# Differential equations
# Computational practicum

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**Problem statement:**

$y' = 2y^{1/2} + 2y$
$y(0) = 1$
$x \in [0;9]$

**Exact solution of the Initial Value Problem**

$$\int \frac{dy}{2(\sqrt{y}+y)} = \int dx$$

$$\int \frac{1}{2} \frac{dy}{(\sqrt{y}+1)\sqrt{(y)}} = x + c$$

$$u = \sqrt{y}+1$$

$$dy = 2\sqrt{y}\, du$$

$$\frac{1}{2}\, 2\int \frac{1}{u}\, du = x + c$$

$$\ln(u) + c = x + c$$

$$\ln(\sqrt{y}+1) = x + c$$

$$\sqrt{y}+1 = e^{x+c}$$

$$\sqrt{y} = e^{x+c} - 1$$

$$y = (e^{x+c} - 1)^2$$

github: https://github.com/greendate/DE-Numerical-methods

By applying initial values we get:

$$(e^{c-1})^2 = 1$$

$$e^{c-1} = 1$$

$$e^{c} = 2$$

$$c = \log(2)$$

And exact solution is:

$$y = (2e^{x-1})^2$$

***There are no points of discontinuity in given range.***

## Implementation

I have chosen python language. With 2 libraries imported, `matplotlib` for plotting graphs and `tkinter` for GUI.

## Program structure

Besides numerical methods and exact solutions, which are straight forward implemented, we have routines for computing approximating errors (plotting difference between method and numerical methods) and investigating convergence. For simplicity *f(x,y)* and *y(x)* are implemented as functions.

### *Euler method*

```
def euler_method(x0, y0, b, n):
	h = (b - x0) / n
	x = []
	for i in range(n):
		x.append(x0 + i * h)
	y = [y0]
	for i in range(1, n):
		y.append(y[i - 1] + h * f(x[i - 1], y[i - 1]))
	return y
```

***Improved Euler method***

```
def improved_euler_method(x0, y0, b, n):
    h = (b - x0) / n
    x = []
    for i in range(n):
        x.append(x0 + i * h)
    y = [y0]
    for i in range(1, n):
        k1 = f(x[i - 1], y[i - 1])
        k2 = f(x[i - 1] + h, y[i - 1] + h * k1)
        y.append(y[i - 1] + (h / 2) * (k1 + k2))
    return y
```

***Runge-Kutta method***

```
def runge_kutta_method(x0, y0, b, n):
    h = (b - x0) / n
    x = []
    for i in range(n):
        x.append(x0 + i * h)
    y = [y0]
    for i in range(1, n):
        k1 = f(x[i - 1], y[i - 1])
        k2 = f(x[i - 1] + h / 2, y[i - 1] + (h / 2) * k1)
        k3 = f(x[i - 1] + h / 2, y[i - 1] + (h / 2) * k2)
        k4 = f(x[i - 1] + h, y[i - 1] + h * k3)
        y.append(y[i - 1] + (h / 6) * (k1 + 2 * k2 + 2 * k3 + k4))
    return y
```

Each method gets same values for x and produces approximations on that points, *Runge-Kutta method* is most accurate, while typical Euler method gives the least precise results.

***** for $k_{i1}$, $k_{i2}$, $k_{i3}$, $k_{i4}$ were used as variables inside loop instead of using arrays to store them

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**Convergence**

Here we check if maximal approximation error of given method using really large N tends to 0, that is, nearest integer of all values have to be 0. With 100 000 as number of grid steps we will get that only Runge-Kutta method is convergent.

**Plotting**

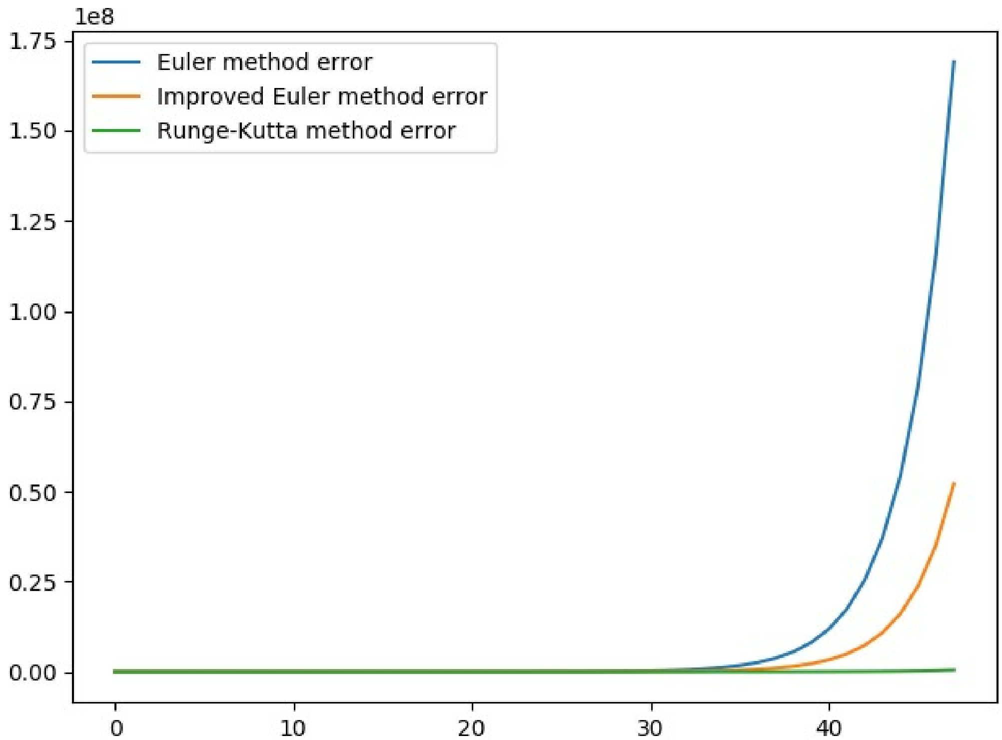


Figure 1: approximation errors

User is allowed to choose the range wherein he wants to observe errors, those values will be used in construction of error plot lines and error arrays.

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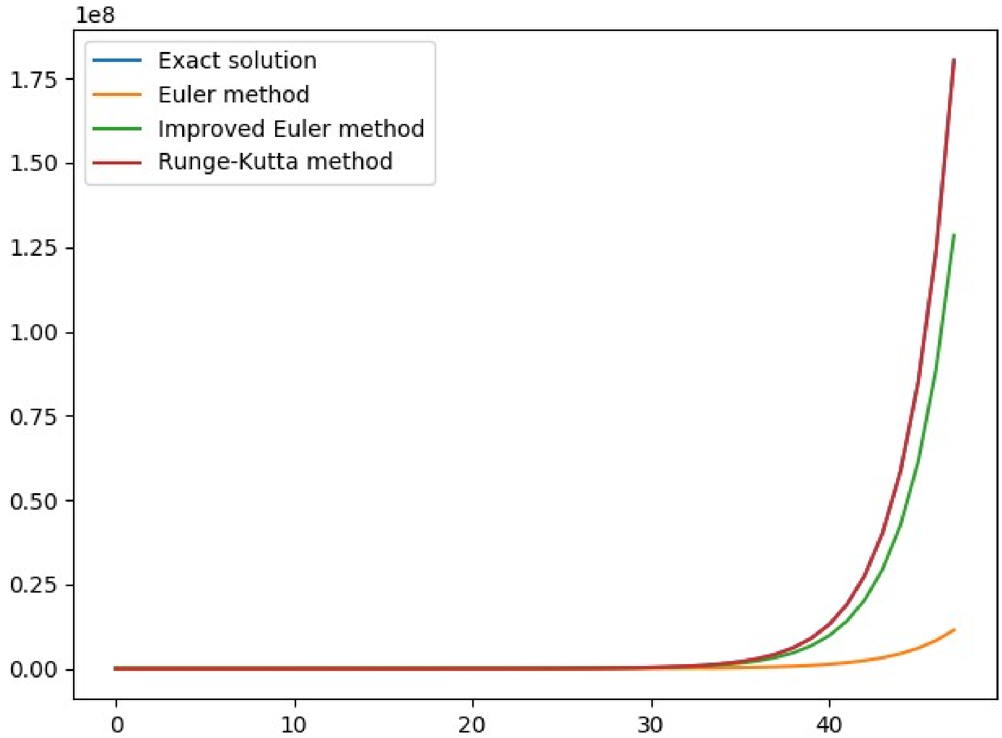


Figure 2 : Exact solution against numerical methods (plots for exact solution and Runge-Kutta method coincide)

***Following functions were used for them:***

```
def plotErrors(error1, error2, error3):
    err_line1, = plt.plot(error1)
    err_line2, = plt.plot(error2)
    err_line3, = plt.plot(error3)
    plt2.legend([err_line1, err_line2, err_line3],
                ['Euler method error', 'Improved Euler method error', 'Runge-Kutta method
                  error'])
    plt2.show()



def plotMethods(exact, euler, improved, runge_kutta):

    line1, = plt.plot(exact)
    line2, = plt.plot(euler)
    line3, = plt.plot(improved)
    line4, = plt.plot(runge_kutta)
    plt.legend([line1, line2, line3, line4],
               ['Exact solution', 'Euler method', 'Improved Euler method', 'Runge-Kutta
                 method'])
    plt.show()
```

```
Here we can see that Runge-Kutta gives best approximation.
```

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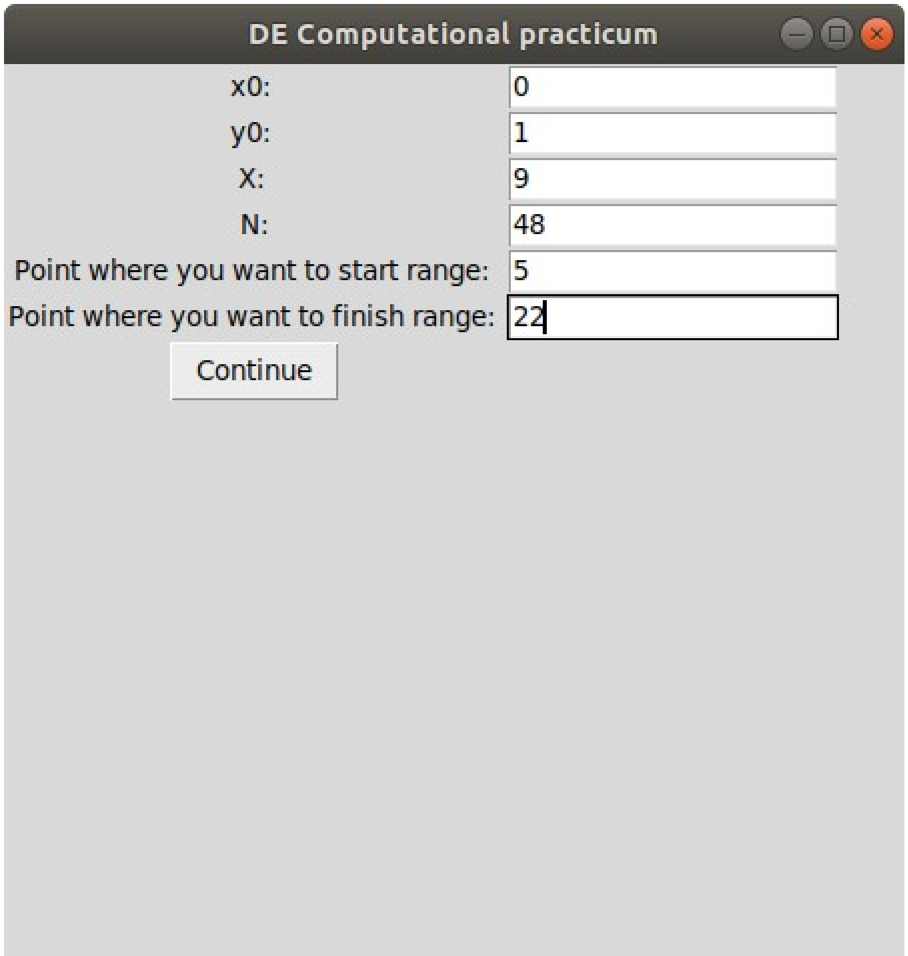
**Graphic User Interface**

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