Measurement of refractive index and thickness of transparent plate by dual-wavelength interference

Hee Joo Choi[1], Hwan Hong Lim[1], Han Seb Moon[1], Tae Bong Eom[2], Jung Jin Ju[3], and Myoungsik Cha[1,*]

[1]*Department of Physics, Pusan National University Busan 609-735, Korea*
[2]*Korea Research Institute of Standards and Science, Daejeon 305-340, Korea*
[3]*Electronics and Telecommunications Research Institute, Daejeon 305-700, Korea.*
**mcha@pusan.ac.kr*

Abstract: We developed an accurate and efficient method for measuring the refractive indices of a transparent plate by analyzing the transmitted intensity versus angle of incidence. By using two different wavelengths, we resolved the 2π-ambiguity inherent to the phase measurement involving a thick medium, leading to independent determination of the absolute index of refraction and the thickness with a relative uncertainty of 10^{-5}. The validity and the accuracy of our method were confirmed with a standard reference material. Furthermore, our method is insensitive to environmental perturbations, and simple to implement, compared to the conventional index measurement methods providing similar accuracy.

OCIS codes: (120.3180) Interferometry; (120.3940) Metrology; (120.5710) Refraction.

References and links

1. R. W. Boyd, *Nonlinear Optics 2nd ed.* (Academic Press, 2003).
2. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, 1989).
3. V. G. Dmitriev, G. G. Gurzadyan, and D. N. Nikogosyan, *Handbook of Nonlinear Optical Crystals*, Vol. 64 of Springer Series in Optical Sciences (Springer-Verlag, 1997).
4. D. J. Gettemy, W. C. Harker, G. Lindholm, and N. P. Barnes, “Some optical properties of KTP, $LiIO_3$, and $LiNbO_3$,” IEEE J. Quantum Electron. **24**(11), 2231–2237 (1988).
5. Schott North America, Inc., “Optical glass,” http://www.us.schott.com/advanced_optics/english/ our_products/materials/optical_ glass.html.
6. G. J. Edwards, and M. Lawrence, “A temperature-dependent dispersion equation for congruently grown lithium niobate,” Opt. Quantum Electron. **16**(4), 373–375 (1984).
7. C. O. Atago, LTD., http://www.atago.net.
8. H. Onodera, I. Awai, and J. Ikenoue, “Refractive-index measurement of bulk materials: prism coupling method,” Appl. Opt. **22**(8), 1194–1197 (1983).
9. G. H. Meeten, “Refractive index errors in the critical-angle and the Brewster-angle methods applied to absorbing and heterogeneous materials,” Meas. Sci. Technol. **8**(7), 728–733 (1997).
10. Metricon Corp, http://www.metricon.com.
11. M. S. Shumate, “Interferometric measurement of large indices of refraction,” Appl. Opt. **5**(2), 327–331 (1966).
12. G. D. Gillen, and S. Guha, “Refractive-index measurements of zinc germanium diphosphide at 300 and 77 K by use of a modified Michelson interferometer,” Appl. Opt. **43**(10), 2054–2058 (2004).
13. J. C. Brasunas, and G. M. Curshman, “Interferometric but nonspectroscopic technique for measuring the thickness of a transparent plate,” Opt. Eng. **34**(7), 2126–2130 (1995).
14. G. D. Gillen, and S. Guha, “Use of Michelson and Fabry-Perot interferometry for independent determination of the refractive index and physical thickness of wafers,” Appl. Opt. **44**(3), 344–347 (2005).
15. G. Coppola, P. Ferraro, M. Iodice, and S. De Nicola, “Method for measuring the refractive index and the thickness of transparent plates with a lateral-shear, wavelength-scanning interferometer,” Appl. Opt. **42**(19), 3882–3887 (2003).
16. F. L. Pedrotti, and L. S. Pedrotti, *Introduction to Optics 2nd ed*. (Prentice Hall, 1996).
17. J. A. Stone, J. E. Decker, P. Gill, P. Juncar, A. Lewis, G. D. Rovera, and M. Viliesid, “Advice from the CCL on the use of unstabilized lasers as standards of wavelength: the helium-neon laser at 633 nm,” Metrologia **46**(1), 11–18 (2009).

#124035 - $15.00 USD Received 9 Feb 2010; revised 26 Mar 2010; accepted 18 Apr 2010; published 21 Apr 2010

18. J. A. Stone, and J. H. Zimmerman, "Index of refraction of air," http://emtoolbox.nist.gov/Wavelength/ Edlen.asp.
19. National Institute of Standards and Technology, Certificate of Standard Reference Material® 1822a, 24 January 2007, http://www.nist.gov
20. Korea Research Institute of Standards and Science, Test Report, Certificate No. 0901–00920–001, 12 August 2009.

1. Introduction

Index of refraction is a fundamental physical quantity that characterizes optical materials in various experiments. Knowledge of accurate dispersion of optical media is essential in understanding various linear and nonlinear optical phenomena. In particular, one must establish accurate dispersion relations in order to predict phase-matching conditions of nonlinear optical crystals [1] and optical pulse propagation in waveguides including optical fibers [2]. Typically, an index accuracy of 10^{-5} is required in analyzing typical optical frequency conversion processes in nonlinear optical crystals [3]. The tolerance becomes more severe for short wavelengths and long crystals.

Several methods for index measurement have been proposed, and utilized in the field. The minimum deviation method, or its modified version [4] is commonly employed in practice for bulk solid optical media, providing an accuracy of $\pm 4 \times 10^{-6}$ in a broad range of optical spectrum for optical glasses [5], and less accuracy for nonlinear optical crystals [6]. A disadvantage of this method is that one has to prepare prism-shaped samples, making the routine measurements difficult.

On the other hand, one can measure the critical angle at which total internal reflection (TIR) begins to occur at the interface between the sample and a high-index prism as reference, to determine the index of refraction. Based on TIR of incoherent illumination, Abbé refractometers usually measure the refractive indices of liquids, but can also be utilized to solids with flat surfaces [7]. One can also use collimated laser beams to measure the critical angle for solid samples [8–10]. Although the TIR methods provide a good accuracy (10^{-4}~10^{-5}) for transparent liquids and solids, a small loss in the sample or incomplete contact between the sample and the prism severely increases the uncertainty in the critical angle and the resulting index evaluation [9].

On the contrary, with interferometric methods, the accuracy in the index measurement is not affected by the small loss in the sample because the real path length difference is measured. Although these methods are usually employed for detecting a very small relative phase difference, they could be extended to measuring the absolute index values. A standard Michelson interferometer (MI) has been utilized to measure the absolute refractive index values of transparent solid plates owing to its simplicity [11,12]. In the MI method, a transparent plate sample is rotated in one of the two arms of the interferometer continuously changing the optical path length difference, and hence producing a fringe pattern with the angle of incidence. By analyzing the fringe pattern with the known value of the sample thickness, one can easily estimate the index. With this method, however, one can guarantee an index uncertainty of only ~10^{-3} which is limited mainly by the accuracy of thickness measurement, because the index n and the thickness d cannot be independently determined from a single set of fringe pattern. The phase difference that determines the fringe pattern is rather sensitive to the multiplication nd. Furthermore, since the MI method is sensitive to air flow in the atmosphere and the small vibration of each component in the setup, the accuracy of the measurement is easily perturbed by the environment.

On the other hand, the 'Fabry-Perot (FP) method' is another interferometric method investigated for the same purpose [13,14]. Since, in this case, the fringe pattern is determined solely by the phase difference between the directly transmitted light wave and the collinearly propagating waves which have suffered internal reflections at the surfaces, the FP fringe pattern is more stable against environmental perturbations than the MI fringes. However, in the analysis of the FP fringes, as well as in the MI fringes, the thickness information limits the accuracy of the index estimation. Attempts were made to obtain both the index and the thickness in the interferometric methods. For example, Gillen and Guha combined the MI and FP methods, successfully determining both the index and the thickness values from the two correlated sets of fringes [14]. However, the accuracy in the estimated index and thickness values did not improve. Coppola et al. also reported a wavelength-scanning FP method to obtain both the index and the thickness [15]. The relative uncertainty was approximately 10^{-4}.

In this work, we developed a modified FP method which employs two lasers with considerably different wavelengths in order to accurately determine the thickness and the refractive indices of a transparent plate. By using the dual-wavelength interferometric method, we resolved the 2π-ambiguity inherent to the phase measurement in a thick medium, leading to the independent determination of the absolute index of refraction and the thickness with a relative uncertainty of 10^{-5}. We confirmed the accuracy with a standard reference material (SRM) for refractive index standard.

2. Characteristics of fringes

If we assume a (semi) transparent sample with perfectly plane-parallel surfaces and collimated monochromatic light impinging on it at an angle of incidence θ as shown in the inset of Fig. 1, the phase difference between the neighboring transmitted light is given by

$$\phi(\theta) = \frac{4\pi d}{\lambda}\sqrt{n^2 - n_a^2 \sin^2\theta}, \tag{1}$$

where n_a is the refractive index of surrounding medium (air in this experiment), d is the thickness of the sample, λ is the vacuum wavelength of light, and n is the (absolute) refractive index of the sample.

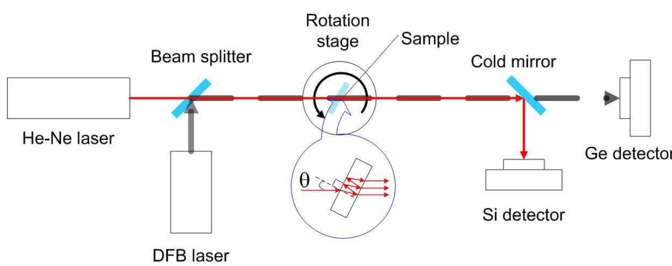

Fig. 1. Schematic diagram of dual wavelength experimental setup. Inset describes multiply reflected/transmitted waves at angle of incidence θ.

If the incident light is an ideal plane wave, the intensity of the transmitted light is obtained by superposition of the electric fields of the directly transmitted light and all the other transmitted ones after multiple internal reflections [16].

$$I_\infty(\theta) = I_0 \frac{[1 - R(\theta)]^2}{R^2(\theta) + 1 - 2R(\theta)\cos\phi(\theta)}, \tag{2}$$

where I_0 is the intensity of the incident light wave, and $R(\theta)$ is the reflectance from the interface at an angle of incidence θ. In practice, since the beam size is finite, the overlap between the neighboring beams becomes poor at a large angle of incidence and not all the beams participate in the superposition. However, we can show that the maxima occur at the angles satisfying $\cos\phi(\theta) = 1$ in this case, too.

Setting $\phi(\theta) = 2m\pi$ (m = integer) in Eq. (1), the peak locations are determined as

$$\theta_m = \sin^{-1}\frac{1}{n_a}\sqrt{n^2 - (\frac{\lambda m}{2d})^2}. \tag{3}$$

#124035 - $15.00 USD Received 9 Feb 2010; revised 26 Mar 2010; accepted 18 Apr 2010; published 21 Apr 2010

In principle, both n and d can be determined by comparing the experimentally measured maxima locations ("peak data") against those obtained by Eq. (3). The first peak is located very close to the normal incidence ($\theta = 0$), associated with the maximum value of m (m_{max}). The spacing between the neighboring peaks gets smaller with increasing angle of incidence, and at the last peak corresponds to the minimum value of m (m_{min}). There can be as many as ~1000 peaks in a typical angular range of the measurement, depending on the thickness and the index. Although it is easy to count the number of peaks ($N = m_{max} - m_{min} + 1$), identifying m_{max} (or m_{min}) is not easy due to the inherent 2π-ambiguity in the interferometric measurement with a thick medium. (Evaluation of index simply by counting the number of peaks typically results in an uncertainty > 10^{-3}.)

Thus, accurate values of n and d cannot be simultaneously determined from a set of fringe data, unless the peak data are extremely accurate. In this work, we used two lasers with different wavelengths in order to resolve the 2π-ambiguity, and to determine accurate values of n and d simultaneously, without performing too costly experiments.

3. Experiment

Our experimental setup is schematically described in Fig. 1. We used a He-Ne laser and a distributed feedback (DFB) diode laser as light sources. The former is a low-power, unstabilized 633 nm laser (Spectra Physics, Model #117), whose vacuum wavelength is 632.991 nm with a relative standard uncertainty of 1.5×10^{-6} [17]. The wavelength of the DFB laser was maintained at 1529.17 nm which was measured with a wavelength meter (Burleigh, Model WA-1000) with a relative resolution of 10^{-7}. The refractive index of air (n_a) is determined mainly by wavelength, atmospheric pressure, temperature and humidity [18]. During the fringe measurement, we monitored the air pressure, humidity and temperature in the laboratory.

For demonstration of our method, we cut and polished a ~0.9 mm thick plate sample out of a SRM, which is a block of uniform soda glass purchased from the National Institute of Standards and Technology, United States. The index dispersion of the SRM between 480.1 and 644.0 nm is provided by the same institute with an uncertainty of ± 1.6×10^{-5} [19]. At first, the thickness of the sample was roughly measured with a digital micrometer which displays thickness reading down to 1 μm. We read d_{rough} = 857 μm for the thickness, but we need better accuracy if we want to determine the index with a smaller uncertainty than 10^{-3}. The thickness would be determined with a better accuracy by analyzing the fringes data, and confirmed by a block gauge measurement later. The sample was rotated by a computer-controlled stepping motor with steps of 0.004 °, given by the vendor. However, the angle measurement was further calibrated by checking the retro-reflection of the input laser beams after 5 full rotations, resulting in an angle accuracy of ± 5.6×10^{-6}. The transmitted beams were separated by a beam splitter, and the powers were detected with a Si and a Ge photodiode for red and IR light, respectively, at each step between −10° and 30 °.

We also note that if the sample is slightly wedged, the fringe data $I(\theta)$ can be asymmetric, affecting the uncertainty in the index determination. The relative phase error due to the imperfect parallelism is estimated to be $\alpha \tan\theta'$ where α is the wedge angle and θ' is the angle of refraction. For our SRM sample α was estimated to be approximately 0.00009 radian. To minimize the wedge effect, we aligned the wedge direction parallel to the rotation axis.

4. Fitting the measured fringe data

A typical set of measured fringe data at 1529 nm is shown in Fig. 2. Similar fringe data were obtained at 633 nm (not shown). The spacing between the neighboring fringes gets smaller with increasing angle of incidence as expected from Eq. (3).

#124035 - $15.00 USD Received 9 Feb 2010; revised 26 Mar 2010; accepted 18 Apr 2010; published 21 Apr 2010

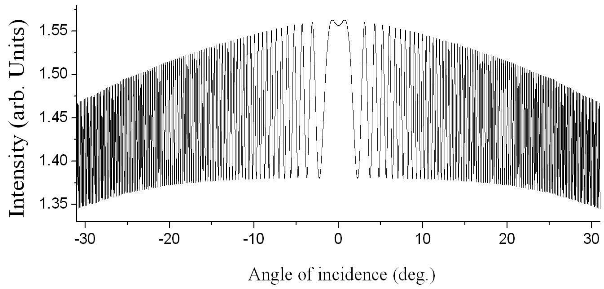

Fig. 2. Transmitted intensity versus angle of incidence for λ = 1529 nm.

Atmospheric temperature, pressure and relative humidity were monitored to be 24 ± 1 °C, 1010 ± 2 hPa, and 20 ± 5%, respectively, during the fringe measurement, giving a refractive index value of air, n_a = 1.00027 ± 0.000001 [18]. A peak detection program was used to find accurate peak locations, $\theta_{exp,i}$ (i = 1, 2, 3, …, N) in the positive angular range, which are compared with the peak locations calculated by Eq. (3), $\theta_{cal,i}$ (i = 1, 2, 3, …, N). For the generation of initial $\theta_{cal,i}$, we start with the previously measured 'rough' thickness d_{rough}, and a 'rough' index n_{rough} obtained by using the total number of fringes N. Typically, the uncertainty in n_{rough} is not better than 10^{-3}. In order to achieve better accuracy, we calculated the sum of the squared error

$$S = \sum_i \left|\theta_{\exp,i} - \theta_{cal,i}\right|^2, \tag{4}$$

and plot the inverse (1/S) as a function of n and d, for those around the roughly determined values, n_{rough} and d_{rough}. Figures 3(a) and 3(b) show the contour plots of 1/S for the fringes measured with the DFB laser and He-Ne laser, respectively. In each plot, provided that the experimental data are perfect, the 1/S plot would indicate a diverging peak at the real values of (n, d).

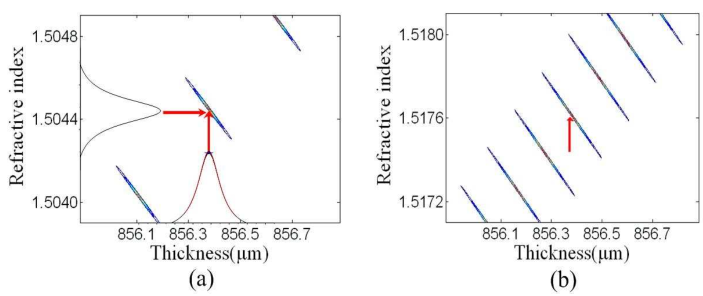

Fig. 3. Contour plots of 1/S versus (n, d) for DFB laser (1529 nm) (a) and for He-Ne laser (633 nm) (b). Because contours are too narrow to view, the projections are shown for one peak in (a). The vertical arrows indicate the peaks with an identical thickness.

In practice, however, due to the experimental angular fluctuations in the fringes maxima, we cannot identify the most pronounced peak among the several peaks within the uncertainty range of the rough thickness measurement (± 1 µm), causing 2π-ambiguity in determining more accurate values of (n, d). However, the peaks indicated by arrows in Figs. 3(a) and 3(b) have their maxima at a common thickness of 856.40 ± 0.01 µm, and we can identify the peaks as indicating the real sets of (n, d), resolving the 2π-ambiguity. As a result, the refractive indices are determined to be 1.504426 ± 0.000011 at 1592 nm (DFB laser) and 1.517681 ± 0.000013 at 633 nm (He-Ne laser), respectively. The uncertainty ranges were set to be 10% of the widths of the 1/S-projections [Fig. 3(a)], where decrease from the peak value is obvious.

#124035 - $15.00 USD Received 9 Feb 2010; revised 26 Mar 2010; accepted 18 Apr 2010; published 21 Apr 2010

For our result at 633 nm, agreement with the NIST's certified value (1.517679 ± 0.000016) is excellent [19].

Furthermore, we had the sample thickness measured by a block gauge, resulting in d = 856.4 ± 0.2 μm [20], which is a more accurate value than the previously measured d_{rough}. The thickness determined by fitting (856.40 ± 0.01 μm) fall in the uncertainty range of the block gauge measurement. Due to the small wedge angle in our SRM sample, however, there will be a thickness change of ~0.05 μm within the experimental beam diameter (~0.5 mm). Here, $I(\theta)$ is determined by Gaussian-weighted average of the fringes with thickness distribution. Because the slant is quite linear, what we obtain from fitting would be the thickness of the sample at the center of the beam. The results were reproducible within an experimental error of 10^{-5}. The factors affecting the uncertainty in our measurement are summarized in Table 1.

We also applied our method to a commercially available fused silica substrate, obtaining n = 1.444432 ± 0.000007 and 1.457187 ± 0.000011 at 633 and 1529 nm, respectively.

Table 1. Uncertainties in measured parameters.

Parameter	Name	Uncertainty	Remarks
θ	Angle of incidence	$\pm 4\text{x}10^{-6}$ (relative)	Two steps (0.008°) for 5 full rotations
λ	Vacuum wavelength	± 1.0 pm @ 633 nm	Smaller at 1529 nm
n_a	Refractive index of atmosphere	$\pm 10^{-6}$	Ref [19].

5. Conclusions

We developed a dual wavelength interferometric method to determine the refractive index of a transparent plate. By analyzing the fringes caused by the Fabry-Perot type interference, we could determine the refractive index and the thickness independently, achieving the relative uncertainty of 10^{-5} or smaller. The validity and the accuracy of our method were confirmed by a SRM. The present experiment was limited to a pair of wavelengths, but extension to a broad spectral range is straightforward provided that lasers with reasonably narrow linewidths are available. Furthermore, because the present method is very easy to implement in a laboratory with simple apparatus, and quite insensitive to the atmospheric turbulence and vibration, we expect that the new method cannot only contribute to establish accurate dispersion formula for newly developed solid optical materials, but also be utilized to the routine check of the refractive indices of produced optical substrates.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation funded by the Ministry of Education, Science and Technology of Korea, 2009-0074213. We thank H. S. Suh for useful discussion on unstabilized He-Ne lasers.

#124035 - $15.00 USD Received 9 Feb 2010; revised 26 Mar 2010; accepted 18 Apr 2010; published 21 Apr 2010