

Alternative Balsam Fir Pine Diameter Growth

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2026-01-27

Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 7 states listed in the native range of Balsam Fir in the Silvics of North America.¹

After subsetting the data to censor observations with missing data, limiting the species to Balsam Fir (FIA species code 12), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Balsam Fir Growth Observations by State

State	Observations
CT	1
MA	40
ME	83061
MN	40793
NH	11573
VT	4172
WV	6

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation² which reduces parameter count while retaining key features of the original model is shown below. The key change is the

¹Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

²Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

term with a ratio of a transformation of diameter at breast height (**dbh**) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (**bal**) serves as the inter-tree competition factor, and site index (**si**) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Balsam Fir site trees,
2. It is estimated using a number of different and not necessarily compatible **si** equations, and
3. The available **si** equations do not all use the same base age.

In the data set **si** is derived from 25 different site index equations for 27 species. Balsam Fir site index comprises 35% of the observations. There are 1 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where **SIBASE** and **SISP** are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0 + \beta_1 \log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7} + \beta_3 \log(si_{s,b} + 4.5))} \quad (1)$$

and

$$\Delta dbh = e^{(\beta_0 + \beta_1 \log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \quad (2)$$

where:

- **dbh** = diameter at breast height (inches),
- **bal** = basal area per acre in larger trees ($feet^2/ac$),
- **cr** = crown ratio (fraction of total height),
- **ht** = total height (feet), and
- $si_{s,b}$ = site index (feet) for species **s** and base age **b**.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending **dbh**. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

Nonlinear mixed-effects model fit by maximum likelihood

Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL, startCR, endCR)

Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset\$SIBASE), as.factor(SIINT)))

	AIC	BIC	logLik
	71704.57	71783.35	-35844.29

Random effects:

Formula: B3 ~ 1 | SIINT

	B3	Residual
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StdDev: 0.01237966 0.3127649

Fixed effects: B0 + B1 + B2 + B3 + B4 + B5 ~ 1

	Value	Std.Error	DF	t-value	p-value
B0	-3.667189	0.03537306	139639	-103.67179	0
B1	-0.367941	0.00351713	139639	-104.61422	0
B2	-0.042757	0.00401374	139639	-10.65266	0
B3	0.129654	0.01194394	139639	10.85522	0
B4	2.527050	0.01916143	139639	131.88214	0
B5	0.698240	0.01727999	139639	40.40744	0

Correlation:

	B0	B1	B2	B3	B4
B1	-0.177				
B2	-0.338	0.491			
B3	-0.573	-0.023	-0.036		
B4	-0.342	0.790	0.613	-0.086	
B5	-0.316	0.465	0.994	-0.030	0.560

Standardized Within-Group Residuals:

	Min	Q1	Med	Q3	Max
	-32.3852603	-0.5899003	-0.1846415	0.4081703	24.9075325

Number of Observations: 139646

Number of Groups: 2

\$SIINT

B3

50.FALSE -0.01236728

50.TRUE 0.01236728

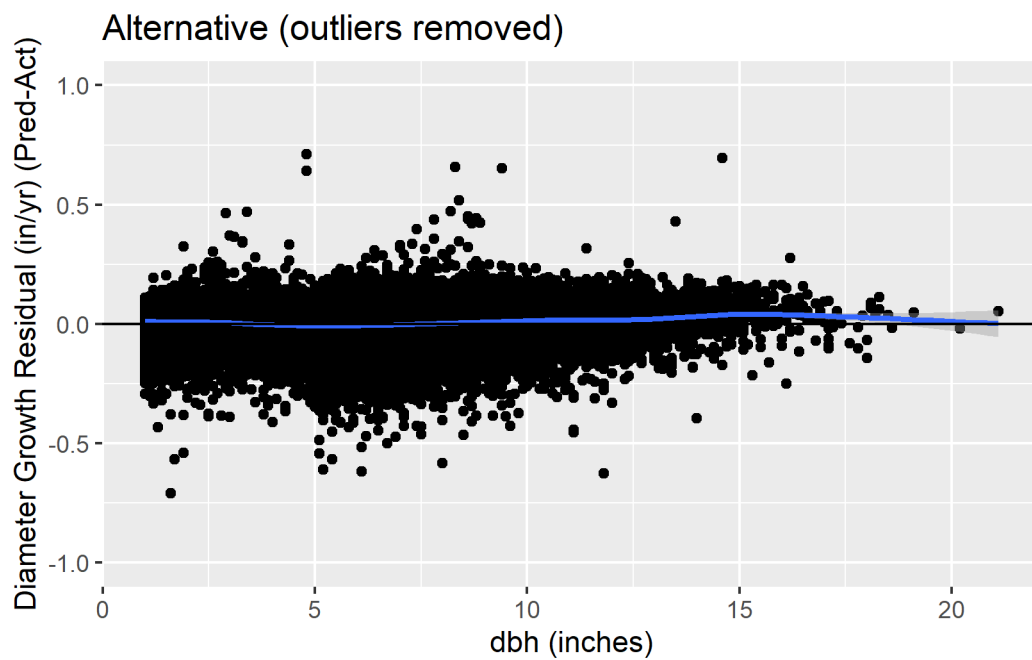
Residual Standard Error: 0.312764862293159 on 139639 degrees of freedom, AIC: 71704.6

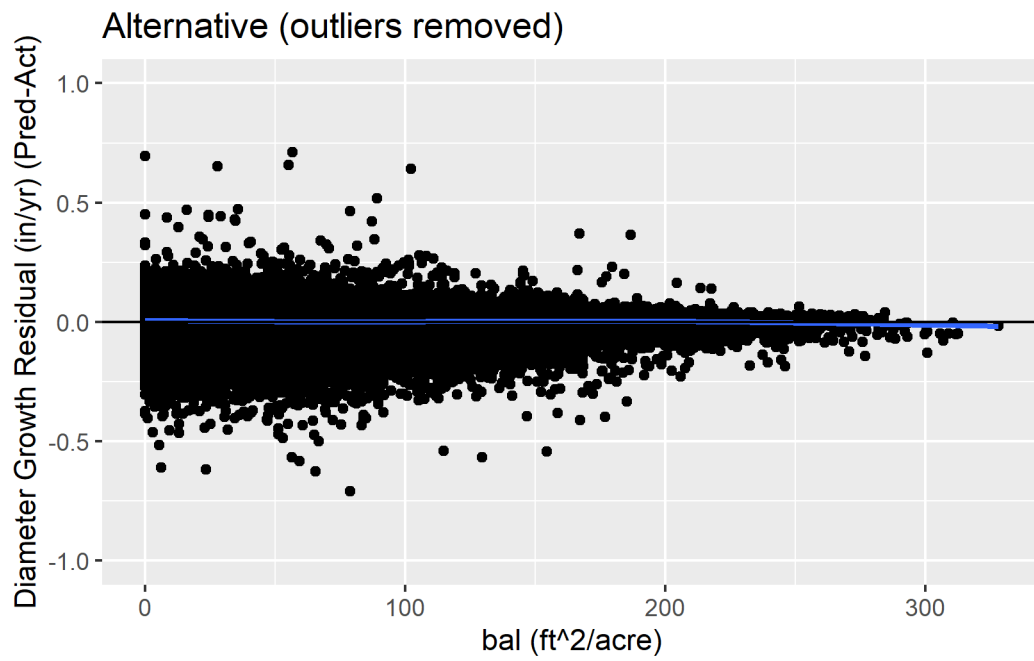
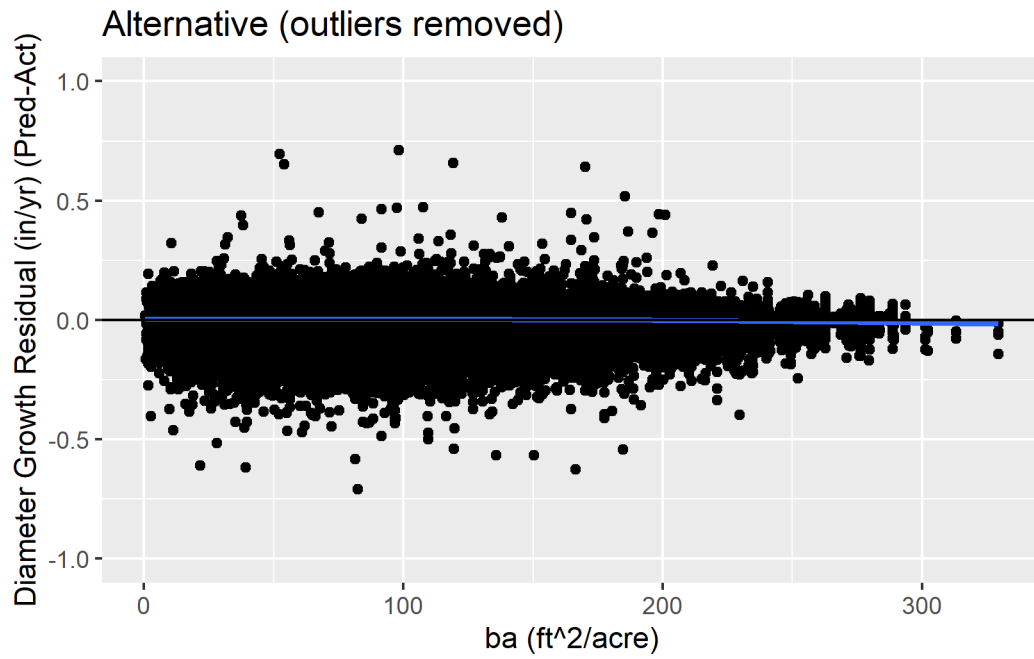
and for Equation 2:

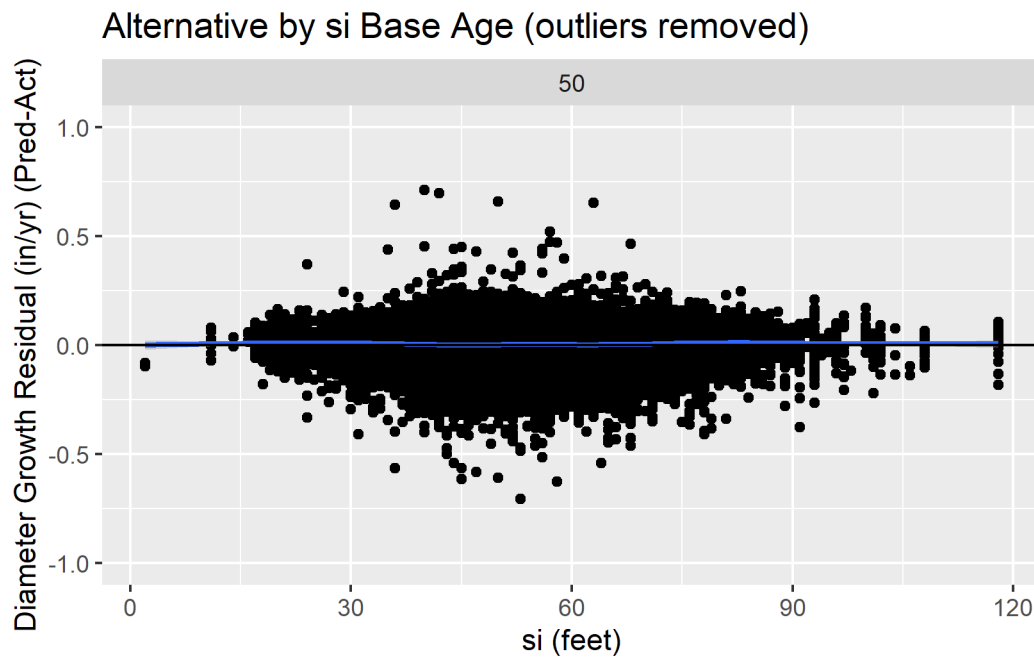
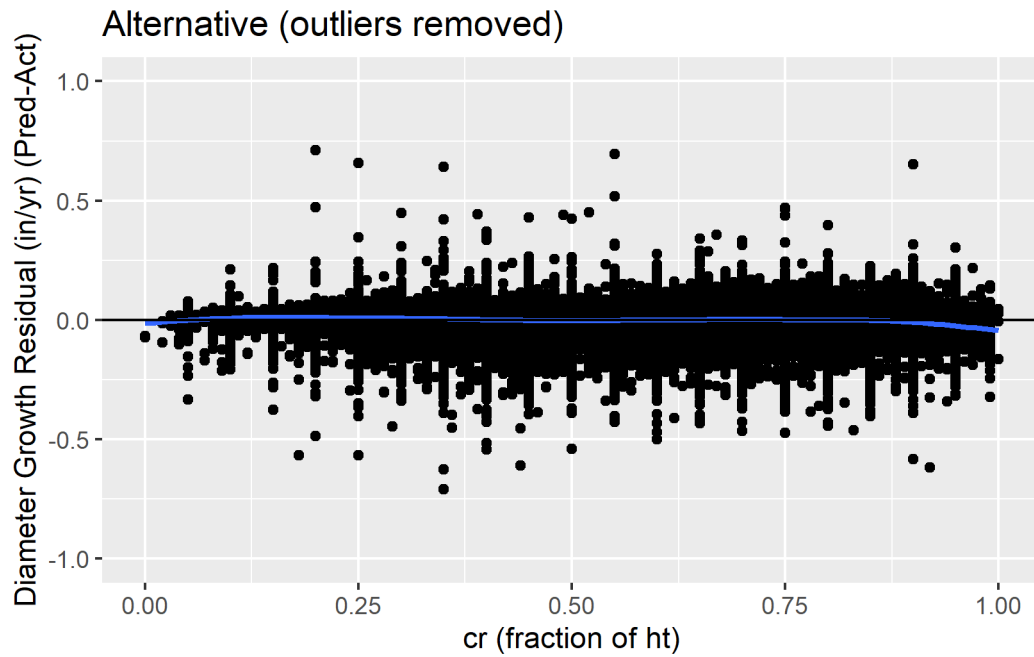
	Coef.	Std. error	t-stat.	p
B0	-3.1801582	0.0188532	-168.68042	0
B1	-0.3525681	0.0034588	-101.93279	0
B2	-0.0387618	0.0036883	-10.50944	0
B4	2.5968033	0.0202114	128.48219	0
B5	0.7168023	0.0175847	40.76286	0

Residual Standard Error: 0.313789812510913 on 139641 degrees of freedom, AIC: 72596.8

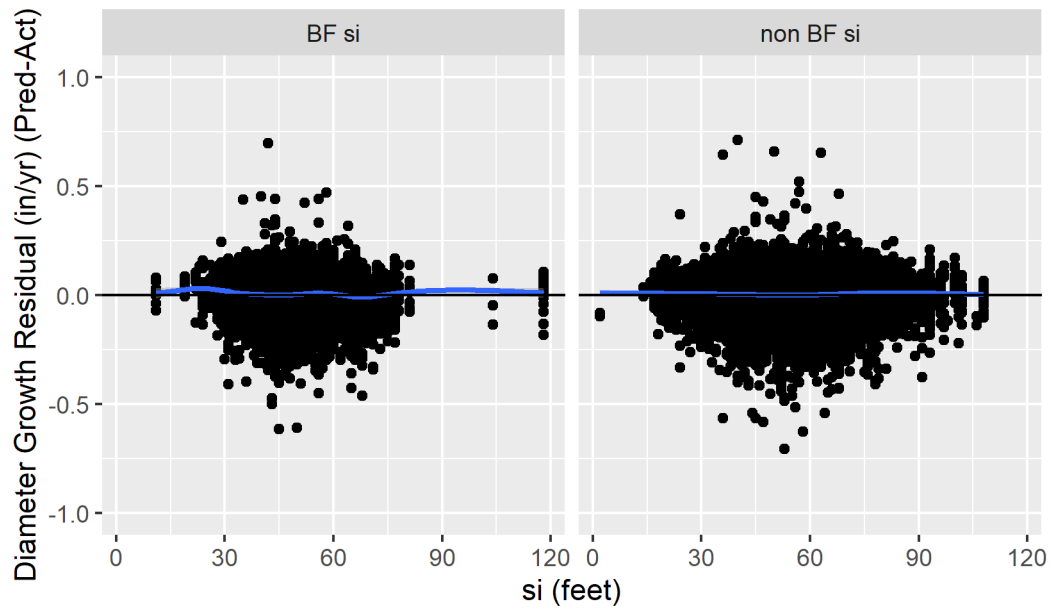
Residual Analysis for Equation 1



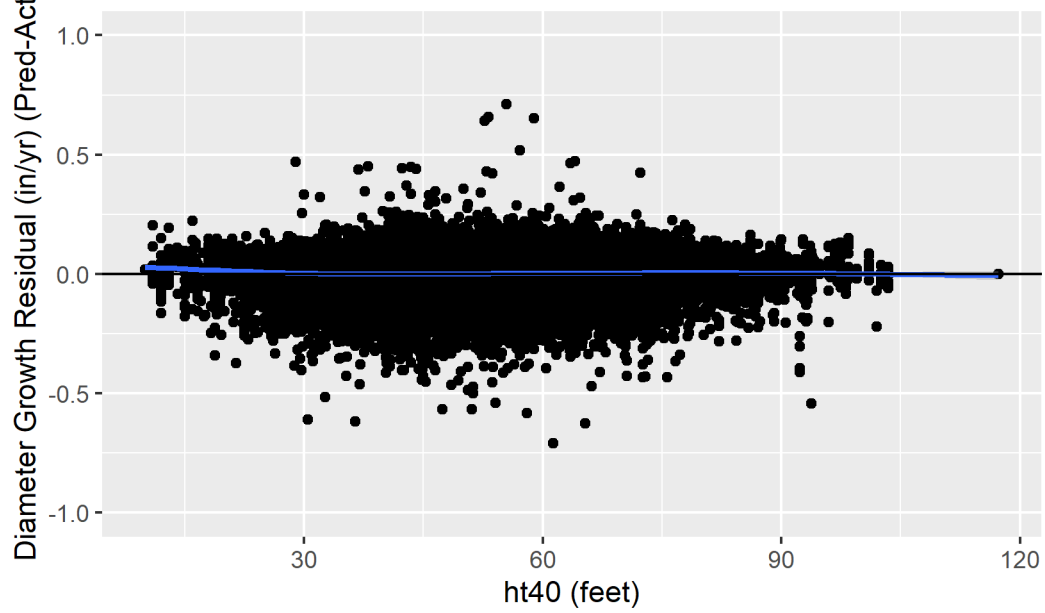


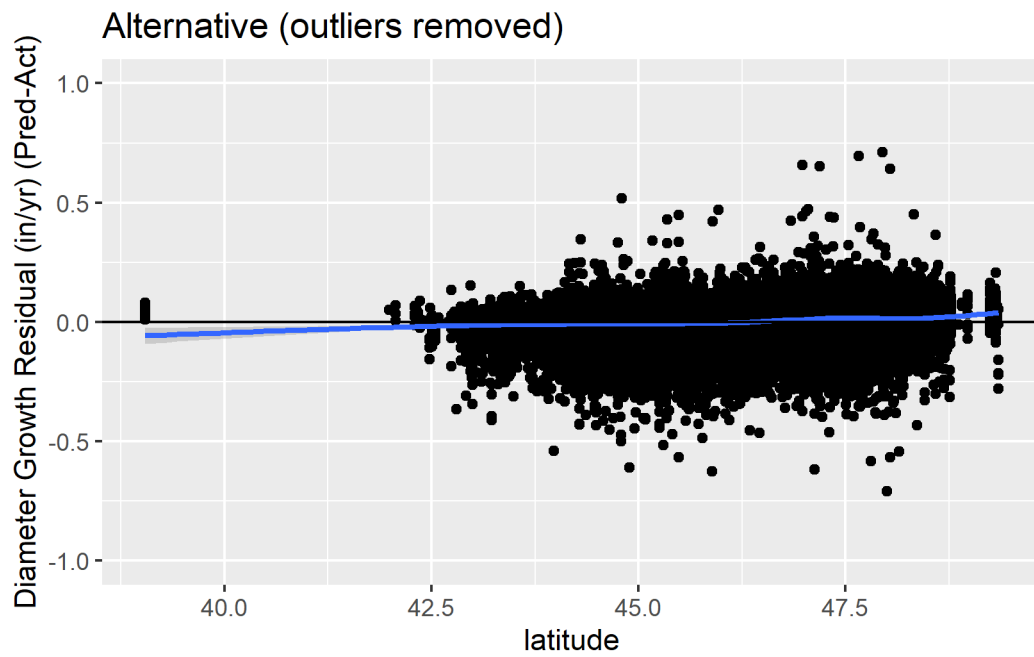
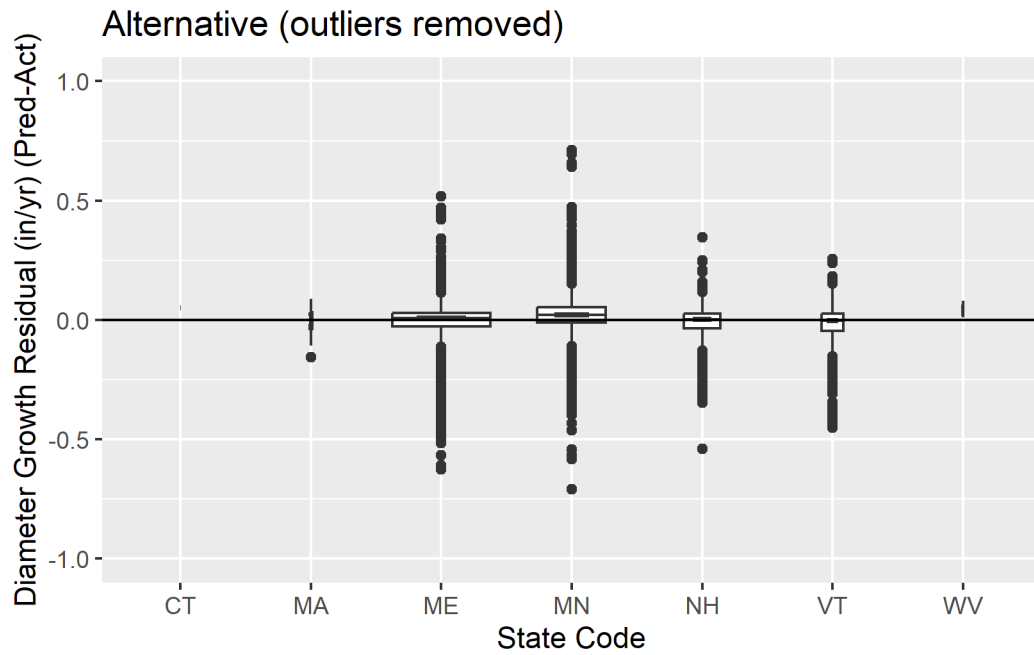


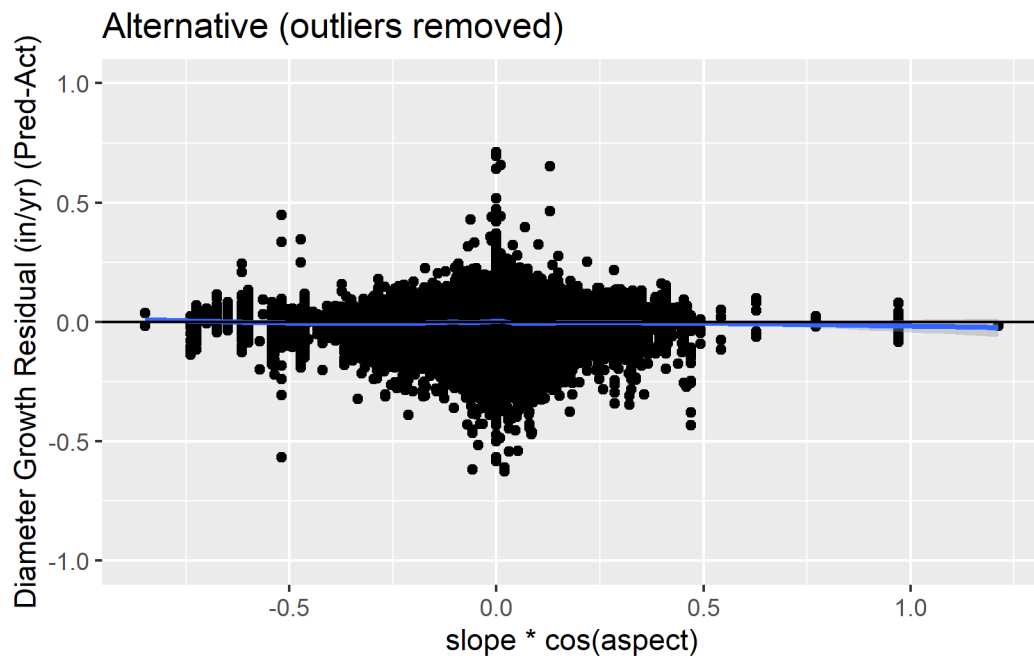
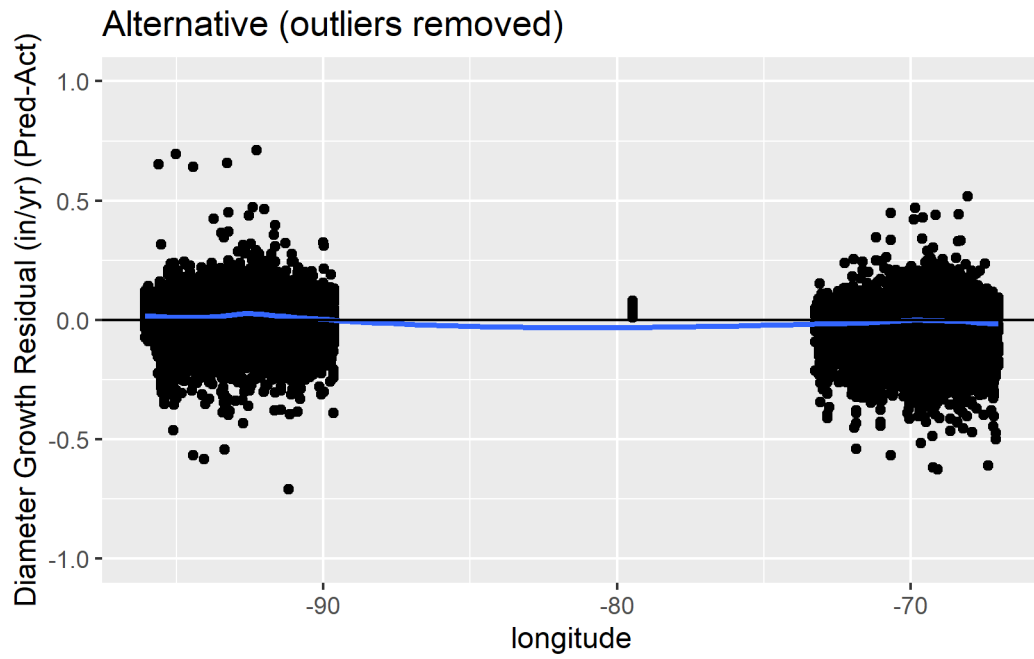
Alternative by si Species (outliers removed)

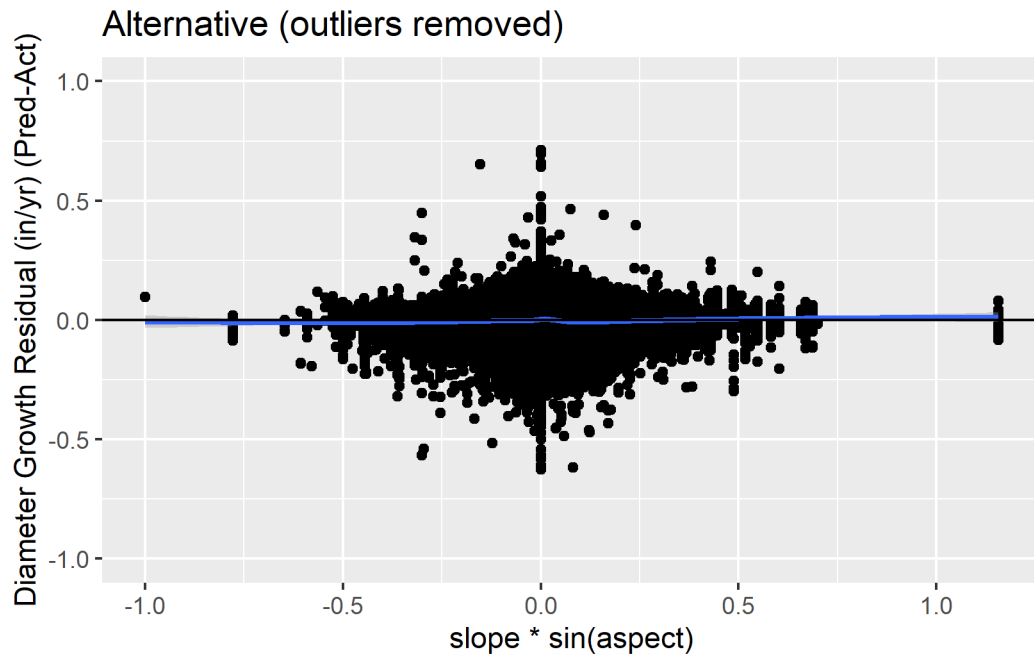


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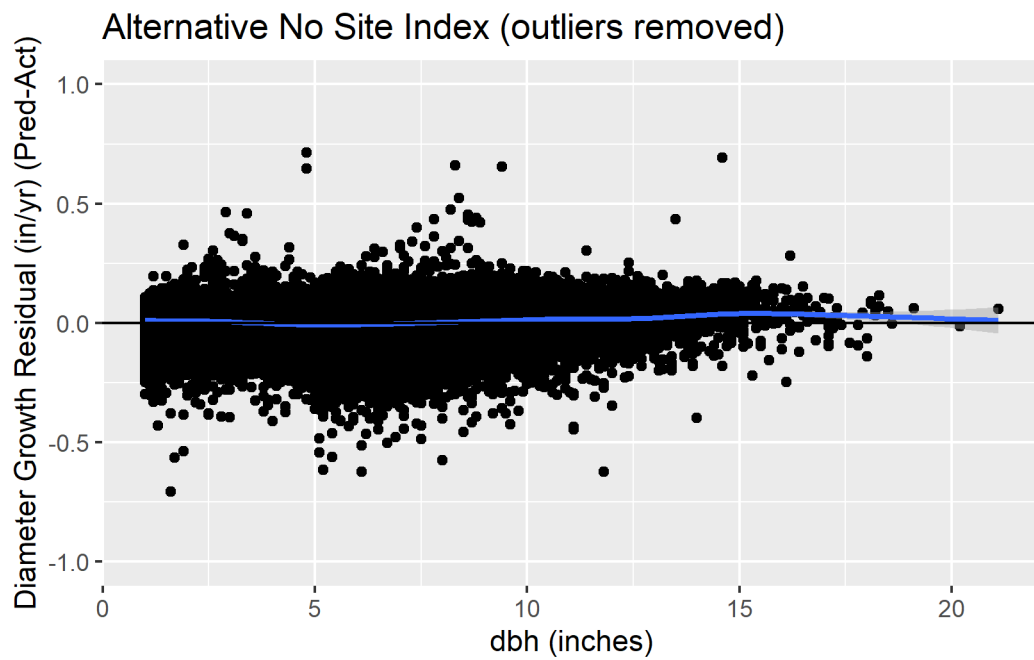


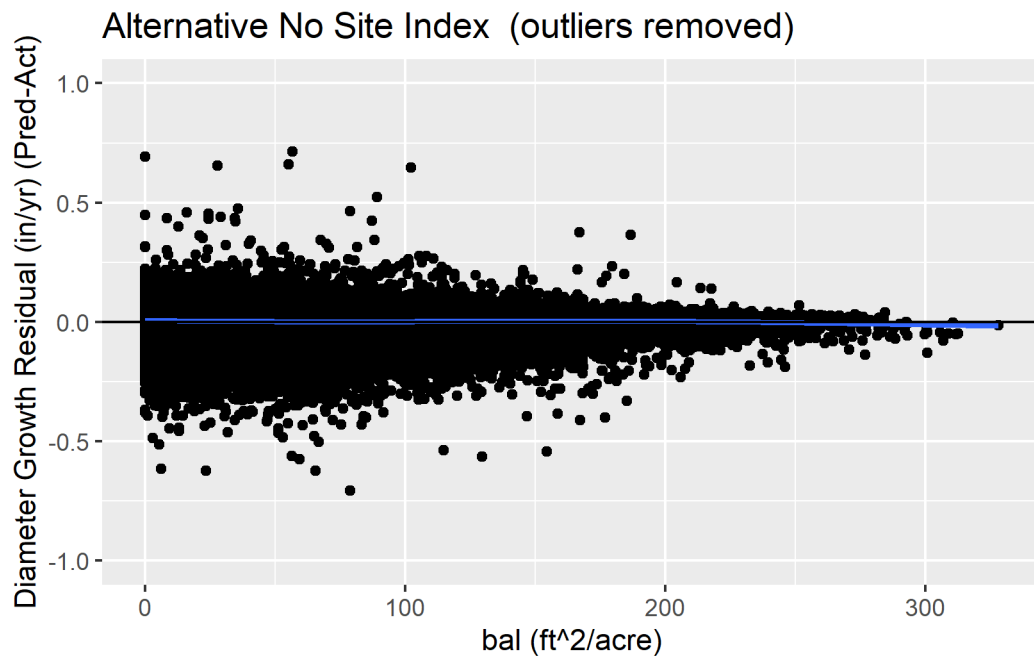
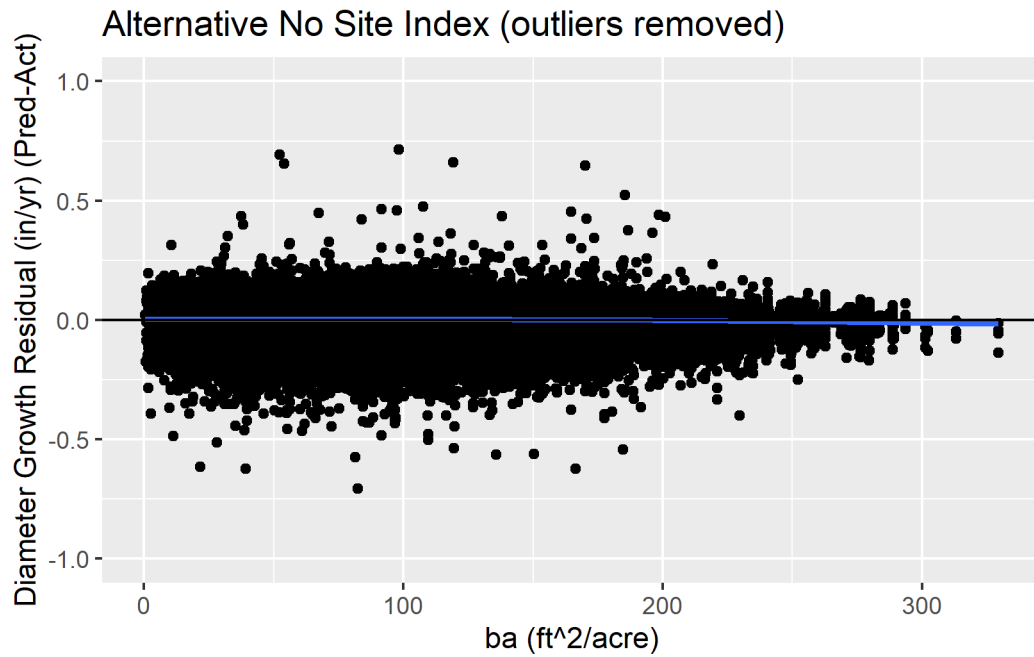


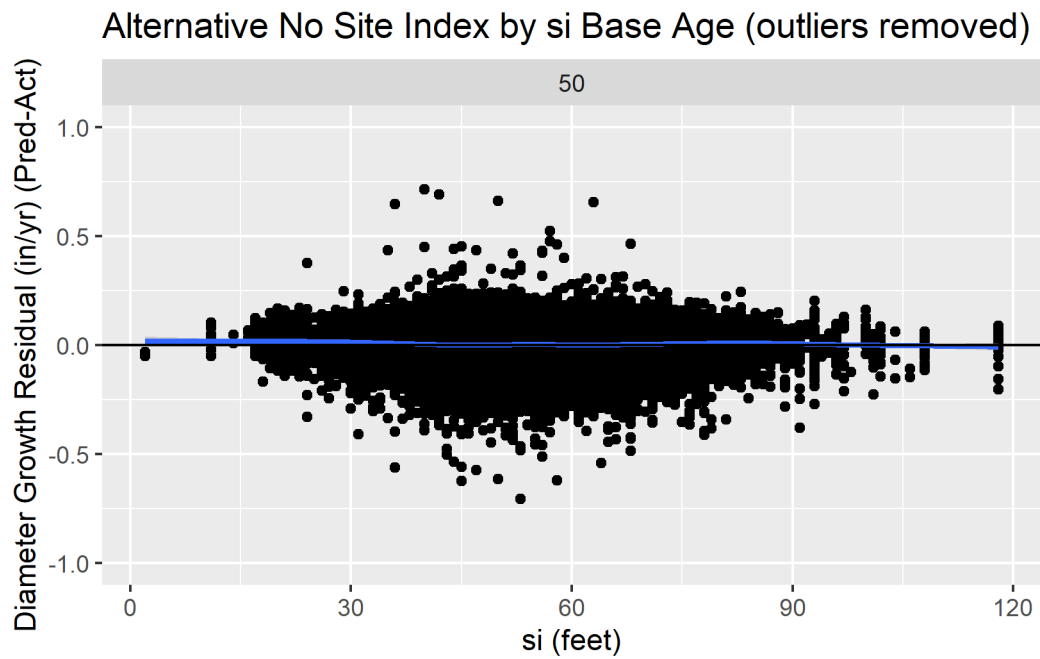
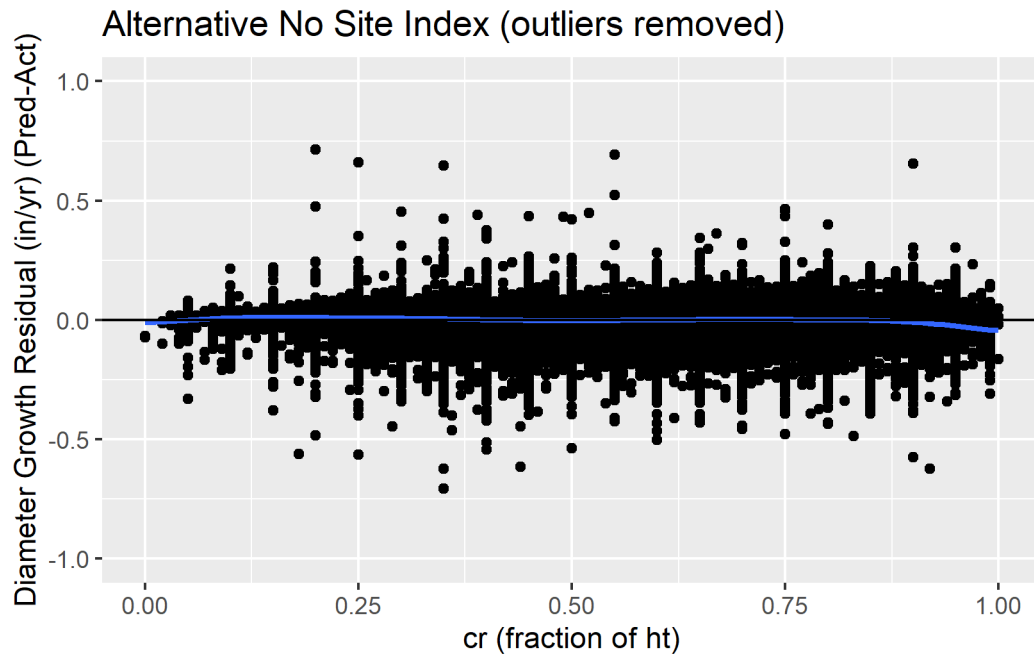




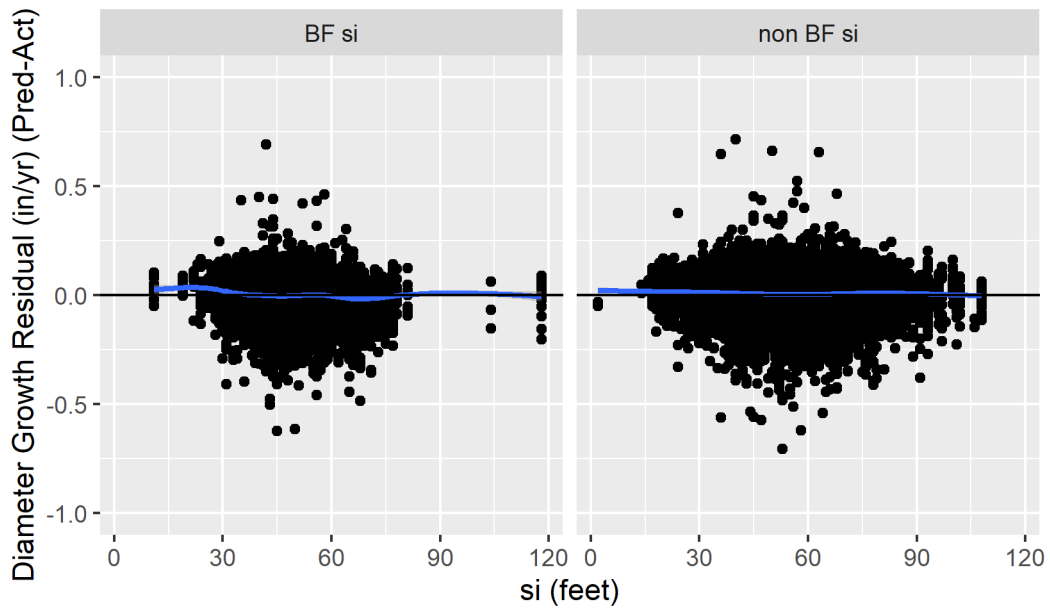
Residual Analysis for Equation 2



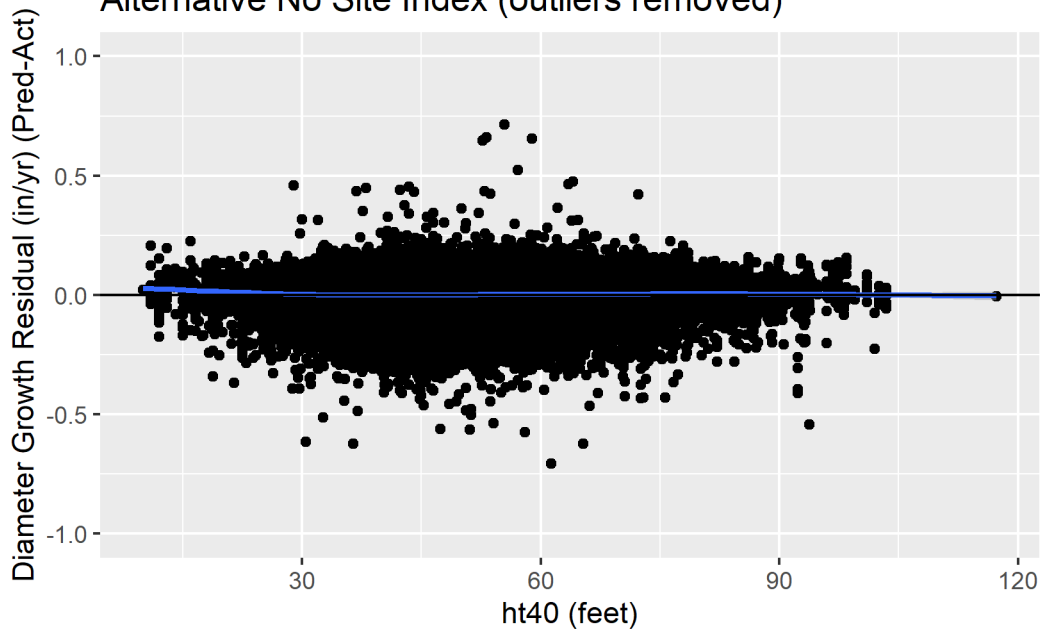


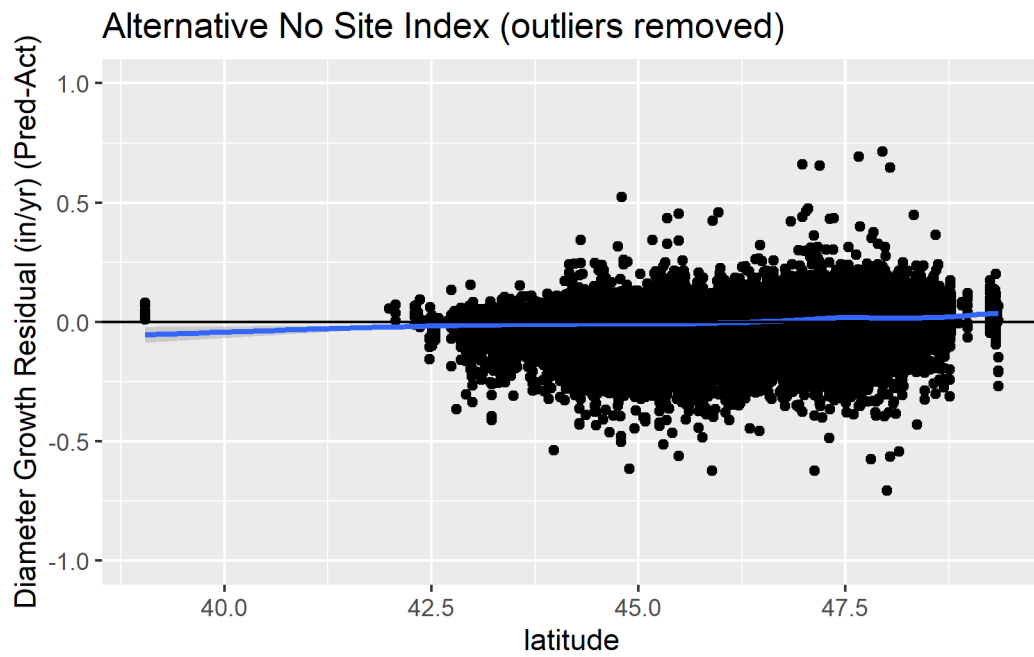
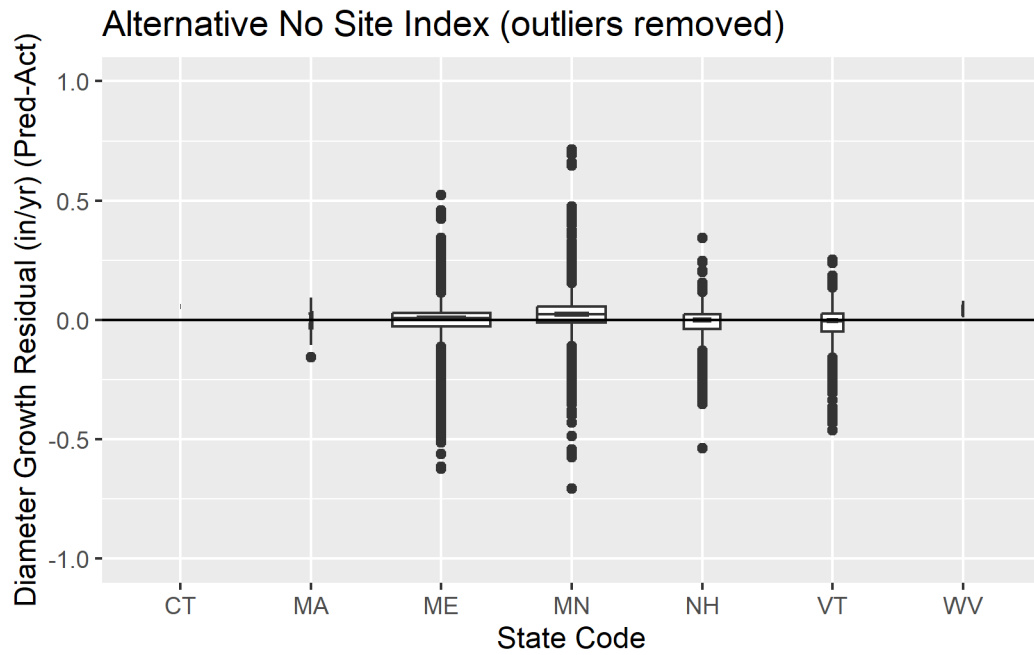


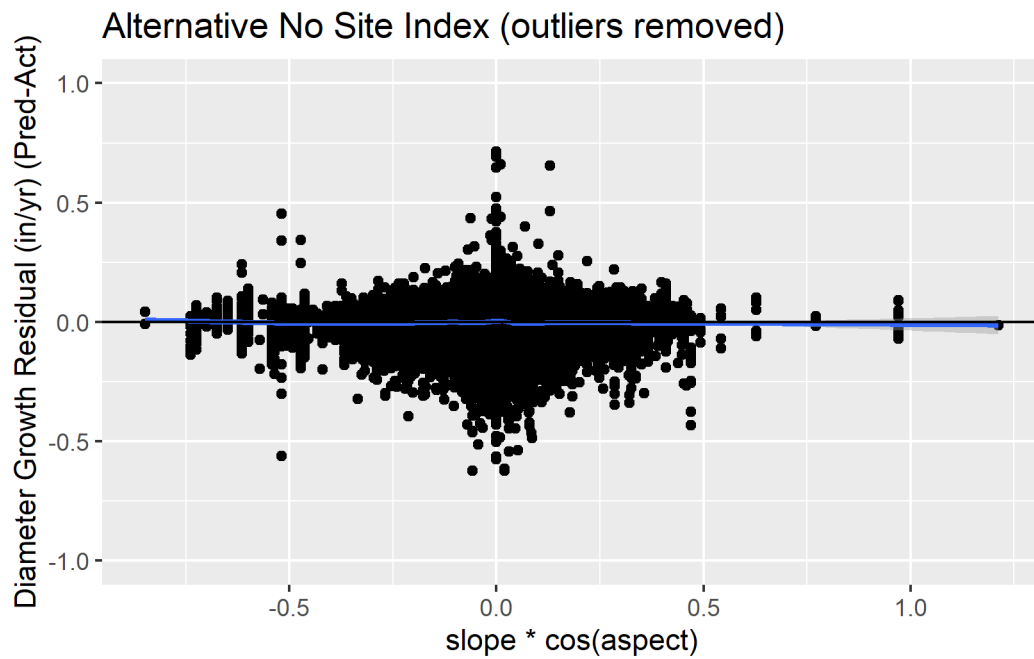
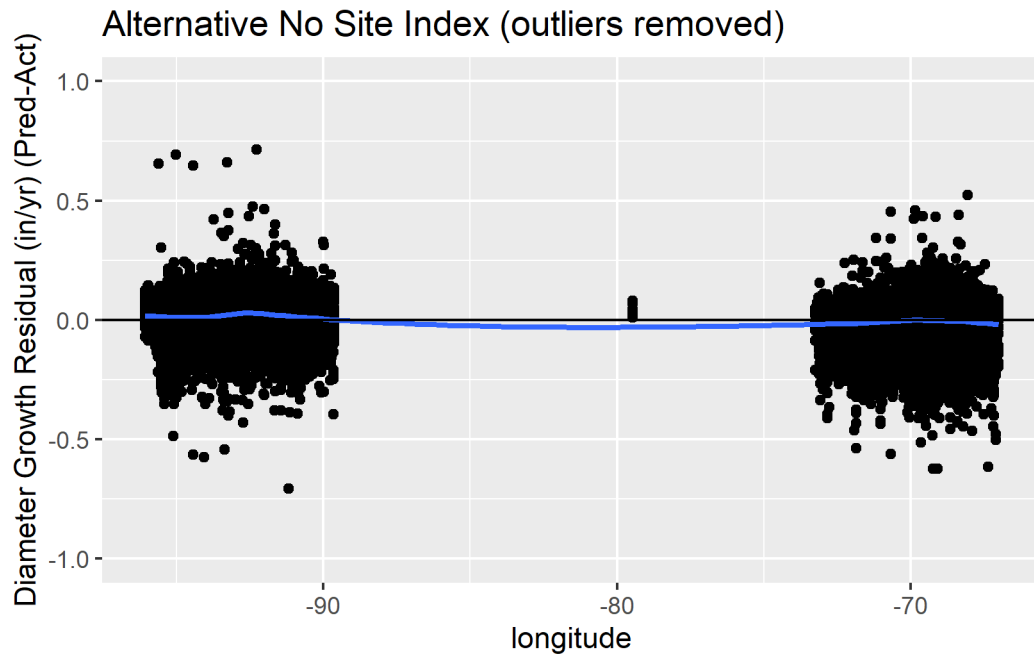
Alternative No Site Index by si Species (outliers removed)

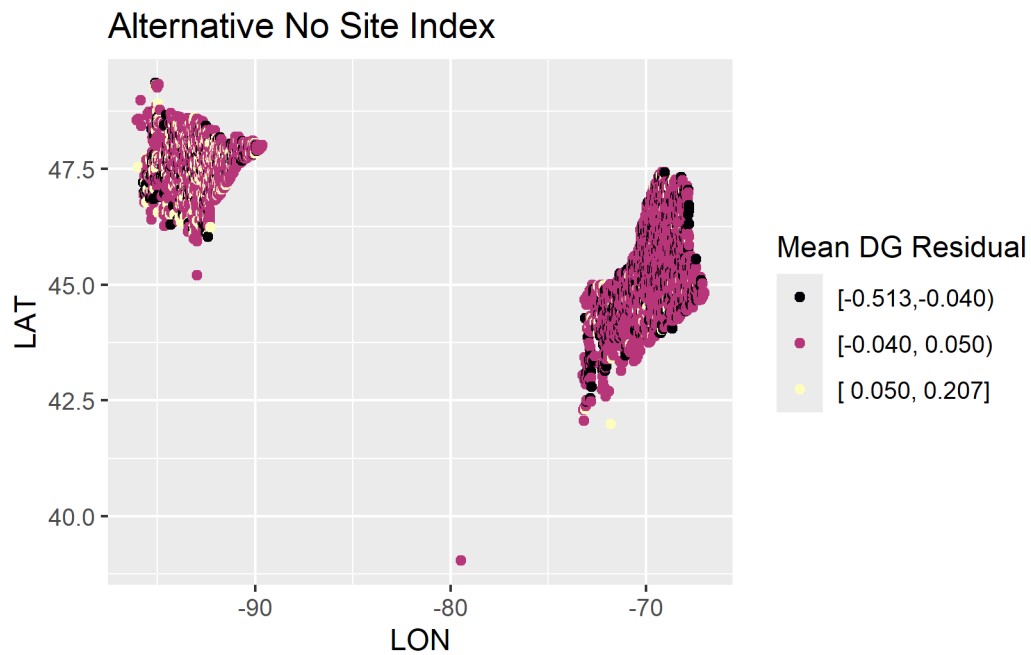
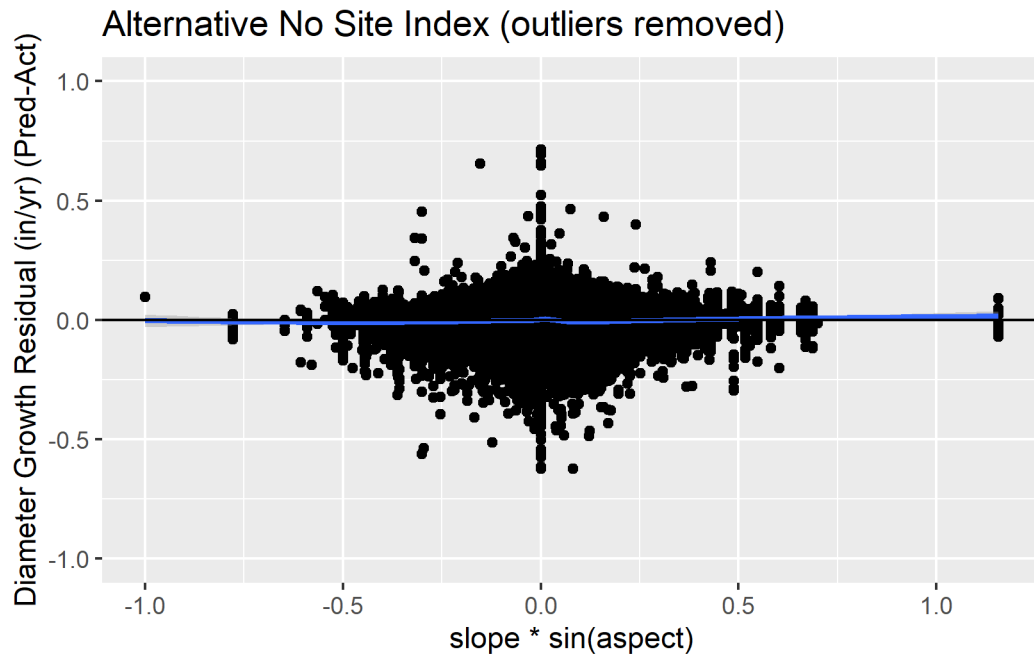


Alternative No Site Index (outliers removed)









Discussion

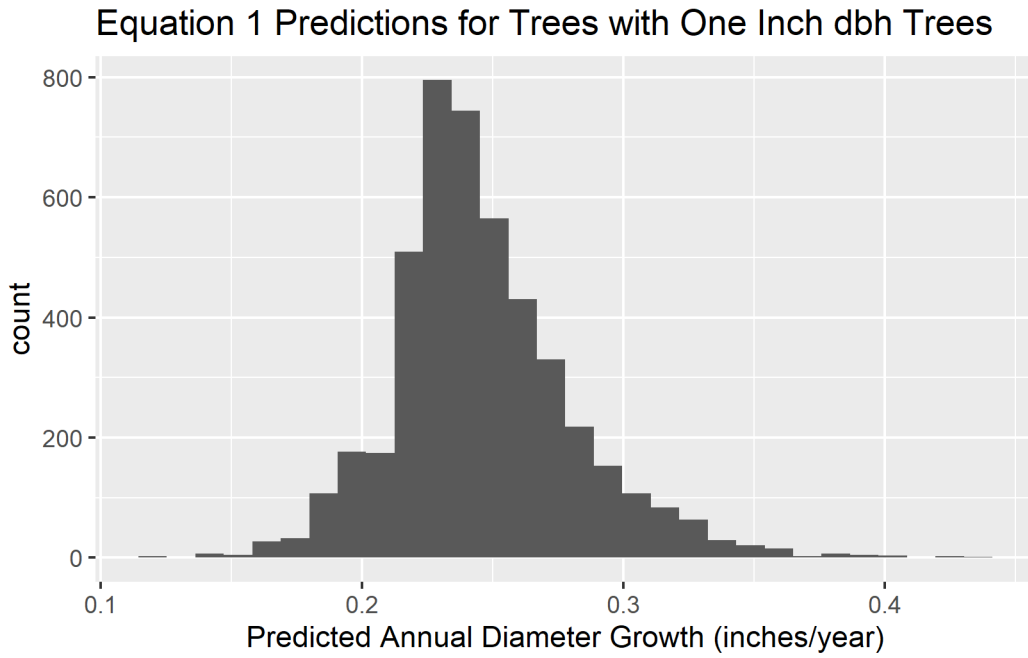
Removing `si` does not degrade the fit significantly. Only MN appears to be over-predicted.

Table 3: Independent Variables for One Inch dbh Trees

Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	4621	111	49	0.41	75	144	313
bal	4621	109	49	0	74	143	313
ht	4621	10	1	5	10	10	25
cr	4621	0.46	0.22	0.03	0.3	0.6	1
si	4621	51	12	17	44	58	108

Equation 1 residual graphs show that there is some indication of residual trends spatially (especially in the western states).

Equation Behavior for Very Small Trees



Equation 2 Predictions for Trees with One Inch dbh Trees

