}\},$$

the expression $\tau[c]$ denotes the induced *collection type*.

Let $(C, \leq_C)$ be a class/data type hierarchy.

– An *attribute declaration*[3] over $(C, \leq_C)$ is of the form $c.p : \tau[c']$ with $c, c' \in C$, τ a collection type constructor, and p an *attribute name*. ~~(~~Additionally, an attribute may be *composite* and we write $c{\blacklozenge}p : \tau[c']$ if this fact plays a rôle. (Attributes and association member ends are distinguished due to their different uses. In UML, both are of class Property. Hence, attribute declarations are a kind of *property declarations*. Another kind of property declaration will be introduced through member end declarations below.)

– A *query operation declaration* over $(C, \leq_C)$ is of the form $c.q(x_1 : \tau_1[c_1], \dots, x_r : \tau_r[c_r]) : \tau[c']$ with $c, c_1, \dots, c_r, c' \in C$, τ a collection type constructor, o an *operation name*, and $x_1, \dots, x_r$ *parameter names*.

– An *association declaration* over $(C, \leq_C)$ is of the form $a(p_1 : \tau_1[c_1], \dots, p_r : \tau_r[c_r])$ with $r \geq 2$, $c_1, \dots, c_r \in C$, $\tau_1, \dots, \tau_r$ classifier annotations, a an *association name*, and $p_1, \dots, p_r$ *member end names*.[4] An association declaration $\mathbf{a} = a(p_1 : \tau_1[c_1], \dots, p_r : \tau_r[c_r])$ *yields* the *property declarations* $\mathbf{a}.p_i : \tau_i[c_i]$ for $1 \leq i \leq r$. An association declaration is *binary* if $r = 2$.[5]

  ~~A *composition declaration* over $(C, \leq_C)$ is of the form $m(p_1 : \mathsf{Set}[c_1], {\blacklozenge}p_2 : \tau_2[c_2])$ with $c_1, c_2 \in C$, τ_2 a collection type constructor, m a *composition name*, and p_1, p_2 *member end names*. A composition declaration $\mathbf{m} = m(p_1 : \mathsf{Set}[c_1], {\blacklozenge}p_2 : \tau_2[c_2])$ *yields* the property declarations $\mathbf{m}.p_1 : \mathsf{Set}[c_1]$~~ For a binary association with $\tau_1 = \mathsf{Set}$, the second member end may be *composite*, and ~~$\mathbf{m}.p_2 : \tau_2[c_2]$.~~ we write $a(p_1 : \mathsf{Set}[c_1], {\blacklozenge}p_2 : \tau_2[c_2])$ if this fact plays a rôle.[6]

~~In UML, each may have . However, such an aggregation kind has no semantic meaning when the property is not a member end of an association: the UML Superstructure Specification 2.4.1 does not mention the aggregation kind in the description of the semantics of , and UML 2.5 explains the use of aggregations for as "to model circumstances in which *one instance* is used to group together a set of instances" (p. 112, our emphasis). Moreover, composite properties, i.e., properties with aggregation kind can only be member ends of binary associations (UML Superstructure Specification 2.4.1, p. 37; UML 2.5, p. 228) and their multiplicity must not exceed one (UML Superstructure Specification 2.4.1, p. 126; UML 2.5, p. 155). Thus, composition declarations are distinguished from general association declarations.~~

[1] Cf. UML Superstructure Specification 2.4.1, p. 128; UML 2.5, p. ~~27.~~34.

[2] UML Superstructure Specification 2.4.1, p. 96; there does not seem to be default in UML 2.5.

[3] We separate attributes from association member ends due to their different uses. In UML, both are of class Property ([?, p. 109]).

[4] The member ends are ordered according to the UML Superstructure Specification 2.4.1, p. 29; UML 2.5, p. ~~206~~197; hence they are represented in a tuple-like notation.

[5] Only binary association may show member ends that are properties not owned by the association (UML Superstructure Specification 2.4.1, p. 37; UML 2.5, p. ~~228~~218). The property declarations induced by a more than binary association result in a query operation.

[6] Composite properties, i.e., properties with aggregation kind composite can only be member ends of binary associations ([?, p. 218]) and their multiplicity must not exceed one ([?, p. 150]).

Class/data type nets (Signatures). A *class/data type net* $\cancel{\Sigma = ((C, \leq_C), P, O, A, M)}$ $\Sigma = ((C, \leq_C), P, O, A)$ comprises a class/data type hierarchy $(C, \leq_C)$ and a set P of attribute declarations, a set O of operation declarations, and a set A of association declarations over $(C, \leq_C)$, ~~and a set M of composition declarations over $(C, \leq_C)$,~~ such that the following properties are satisfied:

- attribute names are unique along the generalization relation: if $c_1.p_1 : \tau_1[c'_1]$ and $c_2.p_2 : \tau_2[c'_2]$ are different property declarations in P and $c_1 \leq_C c_2$, then $p_1 \neq p_2$;
- association ~~and composition~~ names are unique: if d_1 and d_2 are the names of two different association ~~or composition~~ declarations in $M \cup A$, then $d_1 \neq d_2$;
- member end names are unique: if $p_1, \dots, p_r$ are the member end names of an association declaration in A~~or a composition declaration in M~~, then $p_i \neq p_j$ for $1 \leq i \neq j \leq r$;[7]
- the type of a member end[8] owned by a class/data type coincides with its declarations as attribute: We say that a property declaration $\mathbf{a}.p_i : \tau_i[c_i]$ yielded by a binary association $\mathbf{a} = a(p_1 : \tau_1[c_1], p_2 : \tau_2[c_2])$ is *owned by* $c_0 \in C$, if $c_{3-i} \leq_C c_0$ and there is an attribute declaration $c_0.p_i : \tau_i[c_i] \in P$~~; and similarly for property declarations yielded by composition declarations~~, where for the second end $\mathbf{a}.p_2 : \tau_2[c_2]$ of an association declaration $\mathbf{a} = a(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2])$ the property has to be composite, i.e., $c_0 \blacklozenge p_2 : \tau_2[c_2]$. (Note that by the uniqueness of attribute names along the ~~generalization~~ generalisation hierarchy only a single attribute with name p_i may exist.)

A *class/data type net morphism* $\cancel{\sigma = (\gamma, \varphi, \alpha, \mu) : \Sigma = ((C, \leq_C), P, A, M) \to \mathrm{T} = ((D, \leq_D), Q, B, N)}$ $\sigma = (\gamma, \varphi, \alpha) : \Sigma = ((C, \leq_C), P, A$ is given by

- a class/data type hierarchy map $\gamma : (C, \leq_C) \to (D, \leq_D)$;
- an attribute declaration map $\varphi : P \to Q$ such that if $\varphi(c.p : \tau[c']) = d.q : \tau'[d'] \in Q$, then $d = \gamma(c)$, $d' = \gamma(c')$, and $\tau = \tau'$; furthermore, each composite attribute has to be mapped to a composite attribute.
- a query operation declaration map $\rho : O \to R$ such that if $\rho(c.q(x_1 : \tau_1[c_1], \dots, x_r : \tau_r[c_r]) : \tau[c']) = d.r(x_1 : \tau'_1[d_1], \dots, x_r : \tau'_r[d_r]) : \tau[d'] \in R$, then $d = \gamma(c)$, $d_i = \gamma(c_i)$, $d' = \gamma(c')$, $\tau'_i = \tau_i$ and $\tau = \tau'$;
- an association declaration map $\alpha : A \to B$ such that if $\alpha(a(p_1 : \tau_1[c_1], \dots, p_r : \tau_r[c_r])) = b(q_1 : \tau'_1[d_1], \dots, q_s : \tau'_s[d_s]) \in B$, then $r = s$ and $d_i = \gamma(c_i)$ and $\tau_i = \tau'_i$ for $1 \leq i \leq r$, and member ends owned by the association are mapped into owned member ends~~;~~

~~a composition declaration map $\mu : M \to N$ such that if $\mu(m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2])) = n(q_1 : \mathsf{Set}[d_1], \blacklozenge q_2 : \tau'_2[d_2]) \in N$, then $d_1 = \gamma(c_1)$, $d_2 = \gamma(c_2)$, and $\tau_2 = \tau'_2$, and member ends owned by the composition are mapped into owned member ends~~.

Class/data type nets as objects and class/data type net morphisms as morphisms form the category of *class/data type nets*, denoted by Cl.

For the example in Fig. **??** the class/data type net is

Classes/data types: Net, Station, Line, Connector, Unit, Track, Point, Linear,
Boolean, UnlimitedNatural, Integer, Real, String

Generalizations: Point $\leq$ Unit, Linear $\leq$ Unit

Properties: Line.linear : Set[Boolean], Track.linear : Set[Boolean],
Net.station : Set[Station], Net.line : Set[Line],
Station.net : Set[Net], Station.unit : Set[Unit], Station.track : Set[Track],
Line.net : Set[Net], Line.linear : Set[Linear],
Connector.unit : Set[Unit],
Unit.station : Set[Station], Unit.connector : Set[Connector],
Track.station : Set[Station], Track.linear : Set[Linear],
Linear.track : Set[Track], Linear.line : Set[Line]

Associations: ~~l2l(line : Set[Line], linear : Set[Linear])~~L2L(line : Set[Line], linear : Set[Linear]),
~~l2t(linear : Set[Linear], track : Set[Track])~~L2T(linear : Set[Linear], track : Set[Track]),
~~c2u(connector : Set[Connector], unit : Set[Unit])~~C2U(connector : Set[Connector], unit : Set[Unit])

Compositions: ~~n2s(net : Set[Net], ◆station : Set[Station])~~N2S(net : Set[Net], ◆station : Set[Station]),

[7] In UML, member end names need not be unique. However, for (1) a simpler handling of selecting a particular member end in the sentences and avoiding the use of number selectors, and (2) making the notion of member ends "owned" by a class/data type, this constraint is added. An association declaration violating this uniqueness constraints can easily be transformed into an association declaration satisfying it by decorating member end names with the numbers $1, \dots, r$.

[8] All member ends are instances of Property; UML Superstructure Specification 2.4.1, p. 36; UML 2.5, p. 206.

n2l(net : Set[Net], ◆line : Set[Line])N2L(net : Set[Net], ◆line : Set[Line]),

s2u(station : Set[Station], ◆unit : Set[Unit])S2U(station : Set[Station], ◆unit : Set[Unit]),

s2t(station : Set[Station], ◆track : Set[Track])S2T(station : Set[Station], ◆track : Set[Track])

Here all member ends are owned by class/data types.

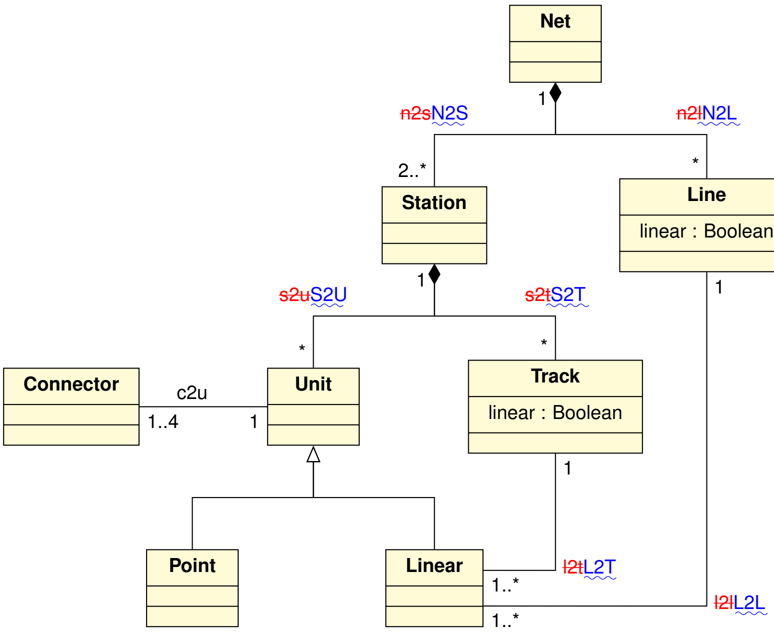


Figure E.1: Sample UML class diagram.

E.4 Models

As stated above, models (in the sense of the term model defined in clause 4) of UML class diagrams are obtained via a translation to Common Logic.

For a classifier net $\Sigma = ((C, \leq_C), K, P, M, A)$, a Common Logic theory $CL(\Sigma)$ is defined consisting of:

- for $c \in C$, a predicate[9] $\mathsf{CL}(c)$, such that
  - $\mathsf{CL}(\mathsf{Boolean}) =$ `buml:Boolean`,
  - $\mathsf{CL}(\mathsf{String}) =$ `buml:String`,
  - $\mathsf{CL}(\mathsf{Integer}) =$ `buml:Integer`,
  - $\mathsf{CL}(\mathsf{UnlimitedNatural}) =$ `form:NaturalNumber`,
  - $\mathsf{CL}(\mathsf{Real}) =$ `buml:Real`,
  - $\mathsf{CL}(\mathsf{c}) = c$, if c is an enumeration type with values $k_1, \ldots, k_n$. In this case, additionally, the Common Logic theory is augmented by `(not (=` $k_i \cdots k_j$`))` for $i \neq j$ and `(forall (x) (if (c x) (or (= x` k_1`)` $\cdots$ `(= x` k_n`))))`,
  - $\mathsf{CL}(\mathsf{List}[c]) =$ `form:Sequence`,
  - $\mathsf{CL}(\mathsf{Set}[c]) =$ `form:Set`,
  - $\mathsf{CL}(\mathsf{OrderedSet}[c]) =$ `form:OrderedSet`,
  - $\mathsf{CL}(\mathsf{Bag}[c]) =$ `form:Bag`,
  - $\mathsf{CL}(\mathsf{c}) = c$, if c a class name which is not one of the above.
- for each relation $c_1 \leq_C c_2$, an axiom `(forall (x) (if (`C_1 `x) (`C_2 `x)))`, where $C_1 = \mathsf{CL}(c_1)$, $C_2 = \mathsf{CL}(c_2)$,
- CL maps each attribute declaration $c.p : \tau[c'] \in P$ to a predicate $\mathsf{CL}(c.p)$ and axioms stating type-correctness and functionality:[23] Note(23

[9] Strictly speaking, this is just a name.

[23] NOTE: shouldn't attributes involving Set only also be represented in a simplified way?

- (forall (x y) (if (c.p x y) (c x)))
- (forall (x y) (if (c.p x y) ($\tau[c']$ y))) [10]
- (forall (x)
  (if (c x) (exists (y) (c.p x y))))
- (forall (x y z)
  (if (and (c.p x y) (c.p x z))
  (= y z)))

- CL maps each query operation declaration $c.q(x_1 : \tau_1[c_1], \ldots, x_r : \tau_n[c_r]) : \tau[c'] \in O$ to a predicate $\mathsf{CL}(c.q)$ and axioms stating type-correctness and functionality:
  - (forall (x x_1 x_2 $\cdots$ x_n y) (if (c.q x x_1 x_2 $\cdots$ x_n y) (c x)))
  - (forall (x x_1 x_2 $\cdots$ x_n y) (if (c.q x x_1 x_2 $\cdots$ x_n y) ($\tau_i[c_i]$ x_i))) for each $i = 1 \ldots n$,[11]
  - (forall (x x_1 x_2 $\cdots$ x_n y) (if (c.q x x_1 x_2 $\cdots$ x_n y) ($\tau[c']$ y)))
  - (forall (x x_1 x_2 $\cdots$ x_n y z)
    (if (and (c.q x x_1 x_2 $\cdots$ x_n y) (c.q x x_1 x_2 $\cdots$ x_n z))
    (= y z)))

  Query operations are modeled as partial functions: they may be undefined for certain arguments due to violation of multiplicity constraints.

- CL maps each composition declaration $m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2]) \in M$ to a constant $\mathsf{CL}(m)$ and axioms stating that $\mathsf{CL}(m)$ is a finite binary relation represented as a sequence of pairs of the correct type:

```
(from:Sequence m)
(forall (p) (if (form:sequence-member p m)
                (and (form:Pair p) (c1 (form:first p)) (c2 (form:second p)))))
```

  In case τ_2 is not present or $\tau_2 = \mathsf{Set}$, this is simplified to a binary relation directly represented as a binary predicate:
  (forall (x y) (if (m x y) (and (c_1 x) (c_2 y))))

- ~~for any pair of~~ [24] for any composition declarations $m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2]), m'(p_1' : \mathsf{Set}[c_1'], \blacklozenge p_2' : \tau_2'[c_2']) \in M$, an axiom stating "each instance has at most one owner": Note(24

```
(forall (o o' i)
        (if (and (form:sequence-member (form:pair o i) m)
                 (form:sequence-member (form:pair o' i) m'))
            (= o o')))
```

  In case `m` is represented in the simplified way, `(form:sequence-member (form:pair o i) m)` is replaced by `(m o i)`, and analogously for `m'`.

- CL maps each association declaration $a(p_1 : \tau_1[c_1], \ldots, p_r : \tau_r[c_r]) \in A$ to a predicate $\mathsf{CL}(a)$ and axioms stating that $\mathsf{CL}(a)$ is a finite relation represented as a sequence of tuples of the correct types (the latter again being represented as sequences)[12]:
  (from:Sequence a)
  (forall (t) (if (form:sequence-member t a)
  (exists (x_1 $\cdots$ x_r)
  (and (c_1 x_1) $\cdots$ (c_r x_r)
  (= t (form:sequence-insert x_1 ($\cdots$ (form:sequence-insert
  x_r form:empty-sequence)))))))

  In case that all the τ_i are omitted (or, equivalently, equal to Set), the representation is simplified to an n-ary predicate:
  (forall (x_1 x_2 $\cdots$ x_n) (if (a x_1 x_2 $\cdots$ x_n) (and (c_1 x_1) $\cdots$ (c_n x_n))))

[10] ($\tau[c]$ x) is an abbreviation of either (if τ is present)
(and (τ x) (forall (m) (if (from:τ-member m x) (c' m)))).
or (if τ is omitted) just (c x).

[11] Note that the $\cdots$ here is meta notation, not a sequence marker.

[24] NOTE: todo: define ownership by a big disjuction over all composition delcarations and composite attributes. each instance has at most one owner: We say that o is +an owner of o' in $\mathcal{I}$ if +$o' \in P^{\mathcal{I}}(c\blacklozenge p : \tau[c'])(o)$ or +$o' \in M^{\mathcal{I}}(m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : +\tau_2[c_2]).p_2)(o)$. +If o_1 and o_2 are owners of o' in $\mathcal{I}$, then +$o_1 = o_2$;

[12] Ignoring the annotations τ_i in the interpretation of an association is intentional, see OMG UML version 2.5~~(ptc/2013-09-05) in section 11.5.3~~, p. 197: "A link is a tuple with one value for each memberEnd of the Association, where each value is an instance whose type conforms to or implements the type of the end. [. . .] When one or more ends of the Association have ~~isUnique =false~~isUnique = *false*, it is possible to have several links associating the same set of instances. In such a case, links carry an additional identifier apart from their end values. When one or more ends of the Association are ordered, links carry ordering information in addition to their end values." Similarly in UML Superstructure Specification 2.4.1, p. 37. The additional information required for links is covered by using sequences of tuples.

- the interpretation of a member end of a binary association declaration owned by a class/data type coincides with the interpretation of the attribute: if for $i \in \{1,2\}$, $\mathbf{a}.p_i : \tau_i[c_i]$ for $\mathbf{a} = a(p_1 : \tau_1[c_1], p_2 : \tau_2[c_2]) \in A$ is owned by $c \in C$ with $c.p_i : \tau_i[c_i] \in P$, then
  ```
  (forall (o s)
       (if (c.p o s) (= s (form:seq2τ_i (form:selecti o a)))))
  ```
  If $\mathbf{a}$ is represented in simplified form, then instead the following is used
  ```
  (forall (o s)
       (if (c.p o s) (forall (x) (iff (member x s) (a o x)))))
  ```
- the interpretation of a member end of a composition declaration[25] owned by a class/data type coincides with the interpretation of the attribute: if for $i \in \{1,2\}$, $\mathbf{m}.p : \tau_i[c_i]$ for $\mathbf{m} = m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2]) \in M$ is owned by $c \in C$ with $c.p : \tau_i[c_i] \in P$, then `(forall (o s)` Note(25
  ```
       (if (c.p o s) (= s (form:seq2τ_i (form:selecti o m)))))
  ```
  Again, if $\mathbf{m}$ is represented in simplified form, then the following is used
  ```
  (forall (o s)
       (if (c.p o s) (forall (x) (iff (member x s) (m o x)))))
  ```

It is straightforward to extend CL from signatures to signature morphisms.

Models. A Σ-model of the UML class diagram institution is just a $\mathsf{CL}(\Sigma)$-model in Common Logic. That is, the UML class diagram institution inherits models from Common Logic. Moreover, model reducts are inherited as well, using the action of CL on signature morphisms.

E.5 Sentences

The set of *multiplicity formulae Frm* is given by the following grammar:

$$
\begin{aligned}
\mathit{Frm} &::= \mathit{NumLiteral} \leq \mathit{FunExpr} \\
&\mid \mathit{FunExpr} \leq \mathit{NumLiteral} \\
\mathit{FunExpr} &::= \#\ \mathit{Attribute} \\
&\mid \#\ \mathit{Association}\ .\ \mathit{End} \\
&\mid \#\ \mathit{Composition}\ .\ \mathit{End} \\
&\mid \#\ \mathit{Operation}\ .\ \mathit{Param} \\
\mathit{Attribute} &::= \mathit{Classifier}\ .\ \mathit{End}{:}\mathit{Type} \\
\mathit{Association} &::= \mathit{Name}\ (\ \mathit{End} : \mathit{Type}(\ ,\ \mathit{End} : \mathit{Type})^*\ ) \\
\mathit{Composition} &::= \mathit{Name}\ (\ \mathit{End} : \mathsf{Set}\ [\ \mathit{Classifier}\ ], \blacklozenge\mathit{End} : \mathit{Type}\ ) \\
\mathit{Operation} &::= \mathit{Name}\ (\ (\ \mathit{NumLiteral} \leq \mathit{Param} \leq \mathit{NumLiteral}{:}\ \mathit{Type},\ )^*\ ) : \mathit{Type} \\
\mathit{Type} &::= \mathit{Annot}\ [\ \mathit{Classifier}\ ] \\
\mathit{Classifier} &::= \mathit{Name} \\
\mathit{End} &::= \mathit{Name} \\
\mathit{Param} &::= \mathit{Name} \\
\mathit{Annot} &::= \mathsf{OrderedSet} \mid \mathsf{Set} \mid \mathsf{Sequence} \mid \mathsf{Bag} \\
\mathit{NumLiteral} &::= \mathsf{0} \mid \mathsf{1} \mid \cdots
\end{aligned}
$$

where *Name* is a set of names and *NumLiteral* is assumed to be equipped with an appropriate function $[\![-]\!] : \mathit{NumLiteral} \to \mathbb{Z}$.

The set of Σ-*multiplicity constraints* $\mathit{Mult}(\Sigma)$ for a class/data type net Σ is given by the multiplicity formulae in *Frm* such that all mentioned elements of *Association* and *Composition* correspond to association declarations and composition declarations of Σ, respectively, and the member end name mentioned in the clauses of *FunExpr* occur in the mentioned association and composition, respectively.

The *translation* of a formula $\varphi \in \mathit{Mult}(\Sigma)$ along a class/data type net morphism σ, written as $\sigma(\varphi)$, is given by applying σ to associations, compositions, and member end names.

Example For the example in Fig. **??** there are the following multiplicity formulas:

$\mathsf{2} \leq \#\mathsf{n2s}(\mathsf{net} : \mathsf{Set}[\mathsf{Net}], \blacklozenge\mathsf{station} : \mathsf{Set}[\mathsf{Station}]).\mathsf{station}\mathsf{2} \leq \#\mathsf{N2S}(\mathsf{net} : \mathsf{Set}[\mathsf{Net}], \blacklozenge\mathsf{station} : \mathsf{Set}[\mathsf{Station}]).\mathsf{station}$

$\#\mathsf{n2s}(\mathsf{net} : \mathsf{Set}[\mathsf{Net}], \blacklozenge\mathsf{station} : \mathsf{Set}[\mathsf{Station}]).\mathsf{net} = \mathsf{1}\#\mathsf{N2S}(\mathsf{net} : \mathsf{Set}[\mathsf{Net}], \blacklozenge\mathsf{station} : \mathsf{Set}[\mathsf{Station}]).\mathsf{net} = \mathsf{1}$

$\#\mathsf{n2l}(\mathsf{net} : \mathsf{Set}[\mathsf{Net}], \blacklozenge\mathsf{line} : \mathsf{Set}[\mathsf{Line}]).\mathsf{net} = \mathsf{1}\#\mathsf{N2L}(\mathsf{net} : \mathsf{Set}[\mathsf{Net}], \blacklozenge\mathsf{line} : \mathsf{Set}[\mathsf{Line}]).\mathsf{net} = \mathsf{1}$

$\#\mathsf{s2u}(\mathsf{station} : \mathsf{Set}[\mathsf{Station}], \blacklozenge\mathsf{unit} : \mathsf{Set}[\mathsf{Unit}]).\mathsf{station} = \mathsf{1}\#\mathsf{S2U}(\mathsf{station} : \mathsf{Set}[\mathsf{Station}], \blacklozenge\mathsf{unit} : \mathsf{Set}[\mathsf{Unit}]).\mathsf{station} = \mathsf{1}$

[25] Note: Adapt to the definition of composition declarations as a special case of association declarations.

$\#\mathsf{s2t}(\mathsf{station} : \mathsf{Set}[\mathsf{Station}], \blacklozenge\mathsf{track} : \mathsf{Set}[\mathsf{Track}]).\mathsf{station} = 1$$\#\mathsf{S2T}(\mathsf{station} : \mathsf{Set}[\mathsf{Station}], \blacklozenge\mathsf{track} : \mathsf{Set}[\mathsf{Track}]).\mathsf{station} = 1$

$1 \leq \#\mathsf{c2u}(\mathsf{connector} : \mathsf{Set}[\mathsf{Connector}], \mathsf{unit} : \mathsf{Set}[\mathsf{Unit}]).\mathsf{unit} \leq 4$$1 \leq \#\mathsf{C2U}(\mathsf{connector} : \mathsf{Set}[\mathsf{Connector}], \mathsf{unit} : \mathsf{Set}[\mathsf{Unit}]).\mathsf{unit} \leq 4$

$\#\mathsf{c2u}(\mathsf{connector} : \mathsf{Set}[\mathsf{Connector}], \mathsf{unit} : \mathsf{Set}[\mathsf{Unit}]).\mathsf{connector} = 1$$\#\mathsf{C2U}(\mathsf{connector} : \mathsf{Set}[\mathsf{Connector}], \mathsf{unit} : \mathsf{Set}[\mathsf{Unit}]).\mathsf{connector} = 1$

$1 \leq \#\mathsf{l2t}(\mathsf{track} : \mathsf{Set}[\mathsf{Track}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{track}$$1 \leq \#\mathsf{L2T}(\mathsf{track} : \mathsf{Set}[\mathsf{Track}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{track}$

$\#\mathsf{l2t}(\mathsf{track} : \mathsf{Set}[\mathsf{Track}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{linear} = 1$$\#\mathsf{L2T}(\mathsf{track} : \mathsf{Set}[\mathsf{Track}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{linear} = 1$

$1 \leq \#\mathsf{l2t}(\mathsf{line} : \mathsf{Set}[\mathsf{Line}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{line}$$1 \leq \#\mathsf{L2T}(\mathsf{line} : \mathsf{Set}[\mathsf{Line}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{line}$

$\#\mathsf{l2l}(\mathsf{line} : \mathsf{Set}[\mathsf{Line}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{linear} = 1$$\#\mathsf{L2L}(\mathsf{line} : \mathsf{Set}[\mathsf{Line}], \mathsf{linear} : \mathsf{Set}[\mathsf{Linear}]).\mathsf{linear} = 1$

"$x = y$" is an abbreviation for the two inequations "$x \leq y$" and "$y \leq x$". "$x \leq y \leq z$" is an abbreviation for the two inequations "$x \leq y$" and "$y \leq z$".

E.6 Satisfaction Relation

The satisfaction relation is inherited from Common Logic, using a translation $\mathsf{CL}(\_)$ of multiplicity formulas to Common Logic. That is, given a UML class and object diagram Σ, a multiplicity formula φ and a Σ-model M (the latter amounts to a $\mathsf{CL}(\Sigma)$-model M in Common Logic):

$$M \models_\Sigma \varphi \text{ iff } M \models_{\mathsf{CL}(\Sigma)} \mathsf{CL}(\varphi)$$

The translation of multiplicity formulas to Common Logic is as follows:

- $\mathsf{CL}(\ell \leq \#c.p : \tau[c']) =$
```
(forall (x y n)
     (if (and (c.p x y) (form:τ-size y n)) (buml:leq [[ℓ]] n))
```
- $\mathsf{CL}(\ell \leq \#a(p_1 : \tau_1[c_1], \ldots, p_r : \tau_r[c_r]).p_i =$
```
(forall (x_1 ··· x_{i-1} x_{i+1} ··· x_r)
     (if (and (c_1 x_1) ··· (c_{i-1} x_{i-1}) (c_{i+1} x_{i+1}) ··· (c_r x_r)
               (form:sequence-size
                    (form:n-select a i [x_1 ··· x_{i-1} x_{i+1} ··· x_r]) n))
          (buml:leq [[ℓ]] n)))
```
  If a is represented in simplified form, the following is used instead:
  $\mathsf{CL}(\ell \leq \#a(p_1 : \tau_1[c_1], \ldots, p_r : \tau_r[c_r]).p_i =$
```
(forall (x_1 ··· x_{i-1} x_{i+1} ··· x_r)
     (if (and (c_1 x_1) ··· (c_{i-1} x_{i-1}) (c_{i+1} x_{i+1}) ··· (c_r x_r))
              (exists (y_1 ··· y_[[ℓ]])
                  (and (not (= (y_1 y_2))) ··· (not (= (y_[[ℓ]]-1 y_[[ℓ]])))
                        (a x_1 ··· x_{i-1} y_1 x_{i+1} ··· x_r)
                        ···
                        (a x_1 ··· x_{i-1} y_[[ℓ]] x_{i+1} ··· x_r) ))))
```
- $\mathsf{CL}(\ell \leq \#m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2]).p_i) =$
```
(forall (x)
     (if (and (c_{3-i} x) (form:τ-size (form:selecti x m) n))
          (buml:leq [[ℓ]] n))
```
  If m is represented in simplified form, the following is used instead:
  $\mathsf{CL}(\ell \leq \#m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2]).p_1) =$
```
(forall (x)
     (if (c_2 x)
              (exists (y_1 ··· y_[[ℓ]])
                  (and (not (= (y_1 y_2))) ··· (not (= (y_[[ℓ]]-1 y_[[ℓ]])))
                        (m y_1 x)
                        ···
                        (m y_[[ℓ]] x) ))))
```
  $\mathsf{CL}(\ell \leq \#m(p_1 : \mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2]).p_2) =$
```
(forall (x)
     (if (c_1 x)
              (exists (y_1 ··· y_[[ℓ]])
                  (and (not (= (y_1 y_2))) ··· (not (= (y_[[ℓ]]-1 y_[[ℓ]])))
                        (m x y_1)
```

```
                        ...
                        (m x y_[[ℓ]]) ))))
```

- $\mathsf{CL}(\ell \leq \#c.q(\ell_1 \leq f_1 \leq \ell'_1 : \tau_1[c_1], \ldots, \ell_k \leq f_k \leq \ell'_k : \tau_k[c_k]) : \tau[c']) =$

```
(forall (x x_1 x_2 ··· x_n)
     (if (and (c.q x x_1 x_2 ··· x_n y)
               (form:τ-size x_1 n_1)
               ...
               (form:τ-size x_k n_k)
               (form:τ-size y n)
               (buml:leq [[ℓ_1]] n_1)
               (buml:leq n_1 [[ℓ'_1]])
               ...
               (buml:leq [[ℓ_k]] n_k)
               (buml:leq n_k [[ℓ'_k]]) )
          (buml:leq [[ℓ]] n)))
```

where $[\![-]\!] : \mathit{NumLit} \to \mathbb{Z}$ maps a numerical literal to an integer, and $[x_1 \cdots x_n]$ abbreviates `(form:sequence-insert` x_1 $\cdots$ `(form:sequence-insert` x_n `form:empty-sequence))`. The translation for $\mathit{FunExpr} \leq \mathit{NumLiteral}$ is analogous. In case of simplified representation, the existence of $[\![\ell]\!]$ distinct individuals would be replaced with a statement expressing that if $[\![\ell]\!] + 1$ individuals have the specified property, at least two of them must be equal.

F Annex: Conformance of TPTP with DOL

(Informative)

TPTP [?, ?, ?] is a language spoken by dozens of first-order theorem provers, and large libraries have been formalized in TPTP. The underlying logic is unsorted first-order logic. In [?], many-sorted first has been formalized as an institution; the single-sorted sublogic (using only a fixed set of sorts $\{s\}$ is isomorphic to unsorted first-order logic).

F.1 Abstract Syntax Conformance of TPTP With DOL

The metaclass `TPTP_file` is a subclass (in the sense of SMOF multiple classification) of `NativeDocument`. The metaclass `annotated_formula` is a subclass (in the sense of SMOF multiple classification) of `BasicOMS`.

G Annex: Conformance of CASL with DOL

(Informative)

CASL [?] extends many-sorted first-order logic with partial functions and subsorting. It also provides induction sentences, expressing the (free) generation of datatypes.

G.1 Abstract Syntax Conformance of CASL With DOL

The metaclass `LIBRARY` is a subclass (in the sense of SMOF multiple classification) of `NativeDocument`. The metaclass `BASIC_SPEC` is a subclass (in the sense of SMOF multiple classification) of `BasicOMS`.

G.2 Semantic Conformance of CASL With DOL

CASL has been presented as an institution in [?, ?]. This ~~annex~~ section presents a sketch of this institution.

CASL signatures consist of a set S of sorts with a subsort relation $\leq$ between them together with families $\{PF_{w,s}\}_{w \in S^*, s \in S}$ of partial functions, $\{TF_{w,s}\}_{w \in S^*, s \in S}$ of total functions and $\{P_w\}_{w \in S^*}$ of predicate symbols. If Σ is a signature, two operation symbols with the same name f and with profiles $w \to s$ and $w' \to s'$, denoted $f_{w,s}$ and $f_{w',s'}$, are in the overloading relation if there are $w_0 \in S^*$ and $s_0 \in S$ such that $w_0 \leq w, w'$ and $s, s' \leq s_0$. Overloading of predicates is defined in a similar way. Signature morphisms consist of maps taking sort, function and predicate symbols respectively to a symbol of the same kind in the target signature, and they must preserve subsorting, typing of function and predicate symbols and totality of function symbols, and overloading.

For a signature Σ, terms are formed starting with variables from a sorted set X using applications of function symbols to terms of appropriate sorts, while sentences are partial first-order formulas extended with *sort generation constraints* which are triples (S', F', σ') such that $\sigma' : \Sigma' \to \Sigma$ and S' and F' are respectively sort and function symbols of Σ'. Partial first-order formulas are translated along a signature morphism $\varphi : \Sigma \to \Sigma''$ by replacing symbols as prescribed by φ while sort generation constraints are translated by composing the morphism σ' in their third component with φ.

Models interpret sorts as nonempty sets such that subsorts are injected into supersorts, partial/total function symbols as partial/total functions and predicate symbols as relations, such that the embeddings of subsorts into supersorts are monotone w.r.t. overloading.

The satisfaction relation is the expected one for partial first-order sentences. A sort generation constraint (S', F', σ') holds in a model M if the carriers of the reduct of M along σ' of the sorts in S' are generated by function symbols in F'.

H Annex: A Core Logic Graph

(Informative)

[26] Note(26

This annex provides a core heterogeneous environment that could be used as a basis for semantics of DOL as defined in Sec. ??.

H.1 Languages

The selected OMS languages are those whose conformance with DOL is established in the preceding annexes (OWL 2 DL in annex ??, Common Logic in annex ??, RDFS in annex ??, CASL in annex ??, UML class diagrams in annex ?? and TPTP in annex ??). The logic graph is shown in Figure ??; the language graph and supports relation in Figure ??. Its nodes refer to the following OMS languages and profiles:

- RDF W3C/TR REC-rdf11-concepts:2014
- RDF Schema W3C/TR REC-rdf11-schema:2014
- EL, QL, RL (all being profiles of OWL) ~~W3C/TR REC-owl2-profiles:2009~~ W3C/TR REC-owl2-profiles:2012
- OWL ~~W3C/TR REC-owl2-syntax:2009~~ W3C/TR REC-owl2-syntax:2012
- CL (Common Logic) ISO/IEC 24707:2007
- UML class diagrams OMG Unified Modeling Language (UML) specification 2.4.1
- CASL [?] and its sublanguage classical first-order logic (FOL)
- TPTP

The list of language translations, given below, comprises standard translations from the literature [?, ?], as well as further translations that are considered useful for logical interoperability:

- EL $\rightarrow$ OWL
- QL $\rightarrow$ OWL
- RL $\rightarrow$ OWL
- RDF $\rightarrow$ RDFS
- RDFS $\rightarrow$ OWL
- OWL $\rightarrow$ CASL.**FOL**
- CASL.**FOL** $\rightarrow$ $TPTP$
- $TPTP$ $\rightarrow$ CASL.**FOL**
- CASL.FOL $\rightarrow$ CL
- CASL.FOL $\rightarrow$ CASL
- $UML\text{-}CD$ $\rightarrow$ CL.

The translations are specified in [?, ?]. Properties of translations have been introduced in section ??. All translations are marked as default translations.

[26] NOTE: Instantiate the whole heterogeneous environment, that is, error logic, unions, etc.

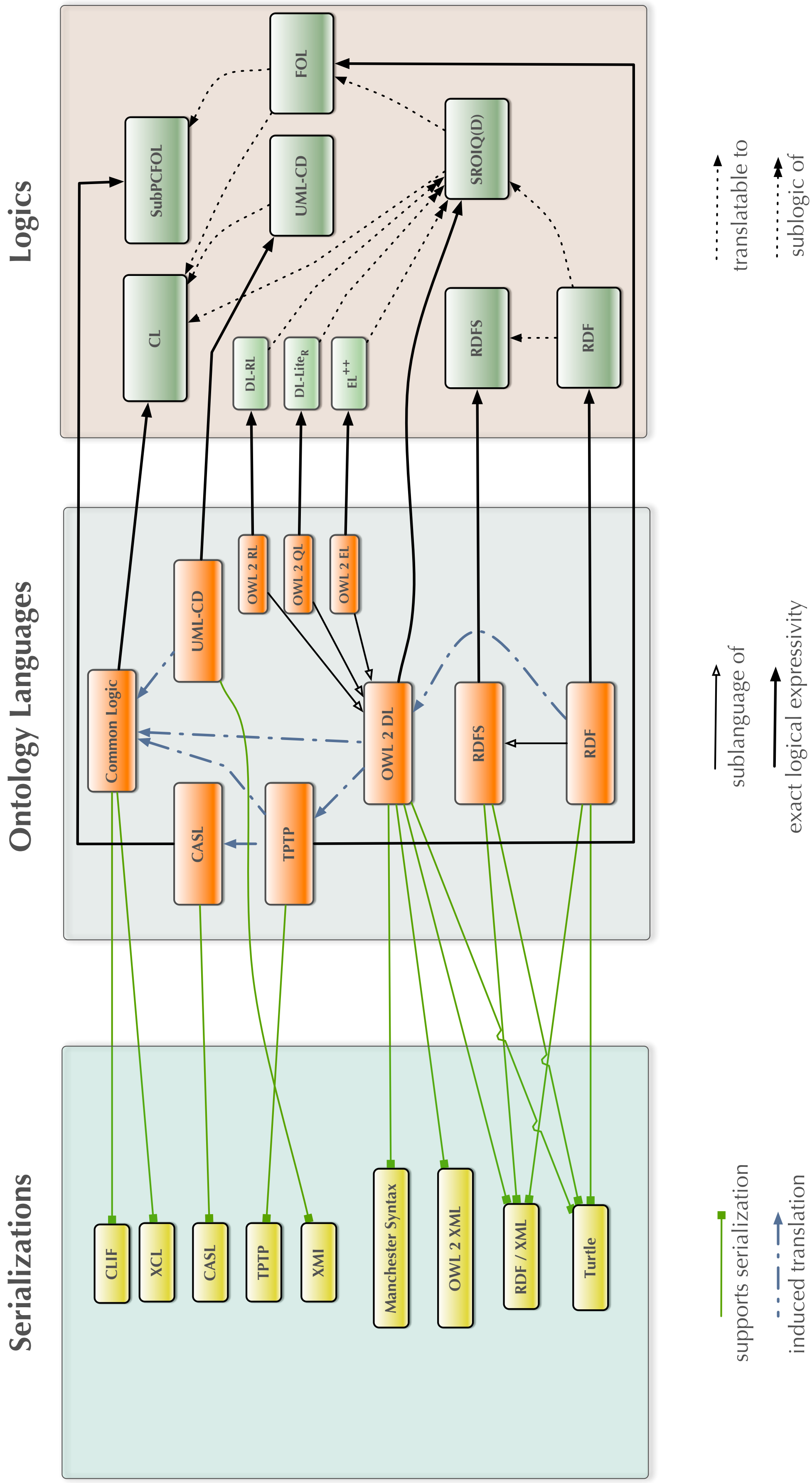


Figure H.1: Subset of the OntoIOp registry, shown as an RDF graph

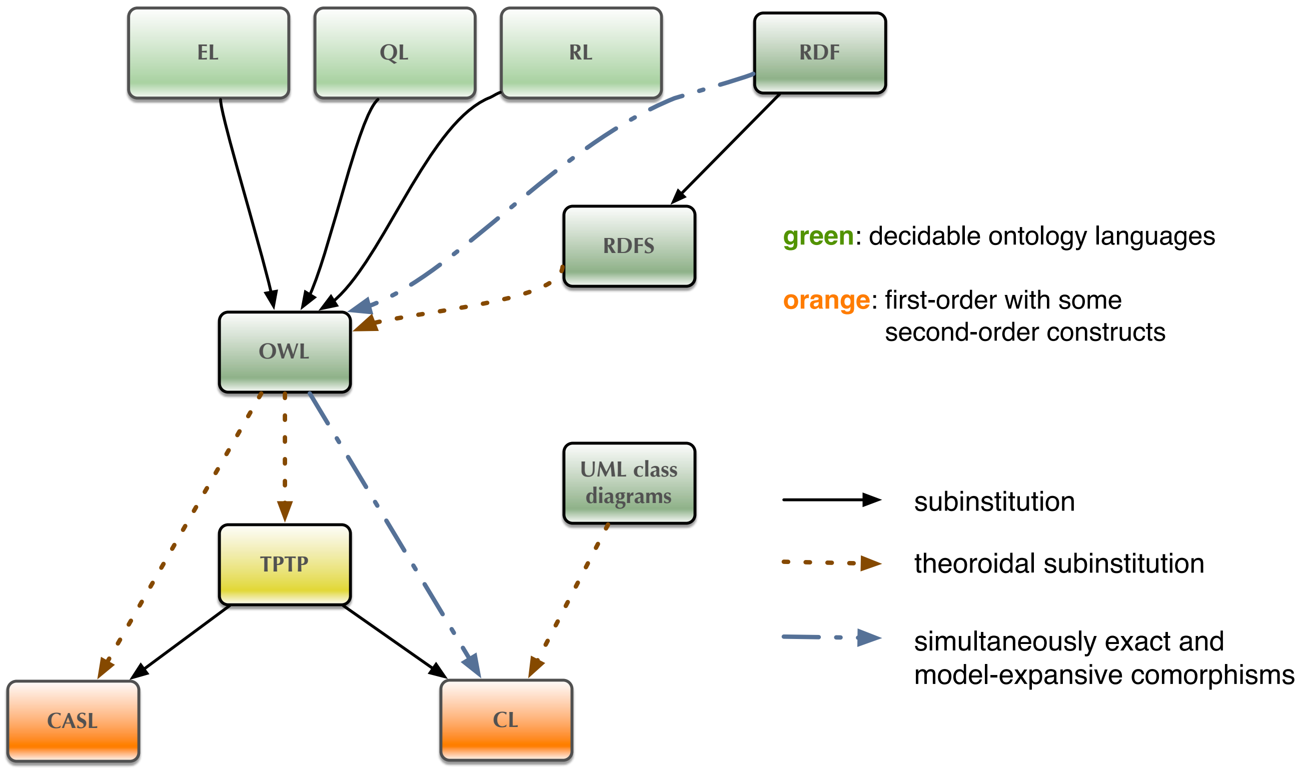


Figure H.2: Translations between conforming OMS languages

H.2 Logics

The logics giving the semantics of these languages are listed below:

- RDF and RDFS, supported respectively by RDF and RDFS
- $\mathcal{EL}$++, supported by the language EL
- DL-Lite$_R$, supported by QL
- RL, supported by RL
- $\mathcal{SROIQ}(D)$, supported by OWL
- CL, supported by CL
- $SubPCFOL^{=}_{ms}$, supported by CASL
- **FOL**, supported by CASL.FOL and $TPTP$
- $UML\text{-}CD$, supported by $UML\text{-}CD$.

The institution comorphisms between these logics are

- $\mathcal{EL}{++} \rightarrow \mathcal{SROIQ}(D)$
- DL-Lite$_R \rightarrow \mathcal{SROIQ}(D)$
- RL $\rightarrow \mathcal{SROIQ}(D)$
- RDF $\rightarrow$ RDFS
- RDFS $\rightarrow \mathcal{SROIQ}(D)$
- $\mathcal{SROIQ}(D) \rightarrow$ CASL.**FOL**
- **FOL** $\rightarrow$ CL
- **FOL** $\rightarrow SubPCFOL^{=}_{ms}$
- $UML\text{-}CD \rightarrow$ CL.

All of them are selected as default logic translations. There are no institution morphisms. The partial union operation between logics is given in the tables below, where $\perp$ denotes undefinedness:

Union	$\mathcal{EL}{++}$	DL-Lite$_R$	RL	RDF	RDFS
$\mathcal{EL}{++}$	$\mathcal{EL}{++}$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$
DL-Lite$_R$	$\mathcal{SROIQ}(D)$	DL-Lite$_R$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$
RL	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	RL	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$
RDF	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	RDF	RDFS
RDFS	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	RDFS	RDFS
$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$
FOL	**FOL**	**FOL**	**FOL**	**FOL**	**FOL**
$SubPCFOL^{=}_{ms}$	$SubPCFOL^{=}_{ms}$	$SubPCFOL^{=}_{ms}$	$SubPCFOL^{=}_{ms}$	$SubPCFOL^{=}_{ms}$	$SubPCFOL^{=}_{ms}$
UML-CD	CL	CL	CL	CL	CL
CL	CL	CL	CL	CL	CL

Union	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	UML-CD	CL
$\mathcal{EL}{++}$	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
DL-Lite$_R$	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
RL	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
RDF	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
RDFS	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
$\mathcal{SROIQ}(D)$	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
FOL	**FOL**	**FOL**	$SubPCFOL^{=}_{ms}$	CL	CL
$SubPCFOL^{=}_{ms}$	$\mathcal{SROIQ}(D)$	**FOL**	$SubPCFOL^{=}_{ms}$	$\perp$	$\perp$
UML-CD	CL	CL	$\perp$	UML-CD	CL
CL	CL	CL	$\perp$	CL	CL

The other assumptions on the logics in the heterogeneous logical environment hold in the expected way.[27] Note(27

H.3 Serializations

The following syntaxes are part of the heterogeneous logical environments:

- Turtle, supported by OWL, EL, QL, RL, RDF, RDFS
- RDF-XML, supported by OWL, EL, QL, RL, RDF, RDFS
- OWL~~2 ~~/XML, supported by OWL, EL, QL, RL
- Manchester Syntax, supported by OWL, EL, QL, RL
- TPTP, supported by TPTP
- CASL, supported by Casl
- XMI, supported by UML-CD
- XCL, supported by CL
- CLIF, supported by CL

H.4 Language and Logic Translations

H.4.1 EL → OWL and $\mathcal{EL}{++} \to \mathcal{SROIQ}(D)$

EL → OWL is the sublanguage inclusion obtained by the syntactic restriction according to the definition of EL, see ~~W3C/TR REC-owl2-profiles:2009~~W3C/TR REC-owl2-profiles:2012. Since by definition, $\mathcal{EL}{++}$ is a syntactic restriction of $\mathcal{SROIQ}(D)$, $\mathcal{EL}{++} \to \mathcal{SROIQ}(D)$ is the corresponding sublogic inclusion.

H.4.2 QL → OWL and DL-Lite$_R$ → $\mathcal{SROIQ}(D)$

QL → OWL is the sublanguage inclusion obtained by the syntactic restriction according to the definition of QL, see ~~W3C/TR REC-owl2-profiles:2009~~W3C/TR REC-owl2-profiles:2012. Since by definition, DL-Lite$_R$ is a syntactic restriction of $\mathcal{SROIQ}(D)$, DL-Lite$_R$ → $\mathcal{SROIQ}(D)$ is the corresponding sublogic inclusion.

[27] Note: @Till: rephrase if need be. TM: I think it is OK, even if quite vague.

H.4.3 RL $\to$ OWL and RL $\to$ $\mathcal{SROIQ}(D)$

RL $\to$ OWL is the sublanguage inclusion obtained by the syntactic restriction according to the definition of RL, see ~~W3C/TR REC-owl2-profiles:2009~~W3C/TR REC-owl2-profiles:2012. Since by definition, RL is a syntactic restriction of $\mathcal{SROIQ}(D)$, RL $\to$ $\mathcal{SROIQ}(D)$ is the corresponding sublogic inclusion.

H.4.4 SimpleRDF $\to$ RDF

SimpleRDF $\to$ RDF is an obvious inclusion, except that SimpleRDF resources need to be renamed if they happen to have a predefined meaning in RDF. The model translation needs to forget the fixed parts of RDF models. Since this part can always reconstructed in a unique way, the result is an isomorphic model translation.

H.4.5 RDF $\to$ RDFS

This is entirely analogous to SimpleRDF $\to$ RDF.

H.4.6 SimpleRDF $\to$ $\mathcal{SROIQ}(D)$

[28] Note(28

A SimpleRDF signature is translated to $\mathcal{SROIQ}(D)$ by providing a class P and three roles *sub*, *pred* and *obj* (these reify the extension relation), and one individual per SimpleRDF resource. A SimpleRDF triple (s, p, o) is translated to the $\mathcal{SROIQ}(D)$ sentence

$$\top \sqsubseteq \exists U.(\exists sub.\{s\} \sqcap \exists pred.\{p\} \sqcap \exists obj.\{o\}).$$

From an $\mathcal{SROIQ}(D)$ model $\mathcal{I}$, obtain a SimpleRDF model by inheriting the universe and the interpretation of individuals (then turned into resources). The interpretation $P^{\mathcal{I}}$ of P gives P_m, and EXT_m is obtained by de-reifying, i.e.

$$EXT_m(x) := \{(y, z) \mid \exists u.(u, x) \in pred^{\mathcal{I}}, (u, y) \in sub^{\mathcal{I}}, (u, z,) \in obj^{\mathcal{I}}\}.$$

RDF $\to$ $\mathcal{SROIQ}(D)$ is defined similarly. The theory of RDF built-ins is (after translation to $\mathcal{SROIQ}(D)$) added to any signature translation. This ensures that the model translation can add the built-ins.

H.4.7 OWL $\to$ FOL

H.4.7.1 Translation of signatures

$\Phi((\mathbf{C}, \mathbf{R}, \mathbf{I})) = (F, P)$ with

- function symbols: $F = \{a^{(1)} | a \in \mathbf{I}\}$
- predicate symbols $P = \{A^{(1)} | A \in \mathbf{C}\} \cup \{R^{(2)} | R \in \mathbf{R}\}$

H.4.7.2 Translation of sentences

Concepts are translated as follows:

- $\alpha_x(A) = A(x)$
- $\alpha_x(\neg C) = \neg\alpha_x(C)$
- $\alpha_x(C \sqcap D) = \alpha_x(C) \wedge \alpha_x(D)$
- $\alpha_x(C \sqcup D) = \alpha_x(C) \vee \alpha_x(D)$
- $\alpha_x(\exists R.C) = \exists y.(R(x, y) \wedge \alpha_y(C))$
- $\alpha_x(\exists U.C) = \exists y.\alpha_y(C)$
- $\alpha_x(\forall R.C) = \forall y.(R(x, y) \to \alpha_y(C))$
- $\alpha_x(\forall U.C) = \forall y.\alpha_y(C)$
- $\alpha_x(\exists R.\mathrm{Self}) = R(x, x)$
- $\alpha_x(\leq nR.C) = \forall y_1, \ldots, y_{n+1}. \bigwedge_{i=1,\ldots,n+1}(R(x, y_i) \wedge \alpha_{y_i}(C)) \to \bigvee_{1\leq i<j\leq n+1} y_i = y_j$
- $\alpha_x(\geq nR.C) = \exists y_1, \ldots, y_n. \bigwedge_{i=1,\ldots,n}(R(x, y_i) \wedge \alpha_{y_i}(C)) \wedge \bigwedge_{1\leq i<j\leq n} y_i \neq y_j$
- $\alpha_x(\{a_1, \ldots a_n\}) = (x = a_1 \vee \ldots \vee x = a_n)$

[28] NOTE: This translation is not really useful. Consider the RDF-OWL-reduct construction instead.

For inverse roles R^-, $R^-(x,y)$ has to be replaced by $R(y,x)$, e.g.

$$\alpha_x(\exists R^-.C) = \exists y.(R(y,x) \land \alpha_y(C))$$

This rule also applies below.

Sentences are translated as follows:

- $\alpha_\Sigma(C \sqsubseteq D) = \forall x.\,(\alpha_x(C) \rightarrow \alpha_x(D))$
- $\alpha_\Sigma(a : C) = \alpha_x(C)[a/x]$[1]
- $\alpha_\Sigma(R(a,b)) = R(a,b)$
- $\alpha_\Sigma(R \sqsubseteq S) = \forall x,y.R(x,y) \rightarrow S(x,y)$
- $\alpha_\Sigma(R_1; \ldots ; R_n \sqsubseteq R) =$
  $\forall x,y.(\exists z_1, \ldots z_{n-1}.R_1(x,z_1) \land R_2(z_1,z_2) \land \ldots \land R_n(z_{n-1},y)) \rightarrow R(x,y)$
- $\alpha_\Sigma(\mathrm{Dis}(R_1,R_2)) = \neg\exists x,y.R_1(x,y) \land R_2(x,y)$
- $\alpha_\Sigma(\mathrm{Ref}(R)) = \forall x.R(x,x)$
- $\alpha_\Sigma(\mathrm{Irr}(R)) = \forall x.\neg R(x,x)$
- $\alpha_\Sigma(\mathrm{Asy}(R)) = \forall x,y.R(x,y) \rightarrow \neg R(y,x)$
- $\alpha_\Sigma(\mathrm{Tra}(R)) = \forall x,y,z.R(x,y) \land R(y,z) \rightarrow R(x,z)$

H.4.7.3 Translation of models

- For $M' \in \mathrm{Mod}^{FOL}(\Phi\Sigma)$ define $\beta_\Sigma(M') := (\Delta, \cdot^I)$ with $\Delta = |M'|$ and $A^I = M'_A, a^I = M'_a, R^I = M'_R$.

Proposition 24 $C^{\mathcal{I}} = \{m \in M'_{Thing} | M' + \{x \mapsto m\} \models \alpha_x(C)\}$

Proof. By Induction over the structure of C.

- $A^{\mathcal{I}} = M'_A = \{m \in M'_{Thing} | M' + \{x \mapsto m\} \models A(x)\}$
- $(\neg C)^{\mathcal{I}} = \Delta \setminus C^{\mathcal{I}} =^{I.H.} \Delta \setminus \{m \in M'_{Thing} | M' + \{x \mapsto m\} \models \alpha_x(C)\} = \{m \in M'_{Thing} | M' + \{x \mapsto m\} \models \neg\alpha_x(C)\}$

□

The satisfaction condition holds as well.

H.4.8 $FOL \rightarrow$ CL

This comorphism maps classical first-order logic (FOL) to Common Logic.

A FOL signature is translated to CL.Fol by turning all constants into discourse names, and all other function symbols and all predicate symbols into non-discourse names. A FOL sentence is translated to CL.Fol by a straightforward recursion, the base being translations of predications:

$$\alpha_\Sigma(P(t_1, \ldots, t_n)) = (P\ \alpha_\Sigma(t_1)\ \ldots\ \alpha_\Sigma(t_n))$$

Within terms, function applications are translated similarly:

$$\alpha_\Sigma(f(t_1, \ldots, t_n)) = (f\ \alpha_\Sigma(t_1)\ \ldots\ \alpha_\Sigma(t_n))$$

A CL.Fol model is translated to a FOL model by using the universe of discourse as FOL universe. The interpretation of constants is directly given by the interpretation of the corresponding names in CL.Fol. The interpretation of a predicate symbol P is given by using $rel^M(int^M(P))$ and restricting to the arity of P; similarly for function symbols (using fun^M). Both the satisfaction condition and model-expansiveness of the comorphism are straightforward.

H.4.9 OWL $\rightarrow$ CL

This comorphism is the composition of the comorphisms described in the previous two sections.

H.4.10 UML class diagrams $\rightarrow$ CL

This translation has been described in annex **??**. Translation of signatures is detailed in section **??**, translation of sentences in section **??**. Models are translated identically.

[1] $t[a/x]$ means “in t, replace x by a”.

H.4.11 $FOL \to$ CASL

This is an obvious sublogic.

H.4.12 UML class diagrams toOWL

Let $\Sigma = ((C, \leq_C), P, O, A, M)$ be a *class/data type net* representing a UML class diagram as described in annex **??**. This net can be translated to OWL2 using the approach described in [**?**]. The ontology is extended by translating parts of this net and its multiplicity constraints $Mult(\Sigma)$:

- For each class $c \in C$ with superclasses $c_1, c_2, ..., c_n \in C$ (i.e. $c \leq_C c_i$ for $i = 1, ..., n$): *@ @* *@

```
\DIFaddend @*  Class: c
        SubClassOf: c1
        ...
        SubClassOf: cn
```

- For each attribute declaration $c.p : c'$ in P *@ @* *@

```
\DIFaddend @*  ObjectProperty: p
        Domain: c
        Range: c'
```

- For each attribute multiplicity $n \leq c.p : \tau[c']$ in $Mult(\Sigma)$ extend the description of class c by: *@ @* *@

```
\DIFaddend @*  SubClassOf: p min n c'
```

- For each attribute multiplicity $c.p : \tau[c'] \leq n$ in $Mult(\Sigma)$ extend the description of class c by: *@ @* *@

```
\DIFaddend @*  SubClassOf: p max n c'
```

- For each unidirectional binary association declaration $a(p_1 : \tau_1[c_1], p_2 : \tau_2[c_2])$ in A: *@ @* *@

```
\DIFaddend @*  ObjectProperty: p
        Domain: c1
        Range: c2
```

- For each bidirectional binary association declaration $a(p_1 : \tau_1[c_1], p_2 : \tau_2[c_2])$ in A: *@ @* *@

```
\DIFaddend @*  ObjectProperty: p1
        Domain: c
        Range: c'

ObjectProperty: p2
        Characteristics: InverseFunctional
        Domain: c
        Range: c'
        InverseOf: p1
```

- For each binary association $n \leq a(p_1 : \tau_1[c_1], p_2 : \tau_2[c_2]).p_i$, with $i \neq j \in \{1, 2\}$ in $Mult(\Sigma)$ extend the description of class c_j by: *@ @* *@

```
\DIFaddend @*  SubClassOf: pi min n ci
```

- For each binary association $a(p_1 : \tau_1[c_1], p_2 : \tau_2[c_2]).p_i \leq n$, with $i \neq j \in \{1, 2\}$ in $Mult(\Sigma)$ extend the description of class c_j by: *@ @* *@

```
\DIFaddend @*  SubClassOf: pi max n ci
```

- For each composition declaration $m(\mathsf{Set}[c_1], \blacklozenge p_2 : \tau_2[c_2])$ in M: *@ @* *@

```
\DIFaddend @*  ObjectProperty: p
        Characteristics:
                Functional,
                Irreflexive
        Domain: c1
        Range: c2
```

- For each binary association $n \leq a(p_1 : \tau_1[c_1], \blacklozenge p_2 : \tau_2[c_2]).p_i$, with $i \neq j \in \{1, 2\}$ in $Mult(\Sigma)$ extend the description of class c_j by: *@ @* *@

```
\DIFaddend @*  SubClassOf: pi min n ci
```

- For each binary association $a(p_1 : \tau_1[c_1], \blacklozenge p_2 : \tau_2[c_2]).p_i \leq n$, with $i \neq j \in \{1, 2\}$ in $Mult(\Sigma)$ extend the description of class c_j by: *@ @* *@

```
\DIFaddend @*  SubClassOf: pi max n ci
```

H.5 Formal Representation of Language and Logic Translations

A formal representation of language and logic translations still needs to be developed. For the syntax aspects of these translations, QVT could be a useful option. However, it would have added value to choose a representation of translations that allows their correctness to be proven easily. Such a representation would have to interact with suitable representations of languages and logics in a logical framework. See [?] for some work in this direction.

I Annex: Extended Logic Graph

(Informative)

This annex extends the graph of logics and translations given in annex ?? by a list of OMS languages whose inclusion in the registry is planned. The graph is shown in Figure ??. Its nodes are included in the following list of OMS languages and profiles (in addition to those mentioned in annex ??):

- PL (propositional logic)
- SimpleRDF (RDF triples without a reserved vocabulary)
- $\mathrm{OBO}^{\mathrm{OWL}}$ and OBO1.4
- RIF (Rule Interchange Format)
- EER (Enhanced Entity-Relationship Diagrams)
- Datalog
- ORM (object role modeling)
- the meta model of schema.org
- different diagram types of the UML (Unified Modeling Language), with possibly different logics according to different UML semantics
- SKOS (Simple Knowledge Organization System; W3C/TR REC-skos-reference:2009)
- $\mathrm{FOL}^{=}$ (untyped first-order logic, as used for the TPTP format)
- F-logic

The actual translations are specified in [?].

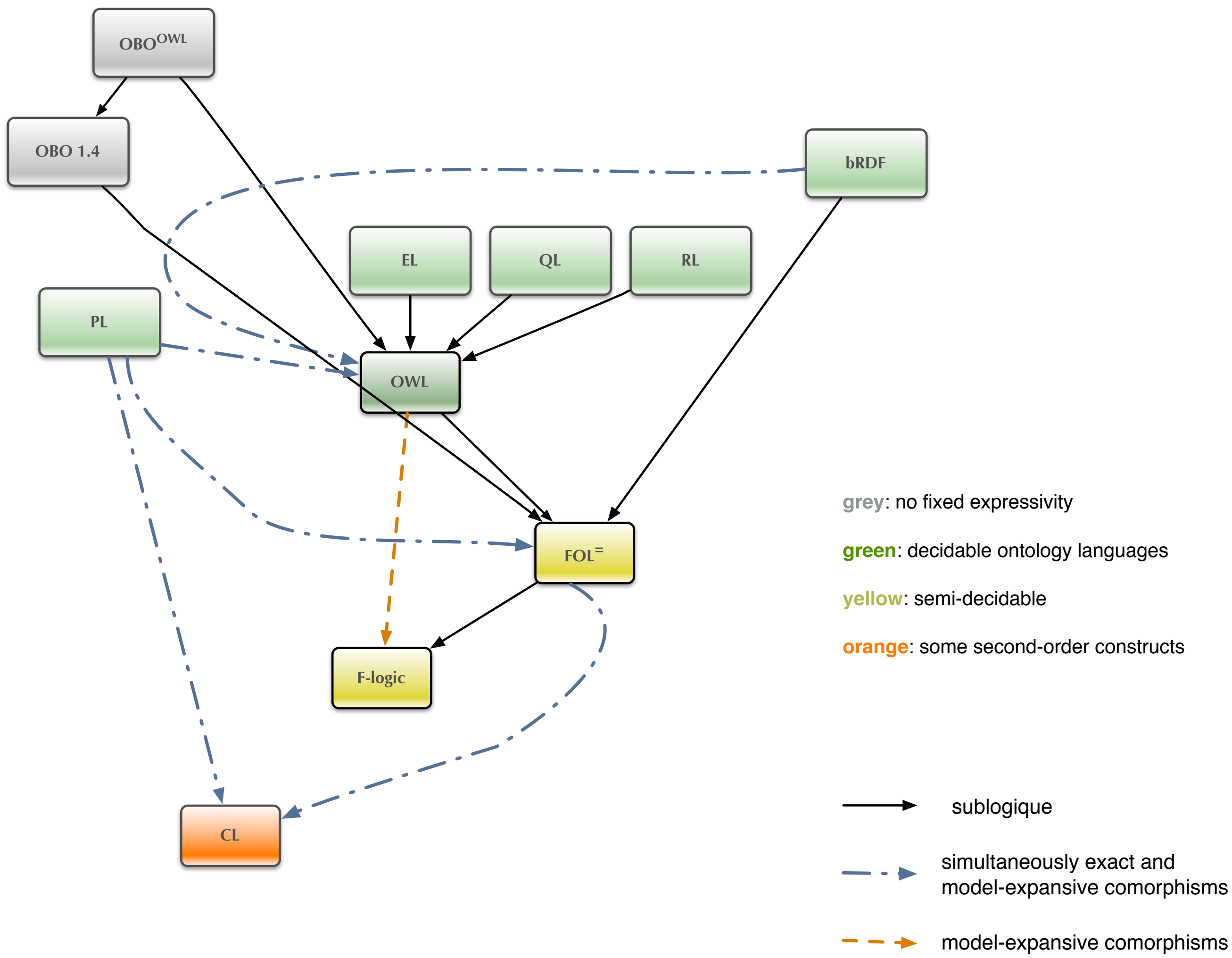


Figure I.1: Translations between conforming OMS languages (extended)

J Annex: DOL Abstract Syntax in EBNF

(Informative)

The following subclauses specify the abstract syntax of DOL in EBNF. Note that it deviates from the EBNF specification in ISO/IEC 14977:1996 in favor of a more concise EBNF syntax. More precisely, ISO/IEC 14977:1996 requires commas between the (non-)terminals of a right-hand side, which are omitted for the sake of better readability. Also, the separator = between left and right hand-side of a rule is replaced with `::=`, and the notation `N+` is used for one or more repetitions of `N`.

J.1 Documents

```
Document            ::= DOLLibrary | NativeDocument
DOLLibrary          ::= library [PrefixMap] LibraryName Qualification
                            LibraryItem*
NativeDocument      ::= <language specific>
LibraryItem         ::= LibraryImport | Definition | Qualification
Definition          ::= OMSDefinition
                      | NetworkDefinition
                      | MappingDefinition
                      | QueryRelatedDefinition
LibraryImport       ::= lib-import LibraryName
Qualification       ::= LanguageQualification
                      | LogicQualification
                      | SyntaxQualification
LanguageQualification ::= lang-select LanguageRef
LogicQualification ::= logic-select LogicRef
SyntaxQualification ::= syntax-select SyntaxRef
LibraryName         ::= IRI
PrefixMap           ::= prefix-map PrefixBinding*
PrefixBinding       ::= prefix-binding Prefix FullIRI [Separators]
Prefix              ::= String
Separators          ::= separators LibraryOMSSeparator OMSSymbolSeparator
LibraryOMSSeparator ::= String
OMSSymbolSeparator ::= String
```

J.2 OMS Networks

```
NetworkDefinition ::= network-definition NetworkName
                                  [ConservativityStrength] Network
NetworkName     ::= IRI
Network         ::= network NetworkElement* ExcludedElement*
NetworkElement  ::= network-element [Id] ElementRef
ExcludedElement ::= PathReference | OMSOrMappingorNetworkRef
PathReference   ::= path OMSOrMappingorNetworkRef OMSOrMappingorNetworkRef
ElementRef      ::= IRI
```

J.3 OMS

```
BasicOMS            ::= <language specific>
ClosableOMS         ::= BasicOMS | OMSReference
OMSReference        ::= oms-reference OMSRef [ImportName]
ExtendingOMS        ::= ClosableOMS | RelativeClosureOMS
RelativeClosureOMS ::= relative-closure ClosureType ClosableOMS
```

```
OMS                  ::= ExtendingOMS
                       | ClosureOMS
                       | TranslationOMS
                       | ReductionOMS
                       | ExtractionOMS
                       | ApproximationOMS
                       | FilteringOMS
                       | UnionOMS
                       | ExtensionOMS
                       | QualifiedOMS
                       | CombinationOMS
                       | ApplicationOMS
ClosureOMS           ::= closure-symbols OMS Closure
TranslationOMS       ::= translation OMS OMSTranslation
ReductionOMS         ::= reduction OMS Reduction
ExtractionOMS        ::= module-extract OMS Extraction
ApproximationOMS     ::= approximation OMS Approximation
FilteringOMS         ::= filtering OMS Filtering
UnionOMS             ::= union OMS [ConservativityStrength] OMS
ExtensionOMS         ::= extension OMS Extension
QualifiedOMS         ::= qualified-oms Qualification* OMS
CombinationOMS       ::= combination Network
ApplicationOMS       ::= application OMS SubstName
Closure              ::= ClosureType CircClosure CircVars
ClosureType          ::= minimize | maximize | free | cofree
CircClosure          ::= Symbol Symbol*
CircVars             ::= Symbol*
OMSTranslation       ::= translate OMSLanguageTranslation* [SymbolMap]
Reduction            ::= reduction RemovalKind OMSLanguageTranslation*
                                   [SymbolList]
SymbolList           ::= Symbol Symbol*
SymbolMap            ::= symbol-map GeneralSymbolMapItem
                                    GeneralSymbolMapItem*
Extraction           ::= extraction RemovalKind InterfaceSignature
Approximation        ::= approx RemovalKind [InterfaceSignature] [LogicRef]
Filtering            ::= filter RemovalKind   ~~BasicOMS~~    BasicOMSOrSymbolListBasicOMSOrSymbolList
::= BasicOMS | SymbolList  Extension           ::= extension [ConservativityStrength]
                                   [ExtensionName] ExtendingOMS
RemovalKind          ::= keep | remove
ConservativityStrength ::= consequence-conservative
                       | model-conservative
                       | not-consequence-conservative
                       | not-model-conservative
                       | implied
                       | monomorphic
                       | weak-definitional
                       | definitional
InterfaceSignature   ::= SymbolList
ImportName           ::= IRI
ExtensionName        ::= IRI
SubstName            ::= IRI
```

J.4 OMS Definitions

```
OMSDefinition        ::= oms-definition OMSName [ConservativityStrength] OMS
Symbol               ::= IRI
SymbolMapItem        ::= symbol-map-item Symbol Symbol
GeneralSymbolMapItem ::= Symbol | SymbolMapItem
Sentence             ::= <an expression specific to an OMS language>
OMSName              ::= IRI
OMSRef               ::= IRI
```

ExtensionRef ::= IRI
LanguageRef ::= IRI
LogicRef ::= IRI
SyntaxRef ::= IRI
OMSLanguageTranslation ::= NamedTranslation | DefaultTranslation
NamedTranslation ::= named-trans OMSLanguageTranslationRef
DefaultTranslation ::= default-trans LanguageRef
OMSLanguageTranslationRef ::= IRI

J.5 OMS Mappings

MappingDefinition ::= InterpretationDefinition
| RefinementDefinition
| EntailmentDefinition
| EquivalenceDefinition
| ~~ModuleRelDefinition~~ ConservativeExtensionDefinition | AlignmentDefinitio
InterpretationDefinition ::= interpretation-definition
InterpretationName
[ConservativityStrength]
InterpretationType
OMSLanguageTranslation*
[SymbolMap]
RefinementDefinition ::= refinement InterpretationName Refinement
InterpretationName ::= IRI
InterpretationType ::= interpretation-type OMS OMS
Refinement ::= RefinementOMS
| RefinementNetwork
| ~~RefinementComposition|~~ SimpleOMSRefinement
| SimpleNetworkRefinement
RefinementOMS ::= refinement-oms OMS
RefinementNetwork ::= refinement-network Network
~~RefinementComposition ::= refinement-composition Refinement Refinement~~ SimpleOMSRefinement ::= sir
~~OMS~~ Refinement OMSRefinementMap Refinement
SimpleNetworkRefinement ::= simple-network-ref ~~Network~~ Refinement NetworkRefinementMap Refineme
OMSRefinementMap ::= oms-refmap [OMSLanguageTranslation] [SymbolMap]
NetworkRefinementMap ::= network-refmap ~~NodeMap*NodeMap ::= node-map OMSName OMSNameOMSLanguageTran~~
~~SymbolMap~~ Refinement* EntailmentDefinition ::= entailment EntailmentName EntailmentType
OMSOMSEntailment ::= oms-oms-entailment OMS OMS
NetworkOMSEntailment ::= network-oms-entailment Network OMSName OMS
NetworkNetworkEntailment ::= network-network-entailment Network Network
EntailmentType ::= OMSOMSEntailment
| NetworkOMSEntailment
| NetworkNetworkEntailment
EntailmentName ::= IRI
EquivalenceDefinition ::= equivalence-definition
EquivalenceName
EquivalenceType
EquivalenceName ::= IRI
EquivalenceType ::= OMSEquivalence | NetworkEquivalence
OMSEquivalence ::= oms-equivalence OMS OMS ~~OMS~~ [OMS] NetworkEquivalence ::= network-equival
~~NetworkModuleRelDefinition~~ [Network]ConservativeExtensionDefinition ::= ~~module-definition~~
~~ModuleName~~ cons-ext-definition ConservativeExtensionName [ConservativityStrength] ~~ModuleTypeInt~~
ConservativeExtensionTypeInterfaceSignatureConservativeExtensionName ::= IRI
~~ModuleType~~ ConservativeExtensionType ::= ~~module-type~~ cons-ext-type OMS OMS
AlignmentDefinition ::= alignment-definition AlignmentName
[~~AlignmentCardinalityPair~~ AlignmentCardinality AlignmentCardinalit
]
AlignmentType Correspondence*
[AlignmentSemantics][1]

[1] Note that this grammar uses “type” as in“the type of a function”, whereas the Alignment API[?] uses “type” for thetotality/injectivity

```
AlignmentName      ::= IRI
  ~~AlignmentCardinalityPair ::= AlignmentCardinalityForwardAlignmentCardinalityBackwardAlignmentCardin~~
~~::= alignment-cardinality-forwardAlignmentCardinalityAlignmentCardinalityBackward ::= alignment-card~~
AlignmentCardinality ::= injective-and-total
                     | injective
                     | total
                     | neither-injective-nor-total
AlignmentType      ::= alignment-type OMS OMS
AlignmentSemantics ::= single-domain
                     | global-domain
                     | contextualized-domain
Correspondence     ::= CorrespondenceBlock
                     | SingleCorrespondence
                     | DefaultCorrespondence
DefaultCorrespondence ::= default-correspondence
CorrespondenceBlock ::= correspondence-block [Relation]
                                             [Confidence] Correspondence
                                             Correspondence*
SingleCorrespondence ::= correspondence  ~~SymbolRef~~  Symbol  [Relation]
                                         [Confidence] GeneralizedTerm
                                         [CorrespondenceID]
CorrespondenceID   ::= IRI
  ~~SymbolRef~~  Symbol  ::= IRI
GeneralizedTerm    ::=  ~~SymbolRef~~  Symbol  Relation          ::= RelationReference | StandardRela
StandardRelation   ::= StandardRelationValues
StandardRelationValues ::= subsumes
                     | is-subsumed
                     | equivalent
                     | incompatible
                     | has-instance
                     | instance-of
                     | default-relation
RelationReference  ::= relation-ref IRI
Confidence         ::= Double

Double ::= < a number ∈ [0,1] >
```

J.6 IRIs and Prefixes

```
IRI     ::= FullIRI | CurieIRI[2]
CurieIRI ::= curie CURIE
FullIRI ::= < as defined by the IRI production in IETF/RFC 3987:2005 >
CURIE   ::= String
```

of the relation/function. For thelatter, this grammar uses "cardinality".

[2]Specified below in clause ??.

K Annex: Extension of DOL with Queries

(Informative)

This annex describes the syntax of queries. A semantics still needs to be developed. DOL's metaclass `LibraryItem` is extended with a new subclass `QueryRelatedDefinition` for definitions related to queries.

K.1 Terms and Definitions

query language OMS language specifically dedicated to queries.
EXAMPLE SPARQL, Prolog
NOTE There are also general purpose OMS languages, which can express both OMS and queries.

query sentence containing query variables that can be instantiated by a substitution.

query variable symbol that will be used in a query and a substitution.
NOTE From an abstract point of view, query variables are just symbols; they are used in a way that they will be substituted using a substitution. Many OMS languages have special notations for (query) variables.
NOTE Usually, query variables are the free variables of a sentence; there can be other (bound) variables.
NOTE If there are no variables in an OMS language, constants can be used as query variables.

substitution OMS mapping that maps query variables of one OMS to complex terms of another OMS.

answer substitution substitution that, when applied to a given query, turns the latter into a logical consequence of a given OMS.

K.2 MOF Abstract Syntax

Queries are a means to extract information from an OMS. DOL's `QueryDefinitions` cover "select"-type queries that deliver an answer substitution for the query variables. (Answer) substitutions can be stored separately, using a `SubstitutionDefinition`. A `ResultDefinition` expresses that certain answer substitutions are the result of a query. Optionally, a result can be expressed to be complete, meaning that it comprises all answer substitutions to the query. Note that by default, OMS are employed with an open world semantics, but using minimizations, (part of) OMS can be equipped with a closed world semantics. The corresponding extension of the DOL metamodel is shown in Fig. ??.

K.3 EBNF Concrete Syntax

```
Term                   ::= <an expression specific to an OMS language>
GeneralizedTerm        ::= Term |   ~~SymbolRef~~    Symbol  QueryRelatedDefinition ::= QueryDefinition
                         | SubstitutionDefinition
                         | ResultDefinition
QueryDefinition        ::= 'query' QueryName '=' 'select' Vars 'where'
                           Sentence 'in' GroupOMS
                           ['along' OMSLanguageTranslation] 'end'
SubstitutionDefinition ::= 'substitution' SubstitutionName ':'
                               GroupOMS 'to' GroupOMS '=' SymbolMap
                               'end'
ResultDefinition       ::= 'result' ResultName '=' SubstitutionName
                            ',' SubstitutionName * 'for' QueryName
                           ['%complete'] 'end'
OMS                    ::= ... | OMS 'with' SubstitutionName
```

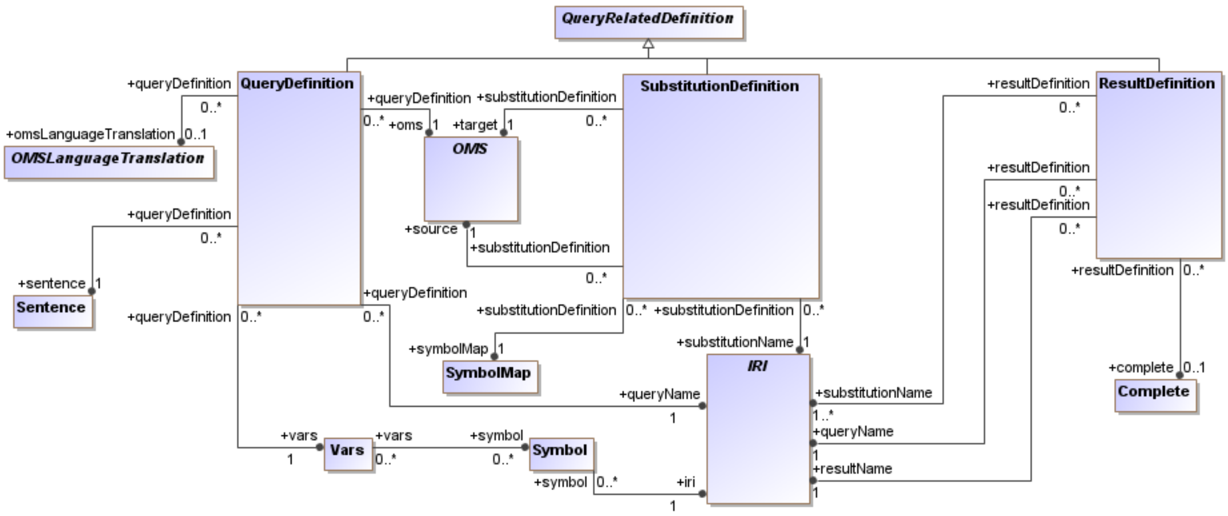


Figure K.1: Extension of DOL metamodel with queries

```
QueryName            ::= IRI
SubstitutionName     ::= IRI
ResultName           ::= IRI
Vars                 ::= Symbol  ',' Symbol *
```

K.4 EBNF Abstract Syntax

```
QueryRelatedDefinition ::= QueryDefinition
                         | SubstitutionDefinition
                         | ResultDefinition
QueryDefinition      ::= select-query-definition
                                               QueryName Vars Sentence OMS
                                               [OMSLanguageTranslation]
SubstitutionDefinition ::= substitution-definition
                                                   SubstitutionName OMS OMS
                                                   SymbolMap
ResultDefinition     ::= result-definition ResultName
                                         SubstitutionName SubstitutionName*
                                         QueryName [Complete]
OMS                  ::= ... | application  OMS  SubstitutionName
QueryName            ::= IRI
SubstitutionName     ::= IRI
ResultName           ::= IRI
Vars                 ::= Symbol*
Complete             ::= complete
```

K.5 Semantics of Queries

While queries are very important from a practical point of view, their semantics so far has been developed only for individual institutions. In [?], three options for an institution-independent semantics of queries and derived signature morphisms (which can map symbols to terms) are discussed. Currently, it is not clear which one would be the best choice. It is expected that after some experience with DOL, a choice will crystallize. This means that in the current version, the semantics of queries is elided, and left for a later version of DOL.

L Annex: Example Uses of all DOL Constructs

(Informative)

This annex provides example uses of DOL constructs. Jointly with clause 7, which contains DOL examples for the usage scenarios, all DOL constructs (although not necessarily all variants of each construct) are covered. The examples follow the DOL Text Serialization (clause ??). The following table provides an overview of which DOL language constructs have been covered where.

Top-level declarations in DOL libraries	
Top-level declaration	**Examples**
library ...	all examples
import IRI	Mereology
language IRI	Alignments, Publications
logic IRI	Alignments, Mereology
serialization IRI	Alignments, Mereology
PrefixMap	Mereology
oms IRI = OMS end	Alignments, Mereology
oms IRI = %consistent OMS end	PropositionalExamples, Mereology
oms IRI = %inconsistent OMS end	PropositionalExamples
oms IRI = %mono OMS end	section ??
oms IRI = %def OMS end	PropositionalExamples
network IRI = IRI, ..., IRI	Alignments
interpretation IRI : OMS to OMS = SymbolMap	Mereology
interpretation IRI : OMS to OMS = %cons SymbolMap	Engine
interpretation IRI : OMS to OMS = translation IRI	Mereology
refinement IRI = OMS refined via SymbolMap to OMS	section ??
refinement IRI = OMS refined via translation IRI to OMS	section ??
refinement IRI = IRI ~~then~~ refined to IRI	section ??
refinement IRI = Network refined to Network	section ??
entailment IRI = OMS entails OMS	PropositionalExamples
entailment IRI = OMSName in Network entails OMS	section ??
entailment IRI = Network entails Network	section ??
equivalence IRI : OMS <-> OMS = OMS end	Algebra
~~module~~ cons-ext IRI : OMS of OMS for Symbols	section 7.3
alignment IRI : OMS to OMS = Correspondences	Alignments
alignment IRI : OMS to OMS = Correspondences assuming SingleDomain	[?]
alignment IRI : OMS to OMS = Correspondences assuming GlobalDomain	[?]
alignment IRI : OMS to OMS = Correspondences assuming ContextualizedDomain	[?]
query IRI = select ars where Sen in OMS	MyQuery
substitution IRI : OMS to OMS = SymbolMap	MyQuery
result IRI = IRIs for IRI	MyQuery

OMS	
OMS notation	**Examples**
BasicOMS	Alignments, Mereology
IRI	Alignments, Mereology
minimize { OMS }	BlocksWithCircumscription
OMS minimize Symbols var Symbols	BlocksWithCircumscription
OMS maximize Symbols var Symbols	BlocksWithCircumscription
free { OMS }	Datatypes
cofree { OMS }	Datatypes
OMS with SymbolMap	Alignments, section **??**
OMS with translation IRI	Mereology
OMS hide SymbolList	Algebra
OMS reveal Symbols	Datatypes
OMS hide along IRI	section **??**
OMS extract Symbols	section 7.3
OMS remove Symbols	All_kinds_of_group_specifications
OMS forget Symbols	All_kinds_of_group_specifications
OMS keep Symbols	All_kinds_of_group_specifications
OMS select BasicOMS	All_kinds_of_group_specifications
OMS reject BasicOMS	All_kinds_of_group_specifications
OMS and OMS	Engine
OMS then OMS	Mereology
OMS then %ccons OMS	[?]
OMS then %mcons OMS	Propositional
OMS then %notccons OMS	[?]
OMS then %notmcons OMS	[?]
OMS then %mono OMS	Sorting
OMS then %def OMS	Persons
OMS then %implied OMS	BlocksWithCircumscription
logic IRI : OMS	all examples
language IRI : OMS	Mereology
serialization IRI : OMS	Mereology
combine NetworkElements	Alignments, Publications

L.1 Simple Examples in Propositional Logic

@ @ *@

```
\DIFaddend * log:  <http://purl.net/DOL/logics/>ser:  <http://purl.net/DOL/serializations/> )logic
log:Propositional syntax ser:Prop/Hetslibrary PropositionalExamplesoms Consistent = props A, B.
A => Bendoms Inconsistent = props A. A / not Aendoms SingleModel = props A, B. A / not Bendentailment
Ent = SingleModel entails  .  not ( A=>B ) endlibrary PropositionalMereologylogic log:Propositional
syntax ser:Prop/Hetsontology Taxonomy = props PT, T, S, AR, PD. S ∨ T ∨ AR ∨ PD ⟶ PT . S ∧
T ⟶ ⊥ .  T ∧ AR ⟶ ⊥ end
```

L.2 Engine Diagnosis and Repair

@ @ *@

`\DIFaddend * library Enginelogic log:Propositionalspec EngineSymptoms =props black`$_e xhaust, blue_e xhaust, lo$
$EngineSymptomsthenpropscarbon_d eposits, clogged_f ilter, clogged_r adiator, defective_c arburetor, worn_r ings, worn_s eals.overheat/$ not
$clogged_r adiator.ping/$ $notincorrect_t iming => carbon_d eposits.low_p ower/$ $notincorrect_t iming => worn_r ingsdefective_c arburetorclog$
$defective_c arburetorclogged_f ilter.blue_e xhaust => worn_r ingsworn_s eals.low_c ompression <=> worn_r ingsendspecEngineRepair =$
$EngineDiagnosisthenpropsreplace_a uxiliary, repair_e ngine, replace_e ngine.worn_r ings => replace_e ngine.carbon_d epositsdefective_c arb$
$repair_e ngine.clogged_f ilterclogged_r adiator => replace_a uxiliaryendspecMyObservedSymptoms = EngineSymptomsthen.overheat.n$
$MyObservedSymptomsandEngineRepairendspecRepair = proprepair.repairendinterpretationrepair1 : RepairtoMyRepair =$
$repair|- > replace_e ngineendinterpretationrepair2 : RepairtoMyRepair = repair|- > repair_e ngineendinterpretationrepair3 :$
$RepairtoMyRepair = repair|- > replace_a uxiliaryend$

L.3 Mereology: Distributed and Heterogeneous Ontologies

@ @ *@

```
\DIFaddend ⋆ owl: <http://www.w3.org/2002/07/owl>lang: <http://purl.net/DOL/languages/>ser:
<http://purl.net/DOL/serializations/>log: <http://purl.net/DOL/logics/>trans: <http://purl.net/DOL/
)library Mereologyimport PropositionalMereologylanguage lang:OWL2 logic log:SROIQ syntax ser:OWL2/Man
BasicParthood =Class: ParticularCategorySubClassOf: ParticularDisjointUnionOf: SpaceRegion,
TimeInterval, AbstractRegion, PerdurantObjectProperty: isPartOfCharacteristics: TransitiveObjectPro
isProperPartOfCharacteristics: Asymmetric SubPropertyOf: isPartOfClass: AtomEquivalentTo: inverse
isProperPartOf only owl:Nothingend interpretation TaxonomyToParthood : Taxonomy to BasicParthood
=translation trans:PropositionalToSROIQ,PT ↦ Particular, S ↦ SpaceRegion,T ↦ TimeInterval, A
↦ AbstractRegion, logic log:CommonLogic syntax ser:CommonLogic/CLIFontology ClassicalExtensionalPart
=BasicParthood with translation trans:SROIQtoCLthen. (forall (X) (if (or (= X S) (= X T) (= X
AR) (= X PD))(forall (x y z) (if (and (X x) (X y) (X z))(and(if (and (isPartOf x y) (isPartOf
y x)) (= x y))(if (and (isProperPartOf x y) (isProperPartOf y z)) (isProperPartOf x z))(iff (overlaps
x y) (exists (pt) (and (isPartOf pt x) (isPartOf pt y))))(iff (isAtomicPartOf x y) (and (isPartOf
x y) (Atom x)))(iff (sum z x y)(forall (w) (iff(overlaps w z)(and (overlaps w x) (overlaps w y)))))(e
(s) (sum s x y))))))). (forall (Set a) (iff (fusion Set a)(forall (b) (iff (overlaps b a)(exists
(c) (and (Set c) (overlaps c a)))))))
```

L.4 Defined Concepts

@ @ *@

```
\DIFaddend ⋆ library Personslanguage lang:OWLontology Persons =Class ⋆ \DIFdelbegin \DIFdel{Person
  Class }\DIFdelend ⋆ ⋆ \DIFaddbegin \DIFadd{: Person
  Class: }\DIFaddend ⋆ Femalethen Class: Woman EquivalentTo: Person and Femaleend
```

L.5 Blocks World: Minimization

[29] *@ @* *@ Note(29)

```
\DIFaddend @⋆ %prefix( lang: <http://purl.net/DOL/languages/>)%

library BlocksWithCircumscription
language lang:OWL

ontology Blocks =
  %% FIXED PART
  Class: Block
  Individual: B1 Types: Block
  Individual: B2 Types: Block DifferentFrom: B1
              %% B1 and B2 are different blocks
then
  %% CIRCUMSCRIBED PART
  minimize {
    Class: Abnormal
    Individual: B1 Types: Abnormal
       %% B1 is abnormal
  }
then
  %% VARYING PART
  Class: Ontable
  Class: BlockNotAbnormal
        EquivalentTo: Block and not Abnormal
        SubClassOf: Ontable
        %% Normally, a block is on the table
```

[29] NOTE: Q-AUT: Here we need the prefixes for registry entries (e.g. logics) once more; they should be reused across examples. Or we need to specify a mechanism that gets rid of *these* prefixes altogether. @TM, could you please comment on my specification enhancement request http://trac.informatik.uni-bremen.de:8080/hets/ticket/1020#comment:33?

```
then %implied
  Individual: B2 Types: Ontable
     %% B2 is on the table
end
```

[30] Note(3

@ @ *@

```
\DIFaddend @*  ontology Blocks_Alternative =
  Class: Block
  Class: Abnormal
  Individual: B1 Types: Block, Abnormal
  Individual: B2 Types: Block DifferentFrom: B1
              %% B1 and B2 are different blocks
              %% B1 is abnormal
  Class: Ontable
  Class: BlockNotAbnormal
        EquivalentTo: Block and not Abnormal
        SubClassOf: Ontable
        %% Normally, a block is on the table
  minimize Abnormal var Ontable, BlockNotAbnormal
then %implied
  Individual: B2 Types: Ontable
     %% B2 is on the table
end

ontology Blocks_Alternative2 =
  Class: Block
  Class: Normal
  Individual: B1 Types: Block, not Normal
  Individual: B2 Types: Block DifferentFrom: B1
              %% B1 and B2 are different blocks
              %% B1 is abnormal
  Class: Ontable
  Class: NormalBlock
        EquivalentTo: Block and Normal
        SubClassOf: Ontable
        %% Normally, a block is on the table
  maximize Normal var Ontable, BlockNotAbnormal
then %implied
  Individual: B2 Types: Ontable
     %% B2 is on the table
end
```

L.6 Alignments

@ @ *@

```
%DIF > prefix( :      <http://www.example.org/alignment#>
         \DIFaddend * owl:  <http://www.w3.org/2002/07/owl>lang:  <http://purl.net/DOL/languages/>se
<http://purl.net/DOL/serializations/>log:  <http://purl.net/DOL/logics/>trans:  <http://purl.net/DOL
)library Alignmentslanguage lang:OWL2 logic log:SROIQ syntax ser:OWL2/Manchesteralignment Alignment1
:    Class:  Woman  to  Class:  Person  =Woman < Personendontology AlignedOntology1 =combine Alignme
Onto1 =Class:  PersonClass:  Woman SubClassOf:  PersonClass:  Bankendontology Onto2 =Class:  HumanBe
Woman SubClassOf:  HumanBeingClass:  Bankendalignment VAlignment :  Onto1 to Onto2 =Person = HumanBe
= Womanendnetwork N =1 :  Onto1, 2 :  Onto2, VAlignmentendontology VAlignedOntology =combine Nendont
VAlignedOntologyRenamed =VAlignedOntology with 1:Bank |-> RiverBank, 2:Bank |-> FinancialBankend
```

[30] NOTE: Instead of Blocks World, perhaps we could specify an ontology that uses inheritance networks with exceptions, and then use circumscription to axiomatize that ontology.

L.7 Distributed Description Logics

@ @ *@

\DIFaddend ⋆ owl: <http://www.w3.org/2002/07/owl>lang: <http://purl.net/DOL/languages/>ser: <http://purl.net/DOL/serializations/>log: <http://purl.net/DOL/logics/>trans: <http://purl.net/DOL/)library Publicationslanguage lang:OWL2 logic log:SROIQ syntax ser:OWL2/Manchesterontology Publicati =Class: PublicationClass: Article SubClassOf: PublicationClass: InBook SubClassOf: PublicationCl Thesis SubClassOf: PublicationClass: MasterThesis SubClassOf: ThesisClass: PhDThesis SubClassOf: Thesisendontology Publications2 =Class: ThingClass: Article SubClassOf: ThingClass: BookArticle SubClassOf: ThingClass: Publication SubClassOf: ThingClass: Thesis SubClassOf: Thingendontology Publications$_C ombined = combine1 : Publications1withtranslationOWL2MS{-}OWL, 2 : Publications2withtranslationOWL2MS$ $OWLwithtranslationMS{-}OWL2DDLthen1 : Publication \xrightarrow{\sqsubseteq}$ 2:Publication1:PhdThesis $\xrightarrow{\sqsubseteq}$ 2:Thesis1:InBook $\xrightarrow{\sqsubseteq}$ 2:BookArticle1:Article $\xrightarrow{\sqsubseteq}$ 2:Article1:Article $\xrightarrow{\sqsupseteq}$ 2:Articleendontology Publications$_E xtended =$ $PublicationswithtranslationDDL2{-}ECOendlibraryMarketlanguagelang : OWL2logiclog : SROIQsyntaxser : OWL2/Manchester$ $combine1 : Class : PurchaseOrder, 2 : ObjectProperty : BuyerObjectProperty : GoodObjectProperty : BoughtBywithtranslationO$ $PurchaseOrder - into- > 2 : BoughtByend$

L.8 Algebra

@ ~~owl: <http://www.w3.org/2002/07/owl>~~ @ *@

%DIF > **prefix**(: <http://www.example.org/algebra#>
\DIFaddend ⋆ lang: <http://purl.net/DOL/languages/>ser: <http://purl.net/DOL/serializatio <http://purl.net/DOL/translations/>)library Algebralanguage lang:CommonLogic syntax ser:CommonLogic/ implicit$_g roup = (forall(xyz)(= (opx(opyz))(op(opxy)z)))(exists(e)(forall(x)(and(= x(opex))(= x(opxe)))))(forall(x)(exists(y)(a$ $x(opx(opxy)))(= x(opx(opyx))))))endspecexplicit_g roup = (forall(xyz)(= (opx(opyz))(op(opxy)z)))(forall(x)(and(= x(opex))(=$ $x(opxe)))))(forall(x)(and(= x(opx(opx(invx))))(= x(opx(op(invx)x))))))endequivalencegroups_e quiv : implicit_g roup <$ $- > explicit_g rouphidee, invend$

@ @ *@

\DIFaddend ⋆ equivalence e : algebra:BooleanAlgebra↔ algebra:BooleanRing =x∧y = x·yx∨y = x+y+x·y¬x = 1+xx·y = x∧yx+y = (x∨y) ∧ ¬(x∧y)end

@ @ *@

\DIFaddend ⋆ language lang:CASLspec InterpolatedGroup =sort Elemops 0:Elem; $_{+:Elem*Elem->Elem;inv:Elem->Elem}$

L.8.1 Groups specified with different forms of hiding and forgetting

L.8.1.1 Groups and hiding

@ @ *@

\DIFaddend ⋆ library All$_k inds_o f_g roup_s pecificationslanguagelang : CASLspecGroup_w ith_i nverse = sortElemops0 :$ $Elem;_{+:Elem*Elem->Elem;inv:Elem->Elemforallx,y,z:elem.x+0=x.x+(y+z)=(x+y)+z.x+inv(x)=0endspecGroup_via_hiding=Group_with_inversehideinvend}$

The semantics of this specification is the class of all monoids that can be extended with an inverse, i.e. class of all groups. The effect is second-order quantification:

@ @ *@

\DIFaddend ⋆ language lang:HasCASLspec Group$_i n_s econd_o rder_l ogic = sortElemops0 : Elem;_{+:Elem*Elem->Elem;existsinv:Elem}$

L.8.1.2 Groups and module extraction

@ @ *@

\DIFaddend ⋆ language lang:CASLspec Group$_v ia_m odule_e xtraction_1 = Group_w ith_i nverseremoveinvend$

The semantics is just Group_with_inverse, since the module needs to be enlarged to the whole specification. This is of course unsatisfactory. A better use of module extraction is the following:

@ @ *@

\DIFaddend ⋆ language lang:CASLspec Group$_w ith_i mplicit_i nverse = sortElemops0 : Elem;_{+:Elem*Elem->Elem;inv:Elem->Elem}$

The semantics of Group_via_module_extraction_2 is just Group_with_implicit_inverse, because adding inv is conservative.

L.8.1.3 Groups via interpolation

@ @ *@

\DIFaddend ⋆ language lang:CASLspec Group$_{v}ia_{i}nterpolation1 = Group_{w}ith_{i}nverse forgetinvendspecGroup_{v}ia_{i}nterpolatio$
$Group_{w}ith_{i}nversekeepElem, 0,_{+_{end}}$

Both specifications are equivalent, and they are equivalent to `Group_with_implicit_inverse`.

L.8.1.4 Groups and filtering

@ @ *@

\DIFaddend ⋆ language lang:CASLspec Group$_{v}ia_{F}iltering_{1} = Group_{w}ith_{i}nverserejectinvendspecGroup_{v}ia_{F}iltering_{2} =$
$Group_{w}ith_{i}nverseselectElem, 0,_{+_{end}}$

Both specifications are equivalent, and they are equivalent to the following theory which just omits the inverse axioms (and hence does not specify groups): *@ @* *@

\DIFaddend ⋆ language lang:CASLspec Group$_{v}ia_{r}eject = sortElemops0 : Elem;_{+:Elem*Elem->Elemforallx,y,z:elem.x+0=x.x+(y+z)}$

L.9 Queries

@ @ *@

```
\DIFaddend ⋆ library MyQuerylanguage lang:CASLspec Person =sort spred Person:sop max,peter:Personendo
MyQuery = select x where Person(x) in Personendsubstitution MySubst :    Person then op x:Person
to Person = x |-> maxendresult MyResult = MySubst for MyQuery
```

L.10 Datatypes

@ @ *@

```
\DIFaddend ⋆ library Datatypeslanguage lang:CASLspec Bag =sort Elemthen free sort Bagops mt:Bag;union,
```

~~This annex sketches scenarios that outline how is intended to be applied. For each scenario, the status of its implementation is described, the features it makes use of are listed, and a brief description is provided.~~

~~DO-ROAM (**D**ata and **O**ntology driven **R**oute-finding **O**f **A**ctivity-oriented **M**obility[1]) is a web service with an interactive frontend that extends OpenStreetMap by an ontology-based search for located activities and opening hours [?] . The service is driven by a set of different OWL ontologies that have been aligned to each other using the Falcon matching tool [?] . The user interface of the DO-ROAM web frontend offers multilingual labels, which are maintained in close connection to the underlying ontologies.~~

~~Porting DO-ROAM to would enable the coherent representation of the aligned ontologies as one OMS network, and it would enable the maintenance of the user interface labels as annotations inside the ontology.~~

~~Consider the following ambient assisted living (AAL) scenario: Clara instructs her **wheelchair** to get her to the **kitchen** (next door to the **living room**. For **dinner**, she would like to take a *pizza* from the **freezer** and bake it in the **oven**. (Her diet is *vegetarian*.) **Afterwards** she needs to rest in **bed**. Existing ontologies for ambient assisted living (the OpenAAL[1] OWL ontology) cover the *core* of these concepts; they provide at least classes (or generic superclasses) corresponding to the concepts highlighted in **bold**. However, that does not cover the scenario completely: Some concepts (here: food and its properties, *italicized*) are not covered. There are separate ontologies for that (such as the Pizza ontology[1]), whereas information about concrete products (here: information about the concrete pizza in Clara's oven) would rather come from Linked Open Datasets than from formal ontologies. Not all concepts (here: space and time, underlined) are covered at the required level of complexity. OpenAAL says that appointments have a date and that rooms can be connected to each other, but not what exactly that means. Foundational ontologies and spatial calculi, often formalized in first-order logic, cover space and time at the level of complexity required by a central controller of an apartment and by an autonomously navigating wheelchair. Thirdly, even description logic might be too complex for very simple devices involved into the scenario, such as the kitchen light switch, for which propositional logic may be sufficient. Thus, an adequate formalization of this scenario has to be heterogeneous. For example, one could imagine the following axioms: "light is switched on if and only if someone is in the room and it is dark outside" – this could be formalized in propositional logic as light_on ≡ person_in_room ∧ dark_outside. "a vegetarian pizza is a pizza whose toppings are all vegetarian" – this could~~

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[1] ~~This is not a fully comprehensive food ontology, but rather a well-known sample OWL ontology;~~

~~be formalized in description logic as VegetarianPizza ≡ Pizza ⊓ ∀hasTopping.Vegetarian "two areas in a house (a working area in a room) are either the same, or intersecting, or bordering, or separated, or one is part of the other" – this could be formalized as an RCC-style spatial calculus in first-order logic as~~

$$\begin{aligned}\forall a_1, a_2.\quad & \mathrm{equal}(a_1, a_2) \veebar \mathrm{overlapping}(a_1, a_2) \veebar \mathrm{bordering}(a_1, a_2) \veebar \mathrm{disconnected}(a_1, a_2)\\ & \veebar \mathrm{part\_of}(a_1, a_2) \veebar \mathrm{part\_of}(a_2, a_1).\end{aligned}$$

~~would be capable of expressing all that within one library of heterogeneous ontologies arranged around an OWL core (here: the OpenAAL ontology), including OMS mappings from OpenAAL to the other ontologies, as well as a re-declaration of a concrete pizza product from a product dataset as an instance of the Pizza OWL class.~~

~~DOLCE is a foundational ontology that has primarily been formalized in the first-order logic ontology language KIF (a predecessor of Common Logic), but also in OWL ("DOLCE Lite") **[?]** . This "properties"). That leaves ambiguities for modeling a view from DOLCE Lite to the first-order DOLCE, as such a view would have to reintroduce the third (temporal) component of such predicates: Should a relation asserted in terms of DOLCE Lite be assumed to hold for *all* possible points/intervals in time, should it be universally quantified? Or should such a relation be assumed to hold for *some* points/intervals in time, should it be existentially quantified? Or should a concrete value for the temporal component be assumed, "0" or "now"?~~

~~would support the formalization of all of these views and, given suitable consistency checking tools, the analysis of whether any such view would satisfy all further axioms that the first-order DOLCE states about temporal parthood.~~

~~The OWL Time ontology[1] covers temporal concepts such as instants and intervals and has been designed for describing the temporal content of Web pages and the temporal properties of Web services. While OWL is suitable for these intended applications, only a first-order axiomatization is capable of faithfully capturing all relevant notions, such as the trichotomy of the "before" relation: One instant is either before another one, or at the same time, or after. Moreover, a relationship between facts expressed in terms of instants and facts expressed in terms of intervals (both of which is, independently, possible in OWL), can only be established via first-order logic, by declaring an interval of length zero equivalent to an instant.~~

~~A separate first-order axiomatization of OWL Time exists **[?]** ,**[?]** . would instead provide the mechanism of modeling OWL Time as one coherent heterogeneous ontology, using OWL and, , Common Logic.[1] For the temporal description logic $\mathcal{DLR}_{\mathcal{US}}$ for knowledge bases and logic-based temporal conceptual data modeling **[?]** ,**[?]** ; $\mathcal{DLR}_{\mathcal{US}}$ combines the propositional temporal logic with the Sinceand Untiloperators and the (non-temporal) description logic $\mathcal{DLR}$ and can be regarded as an expressive fragment of the first-order temporal logic $L^{since,until}$. Within , this would enable one to have 'lightweight' time aspects with OWL Time, which are then properly formalized with $\mathcal{DLR}_{\mathcal{US}}$ or a leaner variant TDL-Lite **[?]** , where notions such as (some time) "before" are given a formal semantics of the intended meaning that the plain OWL Times human-readable object property does not have. The latter, then, would enable the modeler to represent the meaning –hence, restrict the possible models– and check the consistency of the temporal constraints and so-called 'evolution constraints' in the ontology (evolution constraints constrain membership of an object or an individual relation to a concept or relationship over time). For instance, that each divorcee must have been a participant in a marriage before, that boarding only may occur after checking in, and that any employee must obtain a salary increase after two years of employment. It also can be used to differentiate between essential and immutable parthood, therewith being precise in the ontology about, e.g., the distinction how a human brain is part of a human (humans cannot live without it), versus how a hand is part of a human (humans can live without it), versus how the hand is part of, say, a boxer, which is essential to the boxer but only for has long as he is a boxer **[?]** .~~

~~COLORE, the Common Logic Repository[1] is an open repository of more than 150 ontologies as of December 2011, all formalized in Common Logic. COLORE stores metadata about its ontologies, which are represented using a custom XML schema that covers the following aspects[1], without specifying a formal semantics for them: author, date, version, description, keyword, parent ontology[1] name, author, year[1] maps (signature morphisms), definitional extension, conservative extension, inconsistency between ontologies, imports, relative interpretation, faithful interpretation, definable equivalence~~

~~provides built-in support for a subset of the "direct relations" and specifies a formal semantics for them. In addition, it supports the implementation of the remainder of the COLORE metadata vocabulary as an ontology, reusing suitable existing metadata vocabularies such as OMV, and it supports the implementation of one or multiple Common Logic ontologies plus their annotations as one coherent library.~~

1

1 ~~This is also a use case for multiple namespaces: OWL supports namespaces, CL does not.~~

1

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1 ~~Note that this use of the term "module" in COLORE corresponds to the term in this .~~

1 ~~Note that this may cover any sentencesin the sense of this .~~

M Annex: Tools for DOL

(Informative)

M.1 The Heterogeneous Tool Set (Hets)

The Heterogeneous Tool Set (Hets) is an implementation of DOL. Hets is a parsing, analysis and proof tool for OMS, OMS networks and OMS mappings written in DOL and DOL-conforming languages. It supports a wide range of OMS languages and language translations, in particular OWL, RDF, Common Logic, first-order logic and CASL. Support for MOF, UML class diagrams and state machines is in preparation. Hets has been co-developed together with the DOL language presented in this standard, and has been used to test the examples. Hets has been connected to a considerable number of proof tools like theorem provers, supporting various logics. Logics that are not directly supported by any proof tool can be supported indirectly, through a logic mapping into a tool-supported logic.

Hets is open source, licensed under GPLv2 or higher. The sources are available at the following URL `https://github.com/spechub/hets`.

M.2 Ontohub, Modelhub, Spechub

Ontohub/Modelhub/Spechub is another implementation of DOL. It is a repository engine for managing OMS, OMS networks and OMS mappings written in DOL and DOL-conforming languages. It supports the same range of OMS languages and language translations as Hets (indeed, Hets is used for analyzing DOL files). The novel aspect w.r.t. Hets is the provision of git-based repositories and IRIs for DOL libraries, OMS, symbols and mappings (see also Annex ??).

Users of Ontohub/Modelhub/Spechub can upload, browse, search and annotate OMS in various languages via a web frontend, see `https://ontohub.org`, `https://model-hub.org` and `https://spechub.org`. Ontohub/Modelhub/Spechub is open source under GNU AGPL 3.0 license, the sources are available at the following URL `https://github.com/ontohub/ontohub`.

Ontohub/Modelhub/Spechub enjoys the following distinctive features:

- OMS can be organized in multiple repositories, each with its own management of editing and ownership rights,
- private repositories are possible,
- version control of OMS is supported via interfacing the Git version control system,
- OMS can be edited both via the browser and locally with any editor (and in the latter case pushed via Git); Git will synchronize both editing approaches,
- one and the same URL is used for referencing an OMS, downloading it (for use with tools), and for user-friendly presentation in the browser (i.e. Ontohub/Modelhub/Spechub is fully linked-data compliant, see also the end of this section)
- modular and heterogeneous OMS are specially supported,
- OMS can not only be aligned (as in BioPortal and NeOn), but also be combined along alignments (using DOL's `combine` construct),
- logical relations between OMS (interpretation of theories, conservative extensions etc.) are supported,
- support for a variety of OMS languages,
- OMS can be translated to other OMS languages, and compared with OMS in other languages,
- heterogeneous OMS involving several languages can be built,
- OMS languages and OMS language translations are first-class citizens and are available as linked data.

Ontohub/Modelhub/Spechub is not a repository, but a semantic repository engine. This means that Ontohub/Modelhub/Spechub OMS are organized into repositories. The organization into repositories has several advantages:

- Firstly, repositories provide a certain structuring of OMS, let it be thematically or organizational. Access rights can be given to users or teams of users per repository. Typically, read access is given to everyone, and write access only to a restricted set of users and teams. However, also completely open, i.e. world-writeable repositories are possible, as well as private repositories visible only to a restricted set of users and teams. Since creation of repositories is done easily with a few clicks, this supports a policy of many but small repositories (which of course does not preclude the existence of very large repositories). Note that also structuring within repositories is possible, since each repository is a complete file system tree.
- Secondly, repositories are git repositories. Git is a popular decentralized version control system. With any git client, the user can clone a repository to her local hard disk, edit it with any editor, and push the changes back to Ontohub/Modelhub/Spechub. Alternatively, the web frontend can be used directly to edit OMS; pushing will then be done automatically in the background. Parallel edits of the same file are synchronized and merged via git; handling of merge conflicts can be done with git merge tools.
- Thirdly, OMS can be searched globally in Ontohub/Modelhub/Spechub, or in specific repositories. Additionally, user-supplied metadata like categories, formality levels and purposes can be used for searching.

Ontohub/Modelhub/Spechub is linked-data compliant. This means that OMS are referenced by a unique URL of the form `https://ontohub.org/name-of-repository/path-within-repository`. Depending on the MIME type of the request, under this URL, the raw OMS file will be available, but also a HTML version for display in a browser, an XML and a JSON version for processing with tools.

M.3 APIs

Both Hets and Ontohub/Modelhub/Spechub provide APIs for the interchange with other tools[1]. Ontohub/Modelhub/Spechub also provides an API for exchange with other instances, so that e.g. Ontohub and Modelhub can exchange information about available repositories and their OMS.

In the future, these APIs shall be aligned with OMG's standardization effort API4KB.

[1] See `https://github.com/spechub/Hets/wiki/RESTful-Interface` and `https://github.com/ontohub/ontohub/wiki/`.

N Annex: Ontohub loc/id v2

(Informative)

This annex describes the way how Ontohub assigns IRIs to DOL libraries, OMS, symbols etc. Ontohub[1] is an implementation for DOL, and it is suggested that other tools supporting DOL should adopt the same or a similar scheme for IRIs.

N.1 Concept

Generally an Ontohub loc/id (locator/identifier) is just an IRI of a DOL library (contained in a document), an OMS or one of its members (symbols, sentences, mappings). However, Ontohub loc/ids are generated by the Ontohub application and assigned to an OMS. Ontohub tries to infer them from the path of the repository, the path of the OMS and the specific name. Additionally, Ontohub ensures that this specific IRI is actually a locator and not *just* an identifier.

This is quite important as the IRI of an OMS is the general starting interface a user has with the given OMS. When she evaluates the OMS in her tool of choice she'll use the IRI to reference the given OMS. When she wants to work on Ontohub with the given OMS she'll point her browser at the given IRI. As one's familiarity with the Ontohub application increases one will more often want to use the IRI instead of just searching or even browsing for something. This is further intensified if the IRI-schema follows a schema that is easily understood by a user.

N.2 Ontohub-Style

Identifying OMS and their members in Ontohub is a hierarchical task. A DOL document belongs to a repository. An OMS may belong directly to a repository, or indirectly through a DOL library. Mappings, symbols and sentences in turn belong to an OMS. So one could use the hierarchical portion of an IRI instead of the query string. This would mean using a forward slash (/) as separator.

Ontohub loc/ids are specific to an instance of the Ontohub application. However, such an instance might be reachable via multiple multiple FQDNs (fully qualified domain name) and ports. So instead a *qualified loc/id* is expected to be a tuple consisting of the specific application instance, represented by the set of their schema-fqdn-port tuples, and the actual identifying portion beginning with the hierarchical forward slash (/).

N.2.1 qualified loc/id structure

1. Set of Schema + FQDNs + Port for an instance: *INSTANCE*, e.g.
   { `http://ontohub.org`, `http://model-hub.org`, `http://spechub.org` }
2. Identifying portion loc/id with leading forward slash (/)
   - The identifying portion is split into three parts.
   - *HIERARCHY*: is the `path/to/OMS-file`, with elements split by a forward slash (/).
   - *MEMBER*: is the element of the OMS at the specific position. It is being separated from the *HIERARCHY* by two forward slashes (//). These forward slashes are also being used to separate members inside of *MEMBER* (e.g. in the case of an OMS which contains a symbol).
   - *COMMAND*: is not really an element or part of an OMS, but a command the user wishes to execute on the object selected by the previous sections of the loc/id. It is denoted and separated from the rest of the IRI by the use of three consecutive forward slashes (///).

[1] In this annex, "Ontohub" could equally well be substituted by "Modelhub" and "Spechub".

N.2.2 Examples

DOL *document*	
DOL document	`/dol-testing/double_mapped_blendoid`
OMS	`/dol-testing/double_mapped_blendoid//DMB-CommonSource`
Mapping	`/dol-testing/double_mapped_blendoid//SomeMapping`
Symbol	`/dol-testing/double_mapped_blendoid//DMB-CommonSource//KitchenTable`
Sentence	`/dol-testing/double_mapped_blendoid//DMB-CommonSource//Ax02`

OMS	
DOL document	`/dol-testing/double_mapped_blendoid`
OMS	`/default/pizza`
Mapping	`/default/pizza//SomeMapping`
Symbol	`/default/pizza//Veneziana`
Sentence	`/default/pizza//Ax02`

Fully qualified symbols (e.g. $+ : Nat \times Nat \mapsto Nat$) will need to be escaped but will be supported.

N.3 Specification

A qualified loc/id IRI can be specified as a special case of RFC 3987 (IRI, [?]). Code-excerpt **??** on page **??** contains this specification of qualified loc/ids in Augmented Backus-Naur Form (ABNF, [?]). ABNF is used, because RFC 3987 itself specifies IRIs using ABNF and it is desirable to be able to reference rules from the RFC in our specification. Such rules can be easily identified by the `i`-prefix that was used when writing the IRI-rules.

`<Loc-Id-IRI>` represents the start rule for a qualified loc/id and `<Loc-Id>` would be the starting non-terminal for a loc/id without its *INSTANCE* qualifier. The following symbols are non-terminal symbols that represent rules from the IRI-RFC.

- `<iquery>`
- `<ifragment>`
- `<scheme>`
- `<iauthority>`
- `<isegment-nz>`

One should take note that the `<scheme>` rule does not include a `i`-prefix. This is because `<scheme>` is actually taken from RFC 3986 [?], which defines the URI.

```
; Author: Tim Reddehase
; E-Mail: robustus AT rightsrestricted DOT com
; Last-Changed: 2015-02-22
; Version: 0.1.2
;
; This ABNF for Loc/Ids is based on the definition
; of IRIs and as such uses Rules from the RFC-Definition
; of IRIs: http://tools.ietf.org/html/rfc3987#section-2.2
; Rules that represent an IRI-rule usually start with an
; i char.

Loc-Id-IRI = li-instance [ li-ref ] Loc-Id [ "?" iquery ] [ "#" ifragment ]

; Represents an Ontohub-Application instance.
; Semantically multiple <li-instance> values
; can be equivalent and thus forming the
; set of INSTANCE. <scheme> is a rule inside
; of the IRI RFC.
li-instance = scheme "://" iauthority

; a lone repository is also a Loc/Id
Loc-Id = "/" li-repository [ li-hierarchy [ li-member ] ] [ li-command ]

; Represents the path/directory name of the repository
li-repository = isegment-nz

; Represents a ref/ special form
li-ref = "/" "ref/" isegment-nz

; Represents the path inside the Repository to the ontology
li-hierarchy = *( "/" isegment-nz )

; Represents internal 'path' inside of the ontology
; where child-ontologies, mappings, symbols and sentences
; are first-class members.
li-member = *2( "//" isegment-nz )

; Represents a command to be 'executed' on the
; specific resource
li-command = *( "///" isegment-nz )
```

Figure N.1: Specification of loc/id IRIs in ABNF

N.4 ref/ special form loc/ids

There is one additional syntax-element that has not been covered yet. One of the main features that Ontohub provides in its role as an *Open OMS Repository* is versioning of OMS by backing the repositories with git. For many use cases it is important to access such versions and other related files inside of a repository, which can be basically viewed as a directory in a file system. `ref/`-style IRIs accomplish this task.

The `ref/`*`argument`*-form is a prefix of the *HIERARCHY*, *MEMBER* and *COMMAND* components—otherwise referred to as unqualified loc/id, or in short: loc/id.

- Version: `/ref/2/default/pizza//SomeMapping`
- Commit: `/ref/def3ab/default/pizza//SomeMapping`
- Branch: `/ref/master/default/pizza//SomeMapping`
- Date: `/ref/2014-09-07/default/pizza//SomeMapping`
  - would take the latest commit which applies to the Date range.
- MMT: `/ref/mmt/default/pizza?SomeMapping`
  - Does not refer to a specifically designated version of the element, but always refers to the current one instead. This version allows to use MMT-style IRIs [?], which should guarantee basic support for tools which expect the MMT-style.

N.4.1 References inside of the tree

It is important to provide a way to reference files inside a repository, This especially applies to files that do not represent OMS. This will be accomplished by the `tree/` special form. Additionally, Ontohub will support a `treeref` special form which allows to reference a specific version of a files using the *Commit*, *Branch* and *Date* references. MMT is for obvious reasons not supported.

- File: `/tree/default/some_directory/some_child_dir/Foo.txt`
  - applies to HEAD commit of main branch (currently always *master*)
- File at reference: `/treeref/{REF}/default/tree/some_directory/some_child_dir/Foo.txt`
  - where {REF} is any of the above possible ref-types: Commit, Branch or Date

N.5 Disambiguation

If the `path/to/an-OMS` can actually also be a path to a directory – which would be possible if there were a directory named **pizza** and an ontology named **pizza.owl** – will the loc/id be resolved to a disambiguating page.

This page will contain a link to the tree for the directory, e.g. `/tree/default/pizza`, and a link to a `ref/` special form version of the OMS, e.g. `/ref/master/default/pizza`.

If however the loc/id is requested with a *text/plain* content type Ontohub serves the OMS. This is in part because there is no reasonable representation of a directory that could be supported. Another reason is that Ontohub serves OMS as its main objects. And as *text/plain* is the MIME-type that was chosen to always return the textual content of an OMS (the raw file), one needs to serve that, even if the loc/id would be ambiguous in a normal request.

O Annex: Introduction to Category Theory

(Informative)

O.1 Categories

Definition 25 *A category C consists of*

- *a class of* objects*, denoted $|C|$,*
- *for each two objects a and b, a class of* morphisms *(or* arrows*), denoted $C(a, b)$,*
- *for each three objects a, b and c, a* composition *operation, denoted $; : C(a, b) \times C(b, c) \to C(a, c)$ such that the following axioms hold:*
  - *if $f \in C(a, b)$, $g \in C(b, c)$ and $h \in C(c, d)$ for four objects a, b, c, d, then $f; (g; h) = (f; g); h$*
  - *for each object a there is a morphism $id_a \in C(a, a)$ such that for every $f \in C(a, b)$ and every $g \in C(b, a)$ for some object b we have that $id_a; f = f$ and $g; id_a = g$.*

Example Set is the category whose class of objects is the class of all sets, $Set(A, B)$ is the set of all functions from A to B for any sets A and B, id_A is the identity function on a set A and the composition is the usual composition of functions.

Example Rel is the category whose class of objects is the class of all sets, $Rel(A, B)$ is the class of all relations $R \subseteq A \times B$, for any sets A and B, id_A is the diagonal relation $\{(a, a) \mid a \in A\}$ for a set A and the composition of $R \in Rel(A, B)$ with $S \in Rel(B, C)$ for three sets A, B, C is defined as $\{(a, c) \mid$ exists $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S\}$.

Example The category of unsorted first-order signatures has as objects tuples of the form $F = (F_i)_{i \in \mathbb{N}}$ where F_i is a set (of function symbols of arity i, for each natural number i). Given two objects F and G, a morphism $\sigma : F \to G$ is a family of functions $(\sigma_i : F_i \to G_i)_{i \in \mathbb{N}}$, which means that the arities of function symbols are preserved by morphisms. The identity morphism for an object F is the family of identity functions $(id_{F_i})_{i \in \mathbb{N}}$ and the composition is defined component-wise: if $\sigma : F \to G$ and $\tau : G \to H$ are signature morphisms between the signatures F, G and H, then $\sigma; \tau = (\sigma_i; \tau_i)_{i \in \mathbb{N}}$

Example Given an unsorted first-order signature F, a model M of F consists of an universe M_U together with an interpretation of each function symbol $f \in F_i$ as a function M_f taking i arguments in M_U with result in M_U. Given two such models M and N, a model homomorphism $m : M \to N$ is a function $m : M_U \to N_U$ such that for each $i \in \mathbb{N}$ and each $f \in F_i$ we have that $m(M_f(x_1, \ldots, x_n)) = N_f(m(x_1), \ldots, m(x_n))$ for every $x_1, \ldots x_n$ in M_U. The identity function on M_U is a model homomorphism on M and the composition is the usual composition of functions. This gives us the category of first-order models of F.

Definition 26 *Let C be a category. Its dual or opposite category, denoted C^{op}*

- *has the same objects as C: $|C^{op}| = |C|$,*
- *for two objects $a, b \in |C|$, $C^{op}(a, b) = C(b, a)$,*
- *$;^{op} : C^{op}(a, b) \times C^{op}(b, c) \to C^{op}(a, c)$ is defined as $f ;^{op} g = g; f$ for any $f \in C^{op}(a, b) = C(b, a)$ and $g \in C^{op}(b, c) = C(c, b)$. The result $g; f$ is a morphism in $C(c, a) = C^{op}(a, c)$,*
- *for each object a, $id_a \in C^{op}(a, a) = C(a, a)$ is the identity w.r.t. the composition $;^{op}$*

Definition 27 *An object A is called an initial object in a category C if for each object B of C there is exactly one morphism from A to B.*

Definition 28 *An object A is called a terminal object in a category C if for each object B of C there is exactly one morphism from B to A.*

Example In Set, the empty set is the initial object and each singleton set is a terminal object.

O.1.1 Limits and colimits

Definition 29 *A* network[1] *in a category C is a functor $D : G \to C$, where G is a small category*[2]*, and can be thought of as the shape of the graph of interconnections between the objects of C selected by the functor D.*

Definition 30 *A* cocone *of a network $D : G \to C$ consists of an object c of C and a family of morphisms $\alpha_i : D(i) \longrightarrow c$, for each object i of G, such that for each edge of the network, $e: i \longrightarrow i'$ it holds that $D(e); \alpha_{i'} = \alpha_i$.*

Definition 31 *A* colimiting cocone *(or colimit) $(c, \{\alpha_i\}_{i \in |G|})$ has the property that for any cocone $(d, \{\beta_i\}_{i \in |G|})$ there exists a unique morphism $\gamma : c \longrightarrow d$ such that $\alpha_i ; \gamma = \beta_i$.*

By dropping the uniqueness condition and requiring only that a morphism γ should exist, a *weak* colimit is obtained.

When G is the category $\bullet \longleftarrow \bullet \longrightarrow \bullet$, G-colimits are called *pushouts*. When G is a discrete category (i.e. no arrows between objects other than identities), G-limits are called *coproducts*.

Definition 32 *A* cone *of a network $D : G \to C$ consists of an object c of C and a family of morphisms $\alpha_i : c \longrightarrow D(i)$, for each object i of G, such that for each edge of the network, $e: i \longrightarrow i'$ it holds that $\alpha_{i'} = \alpha_i ; D(e)$.*

Definition 33 *A* limiting cone *(or limit) $(c, \{\alpha_i\}_{i \in |G|})$ has the property that for any cone $(d, \{\beta_i\}_{i \in |G|})$ there exists a unique morphism $\gamma : c \longrightarrow d$ such that $\gamma ; \alpha_i = \beta_i$.*

When G is the category $\bullet \longrightarrow \bullet \longleftarrow \bullet$, G-limits are called *pullbacks*. When G is a discrete category, G-limits are called *products*.

O.2 Functors

Definition 34 *Let C and D be two categories. A* functor *$F : C \to D$ is a mapping that*

- *assigns to each object c of C an object $F(c)$ in D,*
- *assigns to each morphism $f \in C(c, d)$ a morphism $F(f) \in D(F(c), F(d))$ such that*
  - *$F(id_c) = id_{F(c)}$ for each $c \in |C|$,*
  - *$F(f; g) = F(f); F(g)$ for each $f \in C(a, b)$, $g \in C(b, c)$ and $a, b, c \in |C|$.*

Example For each category C, the identity functor $id_C : C \to C$ takes each object and each morphism to itself.

Example The forgetful functor F from the category of unsorted first-order models of a signature F to Set takes each model M to the set M_U and each model morphism $m : M \to N$ to its underlying function $m : M_U \to N_U$.

Example The covariant powerset functor $\mathcal{P} : \mathbb{S}et \to \mathbb{S}et$ maps each set A to the set of all subsets of A and each function $f : A \to B$ to the function that takes a subset X of A to the set $\{f(x) \mid x \in X\}$, which is a subset of B.

Example The covariant finite powerset functor $\mathcal{P}_{fin} : \mathbb{S}et \to \mathbb{S}et$ maps each set A to the set of all finite subsets of A and each function $f : A \to B$ to the function that takes a subset X of A to the set $\{f(x) \mid x \in X\}$, which is a subset of B.

O.3 Natural transformations

Definition 35 *Let C, D be two categories and let F and G be two functors between C and D. A* natural transformation *$\eta : F \to G$ assigns to each object $c \in |C|$ a morphism $\eta_c : F(c) \to G(c)$ such that for every $f \in C(c, d)$ we have that $F(f); \eta_d = \eta_c; G(c)$, which means that the following diagram commutes*

$$\begin{tikzcd}
F(c) \arrow[r, "F(f)"] \arrow[d, "\eta_c"'] & F(d) \arrow[d, "\eta_d"] \\
G(c) \arrow[r, "G(f)"'] & G(d)
\end{tikzcd}$$

Example There is an inclusion natural transformation $\iota : \mathcal{P}_{fin} \to \mathcal{P}$, i.e. for each set A, $\iota : \mathcal{P}_{fin}(A) \to \mathcal{P}(A)$ is the inclusion function (each finite subset of a set is also a subset of the set).

[1] A network is called a diagram in category theory texts. This terminology is introduced to disambiguate OMS networks from UML diagrams.

[2] That is, it has a set of objects and sets of morphisms between them instead of classes.