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NEAR THRESHOLD RESPONSE OF A WAVE-SHIFTED CHERENKOV RADIATOR TO HEAVY IONS

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The response of Pilot 425 to heavy ions with energies less than 600 MeV/amu ($\beta \approx 0.8$) is examined both theoretically and experimentally. Measurements are presented from an experiment which employed a ^{20}Ne beam at many energies below 575 MeV/amu. The signal is assumed to come from three sources: (1) Cherenkov light from the heavy ion; (2) Cherenkov light from secondary electrons; and (3) scintillation of the radiator. It is found that the effective index of refraction is 1.518 and that scintillation is present at a level of ~2.7% of the Cherenkov signal for $\beta = 1$ for ^{20}Ne. The first of these values differs from values previously quoted in the literature.

1. Introduction

At medium energies Cherenkov counters can provide accurate velocity measurements for particles with known charge. At relativistic energies they provide accurate charge measurements with important advantages over organic scintillators: they do not saturate with increasing charge, and they do not suffer from large Landau fluctuations. Since Cherenkov emission is feeble, good resolution requires that as much Cherenkov light as possible be collected. One way in which this can be done is to dope the radiator with a wave-shifting material which converts high frequency Cherenkov light into lower frequency light which can more easily escape the radiator and convert to photoelectrons at the photocathodes of photomultiplier tubes. A quenching material must then also be added to the radiator to minimize the scintillation of the wave-shifting fluor. In addition to providing a greater number of visible photons, wave-shifted radiators can make collection of a larger fraction of these photons possible since the wave-shifted photons are emitted isotropically. Roughly half of the total light is wave-shifted and hence isotropized. For particle energies near the Cherenkov threshold, many of these isotropized photons will be totally internally reflected while the directional primary Cherenkov component will not be (for normally incident particles, total internal reflection of this component occurs at $\beta \gtrsim 0.89$ for an index of refraction of 1.5). To capitalize on this advantage, one should use adiabatic light pipes to conduct the light to the photomultiplier tubes since internal reflection is required for efficient light piping. Unfortunately, high spatial uniformity is difficult to attain with light pipes.

The most readily available wave-shifted radiators and the one which will be discussed in this paper, is Pilot 425, manufactured by Nuclear Enterprises, Inc. of San Carlos, California. Several authors have investigated the properties of wave-shifted radiators[1–3]); in each case Pilot 425 was among the samples tested. Sacharidis[1]) investigated the response of light-piped radiators to singly-charged beams and hence was hindered by small quantities of light (less than 1 photoelectron per particle below threshold) and also by a complicated energy and angle dependence of the experimental light-collection efficiency. Atallah and Schmidt[2]) set limits on the amount of scintillation of wave-shifted radiators by taking long exposures with extremely sensitive film while irradiating sample radiators with alpha-particles. They concluded that scintillation contributes less than 5% of the total light emitted by relativistic, singly-charged particles. Cantin et al.[3]) used a ^{14}N heavy ion beam at the Lawrence Berkeley Bevalac to examine the response of a variety of radiators, each housed in a light integration box. For Pilot 425 they quote an index of refraction of $n = 1.49$ and a scintillation fraction of 3% for relativistic ^{14}N ions. Gilman and Waddington[9]) have referred to an index of refraction of $n = 1.44$, a value also communicated to us[10]).

2. Theoretical response

For any Cherenkov radiator there will always be three contributions to the total light output: (1) Cherenkov emission from the primary particle; (2) Cherenkov emission from secondary electrons; and (3) scintillation.

Only the relative contributions of Cherenkov and scintillation light distinguish a Cherenkov radiator from a scintillation detector. We let dL_1/dx, dL_2/dx,

dL_3/dx, respectively, denote these contributions to total light output per unit length. For direct comparison with experiment we use units of photoelectrons per unit length.

We restrict our attention to a 1.27 cm thick Pilot 425 radiator, since our results may easily be extended to other radiators. We also restrict our attention to energies below 600 MeV/amu ($\beta = 0.8$), since we have experimental data only for these energies and since at greater energies prediction of response becomes more difficult because of the increasing complexity of the delta-ray transport problem[4]).

We first consider the Cherenkov emission from the primary particle. For a non-wave-shifted radiator

$$\frac{dL_1}{dx} = \frac{Z^2 e^2}{\hbar c^2} f \int_{n(\omega) > \frac{1}{\beta}} g(\omega)\, q(\omega) \left[1 - \frac{1}{\beta^2\, n^2(\omega)}\right] d\omega, \tag{1}$$

where Ze is projectile charge, βc, projectile speed, f, the light collection efficiency, $q(\omega)$, the quantum efficiency of the phototubes, and $g(\omega)$, the fraction of light of frequency ω which can escape the radiator without being absorbed. For Pilot 425 we adopt the following naive model: for wavelengths longer than λ_1, but shorter than λ_2 primary Cherenkov photons are absorbed by the wave-shifter and converted into photons with a wavelength of 425 nm (the peak of the fluorescence spectrum of Pilot 425; fwhm is 50 nm). For wavelengths longer than λ_2 primary Cherenkov photons are unaffected by the wave-shifter. With $\omega_i = 2\pi c/\lambda_i$ we have:

$$\frac{dL_1}{dx} = \frac{Z^2 e^2}{\hbar c^2} f \left\{ \int_0^{\omega_2} g(\omega)\, q(\omega) \left[1 - \frac{1}{\beta^2\, n^2(\omega)}\right] d\omega + \int_{\omega_2}^{\omega_1} g(\omega_0)\, q(\omega_0) \left[1 - \frac{1}{\beta^2\, n^2(\omega)}\right] d\omega \right\}, \tag{2}$$

where $\omega_0 = 2\pi c/425$ nm.

The known optical properties of Pilot 425 determine ω_1, ω_2 and $g(\omega)$. Nuclear Enterprises gives the following information regarding Pilot 425:

minimum detectable Cherenkov wavelength $= 260$ nm,

index of refraction at 589.2 nm $= 1.490$,

index of refraction at 425 nm $= 1.502$.

The light transmittance of Clinical Perspex (UVT polymethylmethacrylate) and of Pilot 425 were measured by Sacharidis[1]) for 1 cm thick samples. Clinical Perspex was opaque at 295 nm and showed a 90% transmittance at 325 nm. The corresponding wavelengths for Pilot 425 are 390 nm and 414 nm. The difference between Pilot 425 and Clinical Perspex is due to the wave-shifter: for wavelengths shorter than $\sim$400 nm the wave-shifter is absorbing, while for longer than 400 nm wave-lengths the wave-shifter is transparent. So we will take $\lambda_2 = 400$ nm. For λ_1 we will use 260 nm, the shortest detectable Cherenkov wavelength (this is consistent with the transmittance cutoff of Clinical Perspex). Since Pilot 425 is transparent to wavelengths longer than 400 nm we set $g(\omega_0) = g(\omega) = g$. For the index of refraction we assume the form obtained for the elementary classical model of a collection of damped oscillators:

$$n(\omega) = 1 + \frac{C}{\omega_0^2 - \omega^2}. \tag{3}$$

Using the above indices for Pilot 425, we obtain $C = 1.931 \times 10^{32}/\mathrm{s}^2$, $\omega_0^2 = 4.044 \times 10^{32}/\mathrm{s}^2$.

Using the quantum efficiency for RCA 4525 tubes (which were used in the experimental tests) we obtain:

$$\frac{dL_1}{dx} = fg \frac{Z_2 e_2}{\hbar c^2} (7.60 \times 10^{14}/\mathrm{s}) \left[1 - \frac{1}{\beta^2 (1.515)^2}\right] = \eta Z^2 \left(1 - \frac{1}{\beta^2 n_0^2}\right). \tag{4}$$

Here we define η, the system of merit and n_0, the effective Cherenkov index of refraction.

It is a straightforward problem to evaluate the contribution due to the Cherenkov emission of secondary electrons. The number of delta rays produced per unit energy ε per unit length by a heavy particle of charge Ze, velocity βc is:

$$\frac{dn}{d\varepsilon dx} = \frac{2\pi N Z^2 e^4}{m_e c^2 \beta^2 \varepsilon^2} \left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_m}\right), \tag{5}$$

where N is the electron number density and ε_m is the kinematic limit for energy transfer to an electron: $\varepsilon_m = 2 m_e c^2 \beta^2 \gamma^2$. We will assume that each electron of energy ε remains in the radiator at least until its energy drops below the Cherenkov threshold. This is certainly a good approximation for $\beta \lesssim 0.8$ (at this velocity the practical range of the maximum energy delta ray is roughly $\frac{1}{2}$ the thickness of a 1.27 cm radiator). The relatively small number of delta rays which escape the radiator above threshold will be replenished by delta rays produced in the material above the radiator. For velocities in excess of $0.8c$, delta rays are produced which can easily escape the radiator. Evanson[4]) treats this problem by introducing the concept of a cutoff energy E_c below which delta rays contribute according to our prescription and above which they behave as though they have energy E_c. For our purposes this additional complication is unnecessary. Hence, we have that each secondary electron of energy ε contributes an amount of light

$$\eta \int_{\varepsilon}^{\varepsilon_0} \left(1 - \frac{1}{\beta_e^2 n_0^2}\right) \frac{d\varepsilon}{d\varepsilon/dx},$$

ε_0 being the threshold energy for Cherenkov emission by an electron. For $d\varepsilon/dx$ we use the tabulated values of Berger and Seltzer[5]). These results include bremsstrahlung losses and follow the higher energy electron for high energy transfer collisions. For a given medium and index of refraction,

$$\frac{dL_2}{dx} = \eta Z^2 f(\beta). \tag{6}$$

$f(\beta)$ is sketched in fig. 1 for $n_0 = 1.5$ and the value of N corresponding to polymethylmethacrylate ($N = 3.88 \times 10^{23}/\text{cm}^3$). It is seen that between $\beta = 0.575$ and $\beta = 0.8$, dL_2/dx is a linear function of β.

It is impossible to calculate the scintillation component dL_3/dx a priori since scintillation response varies with both dE/dx and Z, i.e., scintillators "saturate". The total "pure" Cherenkov yield is obtained by integrating $dL_1/dx + dL_2/dx$ through the radiator thickness.

Consider for a moment dL/dx only. By changing the variable of integration to a dimensionless momentum[6]) $p = \beta\gamma$ we have:

$$\int \frac{dL_1}{dx}\, dx = L_1 = \eta Z^2 \left(1 - \frac{1}{n_0^2}\right) \times$$
$$\times \int_{p_1}^{p_0} \left(1 - \frac{1}{(n_0^2 - 1)\, p^2}\right) \left(\frac{pAm_p c^2}{\gamma dE/dx}\right) dp, \tag{7}$$

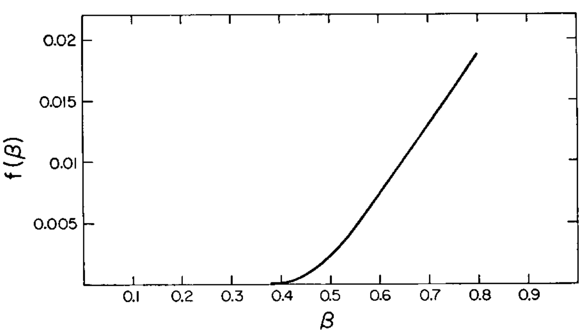

Fig. 1. Light output due to Cherenkov emission from delta rays. The number of photoelectrons collected per unit distance traveled by the primary particle of charge Ze is equal to $\eta Z^2 f(\beta)$, where η is the light collection system figure of merit.

where m_p is the proton mass, A is the mass number of the nucleus and where $p_i(p_0)$ is the initial (final or threshold) momentum. Considering the quantity $p/(\gamma dE/dx)$ to be constant one obtains:

$$L_i = \eta Z^2 \left(1 - \frac{1}{n_0^2}\right) t \left[1 - \frac{1}{(n_0^2 - 1)\, p_0 p_i}\right], \tag{8}$$

where t is the radiator thickness (or the depth of penetration at which the particle drops below threshold). One can show that if α represents the peak-to-peak fractional variation of the quantity $pAm_p c^2/(\gamma dE/dx)$ then

$$L_i \cong \eta Z^2 \left(1 - \frac{1}{n_0^2}\right) t \left[1 - \frac{(1+\alpha^3)}{(n_0^2 - 1)\, p_0 p_i}\right]. \tag{9}$$

For ^{20}Ne ions this implies an accuracy of 0.1% for eq. (8).

Unfortunately, we know of no adequate approximation for $L_2 = \int_0^t (dL_2/dx)dx$, which must be evaluated numerically. In fig. 2 we plot $(L_1 + L_2)$ for ^{20}Ne incident normally on a 1.27 cm thick Pilot 425 radiator for various indices of refraction. Similar curves are presented for ^{56}Fe. Note that the extrapolated cutoff energy for iron is enhanced relative to neon. This is due to the greater slowing rate of the heavier nuclei.

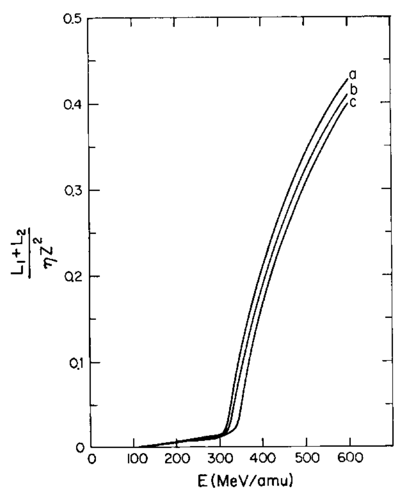

Fig. 2. Pure Cherenkov emission from both the primary particle and secondary electrons for ions traversing a 1.27 cm thick Pilot 425 radiator. Several indices of refraction have been used for the calculations to illustrate the nature of the dependence of the light output on this parameter. The ordinate is in units of photoelectrons and η is expressed in terms of photoelectrons/cm. The top curve (a) corresponds to $Z = 10$, $n_0 = 1.53$, the middle curve (b) to $Z = 10$, $n_0 = 1.515$, and the bottom curve (c) to $Z = 26$, $n_0 = 1.515$.

3. Experiment

We used the Cherenkov counter described in ref. 7 to perform tests of the Pilot 425 response. This counter consists of a sandblasted radiator placed in a light integration box, viewed by 16 RCA 4525 photomultiplier tubes. The counter is characterized by exceptional spatial uniformity and light collection efficiency and an absence of any angular dependence or energy dependence of the light collection efficiency (in the previous section this was an implicit assumption since g did not depend on β or the angle of incidence of the particle). The tests were performed at the Lawrence Berkeley Bevalac where we had a ^{20}Ne heavy ion beam with an energy of 594 MeV/amu. The beam was focused to a diameter of less than 2 cm. Matter in the beam line and in the top of the light integration box reduced the beam energy to 572.4 MeV/amu at the top of the Pilot 425 radiator. A 1024-channel analyzer was used to accumulate pulse height spectra at this and at lower energies (obtained by degrading the beam with lead absorbers). Fig. 3 displays the response as a function of energy for formal incidence. The energies for the data points were obtained by using the range–energy program of Henke and Benton[8]). The error bars for the low energy data are indicative of the magnitude of a possible systematic error due to uncertainties in the beam energy (or equivalent due to uncertainties in the matter in the beam line) of ± 1.5 MeV/amu at the exit window into the experimental area. For energies above ~ 100 MeV/amu this systematic error is negligible. The ordinate is expressed in units of photoelectrons. The scale was determined from the width of the maximum energy data points (multidynode counting statistics were taken into consideration). Note that below threshold, the response is remarkably constant. This cannot be explained in terms of scintillation alone. However, the combination of a term varying as $\sim 1/\beta^2$ with a term varying as $\sim\beta$ (from delta-ray Cherenkov radiation) produces this flat curve. The effective Cherenkov index of refraction n_0, and the system figure of merit η were obtained by an iterative procedure. We first estimated η to be 37.0/cm from results of spectra obtained with atmospheric muons. We also estimated n_0 to be 1.525 from preliminary analysis of the data. We calculated the pure Cherenkov response (primary particle plus secondary electrons) and substracted this from the observed curve. This gives an estimated scintillation response which is quite accurate in the region below threshold since the delta-ray Cherenkov contribution is insensitive to small changes in η and n_0. We then compared the scintillation response in this accurate region with the response from a Pilot Y scintillator and found the saturation properties to be quite similar. This enabled us to extend the estimated scintillation response out to an energy of 600 MeV/amu. Subtracting this response from the observed response produced an experimentally determined "pure" Cherenkov curve which was then fit very nicely with an index of $n_0 = 1.515$. By repeating this procedure one more time we obtained the following values for n_0 and η:

$$n_0 = 1.518 \pm 0.005,$$

$$\eta = 35.3/\text{cm}.$$

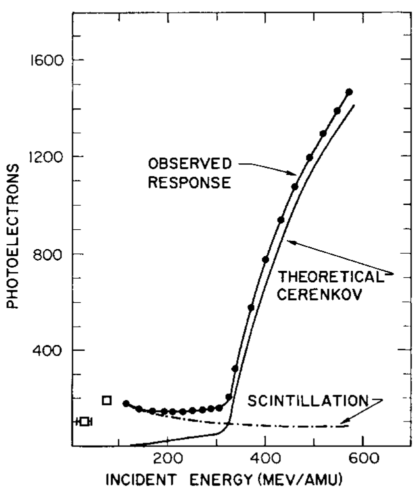

Fig. 3. Observed response of the Cherenkov radiator as a function of incident energy. The theoretical pure Cherenkov response and the scintillation component are plotted separately.

In fig. 3 we plot the scintillation and Cherenkov response separately.

Our value of n_0 does not agree with either of the values 1.44[9,10]) and 1.49[3]) which are often quoted. If one extrapolates the above threshold response for the proton results of ref. 1 one obtains an effective index of refraction of 1.40. This latter result can be explained by inefficient light collection at near threshold velocities where much of the light is in the forward direction and hence has little chance of being totally internally reflected and subsequently collected. Uniform light collection is essential in any attempt to ascertain the energy-dependence of the response of Cherenkov radiators. Problems such as these perhaps account for the discrepancies of the earlier results from both our measured and theoretical indices (1.518 and 1.515 respectively) which agree very well.

By a comparison of the scintillation response to the Cherenkov response for $\beta \approx 1$ (obtained by extrapolating from low velocities) we find a scintillation-to-Cherenkov emission ratio of 2.7%, consistent with previous results[1–3].

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