

Practice Exercises

Andres Gregori

February 21, 2023

Disclosure

The following exercise were compiled with educational purposes only. All rights reserved.

1 Basic Counting and Basic Probability

1. A space S and three of its subsets are given by $S = [1, 3, 5, 7, 9, 11]$, $A = [1, 3, 5]$, $B = [7, 9, 11]$, and $C = [1, 3, 9, 11]$. Find $A \cap B \cap C$, $A^c \cap B$, $A \cap C$, and $(A \cup B) \cup B$.
2. A space S is defined as $S = [1; 3; 5; 7; 9; 22]$, and three subsets as $A = [1; 3; 5]$, $B = [7; 9; 11]$, $C = [1; 3; 9; 11]$. Assume that each element has probability $1/6$. Find the following probabilities: $\Pr(A)$, $\Pr(B)$, $\Pr(C)$, $\Pr(A \cup B)$, $\Pr(A \cup C)$
3. A collection of 26 letters, a-z, is mixed in a jar. Two letters are drawn at random, one after the other. What is the probability of drawing a vowel (a,e,i,o,u) and a consonant in either order? What is the sample space?
4. A simplified model of the human blood-type system has four blood types: A, B, AB, and O. There are two antigens, anti-A and anti-B, that react with a person's blood in different ways depending on the blood type. Anti-A reacts with blood types A and AB, but not with B and O. Anti-B reacts with blood types B and AB, but not with A and O. Suppose that a person's blood is sampled and tested with the two antigens. Let A be the event that the blood reacts with anti-A, and let B be the event that it reacts with anti-B. Classify the person's blood type using the events A , B , and their complements.
5. One ball is to be selected from a box containing red, white, blue, yellow, and green balls. If the probability that the selected ball will be red is $1/5$ and the probability that it will be white is $2/5$, what is the probability that it will be blue, yellow, or green?

6. If the probability that student A will fail a certain statistics examination is 0.5, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these two students will fail the examination?
7. If 50 percent of the families in a certain city subscribe to the morning newspaper, 65 percent of the families subscribe to the afternoon newspaper, and 85 percent of the families subscribe to at least one of the two newspapers, what percentage of the families subscribe to both newspapers?

2 Conditional Probability

1. Suppose a crime has been committed. Blood is found at the scene for which there is no innocent explanation. It is of a type which is present in 1% of the population. a. The prosecutor claims: "There is a 1% chance that the defendant would have the crime blood type if he were innocent. Thus there is a 99% chance that he is guilty". This is known as the prosecutor's fallacy. What is wrong with this argument? b. The defender claims: "The crime occurred in a city of 800,000 people. The blood type would be found in approximately 8000 people. The evidence has provided a probability of just 1 in 8000 that the defendant is guilty, and thus has no relevance." This is known as the defender's fallacy. What is wrong with this argument?
2. Suppose that 10,000 tickets are sold in one lottery and 5000 tickets are sold in another lottery. If a person owns 100 tickets in each lottery, what is the probability that she will win at least one first prize?
3. Suppose that a person rolls two 20 sided balanced dice three times in succession. Determine the probability that on each of the three rolls, the sum of the two numbers that appear will be 20.
4. A box contains 20 red balls, 30 white balls, and 50 blue balls. Suppose that 10 balls are selected at random one at a time, with replacement; that is, each selected ball is replaced in the box before the next selection is made. Determine the probability that at least one color will be missing from the 10 selected balls.
5. Three students A, B, and C are enrolled in the same class. Suppose that A attends class 30 percent of the time, B attends class 50 percent of the time, and C attends class 80 percent of the time. If these students attend class independently of each other, what is (a) the probability that at least one of them will be in class on a particular day and (b) the probability that exactly one of them will be in class on a particular day?

3 Random Variables

1. Suppose that a random variable X has the binomial distribution with parameters $n = 8$ and $p = 0.7$. Find $Pr(X \geq 5)$
2. Two dice are tossed. Let X be the absolute difference in the number of dots facing up. (a) Find and plot the PMF of X . (b) Find the probability that $X \leq 2$. (c) Find $E[X]$ and $Var[X]$.
3. The Yankees are playing the Red Sox. You're a diehard Sox fan and bet your friend they'll win the game. You'll pay your friend \$30 if the Sox lose and your friend will have to pay you only \$5 if the Sox win. What is the probability you have intuitively assigned to the belief that the Red Sox will win? *Hint: Use expected value*
4. Three men A, B, and C shoot at a target. Suppose that A shoots three times and the probability that he will hit the target on any given shot is $1/8$, B shoots five times and the probability that he will hit the target on any given shot is $1/4$, and C shoots twice and the probability that he will hit the target on any given shot is $1/2$. What is the expected number of times that the target will be hit?
5. An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on the average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat.
6. Suppose that two players A and B are trying to throw a basketball through a hoop. The probability that player A will succeed on any given throw is p , and he throws until he has succeeded r times. The probability that player B will succeed on any given throw is mp , where m is a given integer ($m = 2, 3, \dots$) such that $mp \geq 1$, and she throws until she has succeeded mr times. a. For which player is the expected number of throws smaller?
7. Suppose that in a large lot containing T manufactured items, 30 percent of the items are defective and 70 percent are nondefective. Also, suppose that ten items are selected at random without replacement from the lot. Determine (a) an exact expression for the probability that not more than one defective item will be obtained and (b) an approximate expression for this probability based on the binomial distribution.
8. Suppose that a sequence of independent tosses are made with a coin for which the probability of obtaining a head on each given toss is $1/30$. a. What is the expected number of tails that will be obtained before five heads have been obtained?
9. Suppose that the measured voltage in a certain electric circuit has the normal distribution with mean 120 and standard deviation 2. If three

independent measurements of the voltage are made, what is the probability that all three measurements will lie between 116 and 118?

10. If the temperature in degrees Fahrenheit at a certain location is normally distributed with a mean of 68 degrees and a standard deviation of 4 degrees, what is the distribution of the temperature in degrees Celsius at the same location?