Algorithmic Game Theory

Task 1 [15%]

Five commuters A, B, C, D, E, must choose between their own car with a fixed travel cost, or the public bus, where the charge to each user decreases with the number of users. Let ci denote the cost/travel time of using his own car for commuter i ∈ {A, B, C, D, E} and let cA = 1, cB = 3, cC = 5, cD = 7, cE = 9.

Regarding the bus, if 1 person uses it his cost will be 10, if 2 people use it then each of them will have cost 8, if 3 people use it then each of them will have cost 6, if 4 people use it then each of them will have cost 4, while if all 5 people use it then each of them will have cost 2. Naturally, each player wants to minimize his own cost.

Part a. [7%]

Is there any dominated strategy (ies) in this game? If yes, perform an iterated elimination of all dominated strategies. Explicitly mention the sequence in which the corresponding strategies are eliminated and present the resulting game.

There are dominated strategies in this game.

Commuter A will not take the bus because the cost of their personal vehicle is always lower than the cost of the bus regardless of how many players choose the bus.

Commuter B will choose the bus only if four commuters choose the bus, and only then will the cost of his personal vehicle be higher than the bus. However, if there are only three people then Commuter B will choose to take their personal vehicle.

Commuter C will choose to take their personal vehicle unless three people take the bus. If three people take the bus and he is the fourth, then the cost of the bus will be lower than the cost of using their personal vehicle.

Commuter D will not choose the bus unless two other commuters choose to take the bus with them. If three people take the bus, including commuter D, then the cost is lower than when using their personal vehicle.

Commuter E will choose the bus if someone will take the bus with them, otherwise the cost of using the bus is more than the cost of using their personal vehicle.

Part b. [8%]

Find (and clearly present) all pure Nash equilibria (NE) of the game. What is the Price of Stability? No coding is required.

	A	**B**	**C**	**D**	**E**
1	10,1	10,3	10,5	10,7	10,9
2	8,1	8,3	8,5	8,7	8,9
3	6,1	6,3	6,5	6,7	6,9
4	4,1	4,3	4,5	4,7	4,9
5	2,1	2,3	2,5	2,7	2,9

The price of stability is the value of the best Nash equilibrium divided by the value of the optimal solution. For this game, the best Nash equilibrium value is 25 and the optimal value is 10.

25 / 10 = **2.5** which is the price stability.

The cost of Nash equilibrium is 2.5 times worse than the optimal social cost.

If none of the players choose to ride the bus, they will all use their cars and the combined cost will be the Nash equilibrium.

Task 2 [15%]

Consider the following zero sum game G1:

	A	B
X	4	7
Y	8	6

Part a. [3%]

Is there a pure Nash equilibrium in G? Justify your answer

There is no pure Nash equilibrium but there is a mixed equilibrium.

Part b. [12%]

Compute a mixed equilibrium using the indifference conditions of the players. Present both the equilibrium and the analysis clearly. No coding is required.

$4q + 7(1-q) = 8q + 6(1-q)$
$4q + 7 – 7q = 8q + 6 + 6q$
$-3q +7 = 2q + 6$
$1 = 5q$
$1/5 = q$
$1 - 1/5 = 4/5 = 0.8$

$4q + 8(1-q) = 7q + 6(1-q)$
$4q + 8 – 8q = 7q + 6 + 6q$
$-4q +8 = q + 6$
$2 = 5q$
$2/5 = q$
$1 - 2/5 = 3/5 = 0.6$

$7q + 4(1-q) = 6q + 8(1-q)$
$7q + 4 – 4q = 6q + 8 + 8q$
$3q + 4 = -2q + 8$
$5q = 4$
$q= 4/5$
$1 – 4/5 = 1/5 = 0.2$

$8q + 4(1-q) = 6q + 7(1-q)$
$8q + 4 – 4q = 6q + 7 + 7q$
$4q + 4 = -1q + 7$
$5q = 3$
$q= 3/5$
$1 – 3/5 = 2/5 = 0.4$

Task 3 [25%]

Consider the following normal form game G2:

	A	B	C
X	3,3	0,4	0,0
Y	4,0	1,1	0,0
Z	0,0	0,0	1/2,1/2

Your task is to find the correlated equilibrium that maximizes the sum of players' utilities, using Linear Programming in MATLAB. In your report, you need to present the equilibrium that you have computed, the linear program that you are solving (which should include the equilibrium conditions that are satisfied), and copy and paste your MATLAB input and output.

```
f = [-6 -4 0 -4 -2 0 0 0 -1];

A = [1 1 0 0 0 0 0 0 0;
   -3 0 1/2 0 0 0 0 0 0;
   0 0 0 -1 -1 0 0 0 0;
   0 0 0 -4 -1 1/2 0 0 0;
   0 0 0 0 0 0 3 0 -1/2;
   0 0 0 0 0 0 4 1 -1/2;

   1 0 0 1 0 0 0 0 0;
   -3 0 0 0 0 0 1/2 0 0;
   0 -1 0 0 -1 0 0 0 0;
   0 -4 0 0 -1 0 0 1/2 0;
   0 0 3 0 0 0 0 0 -1/2;
   0 0 4 0 0 1 0 0 -1/2;];

b = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0;];
Aeq = [1 1 1 1 1 1 1 1 1];
beq = 1;
lb = [0; 0; 0; 0; 0; 0; 0; 0; 0];
ub = [1; 1; 1; 1; 1; 1; 1; 1; 1];

[x,fval] = linprog(f, A, b, Aeq, beq, lb, ub);

fval = [-2]
x = [0, 0, 0, 0, 1, 0, 0, 0, 0]
```

Task 4 [30%]

Consider the following sponsored search auction instance I:

- ***3 slots. The top slot has a known click-through rate (CTR) ctr1 = 1, the middle slot has a known click-through rate ctr2 = 0.55071 and the bottom slot has a known click-through rate ctr2 = 0.4704.***
- ***3 advertisers. Let the (private) value-per-click of advertiser 1 be 1 million (in some currency), the (private) value-per-click of advertiser 2 be 555710 and the (private) value-per-click of advertiser 3 be 470400.***
- ***The payoff of bidder i in slot j is ctrj (vi − pj), where pj is the price charged per-click in slot j.***

Under the Generalized Second-Price (GSP) auction rule:

- ***Advertisers are asked to declare their value per click (this doesn't mean that their declarations are truthful!). Advertisers are then ranked according to their declarations and the advertiser with the highest declaration is assigned to the slot with the highest CTR, the advertiser with the second highest declaration is assigned to the slot with the second highest CTR, and, finally, the advertiser with the lowest declaration is assigned to the slot with the smallest CTR. For j = 1, 2, the per-click payment pj at slot j, is set to be equal to the declaration of the advertiser assigned to slot j + 1, while p3 = 0.***

Part a. [4%]

Compute the optimal/highest social welfare in I.

The social warfare is calculated by:

Social welfare = Ctr1 * vpc1 + Ctr2 * vpc2 + Ctr3 * vpc3

1000000 * 1 + 0.55071 * 555710 + 0.4704 * 470400

1000000 * 306035.0541 + 221276.16

= **1527311.2141**

Part b. [26%]

Write MATLAB code that computes the pure Nash equilibrium with the highest Price of Anarchy in I, when only integer declarations are permitted.

…