

Deciding to Disclose: Pregnancy and Alcohol Misuse

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Abstract

Background

We draw together methodologies from game theory, agent based modelling, and decision theory to explore the process of decision making around disclosure. This is framed in the context of pregnant women disclosing their drinking behaviour to their midwives.

Objective

The primary purpose it to demonstrate the potential utility of an approach which it is hoped goes some way towards addressing concerns about the ad hoc character of Agent Based Modelling (ABM), by providing a strong theoretical grounding for the reasoning processes of individual agents. To this end we hope to show that these simple rules, operating in an inescapably artificial scenario are nonetheless capable of producing trends from the literature. We also seek to demonstrate the significance of precisely how the decision making process is formulated, by contrasting four distinct decision rules against one another and exploring a simple form of information sharing, supported by the use of statistical emulators for a full exploration of the parameter space.

Methods

We employ game theory to define a signalling game representative of a scenario where pregnant women decide how far to disclose their drinking behaviours to their midwives, and midwives employ the information provided to decide whether a costly referral should be made. This game is then recast as two games played against nature, to permit the use of a decision theoretic approach where both classes of agent use simple rules to decide their moves. Four decision rules are explored - a lexicographic heuristic which considers only the link between moves and payoffs, a Bayesian risk minimisation agent that uses the same information, a more complex Bayesian risk minimiser, and a Cumulative Prospect Theory (CPT) type.

Using a simulator we have developed in Python, we recreate two key qualitative trends described in the Midwifery literature for all the decision models, and investigate the impact of introducing a simple form of information sharing within agent groups. Finally a global sensitivity analysis using Gaussian Emulation Machines (GEMs) was conducted, to compare the response surfaces of the different decision rules in the game.

Results

Selected results showing the ability of all decision rules to reproduce qualitative trends noted in the literature are provided, together with a sensitivity analysis, and comparative heat maps produced using GEMs demonstrating the significance of the precise implementation of the decision making.

Comments

We note that the scenario omits the overwhelming complexity of the reality, and is presented largely in the spirit of a convenient demonstration of the methodology. Clearly a domain where there is sufficient data to permit a more comprehensive approach to validation of model outcomes is desirable, and will form the basis of our future work.

To aid in replication and extension, the model has been implemented as a Python module, and is freely available under the Mozilla Public License from <https://github.com/greenape/disclosure-game-module>.

1 Introduction

The case in favour of Agent Based Modelling (ABM) as a general approach has been made numerously, and elegantly (e.g. Silverman et al. (2011, 2013); Resnick (1994); Gilbert (1999); Epstein and Axtell (1994); Macy and Willer (2002); Axelrod (1997), amongst others). As such we will not belabour the point, and instead turn to addressing some of the concerns expressed about the method. In this instance we focus on the perception of ABM as ad hoc in nature, tending to be a reflection of the assumptions of the modeller rather than empirically or theoretically grounded (Waldherr and Wijermans, 2013). To ameliorate this concern, we extend previous work by Gray (2013) drawing on decision theory to produce simple rule based, learning, decision making agents and show that they are able to play a form of signalling game (Kreps and Cho, 1987) with a basic form of intragroup information sharing. Four decision models of varying complexity, and behavioural plausibility are contrasted, by way of demonstrating the significance of the operationalisation of decision making in ABM.

This is framed in the context of disclosure decisions, and a scenario examining drinking patterns in pregnant women which is presented in the spirit of a motivating example, rather than claimed as an accurate representation of reality. Alcohol consumption in the ante natal period is a significant issue in itself, and has been associated with many potentially negative consequences. For example, Andersen et al. (2012) report results from a large scale Danish cohort study suggesting that even low levels of consumption in early pregnancy increase the risk of spontaneous abortion, although Savitz (2012) has suggested this may be attributable to a previously known link to absence of morning sickness. Risk continues to the point of birth - Kesmodel et al. (2002) found a heightened risk to the infant - into childhood, with a metastudy by Latino-Martel et al. (2010) finding evidence on an increased risk of childhood acute myeloid leukemia (AML). Harm may also extend even further, and a review by Huijink and Mulder (2006) concluded that maternal alcohol consumption could be a contributing factor to Attention Deficit Hyperactivity Disorder (ADHD), and other learning impairments, but note methodological issues in a number of the papers. There is not, however, a clear consensus, with Gray and Henderson (2006) finding no evidence of harm below 1.5 UK units per day. In terms of official guidance, National Institute for Health and Care Excellence (NICE) acknowledge that evidence of harm to the fetus is less than conclusive, but advise not drinking at all, or significant moderation (National Institute for Health and Care Excellence, 2010), with similar advice from the Department of Health (2008).

Turning more specifically to disclosure of alcohol use during pregnancy, research is relatively sparse, although qualitative trends are reported by Phillips et al. (2007), and Alvik et al. (2006). The former explored factors impacting disclosure through a small case study, highlighting the need to build up rapport over several appointments; the latter compared post partum reports of consumption with contemporaneous accounts, finding apparent underreporting during pregnancy which was amplified by increased drinking. The simulation model described in this paper is able to replicate both qualitative trends, i.e. an increase in disclosure over appointments, and more honest behaviour by moderate as compared to heavier drinkers.

This scenario is of substantial independent interest, and shows the potential utility of a simulation approach in arenas where randomised control trials (RCTs) are not viable for ethical, or financial reasons. With this said, the lack of a strong quantitative evidence base against which to validate the behaviour of the model augers for caution in interpreting the results, and a necessary reminder that in this instance the

model is primarily a tool for thinking, rather than a machine for predicting.

A game theoretic approach to generating an abstract form of the problem gives a convenient, and well known framework to reason about the processes involved in the scenario. While scenarios may well map to a plurality of games, this still allows for a principled comparison between interpretations and enforces explicit assumptions. Relating this to decision theory shifts the emphasis away from analytical equilibrium-seeking, and heightens the importance of behaviour change. This is clearly desirable where the focus is on the behavioural processes driving a system in motion, and how they change in response to that movement. Fundamentally, the shift of emphasis is from the process of acquiring and inferring the information needed to make choices, to the process of decision. Naturally, this does not preclude the incorporation of strategic refinement, since decision rules are to a great extent modular, and as demonstrated in this paper can be exchanged without altering the underpinning decision problem. In addition, rules are agnostic as to where the information used derives from, suggesting room for multi-stage processes. As a corollary, the decision problem agents attempt to answer can change, allowing agents' behaviour in novel problems to be informed by beliefs derived under other conditions.

While there is no universal theory of human behaviour to sit at the center of ABM as a method, a key motivation for decision rules is their claim to provide an account of decision making that is behaviourally and cognitively plausible. Their mooted capability in this regard is to some extent supported by work from neuroeconomics, which aims to empirically test theories of decision making (Rustichini, 2009). Many key aspects common to decision rules, for example the idea that a common currency is used by the brain to compare outcomes (Padoa-Schioppa and Assad, 2006, 2008), are supported by neurological findings. In addition, decision rules represent a pleasingly parsimonious alternative to explicit behavioural rules covering all possible eventualities.

Given these features, the application of decision, and game theory to ABM is an attractive approach to computational social science, where the locus of interest is decision making. Taking a balance between the strongly biological, e.g. neural networks, and the more abstract threshold, or microsimulation like models yields a computationally tractable approach. Despite the relative simplicity, it nonetheless captures some of the nuance and sophistication of human decisions.

The remainder of this paper proceeds to provide a brief review of the methodological context (2), before outlining the model (3), and experiments (4), with selected results (5), then closing with a discussion contrasting the decision models (6).

2 Previous Research

This section presents a brief overview of previous research, addressing in turn signalling games, normative decision theory, heuristic decision making, and descriptive decision theory.

2.1 Signalling Games

The preponderance of classical game theory focuses on strategic decision making, in scenarios where all players have complete information about all aspects of the game. An alternative, perhaps more common situation, is that players have incomplete information, i.e. their knowledge of the rules of play is in some way deficient. Harsanyi (1967) introduced the concept of a Bayesian game, resolving the problems introduced by the incomplete information scenario by allowing the possible variations on the rules to be treated as subgames. This adds an additional player - nature, to the game, where nature takes the first move thereby deciding which subgame is played. Nature is assumed to make their move by lottery, and where the probability distribution governing the lottery is known to all players this permits the game to be formulated as one of complete information. Here, we are specifically interested in signalling games (Kreps and Cho, 1987; Spence, 1973), where one player holds some private information which may be communicated (or not) by means of a signal.

This basic form has been widely applied, with substantial interest in what conditions permit honest signalling as Nash equilibria or Evolutionarily Stable Strategies (ESS). Grafen (1990), following from a

suggestion by Zahavi (1975), proposed that if signals intended to indicate mate quality exacted a cost on the signaller (e.g. peacock tail feathers), then honest signalling would constitute an ESS. Similar results have also been demonstrated in a game of job market signalling, where signal cost was differentiated by type (Spence, 1973). Costly signalling has also been suggested as an explanation of behaviour that at first gasp appears counter intuitive, for example Godfray (1991) applied the idea to the food solicitation behaviour of chicks, where a stronger signal carries a risk of being eaten. Moving beyond animal behaviour, Sosis (2003) considered the implications if ritual behaviour, in the context of religion, represented a costly signal, an idea subsequently extended by Henrich (2009) to include cultural transmission, and Wildman and Sosis (2011) to introduce group differentiation.

Other work augments the signalling game model, for example Austen-Smith and Fryer Jr. (2005) adds a second ‘peer group’ audience signalling game to the original Spence game in an effort to explain poor academic performance in some social groups, with some subsequent empirical support for the idea from Fryer Jr. and Torelli (2010). On a similar tack, Feltovich et al. (2002) introduced additional noisy type information, finding that this effectively explained counterintuitive observed behaviour where actors with every right to boast of their quality fail to do so.

2.2 Normative Decision Theory

Where game theory addresses strategic decision making, decision theory deals instead with rational decision making (Peterson, 2009). Taken literally, this leads to normative decision theory, where the focus is on giving the rational answer to a decision problem. An alternative view - descriptive, or behavioural decision theory, holds that the focus should instead be on giving an account of human decision making, complete with observed deviations from perfect rationality, which we address in section 2.4. Finally a third perspective, which to some extent overlaps this division, suggests that decisions are heuristic in nature and rational in ecological context (section 2.3).

The conceptual underpinning of all of these is the central idea of expected utility, originated by Bernoulli (1954) and later formalised by Von Neumann and Morgenstern (1953), which treats all decisions as gambles defined in terms of payoffs and probabilities.

Recently, several studies have explored biological correlates to aspects of expected utility. The fundamental concept, that all outcomes are comparable in a universal currency has been supported by evidence of neural correlates of decision variables (Platt and Glimcher, 1999), and following from this results from Padoa-Schioppa and Assad (2006, 2008) showing neuronal firing in the orbito-frontal cortex (OFC) corresponding to revealed preferences in monkeys. Additionally, some support for neural representation of value, and risk aversion was found by Christopoulos et al. (2009). The model presented in this paper makes an explicit assumption that social decisions utilise the same process, and while this is less well supported there is some evidence to suggest involvement by the same brain region, since damage to the OFC has been shown to impair social judgements in both primates (Watson and Platt, 2012), and humans (Willis et al., 2010).

An alternative normative model of decision making is Bayesian decision theory, proposed by Robbins (1964), which is essentially the application of Bayesian style probabilities to the expected utility model. This allows probabilities used in reasoning to be subjective, which may allow for a better account of decisions from experience (see Hau et al. (2008); Hertwig et al. (2004) for results elucidating the distinction, and comparing the performance of several non-Bayesian models). This model has seen notable successes in practical problems (McNamara and Houston, 1980; Dorazio and Johnson, 2003; Kristensen, 1997), but suggestions by several authors that it could constitute an effective (top-down) model of learning (Tenenbaum et al., 2006; Griffiths et al., 2010), or induction (Gallistel, 2012) in the brain have attracted substantial criticism, e.g. Bowers and Davis (2012) responding to Tenenbaum et al., and Griffiths et al.; and Miller (2012) addressing Gallistel.

2.3 Heuristic Decision Making

As noted, heuristic decision making stems from a contention that Von Neumann and Morgenstern type rationality ignores the both context of decision making, and a lack of correspondence between predicted and actual human decisions (see, for example the Allais paradox (Allais, 1953), and subsequent empirical support

(Oliver, 2003; Burke et al., 1996)). Arguably, this begins with Simon (1956), who suggested that humans do not attempt to make optimal choices, but to satisfice and choose the first ‘good enough’ option. While noting that this will often achieve the same result, the claim is that humans exhibit bounded rationality (Simon, 2000) arising from inherent limits to cognition.

Gigerenzer and Goldstein (1996) take the concept of bounded rationality further, and argue for what they term Fast and Frugal Heuristics (FFHs). This recasts rationality as bound to the context of the behaviour: a rational approach to choosing the right mate might well require checking every possible partner, but given finite time, memory, and so on rapidly becomes nonsensical. On this basis, they contend that the rationality of any given decision rule can only be determined in the context of the environment (Todd and Gigerenzer, 2003), which implies that heuristics are task specific. They provide a number of heuristics for varying decision problems, e.g. recognition (Goldstein and Gigerenzer, 2002), cue ordering (Gigerenzer and Goldstein, 1999; Todd and Dieckmann, 2005), and binary decisions (Brandstätter et al., 2006).

2.4 Descriptive Decision Theory

While heuristic theories arguably fall under the purview of the descriptive, the wider tendency is towards what are in essence patches to normative models. The most influential models in this class derive from Prospect Theory (PT) (Kahneman and Tversky, 1979), which combines a set of heuristics based on observed decision behaviour (Tversky and Kahneman, 1974), with distortions to the perception of probability, and the value of outcomes (Kahneman and Tversky, 1984; Tversky and Kahneman, 1986). Tversky and Kahneman (1992) subsequently addressed issues present in their original formulation by introducing Cumulative Prospect Theory (CPT), which allows for non-binary decisions, at the expense of the heuristics. The essence then, is that high and low probabilities are treated differently, and the subjective value of a loss differs from the equivalent gain (losing your shirt is perceived as more of a loss than winning a shirt is a gain). This last, known as the framing effect is particularly significant, see for example work by Toll et al. (2007) examining the relationship between loss and gain framings and success rates in giving up smoking, and NICE guidance on framing of treatment options (National Institute for Health and Care Excellence, 2007).

CPT has been successful in explaining a number of anomalous results in decision tasks (see Camerer (2000) for a review), and Thaler (2000) comments to the effect that the theory is promising, albeit incomplete, lacking for example any explanation of how frames are constructed. While an effective account of decision behaviour under risk, the theory does not attempt to resolve apparent inconsistencies that arise when outcomes are delayed, i.e. in situations of intertemporal choice. Historically, Discounted Utility (DU) (Samuelson, 1937), which effectively claims that the value of a thing now is exponentially greater than the promise of the same thing at some future date, has been applied to explain this. More recently, Ainslie (1991) has suggested that discounting of future outcomes is hyperbolic, rather than exponential. However neither model is complete, in that both fail to account for results from Thaler (1981) showing differing temporal discounting rates for losses and gains. Loewenstein and Prelec (1992) report additional failings in classic DU models, and propose a modified form of CPT which they suggest is able to handle both immediate, and intertemporal choice.

3 Disclosure Game Model

In this section we outline the disclosure game model, and give details of the four decision rules, but begin with a brief sketch of a pregnancy in terms of encounters between a woman and a midwife.

Typically women will have 12 appointments with a midwife during the antenatal period. Outside of caselowering teams, a woman does not generally have a named midwife, and may see a different practitioner at each appointment. In the UK, and unlike most healthcare scenarios, maternity notes are patient held, so midwives do not have extensive information prior to an appointment unless they have encountered the woman previously. Maternity notes are not generally linked to extra-departmental records, meaning that a history of alcohol related admissions to another service may remain unknown unless revealed by the woman.

According to NICE guidance (National Institute for Health and Care Excellence, 2010; National Institute for Health and Clinical Excellence, 2010) substance misuse should be raised at the initial booking appointment, and subsequent action if a concern is raised is at the discretion of the midwife. This may take the form of specific guidance to reduce intake, or if deemed necessary a referral to a specialist midwife and relevant interdisciplinary team. On alcohol consumption, policy regarding how to determine the level of consumption is generally at the trust level, or according to the best judgement of the individual midwife, with no guidance provided by NICE. This commonly takes the form of average units per week, but may include Tolerance, Annoyance, Cut down, Eye-opener (T-ACE) and similar measures.

Beyond the booking appointment, the onus is on women to raise concerns about their drinking behaviour, or the midwife to probe further if they feel it is warranted. In either case, once a concern has been raised the midwife must respond clinically, and inevitably personally, to the information.

In an ideal world, all interactions with healthcare providers would be immediately and fully disclosive, with no repercussions for the patient. However, alcohol misuse by women is known to attract stigma (Gomberg, 1988), and is a recognised barrier to appropriate treatment in the maternity context (Radcliffe, 2011; National Institute for Health and Clinical Excellence, 2010).

3.1 Disclosure Game

In order to translate the scenario sketched above into a more abstract, tractable form, we cast it as a signalling game, and assume that women's disclosures (or not), are signals. We also make the simplifying assumption that a woman may have one of only three drinking patterns - light, moderate, or heavy. Correspondingly, they are limited in what signals they may send to claiming to be one of these three types.

Midwives are treated in a similar fashion, where their type corresponds to how negatively they regard a drinking pattern - non-judgemental, moderately judgemental, and harshly judgemental. The expression of this judgement is not a matter of choice on their part, and is assumed to have no impact on their response, which is to either refer the woman for specialist treatment, or do nothing.

At the end of a game, each player receives a payoff dependent on the actions and types of both players, which has a partially common interest component. Women receive a payoff based on the health of their eventual baby, with a social cost dependent on the signal they sent and the midwife's reaction to it. Midwives receive the same health payoff as the women, but pay a cost for referring to a specialist, mirroring the organisational cost of non-routine care. Table 1 shows the three payoff matrices which together describe the game.

Taken together, this leads to a game tree that is relatively complex (figure 1 shows a simplified representation) even at the subgame level. Rather than attempt to solve for equilibria, agents treat this two player game as taking place against nature, along the lines of adversarial risk analysis (Insua et al., 2009). This effectively translates the game to a pair of decision problems (figure 2), which agents attempt to resolve at each turn using a simple decision rule, given their prior beliefs and experience of play.

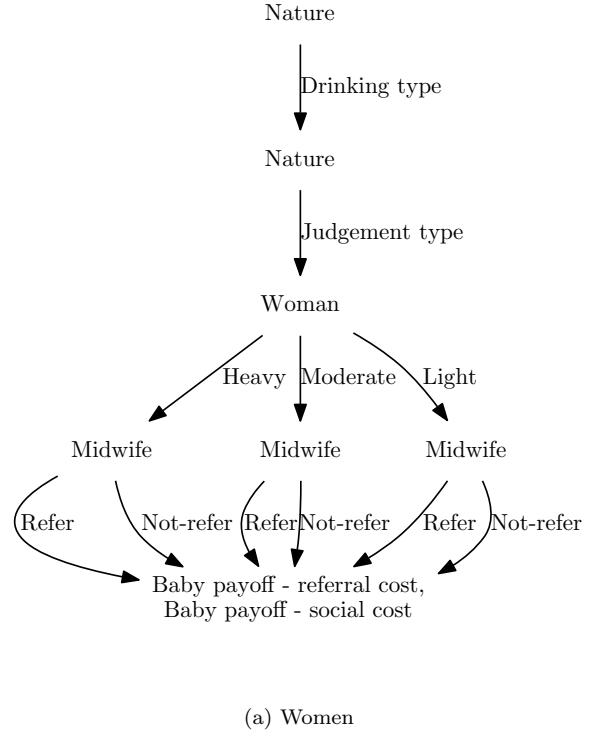


Figure 1: Simplified game tree.

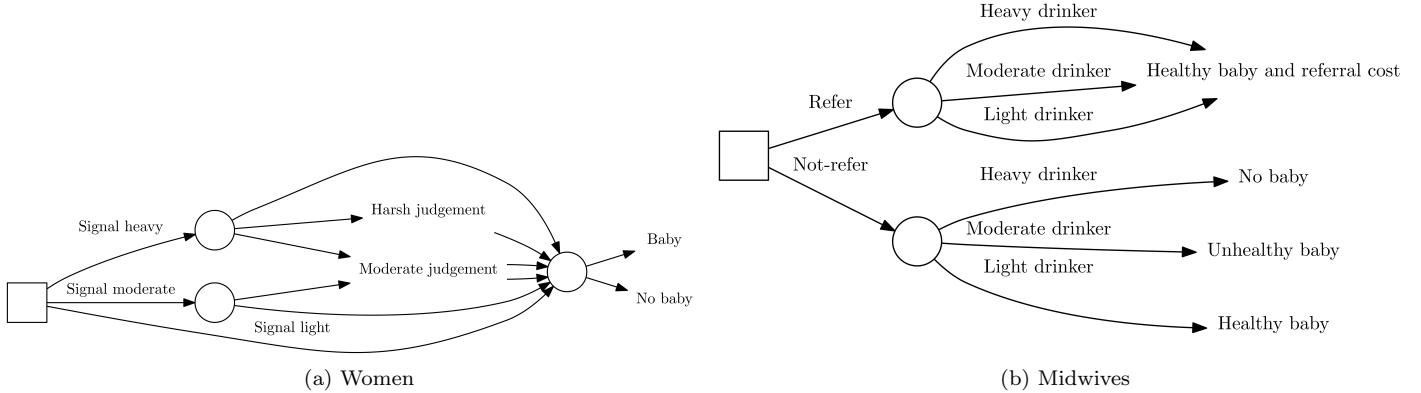


Figure 2: Influence diagrams, showing the game broken into two decision problems.

Women are drawn in order from a queue, and play against a midwife chosen at random. They play for a maximum of r_w rounds ($r_w = 12$ following the routine number of ante-natal appointments in the UK (National Institute for Health and Care Excellence, 2010)) or until they are referred. At which point a new player is drawn from the same distribution that produced the original players to replace them. If they are not referred, they rejoin the back of the queue after their appointment. In either case, they are informed of their payoff after each round and update their beliefs accordingly.

		Woman		
		Heavy	Moderate	Light
Midwife	Harsh	0, -2	0, -1	0, 0
	Medium	0, -1	0, 0	0, 0
	Low	0, 0	0, 0	0, 0

(a) Social cost, X_s

		Woman		
		Heavy	Moderate	Light
Midwife	Refer	10, 10	10, 10	10, 10
	Don't refer	-2, -2	-1, -1	10, 10

(b) Health outcome, X_h

		Woman		
		Heavy	Moderate	Light
Midwife	Refer	-9, 0	-9, 0	-9, 0
	Don't refer	0, 0	0, 0	0, 0

(c) Referral cost, X_c

Table 1: Payoff matrices

Midwives play for r_m rounds ($r_m = 1000$ in all experiments), and conduct appointments in parallel, i.e. if there are 5 midwives, then five women are drawn from the queue and assigned at random to the midwives. Unlike women, midwives are only informed of their payoff if they choose to make a referral. Both groups of agents have perfect recall, and midwives are assumed to retrospectively update their observations if they make a referral after a number of appointments.

Formally then, let $N = \{m, w\}$ be the set of players each with a private type $\theta_i \in \Theta$, and a set of types $\Theta = \{l, m, h\}$, with pure strategies $A_m = \{r, n\}$, $A_w = \{l, m, h\}$. Additionally define a utility function $u_i(s_w, s_m, \theta_w, \theta_m) = X_{s, s_w, \theta_m} + X_{h, \theta_w, s_m} + X_{c, \theta_w, s_m}$, and distributions over types $p_w(l, m, h)$, $p_m(l, m, h)$.

3.2 Agent Models

While in principle a wide variety of agent models are possible, given that decision rules operate on essentially the same information, and produce the same outputs, we limit ourselves here to four. The simplest is a lexicographic rule (1), motivated as in the spirit of a FFH (Gigerenzer, 2004) which uses only information about payoffs given actions; a Bayesian risk minimisation rule using the same information (2); a second Bayesian risk rule (3) which uses information about the underlying lottery; and a two-stage CPT (Hau et al., 2008) agent (4) which is identical with 3, but uses the CPT decision rule from Tversky and Kahneman (1992). Hence, each successive decision model adds a layer of sophistication to the problem representation while retaining the same input-output characteristics.

As noted in section 3.1, agents have perfect recall, and midwives recognise women if they encounter them subsequently. While agents recall perfectly and make use of the new information for retrospective updates, all four agent models make decisions ‘as-if’ they were always facing a new opponent.

A simplifying assumption is made that all midwives have just qualified after receiving identical training. As a result, they have homogenous beliefs about their women and assume to some extent that they are honest. Women are heterogeneous in their prior observations, which are assigned stochastically and constrained such that they have encountered each scenario possible at least once, with exactly k encounters overall.

3.2.1 Lexicographic Heuristic

The lexicographic heuristic (algorithm 1) follows the form of that used in Hau et al. (2008), and assumes a simplified, where an action is a choice between combined lotteries. Functionally, the heuristic maintains a count of the number of times that each action was followed by a payoff, and chooses the action which most commonly has the best payoff, i.e. one reason decision making. This approach requires minimal computation, and does not assume that u_i is static, or known.

Women resolve this by approximating the utility function, as a function $f(s_w, \sigma)$ on their choice of signal and an unknown distribution, which maps to u_w - i.e. s_w is a choice between simple lotteries. The algorithm maintains a count, n , of the number of occurrences of each outcome given the choice from s_w .

Midwives solve a slightly different problem with more information, where s_w is known, and s_m is the lottery choice - $f(s_w, s_m, \sigma)$. This is resolved by maintaining a separate count for each signal (i.e. n_{s_w, s_m}), and otherwise following the same algorithm.

Algorithm 1 Lexicographic heuristic

```

n=1, action=none
while action is none do
    Calculate the nth most common outcome following each action.
    Sort actions by the value of the nth most common outcome.
    if clear winner then
        action = best
    end if
    n = n + 1
end while
return action

```

3.2.2 Bayesian Payoff

The Bayesian payoff agent uses the same subset of information as the lexicographic method, but updates beliefs on the link between actions and payoffs using Bayes rule, and attempts to choose the action which minimises risk.

Given the discrete nature of actions and payoffs, coupled with a desire for tractability of the simulation, the Dirichlet distribution is employed to represent these beliefs. The probability density function takes the form -

$$D(\Theta|\alpha) = \frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} \prod_{i=1}^k \Theta_i^{\alpha_i-1}$$

Where $\alpha = \{\alpha_1 \dots \alpha_k\}$, k is the number of signal-payoff pairs, $\Theta = \{x_1, \dots, x_{k-1}\}$ all more than zero and summing to less than 1, and α_i is the psuedo-count of prior observations for a pair i .

The distribution is particularly convenient, in that to infer the probability of a signal implying a payoff becomes simply -

$$P(x = j|D, \alpha) = \frac{\alpha_j + n_j}{\sum_j (\alpha_j + n_j)} \quad (1)$$

Where n_j is simply the count of occurrences of pair j , so that the belief that a signal j the number of times that type has been observed (including the pseudo-count), over the total number of observations thus far. This makes computation of beliefs fast and simple, since all that must be maintained is a count of observations with no particular concern as to their order. As before, midwives follow a similar pattern but per signal.

Agents then choose s_i to minimise R_i , which is simply -

$$R_w(s_w) = \sum_{x \in X} -xp(x|s_w) \quad (2)$$

$$R_m(s_w, s_m) = \sum_{x \in X} -xp(x|s_w \wedge s_m) \quad (3)$$

Where X is the set of payoffs the agent has observed to follow s .

3.2.3 Bayesian Risk Minimisation

The second Bayesian agent augments the reasoning of the simple payoff model, making the stronger assumption that the utility function is static, and known. Women maintain two sets of beliefs, corresponding respectively to p_m , and the probability of referral given signal choice. This leads to the risk function -

$$R_w(s_w, \theta_w) = \sum_{i \in A_m} \sum_{j \in \Theta} -u_w(s_w, i, \theta_w, j)p(j)p(i|s_w) \quad (4)$$

So that the risk of a signal is the sum of the products of all payoffs with the probabilities of their entailed midwife types and responses.

Midwives reasoning centers on determining the meaning of signals, since given the knowledge of what some signal i conveys about the true type of the sender, the payoff for an action is known. As such, their inference process is the same as for the simple Bayesian agent but over signal-type pairs, and they attempt to minimise -

$$R_m(s_w, s_m) = \sum_{i \in \Theta} -u_w(s_w, s_m, i, \theta_m)p(i|s_w) \quad (5)$$

3.2.4 Descriptive Decision Theory

The most complex decision rule used is CPT, which attempts to reproduce a number of systematic deviations from rationality observed in humans. While CPT has primarily been applied in the context of decisions from description, it has been modified to deal with decisions from experience by incorporating a first stage where probabilities are estimates from observations as in Fox and Tversky (1998). In this instance the Bayesian inference process fills the first stage role.

Rather than the psychologically more interesting PT, the CPT decision rule is used in this instance, because of the requirement for women to evaluate more than two ‘prospects’.¹ CPT uses transformed probabilities, underweighting small probabilities, and overweighting large ones. This is intended to reflect the observed behaviour of humans, where sufficiently high likelihoods are treated as certain, and contrastingly low probabilities as impossible. The correct weighting function is subject to some debate, but here we have used that of Tversky and Kahneman (1992), which treats probabilities differently for gains and losses -

$$\begin{aligned} w^+(p) &= \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \\ w^-(p) &= \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}} \end{aligned}$$

Where p is the unweighted probability, and γ and δ are the weights for gain and loss probabilities respectively. Along similar lines, the values of losses and gains are transformed to reflect a tendency to regard a loss as more significant than a gain -

¹Where a prospect is a payoff-probability pair, with the set of prospects for an action defining the possible outcomes for it.

$$v(x) = \begin{cases} f(x) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ g(x) & \text{if } x < 0 \end{cases}$$

Where,

$$f(x) = \begin{cases} x^\alpha & \text{if } \alpha > 0 \\ \ln(x) & \text{if } \alpha = 0 \\ 1 - (1 + x)^\alpha & \text{if } \alpha < 0 \end{cases}$$

$$g(x) = \begin{cases} -(-x)^\beta & \text{if } \beta > 0 \\ -\ln(-x) & \text{if } \beta = 0 \\ (1 - x)^\beta - 1 & \text{if } \beta < 0 \end{cases}$$

And α , and β are respectively the power of a gain, and a loss, and $x = u_i$. The CPT value of outcome x is $v(x)w^+(x)$ if $x \geq 0$, and $v(x)w^-(x)$ otherwise. For an action the CPT value is the sum of the value of the prospects of that action, as in the Bayesian risk model, and the agent chooses the option which maximises this quantity.

3.3 Information Sharing

It would seem unreasonable to suppose that neither party recounts their experiences to their peers, and to explore the impact of this we also modify the game to introduce a simple form of information sharing within agent groups. This takes the form of having each agent share their memories with their colleagues with some probability q . Individuals then incorporate shared information into their beliefs using weighted updates, such that a shared observation of a low type signal contributes to their beliefs by w , and $0 \leq w \leq 1$ (i.e. $n_j = n_j + w$). Women share only when they have finished play, and provide their complete history of games, because they have accurate information about the outcomes. By the same rationale, midwives share only their history with the most recent woman they referred. Sharing occurs simultaneously for all players at the end of each round, and all memories are either shared immediately or discarded.²

Because of their differing problem representations, the simple payoff reasoners and their more complex counterparts incorporate this exogenous information differently. The simple payoff based rule relies on a belief structure relating actions directly to rewards. Because payoffs differ by the agent's private type, the information shared may not correspond to the experience of the listening agent in the same scenario. As a result, payoff reasoners have a belief bias towards the most common player type, and can believe in outcomes that are, for them, impossible.

By contrast, representing the problem in terms of the probabilities of the individual lotteries yields a structure that abstracts the new information from payoffs, and allows the agent discount implausible outcomes. This stronger assumption as to the static and known qualities of payoffs does however reduce the flexibility of the decision rule.

4 Method

This section provides details of experiments conducted to examine the ability of the model to reproduce qualitative trends reported in the midwifery literature by Alvik et al. (2006), and Phillips et al. (2007); as well as a global sensitivity analysis and construction of statistical emulators to explore, and contrast the response surfaces of the four decision rules.

²Memories of games remain, but it is assumed that only current news is relevant.

4.1 Qualitative Trends

Throughout this paper, parameters for the CPT model were as used in Tversky and Kahneman (1992) (table 2). While there has been significant work on determining appropriate parameterisation for the model (e.g. Neilson and Stowe (2002); Glöckner and Pachur (2012); Nilsson et al. (2011), and particularly Byrnes et al. (1999); Booij et al. (2009) addressing risk aversion and gender), a full exploration of the impact of these parameters, or heterogeneous values within populations is beyond the scope of this work. For simplicity, it was assumed that all three drinking types are equally prevalent within the population, although results derived from the Avon Longitudinal Study of Parents and Children (ALSPAC) suggest that the reality is far more positive (Humphriss et al., 2013). The scenario was biased towards disclosure as the better option by presuming a distribution of midwives strongly skewed towards non-judgemental types, with beliefs initially favouring honesty. Payoffs were as in table 1, which ensure that it is always strictly preferable to refer drinkers, and together with the initial belief that signals will be honest, not refer those claiming otherwise.

Two key measures were used: the fraction of the subpopulation who had ever signalled honestly, and the proportion referred. Both measures were taken after every round of play, and were taken relative to the agent's position in their sequence of appointments giving the probability of signalling honestly, or being referred having had a given number of appointments.

Name	Description	Value
n_w	Number of women	1000
n_m	Number of midwives	100
r_m	Number of appointments per midwife	1000
r_w	Maximum number of appointments per woman	12
Runs	Simulation runs	1000
$p_w(h)$	Proportion of heavy drinkers	1/3
$p_w(m)$	Proportion of moderate drinkers	1/3
$p_w(l)$	Proportion of light drinkers	1/3
$p_m(h)$	Proportion of harsh midwives	5/100
$p_m(m)$	Proportion of moderate midwives	10/100
$p_m(l)$	Proportion of non-judgemental midwives	85/100
q_w	Probability of women sharing	0.
w_w	Weight of shared information for women	0.
q_m	Probability of midwives sharing	0.
w_m	Weight of shared information for midwives	0.
$s_i[a_i] : s_i[a_{-i}]$	Pseudo-count favouring honesty	10:1

Table 2: Model parameters.

4.2 Information Sharing

In addition to assessing the adequacy of the rules in capturing qualitative trends, we also examined the impact of simple information sharing within the population of women (section 3.3) on the robustness of these trends. The original experiment was repeated at $q_w = w_w = 0.25, 0.5, 0.75, 1$, with 100 runs under each condition.

4.3 Global Sensitivity Analysis

In general, we have followed the example of Bijak et al. (2013) for global sensitivity analysis of stochastic agent based models, although see Thiele et al. (2014) for a review of alternative techniques. For this purpose the Gaussian Emulation Machine for Sensitivity Analysis (GEM-SA) software (Kennedy, 2004) was used, which implements the Bayesian Analysis of Computer Code Outputs (BACCO) method developed by Oakley

Name	Description	Min	Max
$p_w(m)$	Proportion of moderate drinkers	0	1
$p_w(l)$	Proportion of light drinkers	0	1
$p_m(m)$	Proportion of moderate midwives	0	1
$p_m(l)$	Proportion of non-judgemental midwives	0	1
q_w	Probability of women sharing	0	1
w_w	Weight of shared information for women	0	1
q_m	Probability of midwives sharing	0	1
w_m	Weight of shared information for midwives	0	1
x_h	Health payoff for healthy delivery	1	100
x_r	Cost for referral	$-(x_h - 1)$	
$s_i[a_i] : s_i[a_{-i}]$	Psuedo-count favouring honesty	1:1	100:1

Table 3: Parameter ranges.

and O’Hagan (Oakley and O’Hagan, 2002, 2004; Oakley et al., 2006). This is a form of variance based sensitivity analysis, which assumes that the model output is an unknown, smooth function of the inputs. The unknown function can then be approximated as a Gaussian Process, which is fitted to the training data using Bayes’ Theorem. The emulator is then able to provide an indication about the extent to which uncertainty in a parameter propagates to uncertainty in the output, and how sharply the output responds to change in each input.

Parameters for training were generated in R (R Core Team, 2014) using Latin Hypercube Sampling (Carnell, 2012) over the space of inputs given in table 3, giving 10 free parameters which were treated as uniformly distributed in the range given. Initially a unit hypercube was generated, then the margins transformed appropriately to cover those regions where the inputs are not bounded between 0 and 1, and for proportions of agent types which necessarily sum to one across the three parameters. Given the limitation of 400 design points for the GEM-SA software, we produced exactly that many parameter combinations and collected results for 100 runs of each, with emulator quality assessed by leave-one-out cross validation. A fixed set of 100 random seeds was used, such that each parameter set was run once with each seed, for every decision rule.

To capture the response characteristics for the model, we measured four outcome variables - (1) the interquartile range of the average signal sent by each type of agent in a run, (2) the average signal of moderate drinking agents in a run, and (3, 4) the interquartile range of the average signal, and interquartile range between simulation runs. Together these four metrics give an indication of how far women are separable by their signalling behaviour (1), the behaviour of the at risk drinking groups³ (2), and finally the variability in the system in response to changes to the parameters (3 & 4).

Measurements were taken at the end of 1000 rounds of play, and emulators were built against the median, and interquartile range of each measure to assess both the overall trend, and the extent to which the parameters contribute to variance between runs.

Sixteen emulators were built, covering each of the four outputs on all four decision models. These emulators were used to conduct a probabilistic sensitivity analysis using GEM-SA to assess the impact of parameters individually, and in combination.

In addition to the sensitivity analysis, we also employed the resulting emulators to rapidly⁴ explore the parameter space. While emulated results are subject to inaccuracy, they do provide an indication of the interest, and plausibility, of regions of the parameter space. Results for the outcomes of the interactions of $s_i[a_i] : s_i[a_{-i}]$ with x_h , and q_w with w_w are given in section 5.3.

³Under most conditions, the behaviour of heavy drinkers tracks closely with their moderate counterparts.

⁴Once constructed, the emulator is sufficiently quick that results requiring several days of runtime on an HPC can be generated in minutes on a desktop computer.

5 Results

5.1 Qualitative Trends

As shown in figure 3a, all four decision rules were able to reproduce both qualitative trends towards more disclosure as women experience more appointments (Phillips et al., 2007), and a greater tendency towards underreporting of consumption by heavier drinkers (Alvik et al., 2006). Trends for all four rules are broadly similar, exhibiting a gradual increase across appointments which subsequently levels off. This levelling can in part be explained by the referral results (figure 3b), which show that the majority of drinkers are referred, even with substantial concealment. Referrals continue to occur, in the absence of honest signals, because drinkers are able to achieve a referral by pooling with higher or lower types, dependent on how their initial beliefs are biased. Despite this the results suggest that a minority of risky drinkers will evade detection altogether, with no notable distinction between heavy and moderate types. Under these parameters, light drinkers always signal honestly and are never referred since there is no perceived advantage in doing so, and the evidence of deceptive signalling is insufficient to outweigh the biased priors of the midwives.

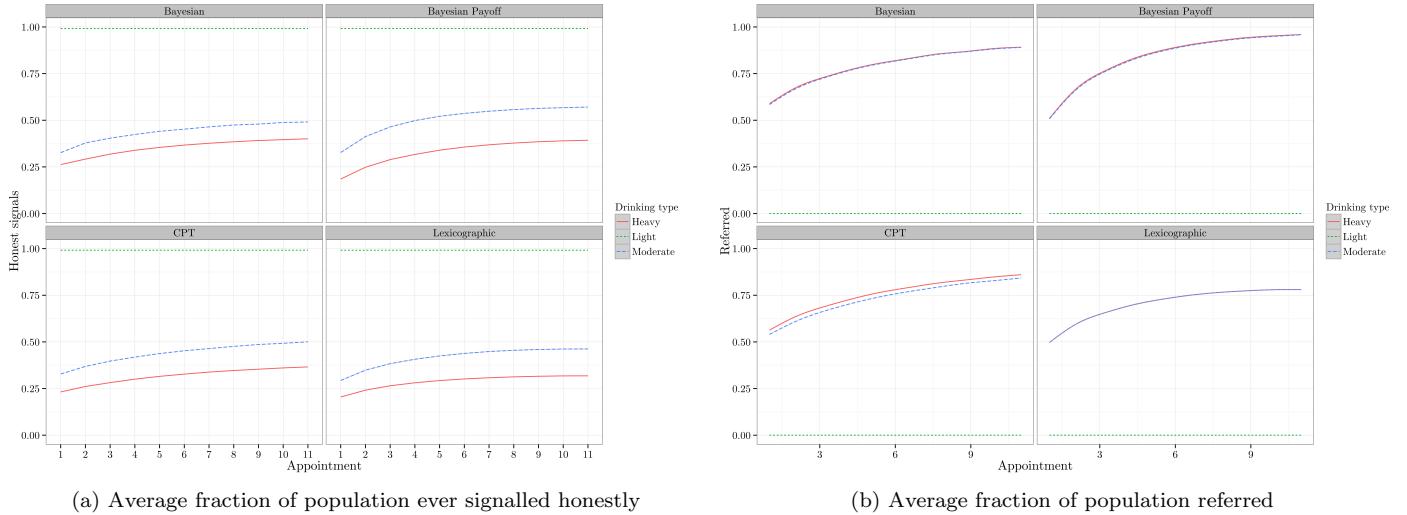


Figure 3: Qualitative trends after 1000 rounds, mean with 95% confidence limit over 1000 runs.

5.2 Information Sharing

Introducing information sharing amongst women leads the behaviour of the decision rules to diverge markedly. Figure 4a shows the proportion of women who have signalled honestly at least once by their final appointment, under four sharing conditions. Aside from the lexicographic decision rule, the general tendency is towards less honest signalling by heavy drinkers, but figure 4b shows a slight increase in referrals for the Bayesian, and CPT rules. For these decision models, this is because information sharing exacerbates the existing tendency to pool with moderate drinkers, who behave more honestly as heavy drinkers become less so.

Particularly notable, is the decline in honest signalling by light drinkers visible in both heuristic type rules at the .25 level, and the associated increase in false positives. This arises because of the lack of homophily in information sharing, as light drinkers become informed about negative outcomes associated with concealment, despite having nothing to conceal. The relatively high referral rates of drinkers heighten the effect further, because shared information becomes dominated by their experiences.

The relationship is not, however, entirely straightforward, in that increasing information sharing leads to greater variance between runs. A linear model was used to predict the between runs interquartile range of the average signal sent by moderate drinkers, with decision rule, and level of information sharing, together

with the interaction between the two as predictors. The regression results were significant ($F_{7,12} = 25, p < 2.9 \times 10^{-6}$) with $R^2 = 94\%$, and intercept 0.07. The only significant coefficients were for the interaction terms, which were 0.44 ($p < 0.05$) for the Bayesian payoff rule, and 0.69 ($p < 0.005$) for the lexicographic. This suggests that information sharing, for the heuristic style decision rules introduces considerable uncertainty to the model, which is explored further in the sensitivity analysis.

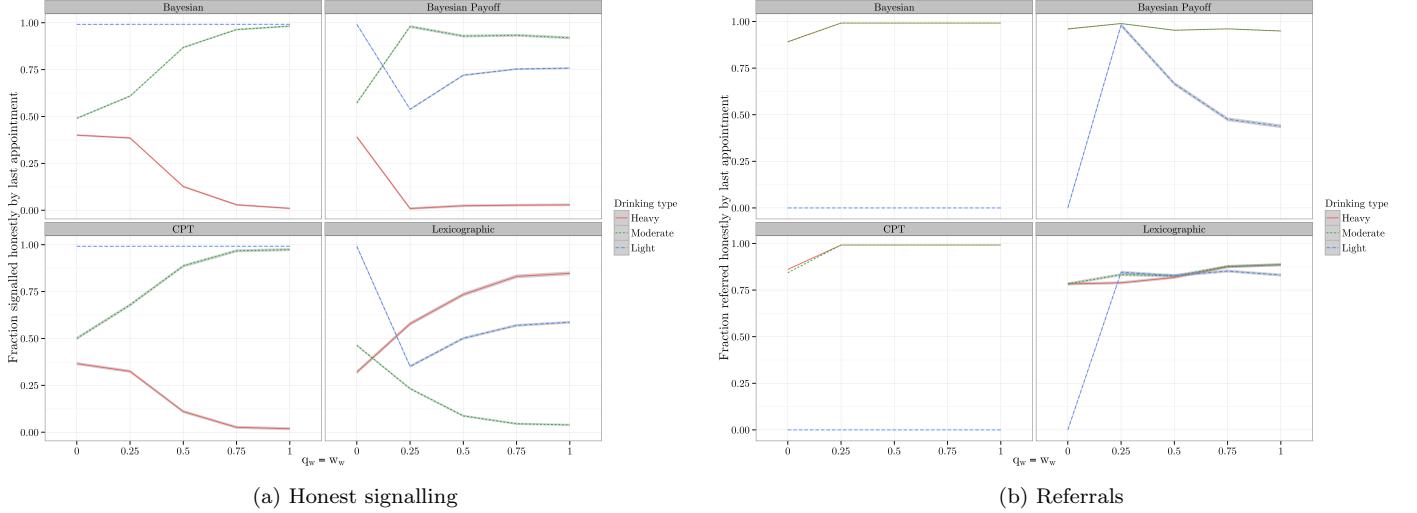


Figure 4: Impact of information sharing on qualitative trends after 1000 rounds, mean with 95% confidence interval over 100 runs.

5.3 Sensitivity Analysis

In this section we present a brief overview of the sensitivity analysis, followed by selected results highlighting the global effect of changes to perceived payoffs and degree of bias towards honesty, as well as information sharing within women. The full results for the sensitivity analysis covering all sixteen emulators are available in appendix A.

For the median signal choice of moderate drinkers, the emulator results suggest that the proportion of light drinkers has a significant effect for all decision rules, accounting for 10%, 38%, 24%, and 5% of the variance in output for the Lexicographic, Bayesian Payoff, Bayesian, and CPT rules respectively. For the Lexicographic rule, the overwhelming majority of variance in signalling behaviour is attributable to the proportions of midwives (44% $p_m(m)$, 7% $p_l(m)$, and a further 15% for their interaction). The proportions of midwives are also key drivers in group separation, and the between run IQR of both measures for this rule.

Perhaps surprisingly, variance attributable to information sharing between midwives is relatively low, with neither the weight nor probability accounting for more than 5% of variance in any measure. While there are small contributions to variance in interaction with other parameters (e.g. 4% to between groups IQR, for the interaction with the proportion of light drinkers under the Bayesian rule), this may suggest that the implementation is lacking, which we touch on in section 6.

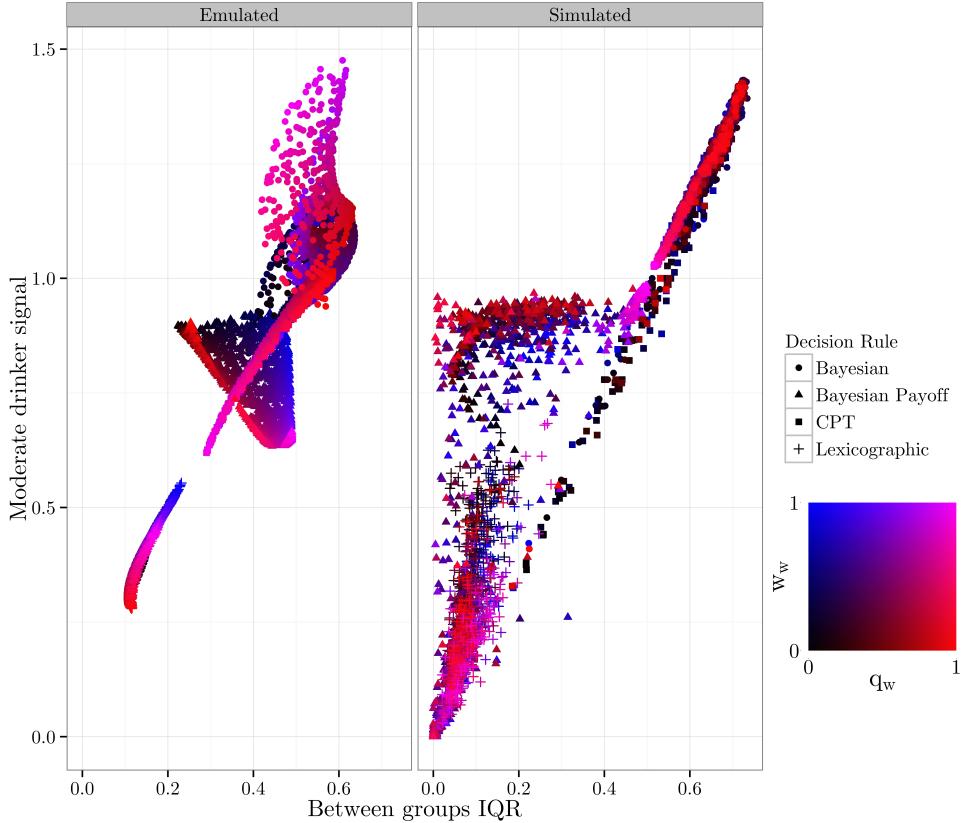


Figure 5: Median moderate drinker signal vs median between drinking type IQR for all decision rules.

Figure 5 gives a qualitative picture of both emulator quality, and the divergent response surfaces of the decision rules in response to variations in information sharing parameters. Emulator fit is clearly imperfect, but overall behaviour is qualitatively similar, with both emulated and simulated plots demonstrating separation in outcome space for the decision rules.

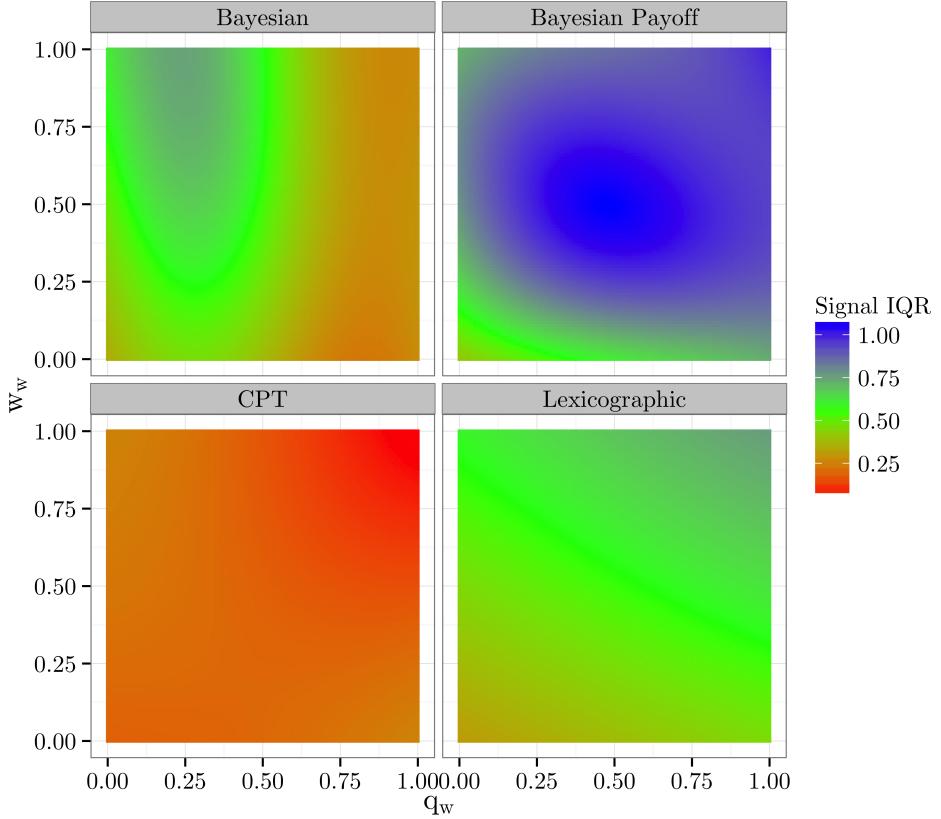


Figure 6: Emulated moderate drinker signal IQR in response to varying q_w , and w_w .

Following from the suggestive results for information sharing introducing uncertainty (section 3.3), figure 6 shows emulated points covering the parameter space in high resolution. These plots reflect the increase in uncertainty of outcome shown for the heuristic type rules, which is especially severe for the Bayesian payoff rule. They also suggest that the Bayesian decision rule is less stable under conditions where the weight of shared information is substantially higher than the probability of sharing. Also of interest is the large region of instability that can be discerned for the Bayesian rule under modest probability of information sharing and higher weights. This suggests that, perhaps unsurprisingly, that what information is shared is significant to outcomes.

For the CPT, and Bayesian decision models, the interaction of bias towards honesty, and distinction between payoffs has a significant, and non-linear effect on instability, and separability of groups. Figure 7 shows the effects, and also highlights the tendency towards poor separability of groups for both the heuristic type decision rules. The response surface of the Bayesian payoff rule is slightly more nuanced than the simple Lexicographic rule. Figure 7 shows better separation close to partial pooling at high payoff distinction, with relatively modest honesty bias, which is reflected by the variance contributions of 11% and 8% respectively. For the more complex rules, the general tendency is towards less pooling for higher values of both, but with pockets where full pooling occurs. The plots also suggest that the sensitivity of the CPT rule is marginally greater, which is supported by the significant contribution to variance of close to 15% for all measures of x_h .

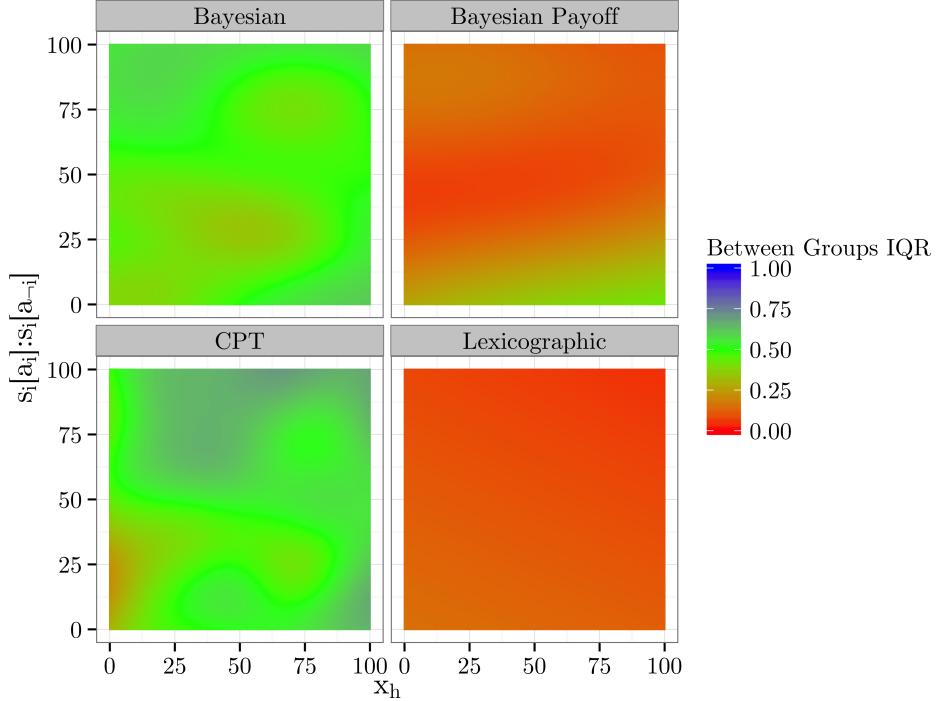


Figure 7: Emulated between groups IQR in response to varying $s_i[a_i] : s_i[a_{-i}]$, and x_h .

6 Discussion and Conclusions

From a pragmatic perspective, the differing response characteristics of the decision rules are substantial and significant. The lack of a mechanism of homophily in the very simple rules leads to a high level of uncertainty in the overall dynamics, since even irrelevant information is taken at face value and to be equally valuable. By contrast, the belief structure of the CPT, and Bayesian models represents a useful abstraction which allows the use of the relevant aspects of the information. Naturally, incorporating homophily, by, for example weighting shared information by the type of the sharer, would represent a relatively trivial modification to the heuristic models. While to some extent this highlights the flexibility of the decision rule approach, it would of course sacrifice the parsimony of the model to a degree. This is an important consideration, given that part of the argument in favour of a decision theoretic approach to agent building lies in the minimal nature of the behavioural rules, which can be seen as occupying a middle ground between parameter and model.

One of the notable features of the results is the similarity in behaviour of the two classes of decision rule. To some extent this reflects poorly on the most complex rule, CPT, which diverges only relatively minorly in behaviour from the Bayesian model. To some extent this might be anticipated, in that the payoffs are clearly unrealistic, which limits the utility of the CPT approach. Additionally, work by Glöckner

and Pachur (2012) has shown that there is considerable variation in individual parameters for the decision model, whereas we have let them remain homogenous here. In the vein, utility functions should arguably vary between individual agents, which could potentially be addressed by replacing the fixed payoffs used here with a distribution. With this said, the significant increase in complexity, which entails both additional parameters and increases to simulation time may auger for a middle ground, particularly where elicitation of payoffs is impractical. This, together with the variability associated with the heuristic type decision rules speaks to a tradeoff between capturing reality, and replicating it.

Continuing the discussion of the issues raised by the representation of payoffs, the temporal aspect is significant, in that there is clearly a timing difference in payoffs, since while the potential social pain of disclosure is immediate, the health outcome comes only later. In light of this, that there is a known impact of time on perceived utility Thaler (1981) suggests that incorporating some form of temporal discounting (e.g. exponential (Samuelson, 1937), or hyperbolic (Ainslie, 1991)), or a decision model which explicitly treats intertemporal choice, such as the CPT like model of Loewenstein and Prelec (1992), is warranted.

As noted in section 4.3, the impact of information sharing between midwives is surprisingly minimal, where it might be expected to play a more significant role in reality. A possible explanation for this lies in the implementation, which may place an excessive constraint on how much information midwives can share. The restriction to sharing only after a referral, together with the disparity in population sizes, and random allocation of appointments leads to midwives rarely having more than a single interaction with woman to pass on to their colleagues. Furthermore, because midwives are only informed of the true type if a referral occurs, they have an inherent myopia, since until they have evidence of deception they will not refer, with said evidence difficult to obtain without a referral.

In reality it might be anticipated that midwives would not withhold judgement so completely, and would pass on concerns about particular women to their colleagues, or that particularly dramatic stories would persist and be passed. This might be addressed by incorporating noisy type information (Feltovich et al., 2002), capturing the unintentional information transmitted during appointments, together with a relaxation of the assumptions about when information may be shared and more sophisticated model of information transmission in general. This also highlights an advantage of the BACCO approach, in that it aids in diagnosing issues with model design by giving insight into parameters which are contributing inappropriately to variance in output.

The conclusions that can be drawn about the behaviours of real life women, and their midwives, are necessarily limited by the paucity of data available to validate the model. While qualitative trends offer a some indication, they are clearly very limited in scope, and do not permit strong claims about the drivers of disclosure, and auger for a sceptical eye as to any recommendations targetted at improving outcomes. As such, further work will focus on applying the model to domains where validation data is more available, which will support a more comprehensive evaluation of the model discrepancy. With this said, the trends reported by Alvik et al. (2006), and Phillips et al. (2007) are borne out by the model, and predictions from the two more complex rules suggest that encouraging information sharing between women may encourage disclosure, but at the expense of reducing accuracy. By contrast, if one takes the view that a Lexicographic model is a better approximation of real behaviour, then outcomes can best be influenced by controlling how far midwives punish their women socially. We would however suggest that there are better reasons than the outputs of a simulation for doing so.

More broadly, the results demonstrate the logistical feasibility, and utility as a ‘tool for thinking’, of an agent model grounded in decision theory. The results also make clear that deciding the operationalisation of the decision making is of key significance.

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A Sensitivity Analysis

A.1 Median Moderate Drinker Signalling

Parameter	Description	Lexicographic	Bayesian Payoff	Bayesian	CPT
$p_w(m)$	Proportion of moderate drinkers	0.367	1.145	0.801	0.614
$p_w(l)$	Proportion of light drinkers	10.080	37.750	23.968	5.137
$p_m(m)$	Proportion of moderate midwives	6.715	13.017	0.894	1.485
$p_m(l)$	Proportion of non-judgemental midwives	43.942	1.655	1.602	2.618
q_w	Probability of women sharing	0.198	5.527	4.460	1.159
w_w	Weight of shared information for women	0.355	13.025	2.716	0.888
q_m	Probability of midwives sharing	0.145	0.667	0.368	0.157
w_m	Weight of shared information for midwives	0.118	0.376	0.176	0.200
x_h	Health payoff for healthy delivery	0.457	9.618	1.912	15.355
$s_i[a_i] : s_i[a_{\neg i}]$	Psuedo-count favouring honesty	0.140	7.537	10.427	7.795
Total	All parameters and two way interactions	86.777	96.527	85.529	74.123

Table 4: Median moderate drinker signalling parameter sensitivity.

Rule	σ^2	Nugget σ^2	Mean output	Total output variance	Code uncertainty	RMSSE
Lexicographic	0.834	0.131	0.817	0.012	0.252	1.746
Bayesian Payoff	1.667	0.475	0.662	0.003	0.181	3.12
Bayesian	3.352	0.534	1.160	0.001	0.068	2.423
CPT	1.503	0.331	1.241	0.002	0.101	1.842

Table 5: Median moderate drinker signalling emulator statistics.

Parameter	Variance
$x_h * s_i[a_i] : s_i[a_{\neg i}]$	20.814
$p_w(l) * s_i[a_i] : s_i[a_{\neg i}]$	5.698
$p_w(l) * x_h$	2.895
$s_i[a_i] : s_i[a_{\neg i}] * w_w$	2.799
$s_i[a_i] : s_i[a_{\neg i}] * q_m$	1.686

Table 6: Top 5 Interactions terms for CPT decision rule.

Parameter	Variance
$p_w(l) * s_i[a_i] : s_i[a_{\neg i}]$	17.270
$p_w(l) * q_m$	6.054
$q_m * w_w$	3.814
$s_i[a_i] : s_i[a_{\neg i}] * q_m$	3.538
$s_i[a_i] : s_i[a_{\neg i}] * w_w$	3.084

Table 7: Top 5 Interactions terms for Bayesian decision rule.

Parameter	Variance
$p_m(l)^*p_m(m)$	15.331
$p_m(m)^*p_w(l)$	3.682
$p_m(l)^*p_w(l)$	3.581
$p_m(m)^*q_m$	0.349
$p_m(l)^*q_m$	0.279

Table 8: Top 5 Interactions terms for Lexicographic decision rule.

Parameter	Variance
$p_w(l)^*w_w$	4.045
$x_h^*s_i[a_i] : s_i[a_{\neg i}]$	1.856
$p_w(l)^*q_m$	1.231
$s_i[a_i] : s_i[a_{\neg i}]^*q_m$	0.997
$q_m^*w_w$	0.929

Table 9: Top 5 Interactions terms for Bayesian Payoff decision rule.

A.2 Median Between Groups IQR

Parameter	Description	Lexicographic	Bayesian Payoff	Bayesian	CPT
$p_w(m)$	Proportion of moderate drinkers	0.327	0.688	0.457	0.586
$p_w(l)$	Proportion of light drinkers	11.223	20.123	11.046	4.081
$p_m(m)$	Proportion of moderate midwives	36.630	1.160	0.364	1.945
$p_m(l)$	Proportion of non-judgemental midwives	6.228	4.487	0.0964	2.627
q_w	Probability of women sharing	0.498	0.235	2.537	1.812
w_w	Weight of shared information for women	1.018	2.307	1.889	0.740
q_m	Probability of midwives sharing	0.158	0.343	0.387	0.156
w_m	Weight of shared information for midwives	0.076	0.973	0.125	0.213
x_h	Health payoff for healthy delivery	0.317	10.960	3.305	16.493
$s_i[a_i] : s_i[a_{\neg i}]$	Pseudo-count favouring honesty	1.107	8.411	2.890	6.729
Total	All parameters and two way interactions	81.702	83.693	47.449	71.032

Table 10: Median between groups IQR parameter sensitivity.

Rule	σ^2	Nugget σ^2	Mean output	Total output variance	Code uncertainty	RMSSE
Lexicographic	0.930	0.240	0.249	0.002	0.040	1.832
Bayesian Payoff	1.242	0.417	0.232	0.001	0.0034	2.308
Bayesian	1.254	0.131	0.644	0.000	0.019	1.167
CPT	1.190	0.313	0.659	0.000	0.024	1.701

Table 11: Median between groups IQR emulator statistics.

Parameter	Variance
$x_h * s_i[a_i] : s_i[a_{\neg i}]$	19.551
$p_w(l) * s_i[a_i] : s_i[a_{\neg i}]$	3.838
$s_i[a_i] : s_i[a_{\neg i}] * w_w$	2.450
$p_w(l) * x_h$	2.337
$s_i[a_i] : s_i[a_{\neg i}] * q_m$	2.046

Table 12: Top 5 Interactions terms for CPT decision rule.

Parameter	Variance
$s_i[a_i] : s_i[a_{\neg i}] * q_m$	4.284
$p_w(l) * q_m$	3.866
$p_w(l) * s_i[a_i] : s_i[a_{\neg i}]$	2.943
$x_h * s_i[a_i] : s_i[a_{\neg i}]$	2.680
$q_m * w_w$	2.282

Table 13: Top 5 Interactions terms for Bayesian decision rule.

Parameter	Variance
$p_m(l) * p_m(m)$	12.046
$p_m(m) * p_w(l)$	5.054
$p_m(l) * p_w(l)$	3.005
$p_m(l) * w_w$	0.819
$p_m(m) * w_w$	0.757

Table 14: Top 5 Interactions terms for Lexicographic decision rule.

Parameter	Variance
$p_w(l) * s_i[a_i] : s_i[a_{\neg i}]$	12.883
$p_w(l) * w_w$	5.667
$p_w(l) * x_h$	2.447
$x_h * s_i[a_i] : s_i[a_{\neg i}]$	2.360
$p_m(m) * p_w(l)$	1.919

Table 15: Top 5 Interactions terms for Bayesian Payoff decision rule.

A.3 Median Moderate Drinker Signalling IQR

Parameter	Description	Lexicographic	Bayesian Payoff	Bayesian	CPT
$p_w(m)$	Proportion of moderate drinkers	0.428	9.828	3.816	2.068
$p_w(l)$	Proportion of light drinkers	8.369	13.791	4.045	2.400
$p_m(m)$	Proportion of moderate midwives	13.416	0.712	0.676	0.583
$p_m(l)$	Proportion of non-judgemental midwives	21.079	0.648	0.659	0.373
q_w	Probability of women sharing	2.307	3.481	0.891	0.600
w_w	Weight of shared information for women	6.021	6.009	0.562	0.937
q_m	Probability of midwives sharing	0.315	1.829	0.114	0.117
w_m	Weight of shared information for midwives	1.652	1.354	0.260	0.0.139
x_h	Health payoff for healthy delivery	0.253	0.612	4.889	15.146
$s_i[a_i] : s_i[a_{-i}]$	Pseudo-count favouring honesty	0.504	3.096	19.863	25.999
Total	All parameters and two way interactions	84.9968	77.413	57.125	83.322

Table 16: IQR of median moderate drinker signalling parameter sensitivity.

Rule	σ^2	Nugget σ^2	Mean output	Total output variance	Code uncertainty	RMSSE
Lexicographic	1.425	0.436	0.549	0.008	0.114	2.719
Bayesian Payoff	1.223	0.496	0.747	0.012	0.207	2.034
Bayesian	1.065	0.000	0.230	0.002	0.088	1.015
CPT	0.874	0.213	0.233	0.001	0.066	1.806

Table 17: IQR of median between groups IQR emulator statistics.

Parameter	Variance
$x_h * s_i[a_i] : s_i[a_{-i}]$	17.377
$p_w(m) * s_i[a_i] : s_i[a_{-i}]$	3.356
$s_i[a_i] : s_i[a_{-i}] * w_w$	3.036
$s_i[a_i] : s_i[a_{-i}] * q_m$	2.067
$p_w(l) * s_i[a_i] : s_i[a_{-i}]$	1.721

Table 18: Top 5 Interactions terms for CPT decision rule.

Parameter	Variance
$p_w(m) * s_i[a_i] : s_i[a_{-i}]$	4.188
$x_h * s_i[a_i] : s_i[a_{-i}]$	3.120
$s_i[a_i] : s_i[a_{-i}] * q_m$	2.423
$p_w(l) * s_i[a_i] : s_i[a_{-i}]$	2.279
$p_w(l) * q_m$	1.489

Table 19: Top 5 Interactions terms for Bayesian decision rule.

Parameter	Variance
$p_m(m)*p_w(l)$	12.068
$p_m(l)*p_w(l)$	6.794
$p_m(l)*p_m(m)$	5.567
$p_m(m)*q_m$	0.886
$p_m(l)*q_m$	0.692

Table 20: Top 5 Interactions terms for Lexicographic decision rule.

Parameter	Variance
$p_w(l)*p_w(m)$	8.357
$p_w(l)*w_w$	7.431
$p_w(l)*s_i[a_i] : s_i[a_{-i}]$	4.882
$p_w(l)*q_m$	2.346
$p_w(l)*w_m$	2.025

Table 21: Top 5 Interactions terms for Bayesian Payoff decision rule.

A.4 Between Groups IQR IQR

Parameter	Description	Lexicographic	Bayesian Payoff	Bayesian	CPT
$p_w(m)$	Proportion of moderate drinkers	0.691	5.926	1.053	1.265
$p_w(l)$	Proportion of light drinkers	3.664	17.047	4.877	3.656
$p_m(m)$	Proportion of moderate midwives	41.369	1.124	0.814	0.591
$p_m(l)$	Proportion of non-judgemental midwives	7.109	0.739	0.496	0.378
q_w	Probability of women sharing	1.963	2.038	0.733	0.589
w_w	Weight of shared information for women	7.932	11.193	2.289	1.960
q_m	Probability of midwives sharing	0.413	1.972	0.267	0.069
w_m	Weight of shared information for midwives	0.120	2.902	0.150	0.123
x_h	Health payoff for healthy delivery	0.228	3.190	6.308	14.777
$s_i[a_i] : s_i[a_{-i}]$	Pseudo-count favouring honesty	0.673	10.411	22.901	26.340
Total	All parameters and two way interactions	85.740	88.611	68.640	84.210

Table 22: IQR of median between groups IQR parameter sensitivity.

Rule	σ^2	Nugget σ^2	Mean output	Total output variance	Code uncertainty	RMSSE
Lexicographic	0.826	0.409	0.259	0.002	0.034	2.364
Bayesian Payoff	3.202	0.520	0.328	0.002	0.032	2.452
Bayesian	1.177	0.041	0.133	0.000	0.018	1.152
CPT	0.874	0.118	0.126	0.000	0.017	1.570

Table 23: IQR of median between groups IQR emulator statistics.

Parameter	Variance
$x_h^*s_i[a_i] : s_i[a_{\neg i}]$	18.626
$p_w(l)^*s_i[a_i] : s_i[a_{\neg i}]$	3.312
$s_i[a_i] : s_i[a_{\neg i}]^*w_w$	2.823
$s_i[a_i] : s_i[a_{\neg i}]^*q_m$	2.694
$x_h^*q_m$	1.022

Table 24: Top 5 Interactions terms for CPT decision rule.

Parameter	Variance
$p_w(l)^*s_i[a_i] : s_i[a_{\neg i}]$	7.947
$x_h^*s_i[a_i] : s_i[a_{\neg i}]$	4.048
$s_i[a_i] : s_i[a_{\neg i}]^*q_m$	3.134
$p_w(l)^*q_m$	2.307
$p_m(m)^*s_i[a_i] : s_i[a_{\neg i}]$	2.232

Table 25: Top 5 Interactions terms for Bayesian decision rule.

Parameter	Variance
$p_m(l)^*p_m(m)$	8.659
$p_m(m)^*p_w(l)$	3.726
$p_m(l)^*p_w(l)$	3.237
$p_m(l)^*w_w$	1.564
$p_m(m)^*w_w$	1.558

Table 26: Top 5 Interactions terms for Lexicographic decision rule.

Parameter	Variance
$q_m^*w_w$	3.830
$p_w(l)^*p_w(m)$	3.808
$p_w(l)^*s_i[a_i] : s_i[a_{\neg i}]$	3.401
$p_w(l)^*q_m$	2.385
$x_h^*s_i[a_i] : s_i[a_{\neg i}]$	2.294

Table 27: Top 5 Interactions terms for Bayesian Payoff decision rule.