Alternative Bigleaf Maple Diameter Growth

Greg Johnson and David Marshall

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Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 3 states listed in the native range of Bigleaf Maple in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Bigleaf Maple (FIA species code 312), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Bigleaf Maple Growth Observations by State

State	Observations
CA	688
OR	2504
WA	1551

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Bigleaf Maple site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 12 different site index equations for 13 species. Bigleaf Maple site index comprises 0% of the observations. There are 2 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7} + \beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, en
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
       AIC      BIC    logLik
```

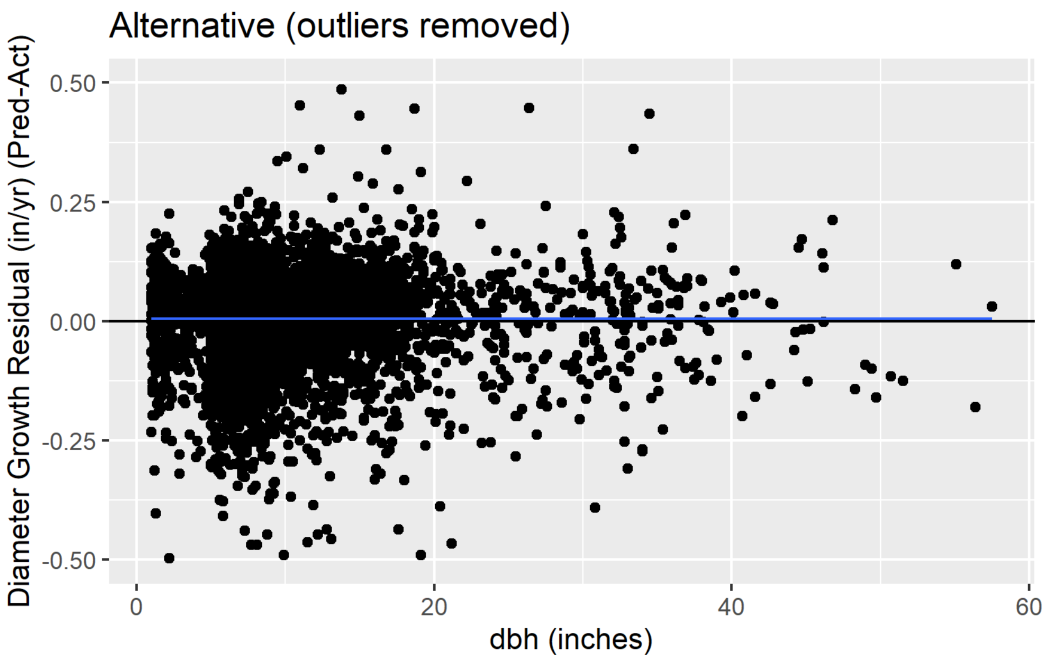
```
  15233.04 15284.75 -7608.518

Random effects:
 Formula: B3 ~ 1 | SIINT
              B3 Residual
StdDev: 0.0880905 1.201692

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value Std.Error   DF    t-value p-value
B0 -5.837576 0.3883409 4734 -15.032093  0.0000
B1 -0.409053 0.0270049 4734 -15.147344  0.0000
B2 -0.060028 0.0299642 4734  -2.003338  0.0452
B3  0.818774 0.0915478 4734   8.943679  0.0000
B4  1.800755 0.1055061 4734  17.067782  0.0000
B5  0.729572 0.0841719 4734   8.667646  0.0000
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.134
B2 -0.300  0.597
B3 -0.759 -0.104 -0.071
B4 -0.281  0.558  0.690 -0.152
B5 -0.295  0.565  0.996 -0.064  0.663

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-14.2288735  -0.5122634  -0.1605568   0.3761722  13.4284111

Number of Observations: 4743
Number of Groups: 4

$SIINT
                   B3
50.FALSE   0.09518358
100.FALSE -0.08651506
50.TRUE    0.06721651
100.TRUE  -0.07588503

Residual Standard Error: 1.20169168149939 on 4734 degrees of freedom, AIC: 15233
```

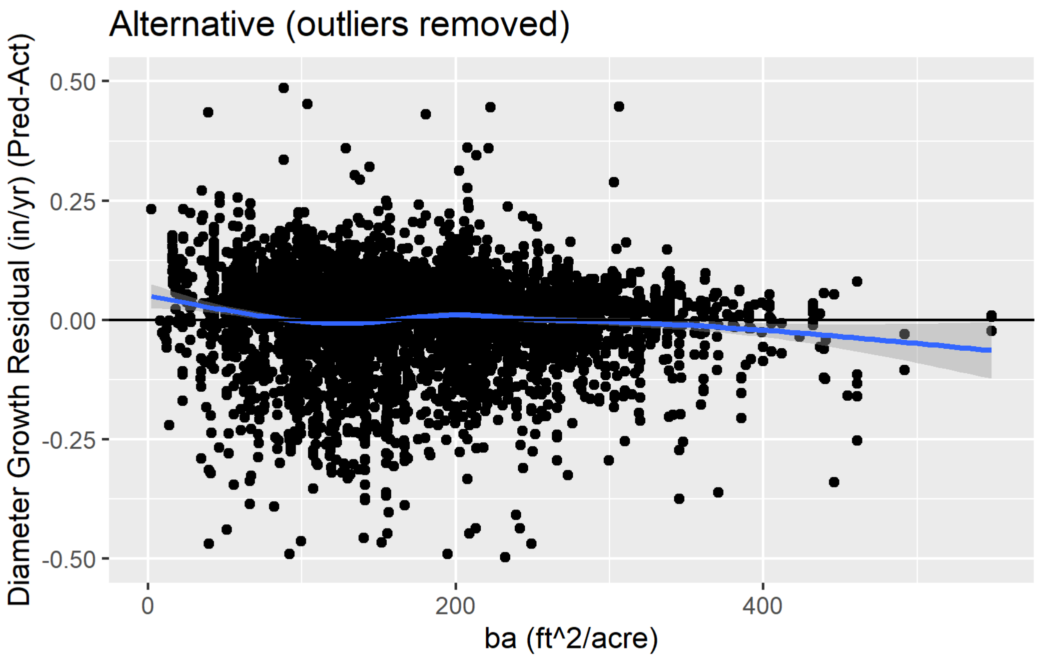
and for Equation 2:

	Coef.	Std. error	t-stat.	p
B0	-2.2311126	0.1680465	-13.276755	0.0000000
B1	-0.3754376	0.0261855	-14.337631	0.0000000
B2	-0.0271598	0.0165401	-1.642062	0.1006436
B4	2.1887029	0.1226651	17.842909	0.0000000
B5	0.8535887	0.1047165	8.151429	0.0000000

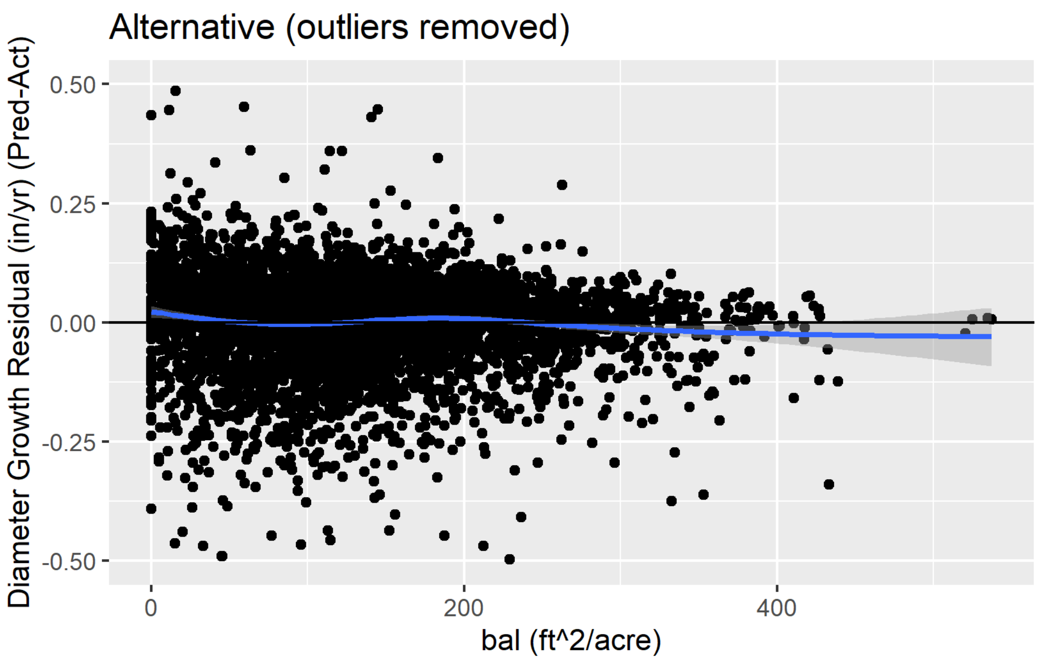
```
Residual Standard Error: 1.22526259996152 on 4738 degrees of freedom, AIC: 15394.2
```

Residual Analysis for Equation 1

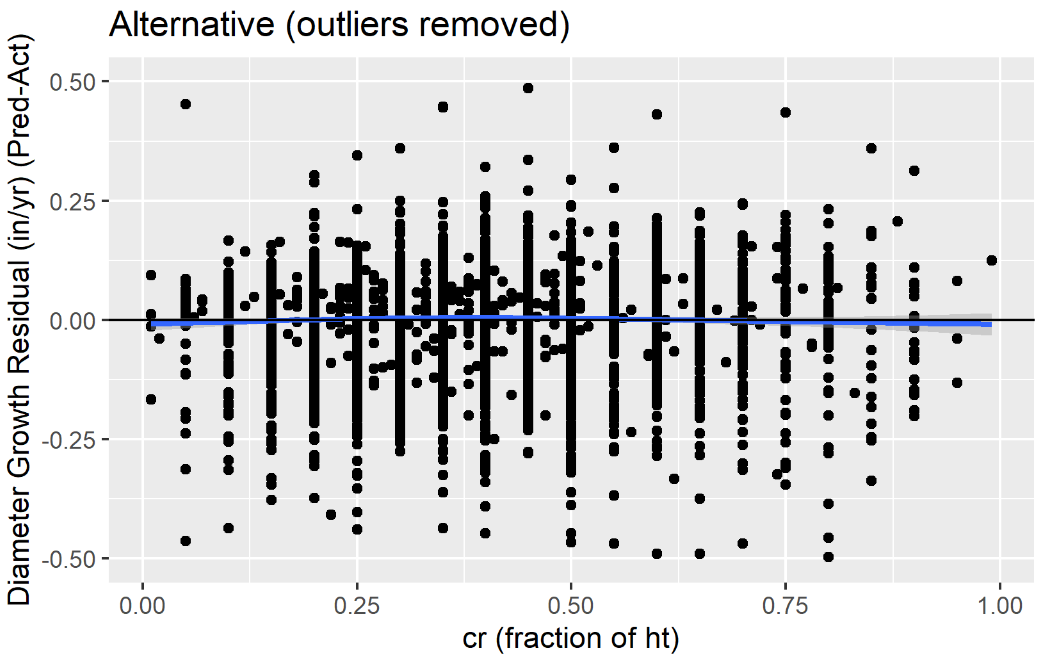




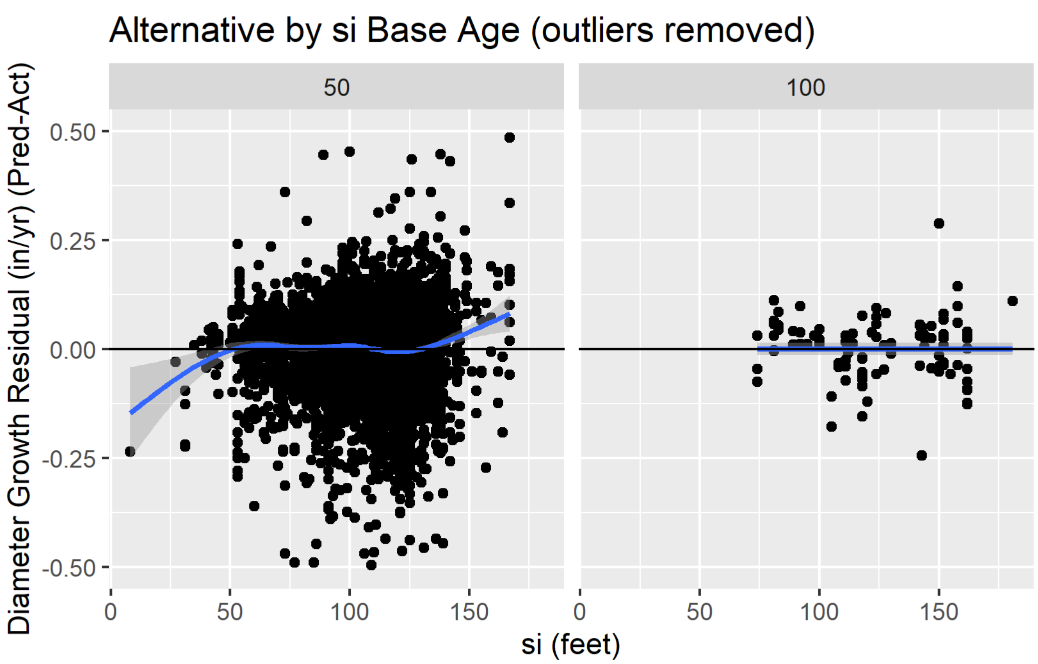
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
ba (ft^2/acre)



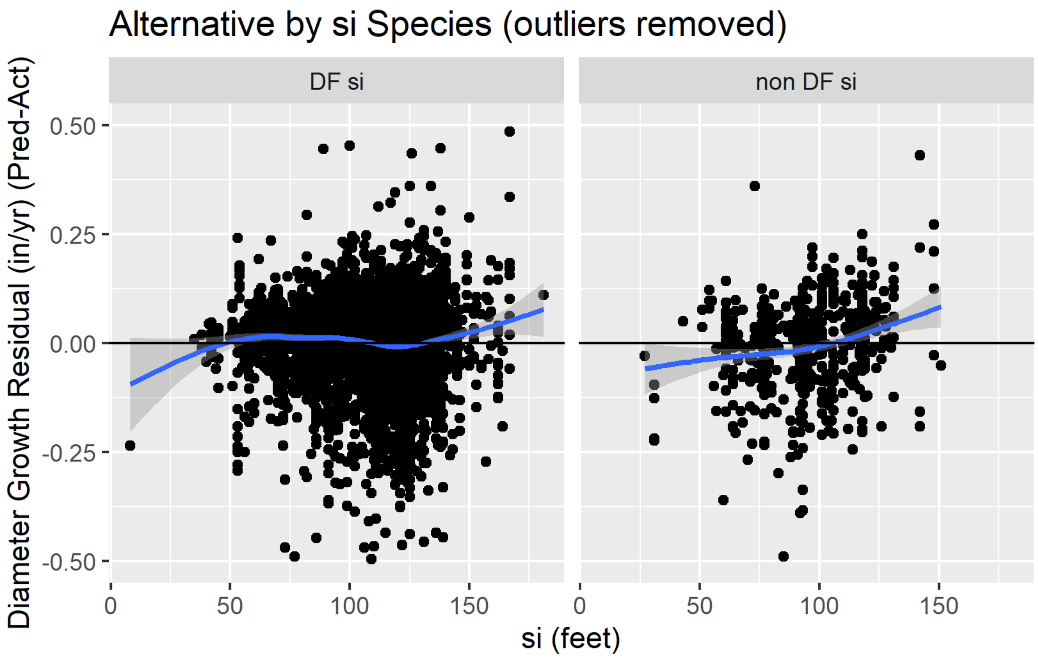
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
bal (ft^2/acre)



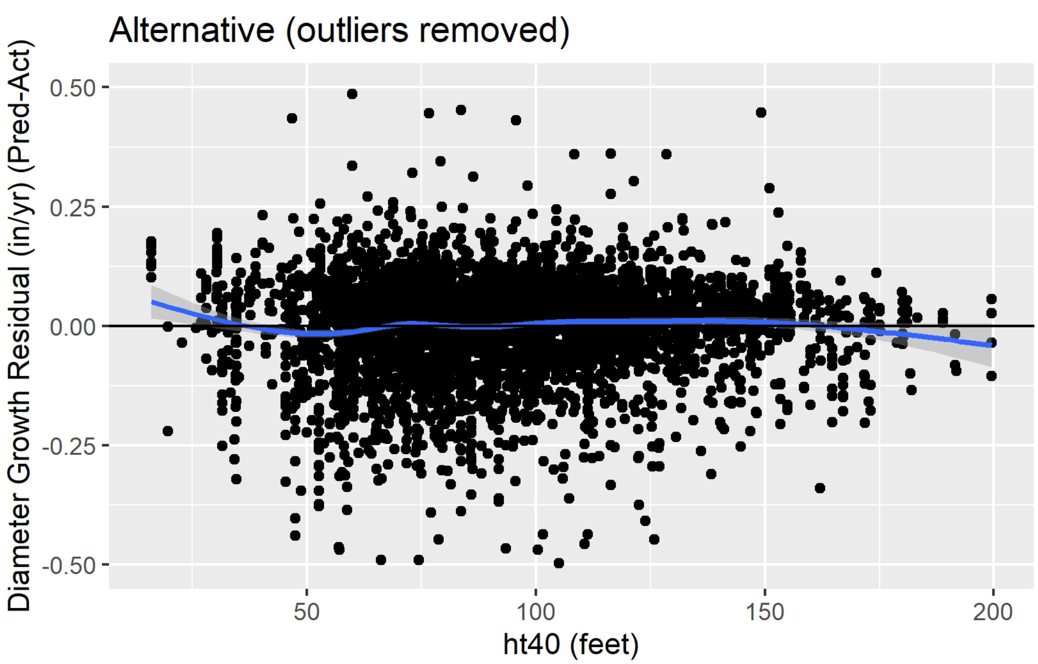
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)



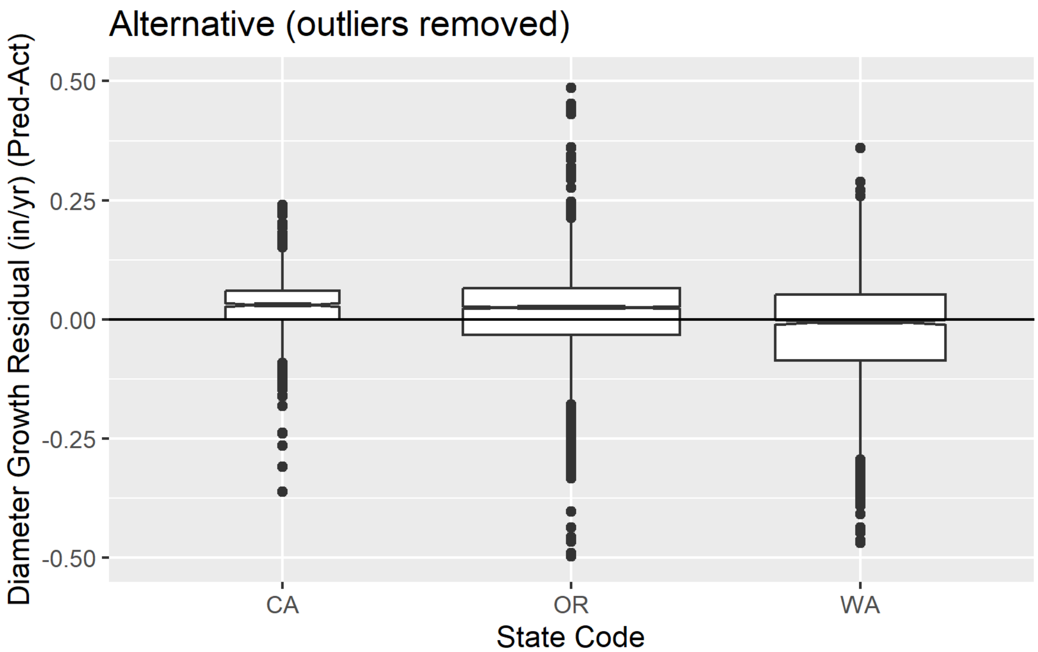
Alternative by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

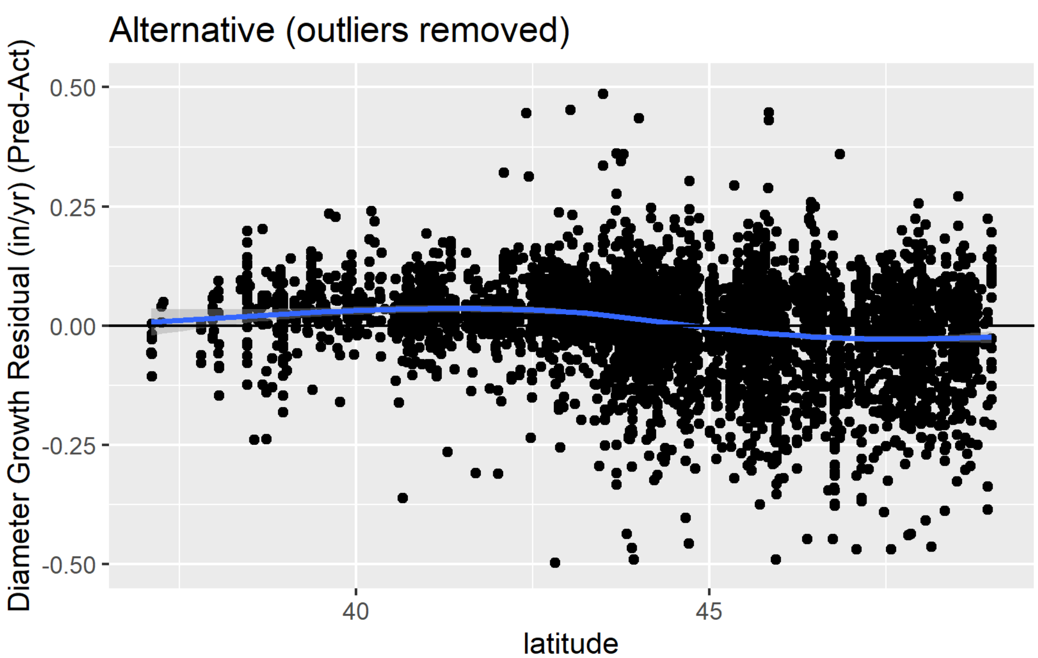


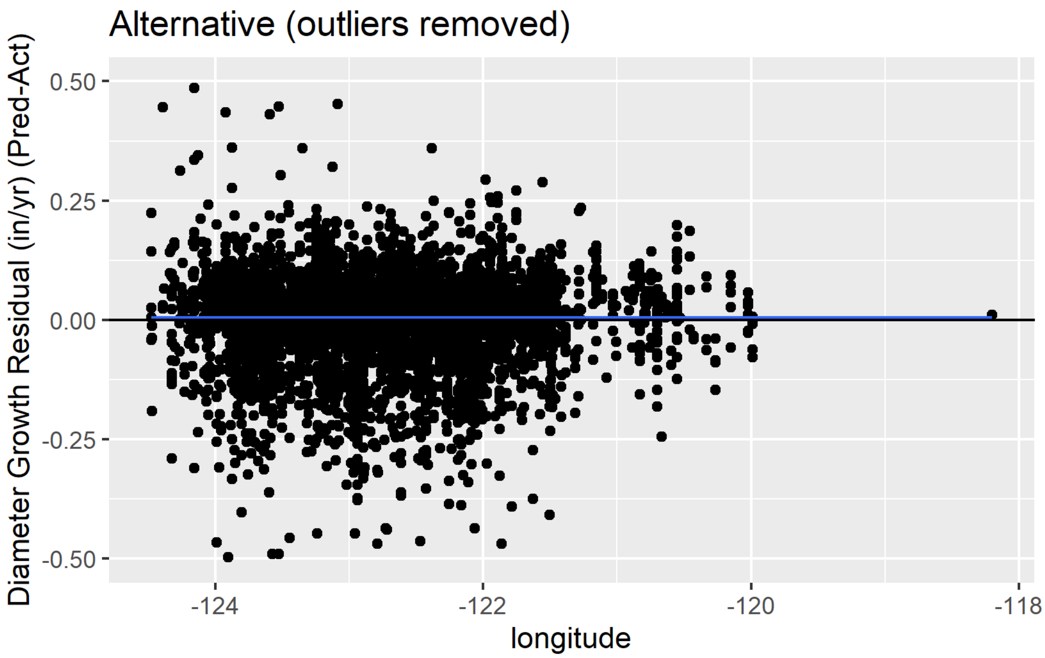
Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

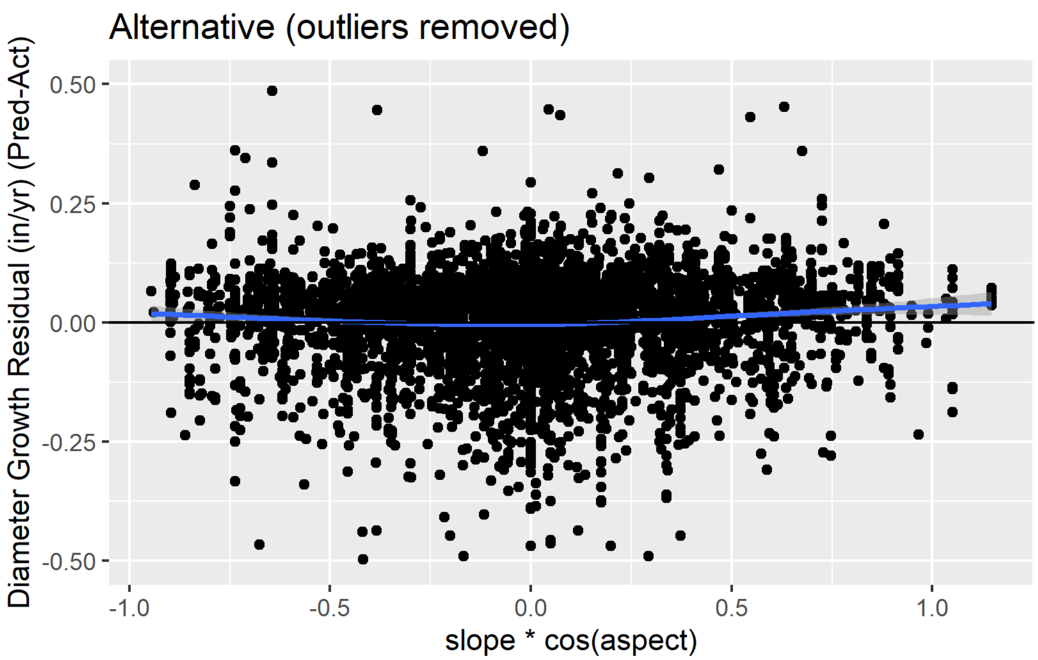


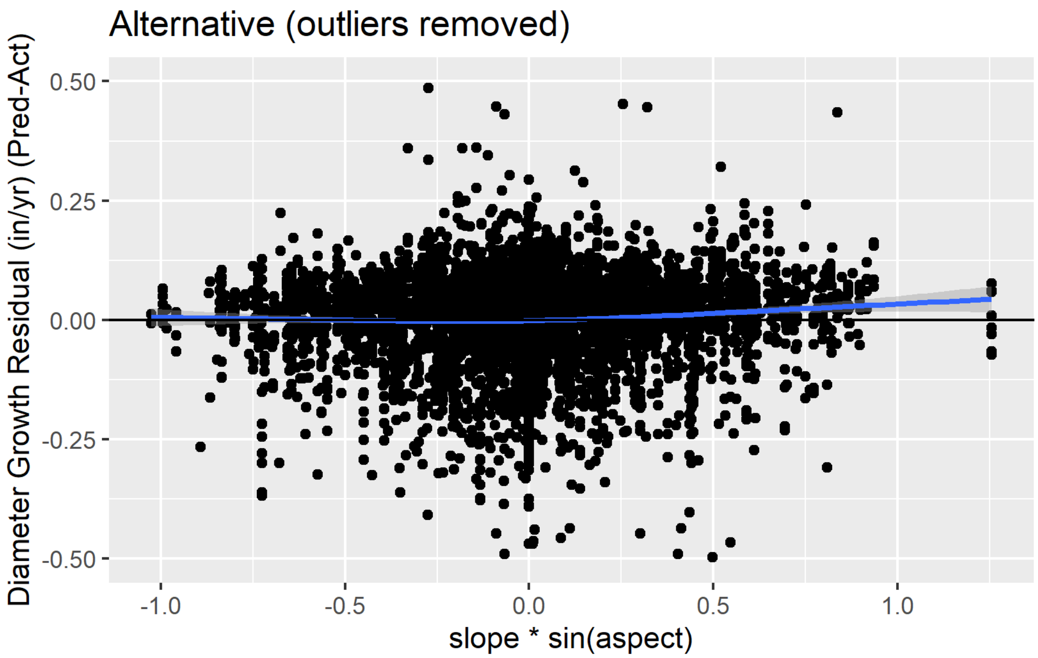
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
200
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
CA
OR
WA
State Code

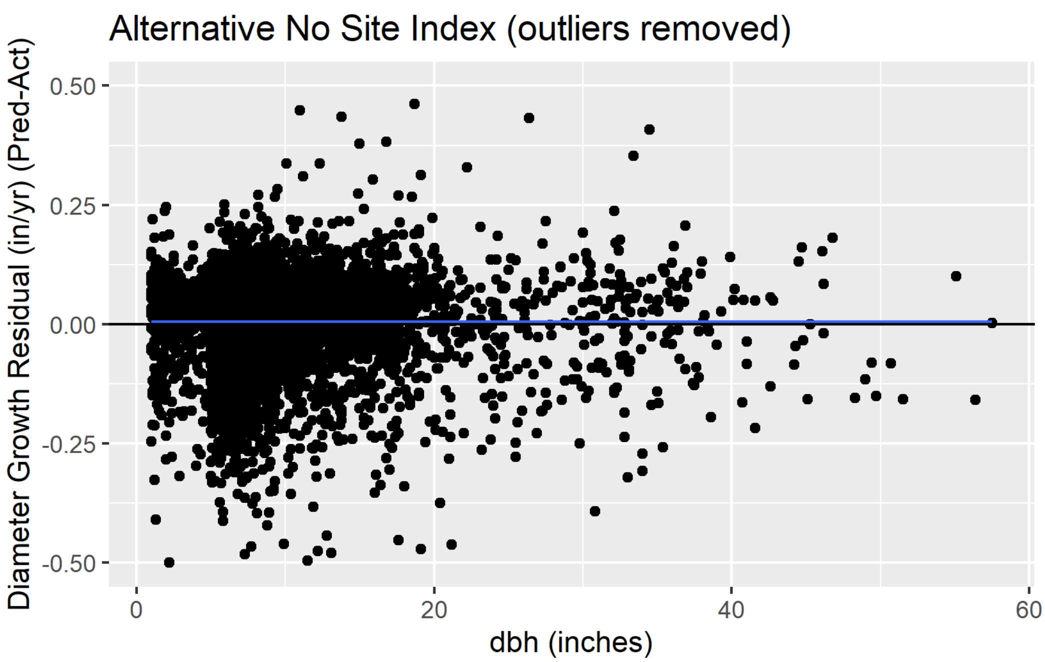

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
45
latitude

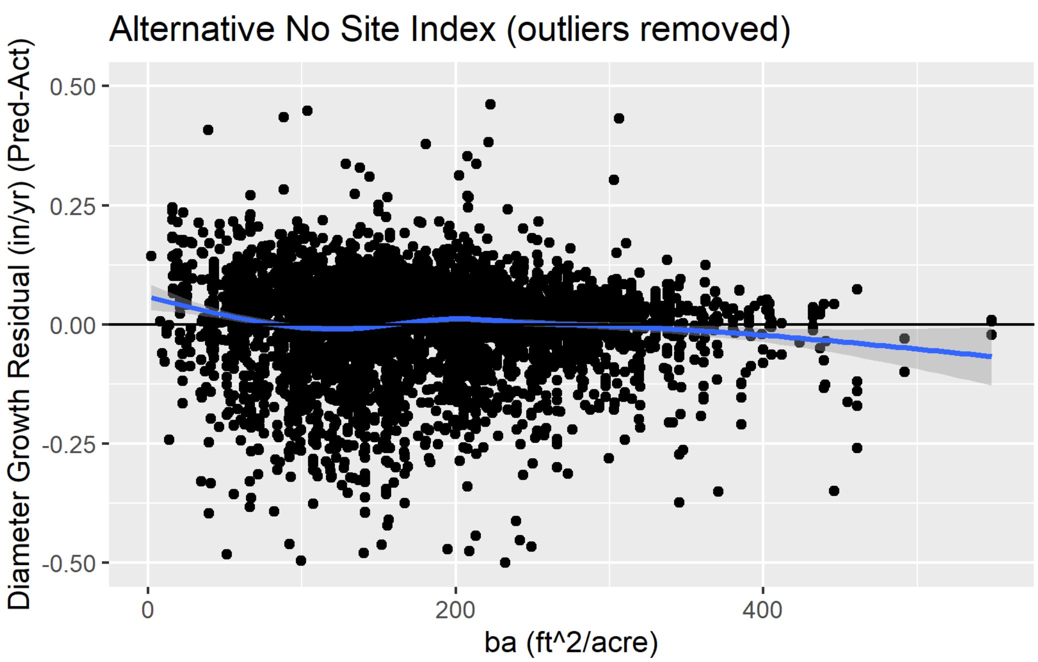

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-124
-122
-120
-118
longitude

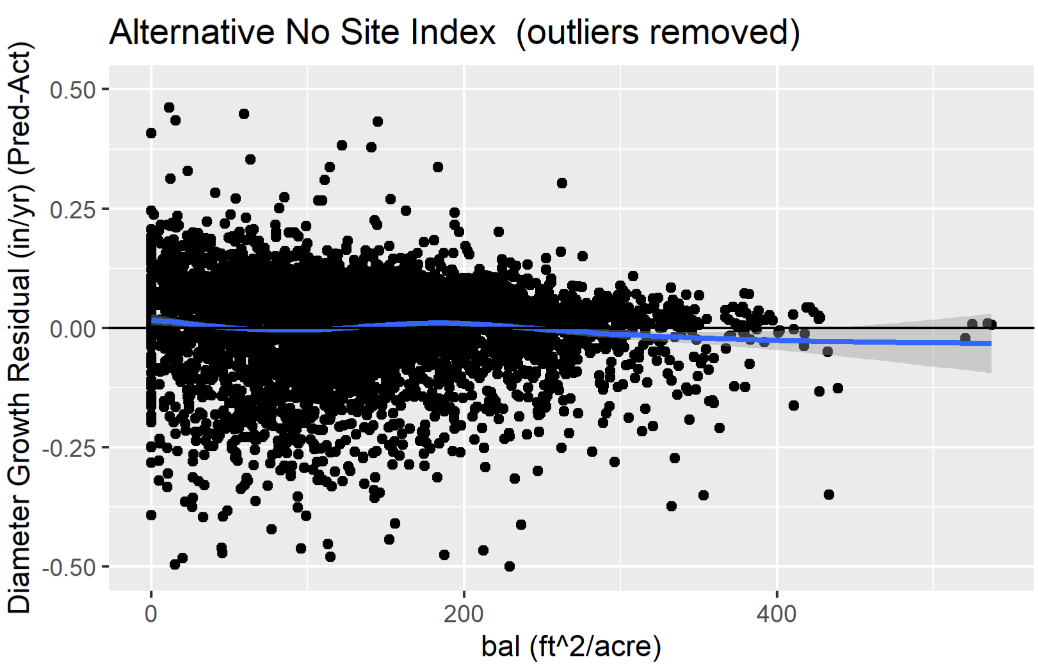

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)

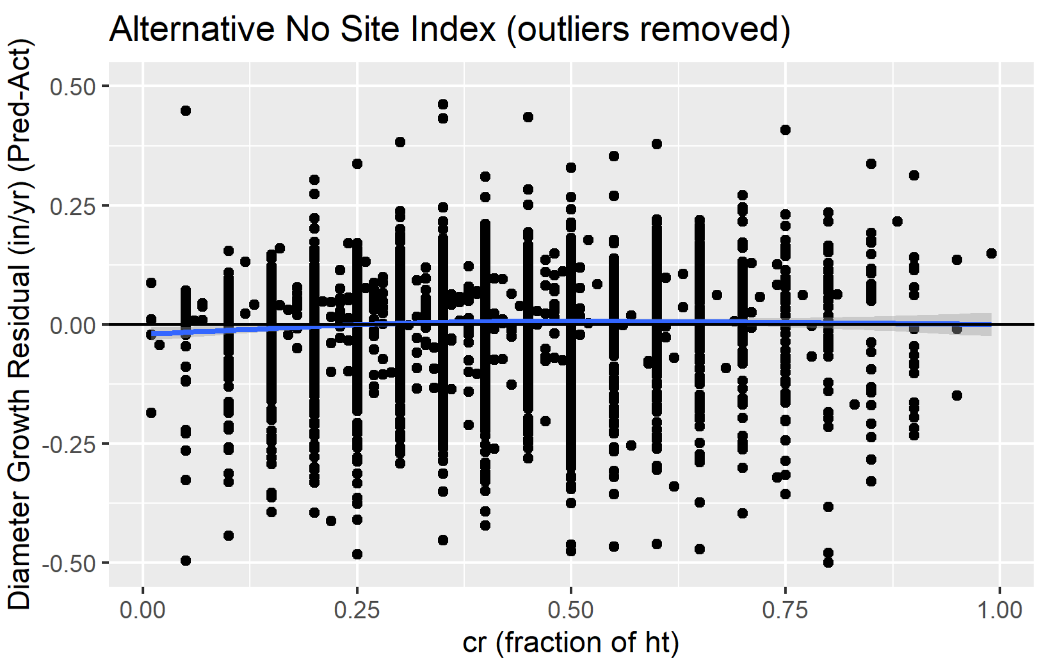


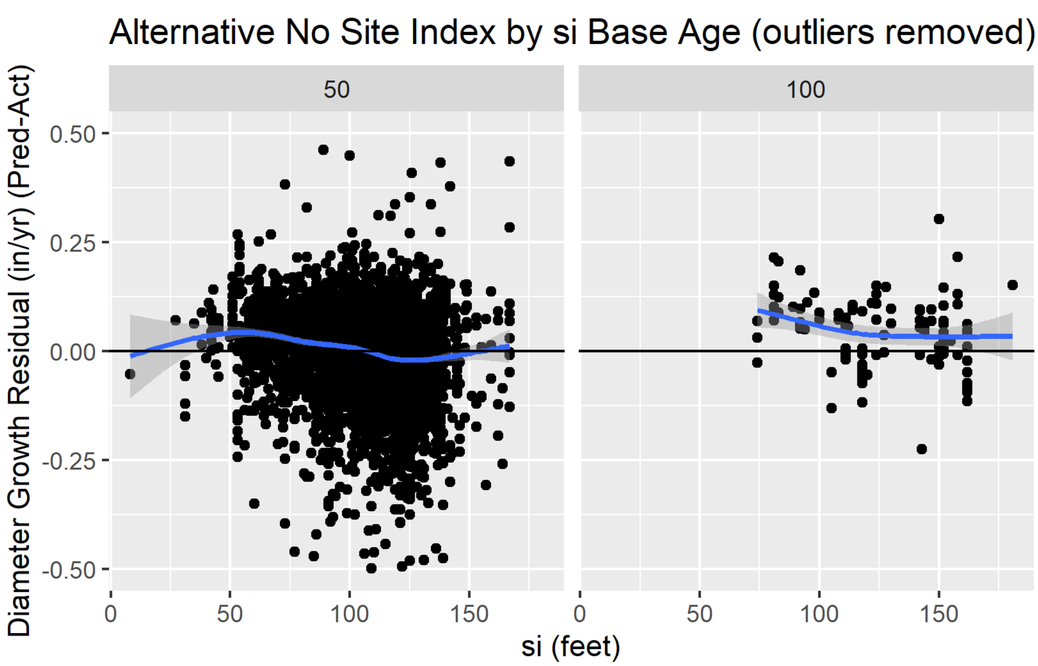
Residual Analysis for Equation 2

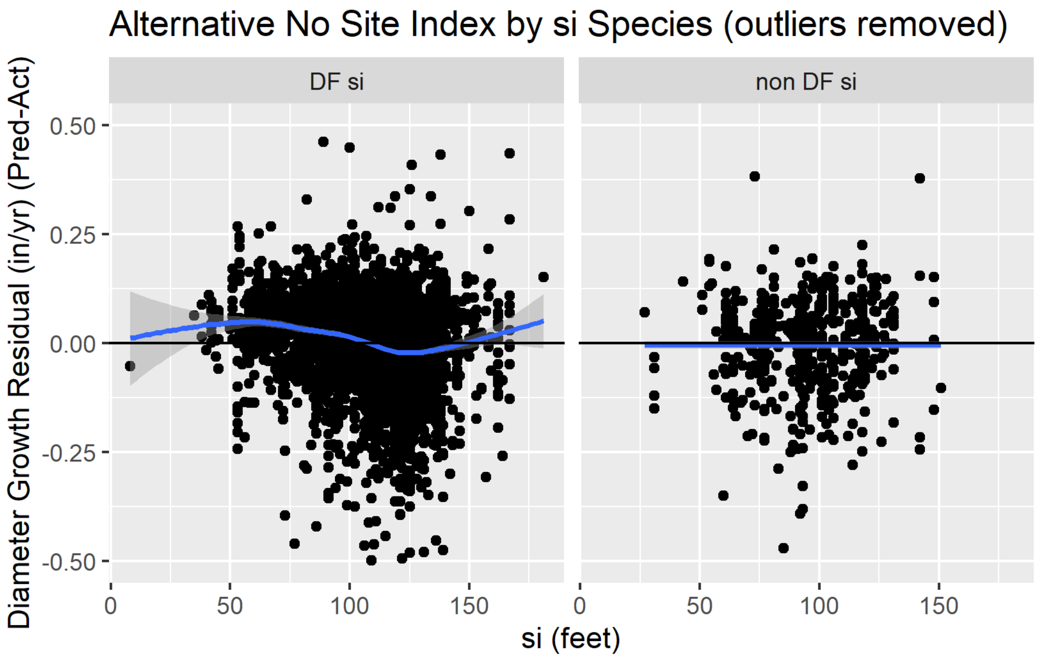


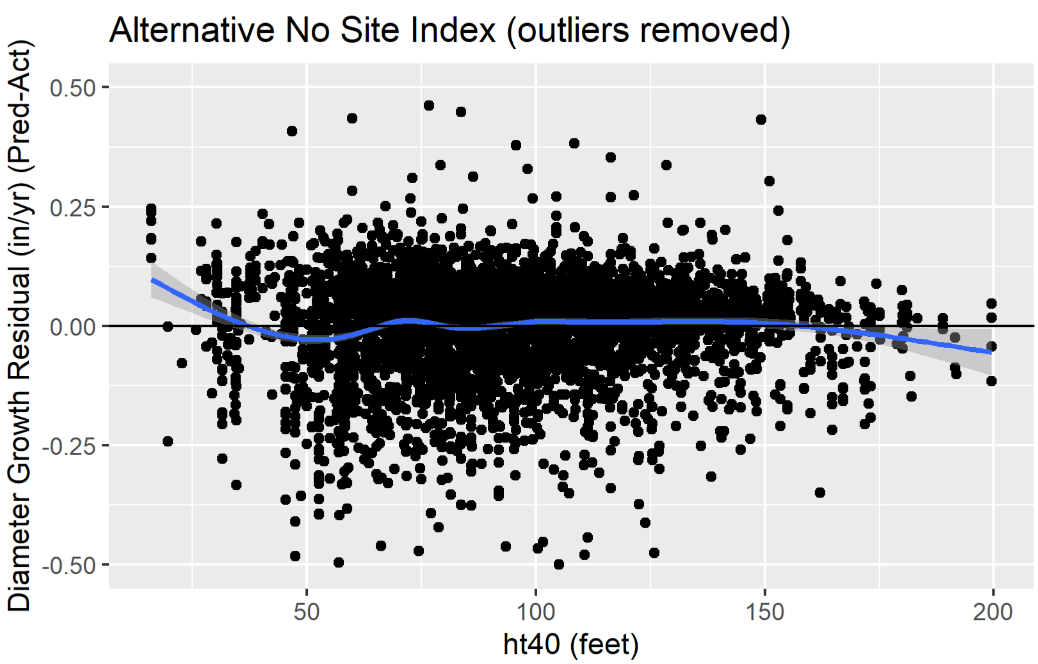

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
ba (ft^2/acre)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
bal (ft^2/acre)

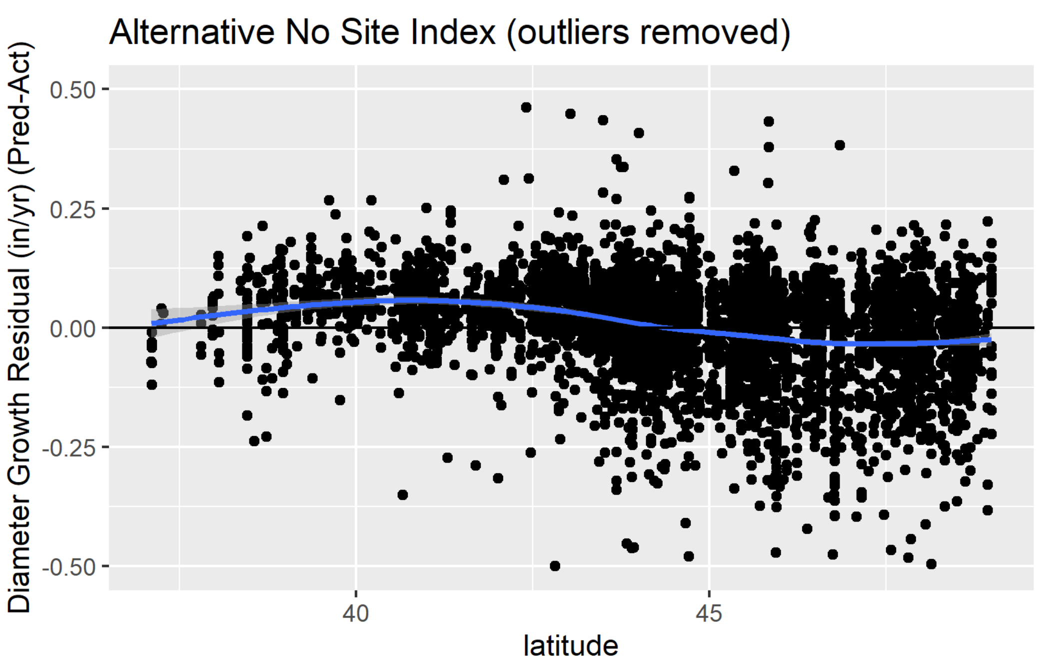

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

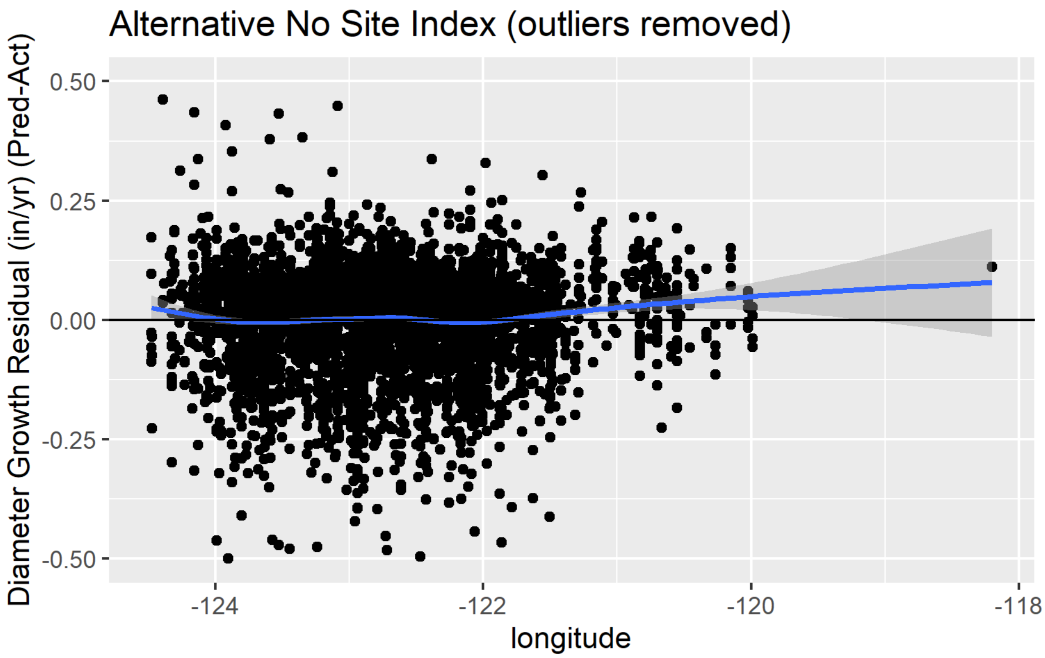

Alternative No Site Index by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

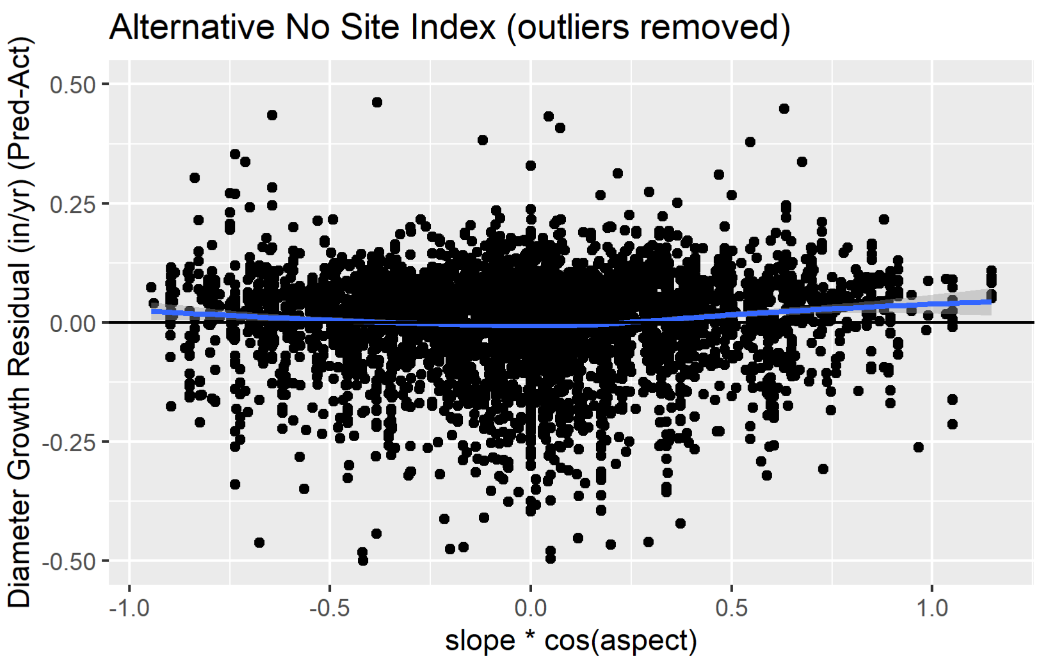

Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

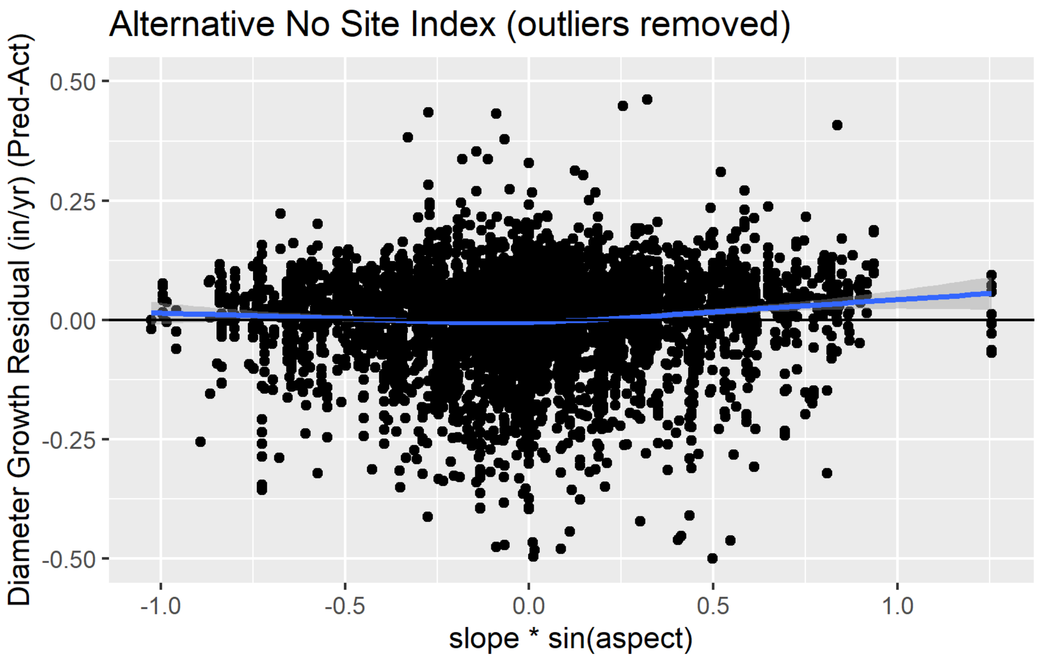

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
200
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
CA
OR
WA
State Code

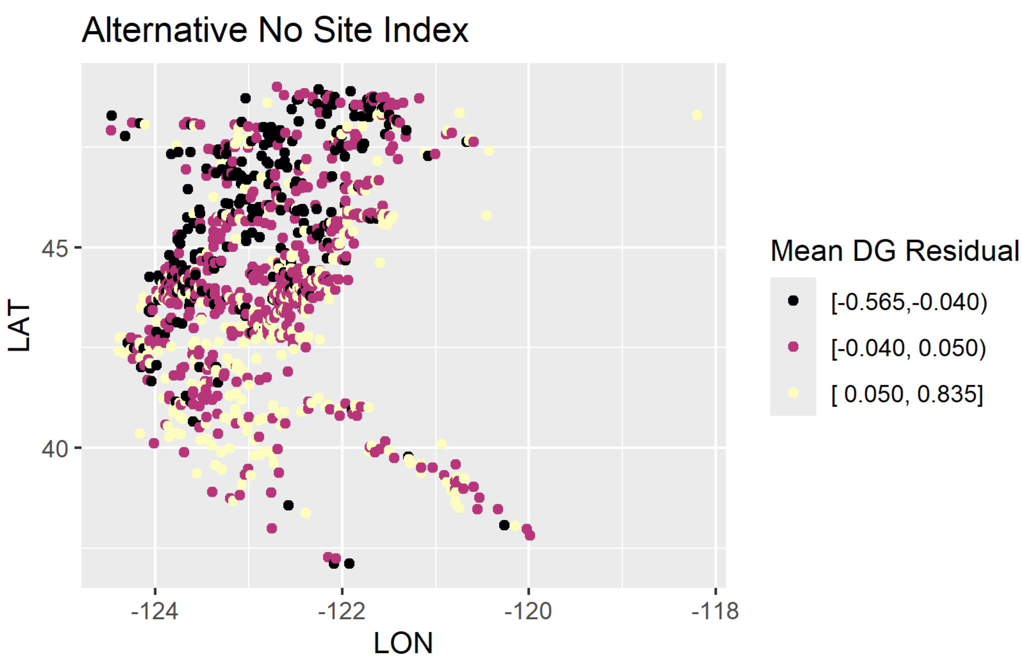

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
45
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-124
-122
-120
-118
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)



Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)



Alternative No Site Index
LAT
45
40
-124
-122
-120
-118
LON
Mean DG Residual
[-0.565,-0.040)
[-0.040, 0.050)
[0.050, 0.835]

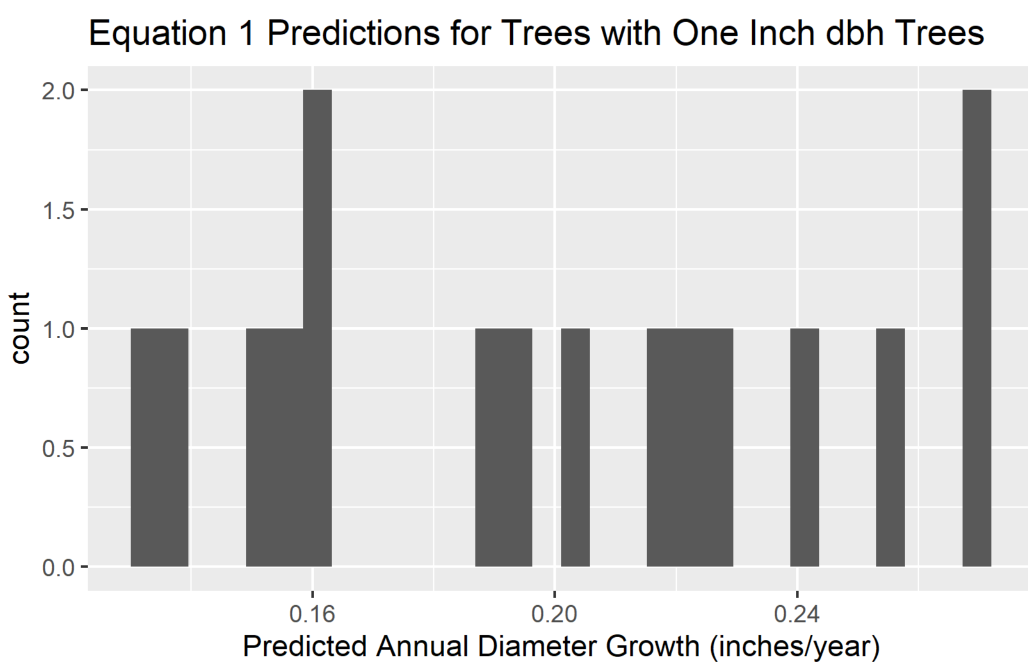
Table 3: Independent Variables for One Inch dbh Trees

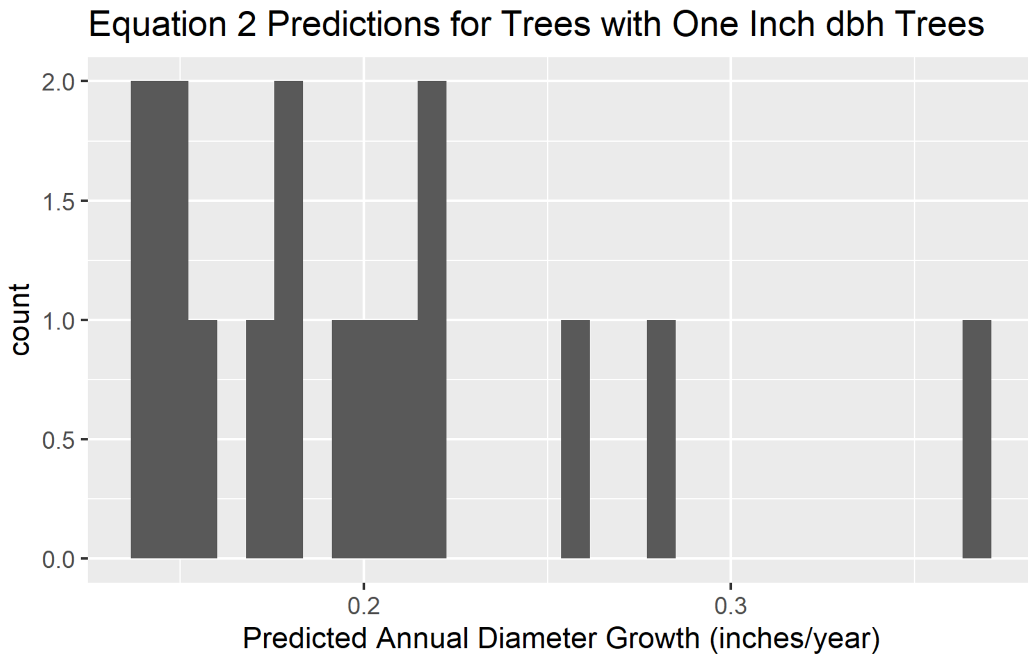
Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	16	122	76	16	67	157	268
bal	16	121	76	15	67	157	267
ht	16	16	2.9	8	15	17	22
cr	16	0.43	0.24	0.1	0.25	0.6	0.8
si	16	97	32	27	76	122	140

Discussion

Removing `si` degrades the fit marginally and probably not significantly. The alternative model fits the data well given the limited data. There is a trend with latitude that seems to be significant.

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
count
2.0
1.5
1.0
0.5
0.0
0.2
0.3
Predicted Annual Diameter Growth (inches/year)