Probability And Statistics

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Abstract

This is a brief summary about the course of Probability and Statistics conducted at the Engineering School of Universidad Panamericana. The objective is to help the student understand basic theoretical concepts reviewed in class and to reinforce specific subjects the student may have missed. For any mistake found in this document, please feel free to reach out. This document is compiled for educational purposes only.

0 Set Theory

Definition A set is a collection of elements. Let: $S = \{\theta_1, \theta_2, ..., \theta_n\}$ Where θ_n is the nth element of the set. A set can contain all kinds of elements such as: all tropical fruits, the first 35 Fibonacci numbers etc. Some of them are finite or others are infinite or uncountable. Understanding the abstraction of what is a set will be fundamental for this course. On a very harsh stroke we could define Probability as: **the measure of a size of a set.** Chan (2021)

Notation

- $\theta \in S$:Element θ is in set S.
- $M \subset S$: Set M is a subset of set S.
- $A \cup B$: Union of sets A and B
- $A \cap B$: Intersection of sets A and B
- $S = \emptyset$: Set S is null or empty.
- $S = \Omega$: S is the Universal set
- S^c: Compliment of set S

0.1 Set properties

Commutative:

$$A \cap B = B \cap A$$
$$A \cup B = B \cup A$$

Associative:

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan's Law:

$$(A \cap B)^c = A^c \cup B^c$$
$$(A \cup B)^c = A^c \cap B^c$$

0.2 Set Operations

Union: Elements from A or B are combined into one set.
Example:

$$
\begin{gathered}
A \subset S \, and \, B \subset S \\
S = \{All \, NFL \, Teams\} \\
A = \{All \, teams \, in \, the \, AFC \, West\} = \{LAC, DEN, KC, LV\} \\
B = \{All \, teams \, in \, CA\} = \{LAC, LAR, SF\} \\
A \cup B = \{LAC, LAR, SF, DEN, KC, LV\}
\end{gathered}
$$

Intersection: Only elements in common from A and B are selected into one set.
Example:

$$
\begin{gathered}
A \subset S \, and \, B \subset S \\
S = \{All \, NFL \, Teams\} \\
A = \{All \, teams \, in \, the \, AFC \, West\} = \{LAC, DEN, KC, LV\} \\
B = \{All \, teams \, in \, CA\} = \{LAC, LAR, SF\} \\
A \cap B = \{LAC\}
\end{gathered}
$$

Difference: Takes all the elements from A but not the ones that are in common with B. Or viceversa
Example:

$$
\begin{gathered}
A \subset S \, and \, B \subset S \\
S = \{All \, NFL \, Teams\} \\
A = \{All \, teams \, in \, the \, NFC \, East\} = \{NYG, DAL, WAS, PHI\} \\
B = \{All \, teams \, with \, at \, least \, 3 \, Super \, Bowls\} = \{SF, DAL, NYG, WAS, LV, DEN, GB, PIT, NE\} \\
B \setminus A = \{NE, PIT, GB, SF, DEN, LV\}
\end{gathered}
$$

Sample Space: We will define the sample space from now on as S and this is going to be the most important set on the experiments we will make. The sample space is the collection of all possible outcomes of an experiment. As we know, there are finite and infinite sets.

0.3 Counting principles

Suppose we ought to perform any experiment. As we said before, some sample spaces are countable and some of them are uncountable. There are some methods for counting this sample spaces.

Permutations Useful for calculating sample spaces in which replacement matters. For example the possible batting orders for a baseball team is influenced by the first at bat. To calculate you just need to do the numbers factorial.

$$n! = n(n-1)(n-2)\dots(2)(1)$$

Combinations Whenever we want to make groups out of a bigger set and count the possible outcomes of doing this experiment we will need combinations. We can say all the possible combinations of doing a Powerball or Melate.

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

It can be read as from r set take n elements.

1 Probability 101

Definition: Although there are many sources that define probability in a different way. Let's stick to Devore (2019) definition which states: We refer to **probability** as the study of randomness and uncertainty in any given situation in which many outcomes can occur. We always use probability in our everyday activities. Even unconsciously we calculate probabilities, but that matter will be left for another course. *See: Kahneman (2011)*

Notation: We need to understand that **probability** calculates **events** within **sample spaces**. For example the event that a die is rolled the sample space is all the possible outcomes $\{1,2,3,4,5,6\}$ and the probability of each event is $\frac{1}{6}$. *For the next steps, we will refer as the probability of an event as* $Pr(x)$

Experiments: The most common experiments we will contemplate and create examples form this are:

- Drawing a card from the traditional French deck of 52 playing cards $S = \{4A, 4K, 4Q, 4J, 10, 9, 8, 7, 6, 5, 4, 3, 2\}$
- Flipping a fair coin $S = \{H, T\}$
- Tossing a fair die $S = \{1, 2, 3, 4, 5, 6\}$

These will be used for further examples since all of the events have equal probability.

1.1 Axioms of Probability

1. All probabilities must be denoted with a number between 0 and 1. *Uncountable set*

$$0 \leq \ Pr(x) \ \leq 1$$

2. Let S be the sample space

$$Pr(S) = 1$$

   This means that all the events that are plausible within the sample space need to sum 1.

3. On mutually exclusive events the probability of all the events is the sum of each probabilities.

$$Pr\left(\bigcup_{i=1}^{\infty} x_i\right) = \sum_{i=1}^{\infty} Pr(x_i)$$

1.2 Propositions from Axioms

Definition: With the axioms of probability already stated we can talk about some propositions we can infer from this axioms.

1. Let

$$Pr(X) < 1$$

   therefore

$$Pr(X) + Pr(X^c) = 1$$

   we can conclude that we can also calculate the probability of an event by doing

$$1 - Pr(X^c)$$

2. For any event $A \subset S$ $Pr(A) \leq 1$

3. For any two events A and B there are certain approximations we need to make in order to avoid double counting. Here it will come real handy your sets theory knowledge. Let:

$$Pr(A \cup B) = Pr(A) + Pr(B) - P(A \cap B)$$

4. For three events A, B and C

$$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(A \cap C) - Pr(B \cap C) - Pr(A \cap B \cap C)$$

Sample Spaces Having Equally Likely Outcomes We will review some brief examples to understand the experiments that have equally likely outcomes.
Example 1: If two dice are rolled, what is the probability that the sum of the faces will equal 7? **(Catan problem)**
We can define all possible outcomes are equally likely by assuming both dice are fair. Therefore we have 36 possible combinations. The tuples that can yield 7 are: $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$ this means there are $\frac{6}{36} = \frac{1}{6}$ of chances the sum of the dices is 7.

References

Chan, S. H. (2021). *Introduction to probability for data science.* Michigan: Michigan Publishing.
Devore, J. L. (2019). *Probability and statistics for engineering and the sciences.* New York: Brooks/Cole.
Kahneman, D. (2011). *Thinking fast and slow.* Farrar: Strauss and Giroux.