Alternative Douglas-fir Diameter Growth

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Alternative Model Formulation

An alternative to the ORGANON diameter growth equation which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Douglas-fir site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 26 different site index equations for 22 species. Douglas-fir site index comprises 88% of the observations. There are 2 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one wheere `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7}+\beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg_test(B0, B1, B2, B3, B4, B5, startDIA, startBAL,      endBAL, start
  Data: df_tree_subset_with_si %>% mutate(SIINT = interaction(as.factor(df_tree_subset_with_s
     AIC      BIC  logLik
  336942 337019.3 -168463

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3 Residual
StdDev: 0.05242354 1.025054

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value  Std.Error     DF    t-value p-value
B0 -5.772704 0.05143374 116670 -112.23573       0
B1 -0.573883 0.00338476 116670 -169.54921       0
B2 -0.096213 0.00566125 116670  -16.99508       0
B3  0.846555 0.02801245 116670   30.22067       0
B4  1.726687 0.00887363 116670  194.58631       0
B5  0.637447 0.01052555 116670   60.56186       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.016
B2 -0.259  0.431
B3 -0.315  0.006  0.000
B4 -0.424  0.039  0.452 -0.004
B5 -0.235  0.421  0.992  0.000  0.404

Standardized Within-Group Residuals:
```

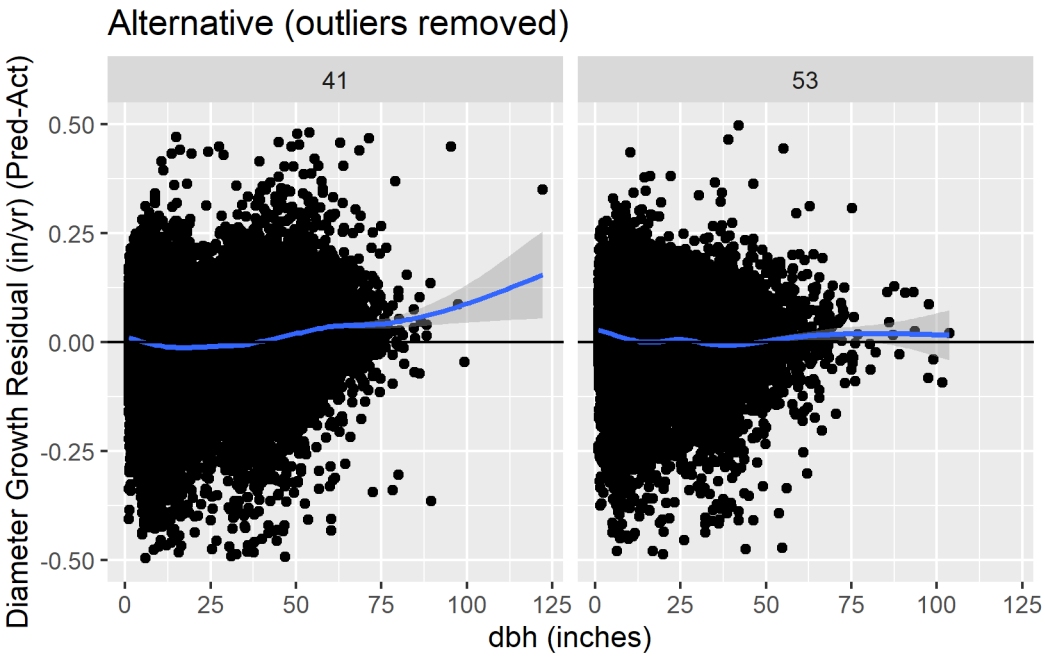
```
         Min           Q1          Med           Q3          Max
-88.51098597  -0.47577890  -0.08630852   0.39896348  19.26745893

Number of Observations: 116679
Number of Groups: 4


$SIINT
                   B3
50.FALSE    0.05796447
100.FALSE -0.02601956
50.TRUE     0.04076369
100.TRUE   -0.07270860

Residual Standard Error: 1.0250542014988 on 116670 degrees of freedom, AIC: 336941.960854266
```

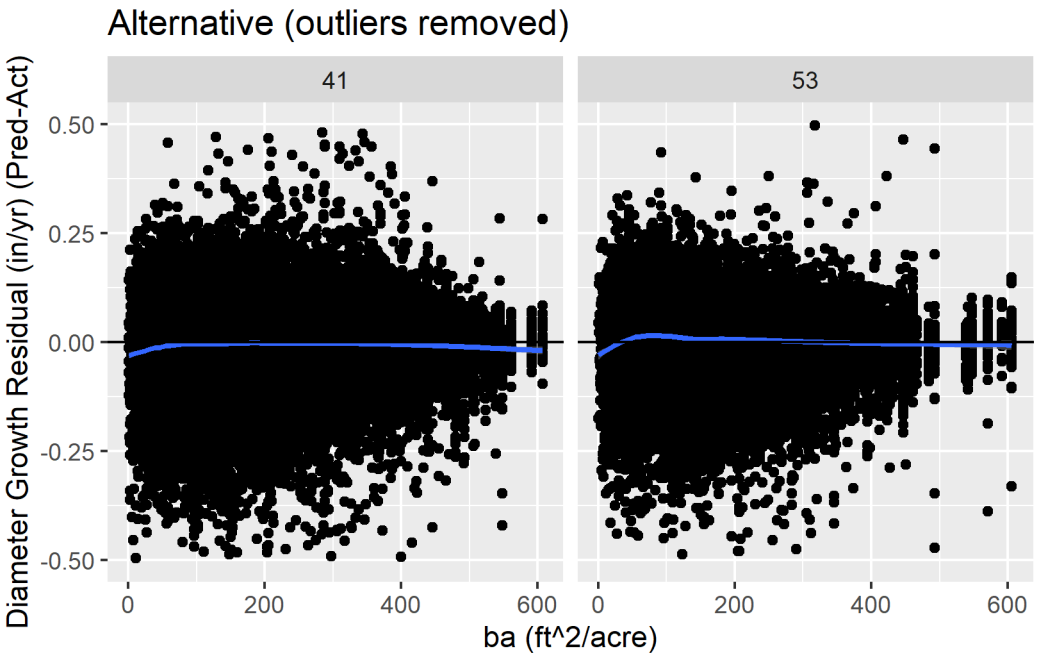
and for Equation 2:

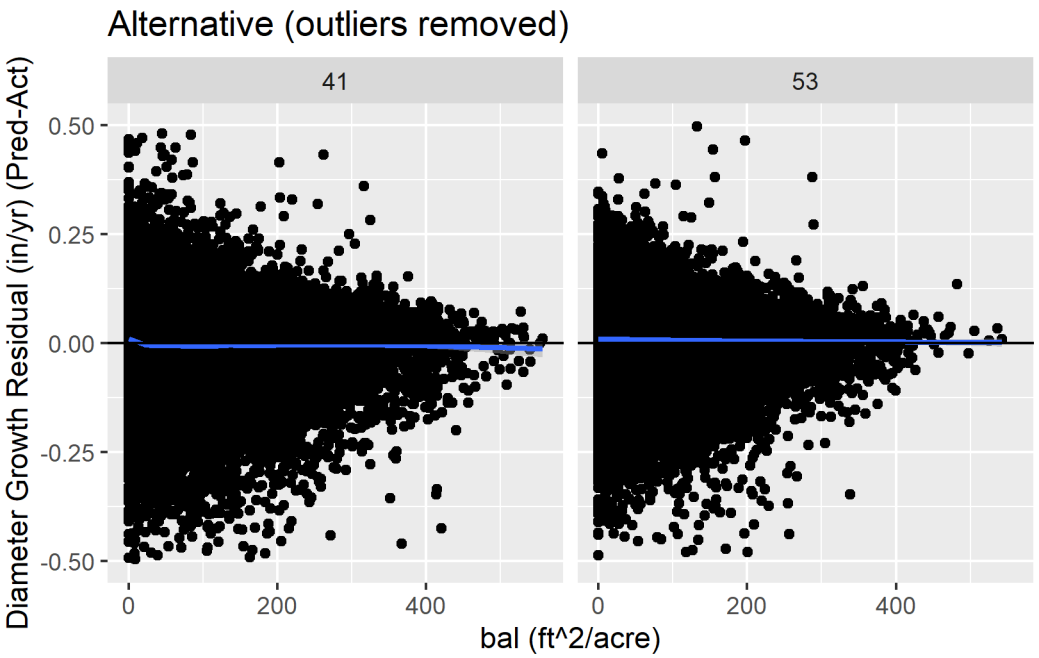
	Coef.	Std. error	t-stat.	p
B0	-2.1157674	0.0269723	-78.44222	0
B1	-0.6394330	0.0038890	-164.42087	0
B2	-0.1324818	0.0088924	-14.89825	0
B4	1.8435476	0.0092577	199.13726	0
B5	0.5546891	0.0117478	47.21659	0

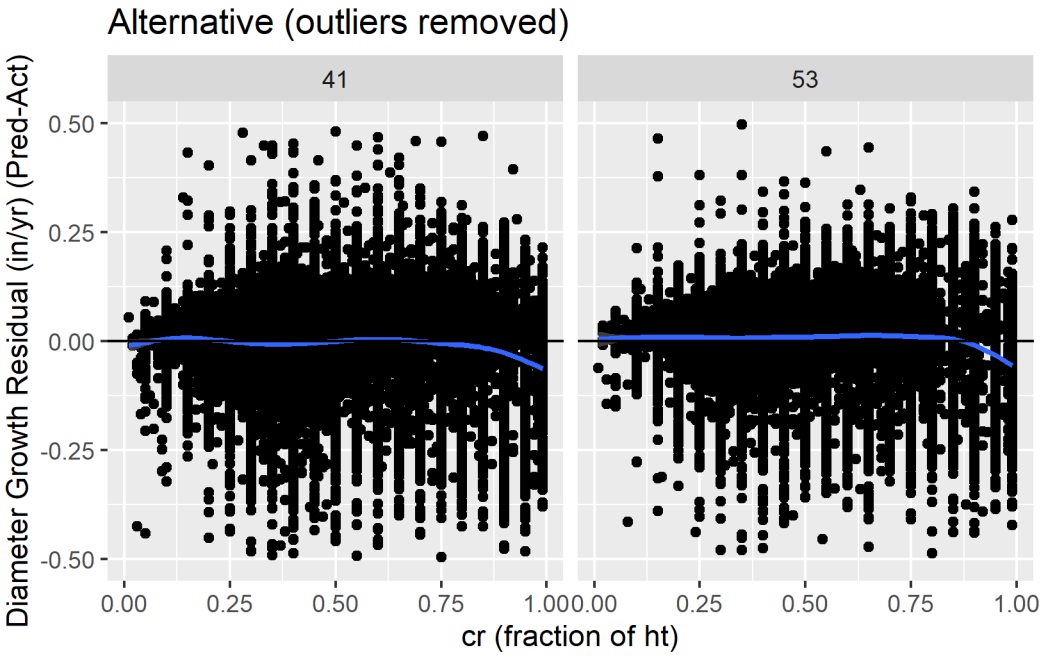
```
Residual Standard Error: 1.11199229005758 on 116674 degrees of freedom, AIC: 355899.37113225
```

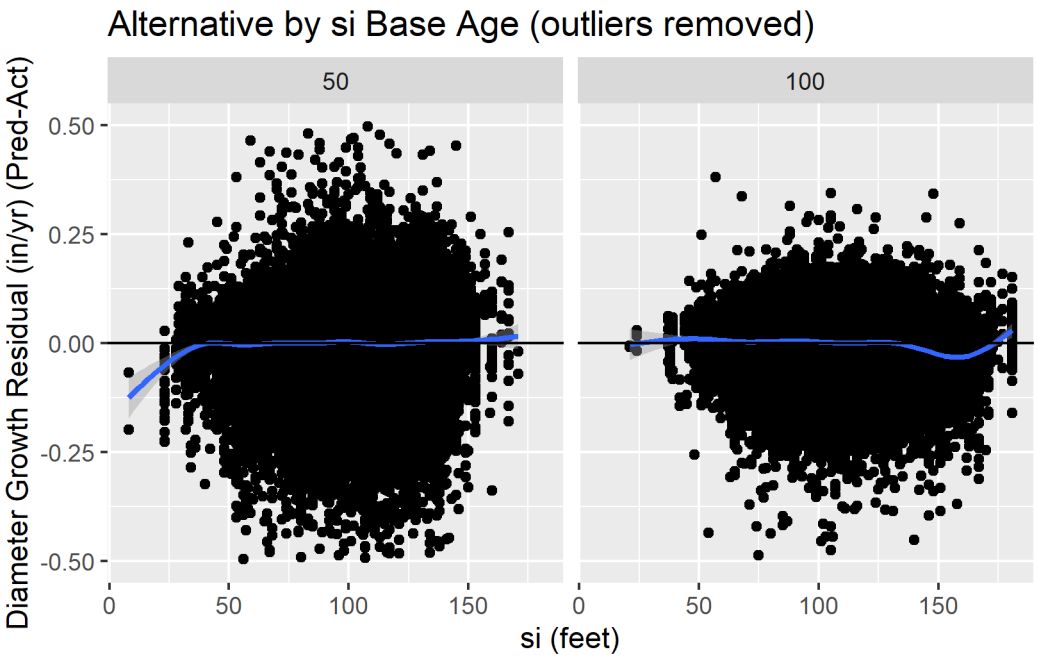
Residual Analysis for Equation 1

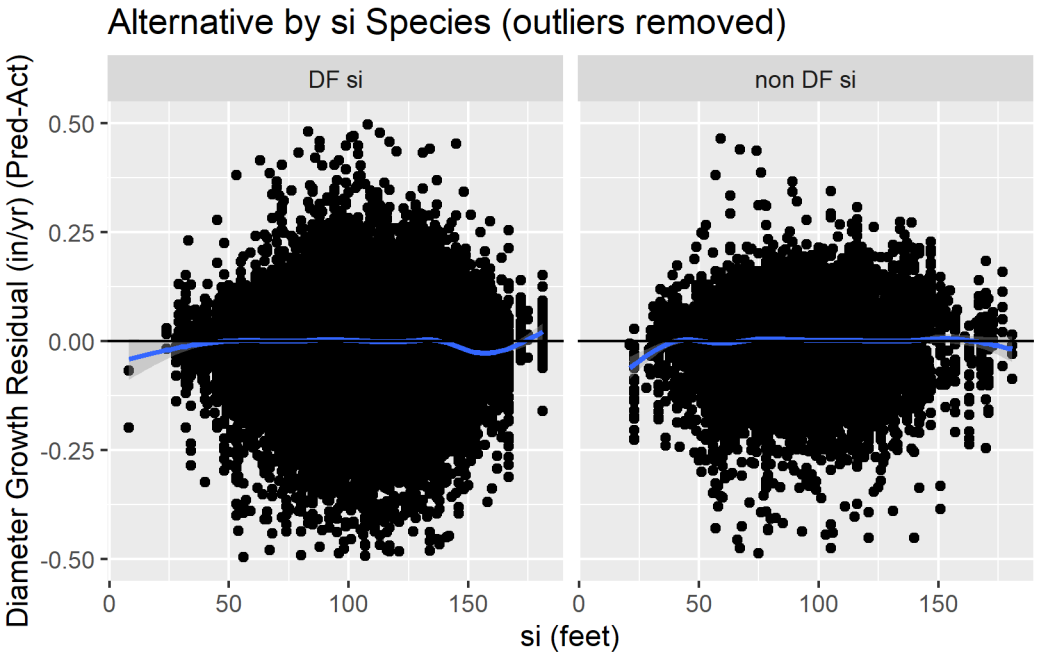


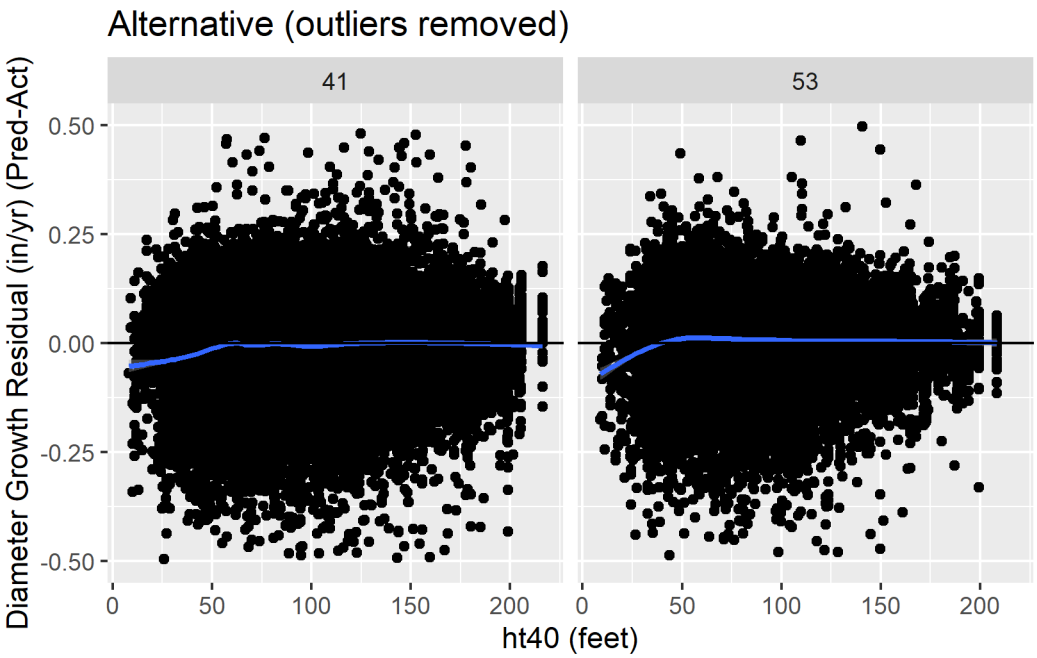


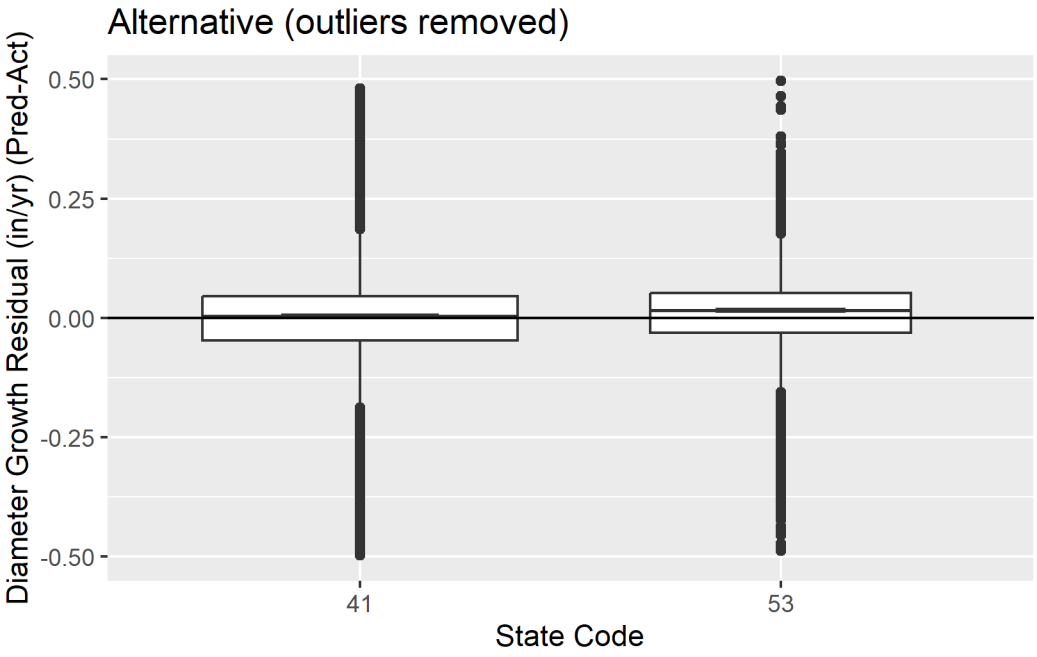

Alternative (outliers removed)
41
53
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
bal (ft^2/acre)


Alternative (outliers removed)
41
53
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

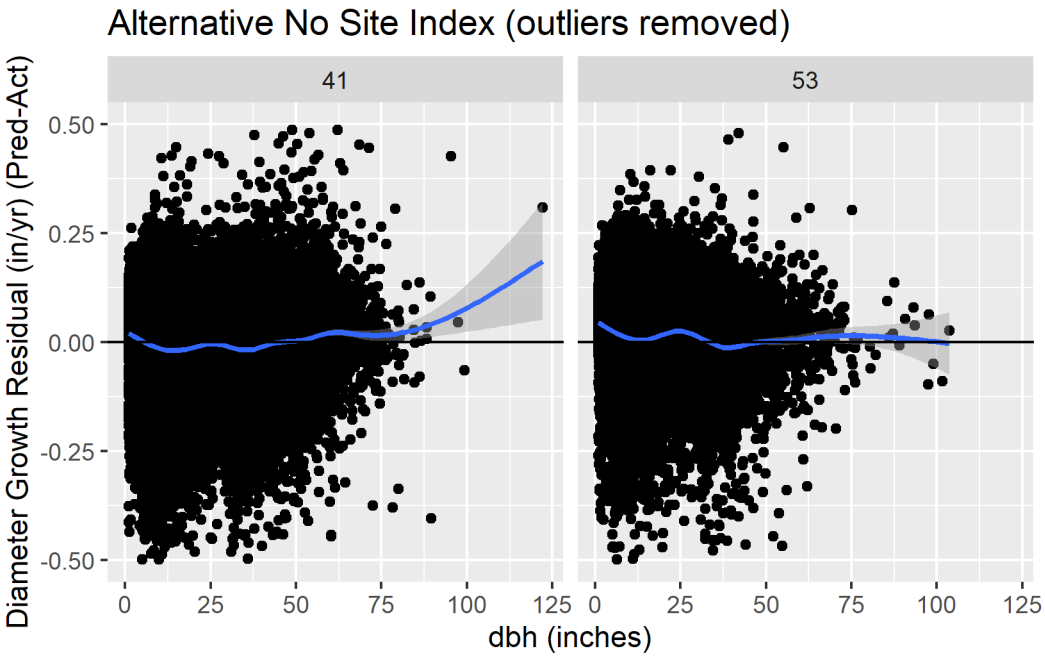

Alternative by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

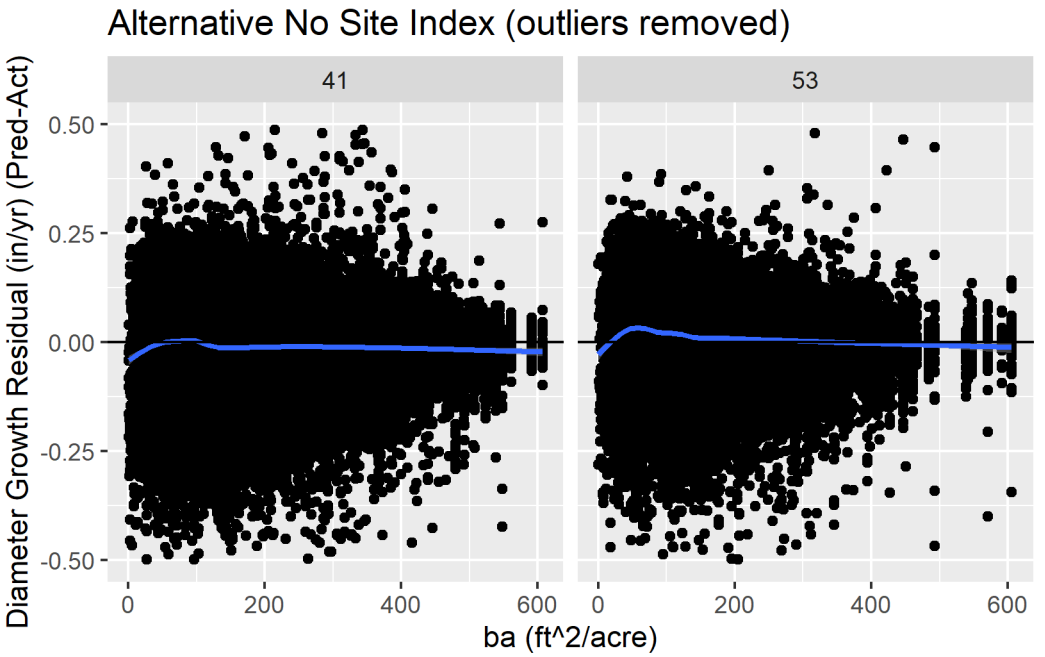

Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

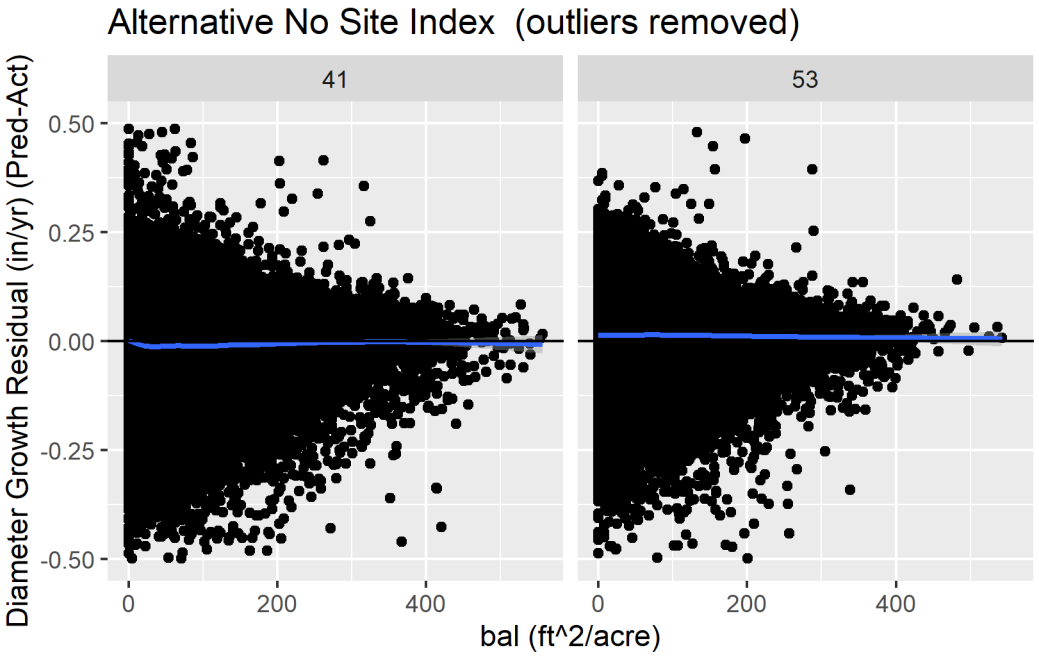

Alternative (outliers removed)
41
53
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
200
ht40 (feet)

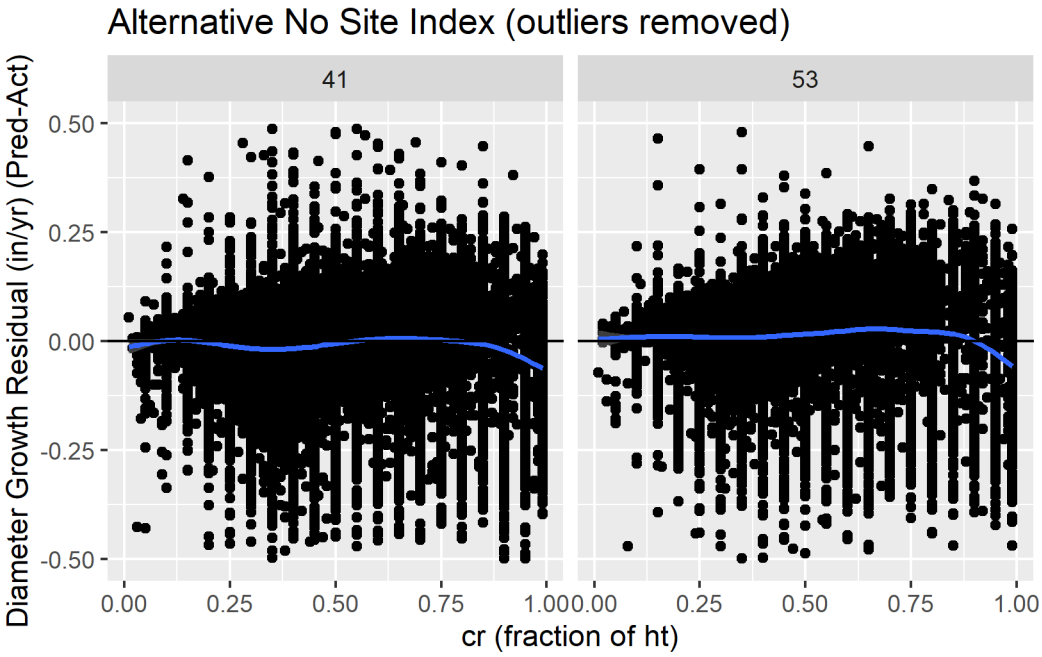

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
41
53
State Code

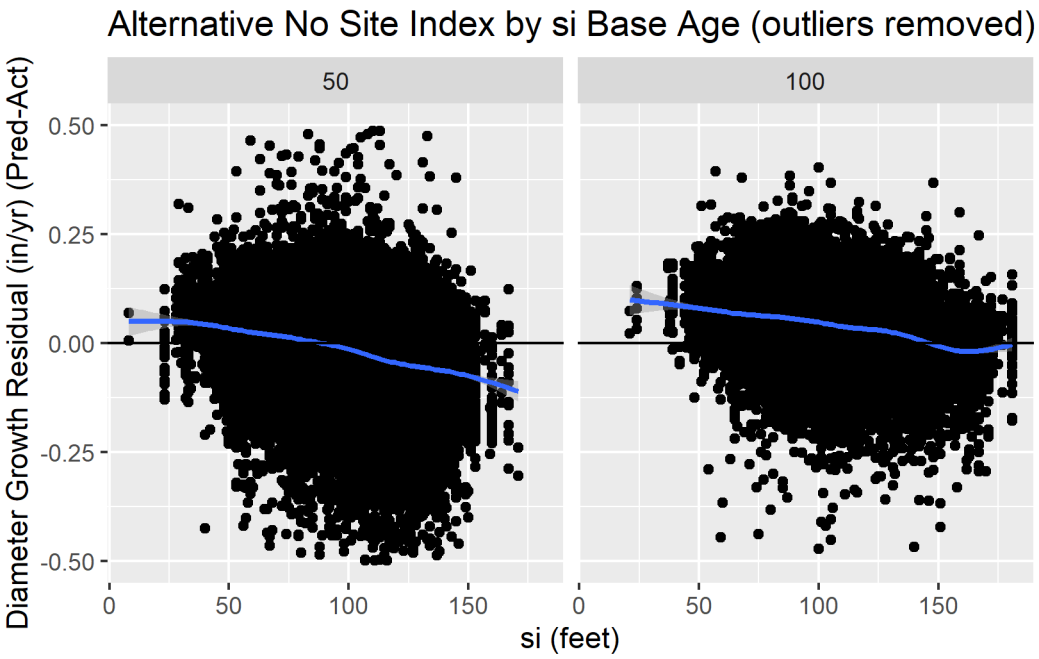
Residual Analysis for ?@eq-nosi

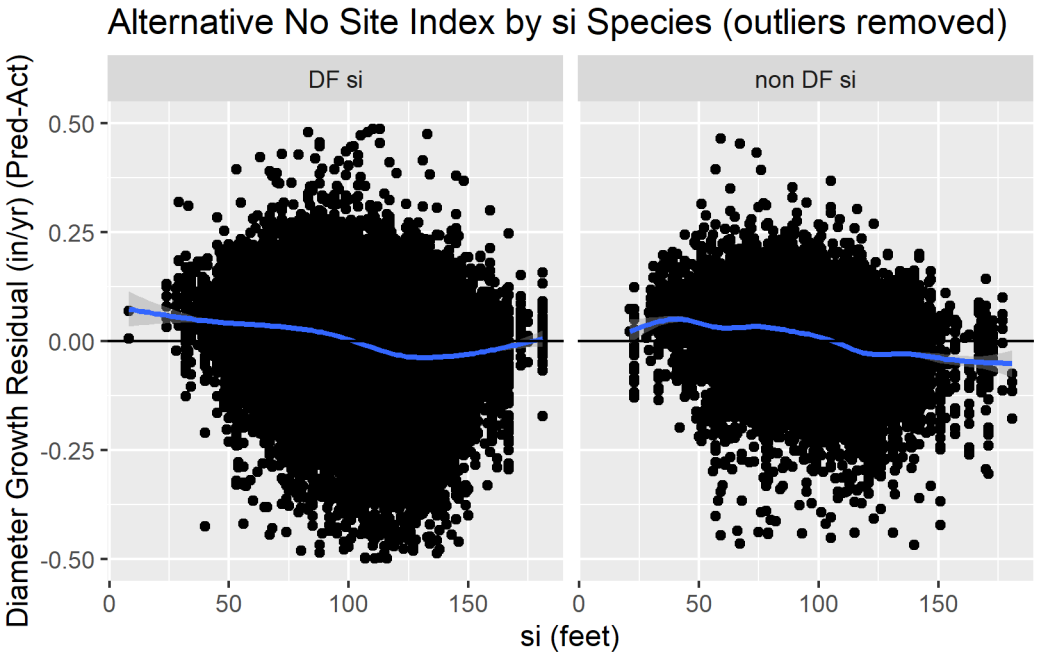


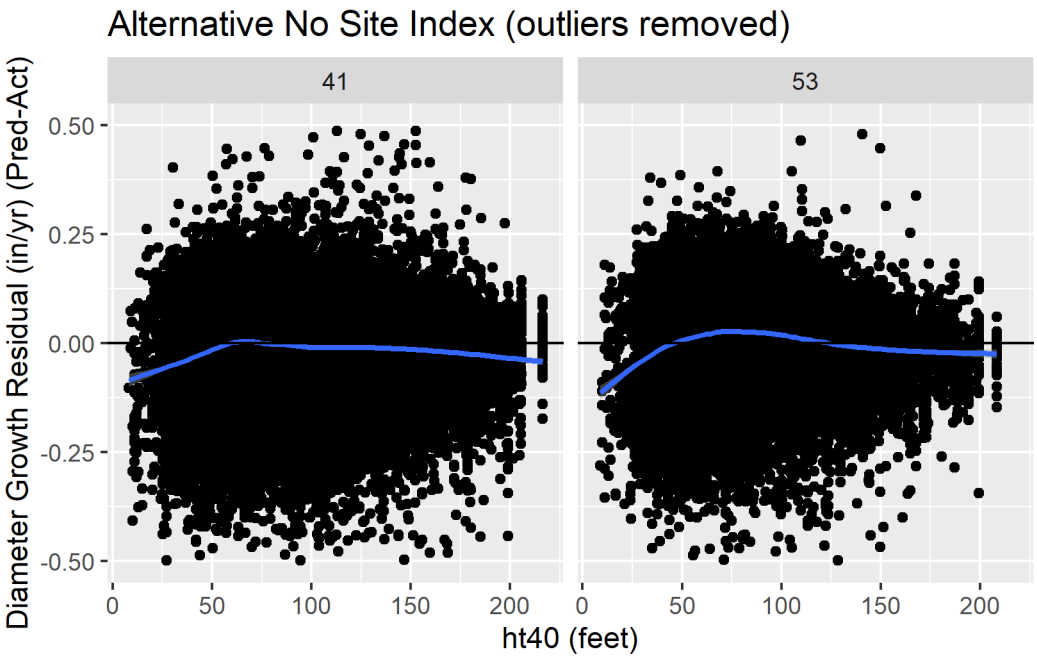



Alternative No Site Index (outliers removed)
41
53
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
bal (ft^2/acre)


Alternative No Site Index (outliers removed)
41
53
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)


Alternative No Site Index by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)


Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)


Alternative No Site Index (outliers removed)
41
53
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
200
ht40 (feet)

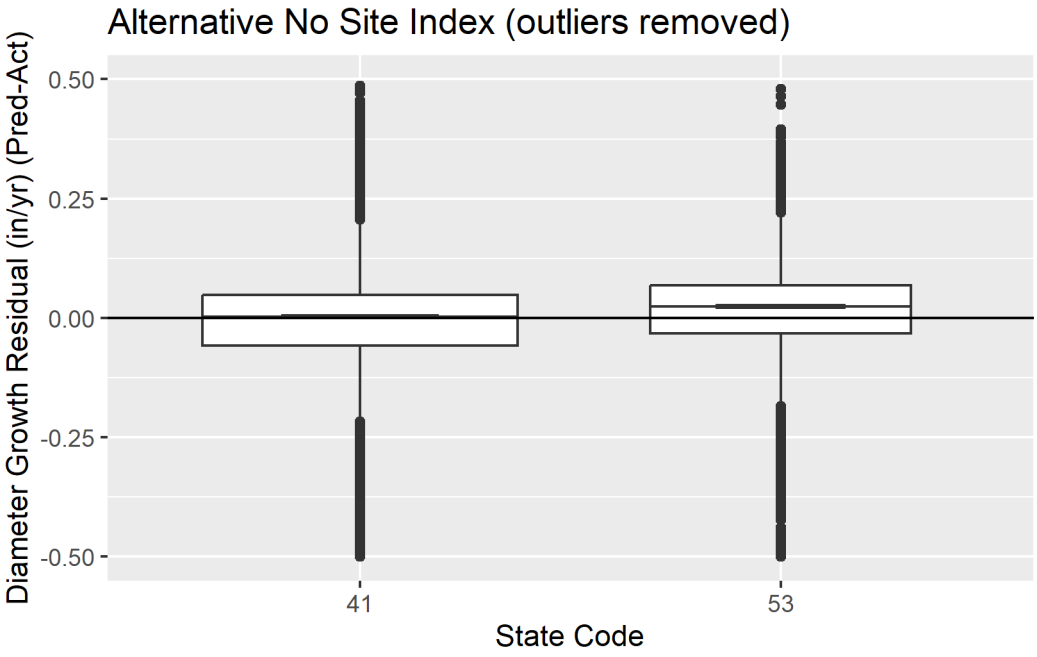

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
41
53
State Code

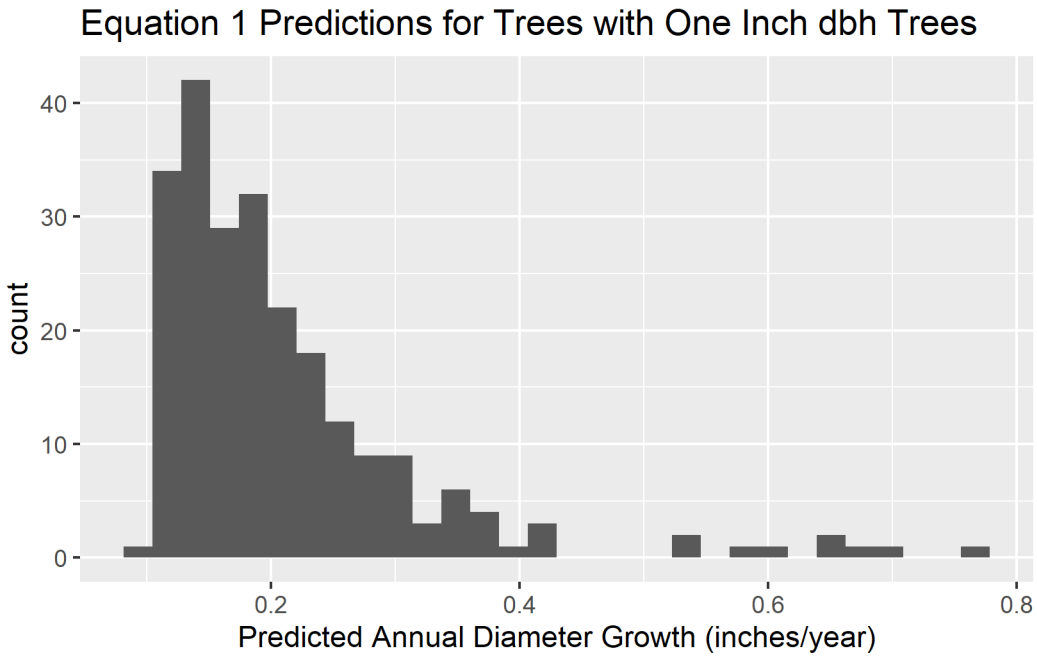
Table 2: Independent Variables for One Inch dbh Trees

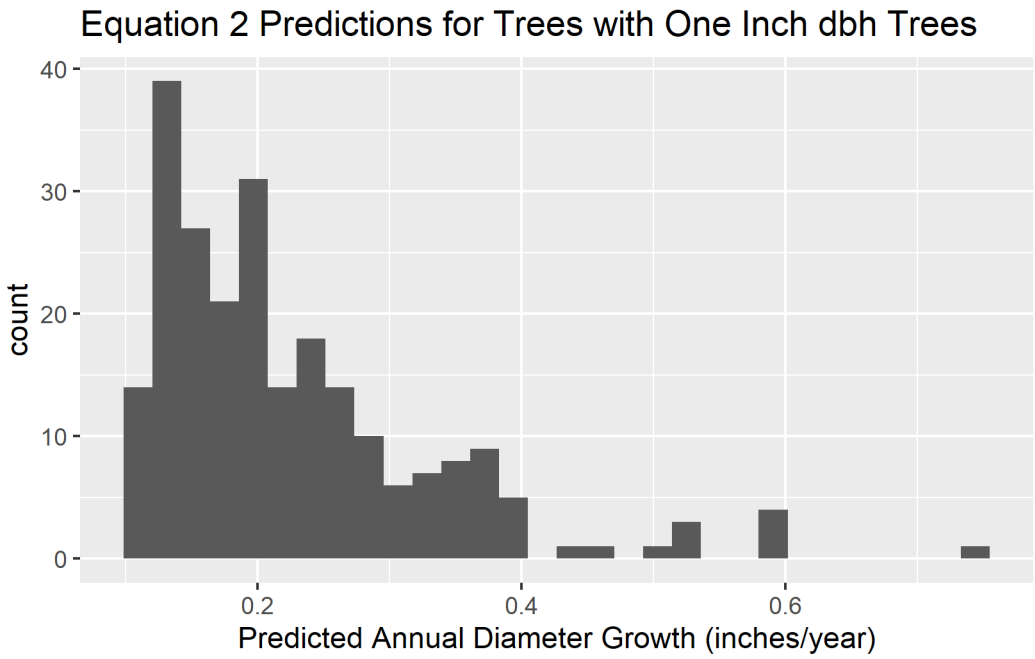
Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	234	118	73	0.41	64	160	413
bal	234	117	73	0	63	159	412
ht	234	9.4	1.9	6	8	10	17
cr	234	0.58	0.27	0.05	0.35	0.8	0.99
si	234	92	22	40	78	107	159

Discussion

Removing `si` degrades the fit significantly. When `si` in its random variable form is used, the bias with the height of the largest 40 trees per acre (`ht40`) is reduced for small values and the bias for Washington (53) is also reduced. **The question remains as to whether `si` is reliable enough to be used.**.

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
count
40
30
20
10
0
0.2
0.4
0.6
Predicted Annual Diameter Growth (inches/year)