Practice Exercises 2

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Disclosure

1 Distributions

1. Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait: a) more than 10 minutes b) between 10 and 20 mins

2. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x > 10 \\ 0 & \text{if } x \leq 10 \end{cases}$$

   (a) Find $Pr(X > 20)$. (b) What is the cumulative distribution function of X? (c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?

3. For some constant c, the random variable X has the probability density function

$$f(x) = \begin{cases} cx^4 & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

   Find (a) E[X] and (b) Var(X).

4. Suppose that 15,000 people in a city with a population of 500,000 are watching a certain television program. If 200 people in the city are contacted at random, what is the approximate probability that fewer than four of them are watching the program?

5. At a certain bank, the amount of time that a customer spends being served by a teller is an exponential random variable with mean 5 minutes. If there is a customer in service when you enter the bank, what is the probability that he or she will still be with the teller after an additional 4 minutes?

6. The annual rainfall in Cleveland, Ohio is approximately a normal random variable with mean 40.2 inches and standard deviation 8.4 inches. What is the probability that (a) next year's rainfall will exceed 44 inches? (b) the yearly rainfalls in exactly 3 of the next 7 years will exceed 44 inches?

2 Central Limit Theorem

1. Suppose that the proportion of defective items in a large manufactured lot is 0.1.What is the smallest random sample of items that must be taken from the lot in order for the probability to be at least 0.99 that the proportion of defective items in the sample will be less than 0.13?

2. Suppose that three girls A, B, and C throw snowballs at a target. Suppose also that girl A throws 10 times, and the probability that she will hit the target on any given throw is 0.3; girl B throws 15 times, and the probability that she will hit the target on any given throw is 0.2; and girl C throws 20 times, and the probability that she will hit the target on any given throw is 0.1. Determine the probability that the target will be hit at least 12 times.

3. Suppose that people attending a party pour drinks from a bottle containing 63 ounces of a certain liquid. Suppose also that the expected size of each drink is 2 ounces, that the standard deviation of each drink is 1/2 ounce, and that all drinks are poured independently. Determine the probability that the bottle will not be empty after 36 drinks have been poured.

4. Each minute a machine produces a length of rope with mean of 4 feet and standard deviation of 5 inches. Assuming that the amounts produced in different minutes are independent and identically distributed, approximate the probability that the machine will produce at least 250 feet in one hour.

3 Confidence Intervals

1. From the following data: Calculate: $\bar{x}$ and s^2 for both groups and provide the quartiles for each of the data.

Age	M	F
0-4	9,595,774	9,166,323
5-9	10,054,322	9,615,416
10-14	10,595,422	10,115,924
15-19	11,415,536	10,907,316
20-24	11,519,287	11,107,006
25-29	11,489,905	11,139,813
30-34	12,003,252	11,718,560
35-39	11,821,516	11,605,307
40-44	11,343,543	11,168,500
45-49	10,505,313	10,505,476
50-54	10,228,535	10,316,869
55-59	10,246,929	10,473,252
60-64	10,299,101	10,928,081
65-69	9,525,197	10,370,041
70-74	7,844,302	8,968,021
75-79	5,487,418	6,727,205
80-84	3,288,516	4,330,345
85-89	1,766,166	2,571,227
90-94	764,240	1,350,121
95-99	208,877	410,977
100+	28,032	76,432

2. In the June 1986 issue of Consumer Reports, some data on the calorie content of beef hot dogs is given. Here are the numbers of calories in 20 different hot dog brands: 186, 181, 176, 149, 184, 190, 158, 139, 175, 148, 152, 111, 141, 153, 190, 157, 131, 149, 135, 132. Assume that these numbers are the observed values from a random sample of twenty independent normal random variables with mean μ and variance σ^2, both unknown. Find a 90% confidence interval for the mean number of calories μ.

3. We are looking for the parasite charge loss μ (watts) from a certain induction motor when current is at 10 amp and at a velocity of 15,000 rpm. Suppose that the parasite charge loss is normally distributed with $\sigma = 3$. Calculate: a) Confidence Interval for μ of 95% when n=25 and $\bar{x} = 58.3$ b) Confidence Interval for μ of 95% when n=100 and $\bar{x} = 58.3$ c) Confidence Interval for μ of 99% when n=100 and $\bar{x} = 58.3$

4 Solutions

4.1 Distributions

1. .368, .233

2. Class discussion

3. 5/32,, 5/3, 5/63
4. .1512
5. $e^{\frac{-4}{5}}$
6. 0.3255, $7C3(0.3255)^5(0.6745)^4$

4.2 Central Limit Theorem

1. 542
2. 0.0555
3. 0.0013
4. 0.001

4.3 Confidence Intervals

1. Class discussion
2. (148.1,165.6)
3. Class discussion