Individual Tree Mortality Equations Incorporating CSI

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Equation Fitting

We fit a survival equation (probability of survival) of a Gompit form to observations for each species with ≥ 3000 observations and remeasurement intervals between 5 and 10 years found in the `tree.csv` data set provided by John Kershaw.

$p_{live} = 1 - e^{-e^{-\beta_1 + \beta_2 \frac{dbh^{\beta_3}}{(bal+1)}}}$

where: dbh = diameter at breast height (cm), and bal = basal area per hectare in larger trees (m^2/ha).

The equation was fit using an integrated fitting algorithm over each year of the remeasurement interval. The error minimized was the trees per hectare (`tph`) prediction error at the end of the remeasurement interval.

Species

The species in Table 1 had enough (≥ 3000) observations to fit the survival equation:

Table 1: Species Available for Survival Equation

Species Code	FIA Code	N	Common Name
BF	12	191518	balsam fir
RM	316	80697	red maple
RS	97	77212	red spruce
BS	95	53381	black spruce
PB	375	41088	paper birch
WS	94	25245	white spruce
SM	318	24257	sugar maple
YB	371	19371	yellow birch
WC	241	18029	arborvitae
QA	746	17922	quaking aspen
WP	129	14044	eastern white pine
AB	531	13663	American beech
EH	261	12501	eastern hemlock
TA	71	6449	tamarack
GB	379	5105	gray birch
RO	833	5103	northern red oak
BT	743	3657	bigtooth aspen
WA	541	3506	white ash

ST	315	3289	striped maple
JP	105	3184	jack pine

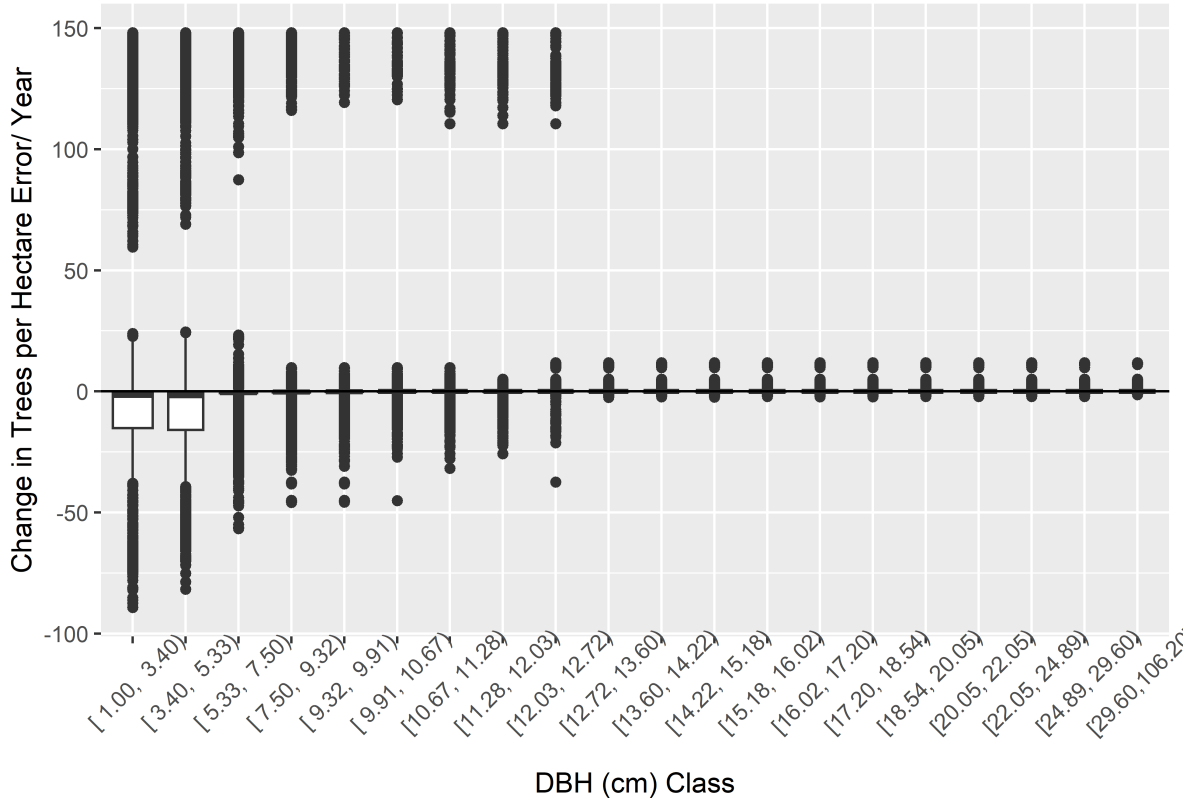
Parameter Estimates

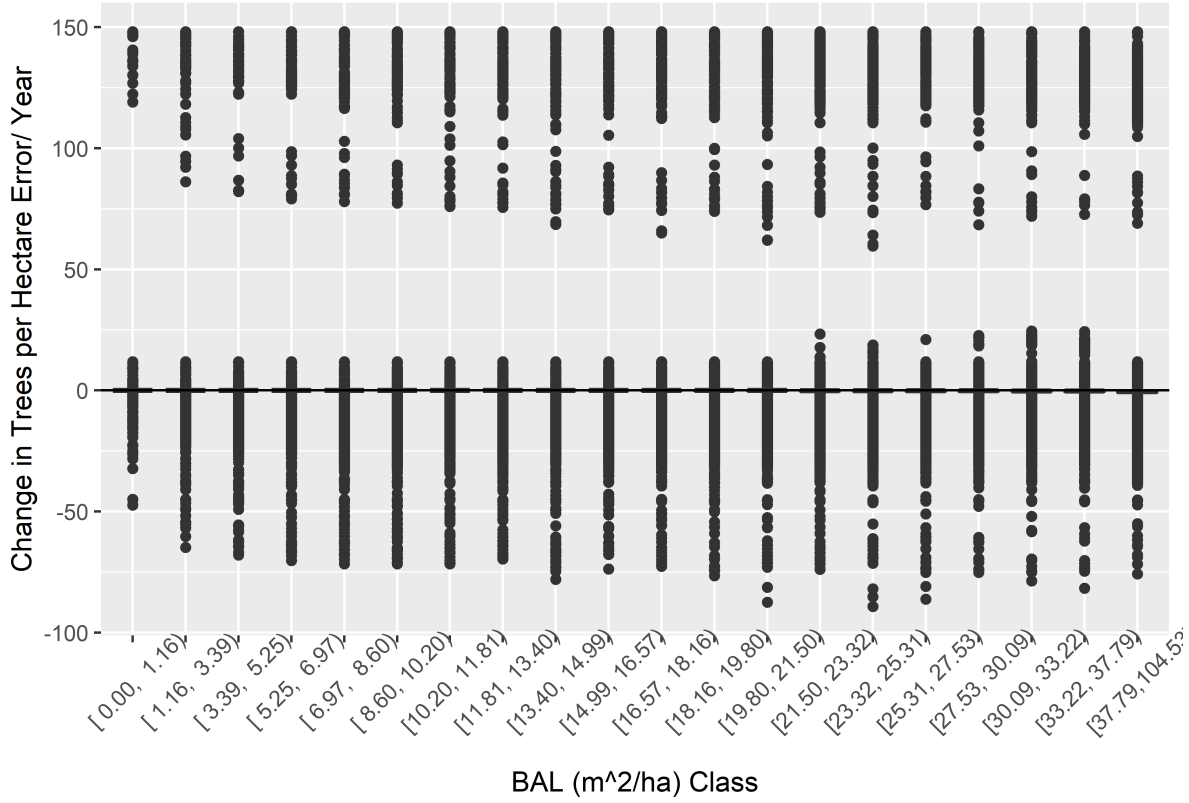
The fit statistics and parameter estimates for the species in Table 1 are in Table 2. All but one species (American Beech) yielded biologically reasonable parameter estimates (`AB`'s β_2 estimate was negative).

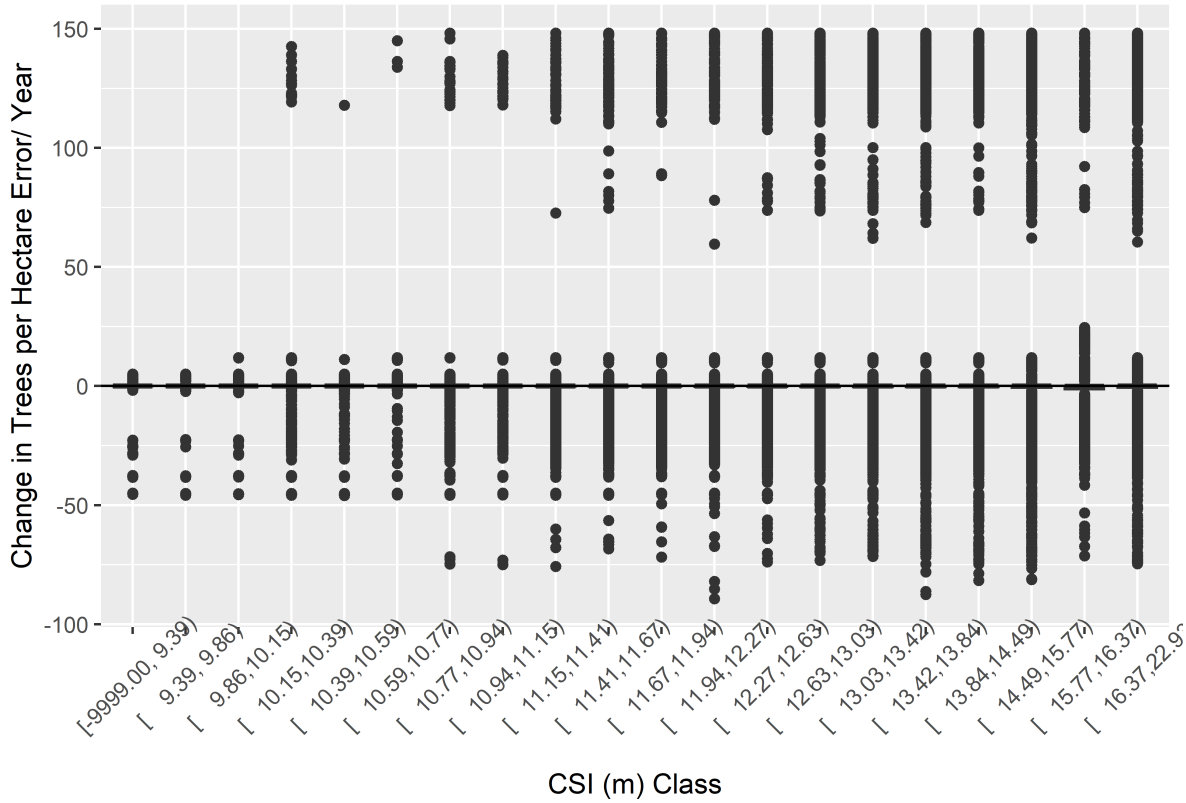
Table 2: Survival Equation Parameter Estimates

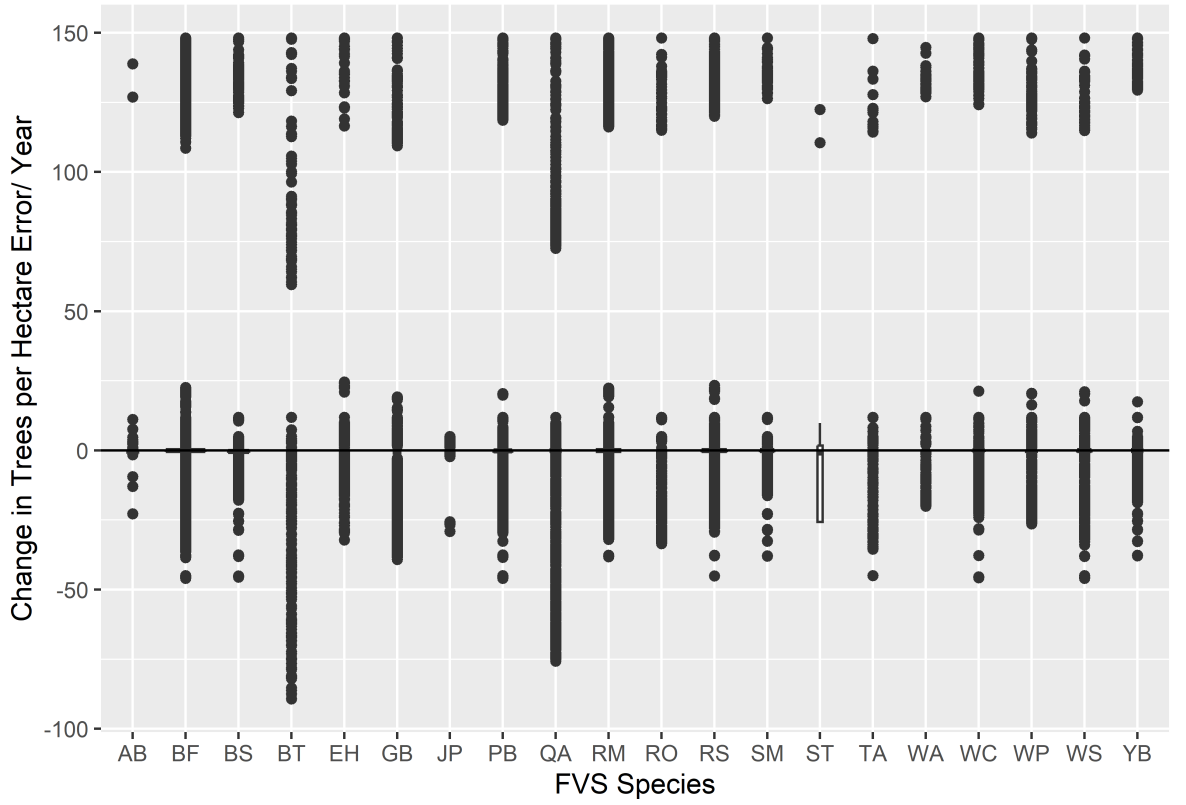
Species	n	MSE	b1	b2	b3
BF	191518	4903.20	-0.9626182	3.7609865	0.7414765
RM	80697	3719.03	-1.1016601	0.0937570	2.4588578
RS	77212	1850.86	-1.0231953	6.9566316	0.2992580
BS	53381	896.90	-1.2832272	0.6224753	0.5341737
PB	41088	5017.29	-1.1304255	0.1575705	1.5784746
WS	25245	721.77	-0.9464213	5.1528610	0.3683826
SM	24257	2084.84	-1.3216928	0.0840536	1.7517667
YB	19371	4704.05	-1.2861758	0.0058181	3.2588545
WC	18029	2295.61	-1.0866627	4.6970194	0.6290715
QA	17922	7118.33	-0.6872042	0.0325129	3.1171555
WP	14044	4036.20	-1.1779610	0.0464297	1.6359074
EH	12501	1520.25	-0.7602993	12.1300823	0.7284034
TA	6449	1079.84	-1.0643472	0.0484539	2.4323037
GB	5105	13004.57	-1.0214240	0.2789377	1.2891585
RO	5103	3356.45	-1.0787602	0.1858255	1.6327093
BT	3657	9697.71	-0.4545934	0.4455582	2.2074701
WA	3506	6945.16	-1.2675468	0.0021850	3.1604358
JP	3184	55.12	-0.5875586	0.7320308	1.2282033

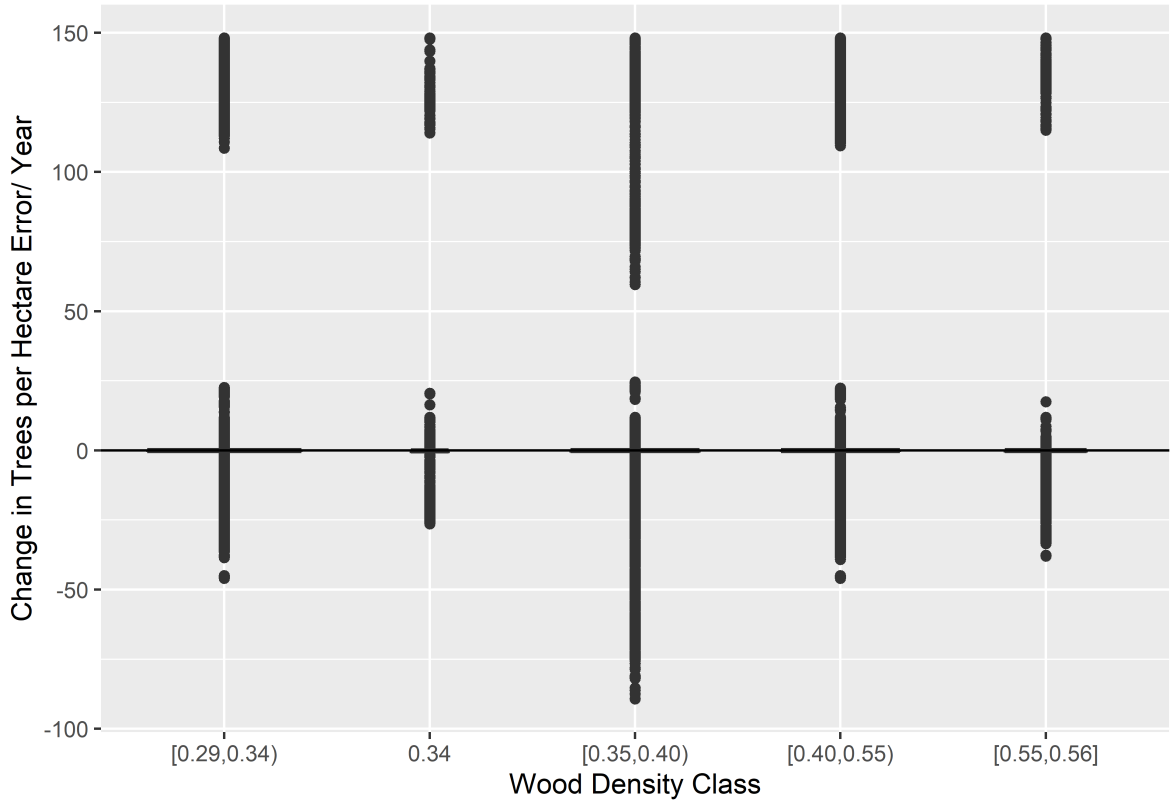
Residual Analysis

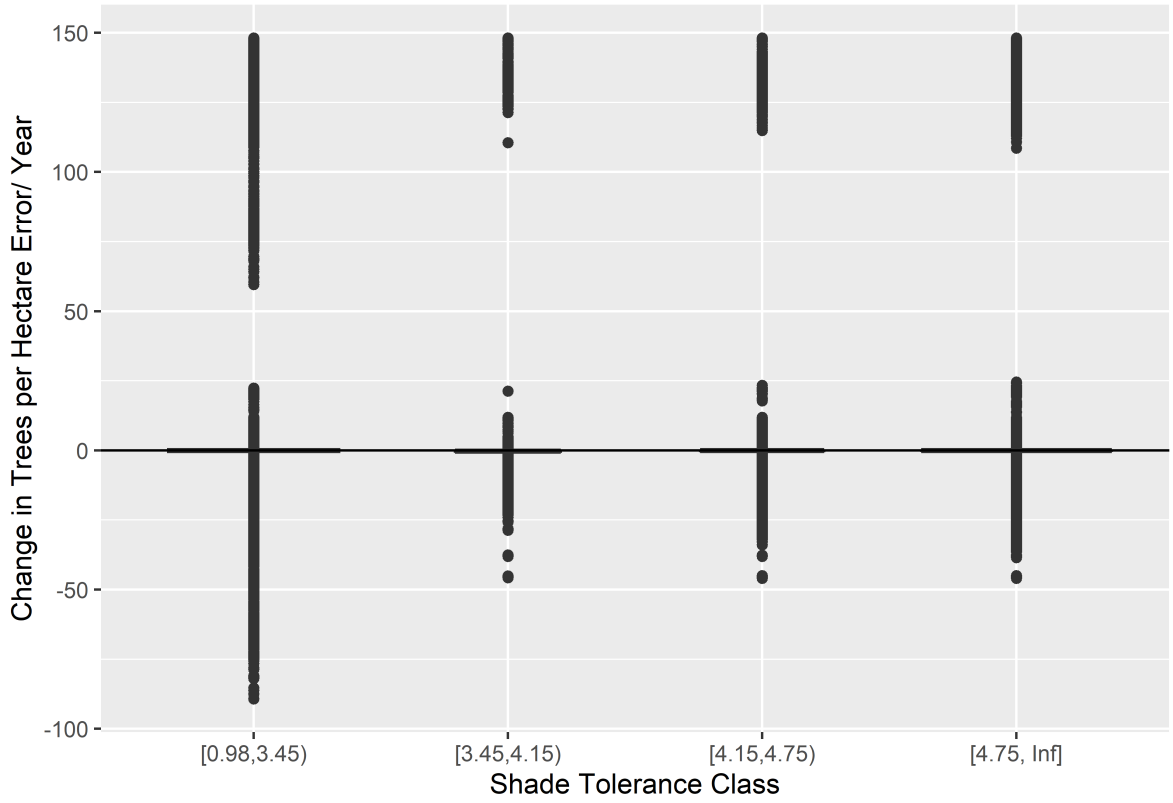


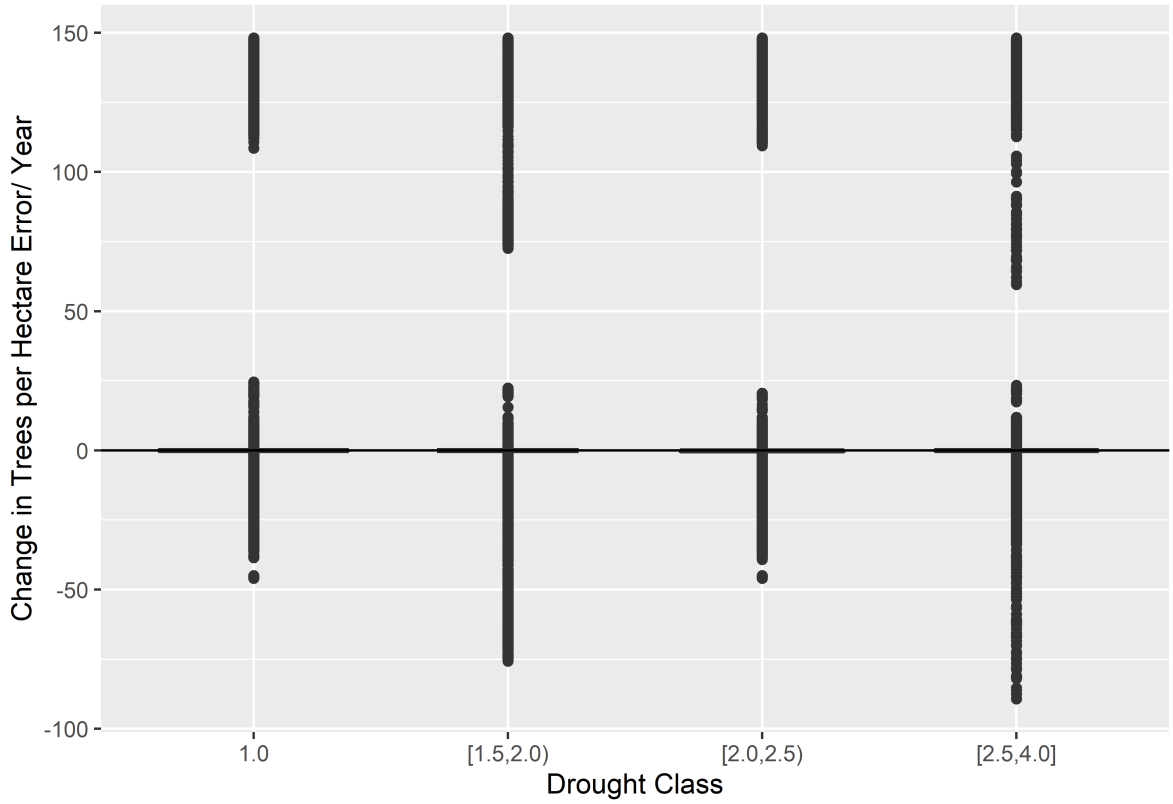


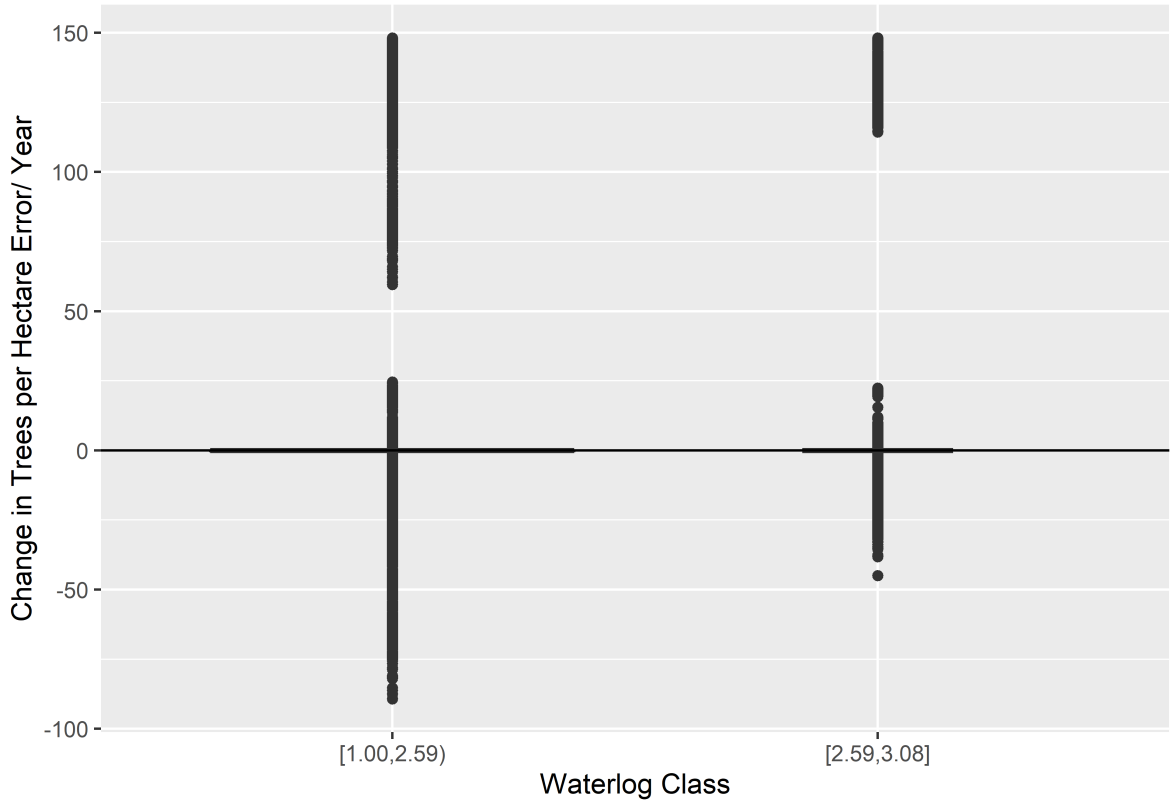

Change in Trees per Hectare Error/ Year
150
100
50
0
-50
-100
[-9999.00, 9.39)
[9.39, 9.86)
[9.86,10.15)
[10.15,10.39)
[10.39,10.59)
[10.59,10.77)
[10.77,10.94)
[10.94,11.15)
[11.15,11.41)
[11.41,11.67)
[11.67,11.94)
[11.94,12.27)
[12.27,12.63)
[12.63,13.03)
[13.03,13.42)
[13.42,13.84)
[13.84,14.49)
[14.49,15.77)
[15.77,16.37)
[16.37,22.9
CSI (m) Class


Change in Trees per Hectare Error/ Year
150
100
50
0
-50
-100
AB
BF
BS
BT
EH
GB
JP
PB
QA
RM
RO
RS
SM
ST
TA
WA
WC
WP
WS
YB
FVS Species


150
100
50
0
-50
-100
Change in Trees per Hectare Error/ Year
[0.29,0.34)
0.34
[0.35,0.40)
[0.40,0.55)
[0.55,0.56]
Wood Density Class

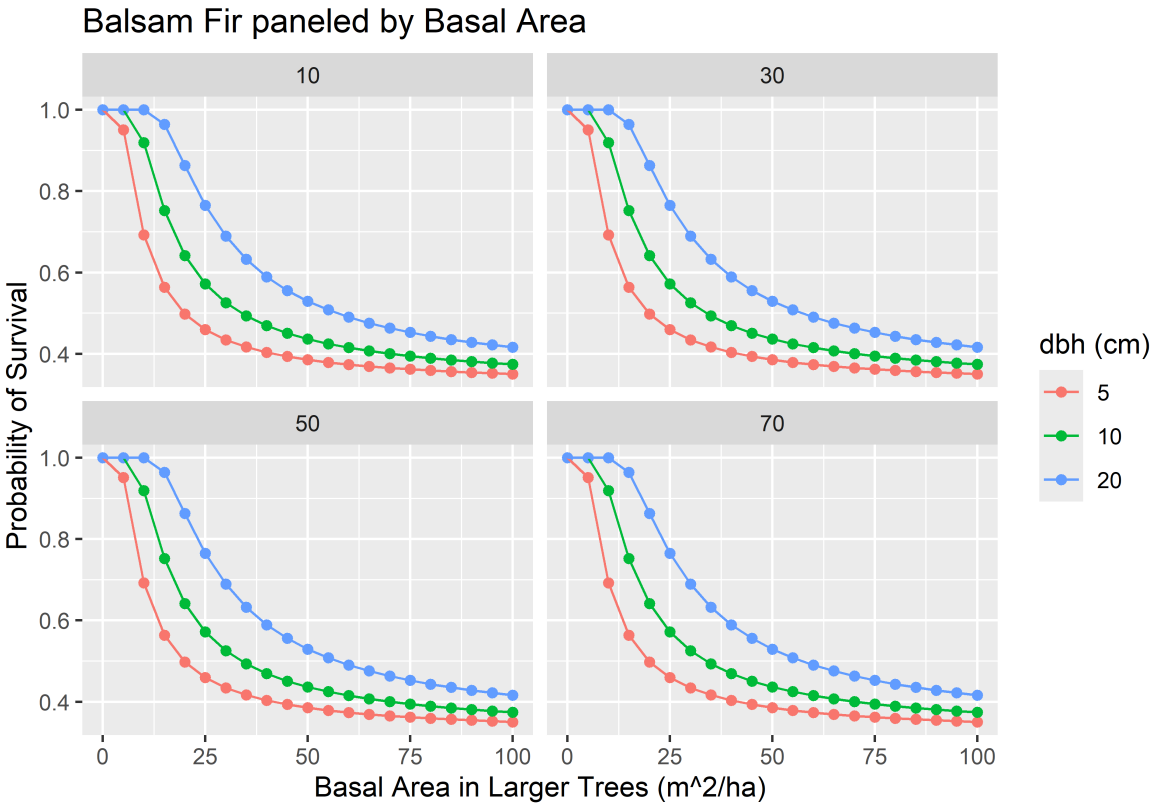

150
100
50
0
-50
-100
Change in Trees per Hectare Error/ Year
[0.98,3.45)
[3.45,4.15)
[4.15,4.75)
[4.75, Inf]
Shade Tolerance Class

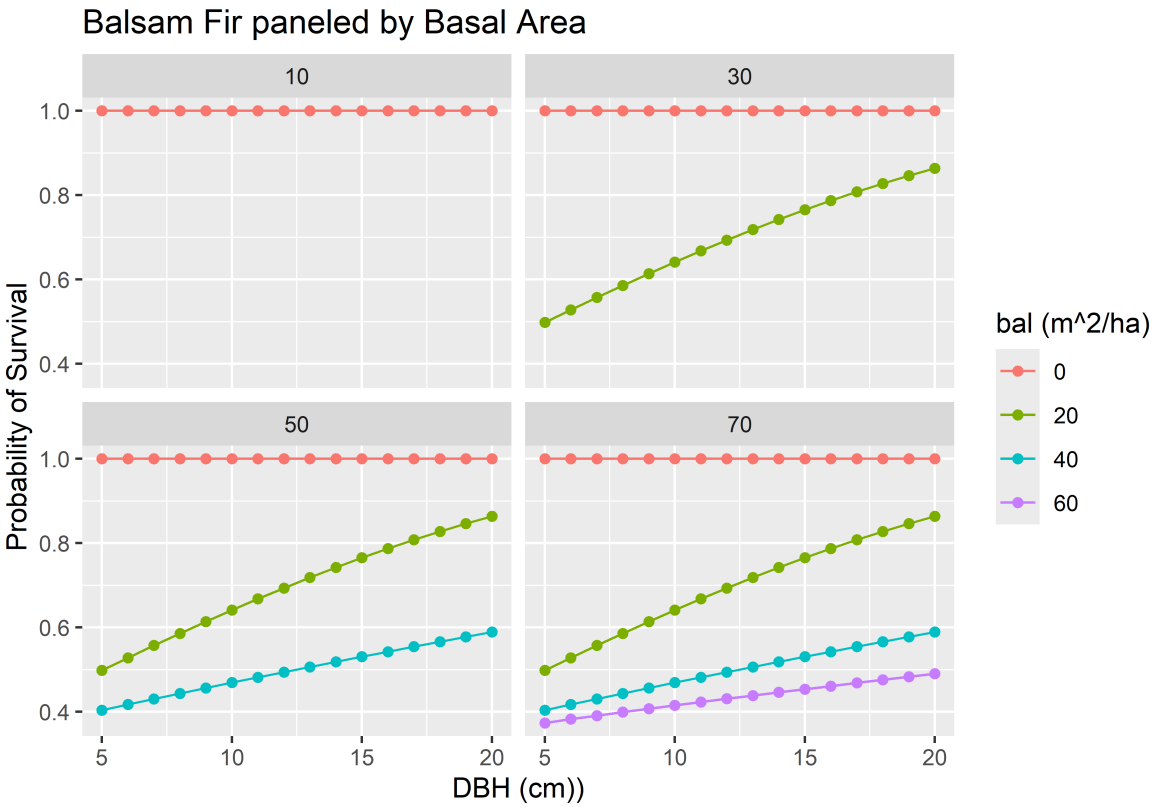

150
100
50
0
-50
-100
Change in Trees per Hectare Error/ Year
1.0
[1.5,2.0)
[2.0,2.5)
[2.5,4.0]
Drought Class


150
100
50
0
-50
-100
Change in Trees per Hectare Error/ Year
[1.00,2.59)
[2.59,3.08]
Waterlog Class

Equation Peformance

The following graph shows survival probability predictions for Balsam Fir to demonstrate the effect of `dbh` and `bal`. The effect of larger trees on survival decreases as trees get larger; a small tree (say 5 cm) with 50 m^2/ha `bal` has a greater chance of dying than a large tree (say 15 cm).




Balsam Fir paneled by Basal Area
10
30
50
70
1.0
0.8
0.6
0.4
Probability of Survival
5
10
15
20
DBH (cm))
bal (m^2/ha)
0
20
40
60