Iterative Inverse Kinematics for Serial Manipulators

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Abstract

Different methods for the inversion of the kinematics for serial manipulators are below discussed, in particular the Jacobian based ones. The algorithms will be evaluated with Matlab simulation and in the real GG1 robot as well, and the one with the best performance and faster convergence rate will be integrated into the system.

Jacobian based Inverse Kinematics

The inversion of the kinematics for serial manipulators has closed form solution only for simple architectures, for redundant and complex robots there are different solutions ($[0, \infty)$), these non-linearities arise from the inverse of the trigonometric relations that map the cartesian space to the joint space. A possible way to solve this problem is to use the differential kinematics relations:

$$x = J\dot{q} \rightarrow \dot{q} = J^{-1}x \tag{1}$$

and then using the knowledge about the initial position getting the joint variables:

$$q_k = \int_0^k \dot{q}(\alpha) d\alpha + q(0) \tag{2}$$

and in the discrete time domain:

$$q_{k+1} = q_k + \dot{q}_k \Delta t \tag{3}$$

thus:

$$q_{k+1} = q_k + J^{-1} \dot{x}_k \Delta t \tag{4}$$

The prerequisite to use the latter algorithm is that the Jacobian is squared and full rank, this condition is generally not satisfied and we need to look for Jacobian inversion techniques.

The first technique analyzed is based on the Lagrangian multipliers, in particular we can consider the minimization of a quadratic kinetic energy functional:

$$g(\dot{q}) = \frac{1}{2} \dot{q}^T W \dot{q} \tag{5}$$

with W symmetric and positive definite, now we can introduce the modified functional with the introduction of Lagrange multipliers:

$$g(\dot{q}, \lambda) = \frac{1}{2} \dot{q}^T W \dot{q} + \lambda^T (\dot{x} - J\dot{q}) \tag{6}$$

then we take the derivative with respect to the parameters:

$$\left(\frac{\partial g}{\partial \dot{q}}\right)^T = W\dot{q} - J^T \lambda = 0 \rightarrow \dot{q} = W^{-1} J^T \lambda \tag{7}$$

$$\left(\frac{\partial g}{\partial \lambda}\right)^T = \dot{x} - J\dot{q} = 0 \rightarrow \dot{x} = J\dot{q} \tag{8}$$

thus substituting (7) in (8):

$$\dot{x} = JW^{-1}J^T\lambda \tag{9}$$

If J is full rank the matrix $JW^{-1}J^T$ is invertible and we can calculate the multiplier:

$$\lambda = (JW^{-1}J^T)^{-1}\dot{x} \tag{10}$$

and if fed into (7):

$$\dot{q} = W^{-1}J^T(JW^{-1}J^T)^{-1}\dot{x} \tag{11}$$

It is worth to notice that when the weight matrix $W = I$ this inversion equals the right pseudo-inverse:

$$\dot{q} = J^{\dagger}\dot{x} \tag{12}$$

In this way even if the Jacobian is not full rank we can get its inverse.

In case the manipulator is redundant we can consider also the null space motions:

$$x = J\dot{q} \tag{13}$$

if $\dot{q}$ is a solution thus $\dot{q} + P\dot{q}_0$ is a solution as well, considering P a matrix that projects an arbitrary velocity $\dot{q}_0$ into the null space of the Jacobian. This way we can add a constraint to the problem and minimize some kind of function that depends on the redundancy of the robot, let's consider the new functional:

$$g'(\dot{q}) = \frac{1}{2}(\dot{q} - \dot{q}_0)^T(\dot{q} - \dot{q}_0) \tag{14}$$

we are trying to minimize this functional and thus we are trying to find the inversion of the Jacobian as before but at the same time trying to minimize the difference between $\dot{q}$ and $\dot{q}_0$ ($\dot{q}_0$ has not been defined yet and it will consider the redundancy of the manipulator).

Again we can repeat what we have done in the previous case:

$$g'(\dot{q}, \lambda) = \frac{1}{2}(\dot{q} - \dot{q}_0)^T(\dot{q} - \dot{q}_0) + \lambda^T(\dot{x} - J\dot{q}) \tag{15}$$

$$\left(\frac{\partial g'}{\partial \dot{q}}\right)^T = \dot{q} - \dot{q}_0 + J^T\lambda = 0 \rightarrow \dot{q} = J^T\lambda + \dot{q}_0 \tag{16}$$

$$\left(\frac{\partial g'}{\partial \lambda}\right)^T = \dot{x} - J\dot{q} = 0 \rightarrow \dot{x} = J\dot{q} \tag{17}$$

thus from (16) and (17):

$$\dot{x} = J(J^T\lambda + \dot{q}_0) \tag{18}$$

$$\lambda = (JJ^T)^{-1}\dot{x} - (JJ^T)^{-1}J\dot{q}_0 = (JJ^T)^{-1}(\dot{x} - J\dot{q}_0) \tag{19}$$

and finally if fed into (16):

$$\dot{q} = J^T((JJ^T)^{-1}(\dot{x} - J\dot{q}_0)) + \dot{q}_0 = J^T(JJ^T)^{-1}\dot{x} - J^T(JJ^T)^{-1}J\dot{q}_0 + \dot{q}_0 = \tag{20}$$

$$\dot{q} = J^\dagger\dot{x} + (I - J^\dagger J)\dot{q}_0 \tag{21}$$

the reader can notice that $(I - J^\dagger J)$ is one of the possible matrix P that we introduced above, if the Jacobian is invertible $J^\dagger J = I$ and thus the second term goes to zero. Basically more $J^\dagger J$ differs from I and more we are redundant in this configuration. Moreover we can have internal motions in the joint space ($\dot{q} \neq 0$) that are not projected into the cartesian space of the end effector ($\dot{x} = 0$) because of the additional term $(I - J^\dagger J)\dot{q}_0$.

The last problem to face is how to choose the vector $\dot{q}_0$, one possible way is:

$$\dot{q}_0 = k_0 \left(\frac{\partial w(q)}{\partial q}\right)^T \tag{22}$$

and if we pick $w(q)$ as measure of manipulability we can maximise the distance of the pose from a singular configuration of the manipulator:

$$w(q) = \sqrt{\det(J(q)J^T(q))} \tag{23}$$

The joint space variables calculated in such a way will not be exactly equal to the ones from $\dot{x}_r = J\dot{q}$, thus $\dot{x}_d \neq \dot{x}_r$, let's consider the error $e = x_d - x_r$ and its time derivative $\dot{e} = \dot{x}_d - \dot{x}_r$, equal to:

$$\dot{e} = \dot{x}_d - J\dot{q} \tag{24}$$

and we can set a Proportional controller with a Feedforward term:

$$\dot{q} = J^{-1}(\dot{x}_d + Ke) \rightarrow \dot{e} + Ke = 0 \tag{25}$$

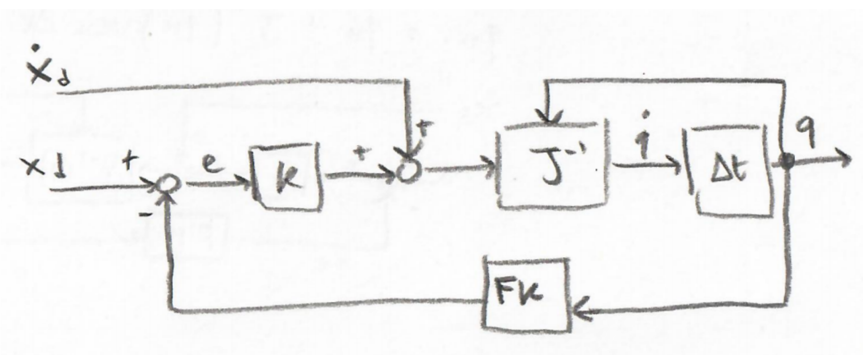

Figure 1: Block diagram of the control scheme for the inverse kinematics of serial manipulators.