Given metapath M, the nodes of given source and target degree $s,\ t$ have randomly distributed DWPC values $\delta_{M,s,t}$ (δ for simplicity) in permuted networks.

$$P(\delta = 0) = 1 - \lambda$$

$$\delta \mid \delta \neq 0 \sim \Gamma(\alpha, \beta)$$

Thus, given an observed DWPC value d, the probability of a more extreme value under the null hypothesis in a one-sided test (p-value) is

$$\begin{aligned} p = P(\delta \geq d) &= P(\delta \neq 0)\ P(\delta \geq d \mid \delta \neq 0) \\ &= \lambda\ \frac{\beta^\alpha}{\Gamma(\alpha)}\ x^{\alpha-1}\ e^{\beta x} \end{aligned}$$

We can estimate the parameters λ, α, and β using the sample mean and sample variance. To compute these quantities, given n, the number of degree-grouped permutations (sample size), let $\{\delta_1, ..., \delta_n\}$ be observed values of DWPC for a single source, target degree combination, and let p be its nonzero values: $p = \{x : x \in \{\delta_1, ..., \delta_n\}$ and $x > 0\}$. Then the permuted DWPC values have positive sample mean $\bar{\delta}^\dagger$ and unbiased sample variance s^2. Note that these are the mean and variance of the nonzero values only.

$$\bar{\delta}^\dagger = \frac{\sum_{=1}^n \delta_i}{|p|}$$

$$s^2 = \frac{\sum_{i=1}^n \delta_i^2 - (\sum_{i=1}^n \delta_i)^2/|p|}{|p| - 1}$$

Since $E[\delta \mid \delta \neq 0] = \frac{\alpha}{\beta}$ and $\mathrm{Var}(\delta \mid \delta \neq 0) = \frac{\alpha}{\beta^2}$, we can solve the two equations to find α and β.

$$\lambda = \frac{|p|}{n}$$

$$\beta = \frac{\bar{\delta}^\dagger}{s^2}$$

$$\alpha = \bar{\delta}^\dagger \beta$$

Thus knowing only the fraction of δ_i that are nonzero, and the mean and variance of nonzero values, we can find all three parameters of the gamma hurdle model.