Indian Institute of Technology, Gandhinagar

Ball and Beam Experiment

ES-245 Control Systems

Group 12

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   2. Consider the design criteria: less than 5% overshoot and settling time less than 3 seconds within a 2% tolerance band. Complete the following:
      - Root Locus: Plot the design criteria using MATLAB (Hint: use sgrid command) and recommend the appropriate compensator to achieve the desired performance.
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   1. Based on the previous recommendations, design a first-order lead/lag compensator to meet the design criteria:
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   2. Comment on the pole-zero placement strategy used in both approaches and explain How does the compensator pole-zero placement affect the root locus and bode plot?
   3. Plot the system's closed-loop response (with the compensator) for a step input. Verify that this system satisfies the design criteria using both the root locus and bode plot Approaches.
3. **Task 8: Demonstration of the Physical Project Setup**

   Choose a specific position for the ball on the beam. Tune the PID controller to keep the ball stable at this location. A final project demonstration will be scheduled towards the end of the semester at a mutually agreed time. Bring your physical setup and demonstrate your PID controller's ability to effectively stabilise the ball and respond to minor perturbations about the desired position along the beam.

1. Task 6: Root Locus and Bode Plot of Ball-Beam System

6.1. Using MATLAB, plot the following for the system's open-loop transfer function:

- **Plot root locus and comment on the system’s stability.**
- **Plot the bode plot and report the system's phase margin and gain margin.**

The open-loop transfer function for the ball-beam system is given as:

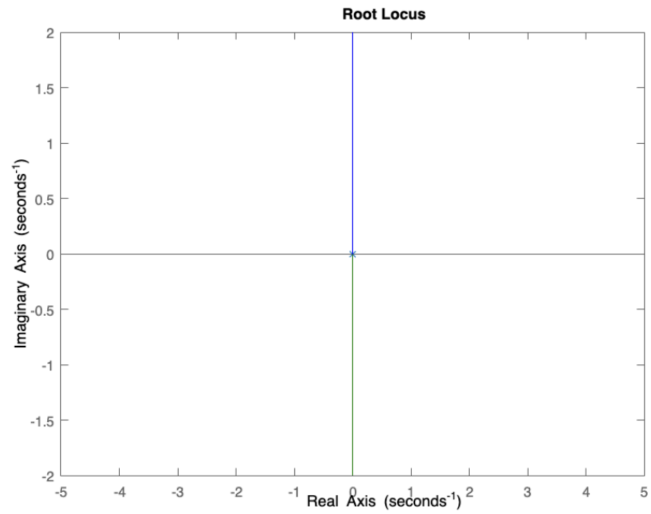
$$P(s) \;=\; -\frac{m.g.d}{L(\frac{J}{R^2}+m)}\frac{1}{s^2}$$

The system dynamics are governed by the physics of the ball-beam mechanism, where the control input (servo angle) directly influences the beam's angle and, subsequently, the ball's position.

The **Root Locus** and **Bode Plot** were generated using MATLAB, focusing on system stability, gain-phase characteristics, and frequency response to analyse the system's behaviour.

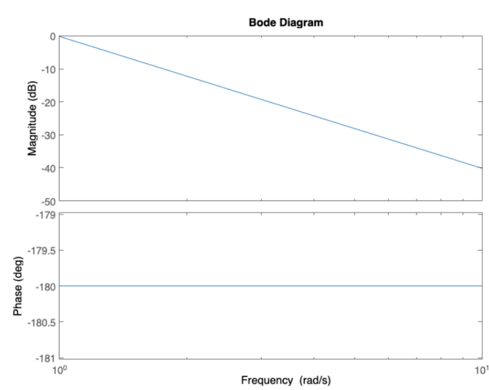
```
clc;
clear all;
close all;

%Task 6.1

% Ball and Beam System Parameters
m = 0.0023267; % mass of the ball (kg)
R = 0.02;      % radius of the ball (m)
g = -9.81;     % acceleration due to gravity (m/s^2)
L = 0.3;       % length of the beam (m)
d = 0.05;     % lever arm offset (m)
J = 6.2045e-7; % moment of inertia for a hollow sphere (kg·m^2)

% Plant Transfer Function
s = tf('s');
P_ball = -m * g * d / (L * (J / R^2 + m)) * 1 / s^2; % Open-loop transfer function

rlocus(P_ball)
axis([-5 5 -2 2])
```

Root Locus code snippet



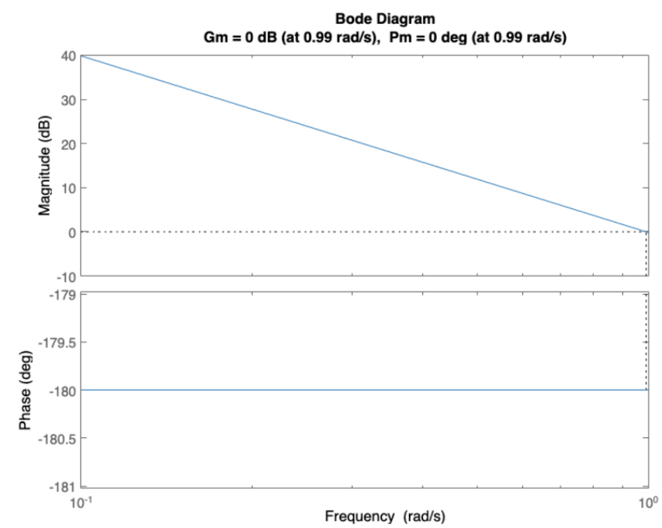
Root Locus plot

```
bode(P_ball)
```



Bode plot of the system’s open-loop transfer function

```
margin(P_ball);
```



Gain Margin and Phase Margin Analysis

Observations:

1. **Root Locus Analysis:**
   - The **root locus plot** shows the system poles on the **imaginary axis**. This is characteristic of an **underdamped or marginally stable system** and reflects the absence of any damping or stabilising force in the open-loop configuration.
   - The double integrator structure, represented by the $\frac{1}{s^2}$ term in the transfer function, results in poles at the origin (s = 0) that move symmetrically along the imaginary axis as the gain is increased. This signifies an **unstable system** in the open-loop state.
   - Notably, the root locus does not cross into the **left half-plane**, meaning the system lacks any tendency toward stability without external compensation.

2. **Bode Plot Analysis:**
   - The **magnitude plot** illustrates a rapid decline in gain at higher frequencies, typical of systems dominated by a double integrator. This suggests poor open-loop frequency response, as the system cannot adequately respond to high-frequency inputs.

- The **phase plot** reveals that the phase approaches **−180°** at critical frequencies. In control theory, this phase behaviour, coupled with low gain margins, indicates a tendency for the system to become unstable when feedback is applied.
- **Gain Margin and Phase Margin:** Both are either **zero or undefined**, which is a direct consequence of the transfer function structure. This further confirms that the open-loop system cannot meet stability requirements or desired performance specifications without the introduction of compensatory measures.

6.2. Root Locus and Bode Plot with Design Criteria

Consider the design criteria: less than 5% overshoot and settling time less than 3 seconds within a 2% tolerance band. Complete the following:

- **Root Locus: Plot the design criteria using MATLAB (Hint: use sgrid command) and recommend the appropriate compensator to achieve the desired performance.**
- **Bode Plot: Analyse the plot to recommend that the compensator meet the design criteria.**

Design Criteria:
To ensure the desired dynamic performance, the following design criteria were established:

- Overshoot (Mp): Less than 5%.
- Settling Time (Ts): Within 3 seconds.

Using standard second-order system equations, the corresponding damping ratio (ζ) and natural frequency (ωn) are derived:

$$\zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2+(\ln M_p)^2}} \quad \text{and} \quad \omega_n = \frac{4}{\zeta T_s}$$

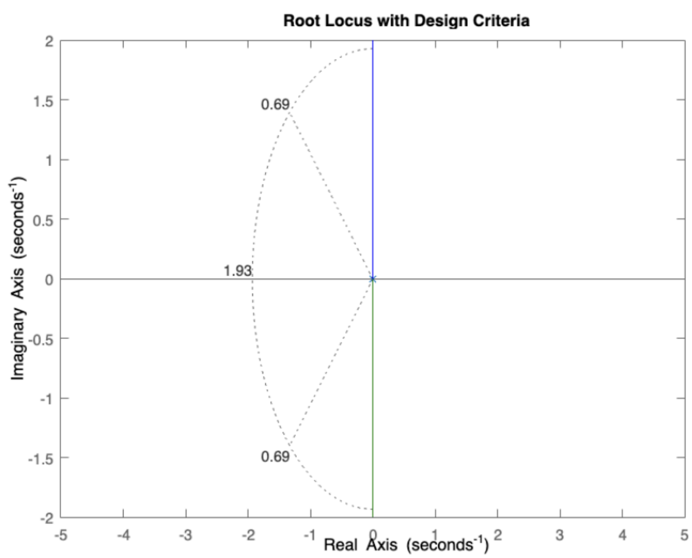
Substituting the values:

$\zeta = 0.69 \ and \ \omega_n = 1.93 \ rad/s$

These parameters define the desired closed-loop pole locations, which must be achieved through compensator design.

```
% Task 6.2: Root Locus and Bode Plot with Design Criteria

% Design Criteria
zeta = sqrt(log(0.05)^2 / (pi^2 + log(0.05)^2)); % Damping ratio for 5% overshoot
wn = 4 / (zeta * 3); % Natural frequency for settling time < 3 seconds

% Root Locus with Design Criteria
figure;
rlocus(P_ball);
sgrid(zeta, wn); % Overlay the design criteria
axis([-5 5 -2 2])
title('Root Locus with Design Criteria');
grid on;
```

Root Locus Snippet



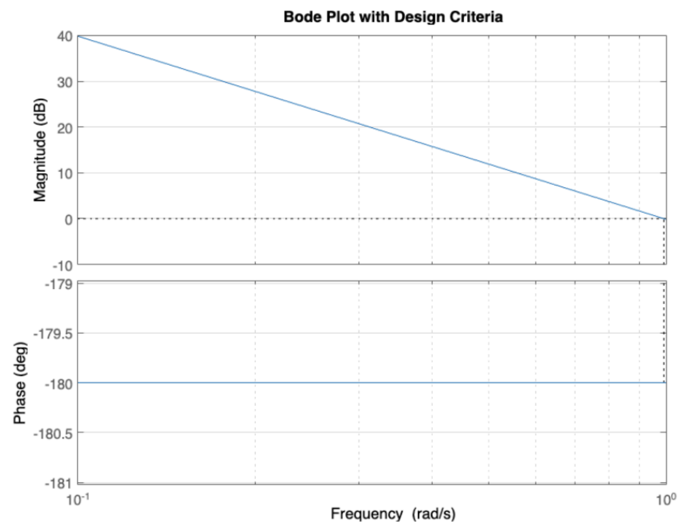
Root Locus Plot with Design Criteria

Root Locus Analysis with Design Criteria:

The root locus was analysed with the design criteria overlaid using MATLAB's **sgrid** command. The following observations were made:

- The design criteria translate to a desired region in the ss-plane, where the closed-loop poles must lie to satisfy the overshoot and settling time constraints.
- The original system's root locus shows no branches passing through the desired region. This indicates that the **uncompensated system** cannot satisfy the performance requirements without external modifications.
- A compensator is required to shift the root locus branches toward the left-half plane (LHP), ensuring the desired damping ratio (ζ) and natural frequency (ωn).

```
% Bode Plot Analysis
figure;
margin(P_ball); % Automatically computes and displays gain/phase margins
title('Bode Plot with Design Criteria');
grid on;
```



Bode Plot with Design Criteria

Bode Plot Analysis with Design Criteria:

The **Bode plot** of the open-loop system was also analysed relative to the design requirements:

- The system exhibits a significant constant phase, insufficiently achieving the desired stability margins. This behaviour arises due to the inherent dynamics of the plant transfer function.
- The phase margin is zero, and the gain margin is either zero or undefined, indicating that the system is on the verge of instability. We need to add some phase to this.
- Without compensation, the system's open-loop Bode plot reveals that it cannot achieve stability or meet the desired performance metrics.

2. Task 7: Controller Design using Root Locus and Bode Plot Approaches

1. **Task 7.1: Based on the previous recommendations, design a first-order lead/lag compensator to meet the design criteria:**

Plot the root locus of the system with the compensator:

The compensator should be designed for specific performance criteria, such as desirable transient response, stability, and gain/phase margins. A first-order lead/lag compensator has the transfer function:

$$G_c(s) = K_c\, \alpha \frac{Ts+1}{\alpha Ts+1}$$

Our original transfer function is

$$G(s) = -\frac{m*g*d}{L(\frac{J}{R^2}+m)}\frac{1}{s^2}$$

The system is inherently unstable since there are no poles in the left half of the s-plane, and its response to any input grows unbounded over time without feedback or compensation. A lead compensator was chosen to improve stability and transient response by adding a zero to shift the root locus into the left half-plane

We can now combine the controller with the plant and plot the root locus. We take the zero of the compensator near the origin to cancel one pole. Furthermore, we can place the compensator pole to the left of the origin to draw the root locus further left in the complex plane.

The design criteria that our system required are less than 5% overshoot and a settling time of less than 3 seconds within a 2% tolerance band.

By Practical/Assumption-based Root locus approach:

Let us assume Zero $Z_0 = 0.01$, and pole $P_0 = 5$ and the value of K=1

Our compensator transfer function will become

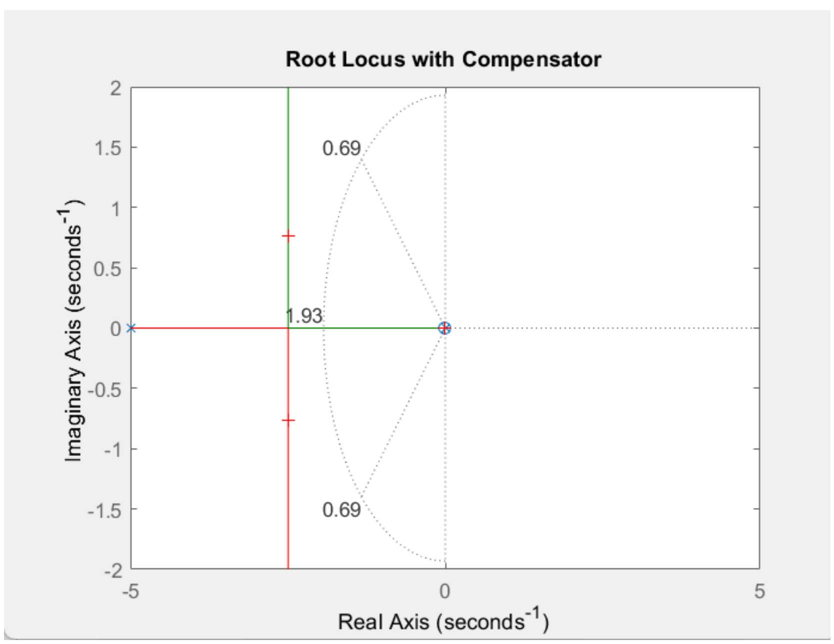
$$G_c(s) = \frac{1+0.01s}{1+5s}$$

The open loop transfer function of the new system becomes

$$G(s)\, G_c(s) = -\frac{m*g*d}{L(\frac{J}{R^2}+m)}\frac{1}{s^2}$$

```
clear;
close all;
clc;
m = 0.0023267; % Mass of the ball (kg)
R = 0.02;      % Radius of the ball (m)
g = -9.81;     % Acceleration due to gravity (m/s^2)
L = 0.3;       % Length of the beam (m)
d = 0.05;      % Lever arm offset (m)
J = 6.2045e-7; % Moment of inertia of the hollow sphere (kg.m^2)

%Transfer function of the ball system
s = tf('s');
P_ball = -m*g*d/L / (J/R^2 + m) / s^2;
%Design of Lead/Lag Compensator
zo = 0.01;
po = 5;
C = tf([1 zo], [1 po]);

% Root locus with compensator
figure;
rlocus(C * P_ball);
title('Root Locus with Compensator');
sgrid(0.69, 1.932);
axis([-5 5 -2 2]);

% Gain selection using root locus
[k, poles] = rlocfind(C * P_ball);
```

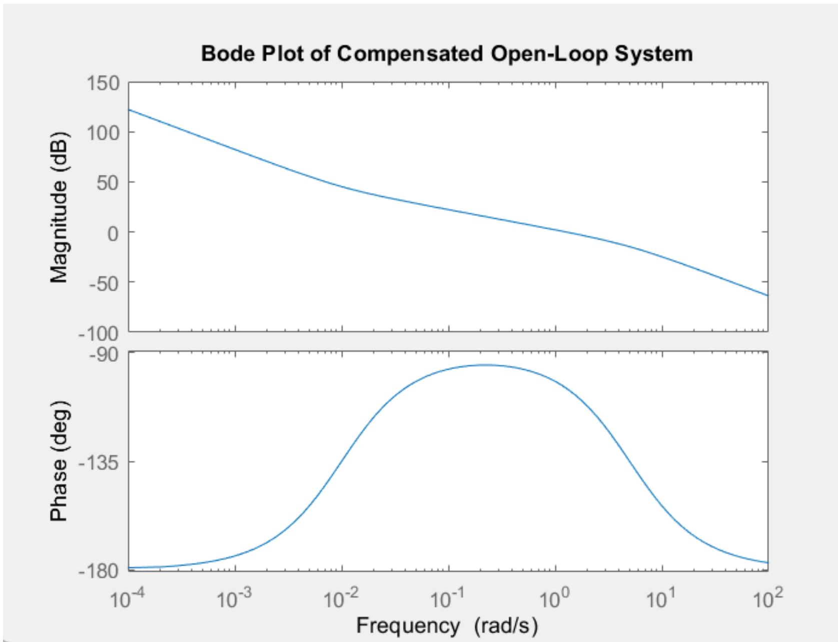
Code for Root locus approach



Plot of root locus with compensator

```
% Gain selection using root locus
[k, poles] = rlocfind(C * P_ball);

% Bode plot of the compensated open-loop system
figure;
bode(k * C * P_ball);
title('Bode Plot of Compensated Open-Loop System');
```

Code snippet for bode plot



Bode plot of compensated open-loop system

By theoretical Root locus approach:

The design criteria that our system required are less than 5% overshoot and a settling time of less than 3 seconds within a 2% tolerance band.

We find the Damping ratio(ζ) and natural frequency (ω_n) using the overshoot and settling time formulas.

$$\zeta = \sqrt{\frac{(ln\, M_p)^2}{\pi^2 + (ln\, M_p)^2}}$$

$$\omega_n = \frac{4}{\zeta T_s}$$

By solving the equations, we get the Damping ratio(ζ) is 0.69 and the natural frequency (ω_n) is 1.932.
The characteristic equation is $S^2 + 2\zeta\omega_n s + \omega_n^2$. By substituting the values, we get the characteristic equation as $s^2+2.622s+3.61$. The roots come out to be $-1.3333 \pm 1.398j$.

The desires pole is $-1.3333+1.398j$. The angle deficiency is -87.6. We need two compensators as lead compensators can compensate up to 50^0.

By calculating it, we get values of T and α . T= 0.821 and $\alpha = 0.1765$
And k = 9.0162 for one compensator and k= 21.574 for two compensators.

The transfer function of the compensator becomes

$$G_c(s) = 9.0162 \frac{s+0.821}{s+4.6} \text{ - } \frac{m*g*d}{L(\frac{J}{R^2}+m)} \frac{1}{s^2}$$

The open loop transfer function of the new system becomes

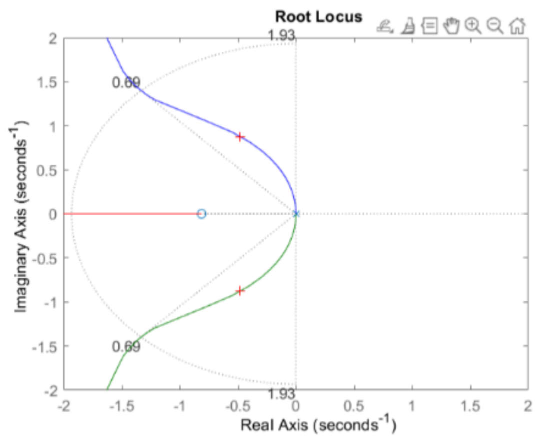
$$G(s)\, G_c(s) = (9.0162) \text{ - } \frac{m*g*d}{L(\frac{J}{R^2}+m)} \frac{1}{s^2} \frac{s+0.821}{s+4.6}$$

```
%zo = 0.001;
%po = 0.5;
K=9.06125
```

```
K = 9.0612
```

```
C=tf([1 0.812],[1 4.6]);

rlocus(C*K*P_ball)
sgrid(0.69, 1.932)
axis([-2 2 -2 2])
[k,poles]=rlocfind(K*C*P_ball)
```

Root locus code snippet after theoretical analysis




```
selected_point = -0.4811 + 0.8734i
k = 0.5042
 poles = 3x1 complex
     -3.6441 + 0.0000i
     -0.4780 + 0.8777i
     -0.4780 - 0.8777i
```

Root Locus code for theoretical analysis

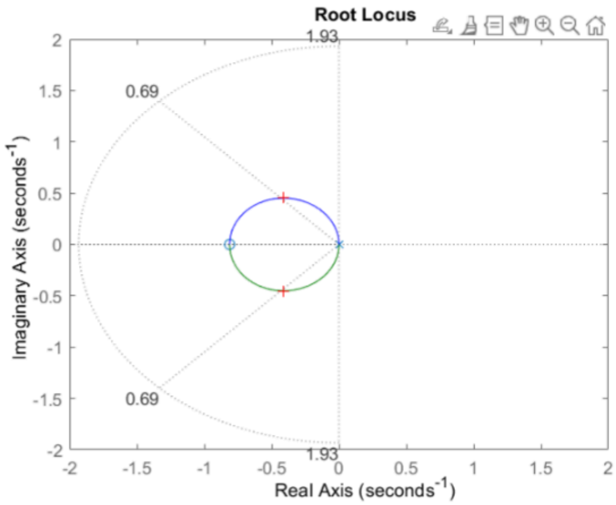
If we add two compensators to our original system, then our new open-loop transfer function becomes

$$G(s)\,G_c(s) = (9.0162)\ -\ \frac{m*g*d}{L(\frac{J}{R^2}+m)}\ \frac{1}{s^2}\ \frac{s+0.821}{s+4.6}$$

```
%zo = 0.001;
%po = 0.5;
K=21.574
```

```
C=tf([1 0.812],[1 4.6]);

rlocus(C*C*K*P_ball)
sgrid(0.69, 1.932)
axis([-2 2 -2 2])
[k,poles]=rlocfind(K*C*C*P_ball)
```

Root locus code snippet for two compensators



Root locus for two compensators

By Practical/Assumption-based Bode Plot approach:

The transfer function of the system is

$$G(s) = -\frac{m*g*d}{L(\frac{J}{R^2}+m)}\frac{1}{s^2}$$

The phase-lead compensator adds a positive phase within a defined frequency range known as the corner frequencies to the system. The amount of phase boost offered by a single lead compensator is 90 degrees. In our controller design, we want to have less than 5% overshoot, which translates to a damping ratio, ζ of 0.7.

Let us assume the phase to 70^0. We can find the α by using the formula

$$\alpha = \frac{1-sin(\Phi)}{1+sin(\Phi)}$$

By calculating it, we get α =0.03109.

To find the frequency at which the phase boost should be applied (center frequency), we face a problem because the phase vs. frequency curve in the Bode plot seems relatively flat. However, there is a known relationship between the bandwidth frequency (ω_bw) and settling time, indicating that it is approximately 1.92 rad/s. Based on this, we want to pick a center frequency a little below this value. We have taken the frequency as 1rad/s.

$$\omega_n = \frac{1}{\sqrt{\alpha}\ T}$$

From this, we get the value of value of T.

The transfer function of the lead compensator becomes

$$G_c(s) = k\frac{1+5.67135s}{1+0.1765s}$$

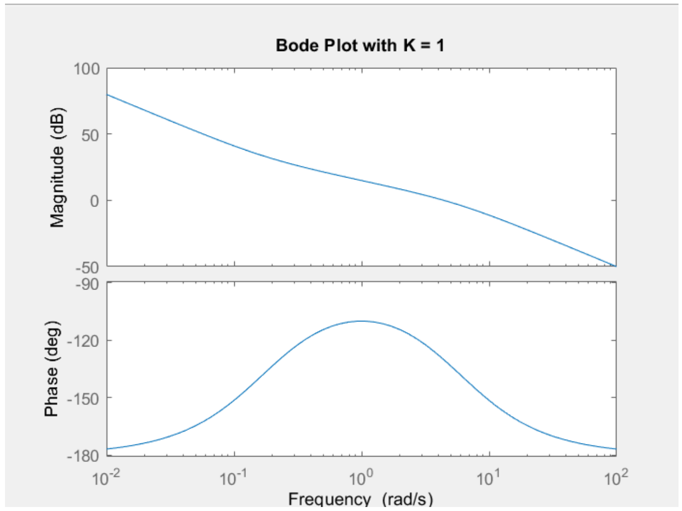
The open loop transfer function of the new system becomes

$$G(s)G_c(s) = k\,\frac{1+5.67135s}{1+0.1765s} \quad - \frac{m*g*d}{L(\frac{J}{R^2}+m)}\,\frac{1}{s^2}$$

Let us assume k=1,

```
% Clear previous data and figures
clear;
close all;
clc;

% System Parameters
m = 0.0023267;         % Mass of the ball (kg)
R = 0.02;        % Radius of the ball (m)
g = -9.81;         % Acceleration due to gravity (m/s^2)
L = 0.3;          % Length of the beam (m)
d = 0.05;         % Lever arm offset (m)
J = 6.2025e-7;      % Moment of inertia of the ball (kg·m^2)

% Transfer Function of the Ball-Beam System
s = tf('s');
P_ball = -m * g * d / L / (J / R^2 + m) / s^2;

% Phase-Lead Compensator Design
angle = 70 * pi / 180;   % Desired phase margin in radians
a = (1 - sin(angle)) / (1 + sin(angle));  % Lead compensator parameter
w = 1;                                   % Crossover frequency (rad/s)
T = 1 / (w * sqrt(a));                   % Time constant for the compensator
K = 1;                                   % Initial compensator gain

% Phase-Lead Compensator Transfer Function
C = K * (1 + T * s) / (1 + a * T * s);

% Plot Root Locus with Compensator
figure;
rlocus(C * P_ball);
title('Root Locus with Compensator');
sgrid(0.69, 1.983);  % Damping ratio and natural frequency
axis([-5 5 -2 2]);

% Bode Plot for Compensated System
figure;
bode(C * P_ball);
title(['Bode Plot with K = ', num2str(K)]);

% Step Response with Initial Compensator
sys_cl = feedback(C * P_ball, 1);  % Closed-loop transfer function
t = 0:0.01:5;                     % Time vector for simulation
figure;
step(0.1 * sys_cl, t);           % Step response for input magnitude 0.25
title('Step Response with K = 1');
```

Bode Plot code snippet for k=1

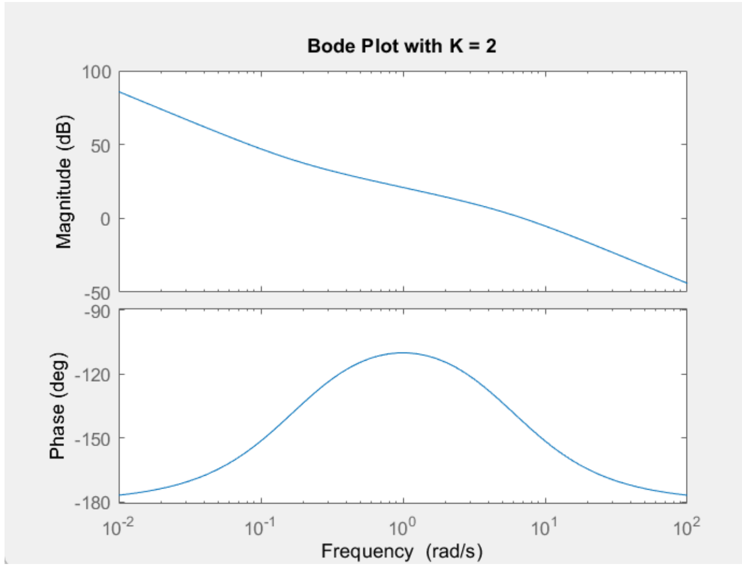


Bode Plot for k=1

Let us assume k=2,

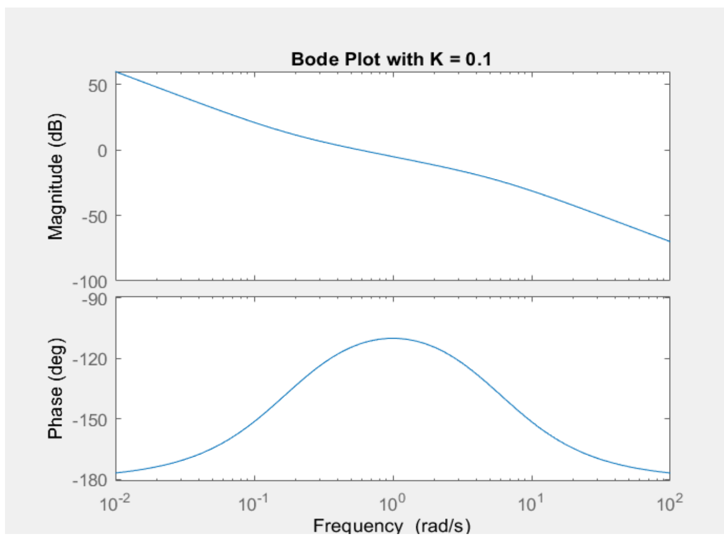
```
%%
% Test the System with Different Gain Values
K = 2;
% Update Compensator with New Gain
C = K * (1 + T * s) / (1 + a * T * s);
sys_cl = feedback(C * P_ball, 1);

% Bode Plot for Compensated System
figure;
bode(C * P_ball);
title(['Bode Plot with K = ', num2str(K)]);

% Step Response for Closed-Loop System
figure;
step(0.1 * sys_cl, t);
title(['Step Response with K = ', num2str(K)]);
```

Bode plot code snippet for k=2



Bode Plot for k=2

Let us assume k=0.1,



Bode plot for k=0.1

2. **Task 7.2: Comment on the pole-zero placement strategy used in both approaches and explain how the compensator pole-zero placement affects the root locus and bode plot.**
   Pole-Zero Placement in the Compensator: A compensator is designed to modify the system's dynamics by carefully placing additional poles and zeros in the transfer function. Placing these poles and zeros determines how the root locus and frequency response (Bode plot) are altered.

Root Locus Approach:

- Adding zeros to the system's transfer function attracts the root locus branches, pulling poles toward those zeros.
- Adding poles repels root locus branches, driving poles away.
- The proper placement of compensator poles and zeros ensures that the dominant poles of the system are shifted into the desired region of the s-plane, satisfying performance requirements like overshoot, settling time, and stability.

Bode Plot Approach:

- The zero has a favourable phasing, which adds to the phase margin of the systems.
- Conversely, a pole reduces the gain of higher frequencies, improving noise rejection and the system's robustness.
- Strategic pole-zero placement ensures that the gain and phase response comply with stability criteria like gain margin (>10dB) and phase margin (>45).

Effect of Compensator Pole-Zero Placement:

Transient and Steady-State Performance:

- The response can be shaped to have more rapid rise times or smaller overshoots by placing zeros close to the desired dominant poles.
- Poles at low frequencies increase low-frequency gain, which increases steady-state error performance.

Root Locus Shape:

- Adding a zero close to the origin bends the root locus toward the left-half plane, improving stability and damping.
- A compensator pole far from the imaginary axis ensures the locus returns to a stable configuration without compromising stability margins.

Bode Plot Shaping:

- Phase Lead Compensators: Zeros precede poles, introducing positive phase shifts that increase phase margins, thus stabilising the system at higher frequencies.
- Phase Lag Compensators: Poles precede zeros, enhancing gain at lower frequencies to reduce steady-state errors but reducing the phase margin, which may require careful placement to maintain overall stability.

Observation:

In compensator design:

- **Dominant Zeros**: These should be strategically placed to influence system dynamics positively without introducing unnecessary high-frequency noise.
- **Poles**: Must be placed with consideration to maintain stability margins while achieving desired gain characteristics.

In both root locus and Bode plot methods, pole-zero placement directly affects the system behaviour and must be aligned with the design objectives, such as stability, speed, or robustness.

3. Task 7.3:Plot the system's closed-loop response (with the compensator) for a step input. Verify that this system satisfies the design criteria using both the root locus and bode plot approaches.

A closed-loop response with the compensator refers to the behaviour of a system when it operates under a feedback loop that includes a compensator

The closed-loop transfer function T(s) of a system with a compensator is given by:

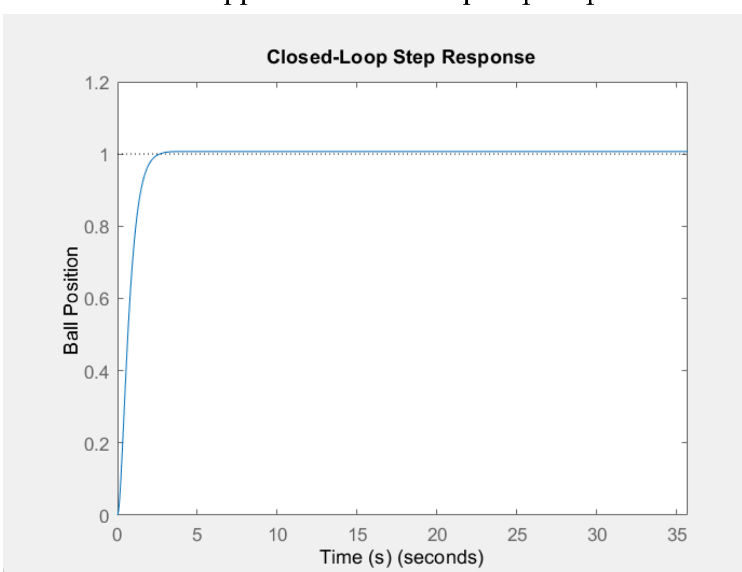
$$T(s)= \frac{G(s)G_c(s)}{1+G(s)G_c(s)H(s)}$$

For the Root locus approach

Closed loop Response using Assumption Method:

```
%% Closed-Loop Response

T = feedback(k * C * P_ball, 1);

% Step response of the closed-loop system
figure;
step(T);
title('Closed-Loop Step Response');
xlabel('Time (s)'); ylabel('Ball Position');

% Verify closed-loop stability using Root Locus and Bode
figure;
rlocus(1 + k * C * P_ball);
title('Root Locus of Closed-Loop System');
figure;
bode(1 + k * C * P_ball);
title('Bode Plot of Closed-Loop System');
```

Code snippet for closed loop step-response



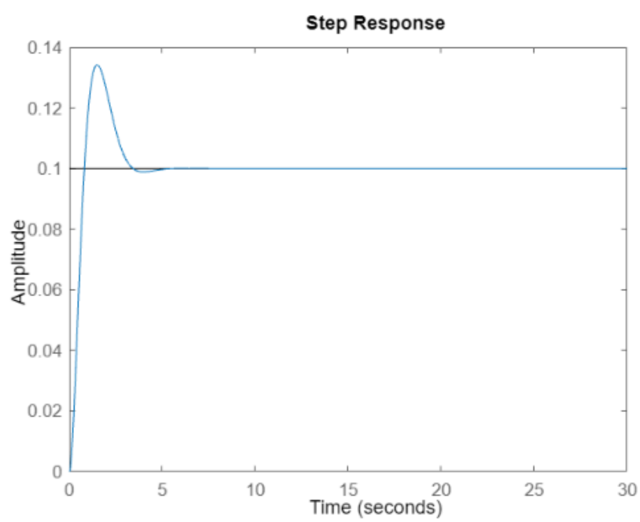
Plot for closed-loop step response

Verification of root locus approach:

- The steady-state value is approximately 1.
- The response does not visibly exceed the steady-state value by more than 5%. So the system meets the overshoot criteria.
- The 2% tolerance band is 1$\pm$0.02i.e, between 0.98 to 1.02.
- The response appears to settle in 3 seconds approximately. So it satisfies the settling time.

Closed loop response for using the analytical method;

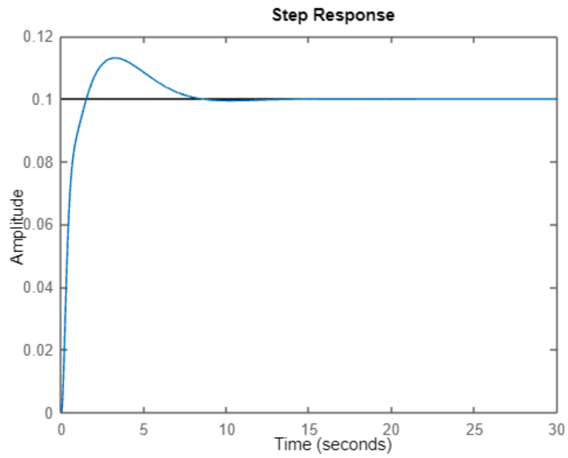
For one compensator,



Plot for step response when one compensator is used

Observation:The system is not stable when we use one compensator

For two compensators:



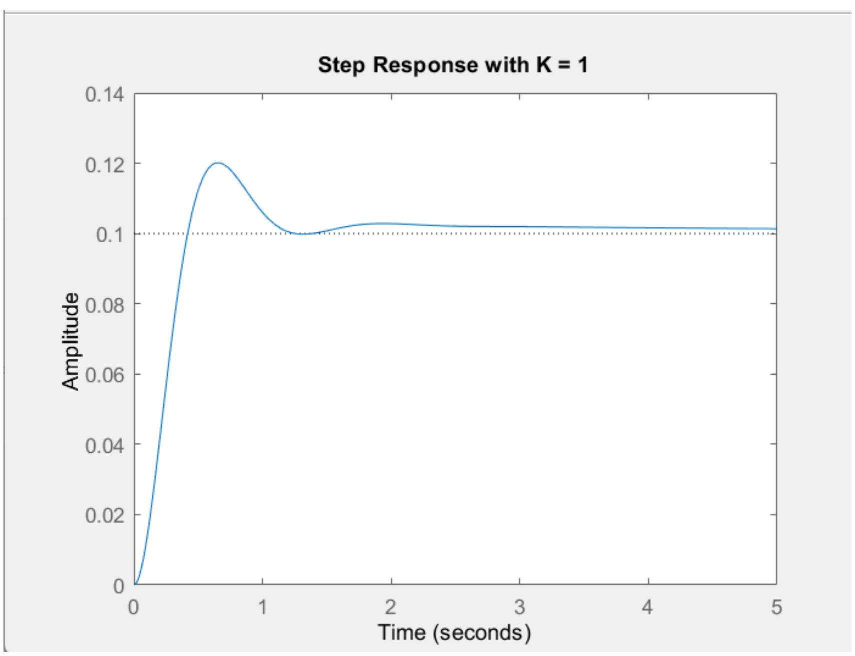
Step response when two compensators are used

Observation: The system is not stable when we use two compensator

For the bode plot approach:

Closed loop response for using the assumption method:

If we take k= 1, Φ= 70;

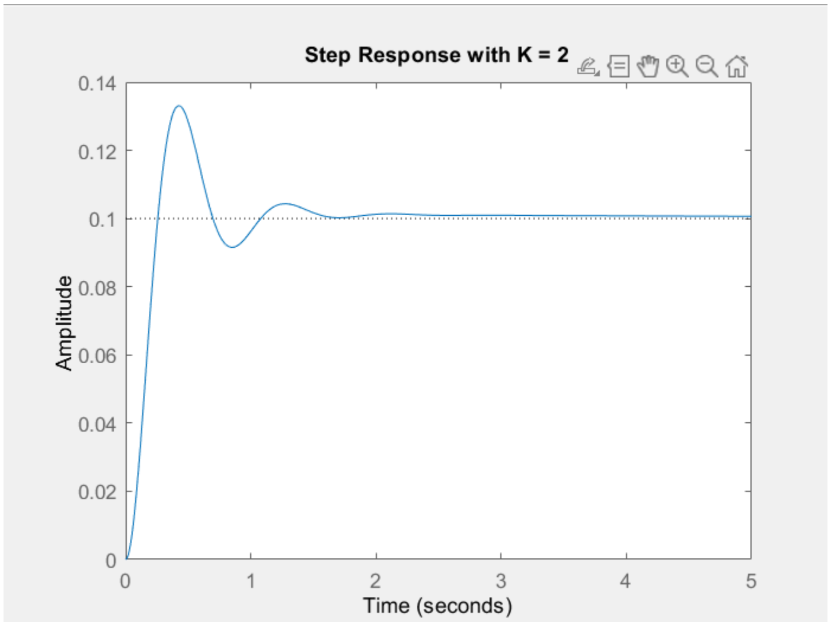


Plot for step response with K=1

Observation: The system is not stable. It has more overshoot.

So let us take the k=2

If we take k=2,Φ= 70;

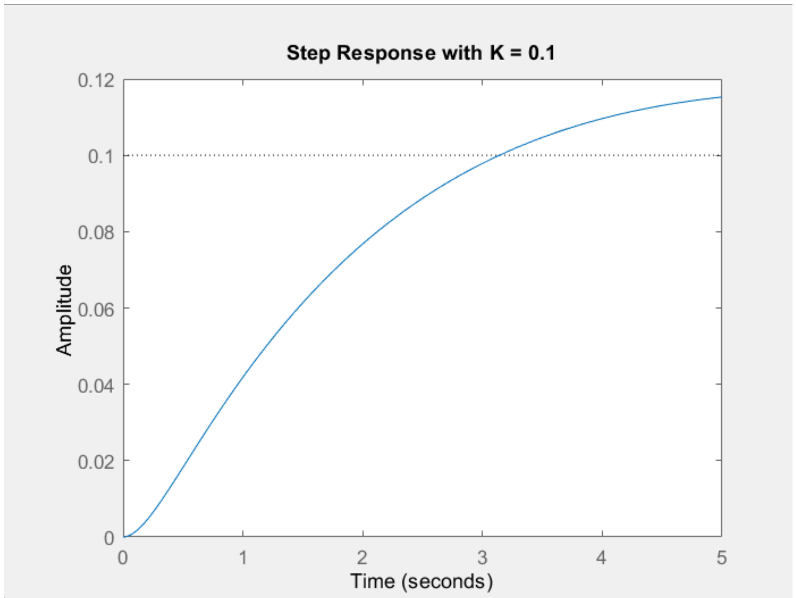


Plot for step response with K=2

Observation:Here, the overshoot is more than the previous one.

So, let's decrease the k value to k =0.1.

If we take k= 0.1, Φ= 70

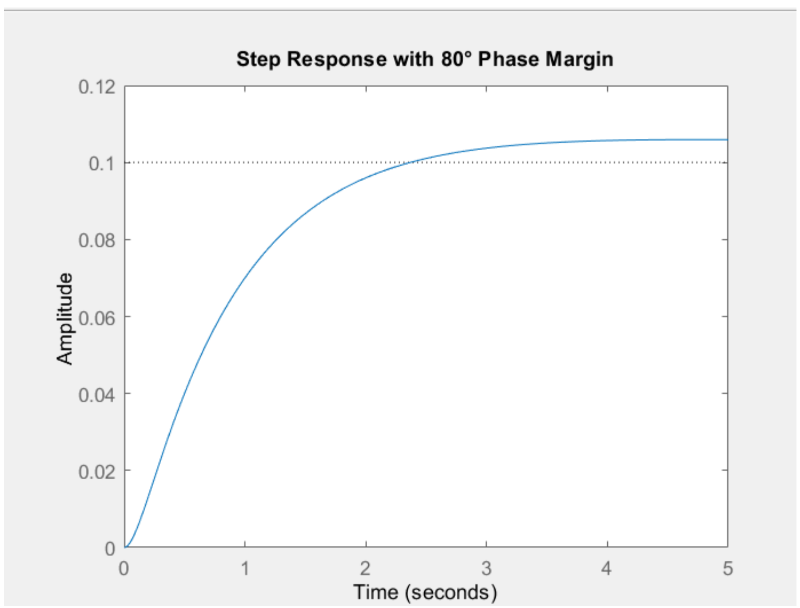


Plot for step response with k=0.1

Observation:The overshoot is less than the previous case

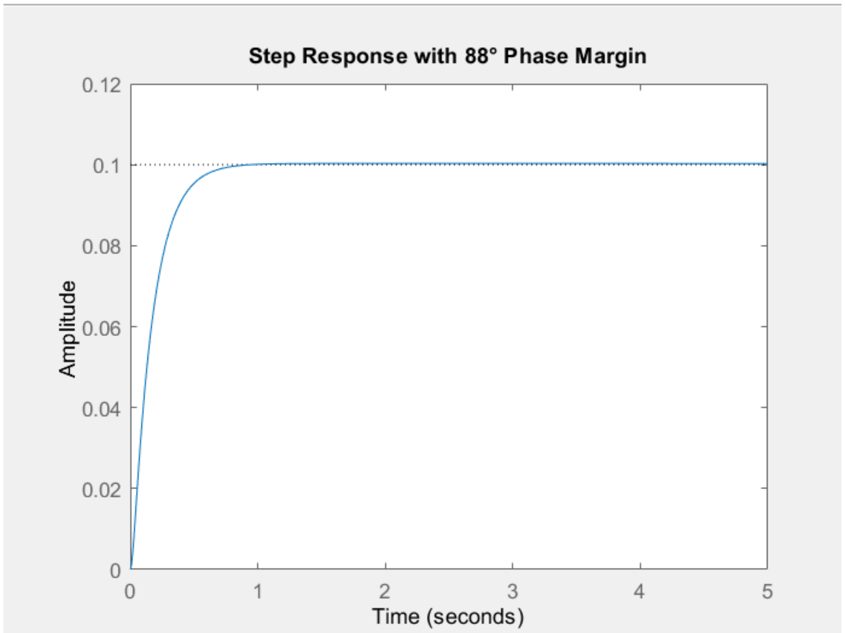
We add a phase to our control system,

If we add a phase Φ= 80, k=0.1



Plot for step response with K=0.1 and phase margin=80

If we add a phase Φ= 88, k=0.1;



Plot for step response with k=0.1 and phase margin=88

Observations:

- If we increased the value of k, the system became faster, and the overshoot was high. If we increase the gain further the overshoot becomes high.
- So, we decrease the gain value, and the system becomes slow. But to decrease the overshoot, we need to increase the phase.
- We added the phase to get less overshoot and the system to be stable.

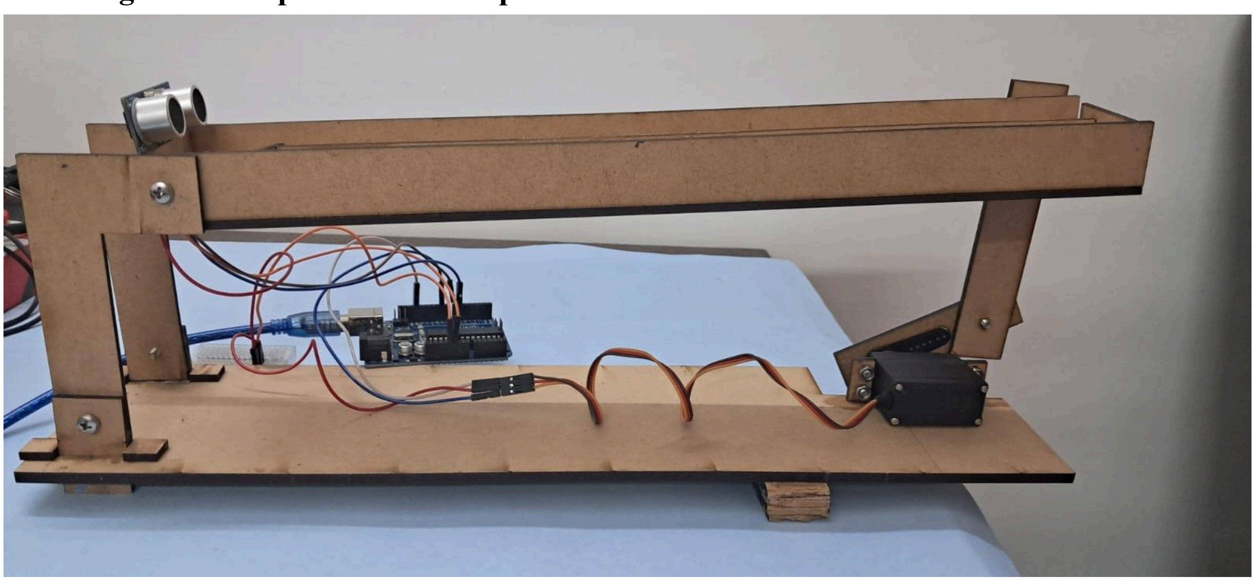
Verification for bode plot approach:

- The steady-state value is approximately 1.
- The response does not visibly exceed the steady-state value by more than 5%. So, the system meets the overshoot criteria.
- The 2% tolerance band is 1 ± 0.02i.e, between 0.98 to 1.02.
- The response appears to settle in less than 1 second. So, it satisfies the settling time.

3. Task 8: Demonstration of the Physical Project Setup

Choose a specific position for the ball on the beam. Tune the PID controller to keep the ball stable at this location. A final project demonstration will be scheduled towards the end of the semester at a mutually agreed time. Bring your physical setup and demonstrate your PID controller's ability to effectively stabilize the ball and respond to minor perturbations about the desired position along the beam.

Following is the setup that we developed:

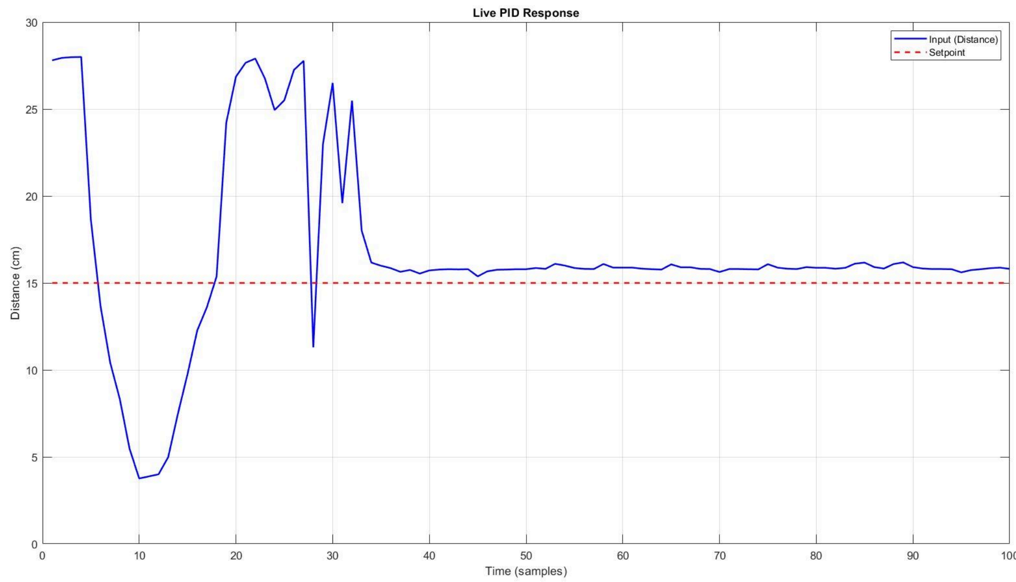


After tuning it, we reached a close level to stabilise it effectively but for a limited number of times. The reasons for some errors are:

1. **Irregular rolling of the ball.**
2. **Higher momentum of the ball, causing overshooting.**
3. **Setup instability error**

4. Air resistance

Following is the real-time plot for the response of the system:



The values of gains are Kp=2.2, Ki=0, Kd=7.7.