Height Increment for the Acadian Variant of FVS (ACD)

Greg Johnson
Greg Johnson Biometrics LLC

2024-03-21

Introduction

We fit new height growth equations for the Acadian Variant of FVS (`ACD`). The equations are designed to:

1. Enforce biologically reasonable long-term behavior (i.e., limit total height attainment to observed maxima),
2. Demonstrate peaking or monotonically decreasing height growth estimates over initial total height, and
3. Incorporate predictor variables capturing social position or competitive status.

Height growth data were obtained from the `ALL_DHT.csv` data set provided by Aaron Weiskittel. The raw data were screened to drop observations with missing crown ratio (`cr`), crown competition factor in larger trees (`ccfl`), and climate site index (`csi`). We limited the data to remeasurement intervals $\geq$ 5 years and $\leq$ 20 years (we initially set the upper bound to 10, but at the suggestion of Ben Rice we extended the limit to 20 to overcome a signal to noise ratio issue with short remeasurement intervals given the slow growth rates of many species). After screening, we had 819311 total observations. The breakdown by remeasurement interval is shown in Table 1.

Because we need equation parameter estimates by species, we limited the equation fitting to species with $\geq$ 3000 observations. Table 2 shows the breakdown by species.

Table 1: Height Growth Remeasurement Intervals

Remeasurement Interval	N Observations
5	346480
6	8941
7	3962
8	5696
9	3934
10	226082
11	334
15	139832
20	84050

Table 2: Height Growth Observations by Species

FVS Spp	FIA Spp	Plant Code	N Observations

BF	12	ABBA	218303
RS	97	PIRU	148464
RM	316	ACRU	126899
BS	95	PIMA	52318
WS	94	PIGL	44443
PB	375	BEPA	40823
SM	318	ACSA3	29398
YB	371	BEAL2	29176
WP	129	PIST	21934
EH	261	TSCA	20593
WC	241	THOC2	17167
AB	531	FAGR	15337
QA	746	POTR5	11060
TA	71	LALA	11039
RO	833	QURU	9278
BT	743	POGR4	5494
WA	541	FRAM2	5119
GB	379	BEPO	3741

Equation Development

`ACDO` the version implemented in R[1] implements a height growth equation fit as a mixed model with random effects on the β_0 and β_2 parameters:

$$\Delta ht = e^{\beta_0 + \beta_1 log(ht) + \beta_2 ht + \beta_3 cr + \beta_4 \frac{ccfl}{100} + \beta_7 csi^2}$$

where ht = total height (m), cr = crown ratio, $ccfl$ is crown competition factor in larger trees (m^2/ha), and csi is climate site index (m).

The equation fits the data reasonably well, but it has several issues:

1. It is not asymptotically bound when integrated over height, which means that it can produce unreasonably tall trees over long projection intervals.
2. It transforms `csi` by squaring it, which does not make sense given the definition of site index and its relationship to height.
3. While the $\beta_1 log(ht) + \beta_2 ht$ construct can produce peaking behavior over `ht`, it is not bound to this constraint, in part leading to issue 1.

To address these issues, we took the basic Chapman-Richards equation form[2] which produces an asymptote as the integrated framework for the height growth equation. The equation differentiated with respect to `ht` is:

$$\Delta ht = abc * e^{-bht}(1 - e^{-bht})^{(c-1)}$$

where `a` is the asymptote, and b and c are the slope and shape parameters. Using this framework to enforce an asymptotic height, we took data provided by Ben Rice to supply an maximum height for each species (`ht_{max}`). Table {tab:maxht} shows the maximum heights used in this analysis.

Using the above differentiated equations as a start point we developed an equation to reflect sensitivity to site productivity (`csi`), and social position (`cr` or `ccfl`). `csi` was transformed to be a ratio of current `csi` to a maximum of 30 meters. In use, any `csi` values over 30 would have to be capped at 30.

[1] R Core Team (2023). *R: A Language and Environment for Statistical Computing.* R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/

[2] Pienaar, L. V. and Turnbull, K. J., 1973. The Chapman-Richards generalization of von Bertalanffy's growth model for basal area growth and yield in even-aged stands. Forest Science 19: 2-22.

$\Delta ht = ht_{max}\beta_1\beta_2 cr^{\beta_3}(csi/30)^{\beta_5}e^{(-\beta_1 ht-\beta_4(ccfl/100))}(1-e^{-\beta_1 ht})^{(\beta_2-1)}$

The `ccfl` term was inserted into the equation in the first exponential term as that term generally defines the decay of height growth past the peak where competition effects would most likely occur. In the fitting process, we found `cr` and `ccfl` do not enter significantly at the same time. Some species are better fit with `cr` and others with `ccfl`. Parameter estimates of 0.0 reflect the selection of one over the other. Several species had negative estimates for β_5. since this is not biologically reasonable, β_5 was set to 0.43 (an average from species with valid estimates) for those species.

Table 3: Maximum Height by Species

FVS Spp	Maximum Height (m)
BF	30.2
RS	35.2
RM	30.1
BS	25.6
WS	30.9
PB	27.9
SM	33.3
YB	25.0
WP	39.6
EH	35.1
WC	31.4
AB	30.8
QA	31.7
TA	26.1
RO	31.9
BT	34.7
WA	36.1
GB	20.7

Equation Fitting

Conifers were fit separately by species, whereas hardwoods were fit with a mixed model where β_4 and β_5 were random effects on species. Other conifers parameters were estimated using a combined data set with all conifer species. Other hardwoods parameters were taken from the fixed effects estimates from the mixed model.

Estimates for White Pine (`WP`) produced errant behavior. `WP` parameters were set to the other conifers estimates with the β_0 (maximum height) set to the `WP` value.

The mixed model for hardwoods resulted in a consistent under-prediction bias. We corrected this by estimating a correction ratio applied to the β_0 parameter for each species i:

$\beta_{0a,i} = \beta_{0,i}\Sigma(\frac{\Delta ht_i}{\widehat{\Delta ht_i}})/n_i$

where $\beta_{0a,i}$ is the adjusted parameter for each species.

The parameter estimates are shown in Table 4.

Table 4: Height Growth Equation Parameter Estimates

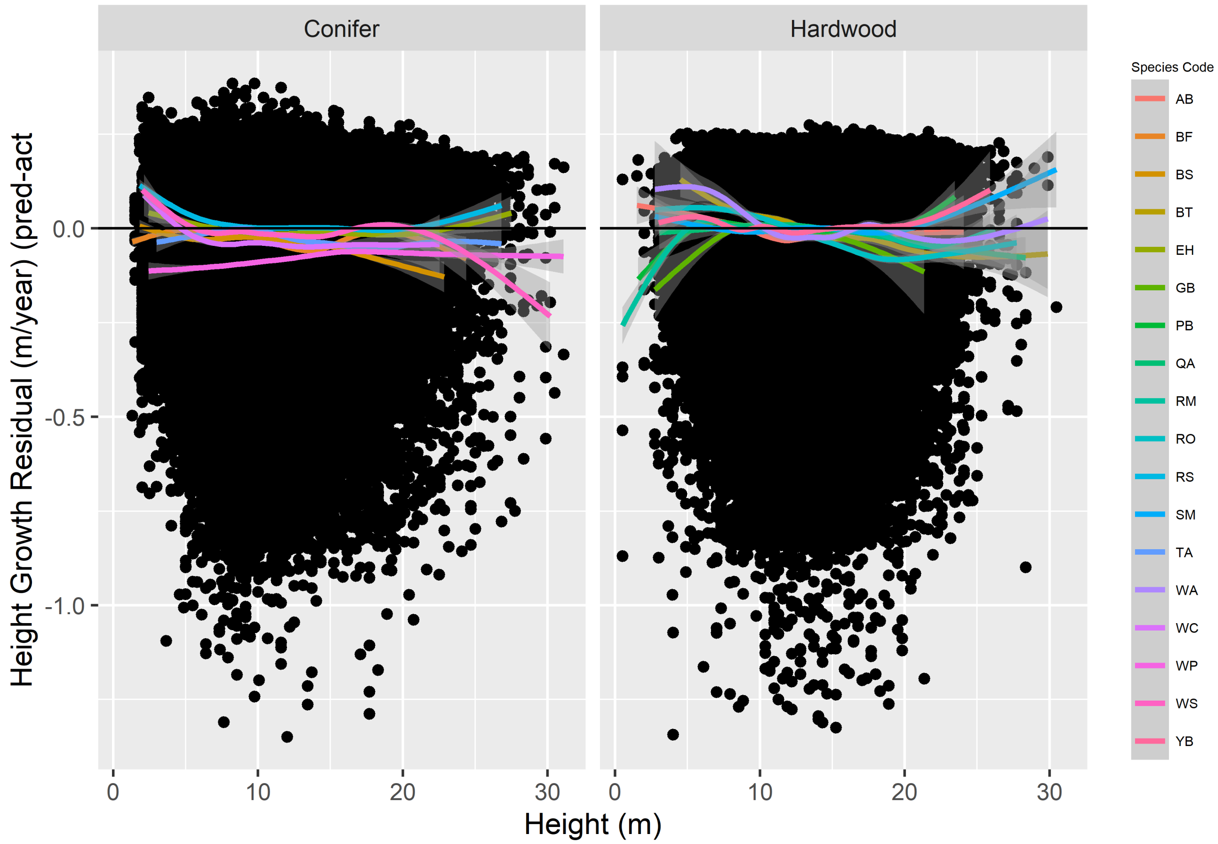
FVS Spp.	FIA Spp	N	MSE	b0	b1	b2	b3	b4	b5

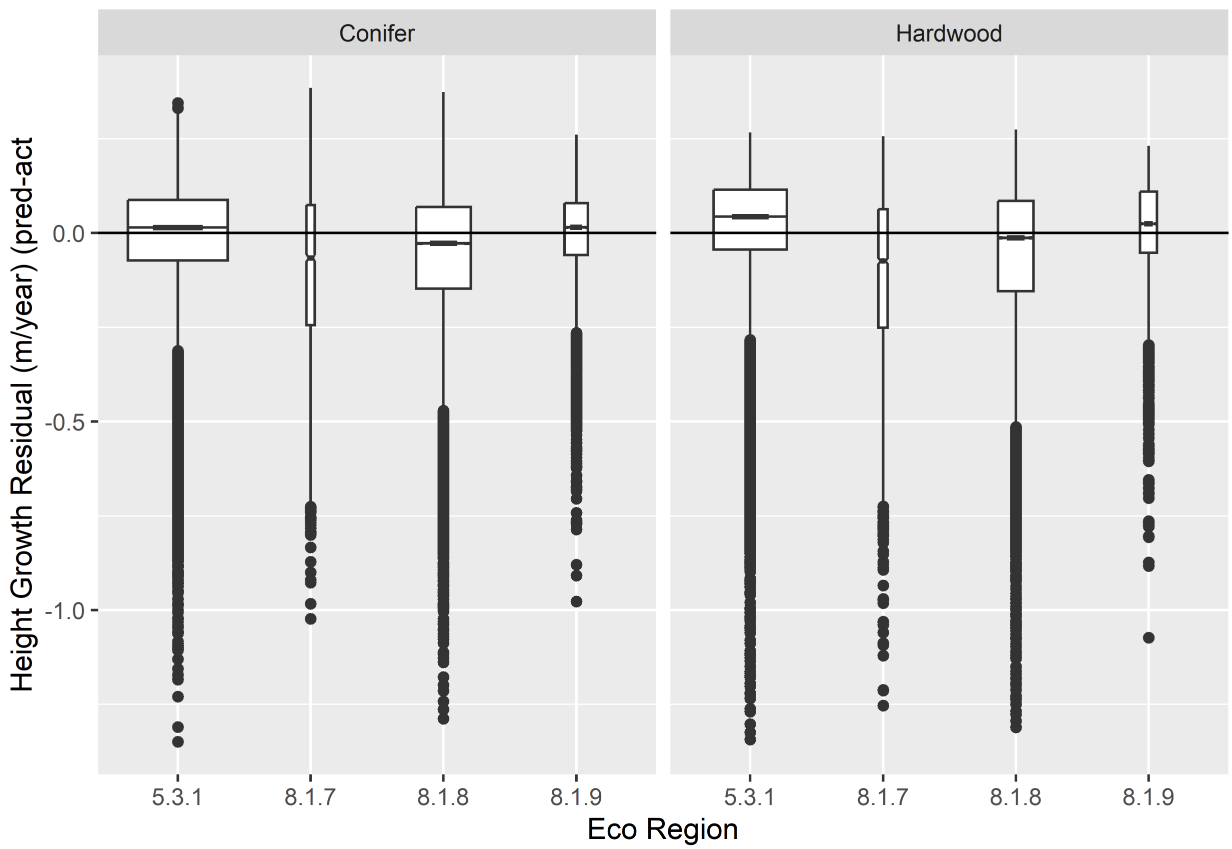
BF	12	218303	1.567659	30.18293	0.0237654	1.7309872	0.3396236	0.0000000	0.4300000
RS	97	148464	1.190058	35.21341	0.0031977	0.6452656	0.2798559	0.0000000	0.0453984
BS	95	52318	0.759038	25.60976	0.0029867	0.8749023	0.1937343	0.0000000	0.1061788
WS	94	44443	1.616540	30.94512	0.0113415	0.9594453	0.2612782	0.0000000	0.4300000
WP	129	21934	2.462007	39.63415	0.0086709	1.0633682	0.2774026	0.0000000	0.1660177
EH	261	20593	1.523230	35.06098	0.0226999	1.0317121	0.2262924	0.0000000	1.3184268
WC	241	17167	1.296424	31.40244	0.0016327	0.5722277	0.1250221	0.0000000	0.0407049
TA	71	11039	1.855388	26.06707	0.0199065	1.6493001	0.3596588	0.0000000	0.4300000
OC	9991	534261	0.000000	31.72659	0.0086709	1.0633682	0.2774026	0.0000000	0.1660177
AB	531	15337	2.265601	40.11061	0.0085548	0.9805647	0.0000000	0.0576520	0.7815698
BT	743	5494	4.982350	40.83646	0.0085548	0.9805647	0.0000000	0.0076990	0.5001779
GB	379	3741	2.130461	23.13511	0.0085548	0.9805647	0.0000000	0.0455226	0.2042321
PB	375	40823	2.496445	33.69588	0.0085548	0.9805647	0.0000000	0.0405947	0.6465336
QA	746	11060	5.453557	37.15198	0.0085548	0.9805647	0.0000000	0.0092467	0.2840619
SM	318	29398	3.166175	38.14175	0.0085548	0.9805647	0.0000000	0.0405047	0.5744540
WA	541	5119	3.676315	46.89629	0.0085548	0.9805647	0.0000000	0.0336856	0.7608173
YB	371	29176	2.991481	30.45648	0.0085548	0.9805647	0.0000000	0.0185188	0.4918140
RM	316	126899	2.591160	34.96240	0.0081491	1.0320316	0.0000000	0.0000000	0.6855181
RO	833	9278	3.424399	41.56113	0.0081491	1.0320316	0.0000000	0.0000000	0.7522605
OH	9990	276325	0.000000	35.41047	0.0085548	0.9805647	0.0000000	0.0164149	0.6245655

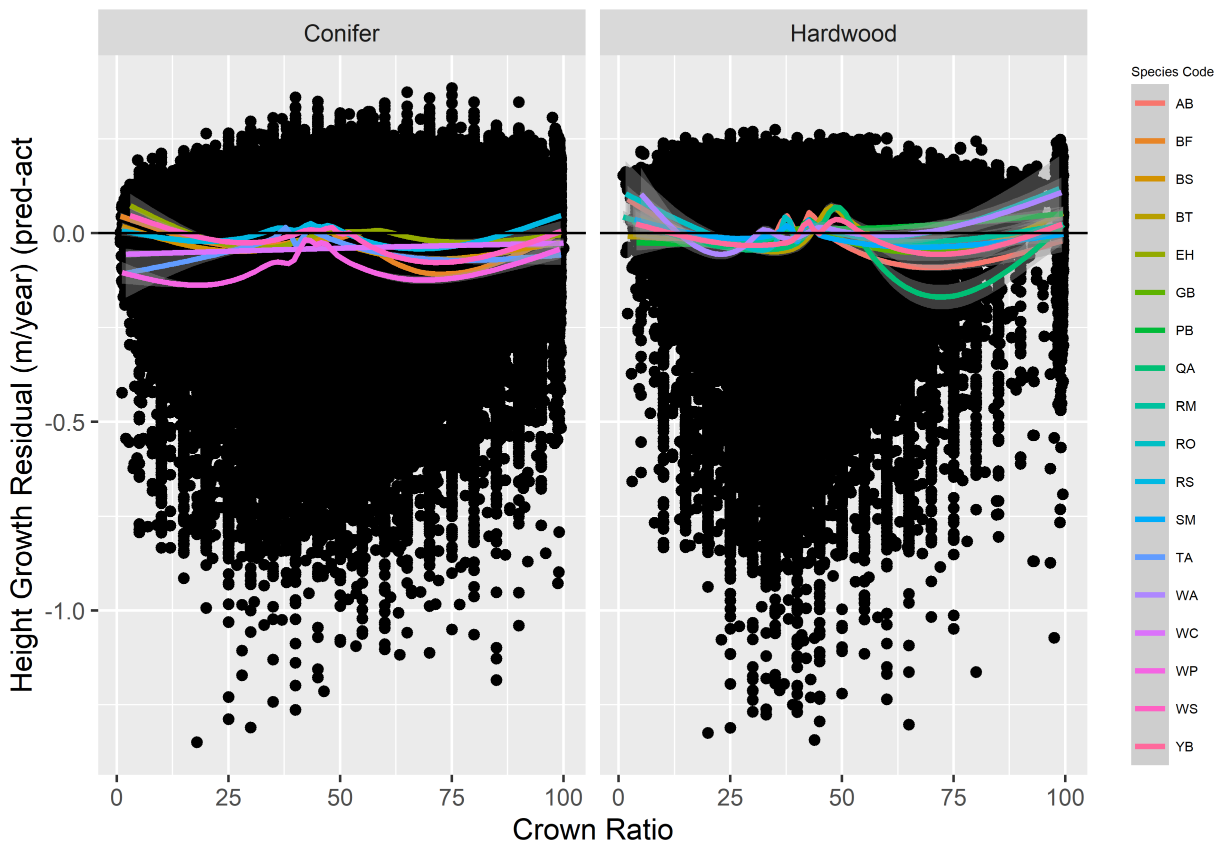
Equation Residual Analysis

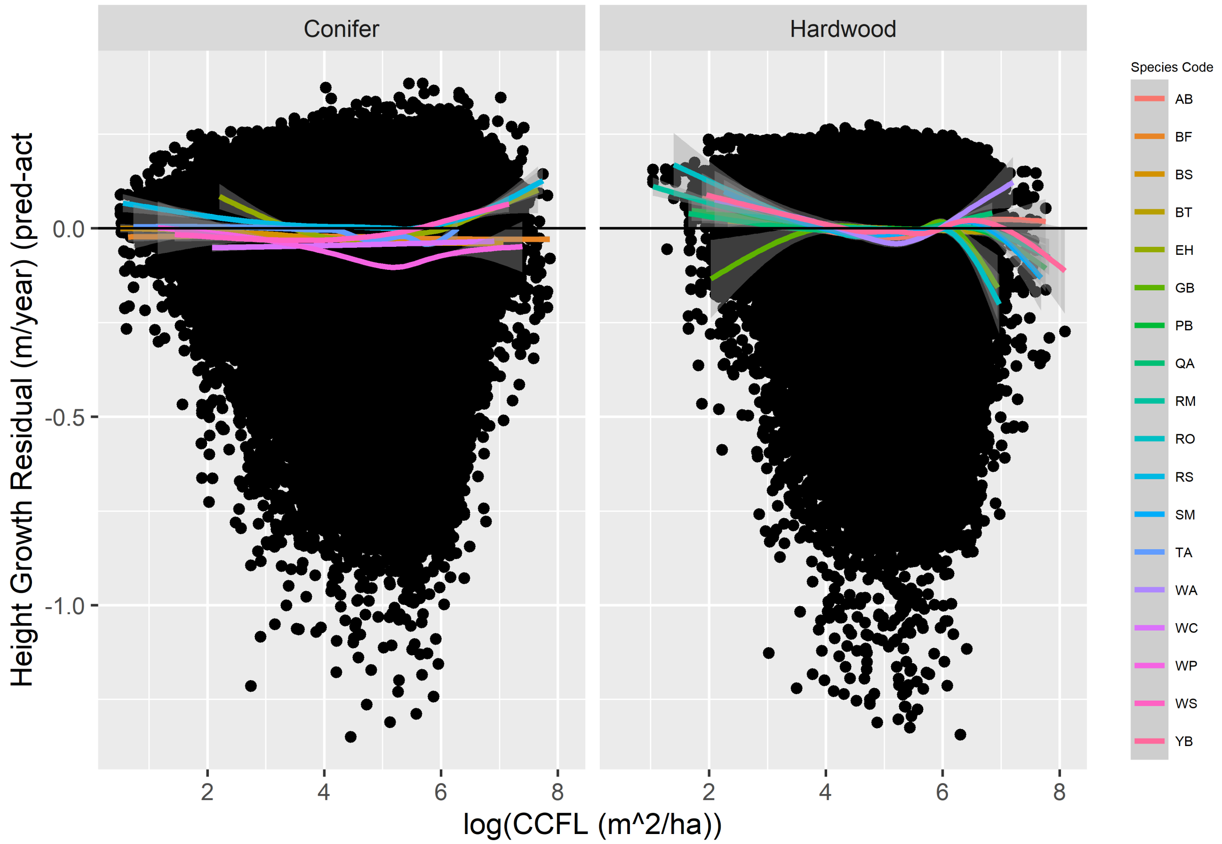
Table 5: Height Growth Equation Residual Summary

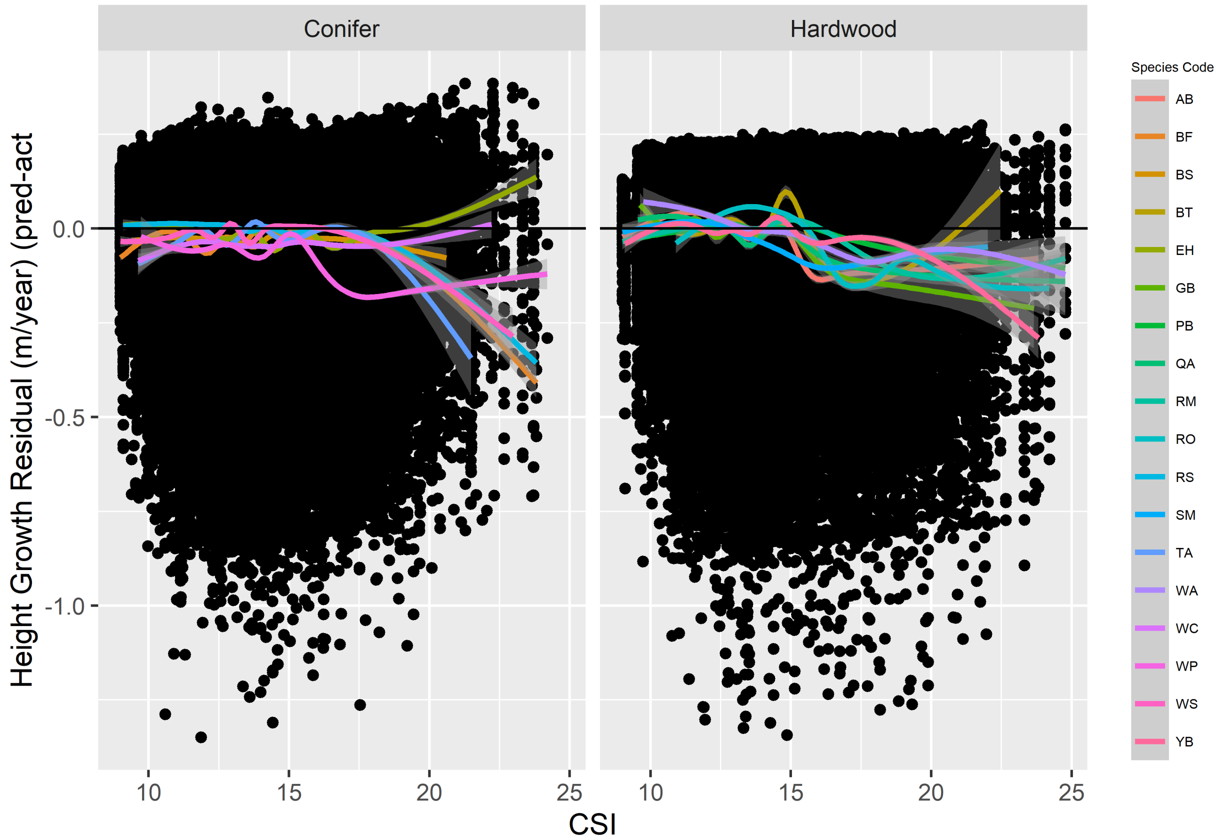
FVS Spp.	FIA Spp	N	Bias (m/yr)	MSE
AB	531	15337	-0.006	0.0300
BF	12	218303	-0.025	0.0244
BS	95	52318	-0.029	0.0115
BT	743	5494	0.000	0.0347
EH	261	20593	-0.010	0.0272
GB	379	3741	-0.001	0.0197
PB	375	40823	0.000	0.0220
QA	746	11060	-0.002	0.0372
RM	316	126899	-0.001	0.0217
RO	833	9278	-0.006	0.0359
RS	97	148464	0.006	0.0172
SM	318	29398	-0.003	0.0237
TA	71	11039	-0.018	0.0234
WA	541	5119	-0.008	0.0397
WC	241	17167	-0.042	0.0320
WP	129	21934	-0.079	0.0452
WS	94	44443	-0.011	0.0211
YB	371	29176	-0.001	0.0259


Conifer
Hardwood
Species Code
AB
BF
BS
BT
EH
GB
PB
QA
RM
RO
RS
SM
TA
WA
WC
WP
WS
YB
Height Growth Residual (m/year) (pred-act
0.0
-0.5
-1.0
0
10
20
30
Height (m)


Conifer
Hardwood
Height Growth Residual (m/year) (pred-act
0.0
-0.5
-1.0
5.3.1
8.1.7
8.1.8
8.1.9
5.3.1
8.1.7
8.1.8
8.1.9
Eco Region


Conifer
Hardwood
Height Growth Residual (m/year) (pred-act
0.0
-0.5
-1.0
0
25
50
75
100
Crown Ratio
Species Code
AB
BF
BS
BT
EH
GB
PB
QA
RM
RO
RS
SM
TA
WA
WC
WP
WS
YB

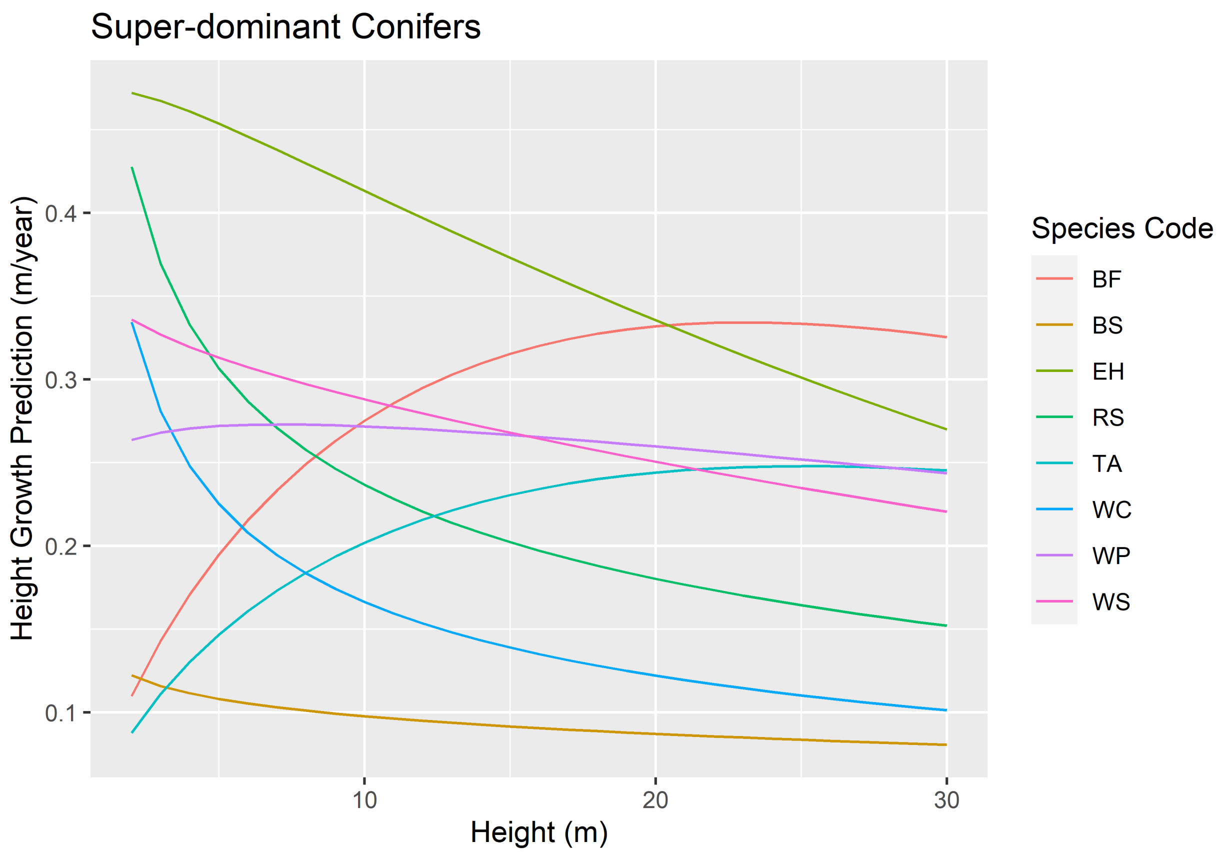

Conifer
Hardwood
Height Growth Residual (m/year) (pred-act
0.0
-0.5
-1.0
2
4
6
8
log(CCFL (m^2/ha))
Species Code
AB
BF
BS
BT
EH
GB
PB
QA
RM
RO
RS
SM
TA
WA
WC
WP
WS
YB

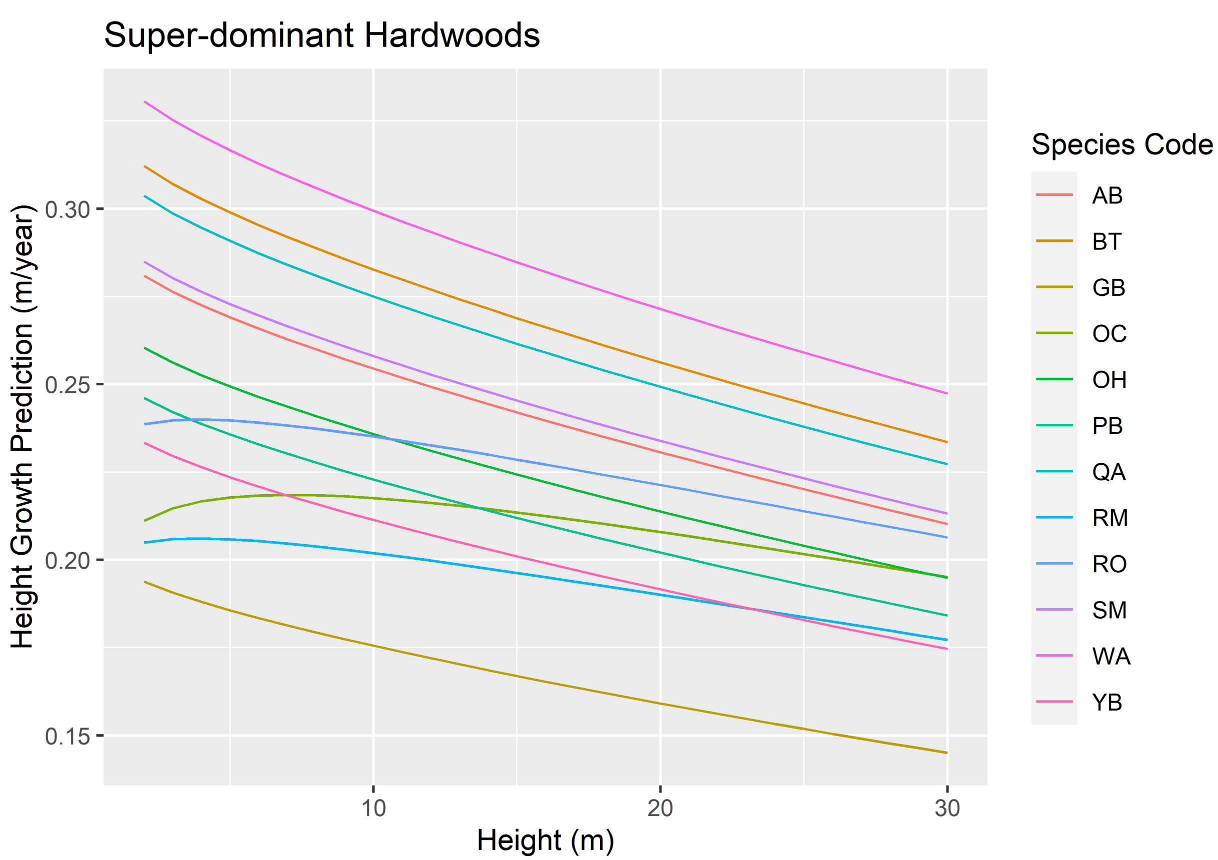


Equation Behavior for Super-Dominant Tree

The graphs below show height growth estimates for:

Tree with 100% `cr`, a `ccfl` of 0.0, and a `csi` of 22 meters.





Trees with a `csi` of 15 meters and a range of `cr` or `ccfl` values.