Acadian FVS Crown Recession Equation

Greg Johnson
Greg Johnson Biometrics LLC

2024-07-08

Data Preparation

We used the total height (`ht`) growth data in the `ALL_dHT.csv` file provided by Aaron Weiskittel.

Tree growth observations were limited to:

- trees with height to crown base (`hcb`) change ≥ 0.0,
- trees with observed height growth (`ht` time 1 $>$ `ht` time 2),
- measured `hcb` at time 1 and time 2.
- crown competition factor (`ccf`) computed,
- remeasurement intervals between 5 and 20 years inclusive, and
- ≥ 1000 tree observations by species.

This yielded 78737 observations for modeling. A breakdown by species is shown in Table 1.

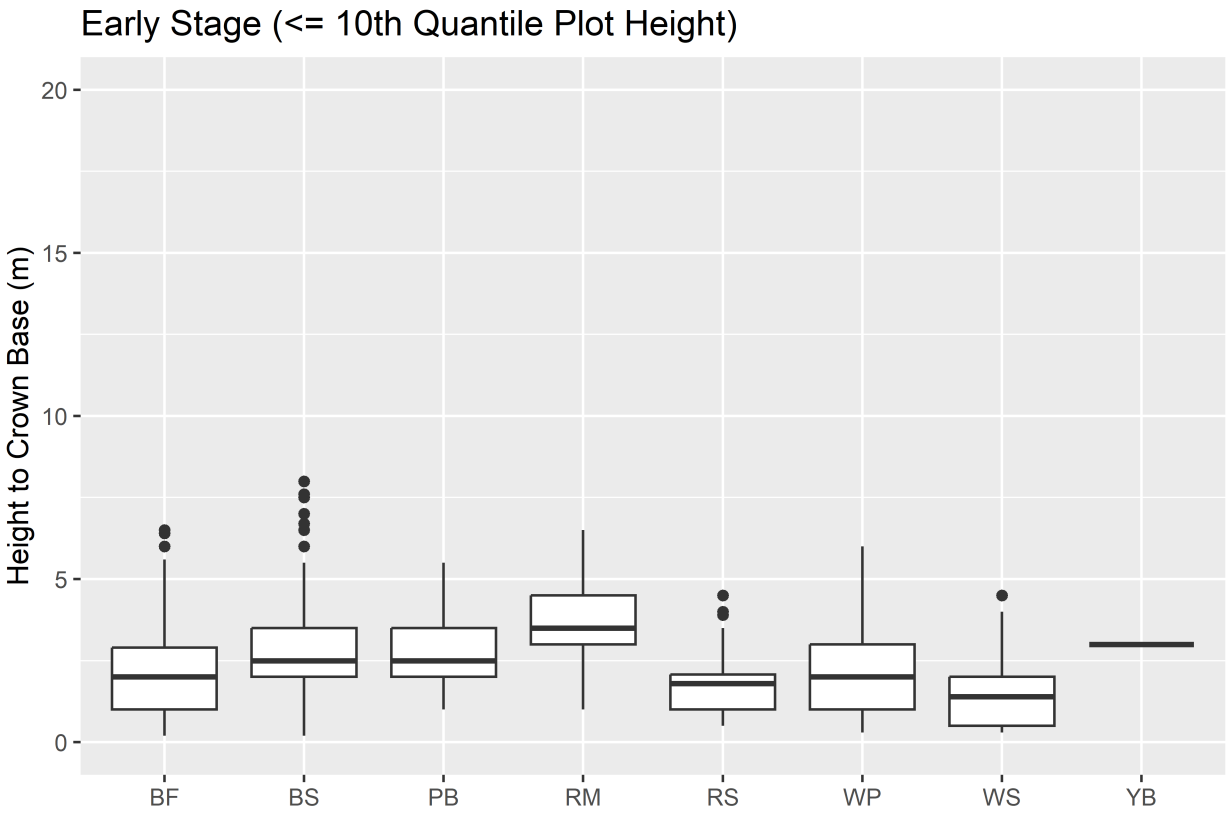
Table 1: Individual Species Available for Height to Crown Base Change Equation Fitting

FVS Sp	N Observations
BF	37047
RS	15724
RM	7799
BS	4166
WS	3512
PB	2049
SM	1397
WP	1278
YB	1264

Early- and Late-Stage Crown Length

To set the lower and upper bounds on crown development, we partitioned the data to isolate plots in the early and late stages of development. For these purposes, we defined the early-stage population as those plots with mean plot height ≤ 6.75 meters (10th quantile) and late-stage as mean plot height ≥ 14.07 meters (90th quantile).

The graphs below show the early- and late-stage individual-tree `hcb` distributions by species.


Early Stage (<= 10th Quantile Plot Height)
Height to Crown Base (m)
20
15
10
5
0
BF
BS
PB
RM
RS
WP
WS
YB

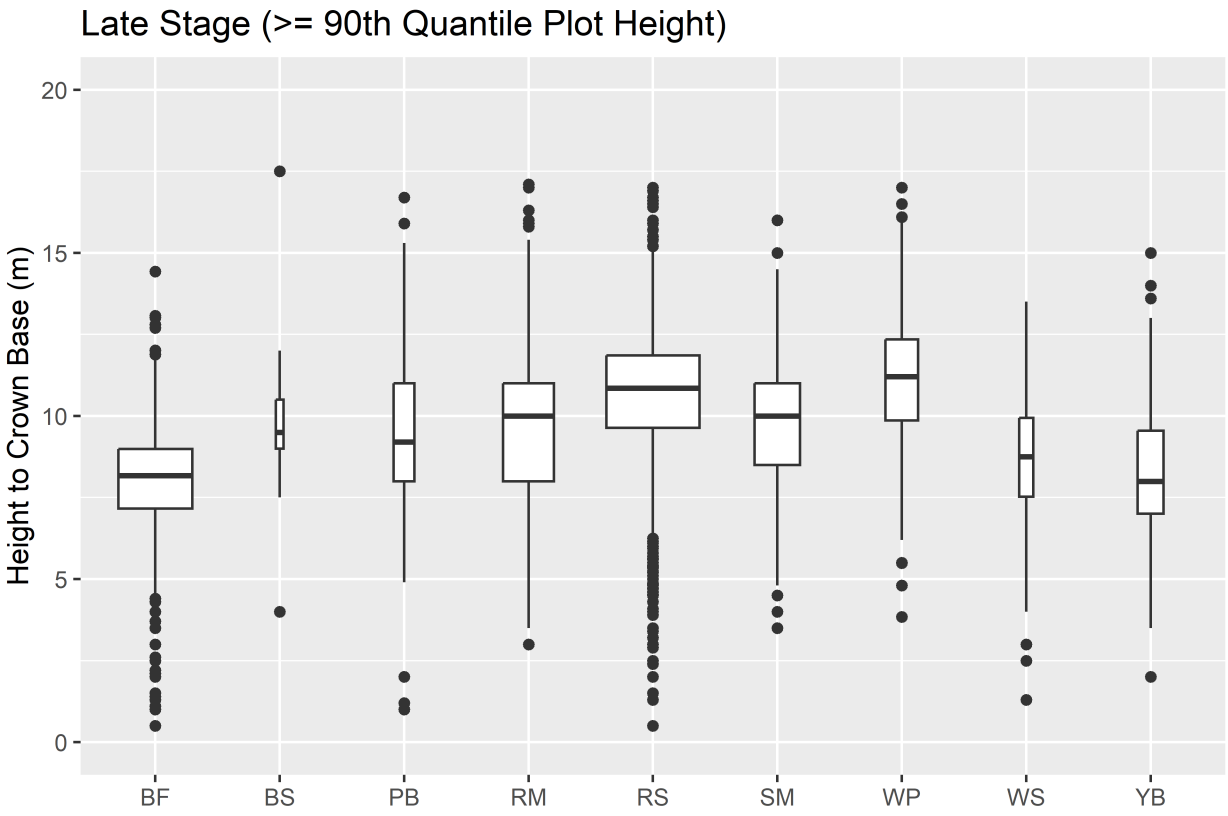
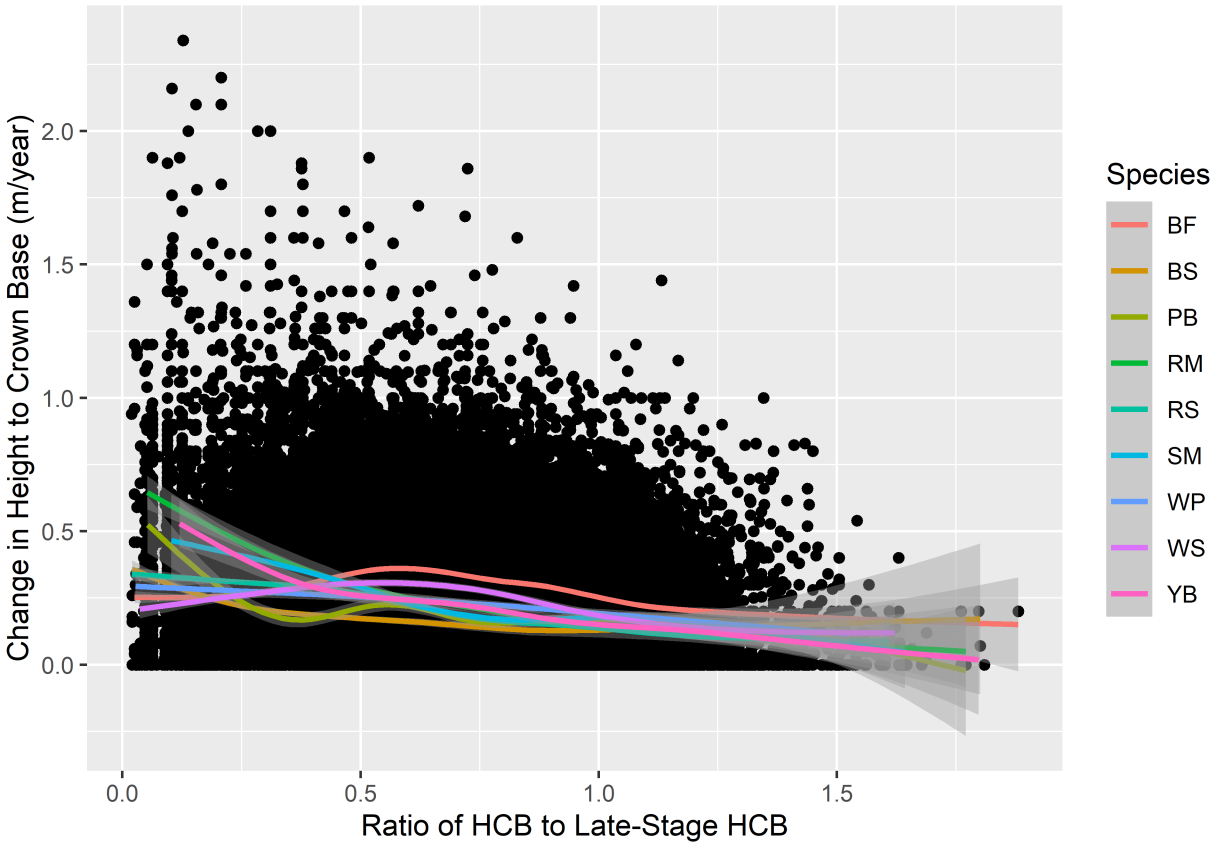


Table 2 shows hcb_{ls} for species and species groups in the data set.

Table 2: Late-stage Height to Crown Base (m)

FVS Sp	N Observations	hcb_ls
RS	2529	10.56
BF	1591	7.98
RM	755	9.66
SM	596	9.74
WP	312	11.13
YB	195	8.34
PB	125	9.43
WS	57	8.33
BS	17	9.72
OC	4506	9.66
OH	1671	9.52

We expect that annual change in height to crown base (Δhcb) would approach a lower asymptote near 0.0 as the ratio of current `hcb` to the late-stage `hcb` (hcb_{ls}) nears or exceeds 1.0. The graph below displays the data and confirms this effect. We would like to construct a Δhcb prediction system that replicates this behavior.



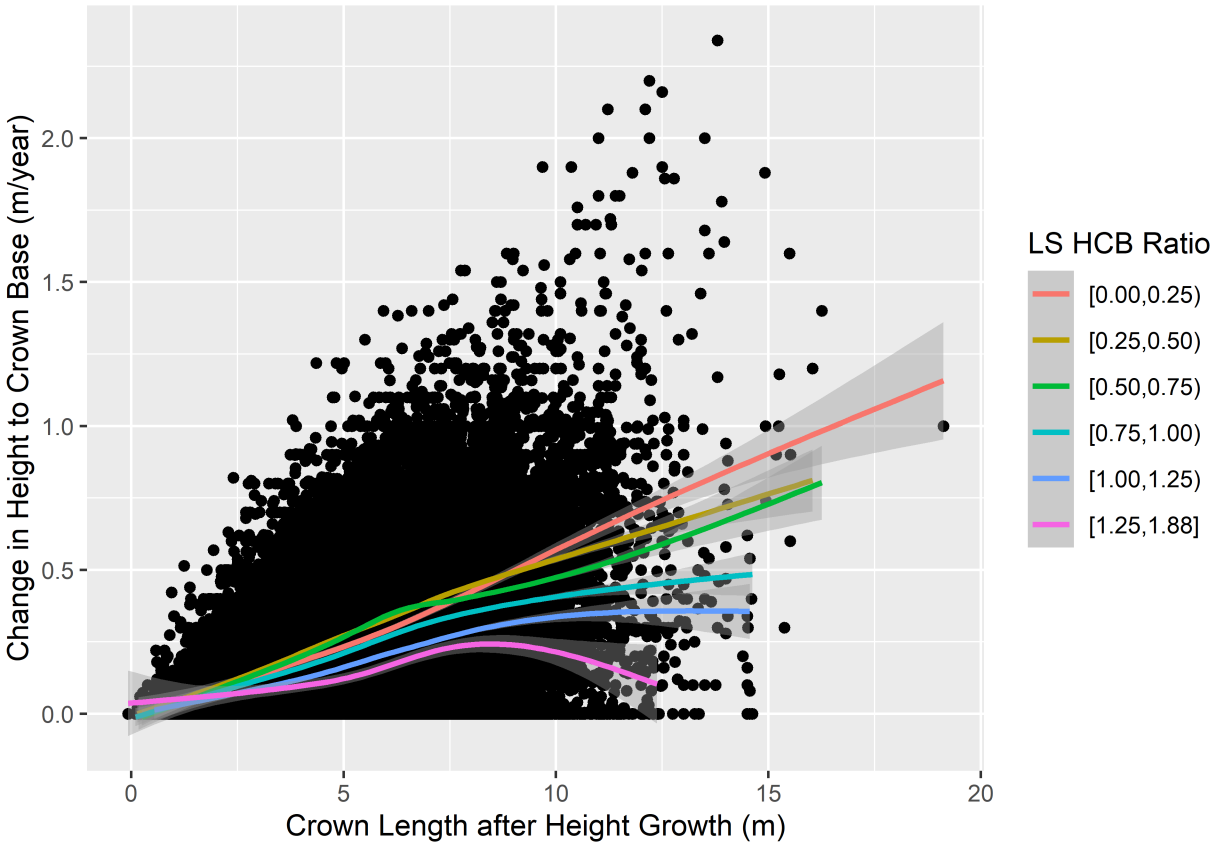
Examination of Height to Crown Base Change

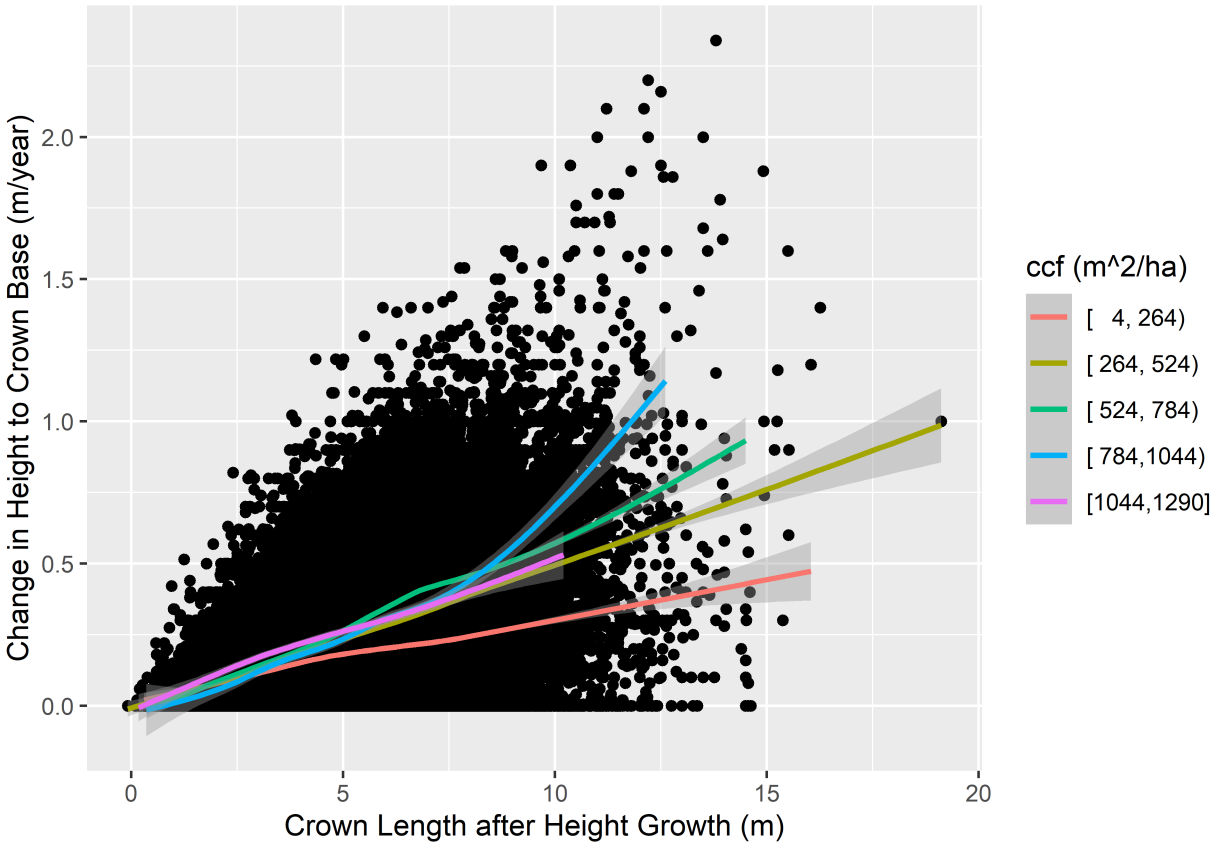
A graphical analysis of `hcb` change (Δhcb) revealed that crown length (`cl`) and height growth (Δht) were useful explanatory variables as the trend with their combination is nearly linear with some among species variation (see first graph).

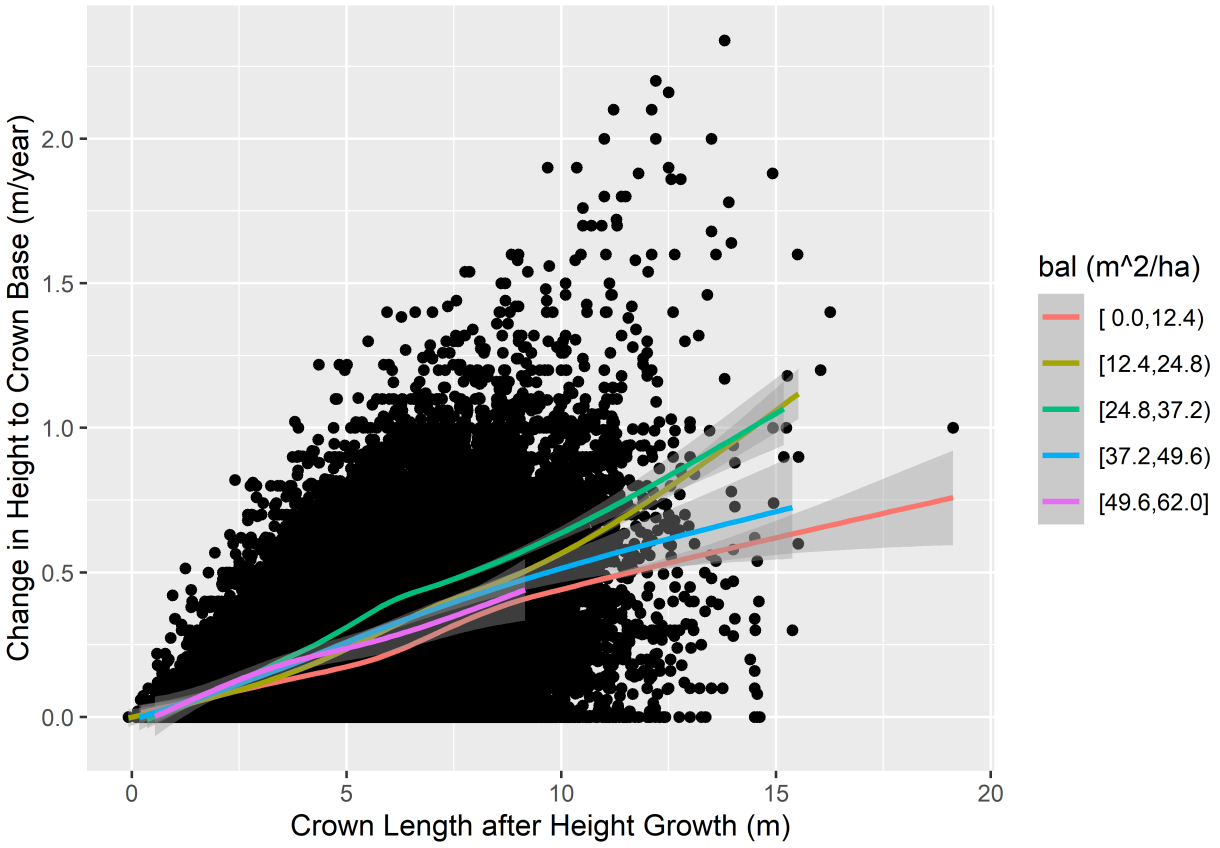
The second graph below shows that Δhcb slows as the ratio of `hcb` to hcb_{ls} increases, and approaches a lower asymptote when the ratio exceeds 1.0.

Crown competition factor (ccf) suggests that crown recession is accelerated upon reaching a threshold for trees with long crown lengths. It is not clear that bal is related to crown recession in a functionally consistent manner. Climate site index (csi) is positively related to (Δhcb). Shade tolerance ($shade$) shows little correlation with Δhcb.

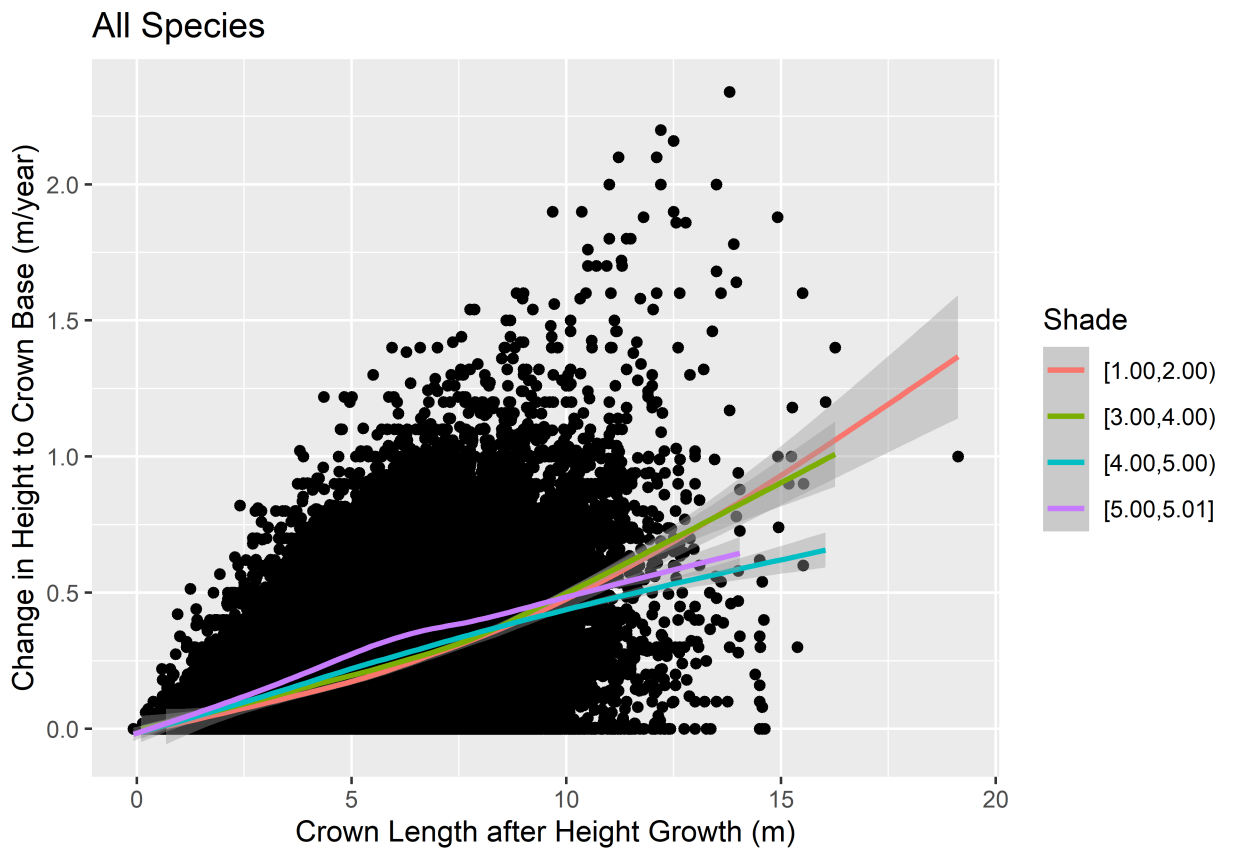

Change in Height to Crown Base (m/year)
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
Crown Length after Height Growth (m)
Species
BF
BS
PB
RM
RS
SM
WP
WS
YB


Change in Height to Crown Base (m/year)
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
Crown Length after Height Growth (m)
LS HCB Ratio
[0.00,0.25)
[0.25,0.50)
[0.50,0.75)
[0.75,1.00)
[1.00,1.25)
[1.25,1.88]


Change in Height to Crown Base (m/year)
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
Crown Length after Height Growth (m)
ccf (m^2/ha)
[4, 264)
[264, 524)
[524, 784)
[784,1044)
[1044,1290]


Change in Height to Crown Base (m/year)
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
Crown Length after Height Growth (m)
bal (m^2/ha)
[0.0,12.4)
[12.4,24.8)
[24.8,37.2)
[37.2,49.6)
[49.6,62.0]


Change in Height to Crown Base (m/year)
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
Crown Length after Height Growth (m)
CSI (m)
[8,10)
[10,12)
[12,14)
[14,16)
[16,18]



Equation Development

We formulated an Δhcb equation using crown length (`cl`), height growth (Δht), late-stage `hcb` ratio ($\frac{hcb}{hcb_{ls}}$) and crown competition factor (`ccf`):

$$\Delta hcb = (\beta_0 * (\frac{hcb}{hcb_{ls}})^{\beta_2})(cl + \Delta ht^{\beta_1})(1 - e^{-\beta_3(ccf+1)})^{\beta_4}$$

The parameter estimates from an integrated fitting process appear in Table 3 where β_0 and β_2 vary by species.

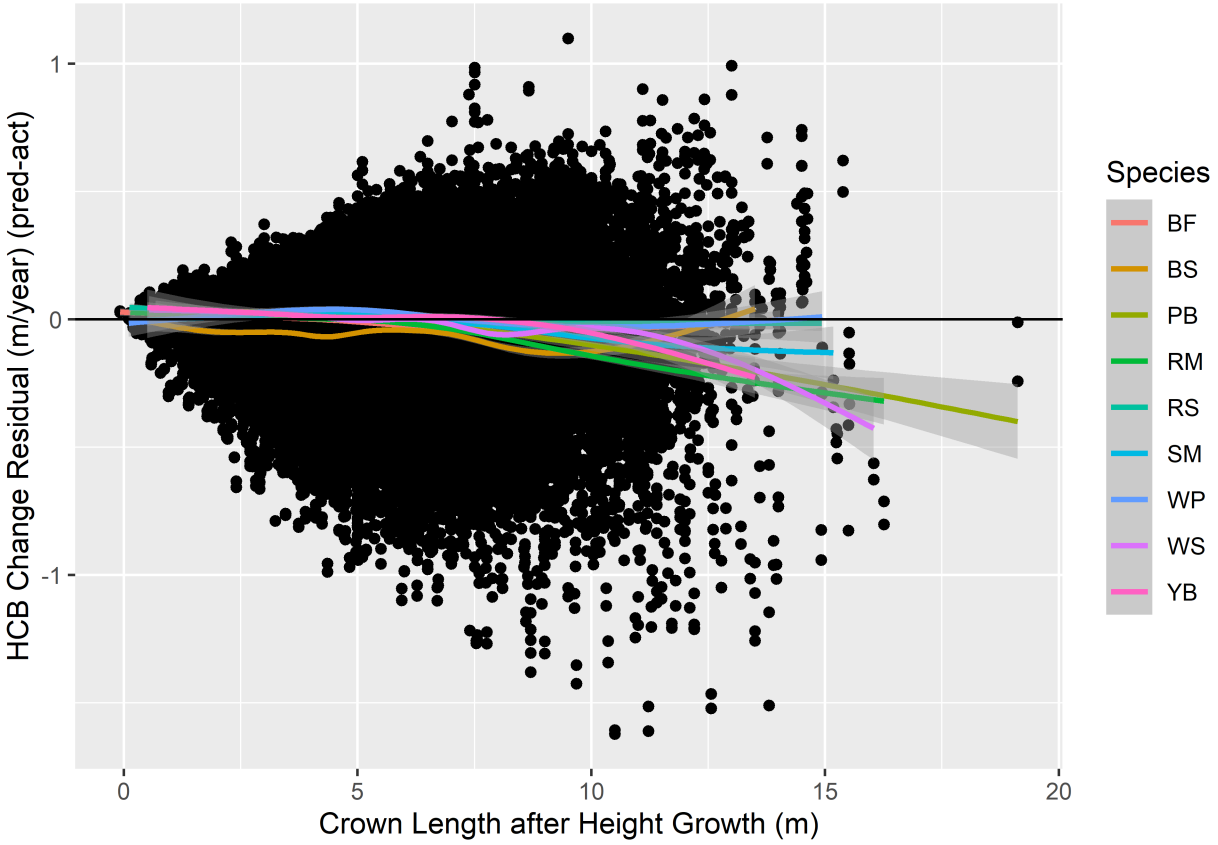
Table 3: Change in Height to Crown Base Equation Parameter Estimates

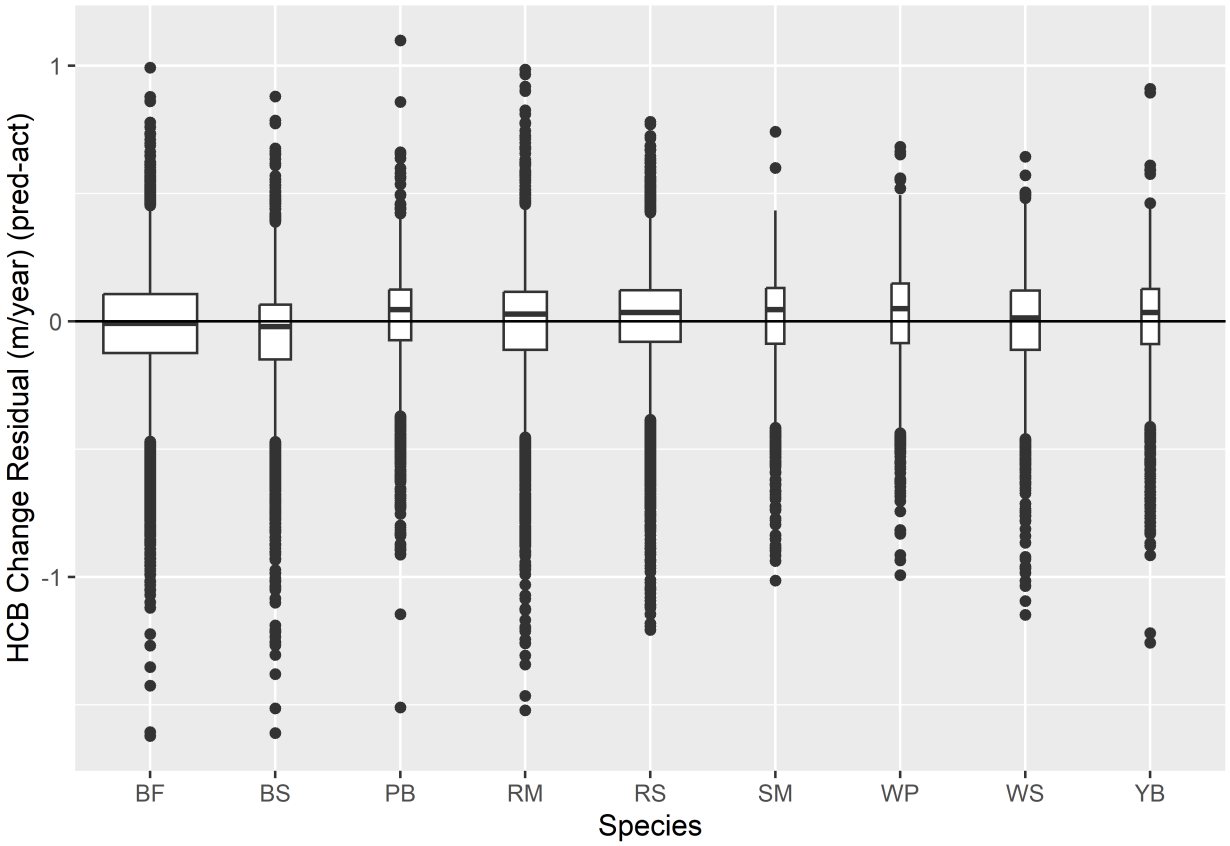
FVS Sp	N Observations	MSE	b0	b1	b2	b3	b4
BF	37047	1.1865409	0.0791502	0.9751972	-0.4395067	0.0025948	1.254663
RS	15724	0.9806985	0.0498006	0.9751972	-0.7721309	0.0025948	1.254663
RM	7799	1.2621017	0.0526865	0.9751972	-1.2027030	0.0025948	1.254663
BS	4166	1.5976061	0.0577257	0.9751972	-0.7228364	0.0025948	1.254663
WS	3512	1.1597765	0.0735299	0.9751972	-0.4912018	0.0025948	1.254663
PB	2049	1.0953531	0.0476873	0.9751972	-0.6341342	0.0025948	1.254663
SM	1397	1.1955312	0.0374392	0.9751972	-1.0113077	0.0025948	1.254663
WP	1278	1.3012331	0.0495131	0.9751972	-0.8048651	0.0025948	1.254663
YB	1264	1.2170852	0.0395582	0.9751972	-1.1962248	0.0025948	1.254663
OC	61727	1.2342459	0.0682109	0.9751972	-0.4130881	0.0025948	1.254663

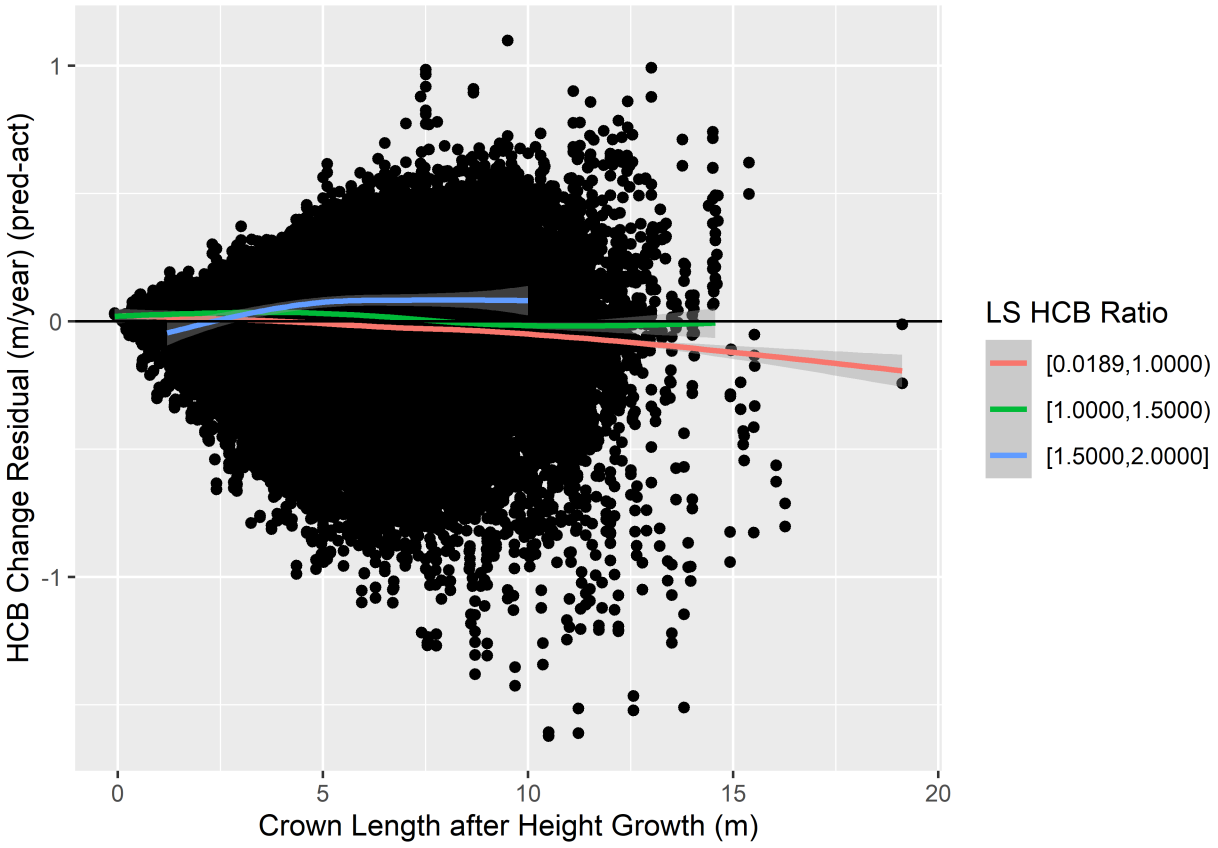
OH	12509	1.2673255	0.0445169	0.9751972	-1.1370584	0.0025948	1.254663

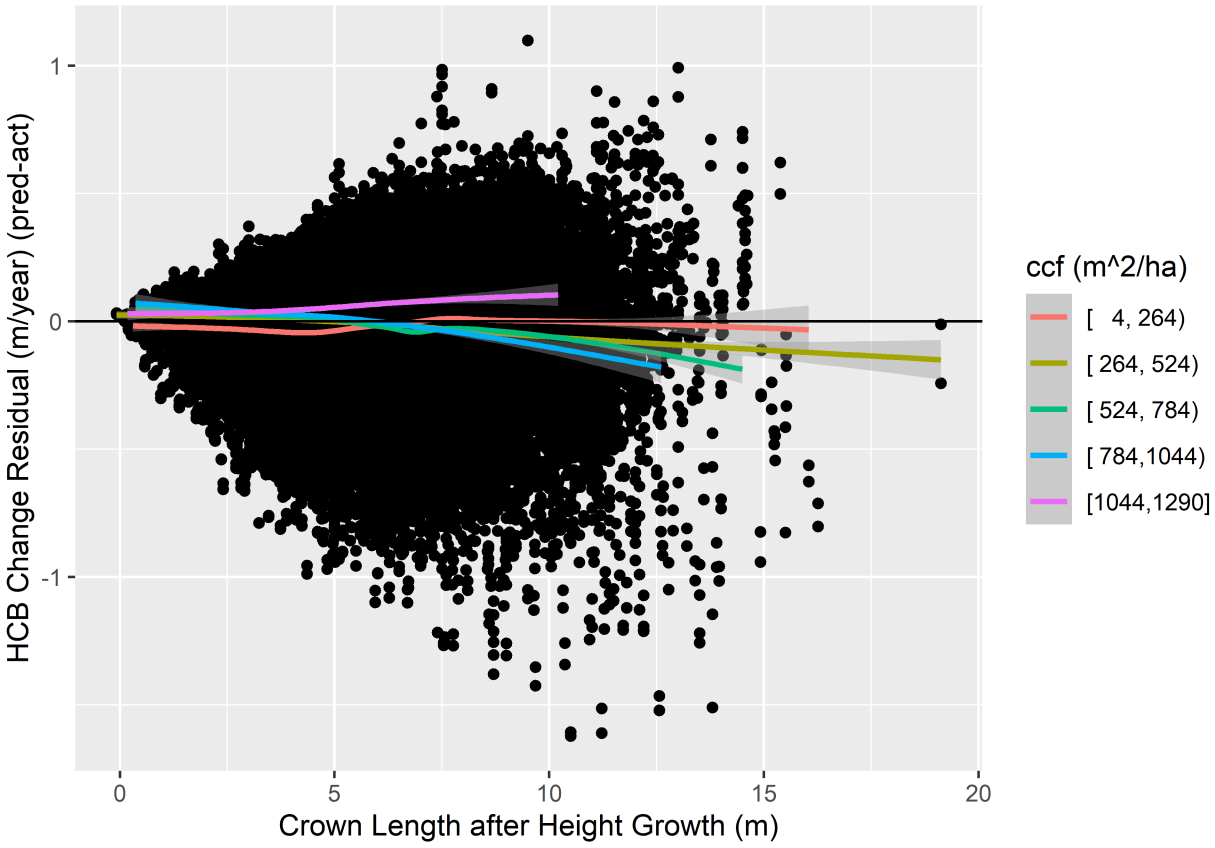
Residual Analysis and Equation Behavior

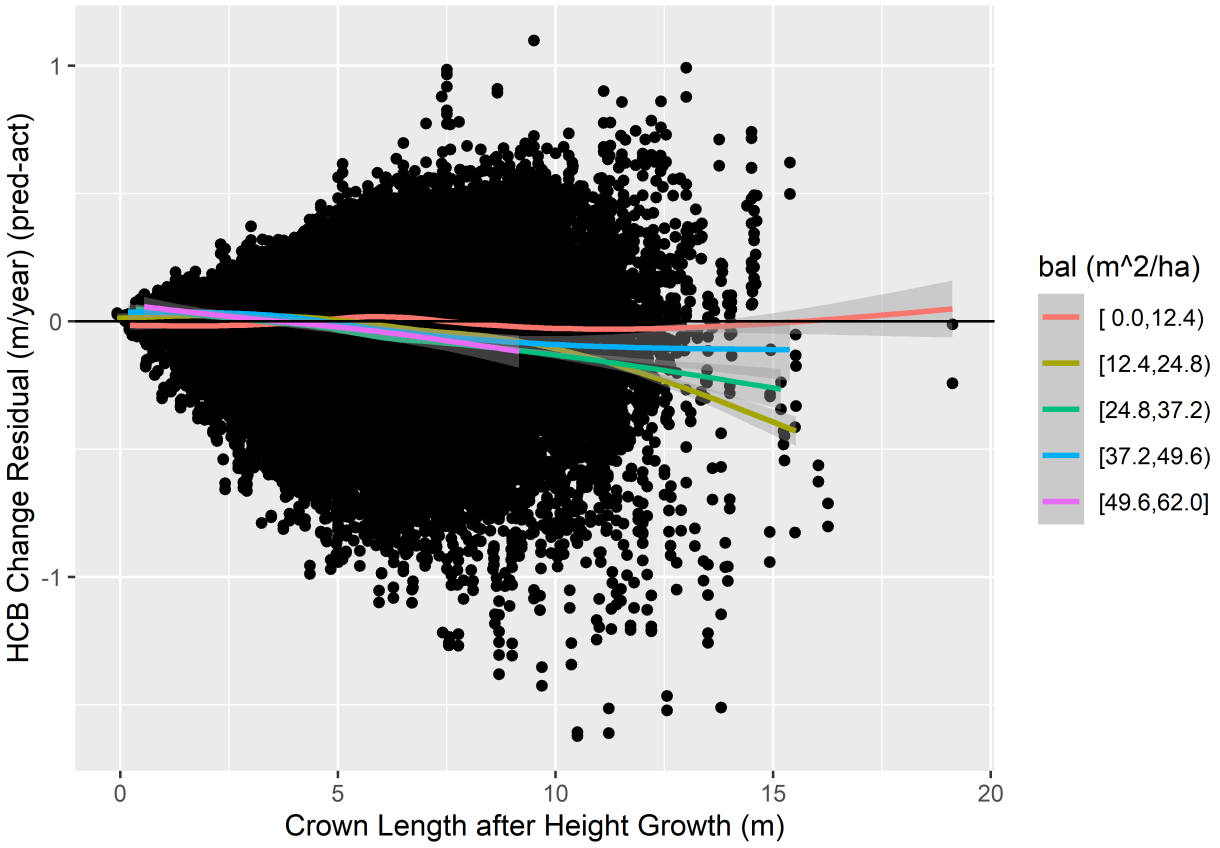
The equation residuals are shown below over explanatory variables. There is a trend in the residuals with height to crown base (`hcb`) and annual height growth (Δht) that should be addressed. The last graph is a demonstration of equation behavior across the range of crown lengths and growth rates.



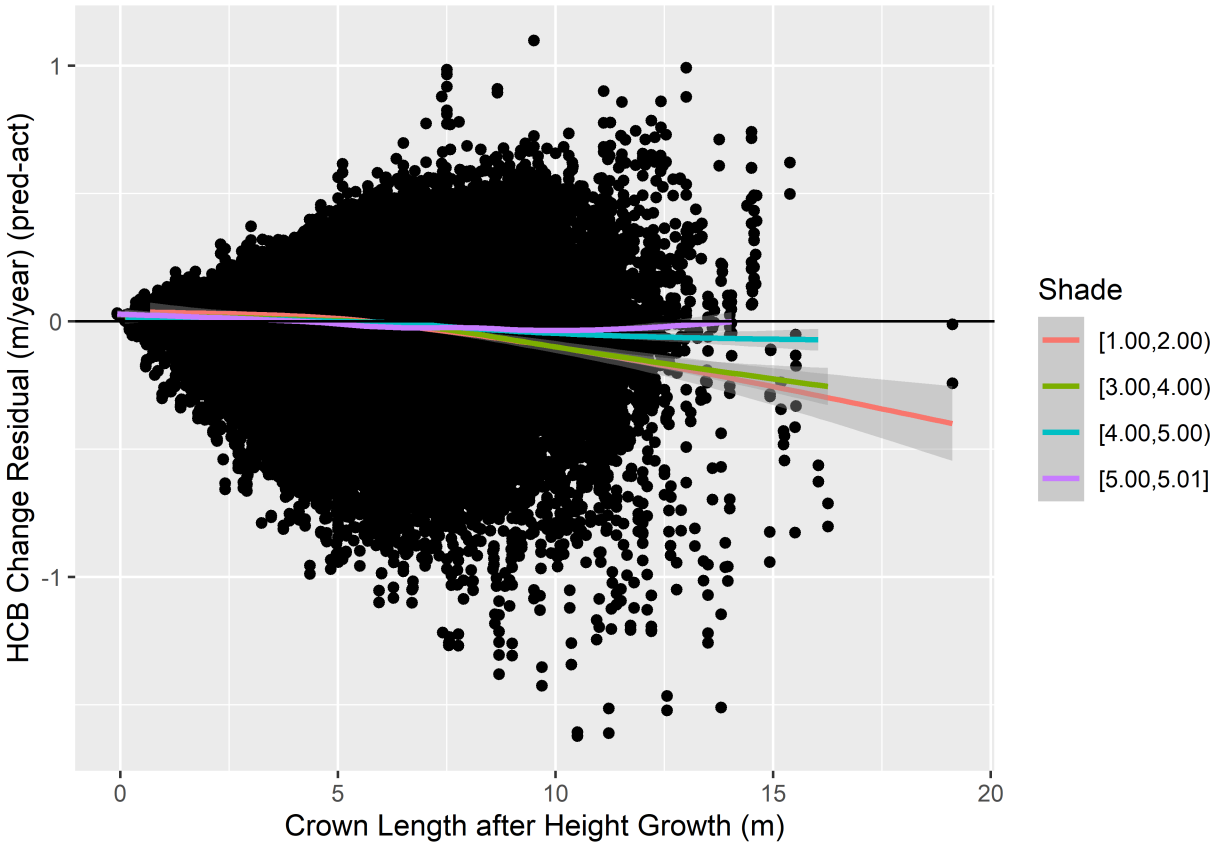

HCB Change Residual (m/year) (pred-act)
1
0
-1
BF
BS
PB
RM
RS
SM
WP
WS
YB
Species

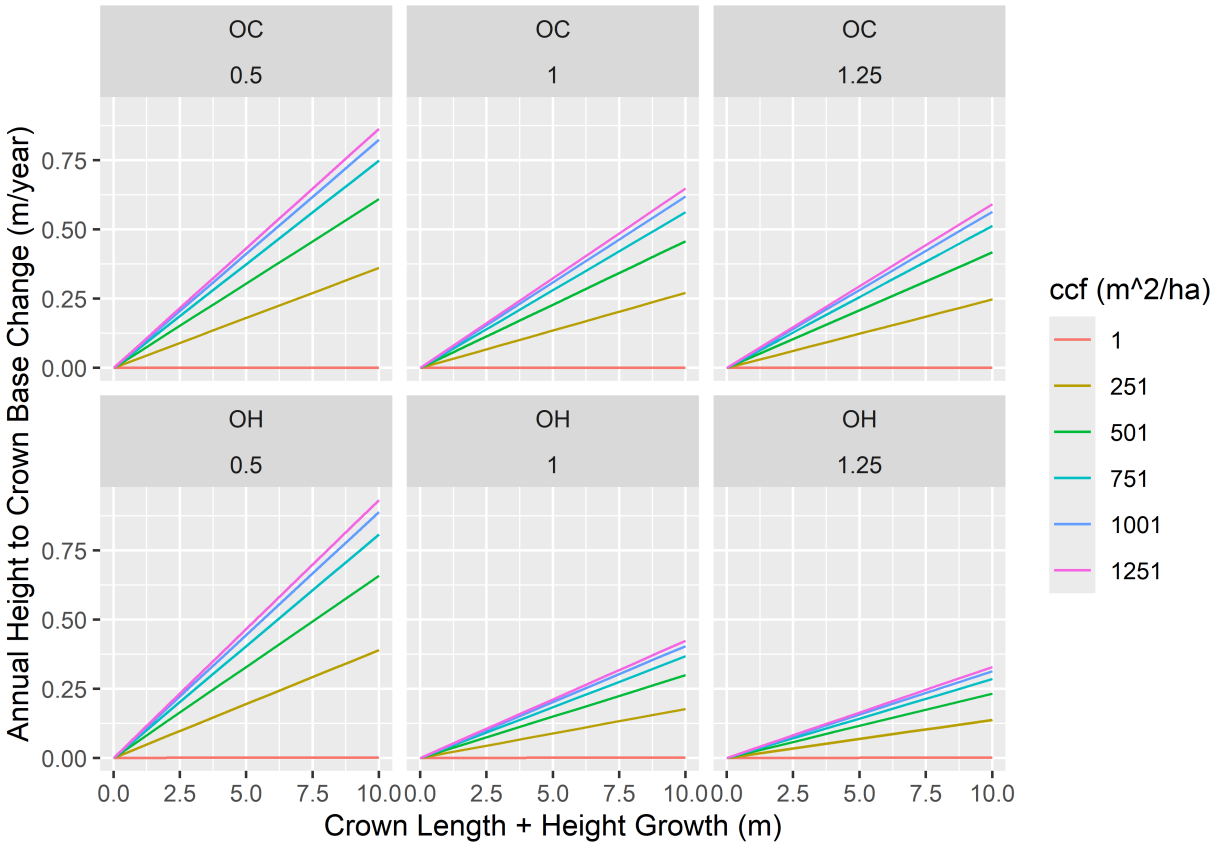

HCB Change Residual (m/year) (pred-act)
1
0
-1
0
5
10
15
20
Crown Length after Height Growth (m)
LS HCB Ratio
[0.0189,1.0000)
[1.0000,1.5000)
[1.5000,2.0000]


HCB Change Residual (m/year) (pred-act)
1
0
-1
0
5
10
15
20
Crown Length after Height Growth (m)
ccf (m^2/ha)
[4, 264)
[264, 524)
[524, 784)
[784,1044)
[1044,1290]


HCB Change Residual (m/year) (pred-act)
1
0
-1
0
5
10
15
20
Crown Length after Height Growth (m)
bal (m^2/ha)
[0.0,12.4)
[12.4,24.8)
[24.8,37.2)
[37.2,49.6)
[49.6,62.0]


HCB Change Residual (m/year) (pred-act)
1
0
-1
0
5
10
15
20
Crown Length after Height Growth (m)
csi (m)
[8,10)
[10,12)
[12,14)
[14,16)
[16,18]


HCB Change Residual (m/year) (pred-act)
1
0
-1
0
5
10
15
20
Crown Length after Height Growth (m)
Shade
[1.00,2.00)
[3.00,4.00)
[4.00,5.00)
[5.00,5.01]


OC
0.5
OC
1
OC
1.25
OH
0.5
OH
1
OH
1.25
Annual Height to Crown Base Change (m/year)
0.00
0.25
0.50
0.75
0.0
2.5
5.0
7.5
10.0
Crown Length + Height Growth (m)
ccf (m^2/ha)
1
251
501
751
1001
1251