Alternative Balsam Fir Diameter Growth

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Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 10 states listed in the native range of Balsam Fir in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Balsam Fir (FIA species code 12), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Balsam Fir Growth Observations by State

State	Observations
CT	1
MA	40
ME	83061
MI	24161
MN	40793
NH	11573
NY	7428
VT	4172
WI	22292
WV	6

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Balsam Fir site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 27 different site index equations for 30 species. Balsam Fir site index comprises 33% of the observations. There are 1 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7}+\beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, er
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
     AIC      BIC    logLik
  105104 105185.4 -52543.99

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3  Residual
StdDev: 0.01097501 0.3174408

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value   Std.Error     DF    t-value p-value
B0 -3.494068 0.029842193 193520 -117.08482       0
B1 -0.414431 0.003233631 193520 -128.16264       0
B2 -0.068861 0.005162066 193520  -13.33987       0
B3  0.088204 0.010242857 193520    8.61125       0
B4  2.356021 0.014610190 193520  161.25874       0
B5  0.609137 0.013398577 193520   45.46285       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.243
B2 -0.402  0.528
B3 -0.523 -0.008 -0.038
B4 -0.405  0.774  0.655 -0.089
B5 -0.377  0.499  0.994 -0.031  0.599

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-31.9347814  -0.5754987  -0.1648738   0.4248148  24.6460943

Number of Observations: 193527
Number of Groups: 2

$SIINT
```

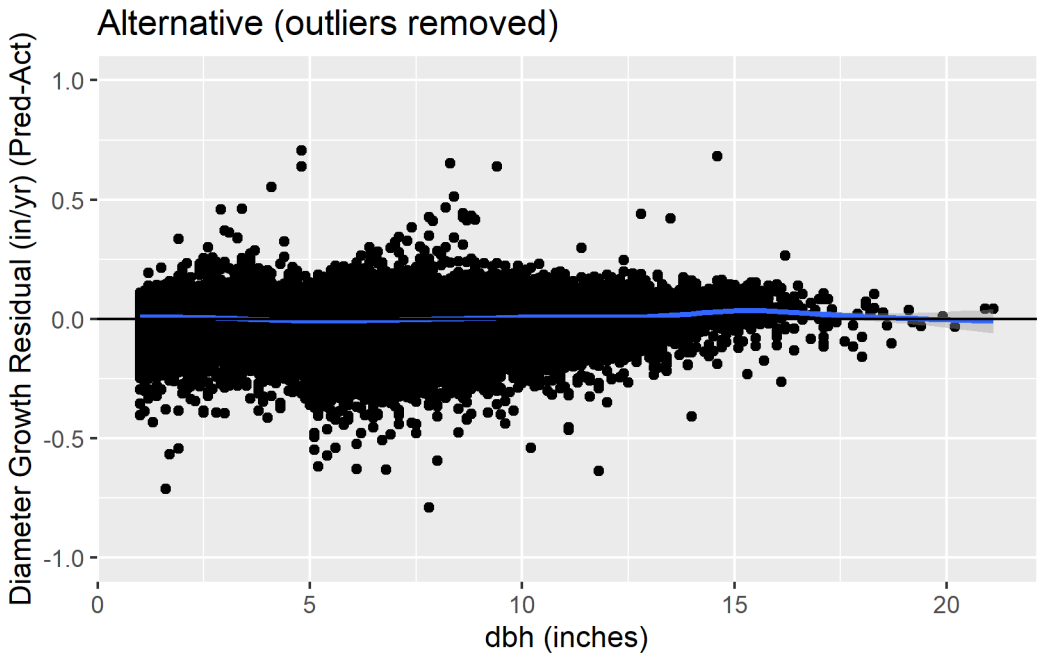
```
                  B3
50.FALSE -0.01096438
50.TRUE   0.01096438

Residual Standard Error: 0.317440841300731 on 193520 degrees of freedom, AIC: 105104
```

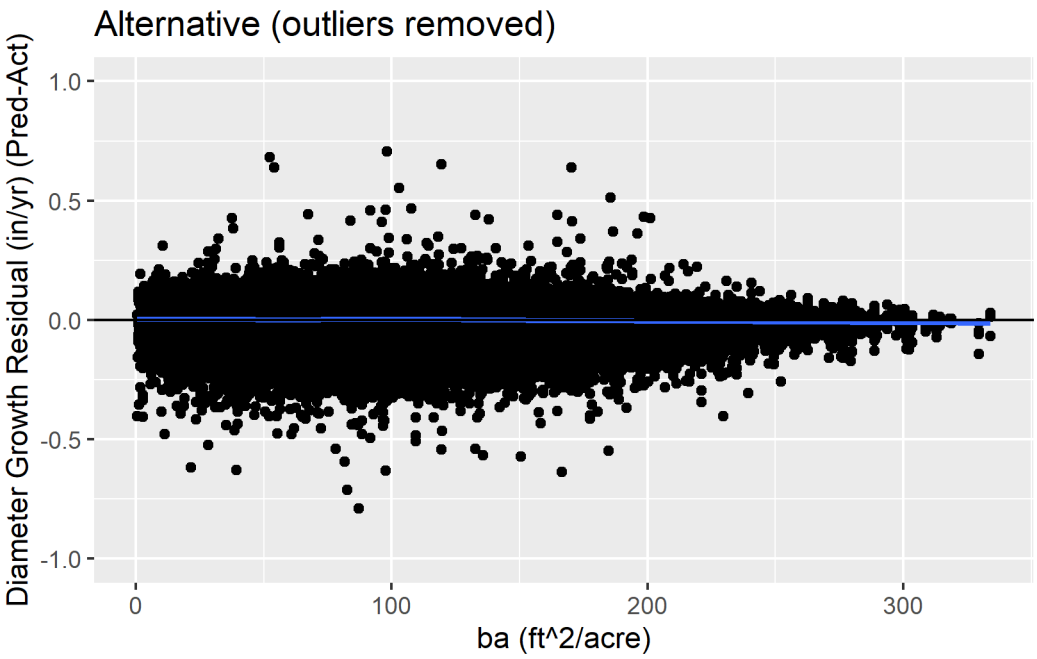
and for Equation 2:

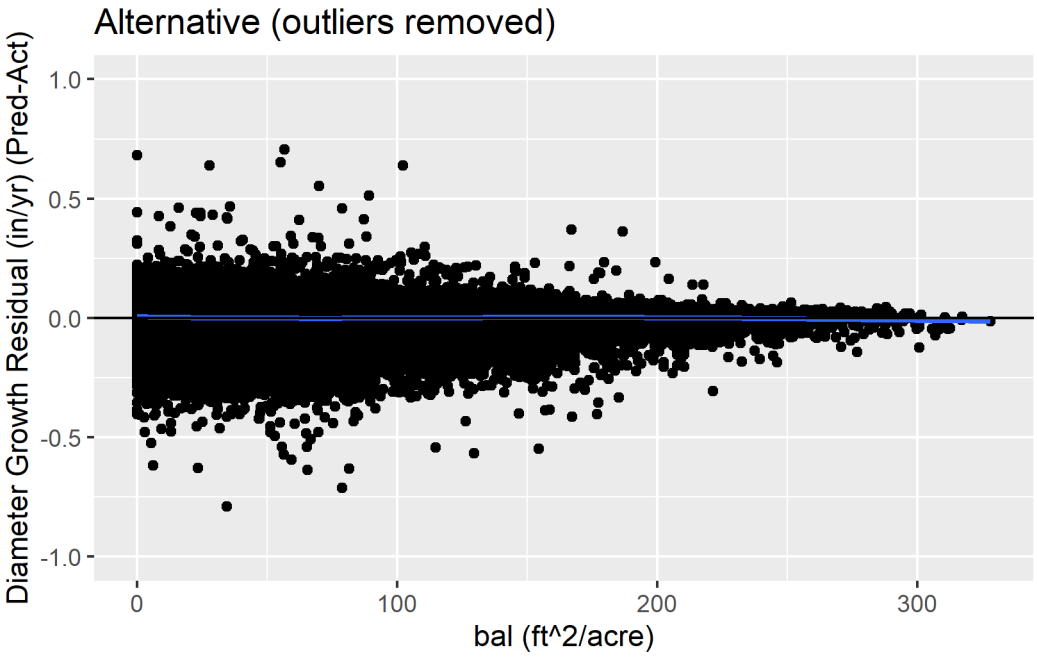
	Coef.	Std. error	t-stat.	p
B0	-3.1635990	0.0176928	-178.80727	0
B1	-0.4015284	0.0031861	-126.02518	0
B2	-0.0650773	0.0048980	-13.28642	0
B4	2.3993930	0.0151128	158.76598	0
B5	0.6206128	0.0134995	45.97285	0

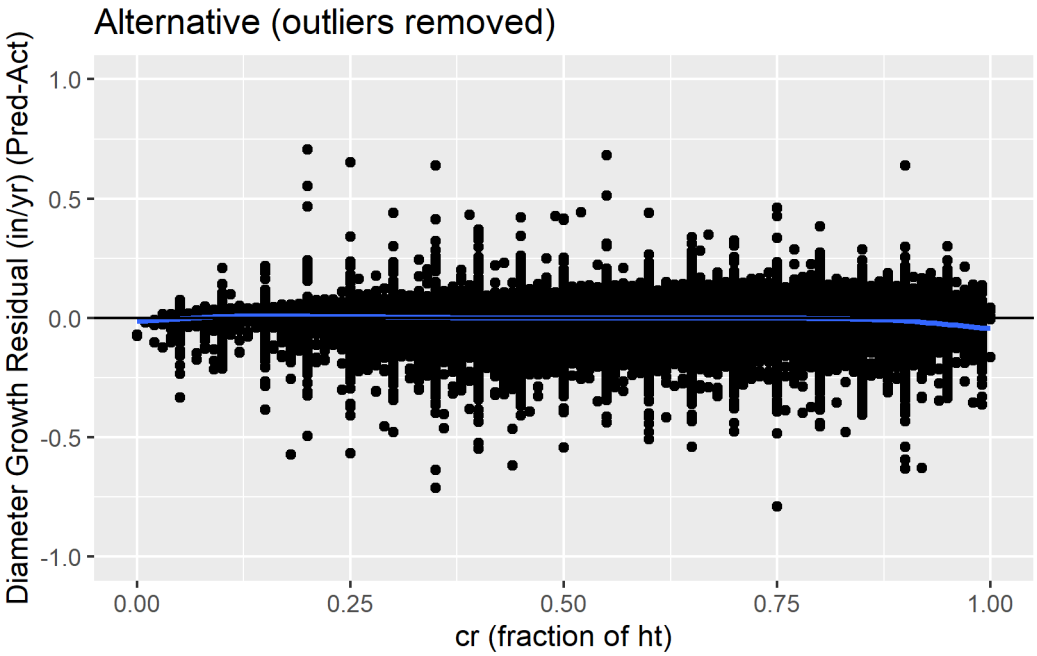
```
Residual Standard Error: 0.318189813937393 on 193522 degrees of freedom, AIC: 105994.5
```

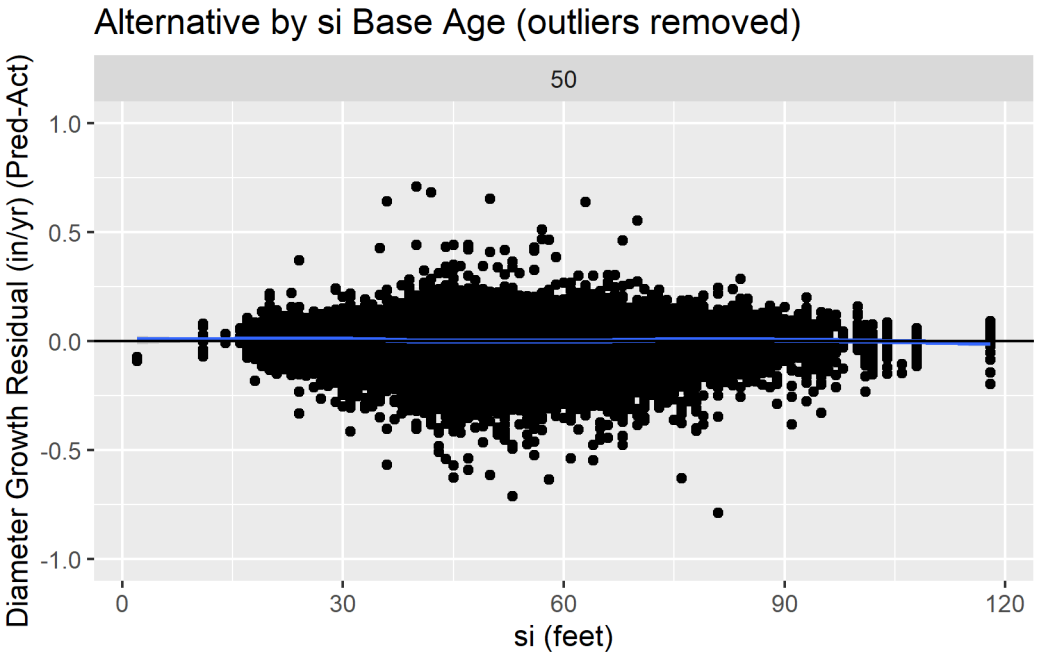
Residual Analysis for Equation 1

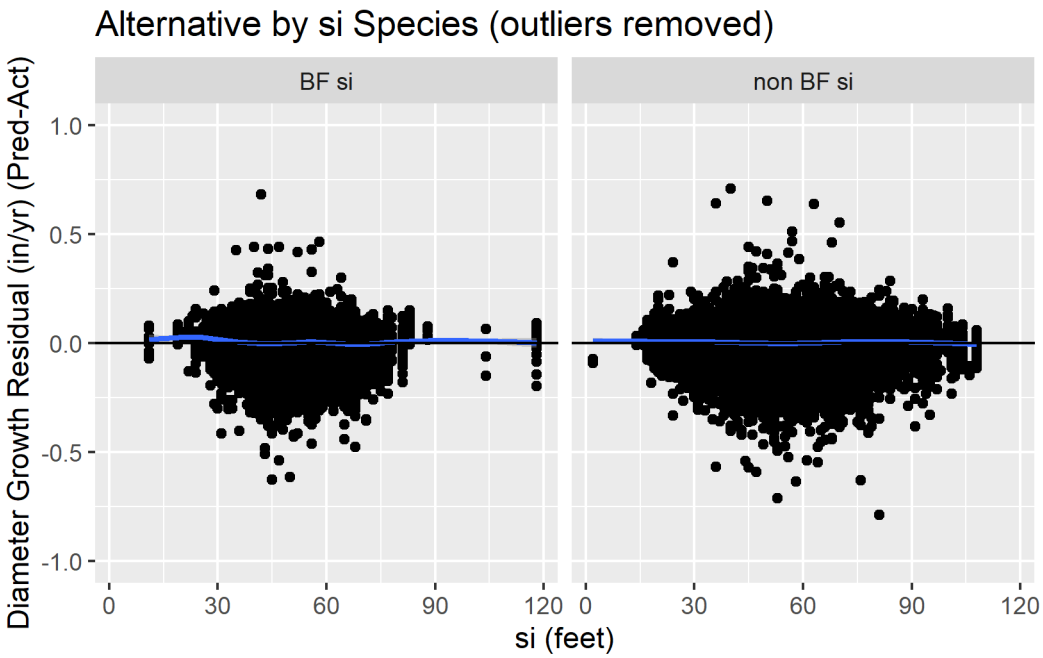


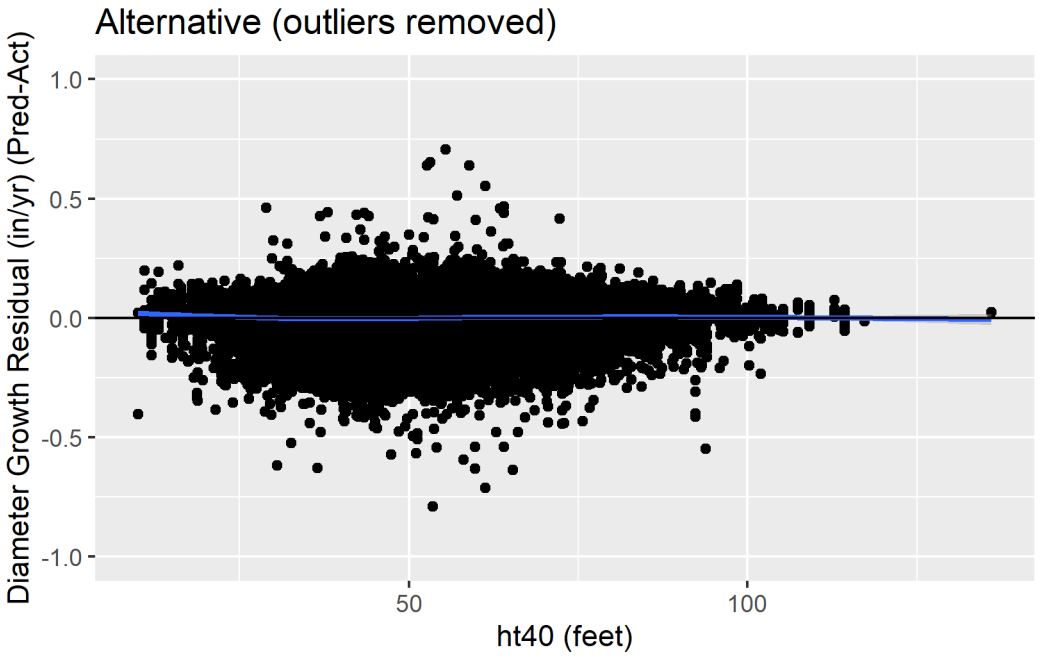

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
ba (ft^2/acre)

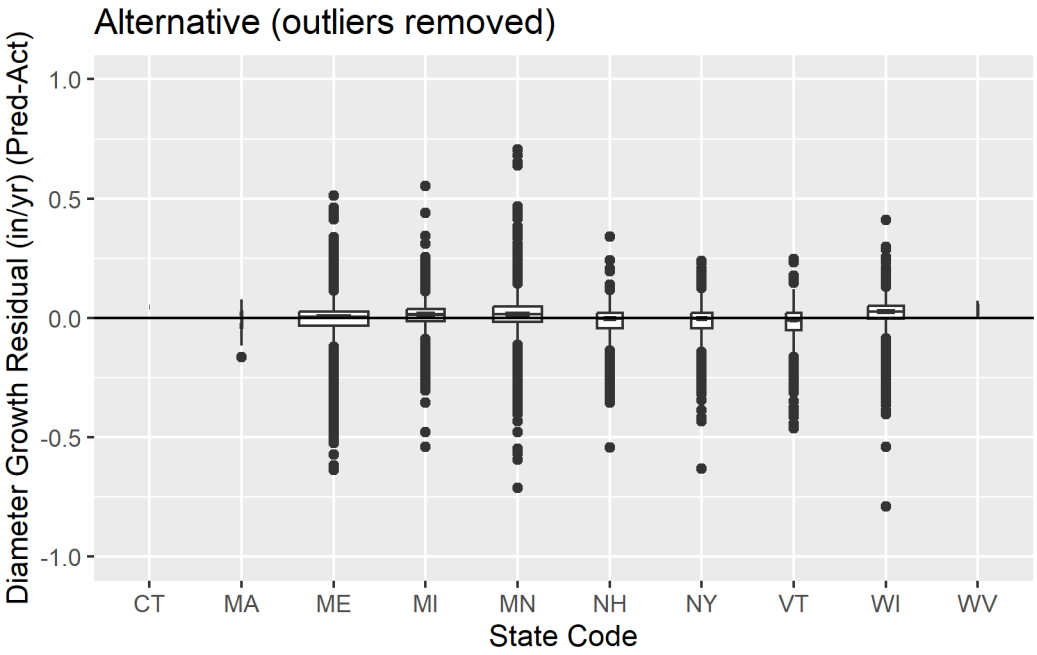

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
bal (ft^2/acre)

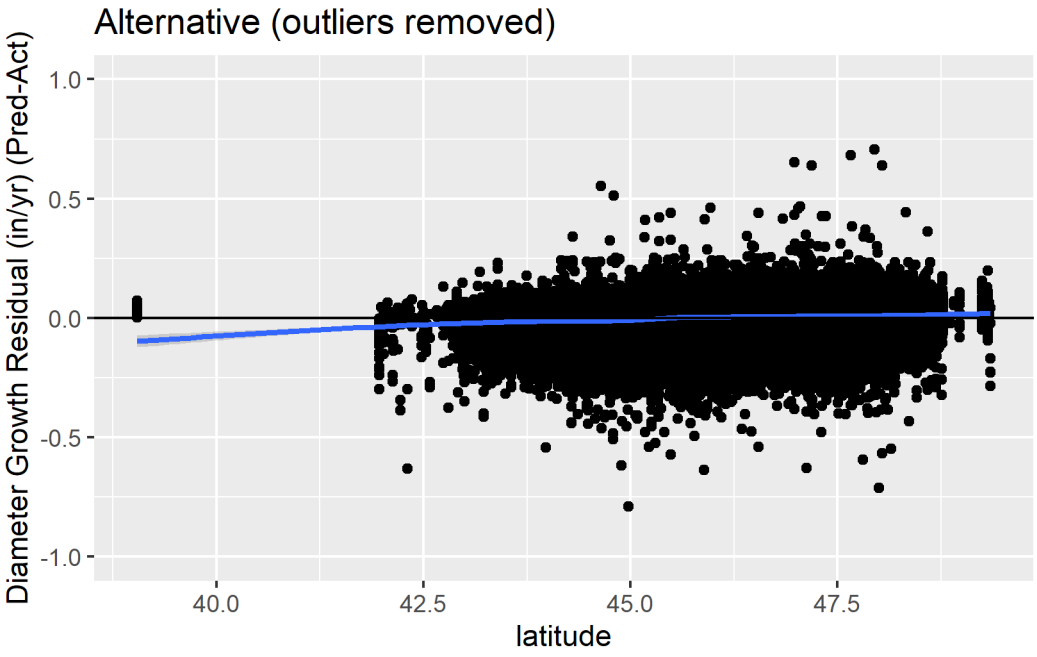

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

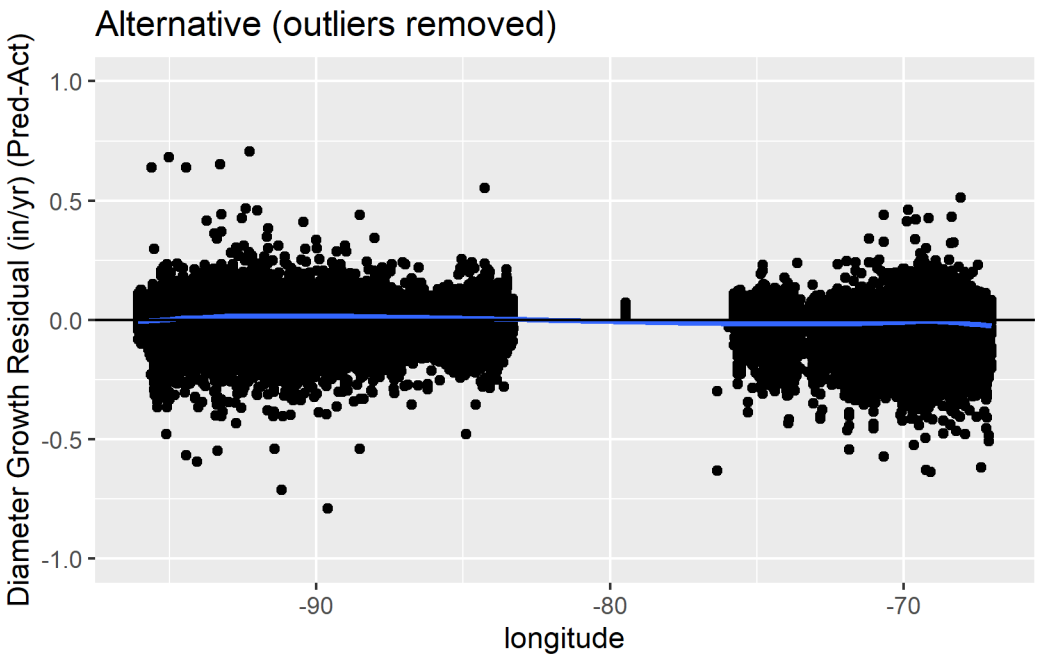

Alternative by si Base Age (outliers removed)
50
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

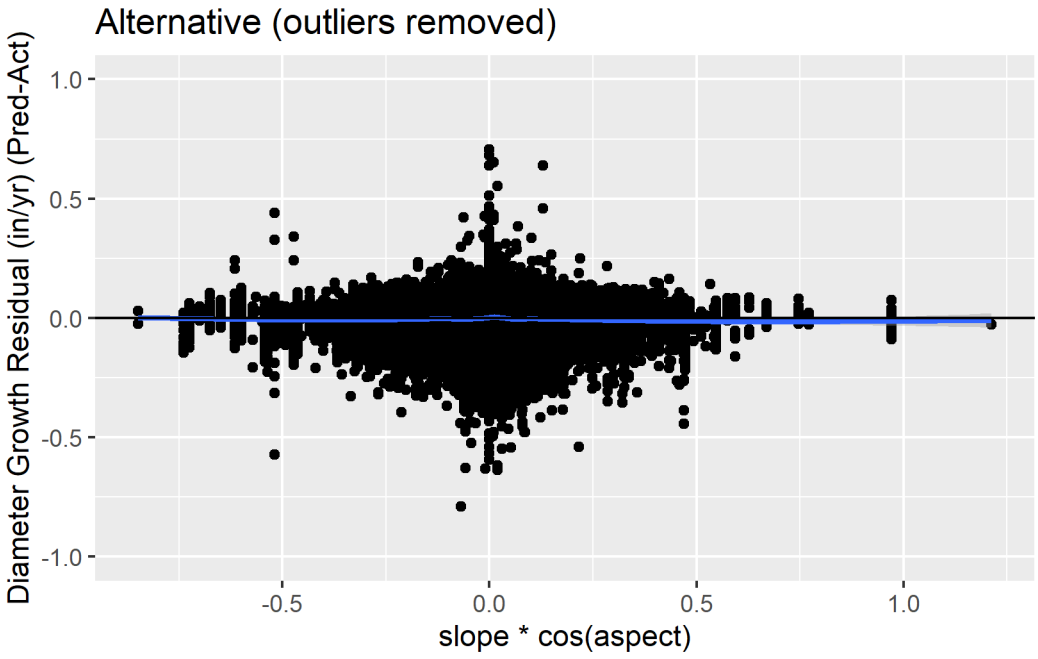

Alternative by si Species (outliers removed)
BF si
non BF si
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

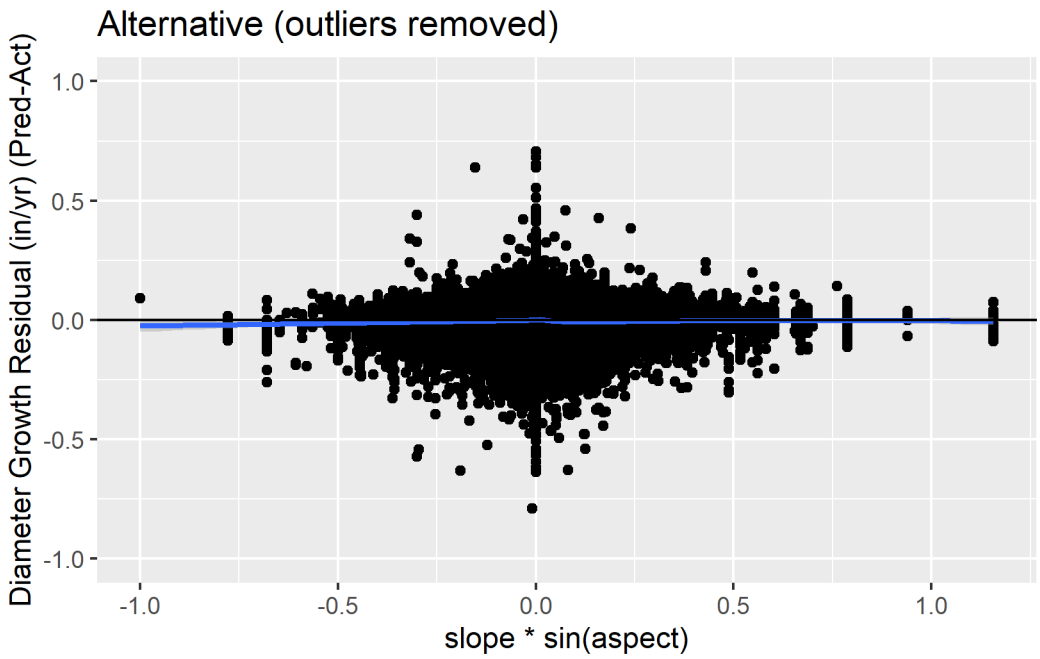

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
50
100
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
CT
MA
ME
MI
MN
NH
NY
VT
WI
WV
State Code

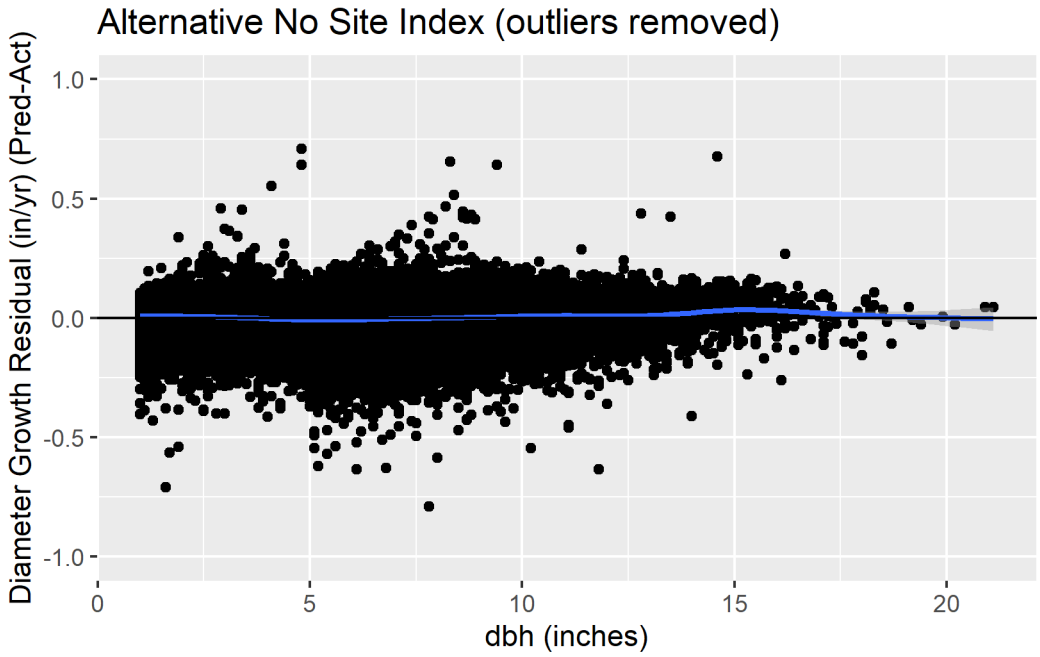

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
40.0
42.5
45.0
47.5
latitude

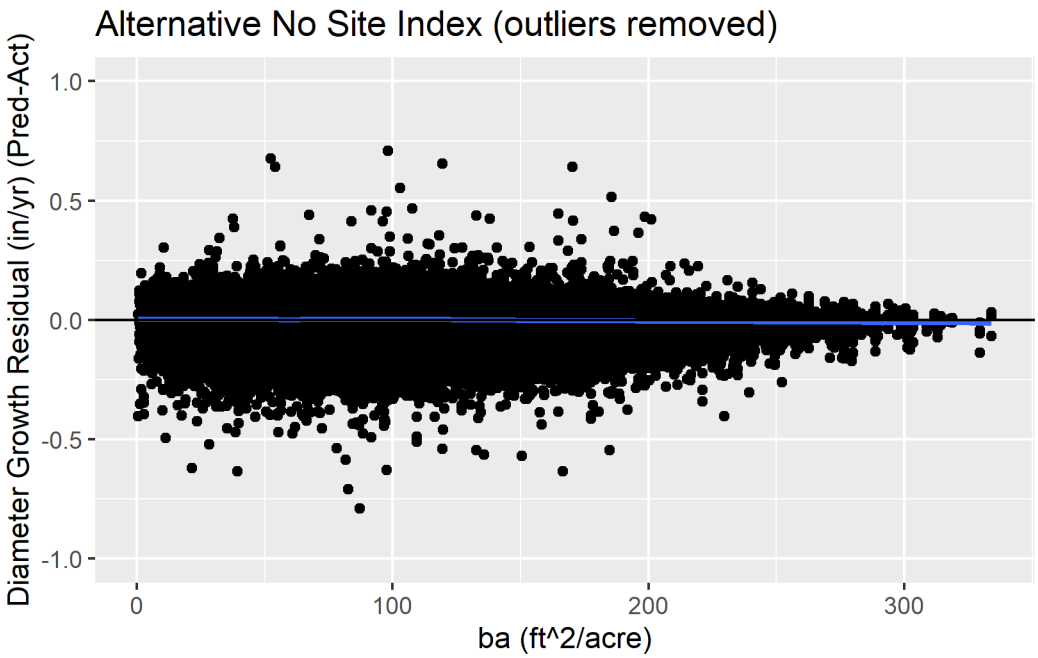

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-90
-80
-70
longitude

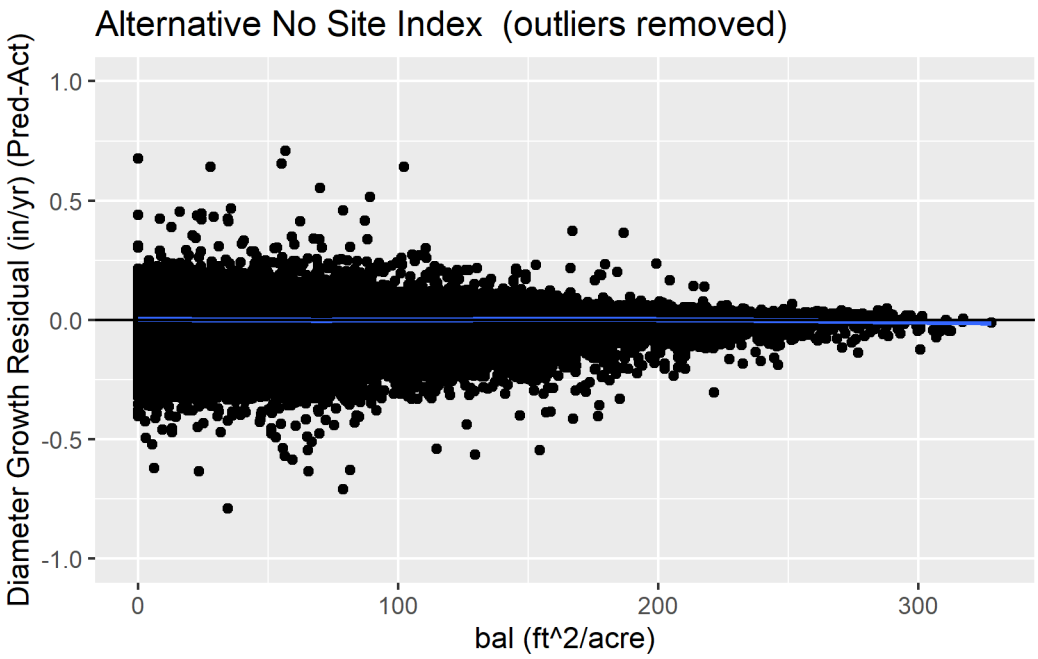

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)

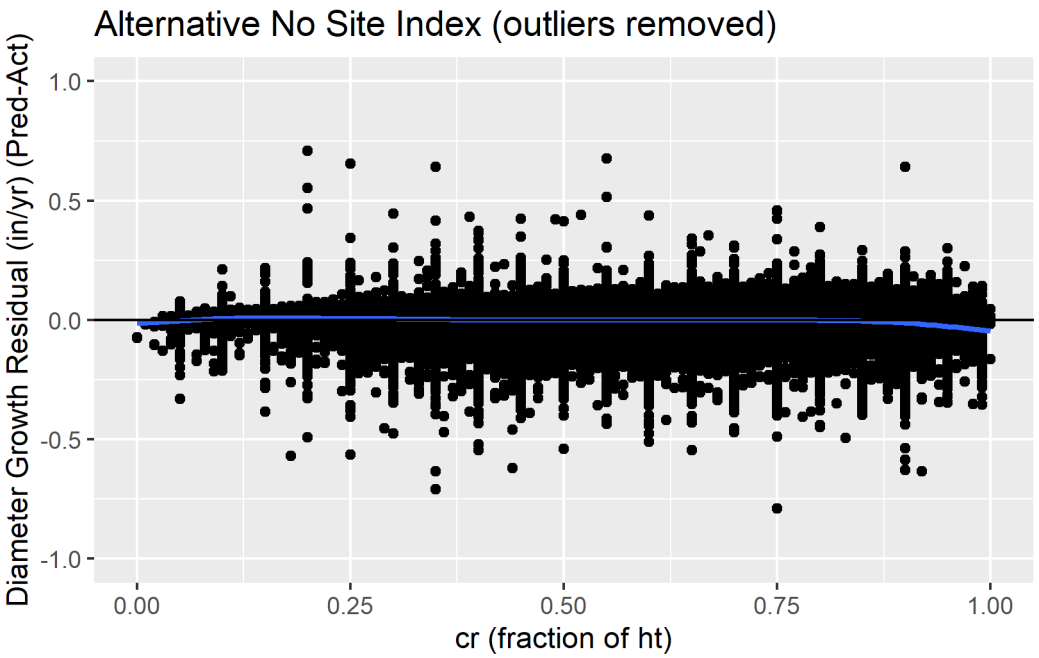


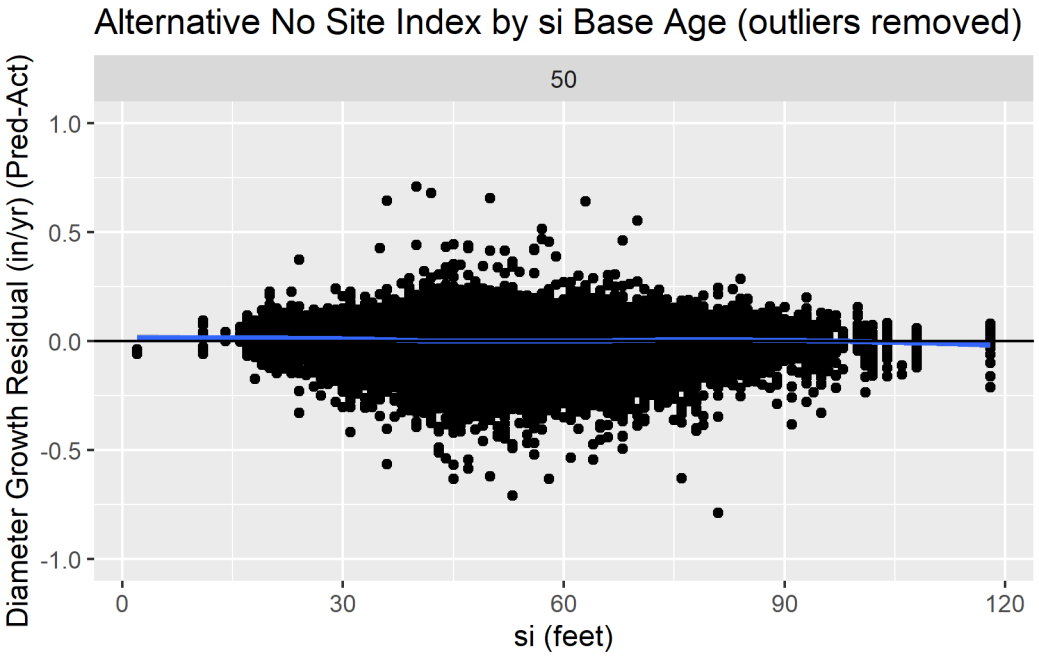
Residual Analysis for Equation 2

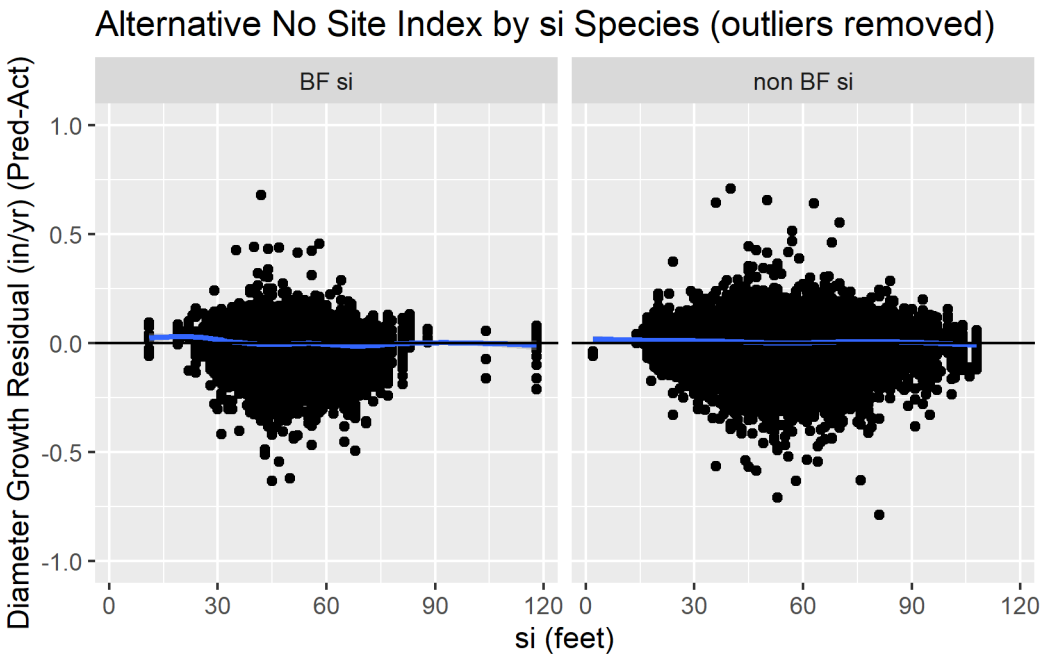


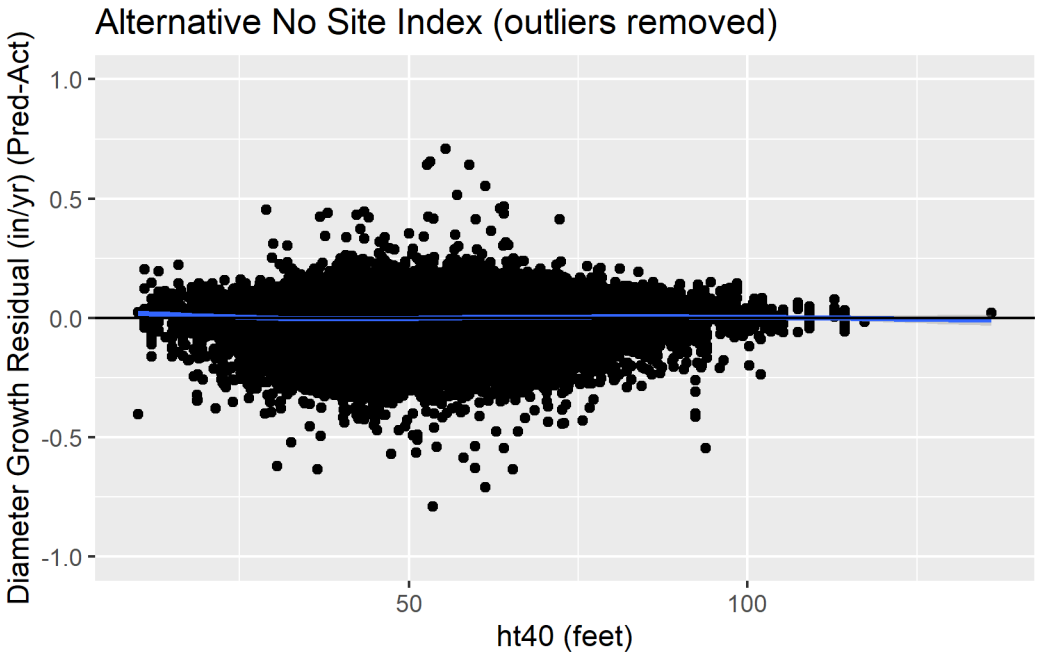

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
ba (ft^2/acre)

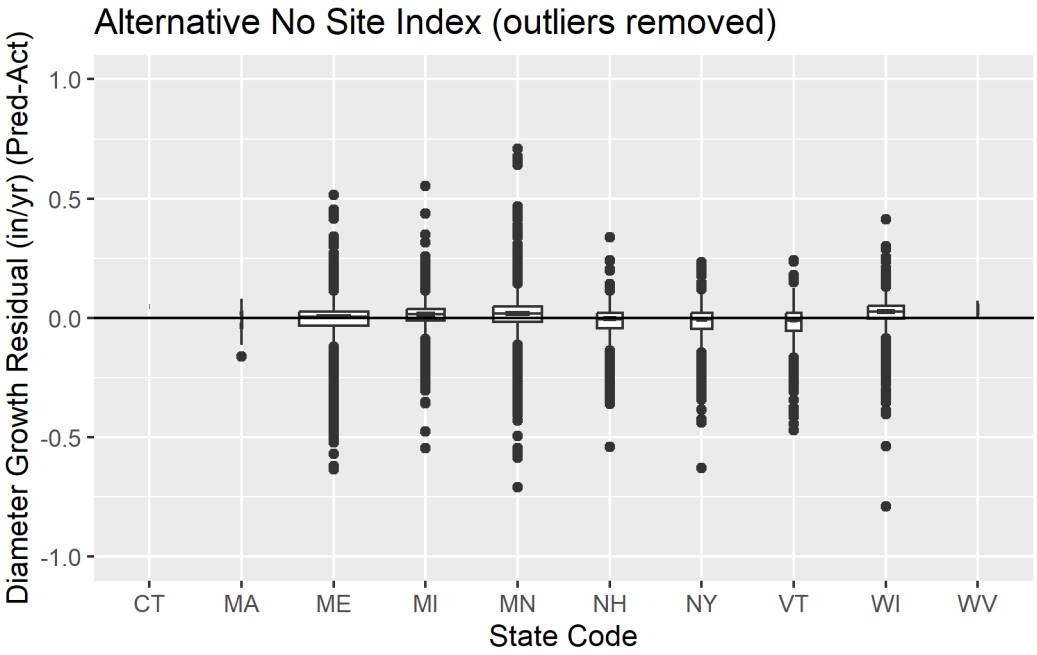

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
bal (ft^2/acre)

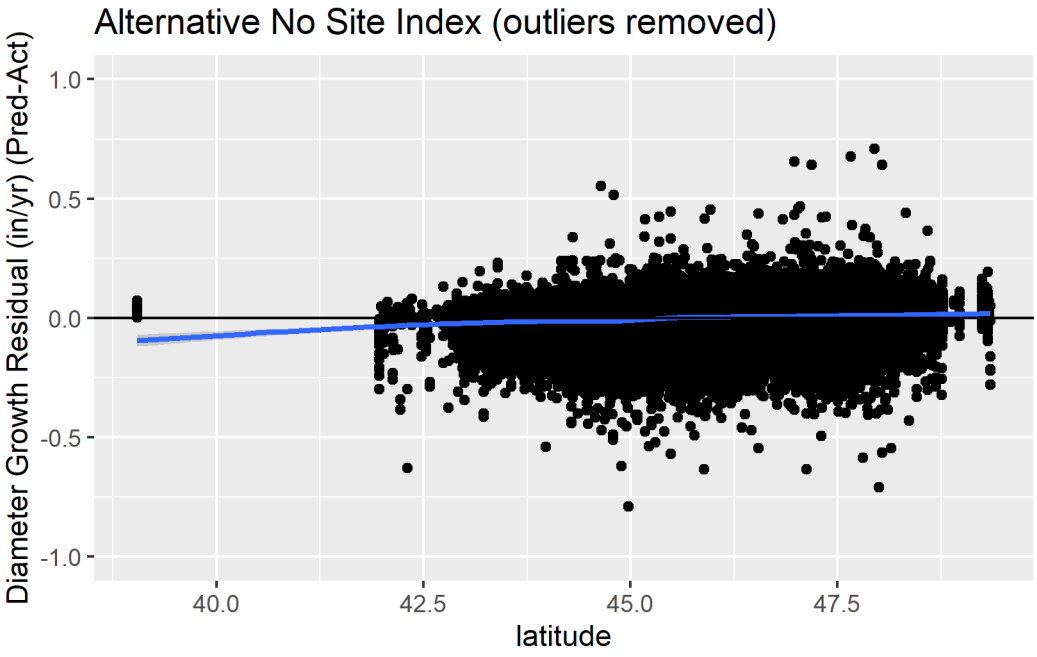

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

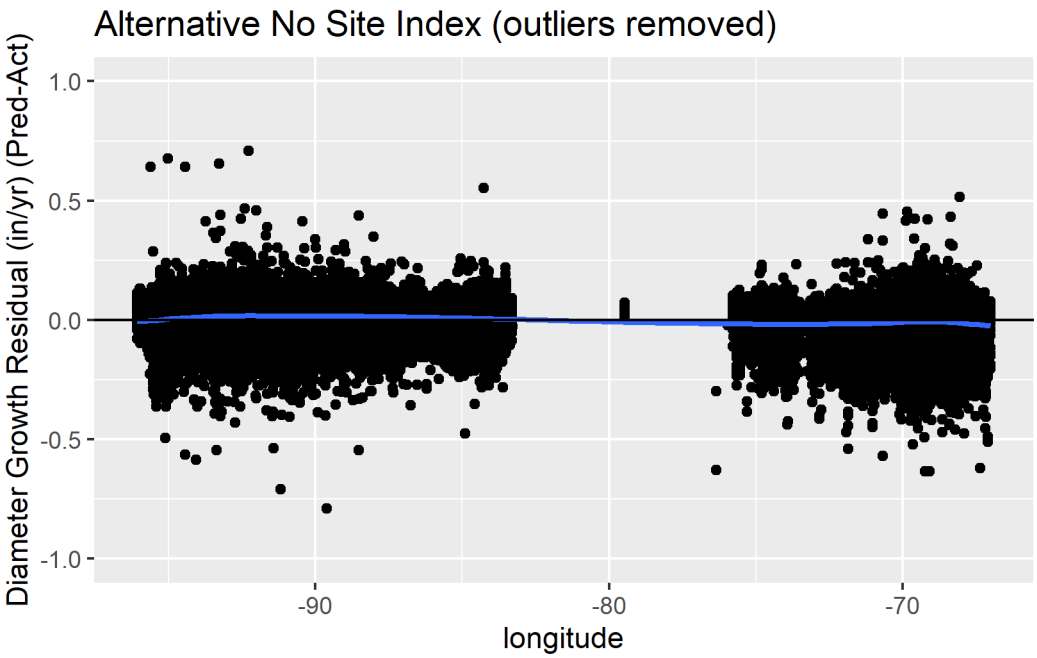

Alternative No Site Index by si Base Age (outliers removed)
50
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

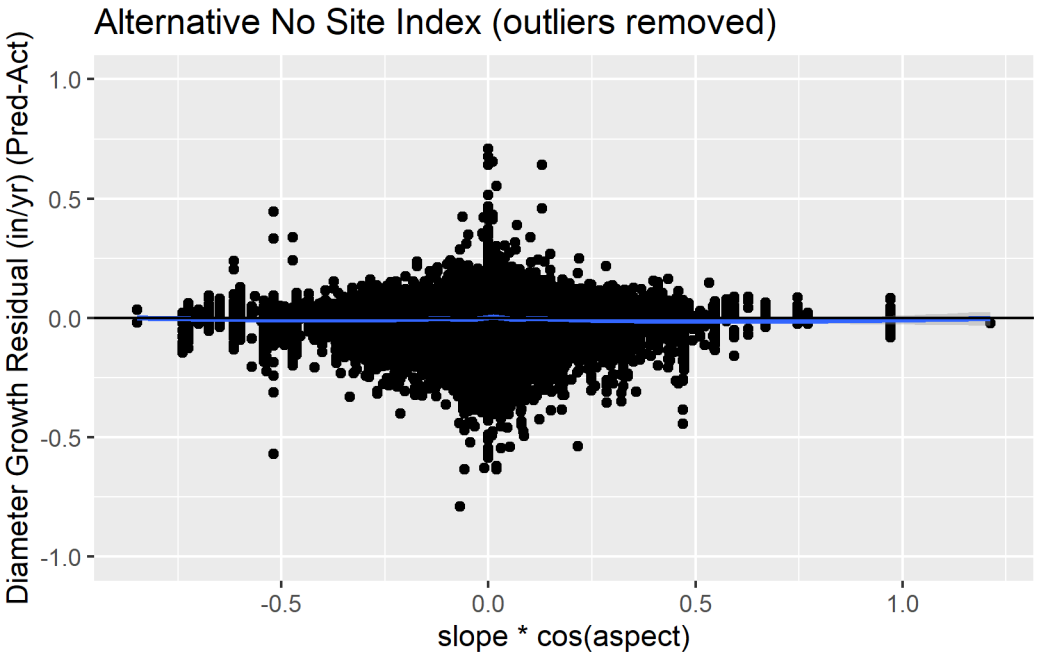

Alternative No Site Index by si Species (outliers removed)
BF si
non BF si
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

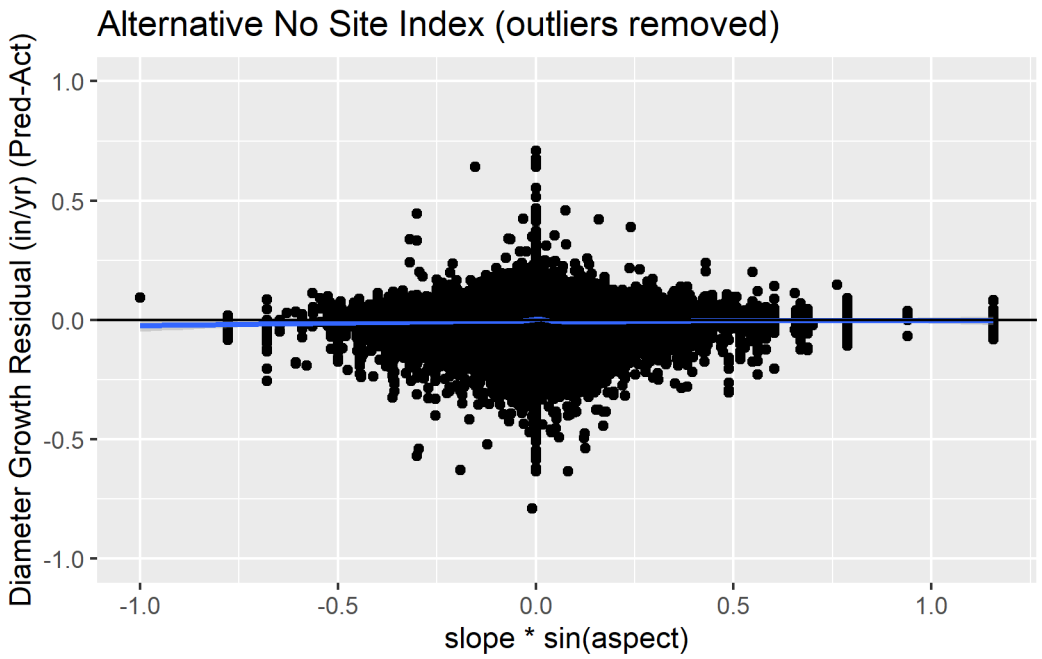

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
50
100
ht40 (feet)

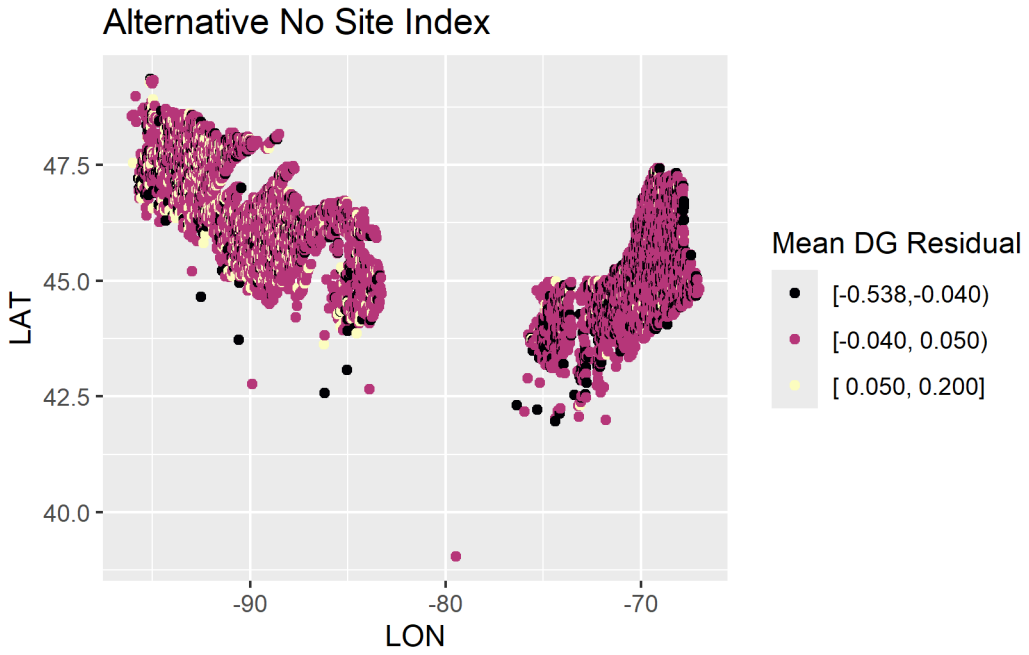

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
CT
MA
ME
MI
MN
NH
NY
VT
WI
WV
State Code


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
40.0
42.5
45.0
47.5
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-90
-80
-70
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)





Discussion

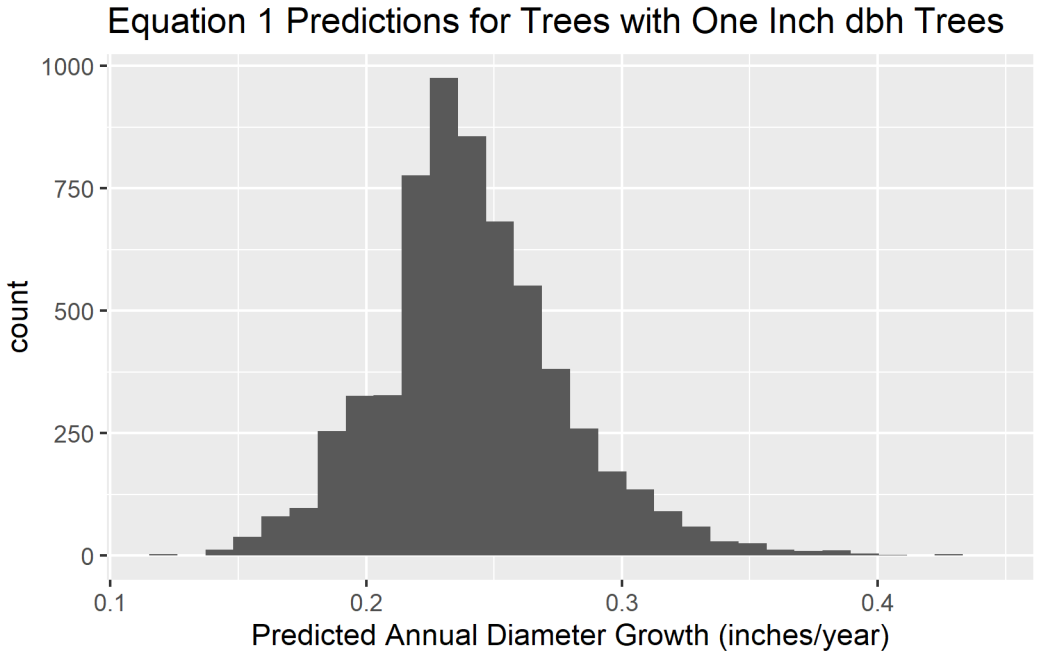
Removing `si` does not degrade the fit significantly. Only MN appears to be over-predicted.

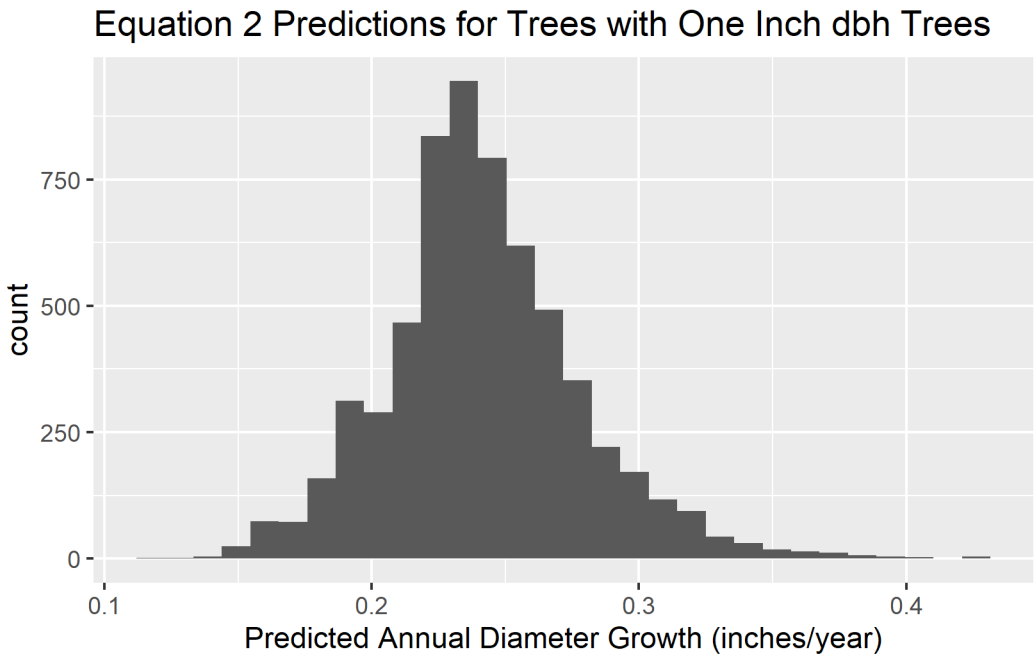
Table 3: Independent Variables for One Inch dbh Trees

Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	6185	111	49	0.41	76	145	313
bal	6185	110	49	0	74	144	313
ht	6185	9.9	0.91	5	10	10	25
cr	6185	0.48	0.22	0.03	0.3	0.65	1
si	6185	51	13	17	44	59	108

Equation 1 residual graphs show that there is some indication of residual trends spatially (especially in the western states).

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
count
750
500
250
0
0.1
0.2
0.3
0.4
Predicted Annual Diameter Growth (inches/year)