Extended Surfaces Heat transfer

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I. Objectives

The Objectives of the experiment are as follows :

- To check the temperature at various points on a surface that has different cross-sectional shapes.
- To see how the measured temperature compares to what theory predicts for fins with an insulated end.
- To compare how well different types of fins work in terms of efficiency and effectiveness.

II. Essential Background

A. *Fins/Extended Surfaces with Application of Fins :*

Fins or extended surfaces are structures attached to solid objects that increase the rate of heat transfer by increasing the contact area with the surrounding environment. They are designed to dissipate heat more efficiently when the heat transfer coefficient of the surrounding fluid is low because the increase in surface area compensates for the low heat transfer coefficient.

- **Heat Exchangers**: Condensers, evaporators, and radiators all employ fins to enhance heat dissipation.
- **Automotive:** To remove heat from the engine coolant, car radiators use fins.
- **Electronics Cooling:** To enhance thermal control, heat sinks are affixed to the heat sinks of CPUs, GPUs, and other electronic equipment.
- **Aerospace Industry:** Fins are utilized to effectively regulate heat in aircraft engines and other high-temperature components.

B. *Equation for fins with constant cross-sectional area assumes a steady state and neglects heat generation.*

The following assumptions have been taken:

- Steady-state heat conduction
- 1-dimensional heat conduction
- No heat generation
- Constant cross-sectional area

The heat conduction equation is given by:

$$\boxed{\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0}$$

where:

- $\theta(x) = T(x) - T(\infty)$ is the temperature difference between the fin at a given location x and the ambient temperature.
- h is the convective heat transfer coefficient (W/m^2·K).
- $T(x)$ is the temperature distribution along the length of the fin.
- P is the perimeter of the fin (m).
- k is the thermal conductivity of the fin material (W/m·K).
- A is the cross-sectional area of the fin (m^2).
- T is the ambient temperature (°C or K).

C. *Plot the temperature distribution for fins with an insulated tip.*

The boundary conditions for a fin with an insulated tip:

- At $x = 0$: $T = T_b$, $\theta = T_b - T_\infty = \theta_b$
- At $x = L$ (insulated tip): $-k\left.\frac{dT}{dx}\right|_{x=L} = 0$, which implies $\left.\frac{dT}{dx}\right|_{x=L} = 0$
- The heat conduction equation is:

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0$$

By solving these equations, we get:

$$\theta(x) = \theta_b \frac{\cosh\left(m(L-x)\right)}{\cosh(mL)}$$

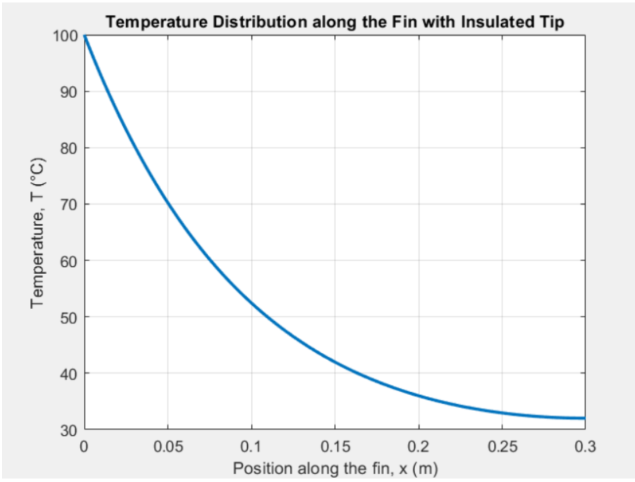

Fig. 1. Temperature distribution for fins with an insulated tip.

D. Fin Efficiency and Effectiveness

Fin Efficiency (η)**:** It is the ratio of the heat transfer from a fin to the maximum possible heat transfer if the entire fin were at its base temperature. Fin efficiency indicates the extent to which the temperature along the length varies from the base temperature. It is given by:

$$\eta = \frac{\text{Actual heat transfer rate from fin}}{\text{Maximum possible heat transfer}}$$

Fin efficiency is always less than 1, indicating that there is some heat loss due to temperature differences along the fin.

Fin Effectiveness (ϵ)**:** It is the ratio of the heat transfer from a fin to the heat transfer without the fin. It indicates how much the fin contributes to enhancing heat transfer. Fin effectiveness can be greater than 1, suggesting that the fin significantly improves heat transfer compared to the surface without fin. It is given by:

$$\epsilon = \frac{\text{Actual heat transfer with fin}}{\text{Heat transfer without fin}}$$

III. Experimental Setup

The experimental setup consists of the following components:

A. Fins

The experiment uses fins with different geometries (circular and square) and with constant cross-sectional area. These fins are made of copper. Specifically:

- Square fins have each side of length 1 cm.
- Circular fins have a diameter of 1 cm.

B. Heater

One end of the fin (the base) has a heat source attached to it. The temperature at the base is kept constant as heat is supplied to replicate a heated surface. A power supply regulates this to ensure that the base temperature remains constant.

C. Thermocouples

To measure the temperature distribution along the fin, thermocouples are placed at various points along its length. A thermocouple is positioned at several locations to record the temperature gradient from the base temperature to the tip. The thermocouples are connected to a temperature data logger to record the temperatures.

D. Insulation

To minimize convective and radiative losses and to simulate real-world conditions, the experimental apparatus is placed inside an insulated chamber.

E. Power Supply

A power supply provides power to the heater and allows for adjusting the voltage and current to control the heat input.

F. Data Logger

The data logger records real-time temperature data from the thermocouples.

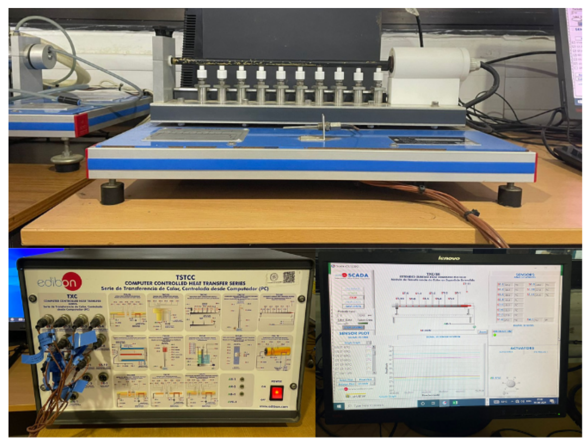

Fig. 2. Experimental Setup

IV. Procedure

1) After the setup was created, the heater and all the 11 thermocouples were interfaced with the software after installation of the required equipment.
2) To record the temperature of the metal rod at different points, ten thermocouples were installed at equal intervals along the length of the metal rod and the eleventh one measured the ambient temperature.
3) The experiment was initiated by starting the timer in the software, which also started the heater.
4) As the heater applied the heat to one end of the rod, the thermocouples recorded the temperature of the rod at their respective points.
5) Temperature readings from each thermocouple were captured every 10 seconds by the software, which automatically collected the data in an Excel sheet.
6) This process continued for 20 minutes and all the temperature readings were recorded in an Excel sheet for further analysis.

V. Calculation and Results

A. Temperature Variation over Time

1. Square Cross-Section:

- The temperature starts at **37.5°C** near the base (0 cm) and gradually decreases as we move towards the tip of the fin (27 cm).
- The temperature profile shows a smooth, gradual decline, which is typical for conductive heat transfer along a solid fin.
- The temperatures at positions farther from the heat source approach an almost constant value of around **25.8°C**, indicating that the tip of the fin is approaching thermal equilibrium with the surroundings.
- The small difference in temperature between points beyond **21 cm** suggests minimal heat transfer beyond this point, highlighting the efficiency loss toward the end of the fin.

2. Circular Cross-Section:

- The temperature at the base of the fin starts significantly higher at **47.8°C**, compared to the square cross-section.
- The temperature decreases rapidly in the first few centimeters, reaching **39.0°C** at the 6 cm mark, which shows a sharp initial heat dissipation.
- As the distance increases, the temperature continues to drop but more gradually, until it stabilizes around **28°C - 27.9°C** toward the tip.
- The more rapid initial heat loss for the circular cross-section may be due to its larger surface area compared to the square cross-section, allowing for more effective heat dissipation in the initial sections.
- The stabilization of the temperature at the fin's tip suggests that, like the square cross-section, the fin's thermal capacity diminishes, but slightly higher temperatures are retained toward the end of the circular fin.

3. Comparison:

- The circular cross-section starts with a higher initial temperature, likely due to its greater ability to absorb heat at the base.
- The square cross-section shows a more gradual decline in temperature, maintaining slightly higher temperatures over longer distances before stabilizing.
- Both cross-sections show a trend of stabilization at the tip, though the circular fin stabilizes at a slightly higher temperature.

B. Temperature Distribution along the Fin

Assuming the fin tip is insulated, the temperature boundary conditions for the fin with square cross section area are:

$$T(0) = 37.5°C, T(\infty) = 24.4°C$$

and the temperature boundary conditions for the fin with circular cross section area are:

$$T(0) = 47.8°C, T(\infty) = 24.8°C$$

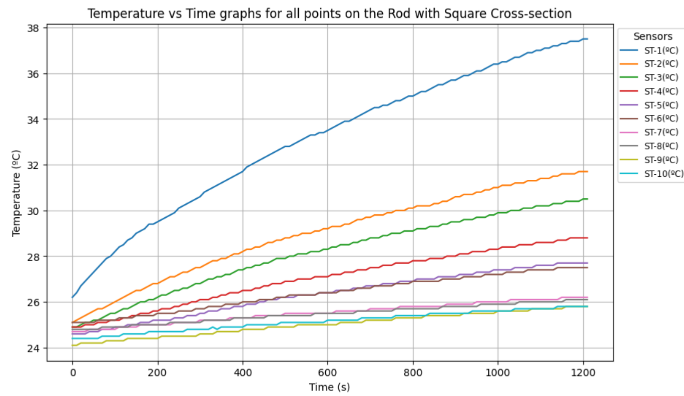

Fig. 3. Temperature vs Time graph for all points [Square Cross-Section]

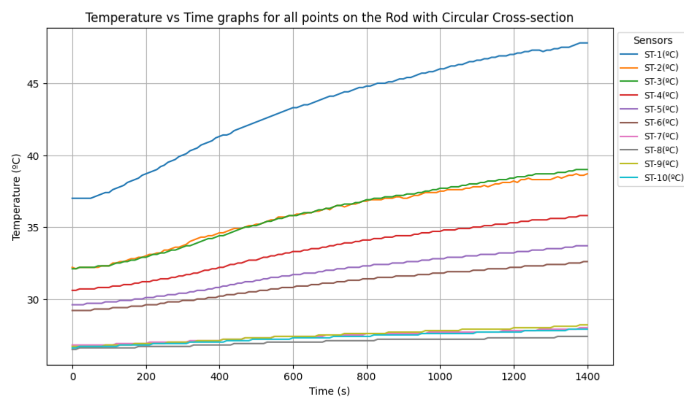

Fig. 4. Temperature vs Time graph for all measurement points [Circular Cross-Section]

Using the principle of energy conservation, the governing differential equation becomes:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where:

$$\boxed{m = \sqrt{\frac{h \cdot P}{k \cdot A_c}}}$$

$$\theta = T(x) - T(\infty)$$

The terms are defined as follows:

- h: Heat transfer coefficient (W/m^2·K)
- P: Perimeter of the copper bar (m)
- k: Thermal conductivity of the material (W/m·K)
- A_c: Cross-sectional area of the copper bar (m^2)
- $\theta(x)$: Temperature difference at position x

To solve this, we make use of the following boundary conditions:

$$\theta(x = 0) = \theta_b$$

$$-k\frac{d\theta(x = L)}{dx} = 0$$

On solving, we get the following temperature distribution:

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh\left(m(L-x)\right)}{\cosh(mL)}}$$

The temperature distribution along the fin over a period of 20 minutes (for square cross section) and 23 minutes (for circular cross section) from the start of the experiment is shown below:

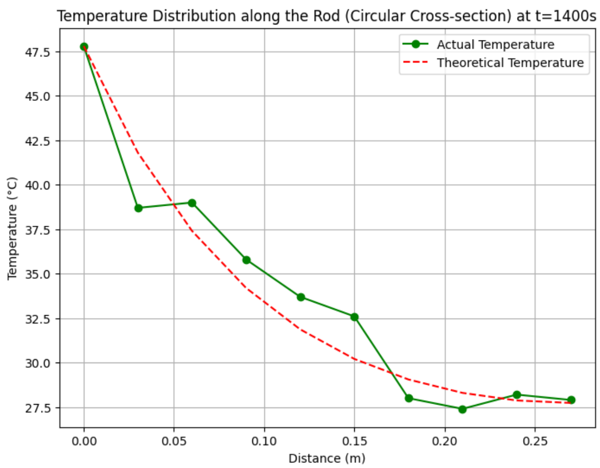

Fig. 5. Temperature vs Distance graph [Circular Cross-Section]

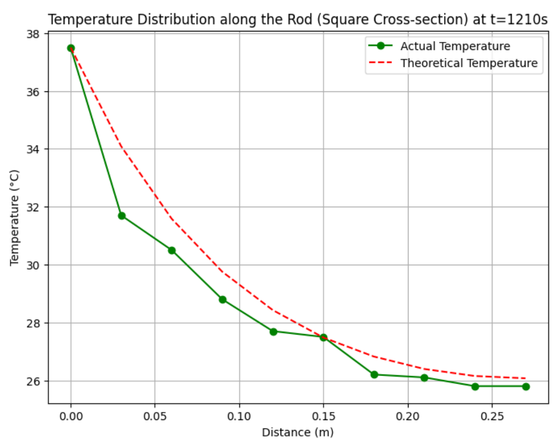

Fig. 6. Temperature vs Distance graph [Square Cross-Section]

C. Fin Parameters

Effectiveness: We know,

$$\varepsilon = \frac{q_{\text{w,fin}}}{q_{\text{wo,fin}}}$$

where:

$$q_{\text{w fin}} = M \tanh(mL) \quad \text{(Actual heat transfer with the fin)}$$

$$q_{\text{wo fin}} = hA_c\theta_b \quad \text{(Heat transfer without the fin)}$$

Here,

$$M = \sqrt{hPkA_c}\theta_b$$

L is the length of the fin

Efficiency: We know,

$$\eta = \frac{q_{\text{w,fin}}}{q_{\text{ideal}}} \times 100$$

where:

$$q_{\text{ideal}} = hA_f\theta_b \quad \text{(Maximum Heat transfer at } \theta = \theta_b\text{)}$$

Here, A_f is the surface area of the fin. In our analysis, the rods/fins were assumed to have the following dimensions:A cylindrical rod with a diameter of 10 mm. A square rod with a side length of 10 mm. Both rods were 270 mm in length.

And in our calculations, we have used the following parameter values:

- Heat transfer coefficient, $h = 100\,\mathrm{W/m^2 \cdot K}$
- Perimeter of the square fin, $P_s = 0.04\,\mathrm{m}$ and perimeter of the circular fin, $P_c = 0.0314\,\mathrm{m}$
- Thermal conductivity of copper, $k = 386\,\mathrm{W/m \cdot K}$
- Cross-sectional area of the square fin, $A_{cs} = 0.0001\,\mathrm{m^2}$ and cross-sectional area of the circular fin, $A_{cc} = 0.0000785\,\mathrm{m^2}$
- Length of the fin, $L = 0.27\,\mathrm{m}$
- Base temperature difference for square fin, $\theta_{bs} = 13.1\,°\mathrm{C}$ and base temperature difference for circular fin, $\theta_{bc} = 23\,°\mathrm{C}$

Parameter	Circular Fin	Square Fin
Heat Transfer Rate (W)	7.0401	5.1055
Effectiveness	38.9730	38.9730
Efficiency (%)	36.09	36.09

VI. Errors

Several sources of errors were identified in the experiment, which could have impacted the results:

TABLE I
COMPARISON OF THEORETICAL AND ACTUAL TEMPERATURES FOR SQUARE FIN

Theoretical Temp (°C)	Actual Temp (°C)	Absolute Error (°C)	Percent Error (%)
37.50	37.50	0.00	0.00
34.09	31.70	2.39	7.00
31.58	30.50	1.08	3.43
29.75	28.80	0.95	3.20
28.43	27.70	0.73	2.56
27.48	27.50	0.02	0.07
26.82	26.20	0.62	2.32
26.39	26.10	0.29	1.11
26.15	25.80	0.35	1.33
26.07	25.80	0.27	1.03

A. Systematic Errors

- **Heat Loss from Fin Tip:** The absence of proper insulation at the fin tip may have resulted in heat loss, leading to deviations from the ideal insulated tip condition assumed in the theoretical model.
- **Early Steady-State Assumption:** If the system was assumed to have reached steady-state prematurely, this could have introduced temperature fluctuations, thereby affecting the results.

TABLE II
COMPARISON OF THEORETICAL AND ACTUAL TEMPERATURES FOR CIRCULAR FIN

Theoretical Temp (°C)	Actual Temp (°C)	Absolute Error (°C)	Percent Error (%)
47.80	47.80	0.00	0.00
41.81	38.70	3.11	7.43
37.41	39.00	1.59	4.25
34.20	35.80	1.60	4.68
31.87	33.70	1.83	5.74
30.21	32.60	2.39	7.92
29.05	28.00	1.05	3.62
28.30	27.40	0.90	3.17
27.87	28.20	0.33	1.18
27.73	27.90	0.17	0.61

B. *Environmental Factors*

- **Room Temperature Variations:** Fluctuations in ambient temperature or airflow could have altered the heat transfer rates, resulting in inconsistencies in the data.

C. *Experimental Setup Issues*

- **Inconsistent Heater Performance:** Any variations in the heater's output could cause inconsistencies in the base temperature, leading to variations in the recorded temperature profiles.
- **Material Imperfections:** Inhomogeneities in the fin material, such as varying thermal conductivity due to defects, may have influenced the heat transfer patterns.

D. *Data Logging Errors*

- **Precision of Data Logger:** Limited accuracy in the data logger could have introduced small errors in the temperature measurements.

VII. CONCLUSION

The following conclusions can be drawn from the experiment:

- **Heat Transfer Comparison**: The square rod transfers more heat compared to the circular rod.
- **Efficiency and Effectiveness**: Both the square and circular rods have equal efficiencies and effectiveness, as the ratios of P/A for both rods are the same. However, their heat transfer rates differ.
- **Temperature Variation**: The point on the rod closest to the heater experiences the highest temperature increase over time, whereas the point farthest from the heater experiences the lowest increase.

The experiment successfully demonstrated the heat flow along the length of the fin, as depicted in the temperature distribution chart. This chart was compared with the theoretical graph, which showed a smoother curve representing ideal heat distribution. The practical graph exhibited some minor deviations, particularly at the beginning.

Overall, the fin exhibited good heat dissipation behavior. The experiment confirmed that the concept of heat dissipation, as applied, aligns well with theoretical expectations and indicates that cooling performance can be improved in various applications.

VIII. REFERENCES

1) F. Incropera and D. DeWitt, *Fundamentals of Heat and Mass Transfer*, 6th ed., Wiley, 2006.
2) Y. A. Cengel, *Heat and Mass Transfer: A Practical Approach*, 5th ed., McGraw-Hill, 2014.

IX. APPENDIX

- Data for Circular Cross section
- Data for Square Cross section
- Code file for plots