Practice Exercises 3

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Disclosure

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1 Distributions

1. Suppose that a fair coin is flipped 200 times. Approximate the probability of getting between 90 and 110 heads using the normal approximation to the binomial distribution.

2. Suppose that a survey of 500 people is conducted, and 40% of them plan to vote for Candidate A. Approximate the probability that between 180 and 220 people plan to vote for Candidate A using the normal approximation to the binomial distribution.

3. Suppose that a quality control inspector checks 1000 light bulbs, and the probability that a bulb is defective is 0.02. Approximate the probability that between 15 and 25 bulbs are defective using the normal approximation to the binomial distribution.

4. Suppose that a drug is effective in treating a disease in 70% of cases. A sample of 400 patients with the disease is treated with the drug. Approximate the probability that between 250 and 300 patients are cured using the normal approximation to the binomial distribution.

2 Central Limit Theorem

1. The heights of adult women in a certain population have a mean of 65 inches and a standard deviation of 3 inches. If a random sample of 100 women is selected, what is the probability that the sample mean height is less than 64 inches?

2. A company produces light bulbs with a mean life of 500 hours and a standard deviation of 50 hours. If a random sample of 25 light bulbs is selected, what is the probability that the sample mean life is more than 520 hours?

3. A teacher wants to estimate the average time her students spend on homework each week. She randomly selects a sample of 36 students and finds that the sample mean time spent on homework is 2.5 hours with a standard deviation of 0.5 hours. What is the 95% confidence interval for the true population mean time spent on homework?

4. A company produces computer chips with a mean processing time of 10 milliseconds and a standard deviation of 1 millisecond. If a random sample of 100 computer chips is selected, what is the probability that the sample mean processing time is between 9.5 and 10.5 milliseconds?

5. The weights of chocolate bars produced by a company have a mean of 100 grams and a standard deviation of 5 grams. If a random sample of 64 chocolate bars is selected, what is the probability that the sample mean weight is less than 98 grams?

3   Confidence Intervals

1. A manufacturer claims that the average weight of a bag of chips is 200 grams. A sample of 50 bags was taken and the mean weight was found to be 195 grams with a standard deviation of 10 grams. Calculate the 95% confidence interval for the true mean weight of a bag of chips.

2. A hospital administrator wants to estimate the average length of stay for patients in the hospital. A sample of 100 patients was taken and the mean length of stay was found to be 4.5 days with a standard deviation of 1.2 days. Calculate the 99% confidence interval for the true mean length of stay.

3. A financial analyst wants to estimate the average return on a particular stock. A sample of 25 returns was taken and the mean return was found to be 8% with a standard deviation of 2%. Calculate the 90% confidence interval for the true mean return.

4. A political pollster wants to estimate the proportion of people who support a particular candidate. A sample of 500 people was surveyed and 220 of them expressed support for the candidate. Calculate the 95% confidence interval for the true proportion of people who support the candidate.

5. A medical researcher wants to estimate the proportion of people who have a particular disease. A sample of 1000 people was tested and 80 of them were found to have the disease. Calculate the 99% confidence interval for the true proportion of people who have the disease.

6. A car manufacturer wants to estimate the variance in the fuel efficiency of a new model of car. A sample of 50 cars was tested and the variance in fuel efficiency was found to be 4.5 miles per gallon. Calculate the 95% confidence interval for the true variance in fuel efficiency.

7. A quality control inspector wants to estimate the variance in the diameter of a particular type of bolt. A sample of 100 bolts was measured and the variance in diameter was found to be 0.02 millimeters. Calculate the 99% confidence interval for the true variance in diameter.

8. A psychologist wants to estimate the variance in the reaction time of people to a particular stimulus. A sample of 30 people was tested and the variance in reaction time was found to be 0.2 seconds. Calculate the 90% confidence interval for the true variance in reaction time.

4 Hypothesis Testing

1. An environmental organization claims that the average temperature in a particular region has increased by at least 1 degree Celsius over the past decade. A sample of 30 temperature readings from the past decade is collected, and the sample mean temperature increase is 0.8 degrees Celsius with a standard deviation of 0.2 degrees Celsius. Test the organization's claim at a 5% level of significance.

2. A bank claims that the average waiting time in their customer service line is less than 5 minutes. A random sample of 100 customers is selected and their waiting times are recorded. The sample mean waiting time is 4.5 minutes with a standard deviation of 1 minute. Test the bank's claim at a 10% level of significance.

3. A restaurant owner wants to test whether a new menu item is popular among customers. A sample of 50 customers is selected and asked whether they liked the new item or not. 40 customers said they liked it while 10 said they did not. Test the hypothesis that the new menu item is popular among customers at a 5% level of significance.

5 Solutions

5.1 Distributions

1. 0.7977

2. 0.8554

3. 0.8708

4. 0.9813

5.2 Central Limit Theorem

1. 0.0004

2. 0.0228

3. (2.337, 2.663)

4. 1

5. 0.0202

5.3 Confidence Intervals

1. (192.16, 197.84)

2. (4.18, 4.82)

3. (6.63, 9.37)

4. (0.391, 0.489)

5. (0.052, 0.108)

6. (6.98, 16.04)

7. (3461.5, 6732)

8. (0.646, 1.545)

5.4 Hypothesis Testing

1. We reject the null hypothesis. There is sufficient evidence to suggest that the average temperature increase is greater than 1 degree Celsius. t=-2.7386

2. We reject the null hypothesis. There is sufficient evidence to suggest that the average waiting time is less than 5 minutes. t=-5

3. We reject the null hypothesis. There is sufficient evidence to suggest that the proportion of customers who like the new menu item is greater than 0.5. z=6.32