

Q17

```
import math
```

```
def iexp(n):
```

```
    return complex(math.cos(n), math.sin(n))
```

```
def is_pow2(n)
```

```
    return False if n==0 else (n==1 or is_pow2(n>>1))
```

```
def dft(xs)
```

```
    "naïve dft"
```

```
    n = len(xs)
```

```
    return [sum(xs[k] * iexp(-2 * math.pi * i * k / n) for k
```

```
        in range(n)) for i in range(n)]
```

```
def dftinv(xs):
```

```
    "naïve dft"
```

```
    n = len(xs)
```

```
    return [sum(xs[k] * iexp(2 * math.pi * i * k / n) for k
```

```
        in range(n)) / n for i in range(n)]
```

```

for i in range(n):
    if name == "main":
        wave1 = [1, 0, 0, 0, 0, 0, 0]
        wave2 = [1, 1, 1, 1, 1, 1, 1]
        wave3 = [1, -1, 1, -1, 1, -1, 1]
        wave4 = [3, 0, 2, 0, 2, 0, 0]
        dfreq1 = [1, 1, 0, 0, 0, 0, 1]
        dfreq2 = [1, 1, 0, 0, 0, 0, 1]
        dfreq3 = [1, 1, 0, 0, 0, 0, 1]
        dfreq4 = [1, 1, 0, 0, 0, 0, 1]
        dfreq5 = [1, 1, 0, 0, 0, 0, 1]
        dfreq6 = [1, 1, 0, 0, 0, 0, 1]
        wave5 = dfreq1
        wave6 = dfreq2
        print(dfreq1)
        print(dfreq2)
        print(dfreq3)
        print(dfreq4)
        print(dfreq5)
        print(dfreq6)
        print(wave5)
        print(wave6)

```

$(1+0j), (1+0j), (1+0j), (1+0j), (1+0j), (1+0j), (1+0j)$
 $[8+0j], (-5.551e-16+2.220e-16j), (-4.286e-16-4.440e-16j), (-2.220e-16+8.881e-16j),$
 $-4.898e-16j, (-2.109e-15-1.221e-15j), (-2.932e-15-6.661e-16j), (3.55e-15+1.11e-15j)]$
 $[0j, (1.11e-16-1.11e-16j), (9.55e-17-1.11e-16j), (8.88e-16-1.55e-15j),$
 $(8+3.42e-15j), (-2.66e-15+1.11e-16j), (-2.93e-15-6.66e-16j), (-5.21e-15-2.66e-15j)]$
 $[9+0j], (0.999-4.44e-16j), (1-4.89e-16j), (1-6.66e-16j), (9+2.93e-15j),$
 $(0.99-1.33e-15j), (1-1.469e-15j), (0.999-1.77e-15j)]$
 $[0+5+0j], (0.3-0.125j), (-5.35e-17+5.55e-17j), (-0.0517+0.124j), 3.061e-17j,$
 $(-0.0517-0.124j), (-8.906e-17+1.387e-16j), (0.301+0.124j)]$
 $[0.375+0j], (0.301-1.387e-17j), (0.124+0j), (-0.0517-1.387e-17j), (-0.125+1.221e-15j),$
 $(-0.0517+1.804e-16j), (0.124+0j), (0.301-3.88e-16j)]$

(請翻面繼續作答)

(3) 指出函數摺積的傅立葉轉換的乘積，即一個域中的摺積對應於另一個域中的乘積。

$$\begin{aligned}\mathcal{L}[f(t) * g(t)] &= \int_0^\infty \int_0^t f(\tau) g(t-\tau) dt e^{-st} dt \\ &= \int_0^\infty f(\tau) \int_\tau^\infty g(t-\tau) e^{-st} dt d\tau\end{aligned}$$

$$\text{令 } t-\tau=x \quad dt=dx$$

$$= \int_0^\infty f(\tau) \int_0^\infty g(x) e^{-s(\tau+x)} dx d\tau$$

$$= \int_0^\infty g(x) e^{-sx} dx \int_0^\infty f(\tau) e^{-s\tau} d\tau$$

$$= G(s) \cdot F(s)$$