1. (a) $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ # $-\infty < x < \infty$

(b) $P(-1 \le Z \le 1)$

$\int_{-1}^{1} e^{\frac{1}{2}x^2} dx = 0.687$ #

(c) $P(-x \le Z \le x) = 95\%$

$97.5\% = 0.975 \Rightarrow Z = 1.96$

(d) $f_\theta(\theta) = \frac{1}{2^{v/2}\Gamma(v/2)} x^{v/2-1} \times e^{-x/2}$, $x > 0$

(e) 1 #

(f) $\sqrt{2}$ #

(g) $P(\theta \le 1) = 0.6829$ #

$\int_0^1 \frac{1}{\beta^2} x e^{-x/\beta} \sqrt{x} = 0.6829$ #

1-a

```
import sympy as sy
z = sy.pi
x = sy.symbols('x')
μ = sy.symbols('μ', real=True)
σ = sy.symbols('σ', positive=True)
fx = sy.exp(-1/2*((x-μ)/σ)**2)/
     σ/sy.sqrt(2*π)
```

1-b

```
μ = 0
σ = 1
Z1 = -1
Z2 = -1
st.norm.cdf(Z2,0,1)
-st.norm.cdf(Z1,0,1)
```

1-e

```
μ = sy.integrate
(x*fx(x,0,sy.oo))simplify()
```

1-f

```
σ**2 = sy.integrate((x-μ)**2)*fx,(x,0,sy.oo))
simplify()
```

2-a

$$f_T(t)=\begin{cases} e^{-t} & x>0 \\ 0 & x\le 0 \end{cases}$$

2-b $E(T)=\beta=1$

2-c $Std(T)=\sqrt{\beta}=1$

2-d $P(T>1)$

$\int_1^{\infty} e^{-t}dt = e^{-1}=0.3679$

2-e $\alpha=3 \quad \beta=1$

$$f_t(T)=\frac{1}{1^3\Gamma(3)}t^{3-1}e^{-t/1}=\frac{1}{2}t^2e^{-t}$$

$$f_t(T)=\begin{cases} \frac{1}{2}t^2e^{-t} & t>0 \\ 0 & t\le 0 \end{cases}$$

2-f $E(T3)=\alpha\beta=3\times 1=3$

2-g $Std[T3]=\sqrt{\alpha\beta}=\sqrt{3}$

2-h $P(T3>3)=0.4232$

2-i $P(T3>7)=0.0296$

使用3.3超过7年用机率

为2.96%很小可接受