test derivations

1 mean shift process

To check that the mean shift process is correct, we apply the process to 8 three dimensional datapoints in the shape of a cube with side length s. Our test example will include arbitrarily many iterations for k = 3, k = 6, and k = 7.

$$\{s, s, s\}$$
$$\{-s, s, s\}$$
$$\{s, -s, s\}$$
$$\{-s, -s, s\}$$
$$\{s, s, -s\}$$
$$\{-s, s, -s\}$$
$$\{s, -s, -s\}$$
$$\{-s, -s, -s\}$$

2 case 1: k = 3

Every point in the cube has 3 equidistant neighbors with a total distance of s from the main point. The following generalizes 1 iteration of the mean shift process transformation for each point.

$$\text{main point: } \{a, b, c\}, a \in \{-s, s\}, b \in \{-s, s\}, c \in \{-s, s\}$$
$$\text{three nearest neighbors: } \{-a, b, c\}, \{a, -b, c\}, \{a, b, -c\}$$
$$\text{3nn average: } \frac{1}{3}\{a, b, c\}$$

So each iteration of the mean shift process multiplies each point in the cube by a factor of 1/3, while all of the points' directions remain unchanged.

3 case 2: k = 6

Every point in the cube has 3 additional equidistant second closest neighbors with a total distance of $(\sqrt{2})s$ from the main point. The following generalizes 1 iteration of the mean shift process transformation for each point.

$$\text{main point: } \{a, b, c\}, a \in \{-s, s\}, b \in \{-s, s\}, c \in \{-s, s\}$$
$$\text{three nearest neighbors: } \{-a, b, c\}, \{a, -b, c\}, \{a, b, -c\}$$

$$\text{three second nearest neighbors: } \{-a, -b, c\}, \{-a, b, -c\}, \{a, -b, -c\}$$

$$\text{6nn average: } \{0, 0, 0\}$$

All points will become the zero vector, and the process will end. Only 1 iteration is possible.

4 case 3: k = 7

The last case includes one additional vector in the set of neighbors, $\{-a, -b, -c\}$. We know that the sum of the first six neighbors is zero, which makes the sum of all seven neighbors $\{-a, -b, -c\}$. This is then divided by 7 because the average is computed. This means that each transformation multiplies the original point by $-\frac{1}{7}$ without changing any of the points' directions.

5 MOD algorithm

To check that the MOD algorithm is correct, we again apply the process to 8 three dimensional datapoints in the shape of a cube with side length s. As before, our test example will include arbitrarily many iterations for k = 3, k = 6, and k = 7. We specifically compute each point's Euclidean distance from the corresponding point in the mean shift process's output.

6 case 1: k = 3

We'll use the derived solution for the mean shift process's output when k = 3 to generalize the distance between each output point and it's corresponding original point. The variables a, b, c, and s are defined as they were previously.

$$\text{dist} = ||\{a, b, c\} - \frac{1}{3^l}\{a, b, c\}||_2^2$$

$$\text{dist} = \Big|\Big|1 - \frac{1}{3^l}\Big|\Big|_2^2 \left(\Big|\Big|\{a, b, c\}\Big|\Big|_2^2 \right)$$

$$\text{dist} = \sqrt{\left(1 - \frac{1}{3^l}\right)^2} \sqrt{3s^2}$$

$$\text{dist} = \sqrt{3}\left(1 - \frac{1}{3^l}\right)s$$

7 case 2: k = 6

We'll use the derived solution for the mean shift process's output when k = 3 to generalize the distance between each output point and it's corresponding original point. The variables a, b, c, and s are defined as they were previously.

$$\text{dist} = ||\{a, b, c\} - 0\{a, b, c\}||_2^2$$

$$\text{dist} = \sqrt{3s^2}$$

$$\text{dist} = \left(\sqrt{3}\right)s$$

8 case 3: k = 7

We'll use the derived solution for the mean shift process's output when k = 3 to generalize the distance between each output point and it's corresponding original point. The variables a, b, c, and s are defined as they were previously.

$$\text{dist} = ||\{a, b, c\} - \frac{1}{(-7)^l}\{a, b, c\}||_2^2$$

$$\text{dist} = ||1 - \frac{1}{(-7)^l}||_2^2\left(||\{a, b, c\}||_2^2\right)$$

$$\text{dist} = \sqrt{\left(1 - \frac{1}{(-7)^l}\right)^2}\sqrt{3s^2}$$

$$\text{dist} = \sqrt{3}\left(1 - \frac{1}{(-7)^l}\right)s$$