Stranded Assets Model

Mistra Financial Services Green Macro Project

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Chapter 1

Introduction

This report is "live" documentation of the stranded assets formal model ("the model") with reference to its implementation using the R package PK-SFC ("the code"). It contains all of the model code and also serves as the source code for the model itself. Note the following generic notation and standards used in the code:

- Uppercase: nominal values; lowercase: real values
- `varname` + `e`: an expected future value (except for prices of equities)
- `varname` + `mean`: a running average of historical values
- `varname` + `gr`: the prior one-period growth rate
- `varname` + `k`, `i`, `c` or `b`: sector-specific variables:
  - `k`: conventional capital
  - `i`: innovative capital
  - `c`: consumption goods
  - `b`: banks

In the equations below, the `mean` is indicated by an overbar, where the average is taken over the previous four periods. For an arbitrary variable x,

$$\bar{x} \equiv \frac{1}{4} \sum_{t=-4}^{-1} x_t$$

The growth rate from the previous period, `gr`, is indicated a "hat". For an abitrary variable x,

$$\hat{x} \equiv \frac{x - x_{-1}}{x_{-1}}$$

Chapter 2

Time and transition

Time is tracked by a variable t, which increments by one in each time step,

```
# Equations begin
t = t(-1) + 1
```

The innovative sector enters after time step n, with transitional dynamics in the first period after entry

```
# firstPeriodInov=(t==(n+1))
```

Moreover, after starting, the innovative sector delays entry into markets for a period `entry` because it is assumed to be too small initially to attract financial flows. Once it has grown enough, firms producing innovative capital goods try to rise funds via an IPO. Model experiments have shown that if the innovators try to enter the financial market too soon it hurts them because they can't raise enough money via the IPO and thus are over indebted.

Investors exit the conventional capital goods sector as soon as either: a) production goes to zero or b) gross profits of the sector go to zero,

```
exitk = ifelse(exitk(-1) == 1, 1, ifelse(yk == 0 | Fk < 0, 1, 0))
```

The loan period is given by `nLoans`, referred to in equations below by n_{loans}.

For several variables, expectations are formed through an adaptive process. A common term in each of these expressions takes historical growth into account when anticipating future growth. The historical average growth rate is multiplied by a parameter η (`eta`), which is zero if historical growth is ignored when forming expectations and one if anticipated future growth is equal to historical average growth.

Chapter 3

Wealth and income

3.1 Wealth

Nominal wealth is given by previous period wealth plus disposable income net of consumption,

$$V_h = V_{h,-1} + (\mathrm{YD}_h - C_h)$$

```
Vh = Vh(-1) + (YDh - Ch)
```

Expected wealth in the next period is calculated using the same formula, but with expected disposable income,

$$V_h^e = V_{h,-1} + (\mathrm{YD}_h^e - C_h)$$

```
Vhe = Vh(-1) + YDhe - Ch
```

Total wealth, V_h, is equal to the value of equity shares plus money held. Denoting the number of shares in sector x by e_x, the equity price by $p_{x,e}$, and money that household wish to hold,

$$V_h = p_{c,e}e_c + p_{k,e}e_k + p_{i,e}e_i + M_h$$

```
Vh = pce * ec + pke * ek + pie * ei + Mh
```

3.2 Labor and wages

Labor is employed in all three productive sectors. The capital-to-labor ratio for conventional and innovative capital are denoted ℓ_k and ℓ_l (`lk` and `li` in the code). The general expression for employment N_y in sector y is the sum of utilization times capital stock divided by the capital per labor ratio for both conventional and innovative capital,

$$N_y = u_{iy}\frac{i_{y,-1}}{\ell_i} + u_{ky}\frac{k_{y,-1}}{\ell_k}$$

Additional conditions ensure that employment is zero in the innovative sector before it starts producing, while employment is zero in the conventional capital goods sector once it has stopped operations. Also, only conventional capital is ever used in the conventional capital goods sector,

```
Nc = uic * ic(-1)/li + ukc * kc(-1)/lk
Ni = uii * ii(-1)/li
Nk = ifelse(exitk(-1) != 1 & inktot > 0, ukk * kk(-1)/lk, 0)
```

Total employment is the sum of employment in each sector,

$$N_{\text{tot}} = N_c + N_i + N_k$$

```
Ntot = Nc + Nk + Ni
```

The size of the labor force is set as an exogenous parameter, LF. The employment rate is then total employment divided by the labor force, N_{tot}/LF.

Average labor productivity in sector x, $\overline{\lambda}_x^N$ (denoted `Aprx` in the code) is given by real output from the sector, y_x, divided by sector employment,

$$\overline{\lambda}_x^N = \frac{y_x}{N_x}$$

After first ensuring that innovative and conventional capital are in use, this is represented in code by

```
Aprc = yc/Nc
Apri = yi/Ni
Aprk = ifelse(exitk(-1) != 1 & Nk > 0, yk/Nk, 0)
```

In each time period, firms set a target real wage, possibly after negotiations with labor. For sector x, this is denoted ω_x^T (`omegaTx` in the code). The target real wage increases with average labor productivity and the employment rate. The general equation is

$$\omega_x^T = \Omega_0 + \Omega_1 \log \overline{\lambda}_x^N + \Omega_2 \log \frac{N_{\text{tot}}}{LF}$$

With a check to ensure that the conventional capital goods sector is still operation, this is represented in code as

```
omegaTc = Omega0 + Omega1 * log(Aprc) + Omega2 * log(Ntot/LF)
omegaTk = ifelse(exitk(-1) != 1 & Nk > 0, Omega0 + Omega1 * log(Aprk) + Omega2 *
    log(Ntot/LF), 0)
omegaTi = Omega0 + Omega1 * log(Apri) + Omega2 * log(Ntot/LF)
```

The nominal wage is then equal to the nominal wage from the previous period plus a correction based on the gap between the target and the realized real wage. For a general sector x, the equation for the real wage W_x is

$$W_x = W_{x,-1} + \Omega_3 \left(\omega_{x,-1}^T - \frac{W_{x,-1}}{p_{x,-1}} \right)$$

With conditional statements taking into account whether the innovative and conventional capital goods sectors are producing, this is represented in code as

```
Wc = Wc(-1) + Omega3 * (omegaTc(-1) - Wc(-1)/pc(-1))
Wk = ifelse(exitk(-1) == 0, Wk(-1) + Omega3 * (omegaTk(-1) - Wk(-1)/pc(-1)),
    0)
Wi = Wi(-1) + Omega3 * (omegaTi(-1) - Wi(-1)/pc(-1))
```

> **Note:** I would feel more comfortable with a more behaviorally-grounded expression. While workers want a real wage, and they notice prices, the negotiation is in terms of the nominal wage. Realness enters in two ways: sharing efficiency gains and adjusting for expected inflation. **Further note:** If we have a productivity story to explain the transition, it is important to deal wth the wage share. We could fix it. To be determined when the scenario will be dealt with.

3.3 Profits and dividends

The basic profit equation is the value of output, less the wage bill, less payments on loans held in the previous period. For a sector x,

$$F_x = Y_x - W_x N_x - r_{lx} L_{x,-1}$$

In the capital goods sectors this expression is modified in the following ways:

- In the conventional capital goods sectors, if there is investment in the sector and firms have not exited the sector then it pays dividends, else dividends are zero;
- In the innovative sector, there are two conditions (**Note:** But at present they contain the same equations, so effectively after innovative capital is introduced the sector begins to pay dividends - **Response:** this was a mistake and has been corrected).

```
Fc = Yc - Wc * Nc - rlc * Lc(-1)
Fi = Yi - Wi * Ni - rli * Li(-1)
Fk = ifelse(exitk(-1) != 1 & inktot > 0, Yk - Wk * Nk - rlk * Lk(-1), 0)
```

The basic equation for expected profits in sector x is given by an adaptive expectations model in which both historical growth and the accuracy of past forecasts influence the result,

$$F_x^e = \left(1 + \eta \hat{F}_x\right) \bar{F}_x + \nu \left(F_{x,-1} - F_{x,-1}^e\right)$$

The exception is for the conventional capital goods sector, where dividends go to zero after exit

```
Fce = Fcmean + Fcmean * Fcgr * eta + nu * (Fc(-1) - Fce(-1))
Fie = Fimean + Fimean * Figr * eta + nu * (Fi(-1) - Fie(-1))
Fke = ifelse(exitk(-1) != 1, Fkmean + Fkmean * Fkgr * eta + nu * (Fk(-1) - Fke(-1)),
    0)
```

The expressions for four-period means and growth rates are given by

```
Fcgr = ifelse(Fc(-2) == 0, 0, (Fc(-1) - Fc(-2))/Fc(-2))
Figr = ifelse(Fi(-2) == 0, 0, (Fi(-1) - Fi(-2))/Fi(-2))
Fkgr = ifelse(Fk(-2) == 0, 0, (Fk(-1) - Fk(-2))/Fk(-2))
Fcmean = mean(c(Fc(-4), Fc(-3), Fc(-2), Fc(-1)))
Fimean = mean(c(Fi(-4), Fi(-3), Fi(-2), Fi(-1)))
Fkmean = mean(c(Fk(-4), Fk(-3), Fk(-2), Fk(-1)))
```

Dividends are distributed to capitalists from productive sectors and the banking sector. For a productive sector x, the basic expression for dividends is profits F_x less autonomous investment I_x, as long as the result is positive,

$$\mathrm{FD}_x = \max\left(F_x - I_x\right)$$

This is then modified to take into account the introduction of innovative capital, as well as a correction for money holdings after "entry". For that last consideration, if previous period money held by the innovative sector, $M_{i,-1}$, plus profits is more than sufficient to pay dividends then they will be paid, otherwise they won't.

```
FDTc = gammaTc * pce(-1) * ec(-1) - CGce
FDTi = gammaTi * pie(-1) * ei(-1) - CGie
FDTk = gammaTk * pke(-1) * ek(-1) - CGcke
FDc = FDc(-1) + iota * (FDTc - FDc(-1))
FDi = FDi(-1) + iota * (FDTi - FDi(-1))
FDk = FDk(-1) + iota * (FDTk - FDk(-1))
```

> **Note:** Alter this for dividend policy.
>
> **Proposal:** Have the firms traget a specific yield ratio (γ^T). In this case, the optimal dividend would be (taking into account expected capital gains): $FD^T = \gamma^T * p_{x,e}(-1) * e_x(-1) - CG_x^e$. We could have a slowly adjusting dividend $FD = FD(-1) + \omega * (FD^T - FD(-1))$ so as to avoid too much fluctuation on dividends.

Banks have zero net worth, so they pay out their income from loan repayments as dividends. However, if the conventional capital goods sector goes bankrupt then banks must write off the loans, which is a loss that offsets income from loan payments,

```
FDb = rlc * Lc(-1) + ifelse(yk != 0 & Fk >= 0, rlk * Lk(-1), -Lk(-1)) + rli *
    Li(-1)
```

Expectations are set, as for other variables, by an adaptive expectations model,

$$\mathrm{FD}_s^e = \left(1 + \eta\widehat{\mathrm{FD}}_s\right)\overline{\mathrm{FD}}_s + \chi\left(\mathrm{FD}_{s,-1} - \mathrm{FD}_{s,-1}^e\right)$$

Expected dividends for the conventional capital goods sector are zero after exit,

```
FDbe = FDbmean + FDbmean * FDbgr * eta + chi * (FDb(-1) - FDbe(-1))
FDie = FDimean + FDimean * FDigr * eta + chi * (FDi(-1) - FDie(-1))
FDce = FDcmean + FDcmean * FDcgr * eta + chi * (FDc(-1) - FDce(-1))
FDke = ifelse(exitk(-1) != 1, FDkmean + FDkmean * FDkgr * eta + chi * (FDk(-1) -
    FDke(-1)), 0)
```

Four-period averages and growth rates are given by

```
FDbgr = ifelse(FDb(-2) == 0, 0, (FDb(-1) - FDb(-2))/FDb(-2))
FDcgr = ifelse(FDc(-2) == 0, 0, (FDc(-1) - FDc(-2))/FDc(-2))
FDigr = ifelse(FDi(-2) == 0, 0, (FDi(-1) - FDi(-2))/FDi(-2))
FDkgr = ifelse(FDk(-2) == 0, 0, (FDk(-1) - FDk(-2))/FDk(-2))
FDbmean = mean(c(FDb(-4), FDb(-3), FDb(-2), FDb(-1)))
FDcmean = mean(c(FDc(-4), FDc(-3), FDc(-2), FDc(-1)))
FDimean = mean(c(FDi(-4), FDi(-3), FDi(-2), FDi(-1)))
FDkmean = mean(c(FDk(-4), FDk(-3), FDk(-2), FDk(-1)))
```

Chapter 4

Consumption

There are two consumer classes in the model, wage and profit earners. Real consumption by wage earners is c_w and by capitalists is c_c. These are calculated in stages. First, planned consumption is calculated for both classes as a function of disposable income and previous-period wealth,

$$c_{w1} = \xi_{w1} y^e_{dw} + \xi_{w2} v_{w,-1}$$

$$c_{c0} = \xi_{c1} y^e_{dc} + \xi_{c2} v_{c,-1}$$

```
chT = xih1 * ydce + xih2 * vh(-1)
# cw1= xiw1*ydwe + xiw2*vw(-1) cc0=xic1*ydce+xic2*vc(-1)
```

Next, because capitalists' income can be negative, planned consumption is set to zero in case the calculation above gives a negative result,

$$c_{c1} = \frac{1}{2}\left(c_{c0} + |c_{c0}|\right) = \max\left(0, c_{c0}\right)$$

```
# cc1=(cc0+abs(cc0))/2
```

It is possible that planned consumption exceeds the capacity of the economy to produce consumption goods. Denote capital productivity of conventional capital goods by κ_k (in the code it is `prk`) and innovative capital goods by κ_i (`pri` in the code). Then, denoting real capital stocks by k_c and i_c in the consumption goods sector, real output of consumption goods c_s from the previous period is

$$c_s = k_{c,-1}\kappa_k + i_{c,-1}\kappa_i$$

Excess production of consumption goods over planned consumption is x_1 (`exc1` in the code),

$$x_1 = c_s - c_{w1} - c_{c1}$$

This might be negative, in which case actual consumption must be less than planned. The deficit (a negative number, if it is exists) is given by a variable x_2 (`exc2` in the code),

$$x_2 = \frac{1}{2}\left(x_1 - |x_1|\right)$$

```
cs = kc(-1) * prk + ic(-1) * pri
ch = min(cs, chT)
exch = chT - ch
# exc1=cs-cw1-cc1 exc2=(exc1-abs(exc1))/2
```

The deficit is then shared by wage earners and capitalists in proportion to their planned expenditure,

$$c_w = c_{w1} + \frac{c_{w1}}{c_{w1} + c_{c1}} x_2$$

$$c_c = c_{c1} + \frac{c_{c1}}{c_{w1} + c_{c1}} x_2$$

```
# cw=cw1+cw1/(cw1+cc1)*exc2 cc=cc1+cc1/(cw1+cc1)*exc2
```

Once real consumption has been calculated, it is converted to nominal values using the price of consumption goods:

```
Ch = ch * pc
# Cw=cw*pc Cc=cc*pc
```

4.1 Purchasing power of wealth

Real purchasing power out of wealth is nominal wealth deflated by the cost of consumption goods,

$$v_i = \frac{V_i}{p_c}$$

```
vh = Vh/pc
# vc=Vc/pc vw=Vw/pc
```

4.2 Wage earners' disposable income

Nominal disposable income for wage earners is given by the sum of wages from all three sectors:

$$\mathrm{YD}_w = W_c N_c + W_k N_k + W_i N_i$$

Real disposable income is given by two terms, one a deflated version of nominal income, the second a correction of previous-period nominal wealth for price inflation of consumption goods,

$$yd_w = \frac{1}{p_c} \left(\mathrm{YD}_w - \pi_c V_{w,-1} \right)$$

```
YDh = YDw + YDc
YDw = Wc * Nc + Wk * Nk + Wi * Ni
YDc = FDc + FDk + FDb + FDi
ydh = YDh/pc - pic * Vh(-1)/pc
# YDw=Wc*Nc+Wk*Nk+Wi*Ni ydw=YDw/pc-pic*Vw(-1)/pc
```

Expected nominal income in the next period is given by an adaptive expectations model in which wage-earners' base expectation incorporates some proportion of growth during the previous period, adjusted by a term proportional to the previous-period deviation from expectations. Using a bar for an average, and a hat to indicate to a growth rate,

$$\mathrm{YD}_w^e = \left(1 + \eta \widehat{\mathrm{YD}}_w \right) \overline{\mathrm{YD}}_w + \epsilon \left(\mathrm{YD}_{w,-1} - \mathrm{YD}_{w,-1}^e \right)$$

Expected real disposable income is then given by a similar expression to current disposable income, but in terms of expected nominal income,

```
YDhe = YDhmean + YDhmean * YDhgr * eta + epsilon * (YDh(-1) - YDhe(-1))
ydhe = YDhe/pc - (pic * Vh(-1))/pc
# YDwe= YDwmean+YDwmean*YDwgr*eta+epsilon*(YDw(-1)-YDwe(-1)) ydwe=
# YDwe/pc-(pic*Vw(-1))/pc
```

The growth rate and four-period averages of nominal disposable income are given by

```
YDhgr = ifelse(YDh(-2) == 0, 0, (YDh(-1) - YDh(-2))/YDh(-2))
YDhmean = mean(c(YDh(-4), YDh(-3), YDh(-2), YDh(-1)))
# YDwgr=ifelse(YDw(-2)==0,0,(YDw(-1)-YDw(-2))/YDw(-2))
# YDwmean=mean(c(YDw(-4),YDw(-3),YDw(-2),YDw(-1)))
```

4.3 Capitalists' disposable income

Disposable income for capitalists is given by the sum of profit income and capital gains,

$$\mathrm{YD}_c = \mathrm{YP}_c + \mathrm{CG}$$

```
# YDc=YPc + CG
```

As for wage income, the purchasing power of capitalists' nominal wealth is corrected for inflation in the price of consumption goods,

```
# ydc=YDc/pc - pic*Vc(-1)/pc
```

Also, expected real and nominal income is calculated similar to that for wage-earners, except that expectations are in terms of dividends and capital gains,

```
# YDce1=YPce + CGce+CGke+CGie YDce=(YDce1+abs(YDce1))/2
# ydce=YDce/pc-(pic*Vc(-1))/pc
```

Profit income is given by dividends FD_x from different sectors $x \in \{c, k, b, i\}$,

$$\mathrm{YP}_c = \mathrm{FD}_c + \mathrm{FD}_k + \mathrm{FD}_b + \mathrm{FD}_i$$

An equivalent sum applies to expected values of these variables,

```
# YPc=FDc+FDk+FDb+FDi YPce=FDce+FDke+FDie+FDbe
```

Capital gains are determined by the expected change in the price per share in different sectors multiplied by the number of shares e_j held in the previous period,

$$\mathrm{CG} = \sum_{j \in \{c,k,i\}} e_{j,-1} \Delta p_j^e$$

There is a special consideration in the first period in which shares are held in the innovative sector,

```
CG = CGi + CGc + CGk
```

In terms of individual sectors, capital gains are given by the terms in the sum above, but with some conditions. For the conventional capital sector, capital gains are zero if real output has gone to zero, while for the innovative sector it is necessary to catch discontinuities when it is first introduced.

> **Note:** This could be modified in future, because I think that, as calculated, CG can differ from the sum of CGc, CGk and CGi.

```
CGc = ec(-1) * (pce - pce(-1))
CGk = ifelse(yk != 0, ek(-1) * (pke - pke(-1)), 0)
CGi = ei(-1) * (pie - pie(-1))
```

Real capital gains are calculated as nominal gains divided by the price of holdings, which is the product of price per share and the number of shares held, with some checks for whether innovative capital has been introduced or not,

```
cgc = CGc/(pce(-1) * ec(-1))
cgk = ifelse(yk != 0 & Fk >= 0, CGk/(pke(-1) * ek(-1)), 0)
cgi = CGi/(pie(-1) * ei(-1))
```

Nominal expected capital gains are given by an adaptive expectations model as used elsewhere. For sector x,

$$\mathrm{CG}_x^e = \left(1 + \eta\widehat{\mathrm{CG}}_x\right)\overline{\mathrm{CG}}_x + \theta\left(\mathrm{CG}_{x,-1} - \mathrm{CG}_{x,-1}^e\right)$$

```
CGce = CGcmean + CGcmean * CGcgr * eta + theta * (CGc(-1) - CGce(-1))
CGke = ifelse(exitk(-1) != 1, CGkmean + CGkmean * CGkgr * eta + theta * (CGk(-1) -
    CGke(-1)), 0)
CGie = CGimean + CGimean * CGigr * eta + theta * (CGi(-1) - CGie(-1))
```

Real expected capital gains are deflated by the same price as for observed capital gains,

```
cgce = CGce/(ec(-1) * pce(-1))
cgke = ifelse(exitk(-1) != 1, CGke/(ek(-1) * pke(-1)), 0)
cgie = CGie/(ei(-1) * pie(-1))
```

Finally, the expressions for growth rates and four-period mean values are given by

```
CGcgr = ifelse(CGc(-2) == 0, 0, (CGc(-1) - CGc(-2))/CGc(-2))
CGigr = ifelse(CGi(-2) == 0, 0, (CGi(-1) - CGi(-2))/CGi(-2))
CGkgr = ifelse(CGk(-2) == 0, 0, (CGk(-1) - CGk(-2))/CGk(-2))
CGcmean = mean(c(CGc(-4), CGc(-3), CGc(-2), CGc(-1)))
CGimean = mean(c(CGi(-4), CGi(-3), CGi(-2), CGi(-1)))
CGkmean = mean(c(CGk(-4), CGk(-3), CGk(-2), CGk(-1)))
```

Chapter 5

Money and banking

Total money in the system is M_s, the sum of money held by wage-earners, M_w, capitalists, M_c, and innovators. Innovators hold a stock M_i of cash after their IPO, which is used to reduce the quantity of loans or pay dividends.

```
Ms = Ld
# Hidden Equation Ms = Mh Ms=Mw+Mc+Mi
```

5.1 Money held by wage earners

For wage earners, their money holdings constitute their wealth,

```
# Mw=Vw
```

Banks do not pay interest on the money holdings of wage-earners and do not issue them loans, so wage-earners' money holdings are passive, and might be held as physical cash or cash balances at the bank.

5.2 Money held by capitalists

Capitalists wish to hold money both for transactions (money for consumption, M_c^D) and as a store of wealth (money for finance, $M_c^{D,f}$). For transactions purposes, capitalists' desired money holdings are given as a fraction β of nominal consumption,

$$M_c^D = \beta C_c$$

```
# McD=beta*Cc
```

Actual holdings of cash are given by the difference between nominal wealth V_c and wealth held as non-money equities,

$$M_c = V_c - \left(V_{ec} - M_c^{D,f}\right) = M_c^{D,f} + V_c - V_{ec}$$

```
# Mc=McDf+Vc -Vec
Mh = Vh - pce * ec - pke * ek - pie * ei
```

Money held for financial purposes is discussed later in the context of portfolio allocation.

5.3 Money held by innovators

After innovative firms go public, they have a store of cash. Initially they have no cash on hand, and they operate with loans, so just before the IPO the value of their money holdings is zero. When they go public, they issue an exogenously specified number of shares of new equity, e_i^s at the market's expected price, p_i^e. This gives a one-time cash injection, while after the IPO they add to cash holdings from their profits, F_i, net of new investment I_i and dividends FD_i. The innovative firms put a priority on growth and paying dividends to investors, so from the time they go public onwards, they use their money for those purposes first,

$$M_{i,\text{left}} = M_{i,-1} + F_i + e_i^s p_i^e - I_i - \mathrm{FD}_i$$

```
# Mileft=Mi(-1)+Fi+esi*pie-Ii-FDi
```

They then pay down their loans if they can, by making a regular payment calculated as total outstanding loans L_i from the previous period divided by the loan term n_{loans}. After that payment, money holdings are

$$M_i = \max\left(M_{i,\text{left}} - \frac{L_{i,-1}}{n_{\text{loans}}}, 0\right)$$

$$L_i = L_{i-1} - \min\left(\frac{L_{i,-1}}{n_{\text{loans}}}, M_{i,\text{left}}\right)$$

```
# Mi=max(Mileft-Li(-1)/nLoans,0) Li=Li(-1)-min(Li(-1)/nLoans,Mileft)
```

Note that $M_{i,\text{left}}$ can be negative, or is insufficient to make a full payment, in which case innovative firms take on new loans.

5.4 Loans

Loans to the innovative sector are discussed above. For the consumer and conventional capital goods sectors, the basic formula is that current outstanding loans equal previous period loans, plus investment net of profits (or zero, if profits exceed investment), less the value of new equity issues. For an arbitrary sector $x \in \{c, k\}$,

$$L_x = L_{x,-1} + \max\left(I_x - F_x, 0\right) - e_x^s p_{x,e}$$

Note that the net effect may be either an addition of new loans or a paying down of prior loans. Also, for the conventional capital goods sector, if output or profits are zero because the innovative sector has superseded it, then there is no demand for loans. These are captured in the following equations:

```
Lc = Lc(-1) + Ic - esc * pce - FUc
Lk = ifelse(yk != 0 & Fk >= 0, Lk(-1) + Ik - esk * pke - FUk, 0)
Li = Li(-1) + Ii - esi * pie - FUi
```

> **Note:** This is one place to alter the behavior towards dividends. Firms should strive to meet dividend payments.
>
> **Proposal:** Have loans as the buffer stock: $L_x = L_x(-1) + I_x - RE_x - e_x^s * p_{x,e}$ where RE_x are retained earnings: $RE_x = F_x - FD_x$

The total demand for loans is the sum of loans to each productive sector,

$$L_d = \sum_{x \in \{c,k,i\}} L_x$$

```
Ld = Lc + Lk + Li
```

Banks follow an accomodationist policy in which the demand for loans is automatically supplied (although at different rates of interest to different sectors),

$$L_s = L_d$$

5.4.1 Interest rates

Interest rates are set at different levels for different sectors depending on their historical average profit rates r_x^a compared to a target (or benchmark) profit rate r_b. There is a base, or reference interest rate r_l, and then sector interest rates are calculated as

$$r_{lx} = r_l \left[1 + \frac{1}{1 + e^{\kappa(r_x^a - r_b)}} \right]$$

```
rlc = rl * (1 + 1/(0.5 + exp(kappa * (rca - rb))))
rlk = ifelse(exitk(-1) == 0, rl * (1 + 1/(0.5 + exp(kappa * (rka - rb)))), 0)
rli = rl * (1 + 1/(0.5 + exp(kappa * (ria - rb))))
```

If realized returns equal the benchmark, then the rate is equal to $(3/2)r_l$. If realized profit rates are higher than the benchmark, then the interest rate is lower, and vice versa.

> **Note:** If the expression used 1/2 instead of 1 in the sum, then `rlx` would equal `rl` when `rxa = rb`.

Real interest rates are deflated by the cost of capital. For the innovative and conventional capital goods sectors that is given by the inflation rate of innovative and conventional capital prices, respectively. For the consumption goods sector it is given by the average inflation rate of capital used in the sector,

```
rrlc = (1 + rlc)/(1 + pic) - 1
rrli = (1 + rli)/(1 + pii) - 1
rrlk = ifelse(exitk(-1) == 0, (1 + rlk)/(1 + pik(-1)) - 1, 0)
```

5.4.2 Leverage

Loan leverage in sector x, λ_x, is calculated in each period as the ratio of outstanding loans to the replacement value of the capital stock at current prices,

$$\lambda_x = \frac{L_x}{p_k k_x + p_i i_x}$$

After checking that the sector is operating, in model code this becomes

```
lambdac = Lc/(kc * pk + ic * pi)
lambdai = Li/(ki * pk + ii * pi)
lambdak = ifelse(exitk(-1) == 0, Lk/(kk * pk), 0)
```

Chapter 6

Investment plans and capital stock

Total nominal investment in sector x is denoted I_x. It is equal to real investment in each type of capital (i_{xk} or i_{xi}), multiplied by the price,

$$I_x = p_k i_{xk} + p_i i_{xi}$$

In the code, i_{xk} is `inxk`, so, ensuring the sector is active, the model equations are

```
Ic = inck * pk + inci * pi
Ik = ifelse(exitk(-1) == 0, ink * pk, 0)
Ii = inii * pi
```

Note that innovative capital is never used by the conventional capital goods sector.

Of the total investment required, some must be financed if profits are insufficient. The excess of investment over profits in sector x, if it is positive, is given by I_{fx}.

```
Ifk = ifelse(exitk(-1) != 1 & inktot > 0, max(Ik - FUke, 0), 0)
Ifi = max(Ii - FUie, 0)
Ifc = max(Ic - FUce, 0)
```

> **Note:** Have to modify here also to get a fixed dividend, because the amount available for needed investment should be after dividends.
>
> **Proposal:** Change the I_{fx} function to take into account the dividend strategy $I_{f,x} = I_x - RE_x$

The share of investment funded by equity emissions in sector x is denoted by Ψ_x, where

$$\Psi_x = \frac{1}{1 + e^{\psi_x (r_x^T - cg_{x,-1} - r_{lx})}}$$

In this expression, r_x^T is the target return on capital, $cg_{x,-1}$ are real capital gains from the previous period, r_{lx} is the interest rate on loans, and ψ_x is a calibration parameter. With some conditionals to ensure the sector is active, the model expressions are

```
Psik = ifelse(exitk(-1) != 1 & inktot > 0, 1/(1 + exp(psik * (rkT - cgk(-1) -
    rlk))), 0)
Psii = 0 + (t > (n + entry)) * (1/(1 + exp(psii * (riT - cgi(-1) - rli))))
Psic = 1/(1 + exp(psic * (rcT - cgc(-1) - rlc)))
```

New equity issue in sector x is e_x^s, and is determined by the nominal value of the needed finance and the previous period value of the equity,

$$e_x^s = \frac{\Psi_x I_{fx}}{p_{e,x,-1}}$$

Also, in its IPO, the innovative sector issues an exogenously specified number of shares,

```
esc = (Psic * Ifc)/pce(-1)
esk = ifelse(exitk(-1) != 1 & inktot > 0, (Psik * Ifk)/pke(-1), 0)
esi = (psii * Ifi)/pie(-1)
```

Total equities are then given by a running total,

$$e_x = e_{x,-1} + e_x^s$$

```
ec = ec(-1) + esc
ek = ek(-1) + esk
ei = ei(-1) + esi
```

> **Note:** How about repos? Does that change anything? It can raise the share price, and less risky than dividends, which are two reasons that it's an increasingly common alternative to dividends, especially in expanding sectors.

6.1 Retained profits

Retained profits are calculated as the difference between total profits and dividends as `REk`, `REi`, `REc`,

```
REk = ifelse(exitk(-1) != 1 & inktot > 0, Fk - FDk, 0)
REi = Fi - FDi
REc = Fc - FDc
REke = ifelse(exitk(-1) != 1 & inktot > 0, Fke - FDk, 0)
REie = Fie - FDi
REce = Fce - FDc
```

6.2 Capital stock

The real capital stock in sector x consists of conventional (k_x) and innovative (i_x) stocks. The change in stocks is given by new real investment i_{xk} and i_{xi}, net of depreciation,

$$k_x = k_{x,-1} + i_{xk} - d(k_x)$$

$$i_x = i_{x,-1} + i_{xi} - d(i_x)$$

Taking into account the fact that innovative capital is not used by the conventional capital goods sector, and ensuring that the sector is active, in the model code the corresponding equations are

```
kc = kc(-1) + inck - depkc
kk = ifelse(exitk(-1) == 0, kk(-1) + ink - depkk, 0)
ic = ic(-1) + inci - depic
ii = ii(-1) - depii + inii
```

6.2.1 Depreciation

In general, depreciation in sector x of capital type k, with a lifetime n, can be written

$$d(k_x) = \sum_{\tau=-1}^{-n} \alpha_\tau i_{xk,\tau}, \text{ where } \sum_{\tau=-1}^{-n} \alpha_\tau = 1$$

In the present model, $n = 19$ and

$$\alpha_{-1} = -e^{-n},$$

$$\alpha_\tau = e^{-(n+\tau)}\left(1 - \frac{1}{e}\right),\ \tau \in \{-2, \ldots, -n\}$$

In code, this is

```
# depkk=ink(-1)*(-5.602796e-09+1.522998e-08)+ink(-2)*(-1.522998e-08+4.139938e-08)+ink(-3)*(-4.139938e-0
# depki=inik(-1)*(-5.602796e-09+1.522998e-08)+inik(-2)*(-1.522998e-08+4.139938e-08)+inik(-3)*(-4.139938
# depii=inii(-1)*(-5.602796e-09+1.522998e-08)+inii(-2)*(-1.522998e-08+4.139938e-08)+inii(-3)*(-4.139938
# depkc=inck(-1)*(-5.602796e-09+1.522998e-08)+inck(-2)*(-1.522998e-08+4.139938e-08)+inck(-3)*(-4.139938
# depic=inci(-1)*(-5.602796e-09+1.522998e-08)+inci(-2)*(-1.522998e-08+4.139938e-08)+inci(-3)*(-4.139938
depkk = inkk(-20)
depkc = inck(-20)
depii = inii(-20)
depic = inic(-20)
```

> **Note:** Some alternative formulations include:
>
> 1. Existing: $\alpha_{-1} = -e^{-n}$; $\alpha_\tau = e^{-(n+\tau)}(1 - 1/e)$, $\tau \in \{-2, \ldots, -n\}$
> 2. Fixed lifetime: $\alpha_{-n} = 1$; $\alpha_\tau = 0$ for $\tau \in \{-1, \ldots, -n+1\}$
> 3. Straight-line depreciation: $\alpha_\tau = 1/n$
> 4. Exponential decay (or perpetual inventory dynamics), $\alpha_\tau = \delta(1-\delta)^{1-\tau}$, $n = \infty$
>
> With option #4, it is not necessary to keep track of prior investment, unless the depreciation rate changes.

6.2.2 Investment

For a sector that is fully established as a going concern, desired investment is the sum of desired capacity expansion plus depreciation. The capacity growth rate in sector x depends on expected capacity utilization u^e_x, the real interest rate $\tilde{r}_{lx}$ (deflated by the price of capital goods), average leverage $\bar{\lambda}_x$, and Tobin's q, q_x,

$$g_x = \eta_{0x} + \eta_1 u^e_x - \eta_2 \tilde{r}_{lx}\bar{\lambda}_x + \eta_3 q_{x,-1}$$

When the innovative capital goods sector first starts, it is not yet established as a going concern, and must set a target capacity expansion rate τ. With checks to make sure the sector is active, the model equations are

```
gk = ifelse(exitk(-1) == 0, eta0k + eta1 * uke - eta2 * rrlk * lambdakprev +
    eta3 * qkprev, 0)
gi = eta0k + eta1 * uie - eta2 * rrli * lambdaiprev + eta3 * qiprev
gc = eta0c + eta1 * uce - eta2 * rrlc * lambdacprev + eta3 * qcprev
```

> **Note:** How responsive is firm investment to the interest rate? My understanding was that it was weak at best. Home construction is responsive, and so are financing decisions, but not (I thought) the volume of investment by firms.

The average leverage is given by the average over the prior four periods,

```
lambdacprev = (lambdac(-4) + lambdac(-3) + lambdac(-2) + lambdac(-1))/4
lambdaiprev = (lambdai(-4) + lambdai(-3) + lambdai(-2) + lambdai(-1))/4
lambdakprev = (lambdak(-4) + lambdak(-3) + lambdak(-2) + lambdak(-1))/4
```

> **Note:** We could introduce a Minskian dynamic by comparing λ to a normal level, which gets looser during a boom.

Proposal: I would simplify the growth function and remove the financial terms, keeping only the capacity utilisation term. But then I would introduce a form of credit constrain by which the banks only allow for investment to be such that the new debt burden cannot be above a certain share of expected gross profits. Gross profits is equal to Sales minus the wage bill

$$CF_x = Y_x - W_x N_x$$

and thus we would have that (τ is the maximum debt burden): $\tau = \frac{i_{l,x} L_x^M}{CF_x}$ where L_x^M denotes the maximum level of loans acceptable for that firm.The whole thing gets somewhat complicated. I've written in out on a piece of paper that I scanned and added to this folder (called antoine_scan_1_out_2 and antoine_scan_2_out_2 (sorry for the red ink). I'm not sure it's the way forward hence my hedging behavior in not investing the time in writing it in this document.

6.2.3 Consumption goods sector

Investment in the consumption goods sector can come from either innovative or conventional capital. Total real gross investment in terms of equivalent output (`invyc` in the model code) is denoted $i^y_{c,\text{tot}}$. It is given as a multiple of prior period real output plus depreciation of the two types of capital (or zero, if that value is negative),

$$i^y_{c,\text{tot}} = \max\left(g_c y_{c,-1} + \kappa_k d(k_c) + \kappa_i d(i_c), 0\right)$$

```
invyc = max(gc * yc(-1) + depkc * prk + depic * pri, 0)
```

Investment in either innovative or conventional capital in the consumption goods sector is given by an expression that depends on total desired investment (in terms of productive capacity, $i^y_{c,\text{tot}}$) and the available supply for sale of innovative (i_s) and conventional (k_s) capital. Those expressions are

$$i_{c,i} = z_{11}\left[z_{21} i_s + (1 - z_{21})\frac{i^y_{c,\text{tot}}}{\kappa_i}\right] + (1 - z_{11}) z_{31}\left[z_{41} i_s + (1 - z_{41})\frac{i^y_{c,\text{tot}} - \kappa_k k_s}{\kappa_i}\right]$$

$$i_{c,k} = z_{11} z_{21}\left[z_{41} k_s + (1 - z_{41})\frac{i^y_{c,\text{tot}} - \kappa_i i_s}{\kappa_k}\right] + (1 - z_{11})\left[z_{31} k_s + (1 - z_{31})\frac{i^y_{c,\text{tot}}}{\kappa_k}\right]$$

They only hold if desired investment is positive, which is checked through the logical expression `invyc>0`, which is equal to zero if it is false, and one if it is true. The model equations are:

```
inci = (invyc > 0) * (z11 * (z21 * is + (1 - z21) * invyc/pri) + (1 - z11) *
    z31 * (z41 * is + (1 - z41) * (invyc - ks * prk)/pri))
inck = (invyc > 0) * (z11 * z21 * (z41 * ks + (1 - z41) * (invyc - is * pri)/prk) +
    (1 - z11) * (z31 * ks + (1 - z31) * invyc/prk))
```

The parameters $z_{i1}, i \in \{1, \ldots, 4\}$ take on values of either zero or one. They play the following roles:

1. z_{11} is the desired fraction from innovative capital, and is equal to zero if innovative capital is more costsly and one if conventional capital is more costly;
2. z_{21} corrects for z_{11} based on total capital requirements compared to available supply of innovative capital, and is equal to zero if total requirements exceed the available innovative capital and equal to one otherwise;
3. z_{31} corrects for $(1 - z_{11})$ based on total capital requirements compared to the available supply of conventional capital, and is equal to zero if total requirements exceed the available conventional capital and equal to one otherswise;
4. z_{41} allows for the possibility that desired capital exceeds the amount available from either innovative or conventional sources.

```
z11 = (invyc > 0) * (costk > costi)
z21 = (invyc > 0) * (invyc > is * pri)
z31 = (invyc > 0) * (invyc > ks * prk)
z41 = (invyc > 0) * (invyc > is * pri + ks * prk)
```

For z_{11}, the cost of buying capital goods is calculated per unit of output; that is, it is the price per unit of capital good divided by capital productivity,

```
costk = pk/prk
costi = pi/pri
```

> **Note:** We could consider more complex portfolio choices for `z11`, rather than just one or zero. (Also, in principle, cost should be discounted over the lifetime of capital, but here, by holding productivity and price fixed, then discounting gives rise to the same factor for each type of capital and it factors out when comparing costs.)

Next, we show some alternatives.

6.2.3.1 There is sufficient capital of both types ($z_{21} = z_{31} = z_{41} = 0$)

$$i_{c,i} = z_{11} \frac{i^y_{c,\mathrm{tot}}}{\kappa_i}$$
$$i_{c,k} = (1 - z_{11}) \frac{i^y_{c,\mathrm{tot}}}{\kappa_k}$$

6.2.3.2 There is sufficient conventional capital but not innovative capital ($z_{21} = 1$, $z_{31} = z_{41} = 0$)

$$i_{c,i} = z_{11} i_s$$
$$i_{c,k} = \frac{i^y_{c,\mathrm{tot}}}{\kappa_k} - z_{11} \frac{\kappa_i}{\kappa_k} i_s$$

6.2.3.3 There is sufficient innovative capital but not conventional capital ($z_{31} = 1$, $z_{21} = z_{41} = 0$)

$$i_{c,i} = \frac{i^y_{c,\mathrm{tot}}}{\kappa_i} - (1 - z_{11}) \frac{\kappa_k}{\kappa_i} k_s$$
$$i_{c,k} = (1 - z_{11}) k_s$$

6.2.3.4 There is insufficient capital of either kind ($z_{21} = z_{31} = z_{41} = 1$)

$$i_{c,i} = i_s$$
$$i_{c,k} = k_s$$

6.2.4 Capital goods sectors

For the capital goods sectors, expected output expansion reflects sales, but the sector must also produce the capital goods it needs for itself in order to produce the capital goods for sale. So, for capital goods sector x,

$$\Delta y_{x,\mathrm{self}} = \frac{1}{\kappa_x} g_x y_x \equiv g_{x,x} y_x$$

The relevant model equations are

```
gkk = ifelse(exitk(-1) == 0, gk/prk, 0)
gii = gi/pri
```

Note: gii does not appear in any equations. **Solution:** no need for gii indeed

6.2.4.1 Conventional capital goods sector

Regardless of its plans, output from the conventional capital goods sector is limited by its existing stock of capital. So, $\kappa_k k_{k,-1}$ sets a floor on its own purchases of conventional capital goods. But it will also not invest more than planned, so $g_{k,k} k_{k,-1} + d(k_k)$ (or zero, if this expression is negative) also sets a floor on its own purchases of conventional capital goods. This gives the model expression

```
ink = ifelse(exitk(-1) == 0, min(kk(-1) * prk, max(0, gkk * kk(-1) + depkk)),
    0)
```

Of the total amount produced by the conventional capital goods sector, some is used for its own investment needs, while a remainder k_s is sold,

```
ks = ifelse(exitk(-1) == 0, kk(-1) * prk - ink, 0)
```

Total investment demand for conventional capital goods is given by the sum across sectors,

```
inktot = ink + inck
```

6.2.4.2 Innovative capital goods sector

There are constraints on the total available innovative capital during and immediately after the first period in which the innovative capital goods sector enters the market, because it must produce for its own needs as well as for the market. To determine how much of its own ouptut to use for next-period investment ($i_{i,i}$) and how much to sell (s_i), the innovative sector identifies a desired market share, ρ, and a productive capacity growth τ.

Initial-period production is determined by the productivity of conventional capital, and is split between the amount needed for investment goods in the sector and the amount sold. Because the amount of conventional capital in the sector in the first period is the first-period investment, $i_{i,k} = k_i$, this is

$$y_i = \kappa_k i_{i,k} = i_{i,i,+1} + s_i$$

The value of the amount sold is equal to the desired market share multiplied by the expected size of the market, which is calculated as the previous-period value of production of conventional capital goods plus the amount the innovative sector expects to sell, or

$$p_i s_i = \rho\left(p_i s_i + Y_{k,-1}\right) \implies s_i = \frac{1}{p_i}\frac{\rho Y_{k,-1}}{1-\rho}$$

The expression for the price is discussed elsewhere. Production of innovative capital for next-period investment is given by the amount of depreciated conventional capital plus the amount for desired expansion. Using the notation above for the general expression for depreciation,

$$i_{i,i,+1} = \alpha_{-1} i_{i,k} + \frac{\tau y_i}{\kappa_i}$$

Substituting this expression into the one for y_i, above and solving for first-period investment in conventional capital gives

$$i_{i,k} = \frac{\kappa_i}{\kappa_k}\frac{s_i}{1-\tau-\alpha_{-1}(\kappa_i/\kappa_k)} = \frac{\kappa_i}{\kappa_k}\frac{1}{1-\tau-\alpha_{-1}(\kappa_i/\kappa_k)}\frac{1}{p_i}\frac{\rho Y_{k,-1}}{1-\rho}$$

Note: This differs from the equation as currently implemented, but I think the units work out, so there might be an error in the code. **Solution:** I've done the computation again and I think we made a mistake in your computation

$$i_{i,i,+1} = \alpha_{-1} i_{i,k} + \frac{\tau y_i}{\kappa_i}$$

implies that

$$i_{i,i,+1} = \frac{\alpha_{-1} i_{i,k} \kappa_i + \tau y_i}{\kappa_i}$$

and thus including it in the above equation we get

$$\kappa_i \kappa_k i_{i,k} = \alpha_{-1} i_{i,k} \kappa_i + \tau \kappa_k i_{i,k} + \kappa_i s_i$$

which leads the following formulation.

$$i_{i,k} = \frac{\kappa_i}{\kappa_k} \frac{s_i}{\kappa_i - \tau - \alpha_{-1}(\kappa_i/\kappa_k)} = \frac{\kappa_i}{\kappa_k} \frac{1}{\kappa_i - \tau - \alpha_{-1}(\kappa_i/\kappa_k)} \frac{1}{p_i} \frac{\rho Y_{k,-1}}{1-\rho}$$

Here is the current implementation:

$$i_{i,i} = 0,\ i_{i,k} = \frac{\rho \kappa_i Y_{k,-1}}{(1-\rho) p_i \kappa_k (\kappa_i - e^{1-n} - \tau)},\ \text{first period}$$

After the initial period, all investment is in innovative capital. The investment function, which says how much the firm wishes to invest, is similar to that for the consumption goods sector, but the sector is constrained by its own production in the previous period, which is $\kappa_i i_{i,-1} + \kappa_k k_{i,-1}$,

$$i_{i,i} = \min\left[\kappa_i i_{i,-1} + \kappa_k k_{i,-1}, \frac{1}{\kappa_i} \max\left(g_i y_{i,-1} + \kappa_k d(k_i) + \kappa_i d(i_i), 0\right)\right],\ i_{i,k} = 0,\ \text{after first period}$$

The sector produces capital goods for itself as well as for sale. The sold component is i_s, which is calculated as the difference between total production and the amount needed for own investment.

```
inii = (gi * yi(-1) + depii * pri)/pri
is = ii(-1) * pri - inii
```

Chapter 7

Production and pricing

7.1 Output

Total real output in sector x, y_x, is given by the sum of demands, with one exception. The exception is that in the first period of production of innovative capital, entrepreneurs are aiming to take market share from conventional capital, and so they produce a fixed amount that will allow them to establish that market share, equal to the amount of conventional capital they have acquired multiplied by capital productivity,

```
yc = cc + cw
yi = inii + inci
yk = ifelse(exitk(-1) != 1 & inktot > 0, ink + inck, 0)
```

Nominal output Y_x is given by real output multiplied by the current price,

```
Yc = yc * pc
Yi = yi * pi
Yk = ifelse(exitk(-1) != 1 & inktot > 0, yk * pk, 0)
```

Expected real output is given by an adaptive expectations model of the form

$$y_x^e = y_{x,-1} + \epsilon\left(y_{x,-1}^e - y_{x,-1}\right)$$

```
yce = yc(-1) + epsilon * (yce(-1) - yc(-1))
yie = yi(-1) + epsilon * (yie(-1) - yi(-1))
yke = ifelse(exitk(-1) == 0, yk(-1) + epsilon * (yke(-1) - yk(-1)), 0)
```

7.2 Utilization

Utilization u_x in sector x is the ratio of actual to potential real output using previous period capital stock,

$$u_x = \frac{y_x}{\kappa_k k_{x,-1} + \kappa_i i_{x,-1}}$$

```
uc = yc/(kc(-1) * prk + ic(-1) * pri)
ui = yi/(ii(-1) * pri)
uk = ifelse(exitk(-1) != 1 & inktot > 0, yk/(kk(-1) * prk), 0)
```

Expected utilization is given by an equivalent expression, but in terms of potential output,

```
uce = yce/(kc(-1) * prk + ic(-1) * pri)
uie = yie/(ii(-1) * pri)
uke = ifelse(exitk(-1) == 0, yke/(kk(-1) * prk), 0)
```

Within each sector there are also utilization rates for specific types of capital $u_{i,x}$ or $u_{k,x}$ in sector x, whether conventional or innovative. As a first step, utilization of innovative capital is computed (and is zero for the conventional capital goods sector by assumption). It is presumed that those sectors using innovative capital will employ it fully if possible, so the core expression is

$$u_{i,x} = \min\left(1, \frac{y_x}{\kappa_i i_{x,-1}}\right)$$

However, some edge cases have to be considered:

1. In the first period of operation in the consumption goods sector, utilization of innovative capital is set to 100%, because it is presumably well below the level needed to complete all orders for consumption goods;
2. The innovative capital goods sector has to actually be operating and it behaves differently in its first period of operation;
3. Orders must be placed for innovative capital goods for next period before the sector will produce anything (as captured by `inii+inci` in the expressions below).

```
uic1 = ifelse(ic(-1) == 0, 1, 0) + ifelse(ic(-1) > 0, max(0, min(yc/(pri * ic(-1)),
    1)), 0)
uic = uic1
uii1 = ifelse(t <= (n + entry) & t > (n + 1), ifelse(ii(-1) != 0, max(0, min(1,
    yi/(pri * ii(-1)))), 1), ifelse(t > (n + entry), max(0, min(1, yi/(pri *
    ii(-1)))), 0))
uii = ifelse(t <= (n + entry) & t > (n + 1), ifelse((inii + inci) != 0, ifelse(ii(-1) ==
    0, 1, uii1), 0), 0) + ifelse(t > (n + entry), ifelse((inii + inci) != 0,
    uii1, 0), 0)
```

Expected values are computed using similar equations, but with fewer checks needed because expectations are formed during and after the awkward first period of introducing innovative capital,

```
uice = min(yce/(pri * ic(-1)), 1)
uiie = ifelse(!firstPeriodInov, ifelse(ii(-1) == 0, 1, min(yie/(pri * ii(-1)),
    1)), 0)
```

Next, utilization of conventional capital is computed as a residual. Generically, it is given by

$$u_{k,x} = \max\left(0, \frac{y_x - u_{i,x}\kappa_i i_{x,-1}}{\kappa_k k_{x,-1}}\right)$$

With checks to ensure that it is actually in use, and keeping in mind that the conventional capital goods sector only using conventional capital, the expressions are

```
ukc = ifelse(kc(-1) == 0, 1, 0) + ifelse(kc(-1) > 0, max(0, (yc - uic * pri *
    ic(-1))/(prk * kc(-1))), 0)
ukk = ifelse(exitk(-1) != 1 & inktot > 0, yk/(prk * kk(-1)), 0)
```

Expectations are given by similar expressions,

```
ukce = min((yce - uice * pri * ic(-1))/(prk * kc(-1)), 1)
ukke = ifelse(exitk(-1) == 0, yke/(prk * kk(-1)), 0)
```

7.3 Goods prices and the markup

Prices are set via a target-return markup. Given a markup ϕ_x in sector x and smoothed expected unit costs NUC_x, the price is given by

$$p_x = p_{x,-1} + \zeta * [(1 + \phi_x) * \mathrm{NUC}_x - p_{x,-1}]$$

With checks that the sector is active, this is given in code by

```
pc = pc(-1) + zeta * ((1 + phic) * NUCc - pc(-1))
pi = pi(-1) + zeta * ((1 + phii) * NUCi - pi(-1))
pk = ifelse(exitk(-1) == 0, pk(-1) + zeta * ((1 + phik) * NUCk - pk(-1)), 0)
```

7.3.1 Unit costs

In this economy, unit costs are labor costs, computed as the ratio of the nominal wage bill to total output. The nominal wage bill is given by the unit wage W_x multiplied by labor employed to operate either conventional or innovative capital, $u_{kx}k_{x,-1}/\ell_k + u_{ix}i_x/\ell_i$. Total output is given by utilization multiplied by capital productivity multiplied by the capital stock for each type of capital, and then summed. So, unit costs are

$$\mathrm{UC}_x = \frac{W_x}{u_{kx}\kappa_{kx}k_{x,-1} + u_{ix}\kappa_i i_{x,-1}}\left(\frac{u_{kx}k_{x,-1}}{\ell_k} + \frac{u_{ix}i_{x,-1}}{\ell_i}\right)$$

In the code, terms are rearranged to form the equivalent expression

$$\mathrm{UC}_x = W_x \frac{u_{kx}k_{x,-1}\ell_i + u_{ix}i_{x,-1}\ell_k}{(u_{kx}\kappa_{kx}k_{x,-1} + u_{ix}\kappa_i i_{x,-1})\,\ell_x\ell_i}$$

With checks for whether the sector is operating, the expressions are

```
UCc = Wc * (uic * ic(-1) * lk + ukc * kc(-1) * li)/((ukc * prk * kc(-1) + uic *
    pri * ic(-1)) * li * lk)
UCi = Wi/(pri * li)
UCk = ifelse(exitk(-1) != 1 & inktot > 0, Wk/(prk * lk), 0)
```

Note: For the green economy, we might want to add energy and materials costs to unit costs.

Expected unit costs in the next period UC^e_x are given by an equivalent expression but using expected utilization.

```
UCke = ifelse(exitk(-1) == 0, Wk(-1)/(prk * lk), 0)
UCie = Wi(-1)/(pri * li)
UCce = Wc(-1) * (uice * ic(-1) * lk + ukce * kc(-1) * li)/((ukce * prk * kc(-1) +
    uice * pri * ic(-1)) * li * lk)
```

Finally, the smoothed expected utilization, which is used in the pricing equation, is given by an adaptive expectations model

$$\mathrm{NUC}_x = \mathrm{NUC}_{x,-1} + \zeta\,(\mathrm{UC}^e_x - \mathrm{NUC}_{x,-1})$$

```
NUCk = ifelse(exitk(-1) == 0, NUCk(-1) + zeta * (UCke - NUCk(-1)), 0)
NUCi = NUCi(-1) + zeta * (UCie - NUCi(-1))
NUCc = NUCc(-1) + zeta * (UCce - NUCc(-1))
```

7.3.2 Markups

Markups ϕ_x are set to achieve a target return on capital. For sector x this is denoted r^T_x (or `rxT` in the code). The generic expression is

$$\phi_x = r^T_x \frac{p_{k,-1}k_{x,-1} + p_{i,-1}i_{x,-1}}{\mathrm{NUC}_x y^e_x}$$

If actual output meets expectations, and unit costs match smoothed expectations, then when this markup is applied to total unit costs it will give a return r_x^T when divided by the value of the capital stock from the previous period. Keeping in mind that only conventional capital is use in the conventional capital goods sector, checking for the special conditions in the first period of operation of the innovative capital goods sector, and ensuring the sector is operating, the equations are

```
phii = riT * (pk(-1) * ki(-1) + pi(-1) * ii(-1))/(NUCi * yie)
phic = rcT * (pk(-1) * kc(-1) + pi(-1) * ic(-1))/(NUCc * yce)
phik = ifelse(exitk(-1) == 0, rkT * (pk(-1) * kk(-1))/(UCke * yke), 0)
```

7.3.3 Inflation rate

Price inflation is given simply by

$$\pi = \frac{p - p_{-1}}{p_{-1}}$$

After checking that the price is defined, the equations in the model code are

```
pic = (pc - pc(-1))/pc(-1)
pii = (pi - pi(-1))/pi(-1)
pik = ifelse(exitk(-1) == 0, (pk - pk(-1))/pk(-1), 0)
```

In the consumption goods sector we are also interested in the inflation in the cost of capital, which is computed as a weighted average of the inflation rates for each type of capital (a Laspeyeres index),

```
# picki=(kc(-1)*pik + ic(-1)*pii)/(kc(-1)+ ic(-1))
```

Chapter 8

Private investment

Capitalists allocate all of their expected financial wealth, V^e_{fc} among the equities in the three sectors and money held for financial purposes. The equation can be written in matrix form as

$$\begin{pmatrix} M_c^{d,f} \\ p_{c,e}e_c \\ p_{i,e}e_k \\ p_{k,e}e_i \end{pmatrix} = \begin{pmatrix} \lambda^a_{10} & \lambda^a_{11} & \lambda^a_{12} & \lambda^a_{13} & \lambda^a_{14} \\ \lambda^a_{20} & \lambda^a_{21} & \lambda^a_{22} & \lambda^a_{23} & \lambda^a_{24} \\ \lambda^a_{30} & \lambda^a_{31} & \lambda^a_{32} & \lambda^a_{33} & \lambda^a_{34} \\ \lambda^a_{40} & \lambda^a_{41} & \lambda^a_{42} & \lambda^a_{43} & \lambda^a_{44} \end{pmatrix} \begin{pmatrix} 1 \\ \mathrm{RR}_m \\ \mathrm{RR}_c \\ \mathrm{RR}_k \\ \mathrm{RR}_i \end{pmatrix} V^e_{fc}$$

Here, $M_c^{d,f}$ is money held for financial purposes, $p_{x,e}$ is the price of equities of sector x, and e_x is the number of equities in circulation. The expressions RR_x are real returns in sector x (discussed below), and the λ^a_{mn} are coefficients.

8.1 Conditions on allocation coefficients

Following Godley and Lavoie (and Caiani et al.), we impose some conditions on the λ^a_{mn} coefficients. The equality between expected wealth and allocation is ensured by requiring

$$\sum_{m=1}^{4} \lambda^a_{m0} = 1, \qquad \sum_{m=1}^{4} \lambda^a_{mn} = 0,\ n \in \{1,\dots,4\}$$

We also impose the symmetry constraint, that

$$\lambda^a_{mn} = \lambda^a_{nm},\ m,n \in \{1,\dots,4\}$$

Because

$$\lambda^a_{mn} = \frac{\partial p_{m,e}e_m}{\partial \mathrm{RR}_n},$$

the symmetry condition means that the effect on holdings of asset n due to a change in the real rate of return of asset m is equal to the effect on holdings of asset m due to a change in the real rate of return of asset n.

> **Note:** Do we want to distinguish socially responsible investors? Also, should we give any weight to simply holding a diverse portfolio? Finally, for stranded assets, particularly for "green" investment, the expected real rate of return may be contingent on personal beliefs about future trajectories that are, strictly speaking, not quantifiable.

Two special conditions must be taken into account. First, if there are no equities issued for the innovative capital goods sector ($e_i = 0$) or the conventional capital goods sector has ceased operation (`exitk(-1)==1`), then the equations reduce by one dimension, and also, because of the summing-up conditions, have different coefficients λ_{mn},

$$\begin{pmatrix} M_c^{d,f} \\ p_{c,e}e_c \\ p_{i,e}e_k \end{pmatrix} = \begin{pmatrix} \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} \begin{pmatrix} 1 \\ \mathrm{RR}_m \\ \mathrm{RR}_c \\ \mathrm{RR}_k \end{pmatrix} V_{fc}^e$$

Second, if the conventional capital goods sector is no longer producing, its share price is zero (that is, $p_{k,e} = 0$ after exit).

Taking these two special conditions into account, equity prices and money held for financial reasons are computed in a straightforward way from the foregoing equations,

```
pce = ifelse(exitk(-1) == 1, lambda20 * Vece/ec + (lambda21 * RRm + lambda22 *
    RRc + lambda23 * RRi) * Vece/ec, lambda20a * Vece/ec + (lambda21a * RRm +
    lambda22a * RRc + lambda23a * RRk + lambda24a * RRi) * Vece/ec)
pie = ifelse(exitk(-1) == 1, lambda30 * Vece/ei + (lambda31 * RRm + lambda32 *
    RRc + lambda33 * RRi) * Vece/ei, lambda40a * Vece/ei + (lambda41a * RRm +
    lambda42a * RRc + lambda43a * RRk + lambda44a * RRi) * Vece/ei)
pke = ifelse(exitk(-1) == 1, 0, lambda30a * Vece/ek + (lambda31a * RRm + lambda32a *
    RRc + lambda33a * RRk + lambda34a * RRi) * Vece/ek)
McD = ifelse(exitk(-1) == 1, lambda10 * Vece + (lambda11 * RRm + lambda12 *
    RRc + lambda13 * RRi) * Vece, lambda10a * Vece + (lambda11a * RRm + lambda12a *
    RRc + lambda13a * RRk + lambda14a * RRi) * Vece)
```

8.2 Setting the base shares

Most of the values for the coefficients are set in a calibration run using a simplified version of the model. This procedure is discussed below. However, the base shares λ_{m0}^a and λ_{m0} for $m > 1$ are set when running the full model.

The base share going to the consumption goods sector is set proportional to its share of total real output. Until the innovative sector makes its IPO (that is, while $e_i = 0$), we expect the conventional goods sector to be operating ($y_k > 0$) and profitable ($F_k > 0$). As λ_{10} (the base share for money) is fixed exogenously, this is equal to

$$\lambda_{20} = (1 - \lambda_{10}) \frac{\kappa_k k_c + \kappa_i i_c}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_k}$$

If the innovative sector is in the market, while the conventional capital goods sector has ceased operation, then

$$\lambda_{20} = (1 - \lambda_{10}) \frac{\kappa_k k_c + \kappa_i i_c}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_i + \kappa_i i_i}$$

If both sectors are on the market, then the three-sector version must be used (that is, λ_{20}^a is set), and $\lambda_{20} = 0$.

In the code, this is implemented with a series of `ifelse` functions that implement the following logic:

$$\lambda_{20} = \tilde{\lambda}_{20} (1 - \lambda_{10}), \qquad \lambda_{30} = 1 - (\lambda_{20} + \lambda_{10})$$

where

$$\tilde{\lambda}_{20} = \begin{cases} \dfrac{\kappa_k k_c + \kappa_i i_c}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_i + \kappa_i i_i}, & y_k = 0 \text{ or } F_k < 0 \\ \dfrac{\kappa_k k_c + \kappa_i i_c}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_k}, & y_k \neq 0 \text{ and } F_k \geq 0 \text{ and } e_i = 0 \\ 0, & \text{otherwise} \end{cases}$$

Note: I am nearly certain there is an error in the denominators of the corresponding expressions in the code. Here I have put $\kappa_i i_c$ (or `ic*pri`), rather than $\kappa_k i_c$ (or `ic*prk`), as it appears in the code below.

```
templambda20 = ifelse(yk == 0 | Fk < 0, (kc * prk + ic * pri)/(kc * prk + ic *
    prk + ki * prk + ii * pri), 0)
lambda20 = templambda20 * (1 - lambda10)
lambda30 = 1 - lambda20 - lambda10
```

When all sectors offer shares (`exitk(-1)==0` and $e_i > 0$), the three-sector version of the coefficients comes into force. The relevant condition in this case is whether or not this is the first time period in which all three sectors are operating (so the previous-period values were zero, $\lambda^a_{20,-1} = \lambda^a_{30,-1} = 0$). The expressions are

$$\lambda^a_{20} = \begin{cases} \left(1-\lambda^a_{10}-\theta_3\right)\dfrac{\kappa_k k_c + \kappa_i i_c - \theta_2\left(\kappa_k k_k + \kappa_k k_c + \kappa_i i_c\right)}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_i + \kappa_i i_i + \kappa_k k_k}, & \text{1st period with 3 sectors} \\ \theta\lambda^a_{20,-1} + (1-\theta)\left(1-\lambda^a_{10}-\theta_3\right)\dfrac{\kappa_k k_c + \kappa_i i_c - \theta_2\left(\kappa_k k_k + \kappa_k k_c + \kappa_i i_c\right)}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_i + \kappa_i i_i + \kappa_k k_k}, & \text{otherwise} \end{cases}$$

$$\lambda^a_{30} = \begin{cases} \left(1-\lambda^a_{10}-\theta_3\right)\dfrac{\kappa_k k_k + \theta_2\left(\kappa_k k_k + \kappa_k k_c + \kappa_i i_c\right)}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_i + \kappa_i i_i + \kappa_k k_k}, & \text{1st period with 3 sectors} \\ \theta\lambda^a_{30,-1} + (1-\theta)\left(1-\lambda^a_{10}-\theta_3\right)\dfrac{\kappa_k k_k + \theta_2\left(\kappa_k k_k + \kappa_k k_c + \kappa_i i_c\right)}{\kappa_k k_c + \kappa_i i_c + \kappa_k k_i + \kappa_i i_i + \kappa_k k_k}, & \text{otherwise} \end{cases}$$

In these expressions, θ, θ_2 and θ_3 represent irrational expectations, and are denoted `irrational`, `irrational2` and `irrational3` in the code:

- When $\theta > 0$ (`irrational` > 0), investors persist in maintaining their prior level of base investment in consumption goods and conventional capital rather than shifting to innovative capital;
- When $\theta_2 > 0$ (`irrational2` > 0), investors are biased towards investments in conventional capital over consumption goods;
- When $\theta_3 > 0$ (`irrational3` > 0), investors are biased towards investments in the innovative capital sector.

  Note: In the code, `ii*pri` and `ki*prk` appear twice in the denominator for the first expression (and `ic*prk` appears, as before, rather than `ic*pri`), for both λ^a_{20} and λ^a_{30}. Also, the multiplicand of θ_2 is different in the first and second expressions. Are these errors? I put in what I think is correct, above, but it is different from what is in the code.

Finally, the base share for the innovative sector is determined as a remainder from 100%,

$$\lambda^a_{40} = 1 - \left(\lambda^a_{30} + \lambda^a_{20} + \lambda^a_{10}\right)$$

```
lambda20a = ifelse(exitk(-1) == 0, ifelse(ei == 0, 0, ifelse(lambda20a(-1) ==
    0, (kc * prk + ic * pri - irrational2 * (kk * prk + kc * prk + ic * pri +
    ii * pri + ki * prk)) * (1 - lambda10a - irrational3)/(kc * prk + ic * prk +
    ki * prk + ii * pri + kk * prk + ii * pri + ki * prk), irrational * lambda20a(-1) +
    (1 - irrational) * (kc * prk + ic * pri - irrational2 * (kk * prk + kc *
        prk + ic * pri)) * (1 - lambda10a - irrational3)/(kc * prk + ic * prk +
        ki * prk + ii * pri + kk * prk))), 0)
lambda30a = ifelse(exitk(-1) == 0, ifelse(ei == 0, 0, ifelse(lambda30a(-1) ==
    0, (kk * prk + irrational2 * (kk * prk + kc * prk + ic * pri + ii * pri +
    ki * prk)) * (1 - lambda10a - irrational3)/(kc * prk + ic * prk + ki * prk +
    ii * pri + kk * prk + ii * pri + ki * prk), irrational * lambda30a(-1) +
    (1 - irrational) * (kk * prk + irrational2 * (kk * prk + kc * prk + ic *
        pri)) * (1 - lambda10a - irrational3)/(kc * prk + ic * prk + ki * prk +
        ii * pri + kk * prk))), 0)
lambda40a = 1 - (lambda20a + lambda30a + lambda10a)
```

8.3 Tobin's q

Tobin's q is computed as the value of equity divided by the market value of the capital stock. With checks to ensure that the sector is operating, and setting $q_i = 1$ before the innovative sector's IPO, the expressions are

```
qc = (ec * pce)/(pk * kc + pi * ic - Lc)
qk = ifelse(yk != 0 & Fk >= 0, (ek * pke)/(pk * kk - Lk), 0)
qi = (ei * pie)/(pi * ii - Li)
```

The four-period average is given by

```
qcprev = (qc(-4) + qc(-3) + qc(-2) + qc(-1))/4
qkprev = (qk(-4) + qk(-3) + qk(-2) + qk(-1))/4
qiprev = (qi(-4) + qi(-3) + qi(-2) + qi(-1))/4
```

8.4 Profit rate

Sector profit rates r_x are given by profits over the value of capital,

$$r_x = \frac{F_x}{p_{k,-1}k_{x,-1} + p_{i,-1}i_{x,-1}}$$

With checks to ensure the sector is operating, and recalling that innovative capital is never used in the conventional capital goods sector, these are implemented in code as

```
rc = Fc/(kc(-1) * pk(-1) + ic(-1) * pi(-1))
ri = Fi/(ii(-1) * pi(-1))
rk = ifelse(exitk(-1) != 1 & inktot > 0, Fk/(kk(-1) * pk(-1)), 0)
```

Note: These expressions do not appear to be used anywhere.

8.4.1 Average historical profit rates

Average profit rates over the previous five periods r_x^a are used elsewhere to calculate bank lending rates. These are calculated the same way as instantaneous rates but with sums over the previous five periods,

$$r_x^a = \frac{\sum_{t=-5}^{-1} F_{x,t}}{\sum_{s=-5}^{-1} (p_{k,s}k_{x,s} + p_{i,s}i_{x,s})}$$

```
rca = ifelse(sumkc > 0, (Fc(-4) + Fc(-3) + Fc(-2) + Fc(-1))/sumkc, 0)
sumkc = pk(-4) * kc(-4) + pi(-4) * ic(-4) + pk(-3) * kc(-3) + pi(-3) * ic(-3) +
    pk(-2) * kc(-2) + pi(-2) * ic(-2) + pk(-1) * kc(-1) + pi(-1) * ic(-1)
ria = ifelse(sumki > 0, (Fi(-5) + Fi(-4) + Fi(-3) + Fi(-2) + Fi(-1))/sumki,
    0)
sumki = pi(-4) * ii(-4) + pi(-3) * ii(-3) + pi(-2) * ii(-2) + pi(-1) * ii(-1)
rka = ifelse(sumkk > 0, (Fk(-4) + Fk(-3) + Fk(-2) + Fk(-1))/sumkk, 0)
sumkk = pk(-4) * kk(-4) + pk(-3) * kk(-3) + pk(-2) * kk(-2) + pk(-1) * kk(-1)
```

8.4.2 Expected profit and dividend rates

Expected profit rates are given by similar expressions to the profit rate, but with expected profits (F_x^e rather than F_x),

```
rcge = Fce/(ec(-1) * pce(-1))
rige = Fie/(ei(-1) * pie(-1))
rkge = ifelse(exitk(-1) != 1, Fke/(ek(-1) * pke(-1)), 0)
```

The expected dividend rate is given by a similar expression but with dividends rather than profits (FD_x^e rather than F_x^e),

```
rce = FDce/(ec(-1) * pce(-1))
rke = ifelse(exitk(-1) != 1, FDke/(ek(-1) * pke(-1)), 0)
rie = FDie/(ei(-1) * pie(-1))
```

8.5 Expected real return

Expected real returns on money are denoted by RR_m and on investment in sector x by RR_x. Cash does not yield any return, so for initial holdings M, the real return is

$$\mathrm{RR}_m = \frac{1}{M}\left(\frac{M}{1+\pi_c} - M\right) = \frac{-\pi_c}{1+\pi_c}$$

In this expression, π_c is the inflation rate for consumption goods. In the model,

```
RRm = -pic/(1 + pic)
```

Expected real return rate on equity holdings in sector x is calculated as a weighted sum of expectations on real capital gains cg_x^e, real dividends r_x^e and real profit rate rg_x^e, computed relative to the sector previous period market capitalization,

$$\mathrm{RR}_x = \chi_{\mathrm{div}}\left(\frac{1+r_x^e}{1+\pi_c} - 1\right) + \chi_{\mathrm{prof}}\left(\frac{1+rg_x^e}{1+\pi_c} - 1\right) + \chi_{\mathrm{gains}}\left(\frac{1+cg_x^e}{1+\pi_c} - 1\right)$$

The coefficients take the following values

$$(\chi_{\mathrm{div}}, \chi_{\mathrm{prof}}, \chi_{\mathrm{gains}}) = \begin{cases} (\chi_1, \chi_1, \chi_{1b}), & cg_x^e \neq 0 \\ (\chi_{1a}, \chi_{1a}, 0), & cg_x^e = 0 \end{cases}$$

```
RRc = ifelse(round(cgce, digit = 6) == 0, chi1a * ((rce + 1)/(1 + pic) - 1) +
    chi1a * ((1 + rcge)/(1 + pic) - 1), (chi1b * ((1 + cgce)/(1 + pic) - 1) +
    chi1 * ((1 + rce)/(1 + pic) - 1) + chi1 * ((1 + rcge)/(1 + pic) - 1)))
RRk = ifelse(yk != 0 & Fk >= 0, ifelse(round(cgke, digit = 6) == 0, chi1a *
    ((rke + 1)/(1 + pic) - 1) + chi1a * ((1 + rkge)/(1 + pic) - 1), (chi1b *
    ((1 + cgke)/(1 + pic) - 1) + chi1 * ((1 + rke)/(1 + pic) - 1) + chi1 * ((1 +
    rkge)/(1 + pic) - 1))), 0)
RRi = ifelse(round(cgie, digit = 6) == 0, chi1a * ((rie + 1)/(1 + pic) - 1) +
    chi1a * ((1 + rige)/(1 + pic) - 1), (chi1b * ((1 + cgie)/(1 + pic) - 1) +
    chi1 * ((1 + rie)/(1 + pic) - 1) + chi1 * ((1 + rige)/(1 + pic) - 1)))
```

Note: Should the condition be $cg_x^e \leq 0$?

Chapter 9

Calibration and timeline

Finally, there is a space to insert the calibrated parameters and a declaration of the time sequence.

```
#Calib
CALIBRATION
#END OF MODEL
timeline 1 500
```

The calibrated parameters are generated from a small steady-state model that runs for a few periods to work out the transients. Most of the relationships in the steady-state model are similar to the ones in the full model. However, the relationships for the asset allocation coefficents λ^a_{mn} and λ_{mn} are fully specified in the steady-state model, and it is where the symmetry and summing-up relationships are enforced. The equations used to determine the coefficients in the steady-state model are reproduced here.

The coefficients in the steady-state model are denoted λ^s_{mn}, and are specified for the two-sector model with consumption and conventional capital goods. The base share for money holdings is set at 10%, while the base shares of consumption and capital goods are set in proportion to the capital stock in each sector,

$$\lambda^s_{10} = 0.1 \qquad \lambda^s_{30} = (1 - \lambda^s_{10})\frac{k_k}{k_c + k_k} \qquad \lambda^s_{20} = 1 - (\lambda^s_{10} + \lambda^s_{30})$$

In the steady-state model there is no inflation, so $\mathrm{RR}_m = 0$. The coefficients λ^s_{12} and λ^s_{13} are set to the same value (which turns out to be essential for ensuring the symmetry conditions under later assumptions), so from the summing-up and symmetry constraints they are both equal to

$$\lambda^s_{13} = \lambda^s_{12} = -\frac{1}{V^e_c}\frac{\lambda^s_{10}V^e_c - M^{D,f}_c}{\mathrm{RR}_c + \mathrm{RR}_k}$$

The coefficient λ^s_{11} is then determined from the adding-up constraint

$$\lambda^s_{11} = -(\lambda^s_{12} + \lambda^s_{13})$$

> **Note:** There seems to be an error in `SSTotal.R`, where the equation is `lambda11s=-lambda13s-lambda12s-lambda13s`. One of the `lambda13s`'s should be struck, I think.

We now have the response to money holdings from a change in the rate of return to either consumption or conventional capital goods, λ^s_{12}, λ^s_{13}. Next, we assume that the response to holdings of the other type of equity from a change in the rate of return to a given equity is one-half the money response,

$$\lambda^s_{32} = \frac{1}{2}\lambda^s_{12} \qquad \lambda^s_{23} = \frac{1}{2}\lambda^s_{13}$$

Note that the specification that $\lambda^s_{12} = \lambda^s_{13}$ ensures that with these definitions the parameters satisfy the symmetry condition that $\lambda^s_{23} = \lambda^s_{32}$. From the adding-up constraint, they then determine the own-response to a change in the rate of return, which is

$$\lambda^s_{22} = -\frac{3}{2}\lambda^s_{12} \qquad \lambda^s_{33} = -\frac{3}{2}\lambda^s_{13}$$

Note: I think the equations in `SSTotal.R` are unnecessarily convoluted. They boil down to what I've written above.

The coefficients in the model are then set in the following way:

$$\lambda_{m0} = \lambda^s_{m0},\ m \in \{1,2,3\}$$

$$\lambda_{nm} = \lambda_{mn} = \lambda^s_{mn},\ m,n \in \{1,2,3\},\ n \geq m$$

Note: Actually, in the calibration procedure λ_{32} is set equal to λ^s_{32}, which is only equal to λ^s_{23} by virtue of additional assumptions, as noted above.

For the three-sector model, the base share for the innovative capital sector is set to zero, so that any investment comes solely through rates of return,

$$\lambda^a_{m0} = \lambda^s_{m0},\ m \in \{1,2,3\}, \quad \lambda^a_{40} = 0$$

Note: This introduces an unrealistic element, in that the base allocation never changes. That's asking a lot from the model. I am wondering if we can introduce a dynamic that reproduces an investor "rebalancing" her portfolio. That is, given past performance, we would adjust the λ^a_{m0}'s. If we could do that in a compelling way, it would be really nice, because it would be a dynamic way of introducing a new sector into a Tobin model. I suspect that would be something new (??) that we could write up all by itself. Normally, rebalancing means bringing a portfolio back to the desired risk profile. We could possibly represent risk through market capitalization (bigger is less risky) and other factors (including political risk from, e.g., an effective climate regime). What do you think?

The change in money holdings with a change in rates of return on either money or equity holdings are kept at their two-sector levels, while the response to the rate of return in the innovative sector is set equal to the value for the conventional capital goods sector,

$$\lambda^a_{1m} = \lambda^s_{1m},\ m \in \{1,2,3\}, \quad \lambda^a_{14} = \lambda^s_{13}$$

The symmetry condition then requires

$$\lambda^a_{m1} = \lambda^a_{1m},\ m \in \{2,3,4\}$$

The own-response for the consumption goods sector is unchanged from the two-sector model,

$$\lambda^a_{22} = \lambda^s_{22}$$

The responses of holdings in the conventional capital goods sector to changes in rates of return in either the conventional or innovative capital goods sectors are one-half the response for the conventional capital goods sector alone,

$$\lambda^a_{24} = \lambda^a_{23} = \frac{1}{2}\lambda^s_{23}$$

From the symmetry conditions we also have

$$\lambda^a_{42} = \lambda^a_{24}, \quad \lambda^a_{32} = \lambda^a_{23}$$

Note: Again, the symmetry condition is implicit and not explicit in the calibration code, where λ^a_{42} and λ^a_{32} are set to $(1/2)\lambda^s_{32}$.

The response of holdings in the conventional capital goods sector to a change in the real rate of return in the innovative capital goods sector is set to one-half the response to a change in the rate of return of the consumption goods sector. Also imposing the symmetry condition, this gives

$$\lambda^a_{43} = \lambda^a_{34} = \frac{1}{2}\lambda^s_{23}$$

> **Note:** Here, again, I take λ^s_{23} to be the "defining" coefficient. In the calibration script λ_{32} is used.

The own-responses in both the conventional and innovative capital goods sectors are set to the own-response for the conventional capital goods sector in the two-sector model,

$$\lambda^a_{44} = \lambda^a_{33} = \lambda^s_{33}$$

> **Note:** There is an error in the `GenCalib.R` script, where the own-response in the conventional capital goods sector is set to one-half the value for the 2-sector model. This leaves the matrix imbalanced.