“Age” model

Hi all - just finished writing but not coding. I’ve expressed it in a form you’ll understand, but not the students. I can redo it in discrete terms when the advection is just a shift of the vector of c for them.

This kind of model tracks the progression of the disease in exposed/ infected individuals.

Let $c(\tau|t)$ be the number of individuals at time t who caught the disease at time $t-\tau$. I.e., τ is an age variable measuring how long they have had the disease. It satisfies

$$\frac{\partial}{\partial t}c + \frac{\partial}{\partial \tau}uc = -[\lambda(\tau) + \alpha(\tau) + \mu]\, c \tag{1}$$

The advective term represents the aging; I’ve put a $u = \frac{\partial}{\partial t}\tau$ in for familiarity, but it is just equal to one. $\lambda(\tau)$ gives the recovery rate, while $\alpha(\tau)$ and μ are the covid and natural mortality, respectively. The advective term has to be discretized without numerical dispersion; a forward difference with $u\,dt = d\tau$ meaning $(dt = d\tau)$ will work fine.

To couple this into the reswt of the dynamocs, we use

$$\frac{\partial}{\partial t}R = \int d\tau\, \lambda(\tau)c(\tau|t) - \mu R \tag{2}$$

$$\frac{\partial}{\partial t}D = \int d\tau\, \alpha(\tau)c(\tau|t) \tag{3}$$

The number of new infections at time t will be determined by $i(\tau)$; this could be zero during the exposed stage, for example. The number of new infections is then

$$I_{new}(t) = \beta(t)\frac{S(t)}{N(t)}\int d\tau\, i(\tau)c(\tau|t)$$

This serves both as removal of S and a flux into c at $\tau = 0$:

$$\frac{\partial}{\partial t}S = -I_{new}(t) + \mu(N_0 - S) \tag{4}$$

and a flux boundary condition for (1)

$$uc(0,t) = I_{new}(t) \tag{5}$$

This ensures that, in the absence of any recovery/ mortality that

$$S(t) + \int d\tau\, c(\tau,t)$$

is conserved.

Equations (1-5), supplemented by a statement about what happens at τ_{max} numerically — I would take any flux across that and add it to the source of D — comprise the model.