Isabel Hernandez-Fuerte

HW 2

1.

Isabel Hernandez-Fuerte

1. 4.1) $\frac{dN}{dt} = rN[1-(N/K)^\theta]$

a)

$0 = rN[1-(N/K)^\theta]$

$0 = 1-(N/K)^\theta$

$-1 = -(N/K)^\theta$

$\ln(1) = \ln(N/K)^\theta$

$0 = \theta \frac{\ln(N/K)}{\theta}$

$e^0 = e^{\ln(N/K)}$

$1 = \frac{N}{K}$

$N = K$ The population = the carrying capacity

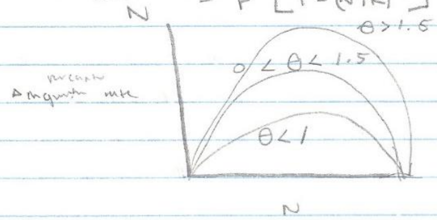
$0 = rN[1-(N/K)^\theta]$ @ $\theta = 0$

$0 = rN[1-0]$

$0 = rN(1)$

$\boxed{0 = N}$

b) $\frac{dN/dt}{N} = r[1-(N/K)^\theta]$



* Should be a parabola
* As the θ increases the value of $(N/K)^\theta$ should increase causing $[1-(N/K)^\theta]$ to decrease. leading to a smaller $\frac{dN}{dt}$

c) The logistic model appears to show how population growth reaches an equilibrium over time. This theta model looks at the stable effects of population on growth rate. The theta model accounts for stability in carrying capacity that may occur in a confined area & space. It is useful for determining the max r of a population. In our homework, we use it for fisheries where the growth rate max would be an important parameter to know.

3. $\frac{dN}{dt} = rN(N-a)[1-(N/k)]$

a) $0 = rN(N-a)[1-(N/k)]$

$0 = [1-(N/k)]$

$-1 = -(N/k)$

$1 = N/k$

$\boxed{1k = N}$

$0 = rN(N-a)[1-(N/k)]$

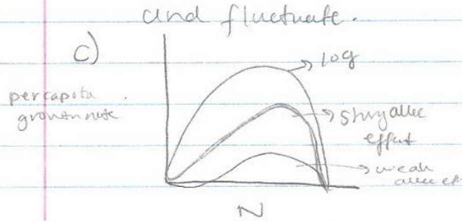
$0 = rN(N-a)$

$0 = N-a$

$\boxed{a = N}$

b) $K = N$ appears to be a stable parameter while the allee effect $a = N$ would change and fluctuate.

c)



According to extra online reading the allee effect essentially skews this graph in comparison to the theta model.

d) The logistic growth curve is a symmetrical parabola. The allee effect curve pulls this graph & skews the highest per capita growth rate.

E.C) $\frac{dN}{dt} = r(1-\frac{N}{k})N$ we solve for $N(t)$?

$\int \frac{dN}{N(1-\frac{N}{k})} = \int r\,dt$

$\frac{1}{N(1-\frac{N}{k})} = \frac{1}{N} - \frac{1}{N-k}$

$\int \frac{dN}{N-N_0} = \ln(N-N_0)$ we need to do $t=0$ to $t=T$

$\int_{t=0}^{t=T} \frac{dN}{N-N_0} = \cancel{\ln(N-N_0)}$

~~$N - k \ln N =$~~

$\int \frac{1}{N(1-\frac{N}{k})} dN = \int r\,dt$

$\int \frac{1}{N} - \frac{1}{N-k}\, dN = rt$

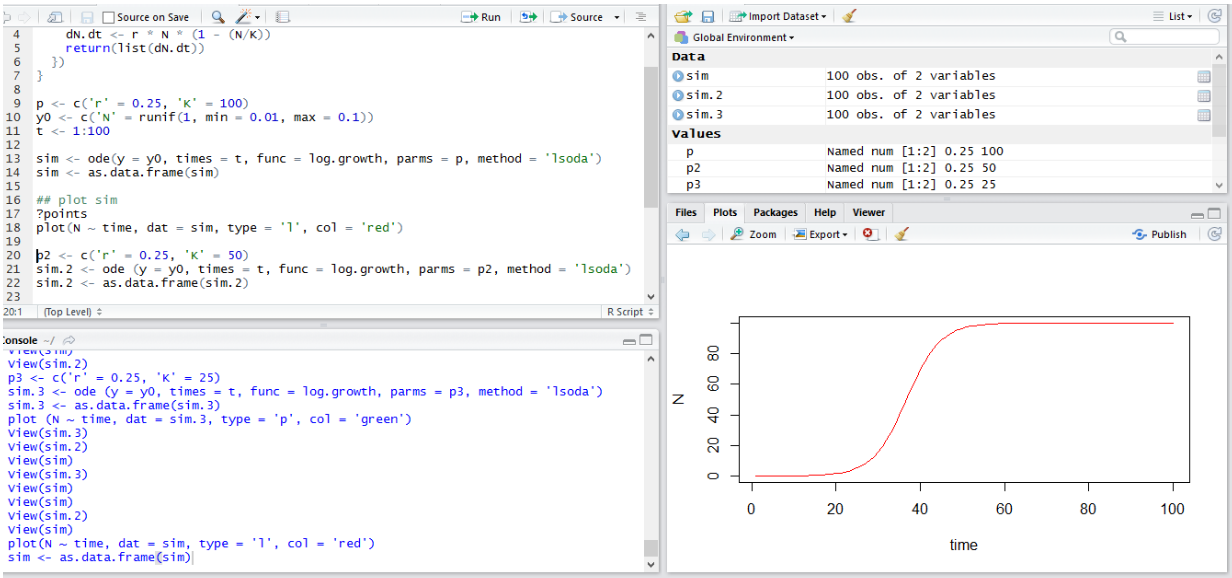
$\ln N - \ln(N-k) = rt$

$\ln \frac{N}{N-k} = rt$

....?

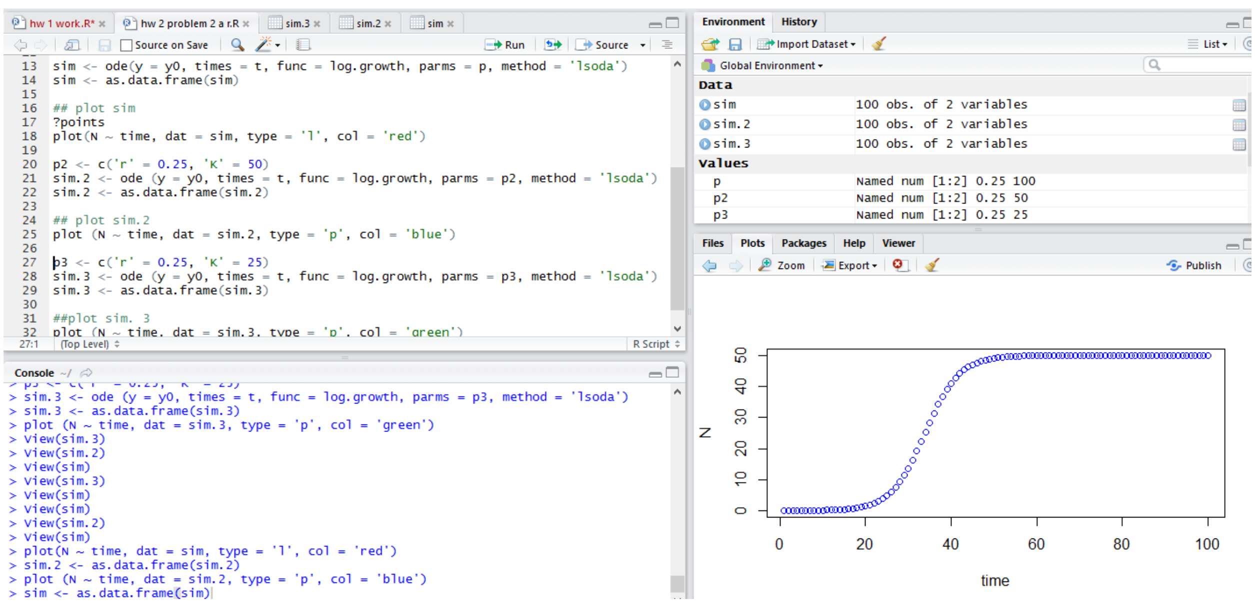
$\boxed{\text{Solution}}$: $N(T) = \dfrac{N(0)e^{rT}}{1 + \frac{N(0)(e^{rt}-1)}{K}}$

2. a)

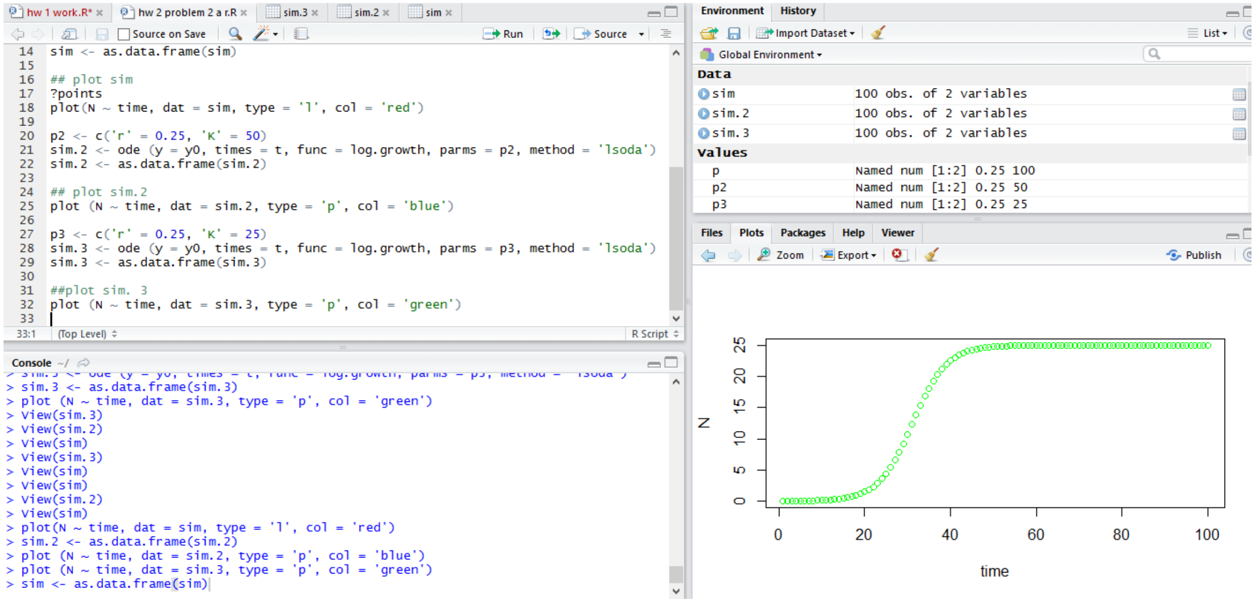


b)

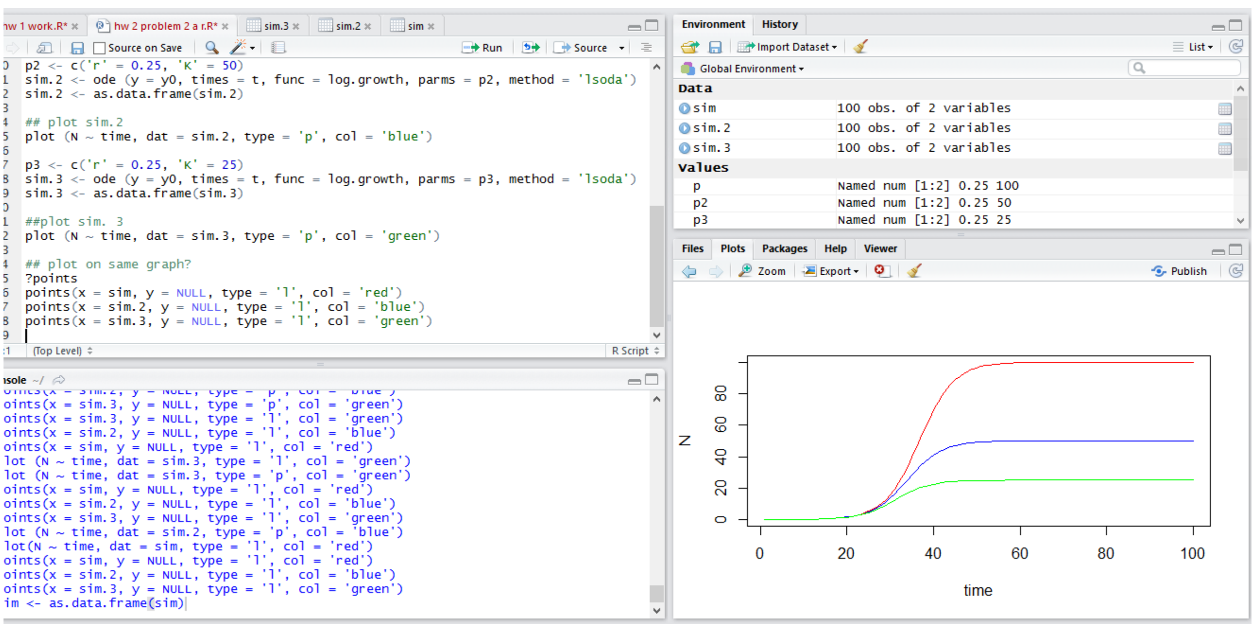
Plot for K= 50



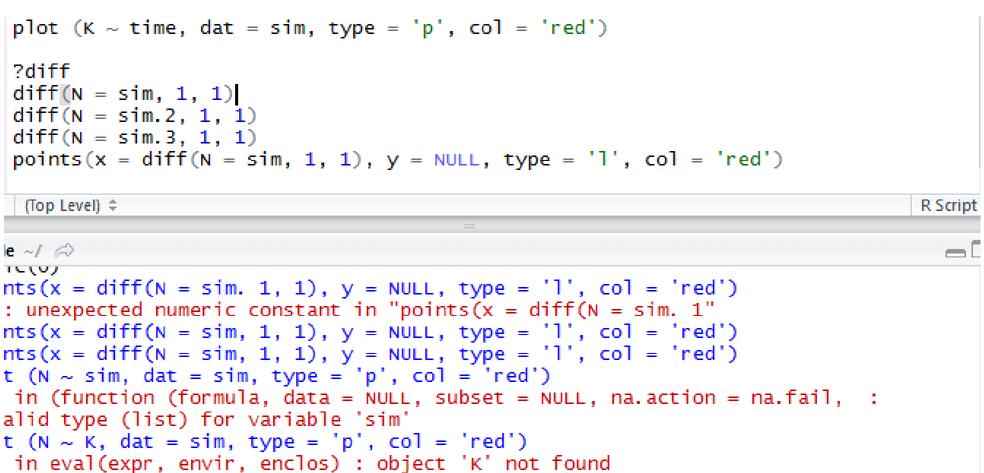
Plot for K = 25



Same Graph



C)

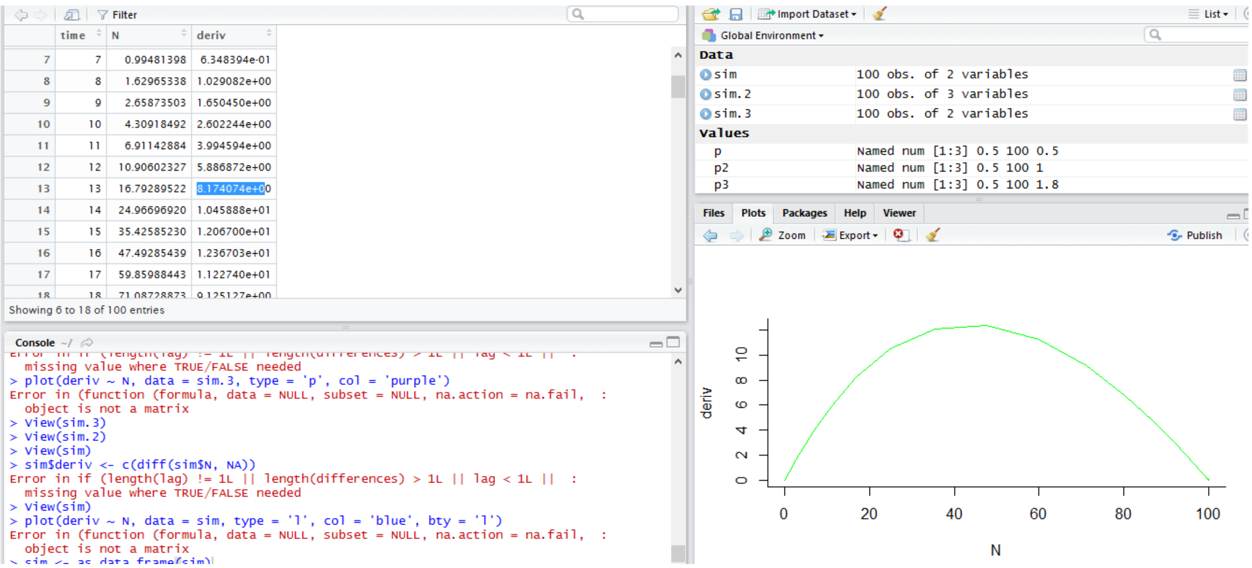


K = 100; r= 0.0694

K = 50; r=0.0281

K = 25; r=0.0155

3. Table showed that the highest derivative for species b was 8.17.



R Code:

```
log.growth <- function(t, y, p) {
  N <- y[1]
  with(as.list(p), {
    dN.dt <- r * N * (1 - (N / K)^theta)
    return(list(dN.dt))
  })
}
p <- c('r' = 0.5, 'K' = 100, 'theta' = 0.5)
y0 <- c('N' = 0.05)
t <- 1:100
sim <- ode(y = y0, times = t, func = log.growth, parms = p, method = 'lsoda')
sim <- as.data.frame(sim)
sim$deriv <- c(diff(sim$N, NA))
plot(deriv ~ N, data = sim, type = 'l', col = 'blue', bty = 'l')
```

```
sim$N[which(sim$deriv == max(sim$deriv, na.rm = TRUE))]


## Species B
p2 <- c('r' = 0.5, 'K' = 100, 'theta' = 1)


sim.2 <- ode(y = y0, times = t, func = log.growth, parms = p2, method = 'lsoda')
sim.2 <- as.data.frame(sim.2)


plot(N ~ time, data = sim.2, type = 'l', col = 'green')


sim.2$deriv <- c(diff(sim.2$N), NA)


plot(deriv ~ N, data = sim.2, type = 'l', col = 'green', bty = 'l')


sim.2$N[which(sim.2$deriv == max(sim.2$deriv, na.rm = TRUE))]


p3 <- c('r' = 0.5, 'k'= 100, 'theta' = 1.8)


sim.3 <- ode(y = y0, times = t, func = log.growth, parms = p3, method = 'lsoda')
sim.3 <- as.data.frame(sim.3)
sim.3$deriv <- c(diff(sim.3$N, NA))
plot(deriv ~ N, data = sim.3, type = 'p', col = 'purple')
```

4. E.C attempt on page 2.