COVID19 SHELTER IN PLACE ORDER OPTIMIZATION:
Limiting the spread of coronavirus disease 2019 while
minimizing the number of days of social distancing practices

ENGR 319
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Introduction

The COVID-19 pandemic, otherwise known as the coronavirus pandemic, is a result of the spread of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) thought to have been contracted from bats originating in Wuhan, China. The first case in the United States was diagnosed in Snohomish County, WA on January 20, 2020. Since then, the number of cases in the US has grown to 1.89 Million with 21,349 in Washington alone. In an effort to slow the spread of the virus, states have issued stay-at-home orders to strongly encourage social distancing. Washington state issued a stay-at-home order from March 23, 2020 to May 31, 2020. The impact the coronavirus has had on the economy has been partially due to the temporary shutdown of businesses. Therefore, reopening as soon as possible without allowing the virus to explode in new cases is an important question to address.

The purpose of this project is to minimize the number of state mandated shelter in place order (SIPO) days in Washington such that the number of infected persons requiring hospitalization is below the hospital capacity and the number of deaths is limited to a defined number. Several variations of the optimization problem will be analyzed: standard case with no SIPO, stay at home order following real life scenario of Washington, and optimizing the duration of SIPO given a minimum start date of March 23. These scenarios will be compared to one another for further analysis of the effectiveness of the SIPO.

System Characterization

In order to formulate this optimization problem, mathematical models for the spread of infectious diseases and current statistics for the coronavirus needed to be considered. I chose to use the Susceptible-Infective-Removed (SIR) Model as my primary mathematical expression for the rate of spread of the disease. The model is represented by 3 equations:

S: $S_{t+1} - S_t = -rS_tI_t$
I: $I_{t+1} - I_t = rS_tI_t - aI_t$
R: $R_{t+1} - R_t = aI_t$

Where a is the recovery rate parameter, determined by the infectious period of the disease. For coronavirus, the infectious period is a combination of the incubation and symptomatic time periods, thus ranging from 12-28 days. Considering the upper range to give us more cushion in our model gives us $a = \frac{1}{t\_infectious} = \frac{1}{24}$. Studies show that the basic reproductive number of coronavirus is somewhere between 3 and 4. I picked 3.5 as the median, then I calculated the standard value of r in the SIR model using the following equation:

$$R_\Phi = 3.5 = \tau c d$$

Where tau is the transmitability, c is the average rate of contact between susceptibles and infectives, and d is the duration of infectiousness. Since r = τc and $a = 1/d$, then R_Φ can also be written as $R_\Phi = \frac{r}{a} = q$. Solving for r, I found r to be 0.1458333. Now, if the SIPO improved social distancing so that the average rate of contact between infectives and susceptibles is reduced by 80%, then the new $R_{\Phi} = q$ value is 0.7 and thus r = 0.0291667. Another important consideration in the problem design was a stopping point. Since a vaccine was predicted to be available within a year, I decided that keeping the objective function bound by the constraints was only necessary for 365 days. Thus, the maximum number of stay at home order days is limited from the start date until day 365. In addition to these considerations, Table 1 in Appendix A outlines the other statistical information important to characterizing this optimization problem.

Using this information to formulate our optimization problem, we have a simple objective function, Δt_{SH}, written in terms of 2 variables, the SIPO start date, t_{0SH} and the stop date, t_{fSH}, and whose minimum is limited by 4 constraints. The optimization problem is written in standard form below:

MIN $\quad f(t) = t_{fSH} - t_{0SH}$

W.R.T. $\quad t = \{ t_{0SH}, t_{fSH} \}$

S.T. $\quad g_1 = 0.0175 \times max(I) \leq 1500$

$$g_2 = t_{0SH} - t_{fSH} \leq 0$$

$$g_3 = -t_{0SH} \leq 62, -t_{fSH} \leq 62$$

$$g_4 = \max(I) \times 0.20 \times 0.3333 \leq 5070$$

It is important to note that g_1 and g_4, while they don't directly involve t_{0SH} or t_{fSH}, are reliant on them through the term $max(I)$, since the maximum number of infectives is depending on the *r* value in the update formula of the SIR model. And this *r* value is dependent on time as illustrated in the following piecewise function:

$$r = \begin{cases} 0.14583 & for \quad t \leq t_{0SH} \\ 0.0291667 & for \quad t_{0SH} \leq t \leq t_{fSH} \\ 0.07 & for \quad t_{fSH} \leq t \end{cases}$$

For the control scenario with no SIPO, Δt_{SH} = 0, I examined how far the function overstepped the constraints. When modeling Washington's current situation, I plotted a graph hardcoded with the SIPO start and stop dates. I will be using this one as a comparison to the optimized problem.

Optimization Algorithm Selection

The optimization problem has a simple continuous objective function with only 2 variables, and several linear and nonlinear constraints, of which some are dependent on non-design variables. This reliance on non-design variables drew me towards using Matlab's built in tool box. I decided to use Matlab's Sequential Quadratic Programming built in function *fmincon* since I needed an algorithm that could handle both linear and non-linear constraints, incorporate the use of non-design variables, and be able to approximate gradients since I was unable to calculate one by hand for the non-linear constraints.

Design Space Exploration

The problem I designed is heavily dependent on the accuracy of the SIR graph. Thus, to evaluate the validity of my model, I graphed the SIR curves without SIPO implementation and the SIR curves based on Washington States actual actions and proceeded to compare them to the statistics provided by the Washington State Department of Health's website.

The plot of the SIR curves without any social distancing measures is shown in *Figure 1*. Based on this graph, we see that the peak of the virus progression would occur around middle of July and predicts that about 35% of the population in Washington will be infected with coronavirus at the peak. By March 23 (day 62), we can see from *Figure 2* that the fraction of population infected is about 0.00005. The number of reported cases in Washington State on March 23 was 369, which correlates to roughly 0.000048. Since 0.00005 ≈ 0.000048, and the peak of the virus roughly correlates to the projected peak of the virus without social distancing measures. Furthermore, upon plotting the actual actions of Washington state, it's apparent that the general trend of the number of infections closely follows the statistics from Washington State Department of Health as shown in *Figure 3*, *Figure 4*, and *Figure 5*. Thus, I concluded that my model was valid.

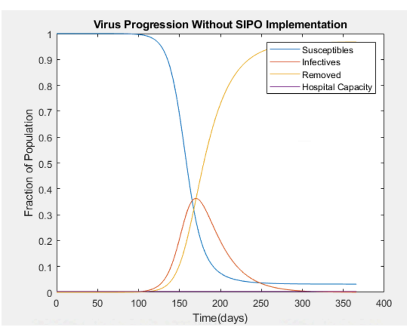


Figure 1. Plot of SIR curves without any social distancing measures implemented

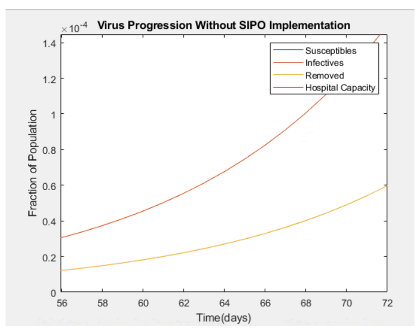


Figure 2. Close up image of Figure 1 centered on day 62, to compare the fraction of population infected with the actual number documented by Washington State on March 23, 2020

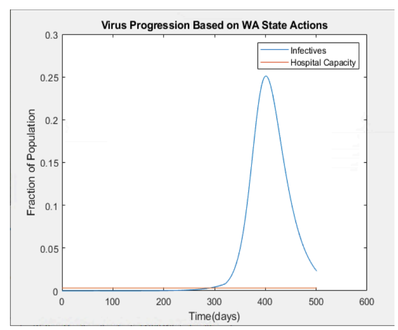


Figure 3. Plot of I curve with social distancing practices as promoted by Washington State.

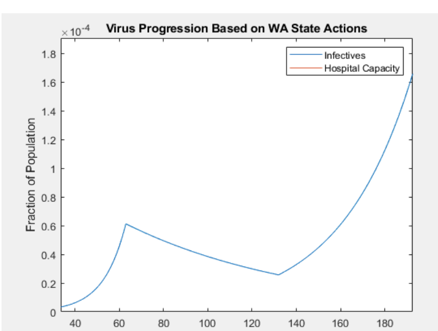


Figure 4. Close up image of Figure 3, noting shape of curve resembles that in Figure 5.

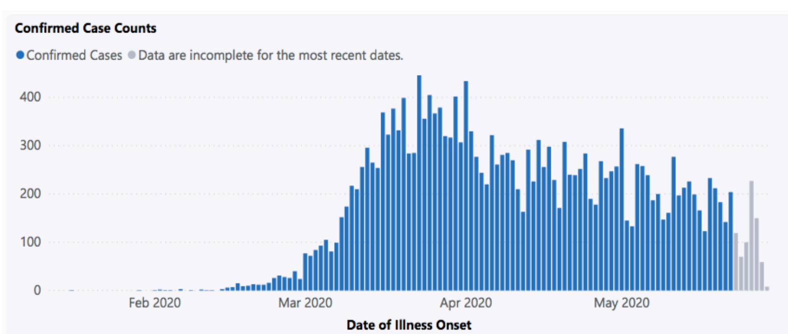


Figure 5. Statistical plot of I curve collected from Washington State Department of Health.

Optimization Results and Validation

Despite numerous efforts in refining the model, *fmincon* was unable to optimize this problem. *Figure 6* is a 3D plot of the objective function. Examining this image, it's clear that the function is continuous, and that the theoretical solution should lie somewhere in the upper half of the plane (the green-yellow region). We can also deduce that there is almost an infinite number of solutions since the minimized point will be a line. However, I expected that the constraints would limit it so that there would only be one optimal solution. To further investigate the reason the algorithm was having trouble optimizing the model, I also graphed 3D plots of the constraints, shown in *Figure 7* and *Figure 8*. Strangely, the region of the constraints I was concerned with is the yellow triangle on each of the graphs. Since it was flat, it meant that the value of the constraint was the same for each combination of t_{0SH} and t_{fSH} in that region. With an unchanging and infeasible constraint value and a line of solutions for the objective function, no wonder the algorithm was having a difficult time optimizing the problem. However, I did not expect the constraint to stay constant. Rather, it should be varying with change in peak of the infectives curve. To verify that the peak of the infective curve was changing, I plotted the I curve given many different t_{0SH} and t_{fSH} combinations. I discovered that the peak of the infective curve did not change as long as the curve was within the 365-day limit. However, Once I started to push the curve past the 365-day limit, then the maximum of the I curve would be the point where the curve crosses the 365-day mark instead of the peak of the curve. In this manner, I was able to manually find a plausible solution to this optimization problem. By visually looking at the graph at varying combinations of t_{0SH} and t_{fSH} until the hospital capacity constraint became active at day 365. The result is shown in *Figure 9*. Here, the start

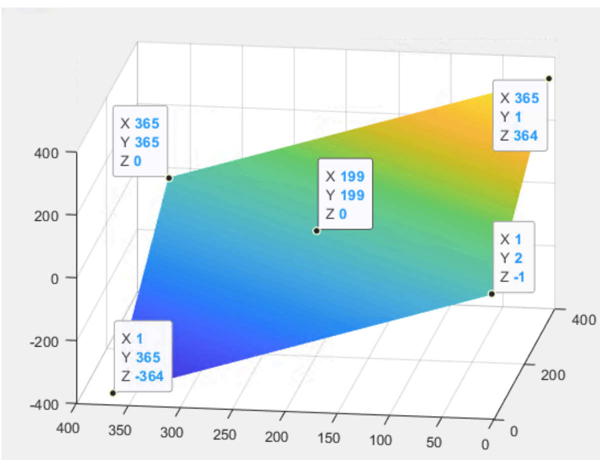


Figure 6. 3D plot of the objective function against the start date (X) and end date(Y) of the stay at home order.

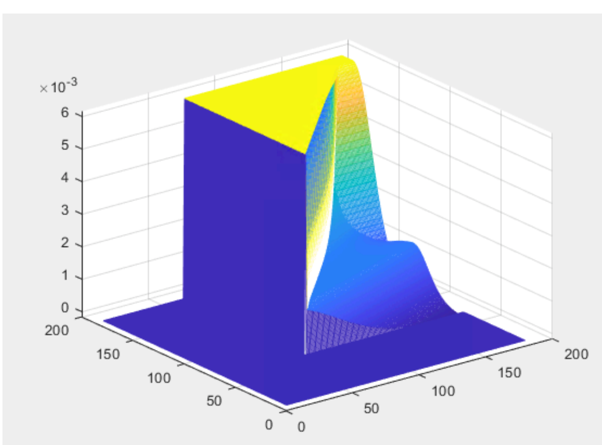


Figure 7. 3D plot of the hospital capacity constraint.

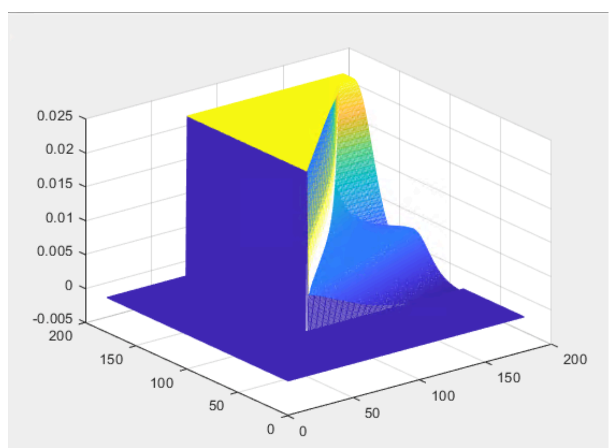


Figure 8. 3D plot of desired death maximum constraint

date was day 62 and the end date was day 141, correlating to March 23 through June 10 for a total of 79 days. The question arises with this finding whether “flattening the curve” is a myth? In this model, the curve is never flattened—the peak is just postponed. This partially explains why the 3D plot of the constraints is flat in my area of interest. However, I am still perplexed at why at some point that flat area doesn’t change as I push the curve past the 365-day upper-bound.

Due to the nature of the objective function and constraints, when using *fmincon* to solve the problem, the result is:

x =

66.8367 *148.1354*

fval =

81.2987

output =

iterations: 3
funcCount: 11

This is the best possible solution I was able to find. I discovered that the algorithm was very sensitive to starting values. If I had a starting value of x = {62, 201}, I get the results stated above and the graph shown in *Figure 10*. We can see in the figure that constraints are violated, and therefore it is an infeasible solution. Now, if I have a starting value of x = {62, 140}, I get an fval of 43.6667, but the constraints are even more violated than the last solution. Tweaking the set *r* values changes the character of the curve. If, for instance, even after the stay-at-home order ends, the R_Φ value remains below 1, the curve will plateau and we will have a feasible solution no matter the duration of the stay-at-home order.

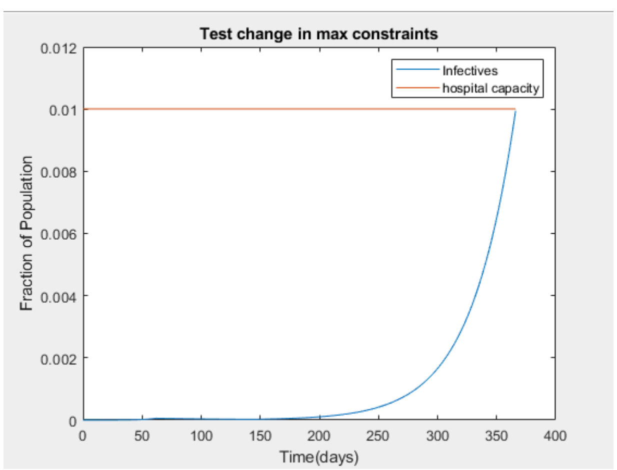


Figure 9. Results of investigation for changing maximum of Infective curve - pseudo optimum found with start date at day 62 and end date at day 141.

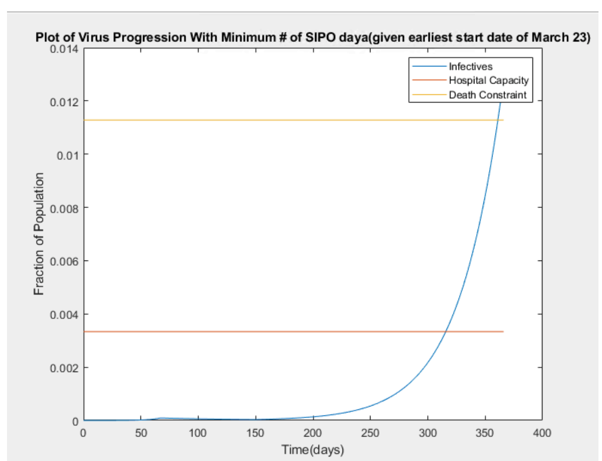


Figure 10. Results from fmincon algorithm. An infeasible solution since it crosses both the hospital capacity and death constraint lines.

In conclusion, we see that a feasible solution should be possible, as illustrated in *Figure 9*. But I have seemed to reach the limits of Matlab's *fmincon* function with this optimization problem. For future work, I would research the use of other optimizer algorithms, consider scaling the constraints, and refining the R_Φ values to resemble a more realistic problem and thus have a better chance of attaining a feasible solution that is less sensitive to a variety of initial guesses. With respect to the importance of social distancing, it's easy to conclude that keeping the reproduction number as low as possible will be the key to limiting the number of positive COVID19 cases. The balance to finding when the best time to open is somewhat ambiguous since it is heavily dependent on the fickle reproduction number. Therefore, I've come to the conclusion that the wisest move is to carefully monitor the reproduction number as the state starts opening up, being careful to keep it as close to 1 if not less than 1 as possible until a vaccine is readily available.

References

https://www.nature.com/articles/s41591-020-0883-7

https://en.wikipedia.org/wiki/COVID-19_pandemic

https://mn.gov/covid19/assets/MNmodel_tech_doc_tcm1148-427724.pdf

https://www.nber.org/papers/w27091.pdf

https://web.stanford.edu/~jhj1/teachingdocs/Jones-on-R0.pdf

https://towardsdatascience.com/social-distancing-to-slow-the-coronavirus-768292f04296

https://www.ncbi.nlm.nih.gov/pubmed/22093478

https://mathinsight.org/discrete_sir_infectious_disease_model

https://www.youtube.com/watch?v=CUzcuiRE15c

https://www.youtube.com/watch?v=NKMHhm2Zbkw

Appendix A

Characteristic	Value	Reference
Population of Washington	7.6 Million	*USA News*
Initial infective number (I_0)	1	*The Herald*
Initial susceptible number (S_0)	7,599,999	*Determined from above information*
Initial removed (R_0)	0	*Assumed*
t_0	January 20, 2020	*The Herald*
Confirmed cases in Washington	21,349	*Washington State Department of Health*
Deaths in Washington	1118	*Washington State Department of Health*
Percent Death Rate in relation to confirmed cases in Washington	5.2%	*Washington State Department of Health*
Hospital Capacity in Washington	13,000	*USA News*
Hospital percent occupancy on a regular basis	61%	*Becker's Hospital Review*
Number of bed available before COVID-19 pandemic	5070	*Determined from above information*
COVID-19 cases requiring hospitalization	20%	*STAT*
Washington declared stay-at-home order start	March 23, 2020	*The News Tribune*
Washington declared stay-at-home order end	May 31, 2020	*The News Tribune*
Number of days SIPO after first case	62 days	*Determined from information above*
Vaccine Release Date	1 year from first case	*News Scientist*
Estimated actual number of infectives	64,047	*Pharmaceutical Technology*

Table 1.Statistical information used in developing the mathematical equations used to characterize the optimization problem.

Appendix B

Contents

initializations

Jordyn Watkins MATH 319 - Optimization Final Project

```
clear all
close all
clc
```

Optimization Problem Defintion

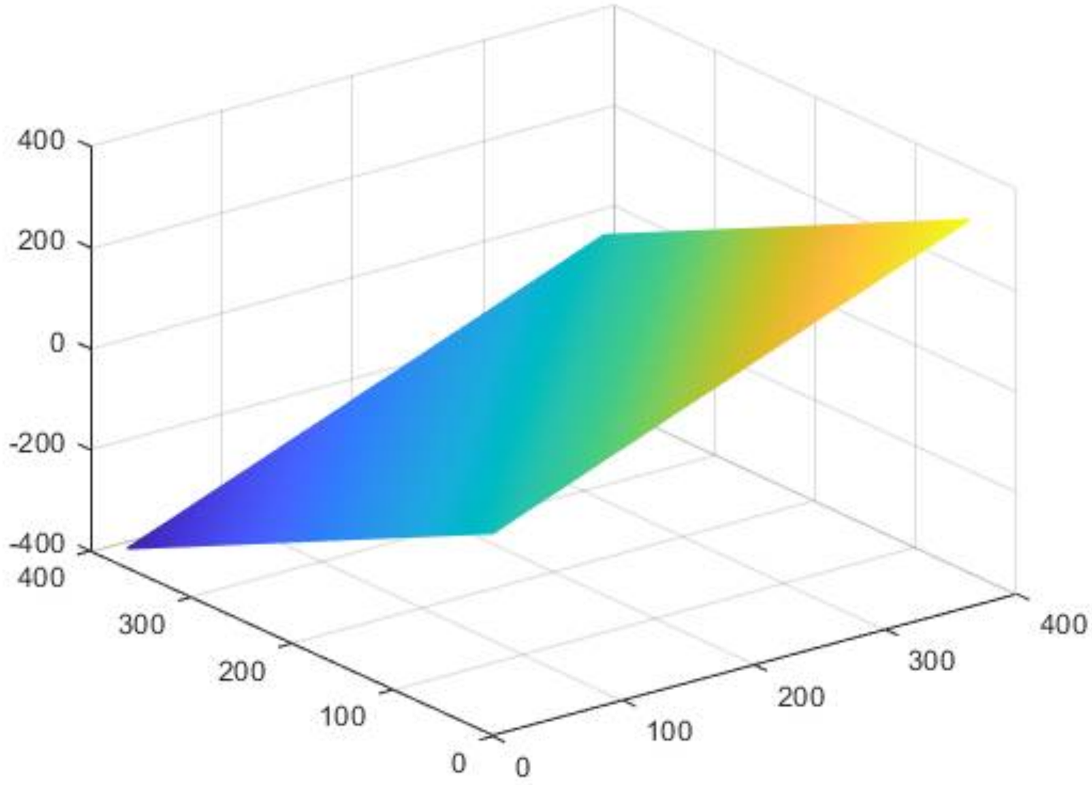
```
% Optmization Problem
% min   f(x) = x2 - x1
% wrt   x = {x1, x2}
% st    0.0175*R <= 1500                * # of deaths < 1500
%       x2 - x1 >= 0                    * # of SIPO days cannot be negative

%       max(I)*0.2*0.33333 <= 5070      * this ensures hospitals remain under
%                                       * their capacity

%       x1, x2 >= 0                     * all variables must be >=0
% =========================================================================
%       S + I + R = 7.6 million       * SIR adds to total population
%                                       * used for error checking - NOT a
%                                         constraint
% =========================================================================
% where x1 = start date of SIPO
%       x2 = the end date of SIPO,
%       S = number of susceptibles
%       I = number of infectives
%       R = number of recovered

% formula used to plot SIR pattern:
%       S(t+1) = S(t) - r*S(t)*I(t)
%       I(t+1) = I(t) + r*S(t)*I(t) - a*I(t)
%       R(t+1) = R(t) + a*I(t)
```

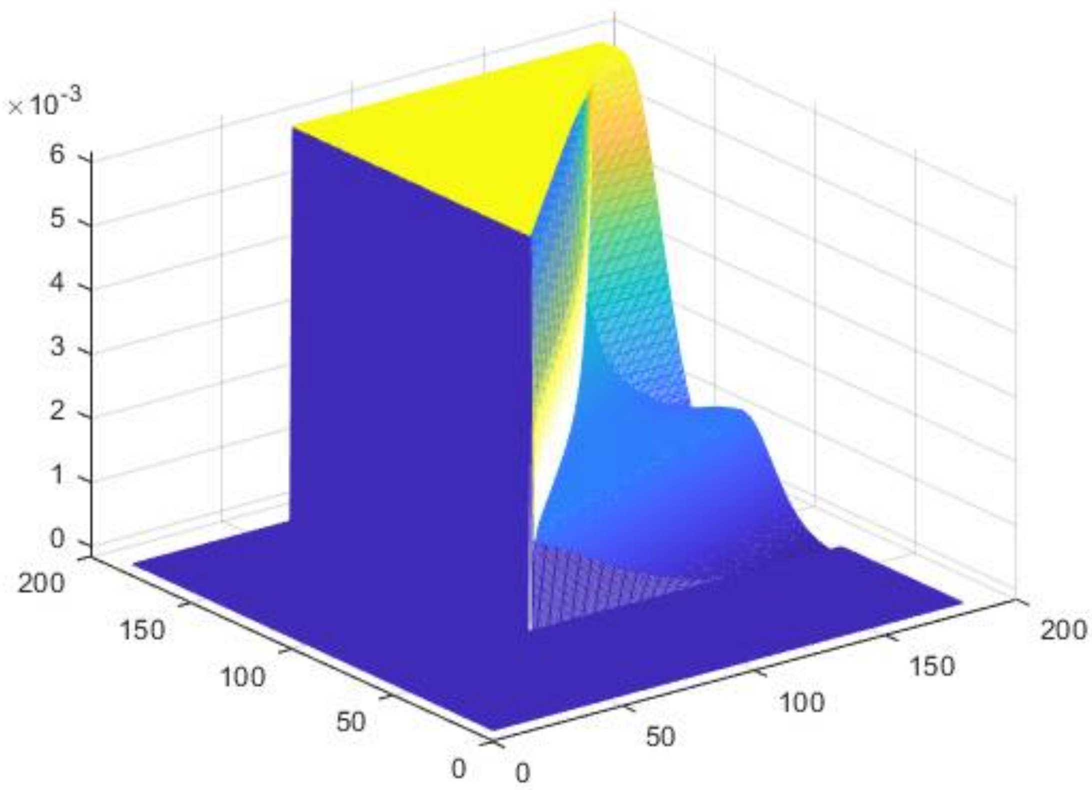
Troubleshooting

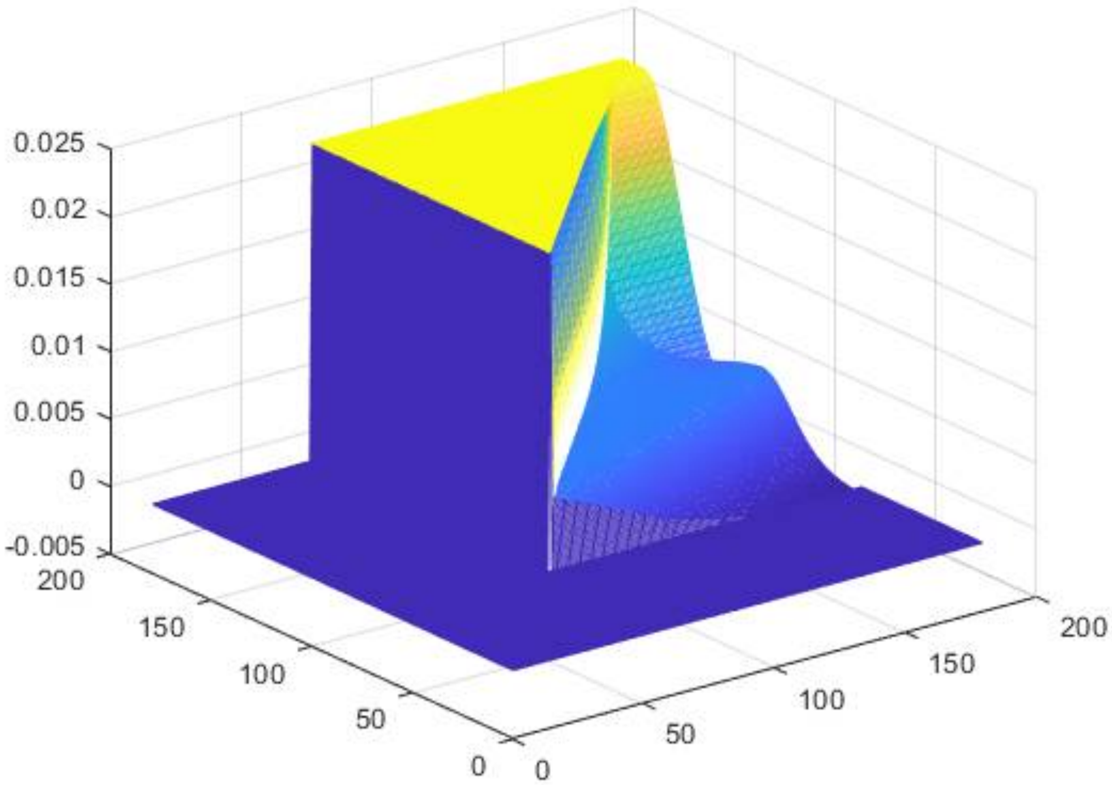
```
%Three dimensional plots of the objective function and major constraints.
%Followed by 2-D plots of the I-curve with respect to hospital capacity to
%see if the maximum of I actually changes or not.

% 3-D plot of objective function
```

```
Z = zeros(365,365);
for X = 1:1:365
    for Y = 1:1:365
        Z(X,Y) = Y-X;
    end
end

figure;
mesh(Z)

% 3-D plot of constraints. Hospital capacity constraint is second plot,
% death constraint is first plot.
for X = 62:1:180
    for Y = 62:1:180
        x = [X,Y];
        g(X,Y,:) = NonLCon(x);
    end
end
figure;
mesh(g(:,:,1))
figure;
mesh(g(:,:,2))


% 2-D plot of I-curve with respect to hospital capacity to see if a
% feasible solution is possible. Appears that a feasible solution is
% possible with x1 = 62 and x2 = 141.
a = 0.04166667;
S(1,1) = 7599999/7600000;
I(1,1) = 1/7600000;
R(1,1) = 0/7600000;
time(1,1) = 0;
hospitalCapacity(1,1) = 76050/7600000;

for x2 = 140:1:142
    for i=1:365

        if(i >= 62)&&(i <= x2)
            r = 0.0291666;
        elseif(i > x2)&&((x2-62) > 0)
            r = 0.07;
        else
            r = 0.14583345;
        end
        S(1,i+1) = S(1,i)- r*S(1,i)*I(1,i);
        I(1,i+1) = I(1,i) + r*S(1,i)*I(1,i) - a*I(1,i);
        R(1,i+1) = R(1,i) + a*I(1,i);
        time(1,i+1) = i+1;
        hospitalCapacity(1,i+1) = 76050/7600000;

    end
    figure;
    plot(time(1,:),I(1,:))
    hold
    plot(time(1,:),hospitalCapacity(1,:))
    legend("Infectives","hospital capacity")
    title('Test change in max constraints')
    xlabel('Time(days)')
    ylabel('Fraction of Population')
end
```

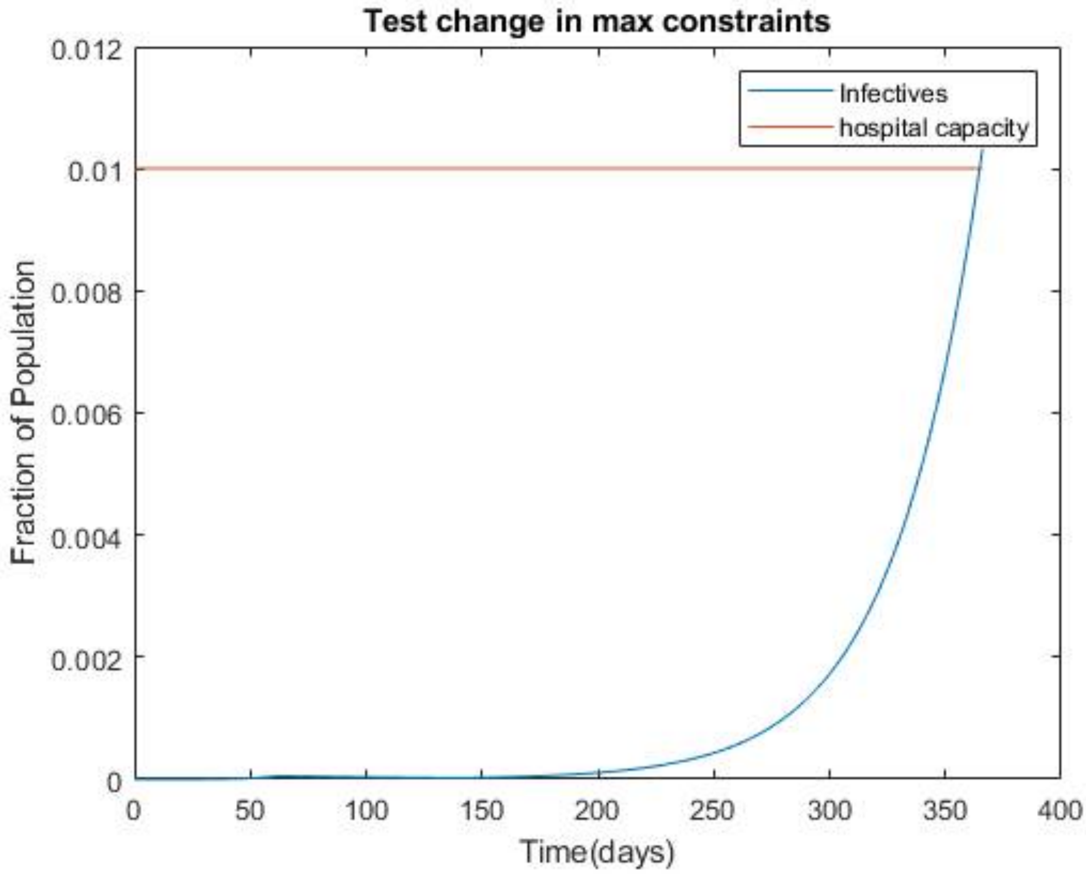
```
Current plot held
Current plot held
Current plot held
```



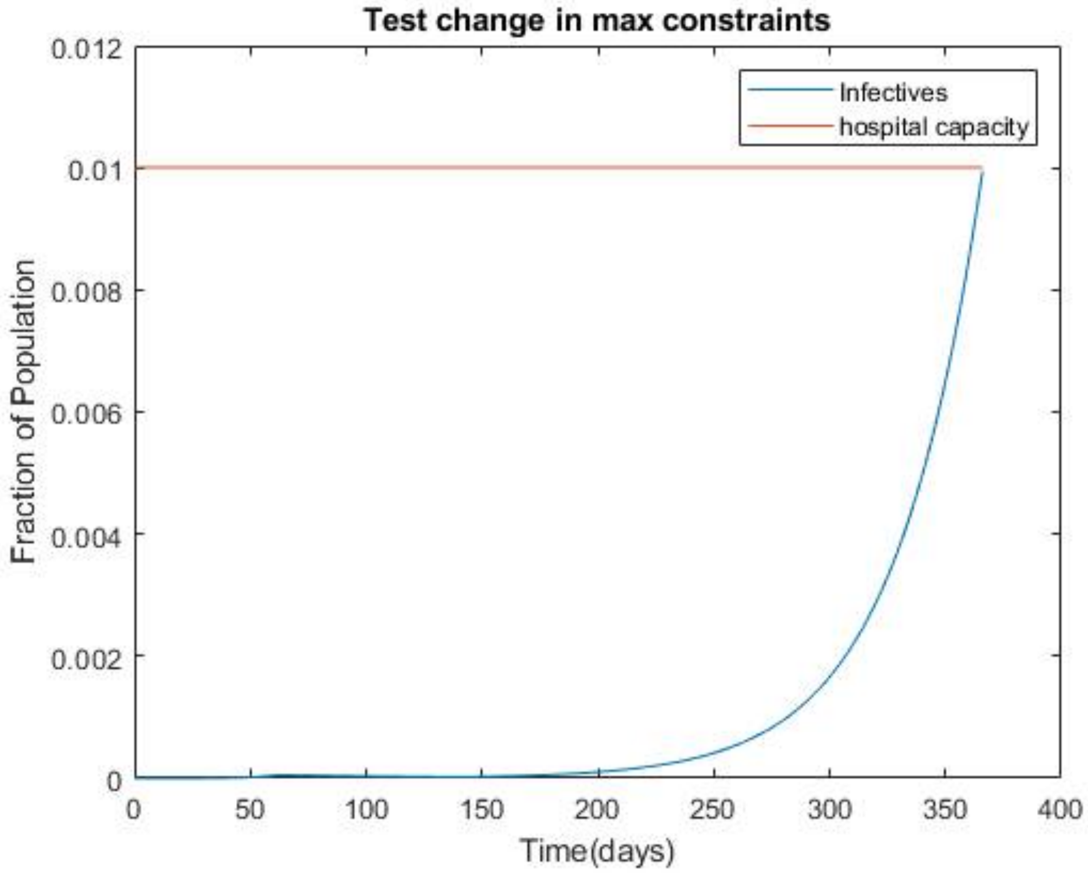


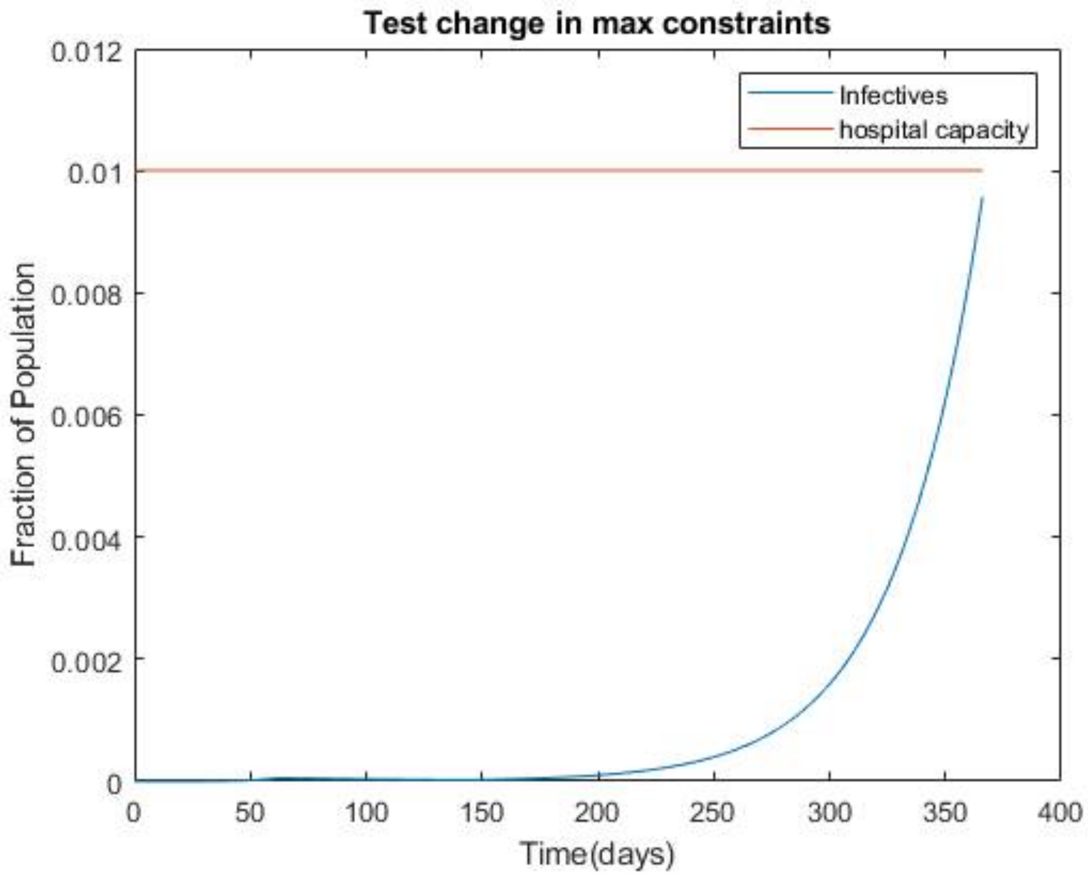


0.025
0.02
0.015
0.01
0.005
0
-0.005
200
150
100
50
0
0
50
100
150
200



Test change in max constraints
Infectives
hospital capacity
Fraction of Population
Time(days)
0.012
0.01
0.008
0.006
0.004
0.002
0
0
50
100
150
200
250
300
350
400



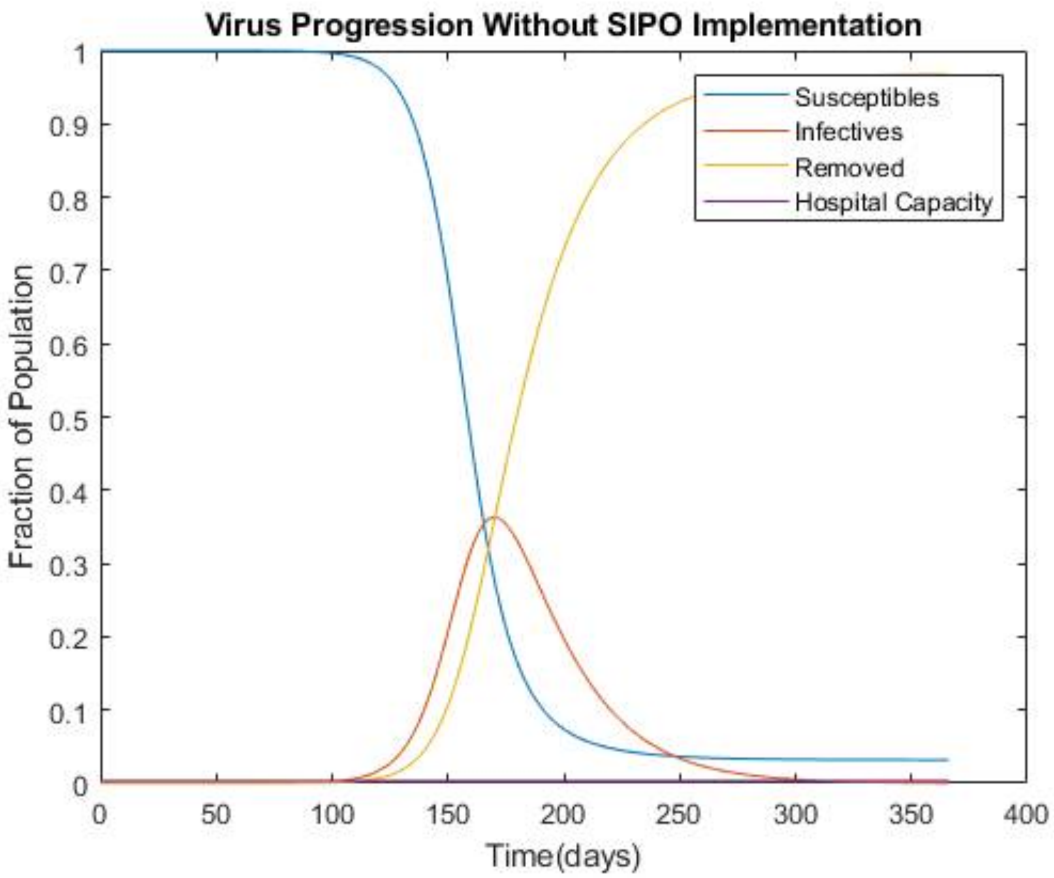


SIR Model WITHOUT SIPO Implementation

This model shows the progression of the coronavirus without any social distancing measures implemented. This serves as a comparison for the optimized problem scenarios.

```
r = 0.14583345;
a = 0.04166667;
S(1,1) = 7599999/7600000;
I(1,1) = 1/7600000;
R(1,1) = 0/7600000;
time(1,1) = 0;
hospitalCapacity(1,1) = 25350/7600000;

for i=1:365

    S(1,i+1) = S(1,i)- r*S(1,i)*I(1,i);
    I(1,i+1) = I(1,i) + r*S(1,i)*I(1,i) - a*I(1,i);
    R(1,i+1) = R(1,i) + a*I(1,i);
    time(1,i+1) = i+1;
    hospitalCapacity(1,i+1) = 25350/7600000;

end

figure
plot(time(1,:),S(1,:))
hold
plot(time(1,:),I(1,:))
plot(time(1,:),R(1,:))
plot(time(1,:),hospitalCapacity)
legend("Susceptibles","Infectives","Removed","Hospital Capacity")
title('Virus Progression Without SIPO Implementation')
xlabel('Time(days)')
ylabel('Fraction of Population')
```

```
Current plot held
```



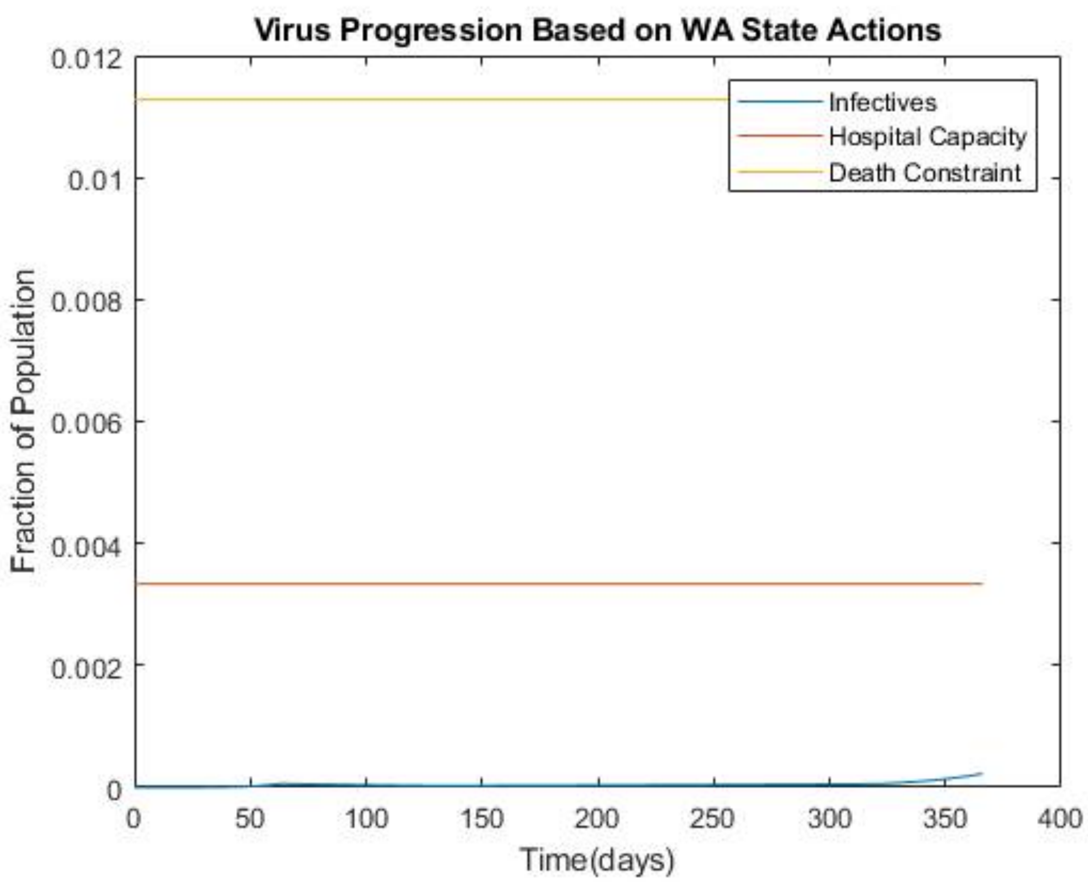
Disease Progression Based on WA State Actions (Actual Model)

This model illustrates the progression of the coronavirus on the population based on Washington State's actual actions - with the implementation date of SIPO, and the phase out stages.

```
a = 0.04166667;
S(1,1) = 7599999/7600000;
I(1,1) = 1/7600000;
R(1,1) = 0/7600000;
time(1,1) = 0;
hospitalCapacity(1,1) = 25350/7600000;
deathcap(1,1) = 85714/7600000;

for i=1:365

    if (i <= 62)                    % R0 of 3.5
        r = 0.14583345;
    elseif (i > 62) && (i <= 131)   % R0 of 0.7
        r = 0.02916667;
    elseif (i > 131) && (i <= 320)  % R0 of 1.1
        r = 0.0458333;
    elseif (i > 320) && (i <= 365)  % R0 of 1.75
        r = 0.072916725;
    end

    S(1,i+1) = S(1,i)- r*S(1,i)*I(1,i);
    I(1,i+1) = I(1,i) + r*S(1,i)*I(1,i) - a*I(1,i);
    R(1,i+1) = R(1,i) + a*I(1,i);
    time(1,i+1) = i+1;
    hospitalCapacity(1,i+1) = 25350/7600000;
    deathcap(1,i+1) = 85714/7600000;

end
```

```
figure
plot(time(1,:),I(1,:))
hold
plot(time(1,:),hospitalCapacity)
plot(time(1,:),deathcap)
legend("Infectives", "Hospital Capacity", "Death Constraint")
title('Virus Progression Based on WA State Actions')
xlabel('Time(days)')
ylabel('Fraction of Population')
```

```
Current plot held
```



Minimum # of SIPO Days (Realistic Model)

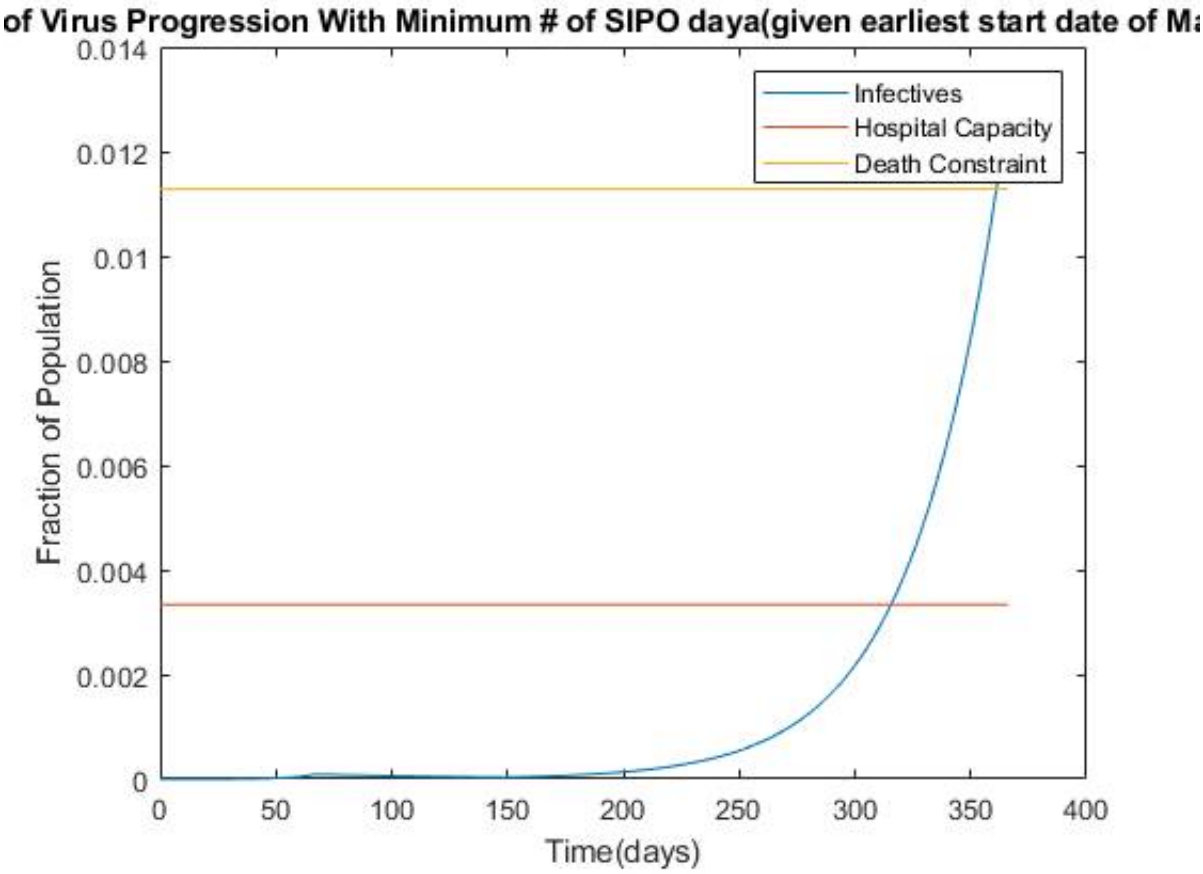
First analysis to find the minimum number of SIPO days modeled after Washington States actual timeline - issuing SIPO March 23 (62 days after first case). Evaluated from t = 0 to t = 365 days since that's when an expected vaccine should be in circulation.

```
% initial conditions
X0 = [62 201];
LB = [62 62];          % Lower bounds on variables
UB = 365*ones(1,2); % Upper bounds on variables

% linear constraints
A = [1 -1];            % Coefficients of linear constraints equations
B = [0];               % Right side vector for linear constraints equations

options = optimset('Display', 'iter', 'LargeScale','off','Algorithm','active-set','GradObj
','on','GradConstr','off');
```

```
[x, fval, flag, output] = fmincon('ObjFcn',X0,A,B,[],[],LB,UB,'NonLCon',options)

a = 0.04166667;
S(1,1) = 7599999/7600000;
I(1,1) = 1/7600000;
R(1,1) = 0/7600000;
time(1,1) = 0;
hospitalCapacity(1,1) = 25350/7600000;
deathcap(1,1) = 85714/7600000;

for i=1:365

    if(i >= x(1))&&(i <= x(2))            % R0 of 0.7
        r = 0.0291666;
    elseif(i > x(2))&&((x(2)-x(1)) > 0)   % R0 of 1.68
        r = 0.07;
    else
        r = 0.14583345;                    % R0 of 3.5
    end
    S(1,i+1) = S(1,i)- r*S(1,i)*I(1,i);
    I(1,i+1) = I(1,i) + r*S(1,i)*I(1,i) - a*I(1,i);
    R(1,i+1) = R(1,i) + a*I(1,i);
    time(1,i+1) = i+1;
    hospitalCapacity(1,i+1) = 25350/7600000;
    deathcap(1,i+1) = 85714/7600000;

end
figure
plot(time(1,:),I(1,:))
hold
plot(time(1,:),hospitalCapacity)
plot(time(1,:),deathcap)
legend("Infectives", "Hospital Capacity", "Death Constraint")
title('Plot of Virus Progression With Minimum # of SIPO daya(given earliest start date of
March 23)')
xlabel('Time(days)')
ylabel('Fraction of Population')
```

```
                                     Max     Line search  Directional  First-order
 Iter  F-count           f(x)   constraint   steplength   derivative   optimality Procedure
    0       3             139            0
    1       7             138            0            1           -1            1
    2      11         81.2987    0.0001883            1        -1.09            1   Hessian mod
ified twice;    dependent

Converged to an infeasible point.

fmincon stopped because the size of the current search direction is less than
twice the value of the step size tolerance but constraints are not
satisfied to within the value of the constraint tolerance.


x =

   66.8367  148.1354


fval =
```

```
   81.2987


flag =

    -2


output = 

  struct with fields:

         iterations: 3
          funcCount: 11
       lssteplength: 1
           stepsize: 0
          algorithm: 'active-set'
      firstorderopt: 1
    constrviolation: 1.8833e-04
            message: '↵Converged to an infeasible point.↵↵fmincon stopped because the siz
e of the current search direction is less than↵twice the value of the step size tolerance
but constraints are not ↵satisfied to within the value of the constraint tolerance.↵↵<sto
pping criteria details>↵↵Optimization stopped because the norm of the current search dire
ction , 0.000000e+00,↵is less than 2*options.StepTolerance = 1.000000e-06, but the maximum
 constraint↵violation, 1.883343e-04, exceeds options.ConstraintTolerance = 1.000000e-06.↵
↵'

Current plot held
```



Minimum # of SIPO Days (Best Case Scenario Model)

This model assumes that the minimum start date can be as early as day 0 (t = 0). The purpose of conducting this optimization problem is to compare this result to that of the realistic model to see how the number of SIPO days can be reduced by mandating the order earlier.

```
% initial conditions
%X0 = [0 140];
%LB = [0 0];          % Lower bounds on variables
%UB = 365*ones(1,2); % Upper bounds on variables

% linear constraints
%A = [1 -1];          % Coefficients of linear constraints equations
%B = 0;               % Right side vector for linear constraints equations

%options = optimset('LargeScale','off','Algorithm','active-set','GradObj','on','GradConstr
','off');
%[x, fval, flag, output] = fmincon('ObjFcn',X0,A,B,[],[],LB,UB,'NonLCon',options)
```

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