

CS388: Natural Language Processing

Lecture 11: Understanding In- Context Learning

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Administrivia

- Project 3 released today
- Project proposals due today
 - Can be >1 page if needed
- Most important: have a detailed plan for models, datasets, and experiments, so we can evaluate for feasibility. Include related work!
- For reproduction: lots of types of papers are okay, just make sure the paper isn't trivial. You can plan for a reproduction with minor extension beyond what was done before



Recap: Dataset Bias

- "Tough" datasets for tasks like QA may feature spurious correlations (e.g., "where" question is always a location and the model can guess a relevant location and do quite well)
- Training strong models such as BERT on these datasets leads to poor generalization
- One debiasing technique:

$$\mathcal{L}(\theta_d) = -(1 - p_b^{(i,c)})y^{(i)} \cdot \log p_d$$

↑ one-hot label vector
↓ log probability
of each label

probability under a copy of the model trained
for a few epochs on a small subset of data (bad model)

This Lecture

- Prompting: best practices and why it works
 - Zero-shot prompting: role of the prompt
 - Few-shot prompting (in-context learning): characterizing demonstrations
- Understanding in-context learning
 - ICL can learn linear regression
 - Induction heads and mechanistic interpretability

Zero-shot Prompting



Zero-shot Prompting

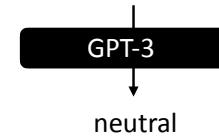
- ▶ Single unlabeled datapoint x , want to predict label y

$x = \text{The movie's acting could've been better, but the visuals and directing were top-notch.}$

- ▶ Wrap x in a template we call a **verbalizer** v

Review: The movie's acting could've been better, but the visuals and directing were top-notch.

Out of positive, negative, or neutral, this review is



Zero-shot Prompting



- ▶ Single unlabeled datapoint x , want to predict label y

$x = \text{The movie's acting could've been better, but the visuals and directing were top-notch.}$

- ▶ Wrap x in a template we call a **verbalizer** v

Review: The movie's acting could've been better, but the visuals and directing were top-notch.

On a 1 to 4 star scale, the reviewer would probably give this movie



Ways to do classification

- ▶ Generate from the model and read off the generation

▶ What if you ask for a star rating and it doesn't give you a number of stars but just says something else?

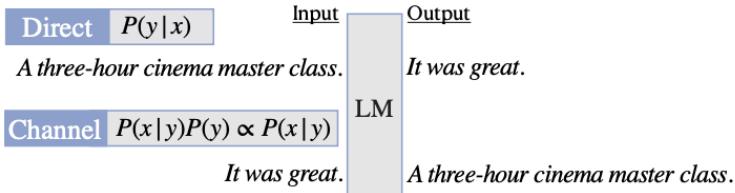
- ▶ Compare probs: *"Out of positive, negative, or neutral, this review is _"*
Compare $P(\text{positive} \mid \text{context})$, $P(\text{neutral} \mid \text{context})$, $P(\text{negative} \mid \text{context})$

▶ This constrains the model to only output a valid answer, and you can normalize these probabilities to get a distribution



Ways to do classification

$(x, y) = ("A \text{ three-hour cinema master class.}", "It was great.")$



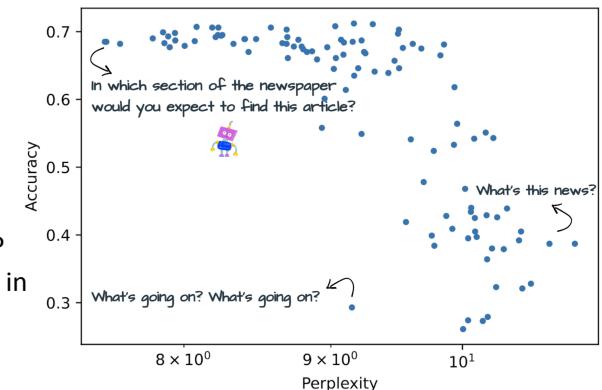
- Can also compute probabilities of **examples given labels** (“noisy channel” method)

Min et al. (2021)



Variability in Prompts

- Plot: large number of prompts produced by {manual writing, paraphrasing, backtranslation}
- x-axis: perplexity of the prompt. How natural is it? How much does it appear in the pre-training data?
- y-axis: task performance



Gonen et al. (2022)



Variability in Prompts

- OPT-175B: average of best 50% of prompts is much better than average over all prompts

Task	Avg Acc	Acc 50%
Antonyms	—	—
GLUE Cola	47.7	57.1
Newspop	66.4	72.9
AG News	57.5	68.7
IMDB	86.2	91.0
DBpedia	46.7	55.2
Emotion	16.4	23.0
Tweet Offensive	51.3	55.8

Gonen et al. (2022)



Prompt Optimization

- A number of methods exist for searching over prompts (either using gradients or black-box optimization)
- Most of these do not lead to dramatically better results than doing some manual engineering/hill-climbing (and they may be computationally intensive)
- Nevertheless, the choice of prompt *is* very important for zero-shot settings! We will see more next time.
- In two lectures: models that are trained to do better at prompts (RLHF)

Few-shot Prompting



Few-shot Prompting

- Form “training examples” from (x, y) pairs, verbalize them (can be lighter-weight than zero-shot verbalizer)
- Input to GPT-3: $v(x_1) v(y_1) v(x_2) v(y_2) \dots v(x_{test})$

Review: *The cinematography was stellar; great movie!*

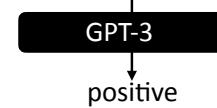
Sentiment (positive or negative): positive

Review: *The plot was boring and the visuals were subpar.*

Sentiment (positive or negative): negative

Review: *The movie's acting could've been better, but the visuals and directing were top-notch.*

Sentiment (positive or negative):



What can go wrong?



Review: *The movie was great!*

Sentiment: positive

Review: *I thought the movie was alright; I would've seen it again.*

Sentiment: positive

Review: *The movie was pretty cool!*

Sentiment: positive

Review: *Pretty decent movie!*

Sentiment: positive

Review: *The movie had good enough acting and the visuals were nice.*

Sentiment: positive

Review: *There wasn't anything the movie could've done better.*

Sentiment: positive

Review: *Okay movie but could've been better.*

Sentiment: positive

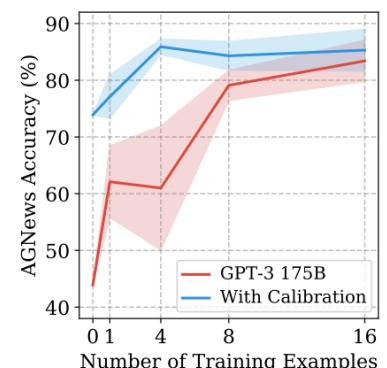


What can go wrong?

- All one training label — model sees extremely skewed distribution

- What if we take random sets of training examples? There is quite a bit of variance on basic classification tasks

- Note: these results are with basic GPT-3 and not Instruct-tuned versions of the model. This issue has gotten a lot better

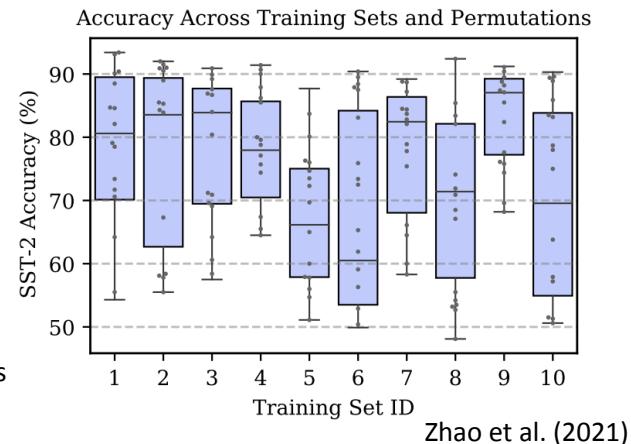


Zhao et al. (2021)



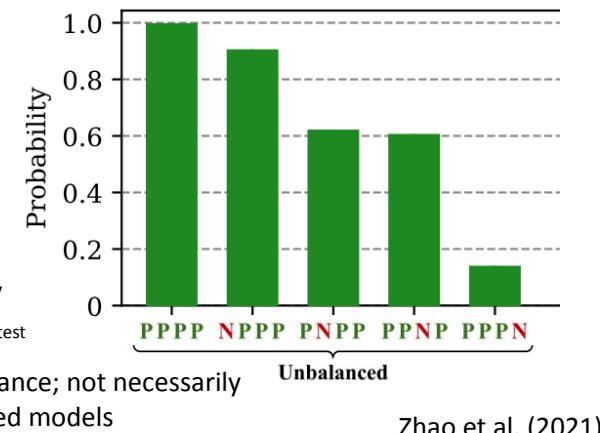
What can go wrong?

- Varies even across permutations of training examples
- x-axis: different collections of train examples.
- y-axis: sentiment accuracy. Boxes represent results over different permutations of the data



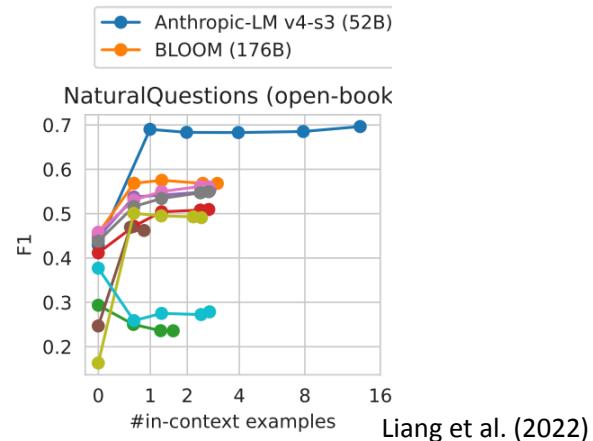
What can go wrong?

- Having unbalanced training sets leads to high “default” probabilities of positive; that is, if we feed in a null x_{test}
- Solution: “calibrate” the model by normalizing by that probability of null x_{test}
- Leads to higher performance; not necessarily crucial with prompt-tuned models

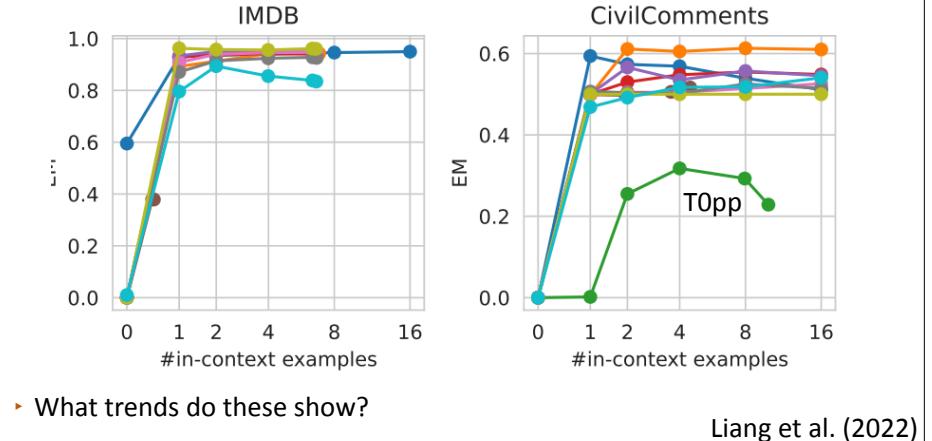


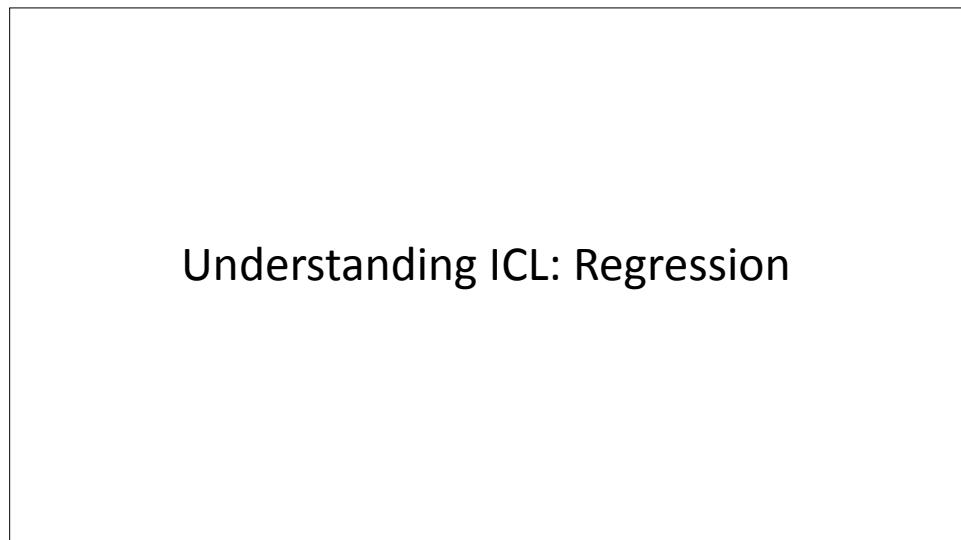
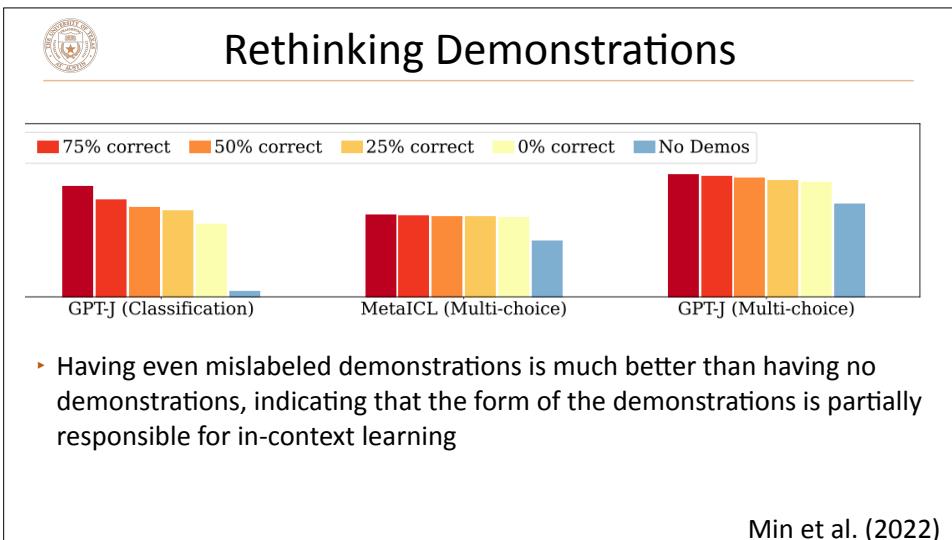
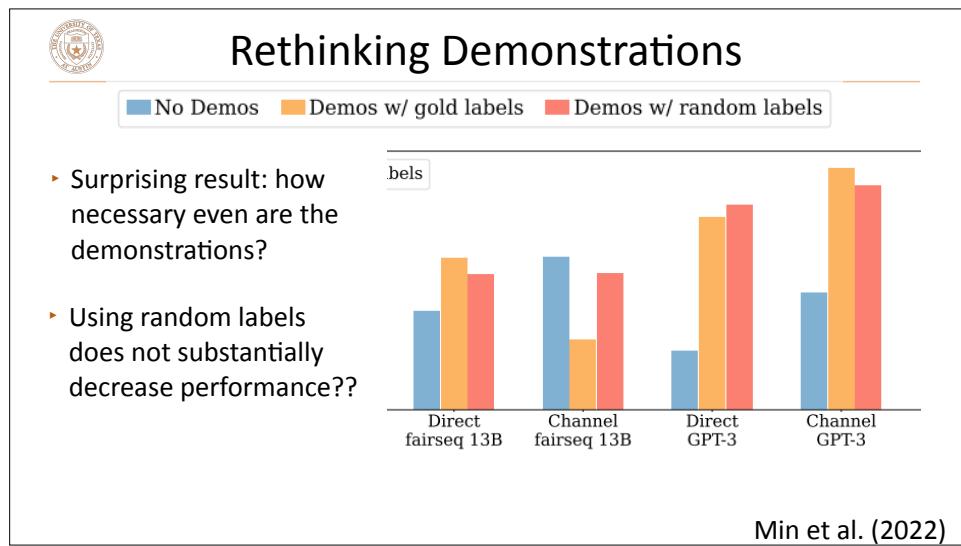
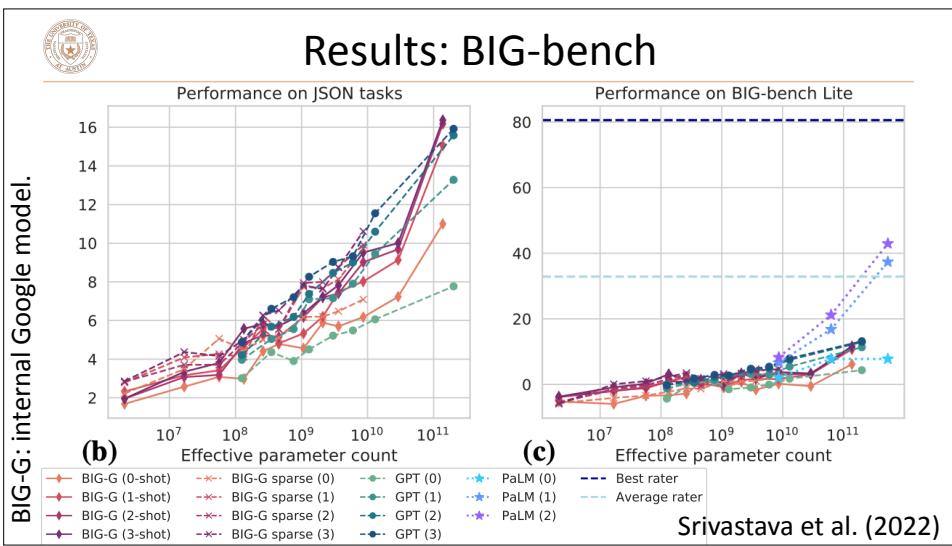
Results: HELM

- So, how much better is few-shot compared to zero-shot?
- Each line is a different LM
- More in-context examples generally leads to better performance
- What do we see here?



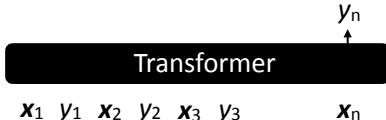
Results: HELM







Linear Regression



- Input space is of the form $[y, \mathbf{x}]$, with the “unused” components set to 0
- See if we can learn regression: given (\mathbf{x}, y) pairs, learn a linear predictor $f(\mathbf{x}) = \mathbf{w}^\top \mathbf{x}$. That is, ground truth is a linear function (synthetic task)
- Equivalent to minimizing the following loss:

$$\sum_i \mathcal{L}(\mathbf{w}^\top \mathbf{x}_i, y_i) + \lambda \|\mathbf{w}\|_2^2$$

minimized by: $\mathbf{w}^* = (X^\top X + \lambda I)^{-1} X^\top y$

Akyürek et al. (2022)



Linear Regression



- Question 1: can a Transformer learn to do linear regression?
- If we train it to do this task on many examples, does it successfully learn to do “ICL” linear regression on new instances?
- If so, there are several different “algorithms” it might correspond to!
- Question 2: can we inspect what algorithm actually gets implemented?

Akyürek et al. (2022)



Linear Regression

- Most of these proofs (and other papers in this space) rely on Transformers being able to perform several kinds of operations

mov($H; s, t, i, j, i', j'$): selects the entries of the s^{th} column of H between rows i and j , and copies them into the t^{th} column ($t \geq s$) of H between rows i' and j' , yielding the matrix:

$$\left[\begin{array}{c|c|c} & H_{:,i-1,t} & \\ \hline H_{:,s,t} & H_{i':j',s} & H_{:,t+1:} \\ \hline & H_{j,t} & \end{array} \right].$$

- How can this be implemented?
What does the attention need to do?

Akyürek et al. (2022)



Linear Regression

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mul($H; a, b, c, (i, j), (i', j'), (i'', j'')$): in each column \mathbf{h} of H , interprets the entries between i and j as an $a \times b$ matrix A_1 , and the entries between i' and j' as a $b \times c$ matrix A_2 , multiplies these matrices together, and stores the result between rows i'' and j'' , yielding a matrix in which each column has the form $[\mathbf{h}_{i''-1}, A_1 A_2, \mathbf{h}_{j''-1}]^\top$.

- Several more operations as well

Akyürek et al. (2022)



Linear Regression

Theorem 1. A transformer can compute Eq. (11) (i.e. the prediction resulting from single step of gradient descent on an in-context example) with constant number of layers and $O(d)$ hidden space, where d is the problem dimension of the input x . Specifically, there exist transformer parameters θ such that, given an input matrix of the form:

$$H^{(0)} = \begin{bmatrix} \dots & 0 & y_i & 0 & \dots \\ & x_i & 0 & x_n & \dots \end{bmatrix}, \quad (12)$$

the transformer's output matrix $H^{(L)}$ contains an entry equal to $w'^\top x_n$ (Eq. (11)) at the column index where x_n is input.

- Also another update possible based on rank-one updates (Sherman-Morrison)

Akyürek et al. (2022)



Proof of Theorem

The operations for 1-step SGD with single exemplar can be expressed as following chain (please see proofs for the Transformer implementation of these operations (Lemma 1) in Appendix C):

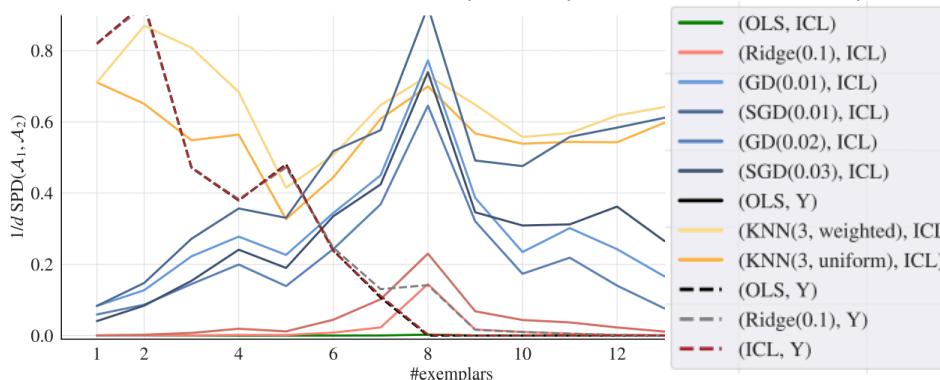
- $\text{mov}(; 1, 0, (1, 1 + d), (1, 1 + d))$ (move x)
- $\text{aff}(; (1, 1 + d), (), (1 + d, 2 + d), W_1 = w)$ ($w^\top x$)
- $\text{aff}(; (1 + d, 2 + d), (0, 1), (2 + d, 3 + d), W_1 = I, W_2 = -I)$ ($w^\top x - y$)
- $\text{mul}(; d, 1, 1, (1, 1 + d), (2 + d, 3 + d), (3 + d, 3 + 2d))$ ($x(w^\top x - y)$)
- $\text{aff}(; (), (), (3 + 2d, 3 + 3d), b = w,)$ (write w)
- $\text{aff}(; (3 + d, 3 + 2d), (3 + 2d, 3 + 3d), (3 + 3d, 3 + 4d), W_1 = I, W_2 = -\lambda) (x(w^\top x - y) - \lambda w)$
- $\text{aff}(; (3 + 2d, 3 + 3d), (3 + 3d, 3 + 4d), (3 + 2d, 3 + 3d), W_1 = I, W_2 = -2\alpha,)$ (w')
- $\text{mov}(; 2, 1, (3 + 2d, 3 + 3d), (3 + 2d, 3 + 3d))$ (move w')
- $\text{mul}(; 1, d, 1, (3 + 2d, 3 + 3d), (1, 1 + d), (3 + 3d, 4 + 3d))$ ($w'^\top x_2$)

Akyürek et al. (2022)



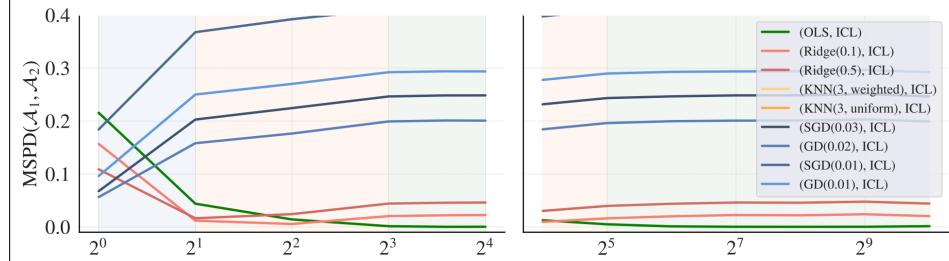
Linear Regression

- Squared prediction difference: L2 between different predictors
- When no noise: ICL matches ordinary least square (OLS) almost exactly



Linear Regression

- Squared prediction difference: L2 between different predictors

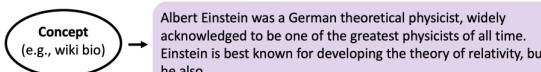


- What gets learned changes with depth. Low-depth: more like GD. Medium-depth: more like ridge. High-depth: OLS



Bayesian Interpretation

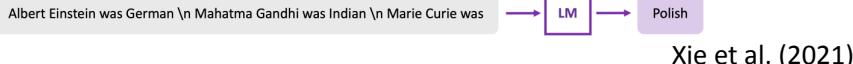
1. Pretraining documents are conditioned on a latent concept (e.g., biographical text)



2. Create independent examples from a shared concept. If we focus on full names, wiki bios tend to relate them to nationalities.



3. Concatenate examples into a prompt and predict next word(s). Language model (LM) implicitly infers the shared concept across examples despite the unnatural concatenation



Xie et al. (2021)

Understanding ICL: Induction Heads and Mechanistic Interpretability



Background: Transformer Circuits

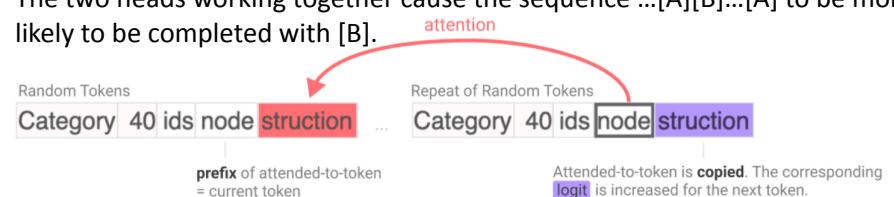
- There are mechanisms in Transformers to do “fuzzy” or “nearest neighbor” versions of pattern completion, completing $[A^*][B^*] \dots [A] \rightarrow [B]$, where $A^* \approx A$ and $B^* \approx B$ are similar in some space
- Olsson et al. want to establish that these mechanisms are responsible for good ICL capabilities
- We can find these heads and see that performance improves; can we causally link these?

Olsson et al. (2022)



Induction Heads

- Induction heads: a pair of attention heads in different layers that work together to copy or complete patterns.
- The first head copies information from the previous token into each token.
- Second attention head to attend to tokens based on what happened before them, rather than their own content. Likely to “look back” and copy next token from earlier
- The two heads working together cause the sequence ...[A][B]...[A] to be more likely to be completed with [B].





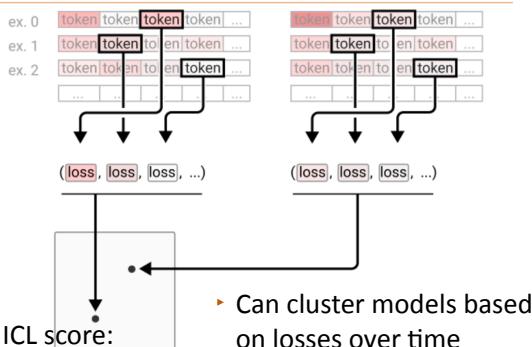
Induction Heads

Step 1: Run each model / snapshot over the same set of multiple dataset examples, collecting one token's loss per example.

Step 2: For each sample, extract the loss of a consistent token. Combine these to make a vector of losses per model / snapshot.

Step 3: The vectors are jointly reduced with principal component analysis to project them into a shared 2D space.

- Characterize performance by ICL score:
 $\text{loss(500th token)} - \text{loss(50th token)} - \text{average}$
 measure of how much better the model is doing later once it's seen more of the pattern



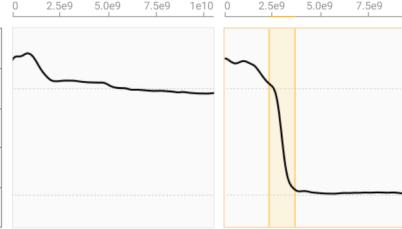
Olsson et al. (2022)

Induction Heads



ONE LAYER (ATTENTION-ONLY)

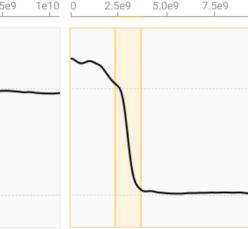
Elapsed Training Tokens



One-layer model has no sudden improvement.

TWO LAYER (ATTENTION-ONLY)

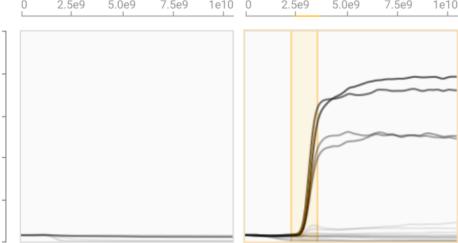
Elapsed Training Tokens



Models with more than one layer have a sudden improvement in ICL.

ONE LAYER (ATTENTION-ONLY)

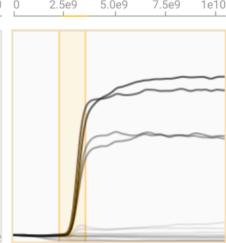
Elapsed Training Tokens



One-layer model has no induction heads.

TWO LAYER (ATTENTION-ONLY)

Elapsed Training Tokens



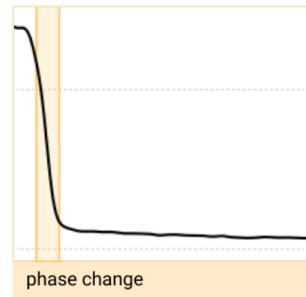
Models with more than one layer have induction heads form during training.

- Improvement in ICL (loss score) correlates with emergence of induction heads

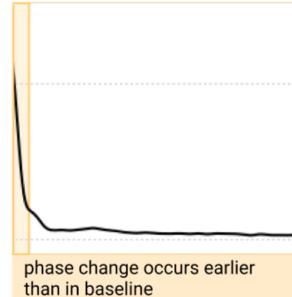


Induction Heads

Elapsed Training Tokens
0 2.5e9 5.0e9 7.5e9 1e10



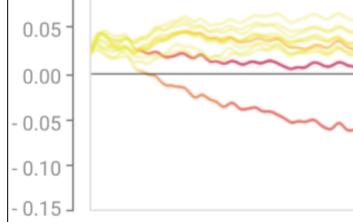
Change architecture to promote induction heads => phase change happens earlier



Induction Heads

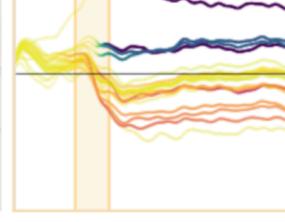
Elapsed Training Tokens

0 2.5e9 5.0e9 7.5e9 1e10



Elapsed Training Tokens

0 2.5e9 5.0e9 7.5e9 1e10



- If you remove induction heads, behavior changes dramatically



Interpretability

- Lots of explanations for why ICL works — but these haven't led to many changes in how Transformers are built or scaled
- Several avenues of inquiry: theoretical results (capability of these models), mechanistic interpretability, fully empirical (more like that next time)
- Many of these comparisons focus on GPT-3 and may not always generalize to other models



Takeaways

- Zero- and few-shot prompting are very powerful ways of specifying new tasks at inference time
- For zero-shot: form of the prompt matters, we'll see more examples next times when we look at chain-of-thought
- For few-shot: number and order of the examples matters, prompt matters a bit less
- Several analyses of why it works: it can learn to do regression and we know a bit about mechanisms that may be responsible for it