Alternative Douglas-fir Diameter Growth

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Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 11 states listed in the native range of Douglas-fir in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Douglas-fir (FIA species code 202), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Douglas-fir Growth Observations by State

State	Observations
AZ	1143
CA	18605
CO	6774
ID	13789
MT	23645
NM	2440
NV	27
OR	69368
UT	2348
WA	47321
WY	865

[1]Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Douglas-fir site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 31 different site index equations for 25 species. Douglas-fir site index comprises 85% of the observations. There are 3 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7} + \beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, en
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
       AIC      BIC    logLik
  542828.5 542909.6 -271406.3

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3 Residual
StdDev: 0.05282021  1.03831

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value   Std.Error     DF    t-value p-value
B0 -6.516475 0.030992908 186315 -210.25698       0
B1 -0.594234 0.003152114 186315 -188.51933       0
B2 -0.091646 0.005197031 186315  -17.63424       0
B3  0.961606 0.024367852 186315   39.46207       0
B4  1.819104 0.007812019 186315  232.85970       0
B5  0.631855 0.010100556 186315   62.55646       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.028
B2 -0.387  0.421
B3 -0.179  0.007 -0.001
B4 -0.520  0.117  0.488 -0.035
B5 -0.360  0.409  0.992  0.001  0.438

Standardized Within-Group Residuals:
         Min           Q1          Med           Q3          Max
-87.36415790  -0.41649943  -0.09128716   0.34421807  19.12625030

Number of Observations: 186325
Number of Groups: 5

$SIINT
```

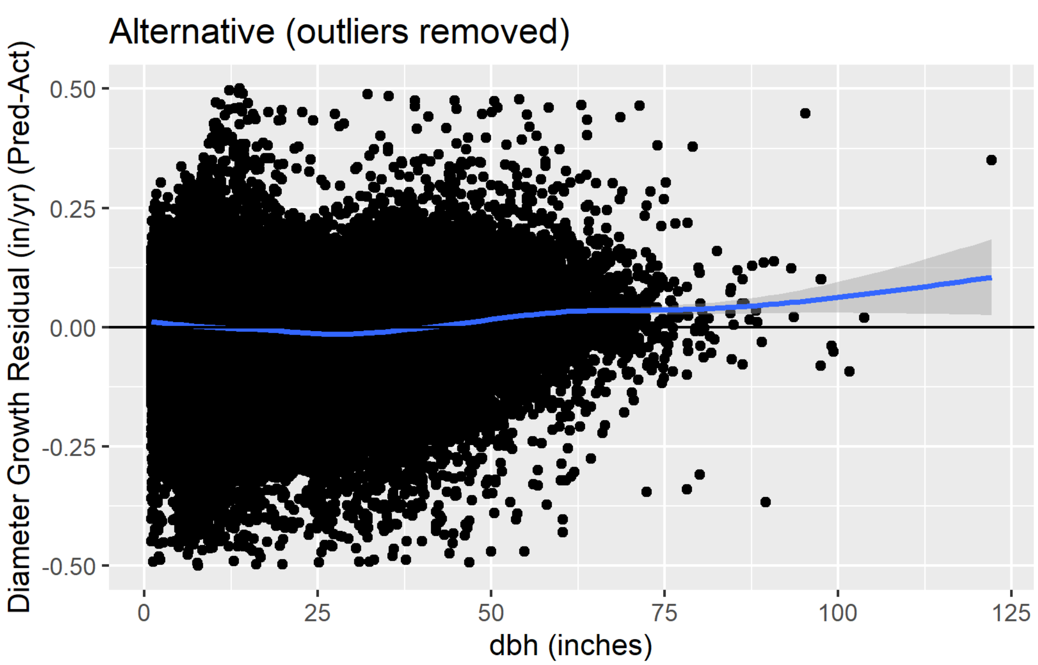
```
                     B3
50.FALSE    0.06108291
80.FALSE    0.02912455
100.FALSE -0.03600569
50.TRUE     0.03012468
100.TRUE  -0.08432644

Residual Standard Error: 1.03831003839467 on 186315 degrees of freedom, AIC: 542828.5
```

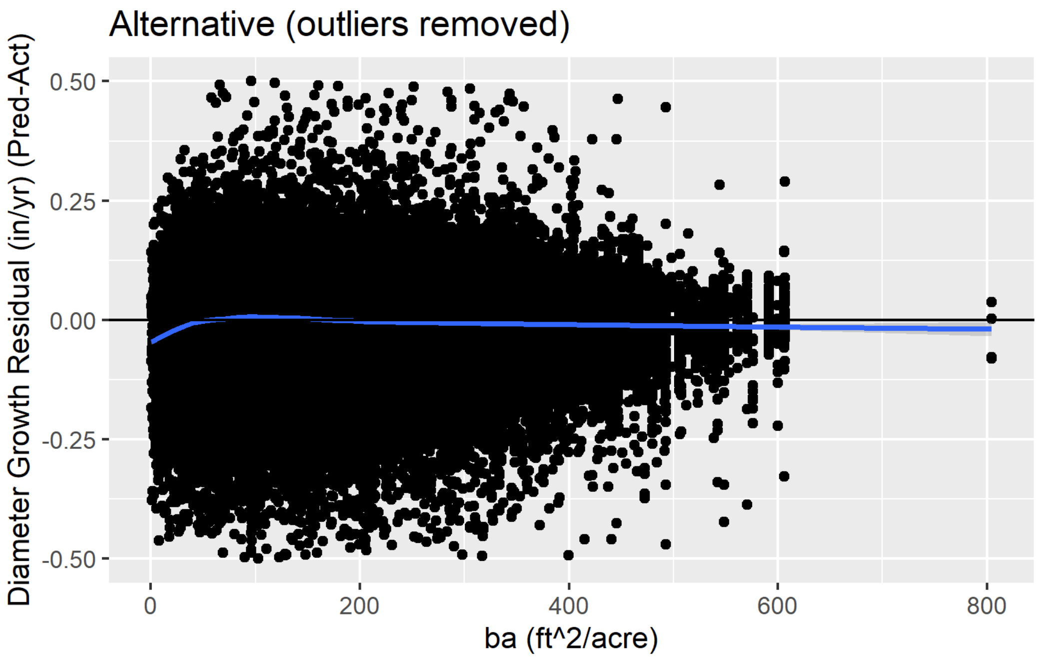
and for Equation 2:

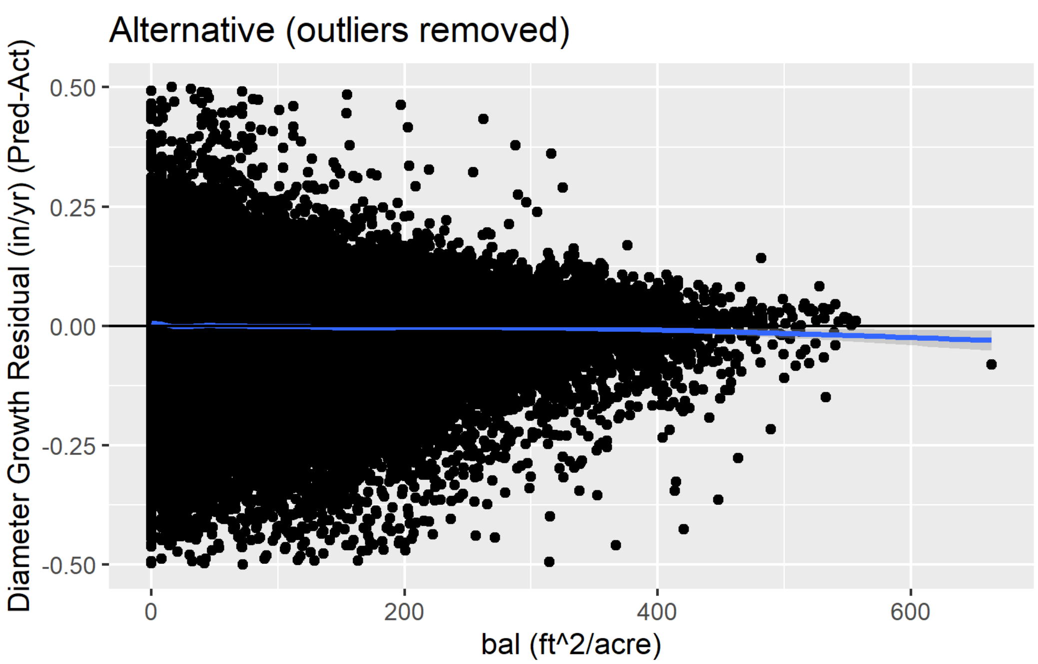
	Coef.	Std. error	t-stat.	p
B0	-3.4527532	0.0248363	-139.02030	0
B1	-0.7036656	0.0038816	-181.28456	0
B2	-0.0880682	0.0079539	-11.07231	0
B4	2.2152039	0.0083456	265.43383	0
B5	0.5636545	0.0158005	35.67311	0

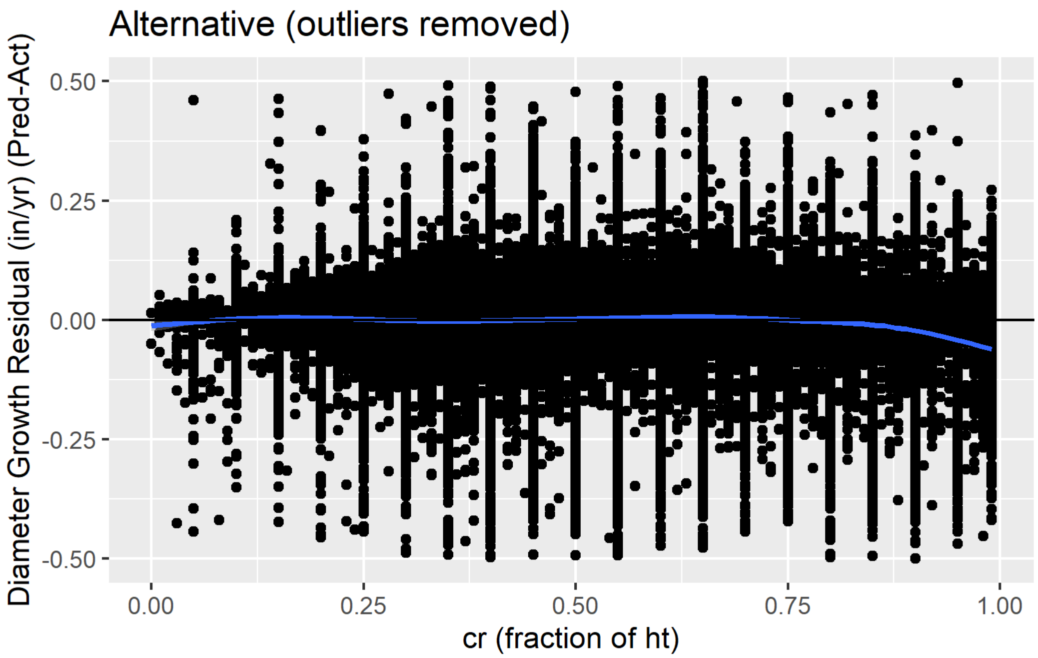
```
Residual Standard Error: 1.14763670588031 on 186320 degrees of freedom, AIC: 580090.1
```

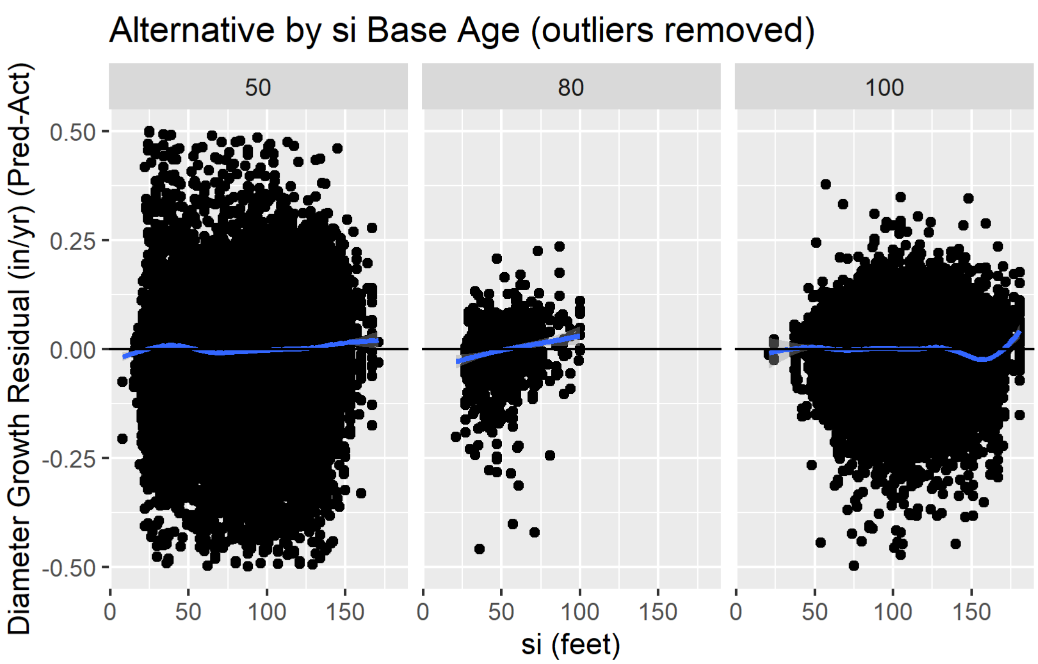
Residual Analysis for Equation 1

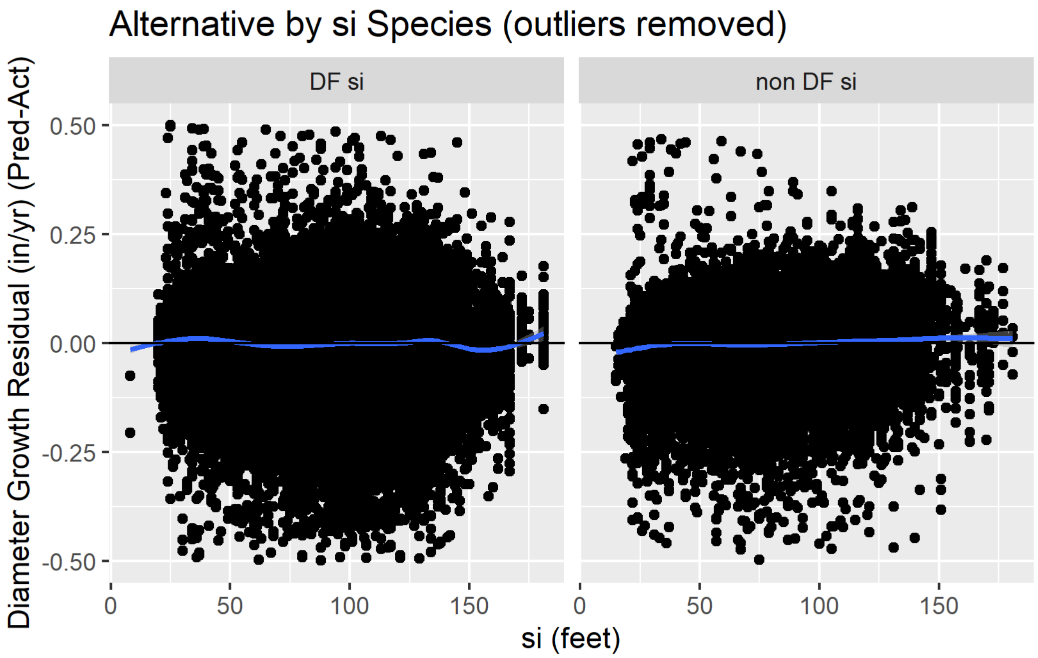


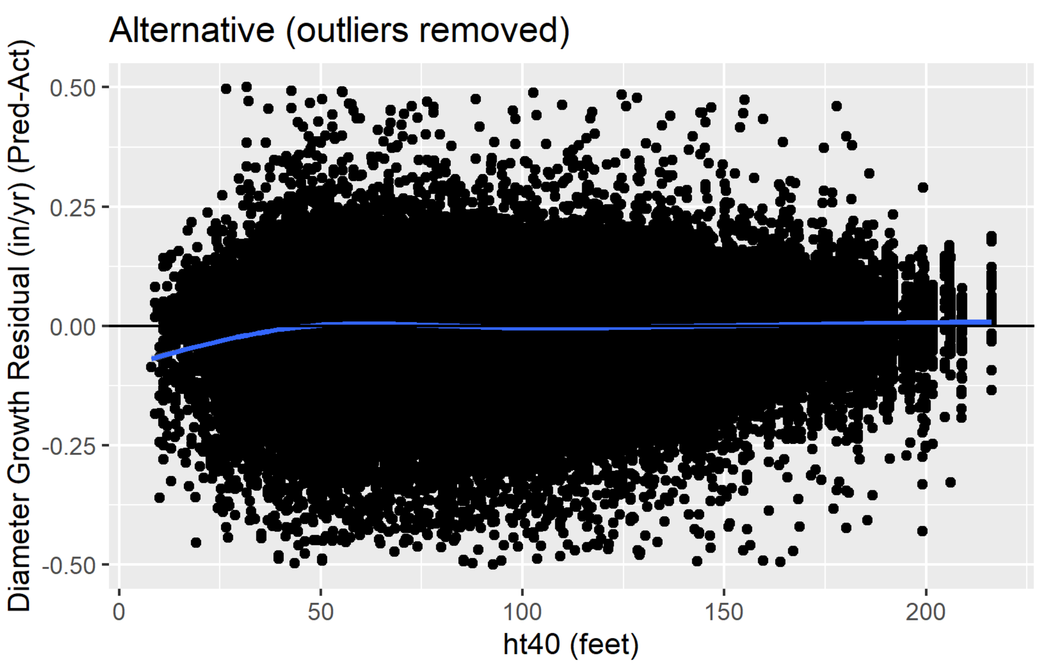

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
800
ba (ft^2/acre)

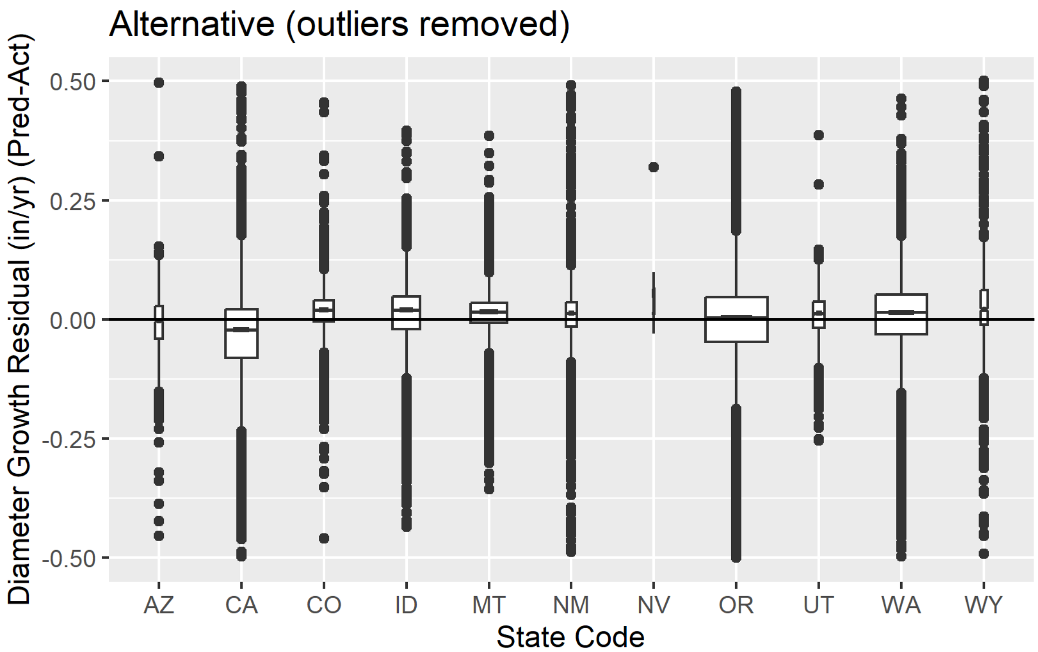

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
bal (ft^2/acre)

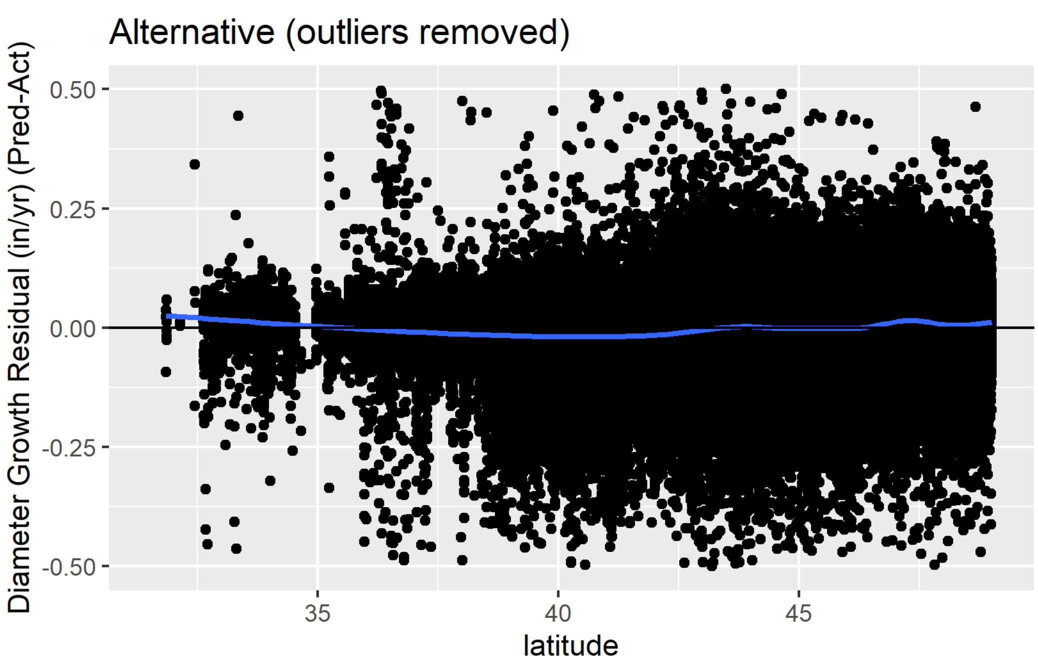

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

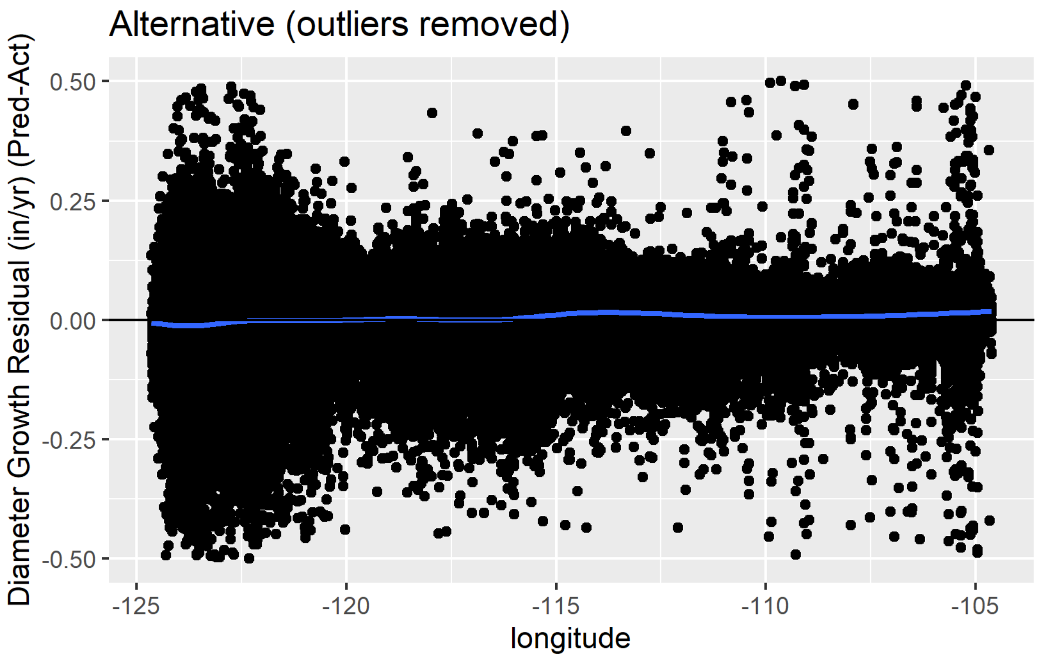

Alternative by si Base Age (outliers removed)
50
80
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

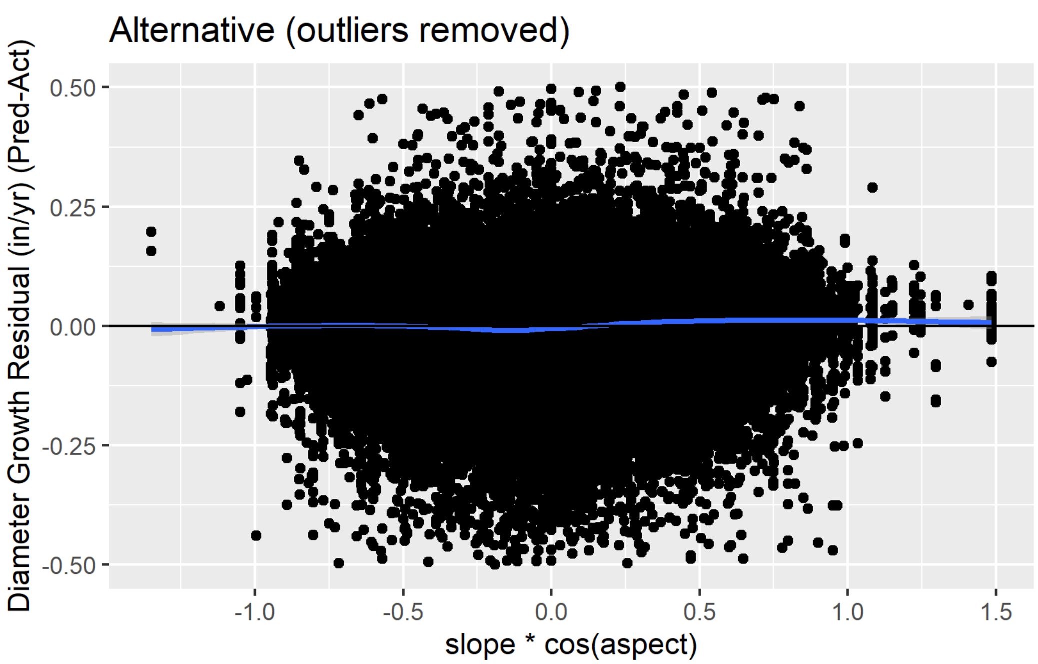

Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

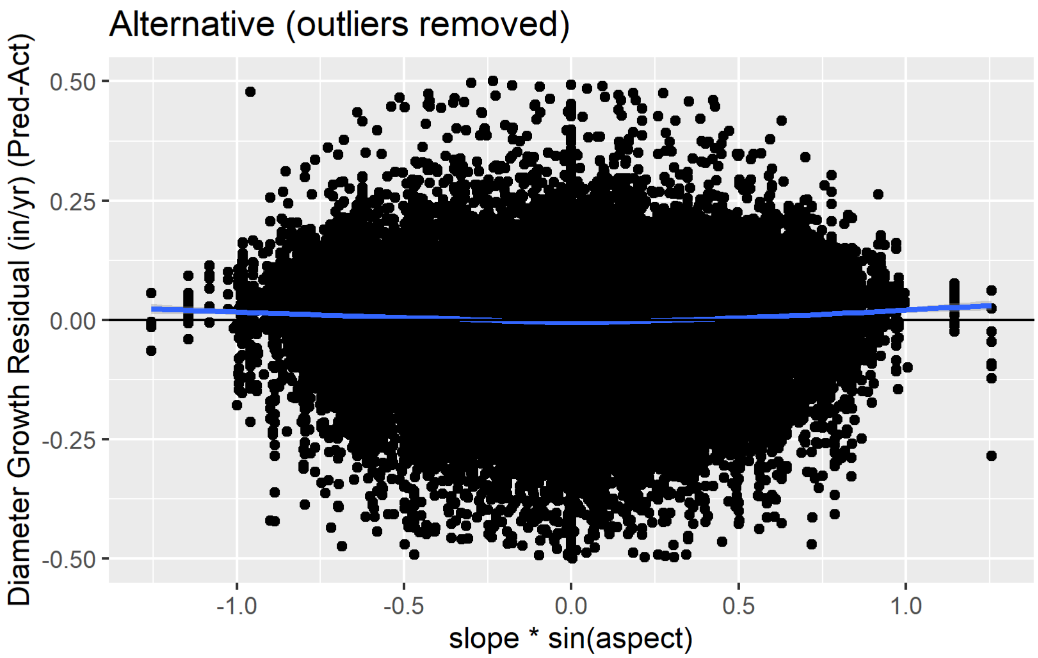

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
200
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
AZ
CA
CO
ID
MT
NM
NV
OR
UT
WA
WY
State Code

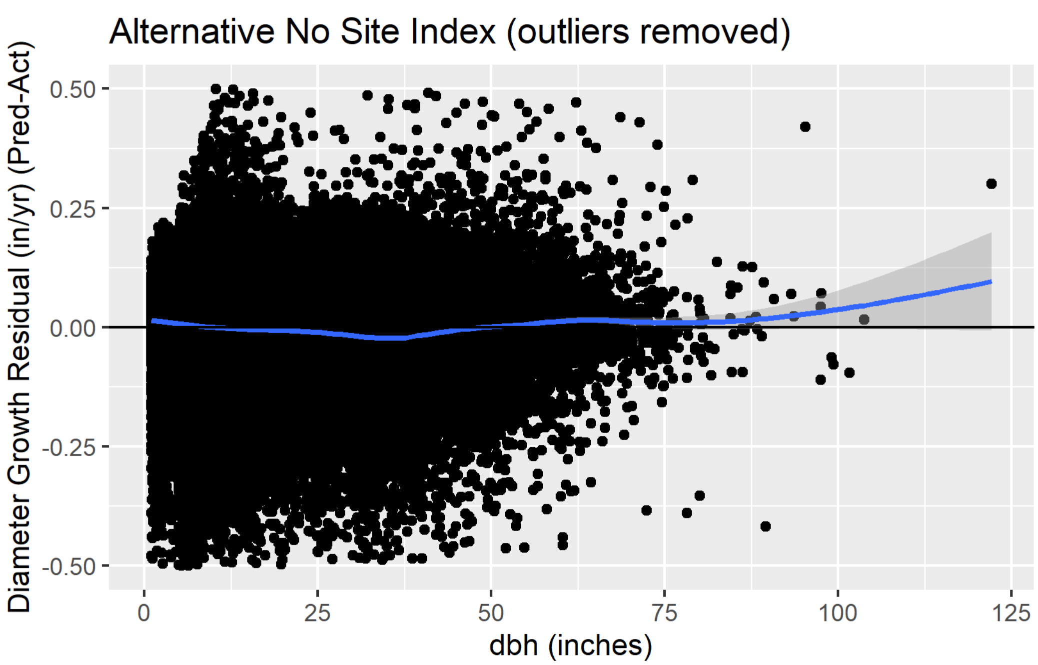

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
35
40
45
latitude

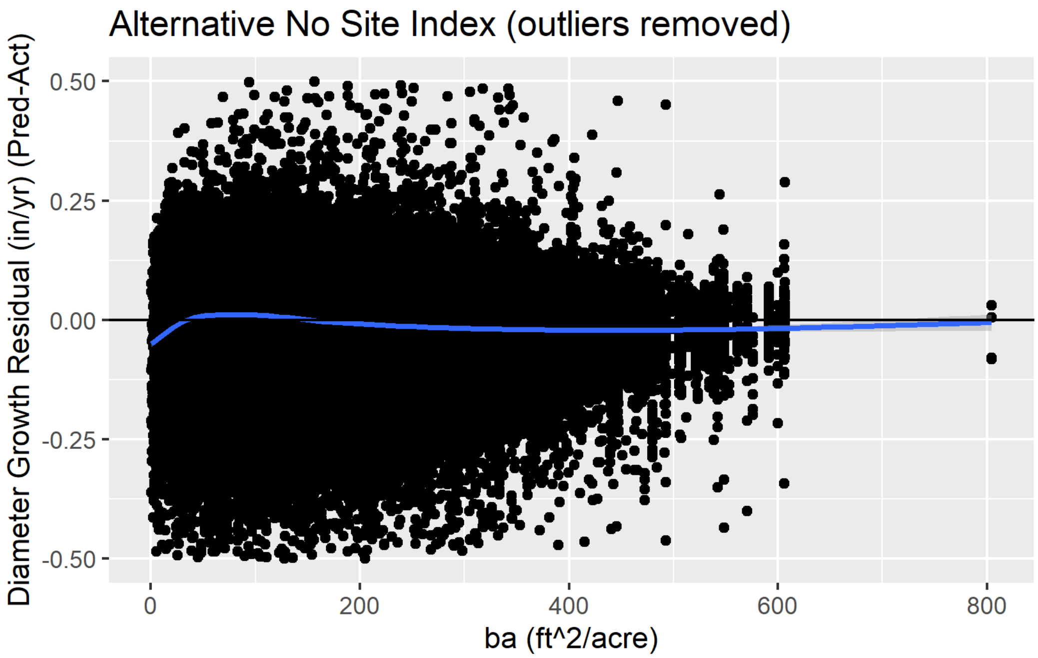

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-125
-120
-115
-110
-105
longitude


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
1.5
slope * cos(aspect)

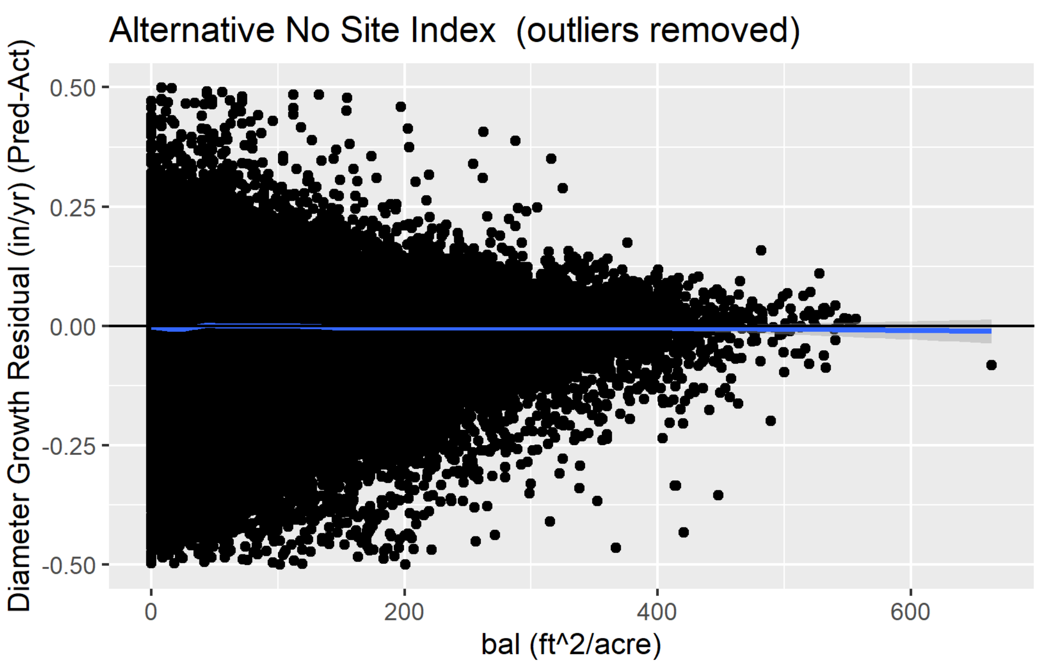


Residual Analysis for Equation 2

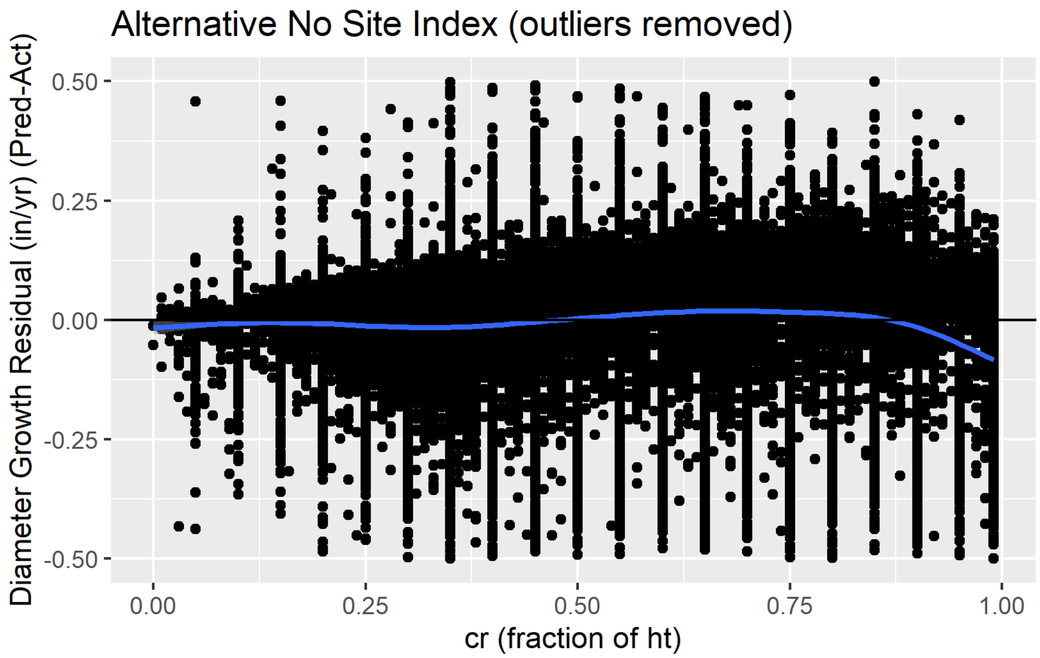


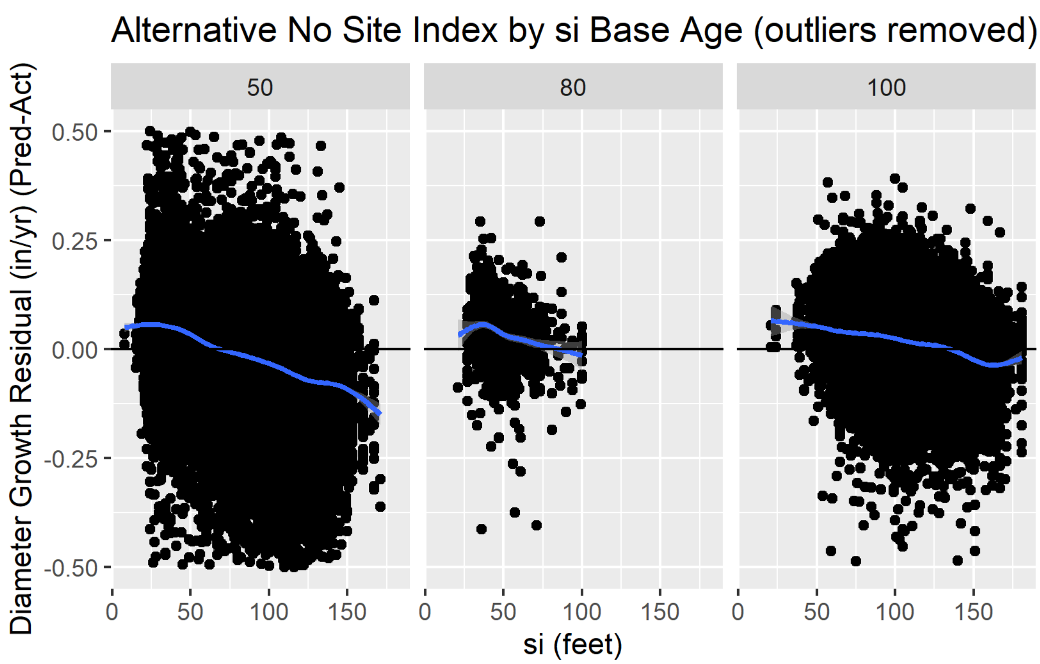


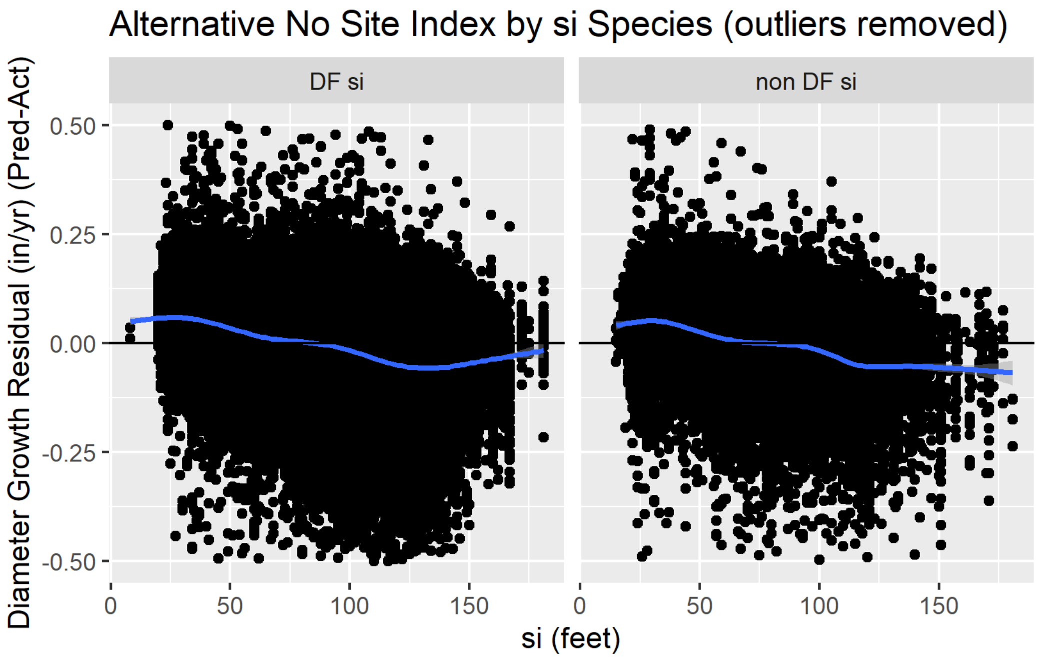
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
800
ba (ft^2/acre)

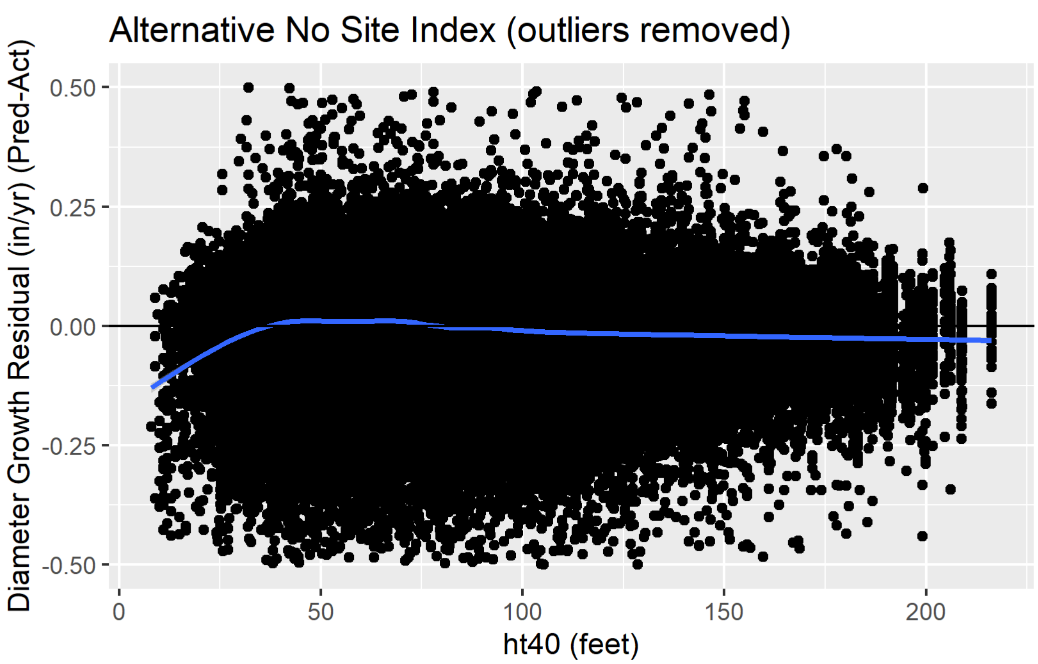


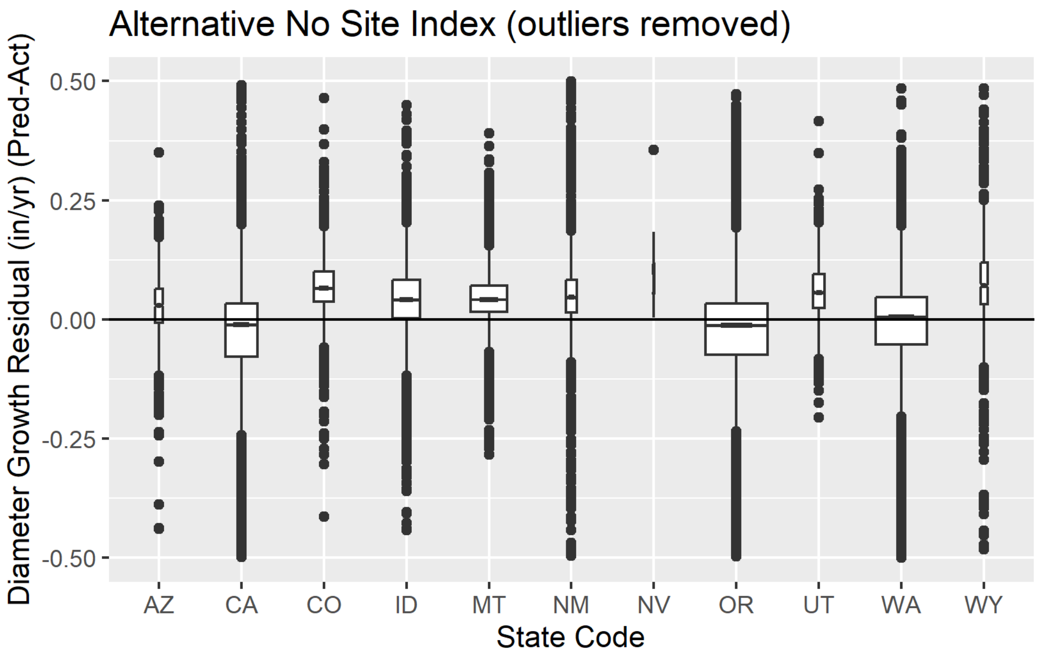
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
bal (ft^2/acre)

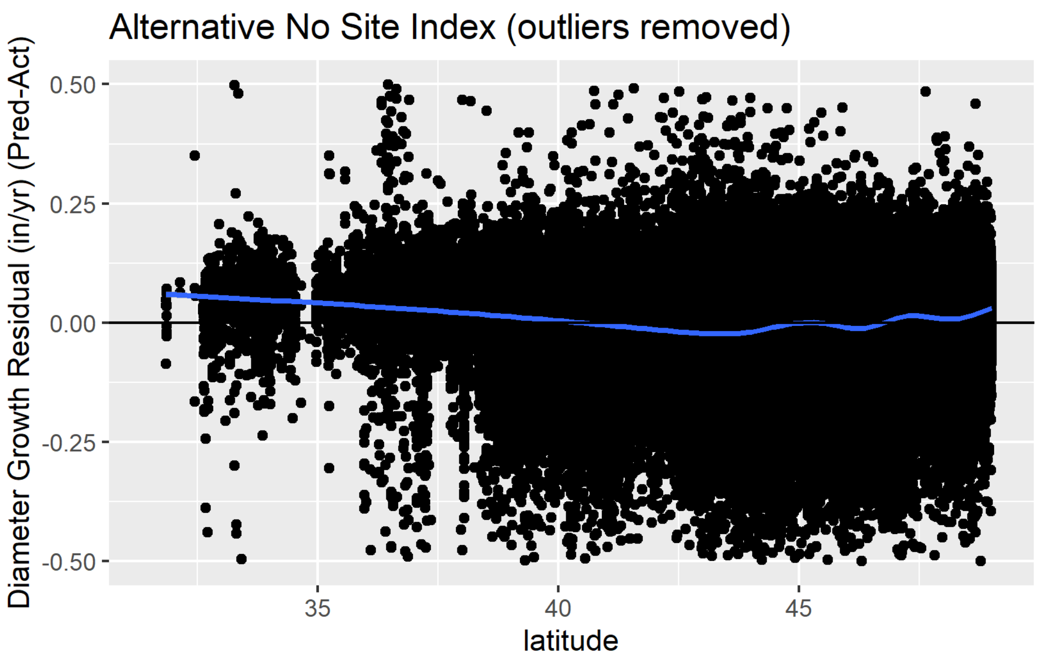

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

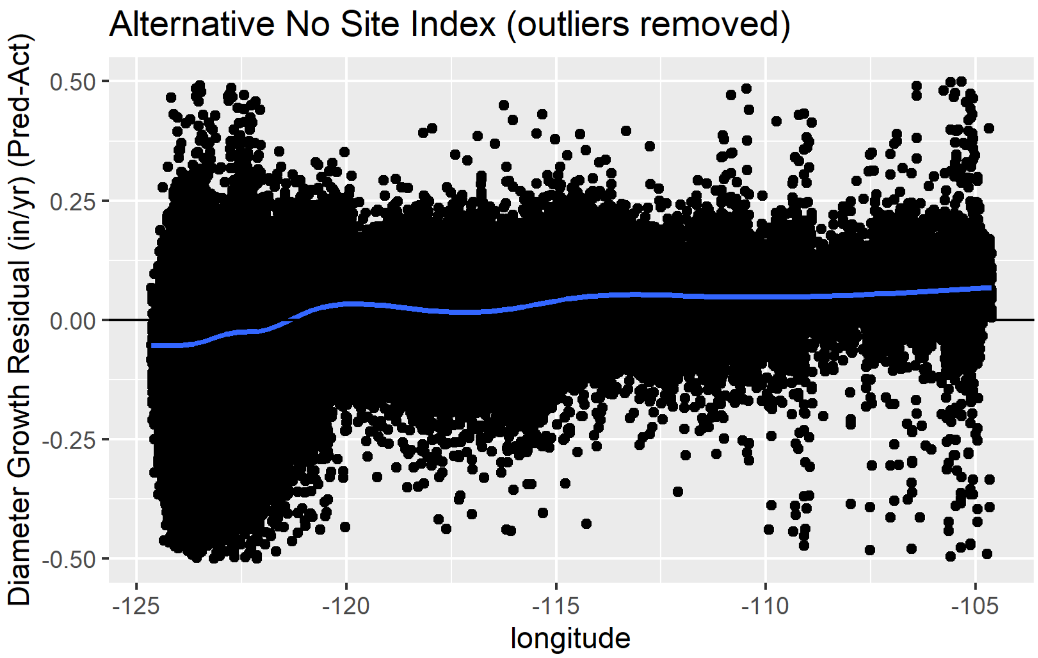

Alternative No Site Index by si Base Age (outliers removed)
50
80
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

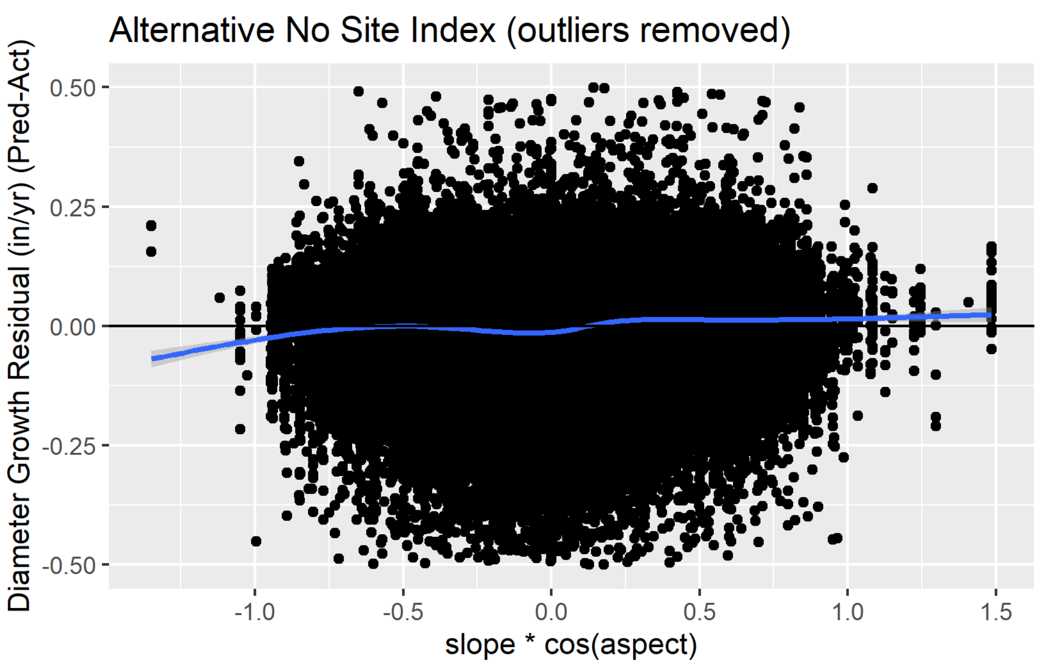

Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

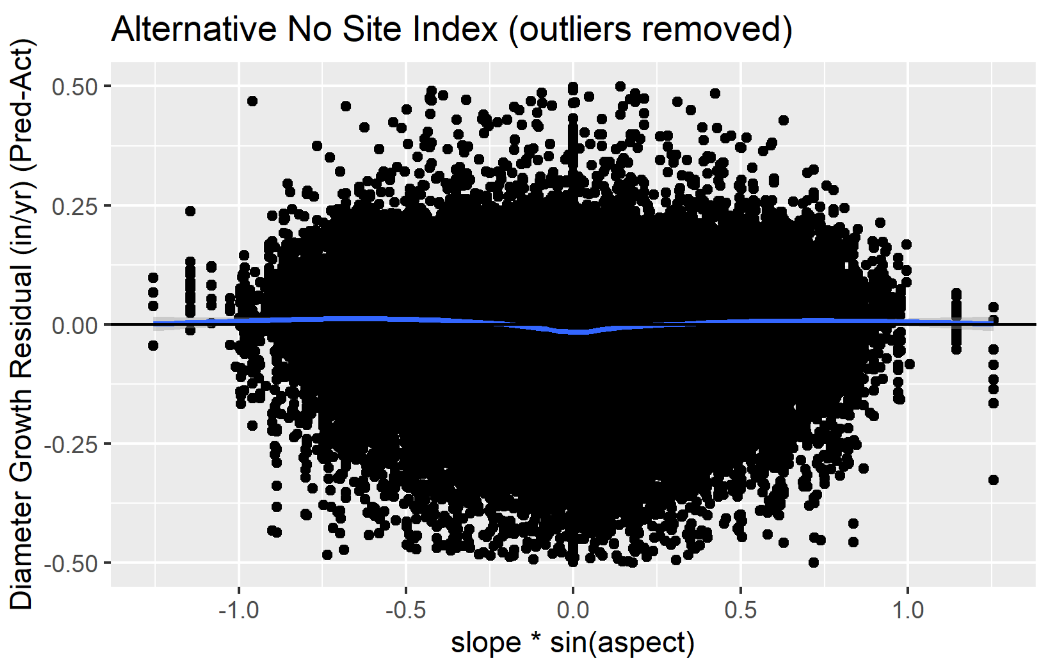

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
200
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
AZ
CA
CO
ID
MT
NM
NV
OR
UT
WA
WY
State Code

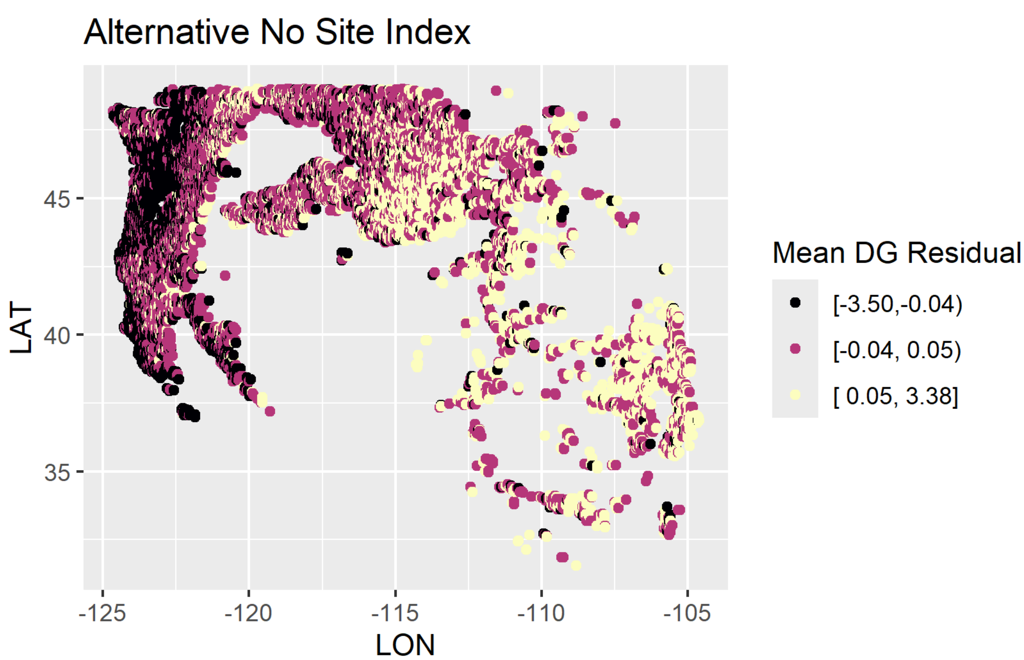

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
35
40
45
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-125
-120
-115
-110
-105
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
1.5
slope * cos(aspect)



Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)



Alternative No Site Index
LAT
45
40
35
-125
-120
-115
-110
-105
LON
Mean DG Residual
[-3.50,-0.04)
[-0.04, 0.05)
[0.05, 3.38]

Table 3: Independent Variables for One Inch dbh Trees

Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	471	113	70	0.41	61	150	472
bal	471	112	70	0	61	150	472
ht	471	9.3	1.9	5	8	10	20
cr	471	0.55	0.25	0.05	0.35	0.8	0.99
si	471	70	30	20	44	90	159

Discussion

Removing `si` degrades the fit significantly. When `si` in its random variable form is used, the bias with the height of the largest 40 trees per acre (`ht40`) is reduced for small values. There is an average bias by state for both equations, but interestingly, the bias for states with small sample sizes are uniformly over-predicted in the equation without site index. We might expect this given that these states are on the edges of Douglas-fir's range.

Equation 1 residual graphs show that residual trends with latitude and longitude are eliminated. This suggests that the admixture of site index values in the data set are doing a reasonable job at localization, albeit in a non-repeatable manner. It also suggests that latitude and longitude could be incorporated into Equation 2.

Equation Behavior for Very Small Trees