

# Deciding to Disclose: Pregnancy and Alcohol Misuse

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# Abstract

## Background

We draw together methodologies from game theory, agent based modelling, and decision theory to explore the process of decision making around disclosure. This is framed in the context of pregnant women disclosing their drinking behaviour to their midwives.

## Objective

The primary purpose is to demonstrate the potential utility of an approach which it is hoped goes some way towards addressing concerns about the ad hoc character of Agent Based Modelling (ABM), by providing a strong theoretical grounding for the reasoning processes of individual agents. To this end we hope to show that these simple rules, operating in an inescapably artificial scenario are nonetheless capable of producing trends from the literature. We also seek to demonstrate the significance of precisely how the decision making process is formulated, by contrasting four distinct decision rules against one another and exploring a simple form of information sharing, supported by the use of statistical emulators for a full exploration of the parameter space.

## Methods

We employ game theory to define a signalling game representative of a scenario where pregnant women decide how far to disclose their drinking behaviours to their midwives, and midwives employ the information provided to decide whether a costly referral should be made. This game is then recast as two games taking play against nature, to permit the use of a decision theoretic approach where both classes of agent use simple rules to decide their moves. Four decision rules are explored - a lexicographic heuristic which considers only the link between moves and payoffs, a Bayesian risk minimisation agent that uses the same information, a more complex Bayesian risk minimiser, and a Cumulative Prospect Theory (CPT) type.

Using a simulator we have developed in Python, we recreate two key qualitative trends described in the Midwifery literature for all the decision models, and investigate the impact of introducing a simple form of information sharing within agent groups. Finally a global sensitivity analysis using Gaussian Emulation Machines (GEMs) was conducted, to compare the response surfaces of the different decision rules in the game.

## Results

Selected results showing the ability of all decision rules to reproduce qualitative trends noted in the literature are provided, together with a sensitivity analysis, and comparative heat maps produced using GEMs demonstrating the significance of the precise implementation of the decision making.

## Conclusions

The ability of all the decision rules to show the qualitative trends suggests that there is some utility associated with this approach.

## Comments

We note that the scenario omits the overwhelming complexity of the reality, and is presented largely in the spirit of a convenient demonstration of the methodology. Clearly a domain where there is sufficient data to permit a more comprehensive approach to validation of model outcomes is desirable, and will form the basis of our future work.

To aid in replication and extension, the model has been implemented as a Python module, and is freely available under the Mozilla Public License from <https://github.com/greenape/disclosure-game>, together with full parameter sets, raw data, and all other code used in producing this paper.

# 1 Introduction

The case in favour of ABM as a general approach has been made numerous, and elegantly REFS. As such we will not belabour the point, and instead turn to addressing some of the concerns expressed about the method. In this instance we focus on the perception of ABM as ad hoc in nature, tending to be a reflection of the assumptions of the modeller rather than empirically or theoretically grounded (Waldherr and Wijermans, 2013). To ameliorate this concern, we extend previous work by Gray (2013) drawing on decision theory to produce simple rule based, learning, decision making agents and show that they are able to play a form of signalling game (Kreps and Cho, 1987) with a basic form of intragroup information sharing. Four decision models of varying complexity, and behavioural plausibility are contrasted, by way of demonstrating the significance of the operationalisation of decision making in ABM.

This is framed in the context of disclosure decisions, and a scenario examining drinking patterns in pregnant women which is presented in the spirit of a motivating example, rather than claimed as an accurate representation of reality. Alcohol consumption in the ante natal period is a significant issue in itself, and has been associated with many potentially negative consequences. For example, Andersen et al. (2012) report results from a large scale Danish cohort study suggesting that even low levels of consumption in early pregnancy increase the risk of spontaneous abortion, although Savitz (2012) has suggested this may be attributable to a previously known link to absence of morning sickness. Risk continues to the point of birth - Kesmodel et al. (2002) found a heightened risk to the infant - into childhood, with a metastudy by Latino-Martel et al. (2010) finding evidence on an increased risk of childhood acute myeloid leukemia (AML) (although they suggest that the rarity of the condition is a limitation). Harm may also extend even further, and a review by Huizink and Mulder (2006) concluded that maternal alcohol consumption could be a contributing factor to Attention Defecit Hyperactivity Disorder (ADHD), and other learning impairments, but note methodological issues in a number of the papers. There is not, however, a clear consensus, with Gray and Henderson (2006) finding no evidence of harm below 1.5 UK units per day. In terms of official guidance, National Institute for Health and Care Excellence (NICE) acknowledge that evidence of harm to the fetus is less than conclusive, but advise not drinking at all, or significant moderation (National Institute for Health and Care Excellence, 2010), with similar advice from the Department of Health (2008).

Turning more specifically to disclosure of alcohol use during pregnancy, research is relatively sparse, although qualitative trends are reported by Phillips et al. (2007), and Alvik et al. (2006). The former explored factors impacting disclosure through a small case study, highlighting the need to build up rapport over several appointments; the latter compared post partum reports of consumption with contemporaneous accounts, finding apparent underreporting during pregnancy which was amplified by increased drinking. The simulation model described in this paper is able to replicate both qualitative trends, i.e. an increase in disclosure over appointments, and more honest behaviour by moderate as compared to heavier drinkers.

This scenario is of substantial independent interest, and shows the potential utility of a simulation approach in arenas where randomised control trial (RCT) are not viable for ethical, or financial reasons. With this said, the lack of a strong quantitative evidence base against which to validate the behaviour of the model augers for caution in interpreting the results, and a necessary reminder that in this instance the focus is primarily methodological.

A game theoretic approach to generating an abstract form of the problem gives a convenient, and well known framework to reason about the processes involved in the scenario. While scenarios may well map to a plurality of games, this still allows for a principled comparison between interpretations and enforces explicit assumptions. Relating this to decision theory shifts the emphasis away from analytical equilibrium-seeking, and heightens the importance of behaviour change. Where the focus is on the behavioural processes driving a system in motion, and how they change in response to that movement, this is clearly desirable. Fundamentally, the shift of emphasis is from the process of acquiring and inferring the information needed to make choices, to the process of decision. Naturally, this does not preclude the incorporation of strategic refinement, since decision rules are to a great extent modular, and as demonstrated in this paper can be exchanged without altering the underpinning decision problem. In addition, rules are agnostic as to where the information used derives from, suggesting room for multi-stage processes. As a corollary, the decision problem agents attempt to answer can change, allowing agents' behaviour in novel problems to be informed

by beliefs derived under other conditions.

A key motivation for decision rules is their claim to provide an account of decision making that is behaviourally and cognitively plausible. Their mooted capability in this regard is to some extent supported by work from neuroeconomics, which aims to empirically test theories of decision making (Rustichini, 2009). Many key aspects common to decision rules, for example the idea that a common currency is used by the brain to compare outcomes (Padoa-Schioppa and Assad, 2006, 2008), are supported by neurological findings.

Given these features, the application of decision, and game theory to ABM is an attractive approach to computational social science, where the locus of interest is decision making. Taking a balance between the strongly biological, e.g. neural networks, and the more abstract threshold, or microsimulation like models yields a computationally tractable approach. Despite the relative simplicity, it nonetheless captures some of the nuance and sophistication of human decisions.

The remainder of this paper proceeds to provide a brief review of the methodological context (2), before outlining the model (3), and experiments (4), with selected results (5), then closing with a discussion contrasting the decision models (??).

## 2 Previous Research

This section presents a brief overview of previous research, addressing in turn signalling games, normative decision theory, heuristic decision making, and descriptive decision theory.

### 2.1 Signalling Games

The preponderance of classical game theory focuses on strategic decision making, in scenarios where all players have complete information about all aspects of the game. An alternative, perhaps more common situation, is that players have incomplete information, i.e. their knowledge of the rules of play is in some way deficient. Harsanyi (1967) introduced the concept of a Bayesian game, resolving the problems introduced by the incomplete information scenario by allowing the possible variations on the rules to be treated as subgames. This adds an additional player - nature, to the game, where nature takes the first move thereby deciding which subgame is played. Nature is assumed to make their move by lottery, and where the probability distribution governing the lottery is known to all players this permits the game to be formulated as one of complete information. Here, we are specifically interested in signalling games (Kreps and Cho, 1987; Spence, 1973), where one player holds some private information which may be communicated (or not) by means of a signal.

This basic form has been widely applied, with substantial interest in what conditions permit honest signalling as Nash equilibria or Evolutionarily Stable Strategy (ESS). Grafen (1990), following from a suggestion by Zahavi (1975), proposed that if signals intended to indicate mate quality exacted a cost on the signaller (e.g. peacock tail feathers), then honest signalling would constitute an ESS. Similar results have also been demonstrated in a game of job market signalling, where signal cost was differentiated by type (Spence, 1973). Costly signalling has also been suggested as an explanation of behaviour that at first gasp appears counter intuitive, for example Godfray (1991) applied the idea to the food solicitation behaviour of chicks, where a stronger signal carries a risk of being eaten. Moving beyond animal behaviour, Sosis (2003) considered the implications if ritual behaviour, in the context of religion, represented a costly signal, an idea subsequently extended by Henrich (2009) to include cultural transmission, and Wildman and Sosis (2011) to introduce group differentiation.

Other work augments the signalling game model, for example Austen-Smith and Fryer Jr. (2005) adds a second 'peer group' audience signalling game to the original Spence game in an effort to explain poor academic performance in some social groups, with some subsequent empirical support for the idea from Fryer Jr. and Torelli (2010). On a similar tack, Feltovich et al. (2002) introduced additional noisy type information, finding that this effectively explained counterintuitive observed behaviour where actors with every right to boast of their quality fail to do so.

## 2.2 Normative Decision Theory

Where game theory addresses strategic decision making, decision theory deals instead with rational decision making (Peterson, 2009). Taken literally, this leads to normative decision theory, where the focus is on giving the rational answer to a decision problem. An alternative view - descriptive, or behavioural decision theory, holds that the focus should instead be on giving an account of human decision making, complete with observed deviations from perfect rationality, which we address in section 2.4. Finally a third perspective, which to some extent overlaps this division, suggests that decisions are heuristic in nature and rational in ecological context (section 2.3).

The conceptual underpinning of all of these is the central idea of expected utility, originated by Bernoulli (1954) and later formalised by Von Neumann and Morgenstern (1953), which treats all decisions as gambles defined in terms of payoffs and probabilities.

Recently, several studies have explored biological correlates to aspects of expected utility. The fundamental concept, that all outcomes are comparable in a universal currency has been supported by evidence of neural correlates of decision variables (Platt and Glimcher, 1999), and following from this results from Padoa-Schioppa and Assad (2006, 2008) showing neuronal firing in the orbito-frontal cortex (OFC) corresponding to revealed preferences in monkeys. Additionally, some support for neural representation of value, and risk aversion was found by Christopoulos et al. (2009). The model presented in this paper makes an explicit assumption that social decisions utilise the same process, and while this is less well supported there is some evidence to suggest involvement by the same brain region, since damage to the OFC has been shown to impair social judgements in both primates (Watson and Platt, 2012), and humans (Willis et al., 2010).

An alternative normative model of decision making is Bayesian decision theory, proposed by Robbins (1964), which is essentially the application of Bayesian style probabilities to the expected utility model. This allows probabilities used in reasoning to be subjective, which may allow for a better account of decisions from experience (see Hau et al. (2008); Hertwig et al. (2004) for results elucidating the distinction, and comparing the performance of several non-Bayesian models). This model has seen notable successes in practical problems (McNamara and Houston, 1980; Dorazio and Johnson, 2003; Kristensen, 1997), but suggestions by several authors that it could constitute an effective (top-down) model of learning (Tenenbaum et al., 2006; Griffiths et al., 2010), or induction (Gallistel, 2012) in the brain have attracted substantial criticism, e.g. Bowers and Davis (2012), and Miller (2012) responding to Tenenbaum et al.; Griffiths et al., and Gallistel respectively.

## 2.3 Heuristic Decision Making

As noted, heuristic decision making stems from a contention that Von Neumann and Morgenstern type rationality ignores the both context of decision making, and a lack of correspondence between predicted and actual human decisions (see, for example the Allais paradox (Allais, 1953), and subsequent empirical support (Oliver, 2003; Burke et al., 1996)). Arguably, this begins with Simon (1956), who suggested that humans do not attempt to make optimal choices, but to satisfice and choose the first ‘good enough’ option. While noting that this will often achieve the same result, the claim is that humans exhibit bounded rationality (Simon, 2000) arising from inherent limits to cognition.

Gigerenzer and Goldstein (1996) take the concept of bounded rationality further, and argue for what they term Fast and Frugal Heuristic (FFH). This recasts rationality as bound to the context of the behaviour - a rational approach to choosing the right mate might well require checking every possible partner, but given finite time, memory, and so on rapidly becomes nonsensical. On this basis, they contend that the rationality of any given decision rule can only be determined in the context of the environment, which implies that heuristics are task specific.

## 2.4 Descriptive Decision Theory

While heuristic theories arguably fall under the purview of the descriptive, the wider tendency is towards what are in essence patches to normative models. The most influential models in this class derive from Prospect Theory (PT) (Kahneman and Tversky, 1979), which combines a set of heuristics based on observed

decision behaviour (Tversky and Kahneman, 1974), with distortions to the perception of probability, and the value of outcomes (Kahneman and Tversky, 1984; Tversky and Kahneman, 1986). Tversky and Kahneman (1992) subsequently addressed issues present in their original formulation by introducing CPT, which allows for non-binary decisions, at the expense of the heuristics. The essence then, is that high and low probabilities are treated differently, and the subjective value of a loss differs from the equivalent gain (losing your shirt is perceived as more of a loss than winning a shirt is a gain). This last, known as the framing effect is particularly significant, see for example work by Toll et al. (2007) examining the relationship between loss and gain framings and success rates in giving up smoking, and NICE guidance on framing of treatment options (National Institute for Health and Care Excellence, 2007).

CPT has been successful in explaining a number of anomalous results in decision tasks (see Camerer (2004) for a review), and Thaler (2000) comments to the effect that the theory is promising, albeit incomplete, lacking for example any explanation of how frames are constructed. While an effective account of decision behaviour under risk, the theory does not attempt to resolve apparent inconsistencies that arise when outcomes are delayed, i.e. in situations of intertemporal choice. Historically, Discounted Utility (DU) (Samuelson, 1937), which effectively claims that the value of a thing now is exponentially greater than the promise of the same thing at some future date, has been applied to explain this. More recently, Ainslie (1991) has suggested that discounting of future outcomes is hyperbolic, rather than exponential, although neither model is complete - both fail to account for results from Thaler (1981) showing differing temporal discounting rates for losses and gains. Loewenstein and Prelec (1992) report additional failings in classic DU models, and propose a modified form of CPT which they suggest is able to handle both immediate, and intertemporal choice.

### 3 Disclosure Game Model

In this section we outline the disclosure game model, and give details of the four decision rules, but begin with a brief sketch of a pregnancy in terms of encounters between a woman and a midwife. Typically women will have 12 appointments with a midwife during the antenatal period. Outside of caseloading teams, a woman does not generally have a named midwife, and may see a different practitioner at each appointment. In the UK, and unlike most healthcare scenarios, maternity notes are patient held, so midwives do not have extensive information prior to an appointment unless they have encountered the woman previously. Maternity notes are not generally linked to extra-departmental records, meaning that a history of alcohol related admissions to another service may remain unknown unless revealed by the woman.

According to NICE guidance (National Institute for Health and Care Excellence, 2010; National Institute for Health and Clinical Excellence, 2010) substance misuse should be raised at the initial booking appointment, and subsequent action if a concern is raised is at the discretion of the midwife. This may take the form of specific guidance to reduce intake, or if deemed necessary a referral to a specialist midwife and relevant interdisciplinary team. On alcohol consumption, policy regarding how to determine the level of consumption is generally at the trust level, or according to the best judgement of the individual midwife, with no guidance provided by NICE. This commonly takes the form of average units per week, but may include Tolerance, Annoyance, Cut down, Eye-opener (T-ACE) and similar measures.

Beyond the booking appointment, the onus is on women to raise concerns about their drinking behaviour, or the midwife to probe further if they feel it is warranted. In either case, once a concern has been raised the midwife must respond clinically, and inevitably personally, to the information.

In an ideal world, all interactions with healthcare providers would be immediately and fully disclosive, with no repercussions for the patient. However, alcohol misuse by women is known to attract stigma (Gomberg, 1988), and is a recognised barrier to appropriate treatment in the maternity context (Radcliffe, 2011; National Institute for Health and Clinical Excellence, 2010).

### 3.1 Disclosure Game

In order to translate the scenario sketched above into a more abstract, tractable form, we cast it as a signalling game, and assume that women’s disclosures (or not), are signals. We also make the simplifying assumption that a woman may have one of only three drinking patterns - light, moderate, or heavy. Correspondingly, they are limited in what signals they may send to claiming to be one of these three types.

Midwives are treated in a similar fashion, where their type corresponds to how negatively they regard a drinking pattern - non-judgemental, moderately judgemental, and harshly judgemental. The expression of this judgement is not a matter of choice on their part, and is assumed to have no impact on their response, which is to either refer the woman for specialist treatment, or do nothing.

At the end of a game, each player receives a payoff dependent on the actions and types of both players, which has a partially common interest component. Women receive a payoff based on the health of their eventual baby, with a social cost dependent on the signal they sent and the midwife’s reaction to it. Midwives receive the same health payoff as the women, but pay a cost for referring to a specialist, mirroring the organisational cost of non-routine care. Table 1 shows the three payoff matrices which together describe the game.

Taken together, this leads to a game tree that is relatively complex even at the subgame level (figure ?? shows the extensive form for a subgame, with information sets). Rather than attempt to solve for equilibria, agents treat this two player game as taking place against nature, along the lines of adversarial risk analysis (Insua et al., 2009). This effectively translates the game to a pair of decision problems, which agents attempt to resolve at each turn using a simple decision rule, given their prior beliefs and experience of play.

Women are drawn in order from a queue, and play against a midwife chosen at random. They play for a maximum of  $r_w$  rounds ( $r_w = 12$  following the routine number of ante-natal appointments in the UK (National Institute for Health and Care Excellence, 2010)) or until they are referred. At which point a new player is drawn from the same distribution that produced the original players to replace them. If they are not referred, they rejoin the back of the queue after their appointment. In either case, they are informed of their payoff after each round and update their beliefs accordingly.

Midwives play for  $r_m$  rounds ( $r_m = 1000$  in all experiments), and conduct appointments in parallel, i.e. if there are 5 midwives, then five women are drawn from the queue and assigned at random to the midwives. Unlike women, midwives are only informed of their payoff if they choose to make a referral. Both groups of agents have perfect recall, and midwives are assumed to retrospectively update their observations if they make a referral after a number of appointments.

Formally then, let  $N = \{m, w\}$  be the set of players each with a private type  $\theta_i \in \Theta$ , and a set of types  $\Theta = \{l, m, h\}$ , with pure strategies  $A_m = \{r, n\}$ ,  $A_w = \{l, m, h\}$ . Additionally define a utility function  $u_i(s_w, s_m, \theta_w, \theta_m) = X_{s, s_w, \theta_m} + X_{h, \theta_w, s_m} + X_{c, \theta_w, s_m}$ , and distributions over types  $p_w(l, m, h)$ ,  $p_m(l, m, h)$ .

### 3.2 Agent Models

While in principle a wide variety of agent models are possible, given that decision rules operate on essentially the same information, and produce the same outputs, we limit ourselves here to four. The simplest is a lexicographic rule (1), motivated as in the spirit of a FFH (Gigerenzer, 2004) which uses only information about payoffs given actions; a Bayesian risk minimisation rule using the same information (2); a second Bayesian risk rule (3) which uses information about the underlying lottery; and a two-stage CPT (Hau et al., 2008) agent (4) which is identical with 3, but uses the CPT decision rule from Tversky and Kahneman (1992). Hence, each successive decision model adds a layer of sophistication to the problem representation while retaining the same input-output characteristics.

As noted in section 3.1, agents have perfect recall, and recognise individual opponents if they encounter them subsequently. While agents recall perfectly and make use of the new information for retrospective updates, all four agent models make decisions ‘as-if’ they were always facing a new opponent.

A simplifying assumption is made that all midwives have just qualified after receiving identical training. As a result, they have homogenous beliefs about their women and assume to some extent that they are honest.

|         |        | Woman |          |       |
|---------|--------|-------|----------|-------|
| Midwife |        | Heavy | Moderate | Light |
|         | Harsh  | 0, -2 | 0, -1    | 0, 0  |
|         | Medium | 0, -1 | 0, 0     | 0, 0  |
|         | Low    | 0, 0  | 0, 0     | 0, 0  |

(a) Social cost,  $X_s$

|         |             | Woman  |          |        |
|---------|-------------|--------|----------|--------|
| Midwife |             | Heavy  | Moderate | Light  |
|         | Refer       | 10, 10 | 10, 10   | 10, 10 |
|         | Don't refer | -2, -2 | -1, -1   | 10, 10 |

(b) Health outcome,  $X_h$

|         |             | Woman |          |       |
|---------|-------------|-------|----------|-------|
| Midwife |             | Heavy | Moderate | Light |
|         | Refer       | -9, 0 | -9, 0    | -9, 0 |
|         | Don't refer | 0, 0  | 0, 0     | 0, 0  |

(c) Referral cost,  $X_c$

Table 1: Payoff matrices

Women are heterogenous in their prior observations, which are assigned stochastically and constrained such that they have encountered each scenario possible at least once, with exactly  $k$  encounters overall.

### 3.2.1 Lexicographic Heuristic

The lexicographic heuristic (algorithm 1) follows the form of that used in Hau et al. (2008), and assumes a simplified decision problem, as in figure ??, where an action is a choice between combined lotteries. Functionally, the heuristic maintains a count of the number of times that each action was followed by a payoff, and chooses the action which most commonly has the best payoff, i.e. one reason decision making. This approach requires minimal computation, and does not assume that  $u_i$  is static, or known.

Women resolve this by approximating the utility function, as a function  $f(s_w, \sigma)$  on their choice of signal and an unknown distribution, which maps to  $u_w$  - i.e.  $s_w$  is a choice between simple lotteries. The algorithm maintains a count,  $n$ , of the number of occurrences of each outcome given the choice from  $s_w$ .

Midwives solve a slightly different problem with more information, where  $s_w$  is known, and  $s_m$  is the lottery choice -  $f(s_w, s_m, \sigma)$ . This is resolved by maintaining a separate count for each signal (i.e.  $n_{s_w, s_m}$ ), and otherwise following the same algorithm.

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#### Algorithm 1 Lexicographic heuristic

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n=1, action=none
while action is none do
    Calculate the nth most common outcome following each action.
    Sort actions by the value of the nth most common outcome.
    if clear winner then
        action = best
    end if
    n = n + 1
end while
return action

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### 3.2.2 Bayesian Payoff

The Bayesian payoff agent uses the same subset of information as the lexicographic method, but updates beliefs on the link between actions and payoffs using Bayes rule, and attempts to choose the action which minimises risk.

Given the discrete nature of actions and payoffs, coupled with a desire for tractability of the simulation, the Dirichlet distribution is employed to represent these beliefs. The probability density function takes the form -

$$D(\Theta|\alpha) = \frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} \prod_{i=1}^k \Theta_i^{\alpha_i - 1}$$

Where  $\alpha = \{\alpha_1 \dots \alpha_k\}$ ,  $k$  is the number of signal-payoff pairs,  $\Theta = \{\Theta_1, \dots, \Theta_{k-1}\}$  all more than zero and summing to less than 1, and  $\alpha_i$  is the psuedo-count of prior observations for a pair  $i$ .

The distribution is particularly convenient, in that to infer the probability of a signal implying a payoff becomes simply -

$$P(x = j|D, \alpha) = \frac{\alpha_j + n_j}{\sum_j (\alpha_j + n_j)} \quad (1)$$

Where  $n_j$  is simply the count of occurrences of pair  $j$ , so that the belief that a signal  $j$  the number of times that type has been observed (including the pseudo-count), over the total number of observations thus far. This makes computation of beliefs fast and simple, since all that must be maintained is a count of observations with no particular concern as to their order. As before, midwives follow a similar pattern but per signal.

Agents then choose  $s_i$  to minimise  $R_i$ , which is simply -

$$R_w(s_w) = \sum_{x \in X} -xp(x|s_w) \quad (2)$$

$$R_m(s_w, s_m) = \sum_{x \in X} -xp(x|s_w \wedge s_m) \quad (3)$$

Where  $X$  is set of payoffs the agent has observed to follow  $s$ .

### 3.2.3 Bayesian Risk Minimisation

The second Bayesian agent augments the reasoning of the simple payoff model, making the stronger assumption that the utility function is static, and known. Women maintain two sets of beliefs, corresponding respectively to  $p_m$ , and the probability of referral given signal choice. This leads to the risk function -

$$R_w(s_w, \theta_w) = \sum_{i \in A_m} \sum_{j \in \Theta} -u_w(s_w, i, \theta_w, j)p(j)p(i|s_w) \quad (4)$$

So that the risk of a signal is the sum of the products of all payoffs with the probabilities of their entailed midwife types and responses.

Midwives reasoning centers on determining the meaning of signals, since given the knowledge of what some signal  $i$  conveys about the true type of the sender, the payoff for an action is known. As such, their inference process is the same as for the simple Bayesian agent but over signal-type pairs, and they attempt to minimise -

$$R_m(s_w, s_m) = \sum_{i \in \Theta} -u_w(s_w, s_m, i, \theta_m)p(i|s_w) \quad (5)$$

### 3.2.4 Descriptive Decision Theory

The most complex decision rule used is CPT, which attempts to reproduce a number of systematic deviations from rationality observed in humans. While CPT has primarily been applied in the context of decisions from description, it has been modified to deal with decisions from experience by incorporating a first stage where probabilities are estimates from observations as in Fox and Tversky (1998). In this instance the Bayesian inference process fills the first stage role.

Rather than the psychologically more interesting PT, the CPT decision rule is used in this instance, because of the requirement for women to evaluate more than two ‘prospects’.<sup>1</sup> CPT introduces the concept of a probability weighting function, which underweights small probabilities, and overweights large ones in an effort to capture the tendency of humans to treat high probability events as sure things, and small probabilities as ‘never going to happen’. A number of different weighting functions have been proposed, but in this instance the original formulation by Tversky and Kahneman (1992) is used. This distinguishes between weighting for gains, and losses -

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}$$

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}$$

Where  $p$  is the unweighted probability, and  $\gamma$  and  $\delta$  are the weights for gain and loss probabilities respectively. Humans have also been observed to value gains and losses differently, with a loss being ‘worse’ than the equivalent gain is ‘good’. This entails a transformed value function -

$$v(x) = \begin{cases} f(x) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ g(x) & \text{if } x < 0 \end{cases}$$

Where,

$$f(x) = \begin{cases} x^\alpha & \text{if } \alpha > 0 \\ \ln(x) & \text{if } \alpha = 0 \\ 1 - (1+x)^\alpha & \text{if } \alpha < 0 \end{cases}$$

$$g(x) = \begin{cases} -(-x)^\beta & \text{if } \beta > 0 \\ -\ln(-x) & \text{if } \beta = 0 \\ (1-x)^\beta - 1 & \text{if } \beta < 0 \end{cases}$$

And  $\alpha$ , and  $\beta$  are respectively the power of a gain, and a loss, and  $x = u_i$ . The CPT value of outcome  $x$  is  $v(x)w^+(x)$  if  $x \geq 0$ , and  $v(x)w^-(x)$  otherwise. For an action the CPT value is the sum of the value of the prospects of that action, as in the Bayesian risk model. The decision rule then requires the agent to choose the action which maximises the prospect theory value.

### 3.3 Information Sharing

It would seem unreasonable to suppose that neither party recounts their experiences to their peers, and to explore the impact of this we also modify the game to introduce a simple form of information sharing within agent groups. This takes the form of having each agent share their memories with their colleagues with some probability  $q$ . Individuals then incorporate shared information into their beliefs using weighted

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<sup>1</sup>A prospect in this instance is a paired outcome and probability, and the set of prospects for an action hence define the outcome space.

updates, such that a shared observation of a low type signal contributes to their beliefs by  $w$ , and  $0 \leq w \leq 1$  (i.e.  $n_j = n_j + w$ ). Women share only when they have finished play, and provide their complete history of games, because they have accurate information about the outcomes. By the same rationale, midwives share only their history with the most recent woman they referred. Sharing occurs simultaneously for all players at the end of each round, and all memories are either shared immediately or discarded.<sup>2</sup>

Because of their differing problem representations, the simple payoff reasoners and their more complex counterparts incorporate this exogenous information differently. The simple payoff based rule relies on a belief structure relating actions directly to rewards. Because payoffs differ by the agent’s private type, the information shared may not correspond to the experience of the listening agent in the same scenario. As a result, payoff reasoners have a belief bias towards the most common player type, and can believe in outcomes that are, for them, impossible.

By contrast, representing the problem in terms of the probabilities of the individual lotteries yields a structure that abstracts the new information from payoffs, and allows the agent discount implausible outcomes. This stronger assumption as to the static and known qualities of payoffs does however reduce the flexibility of the decision rule.

## 4 Method

This section provides details of experiments conducted to examine the ability of the model to reproduce qualitative trends reported in the midwifery literature by Alvik et al. (2006), and Phillips et al. (2007); as well as a global sensitivity analysis and construction of statistical emulators to explore, and contrast the response surfaces of the four decision rules.

### 4.1 Qualitative Trends

Throughout this paper, parameters for the CPT model were as used in Tversky and Kahneman (1992) (table 2). While there has been significant work on determining appropriate parameterisation for the model (e.g. Neilson and Stowe (2002); Glöckner and Pachur (2012); Nilsson et al. (2011), and particularly Byrnes et al. (1999); Booij et al. (2009) addressing risk aversion and gender), a full exploration of the impact of these parameters, or heterogeneous values within populations is beyond the scope of this work. For simplicity, it is assumed that all three drinking types are equally prevalent within the population, although results derived from the Avon Longitudinal Study of Parents and Children (ALSPAC) suggest that the reality is far more positive (Humphriss et al., 2013). The scenario is biased towards disclosure as the better option by presuming a distribution of midwives strongly skewed towards non-judgemental types, with beliefs initially favouring honesty. Payoffs were as in table 1, which ensure that it is always strictly preferable to refer drinkers, and together with the initial belief that signals will be honest, not refer those claiming otherwise.

Two key measures were used - the fraction of the subpopulation who had ever signalled honestly, and the proportion referred. Both measures were taken after every round of play, and were taken relative to the agent’s position in their sequence of appointments giving the probability of signalling honestly, or being referred having had a given number of appointments.

### 4.2 Information Sharing

In addition to assessing the adequacy of the rules in capturing qualitative trends, we also examine the impact of simple information sharing (section 3.3) on the robustness of these trends. Table 3 gives the parameters explored, with a more complete exploration of the effects on the system as a whole performed as part of the global sensitivity analysis.

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<sup>2</sup>Memories of games remain, but it is assumed that only current news is relevant.

| Name                     | Description                               | Value  |
|--------------------------|---|--------|
| $n_w$                    | Number of women                           | 1000   |
| $n_m$                    | Number of midwives                        | 100    |
| $r_m$                    | Number of appointments per midwife        | 1000   |
| $r_w$                    | Maximum number of appointments per woman  | 12     |
| Runs                     | Simulation runs                           | 1000   |
| $p_w(h)$                 | Proportion of heavy drinkers              | 1/3    |
| $p_w(m)$                 | Proportion of moderate drinkers           | 1/3    |
| $p_w(l)$                 | Proportion of light drinkers              | 1/3    |
| $p_m(h)$                 | Proportion of harsh midwives              | 5/100  |
| $p_m(m)$                 | Proportion of moderate midwives           | 10/100 |
| $p_m(l)$                 | Proportion of non-judgemental midwives    | 85/100 |
| $q_w$                    | Probability of women sharing              | 0.     |
| $w_w$                    | Weight of shared information for women    | 0.     |
| $q_m$                    | Probability of midwives sharing           | 0.     |
| $w_m$                    | Weight of shared information for midwives | 0.     |
| $s_i[a_i] : s_i[a_{-i}]$ | Pseudo-count favouring honesty            | 10:1   |

Table 2: Model parameters.

| Name  | Description                               | Min | Max | Step Size |
|-------|---|-----|-----|-----------|
| $q_w$ | Probability of women sharing              | 0   | 1   | 0.25      |
| $w_w$ | Weight of shared information for women    | 0   | 1   | 0.25      |
| $q_m$ | Probability of midwives sharing           | 0   | 1   | 0.25      |
| $w_m$ | Weight of shared information for midwives | 0   | 1   | 0.25      |

Table 3: Information sharing parameter ranges.

### 4.3 Global Sensitivity Analysis

In general, we follow the procedure outlined in Bijak et al. (2013) for stochastic agent based models, although see Thiele et al. (2014) for a review of alternative techniques.

Parameters for training were generated in R (R Core Team, 2014) using Latin Hypercube Sampling (Carnell, 2012) over the space of inputs given in table 4, giving 10 free parameters. Initially a unit hypercube was generated, then the margins transformed appropriately to cover those regions where the inputs are not bounded between 0 and 1, and for proportions of agent types which necessarily sum to one across the three parameters. Given the limitation of 400 design points for the Gaussian Emulation Machine for Sensitivity Analysis (GEM-SA) software, we produced exactly that many parameter combinations and collected results for 100 runs of each. A fixed set of 100 random seeds was used, such that each parameter set was run once with each seed, for every decision rule.

To better capture the response characteristics for the model, we measure three outcome variables - (1) the interquartile range of the average signal sent by each type of agent in a run, (2) the average signal of moderate drinking agents in a run, and (3) the standard deviation of that average signal between simulation runs. Together these three metrics give an indication of how far women are separable by their signalling behaviour (1), the behaviour of the at risk drinking groups<sup>3</sup> (2), and finally the stability of the system in the face of the stochastic elements.

Measurements were taken at the end of 1000 rounds of play, and for 1 and 2, 400 results were selected covering the full hypercube with each chosen randomly from the runs for that design point. This approach, rather than averaging across runs, was taken to avoid obscuring the high degree of variability evident in the

<sup>3</sup>Under most conditions, the behaviour of heavy drinkers tracks closely with their moderate counterparts.

| Name                     | Description                               | Min          | Max   |
|--------------------------|---|--------------|-------|
| $p_w(h)$                 | Proportion of heavy drinkers              | 0            | 1     |
| $p_w(m)$                 | Proportion of moderate drinkers           | 0            | 1     |
| $p_w(l)$                 | Proportion of light drinkers              | 0            | 1     |
| $p_m(h)$                 | Proportion of harsh midwives              | 0            | 1     |
| $p_m(m)$                 | Proportion of moderate midwives           | 0            | 1     |
| $p_m(l)$                 | Proportion of non-judgemental midwives    | 0            | 1     |
| $q_w$                    | Probability of women sharing              | 0            | 1     |
| $w_w$                    | Weight of shared information for women    | 0            | 1     |
| $q_m$                    | Probability of midwives sharing           | 0            | 1     |
| $w_m$                    | Weight of shared information for midwives | 0            | 1     |
| $x_h$                    | Health payoff for healthy delivery        | 1            | 100   |
| $x_r$                    | Cost for referral                         | $-(x_h - 1)$ |       |
| $s_i[a_i] : s_i[a_{-i}]$ | Pseudo-count favouring honesty            | 1:1          | 100:1 |

Table 4: Parameter ranges.

| Name | Description |
|------|-------------|
|------|-------------|

Table 5: Output measures.

output of the payoff reasoning agents in some areas of the parameter space.

Twelve emulators were built, covering each of the three output on all four decision models. These emulators were used to conduct a probabilistic sensitivity analysis using GEM-SA to assess the impact of parameters individually, and in combination.

## 5 Results

LOOK AT MAH GRAFS!

### 5.1 Qualitative Trends

As shown in figure 1, all four decision rules were able to reproduce both qualitative trends towards more disclosure as women experience more appointments (Phillips et al., 2007), and a greater tendency towards underreporting of consumption by heavier drinkers (Alvik et al., 2006). Trends for all four rules are broadly similar, exhibiting a gradual increase across appointments which subsequently levels off. This levelling can in part be explained by the referral results (figure 2), which show that the majority of drinkers are referred, despite substantial concealment. Referrals continue to occur, in the absence of honest signals, because drinkers are able to achieve a referral by pooling with higher or lower types, dependent on how their initial beliefs are biased.

Figure 1: Average fraction of population signalling honestly over appointments, across 100 runs.

Figure 2: Average fraction of population referred over appointments, across 100 runs.

## 5.2 Information Sharing

## 5.3 Sensitivity Analysis

The full results for the sensitivity analysis covering all twelve emulators are available in appendix ??, and in this section we present selected results highlighting.....

# 6 Discussion and Conclusions

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