# Alternative Ponderosa Pine Diameter Growth

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**Data**

We extracted and processed Forest Inventory and Analysis (FIA) data from 14 states listed in the native range of Ponderosa Pine in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Ponderosa Pine (FIA species code 122), and remeasurement intervals $\geq 5$ years we get the observations in Table 1.

Table 1: Ponderosa Pine Growth Observations by State

| State | Observations |
|---|---|
| AZ | 9614 |
| CA | 8941 |
| CO | 4224 |
| ID | 1981 |
| MT | 4841 |
| ND | 36 |
| NE | 1505 |
| NM | 4997 |
| NV | 33 |
| OR | 36093 |
| SD | 9925 |
| UT | 732 |
| WA | 6489 |
| WY | 1317 |

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[1]Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

**Alternative Model Formulation**

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since $\beta_1$ is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Ponderosa Pine site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 25 different site index equations for 31 species. Ponderosa Pine site index comprises 80% of the observations. There are 3 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7} + \beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

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[2]Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending dbh. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,     startCR, e
  Data: pp_subset %>% mutate(SIINT = interaction(as.factor(pp_subset$SIBASE),     as.factor
       AIC      BIC     logLik
  290722.2 290797.5 -145353.1

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3 Residual
StdDev: 0.07393882 1.200779

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value  Std.Error    DF   t-value p-value
B0 -5.308588 0.08171080 90718 -64.96801       0
B1 -1.101697 0.01130919 90718 -97.41608       0
B2 -0.174059 0.01549898 90718 -11.23035       0
B3  1.039123 0.03780961 90718  27.48303       0
B4  1.369188 0.01012419 90718 135.23919       0
B5  0.610714 0.01697746 90718  35.97201       0
 Correlation:
    B0     B1     B2     B3     B4
B1 -0.125
B2 -0.392  0.356
B3 -0.384  0.009  0.004
B4 -0.345 -0.062  0.430 -0.106
B5 -0.368  0.304  0.989  0.013  0.373

Standardized Within-Group Residuals:
        Min           Q1          Med          Q3          Max
-66.30272161  -0.27342453  -0.02141984   0.28765311  19.07109873

Number of Observations: 90728
Number of Groups: 5


$SIINT
```

```
                  B3
50.FALSE   0.07688851
80.FALSE   0.04397941
100.FALSE -0.07395164
50.TRUE    0.05537836
100.TRUE  -0.10229465
```
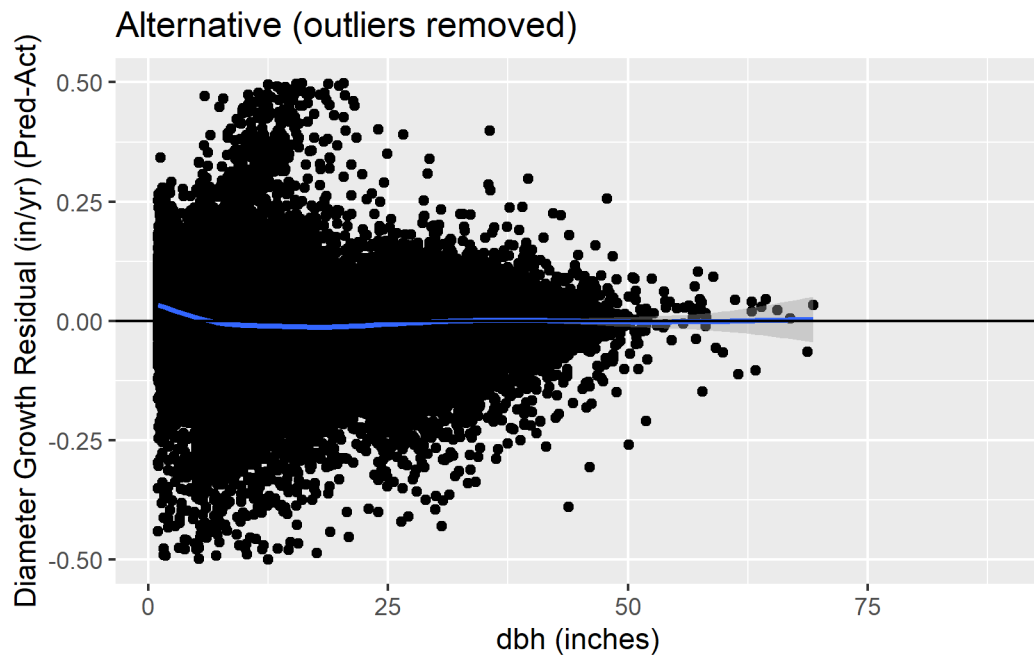
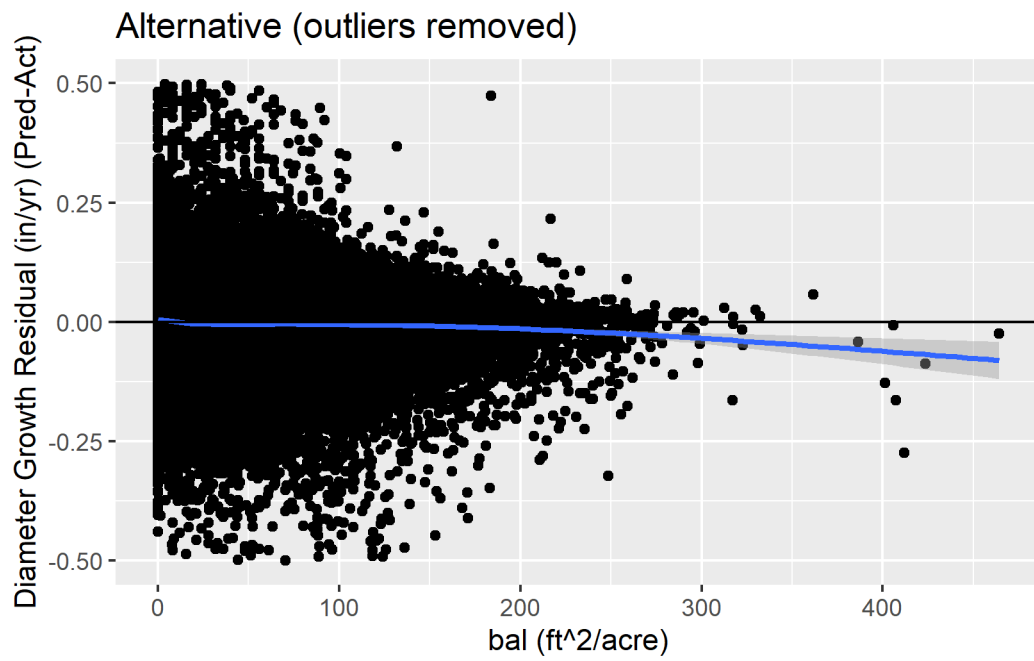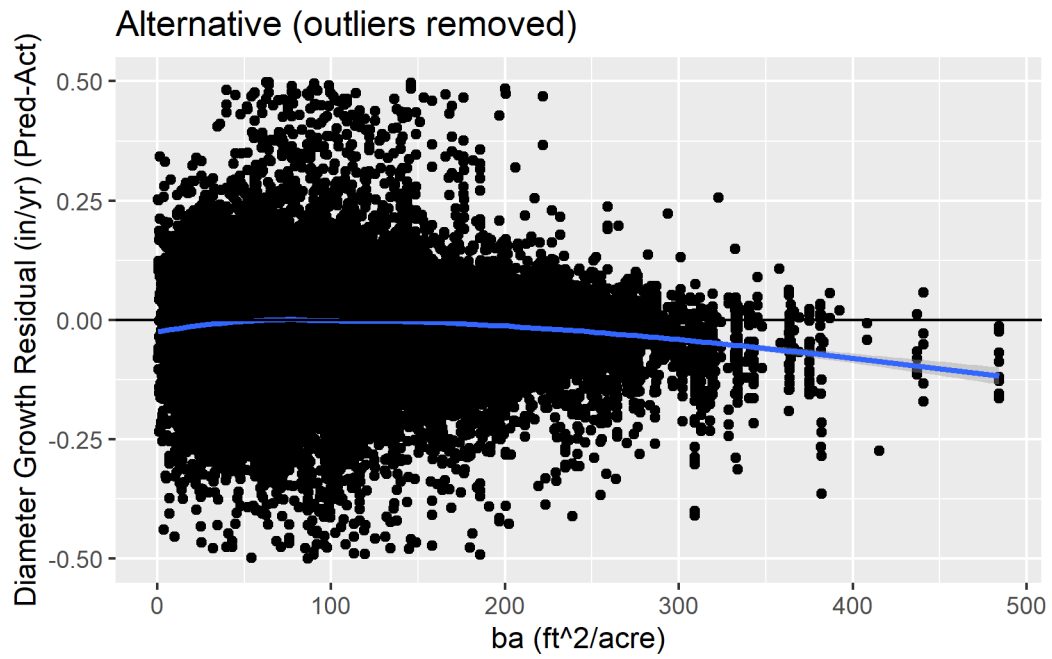Residual Standard Error: 1.20077940391048 on 90718 degrees of freedom, AIC: 290722.2

and for Equation 2:

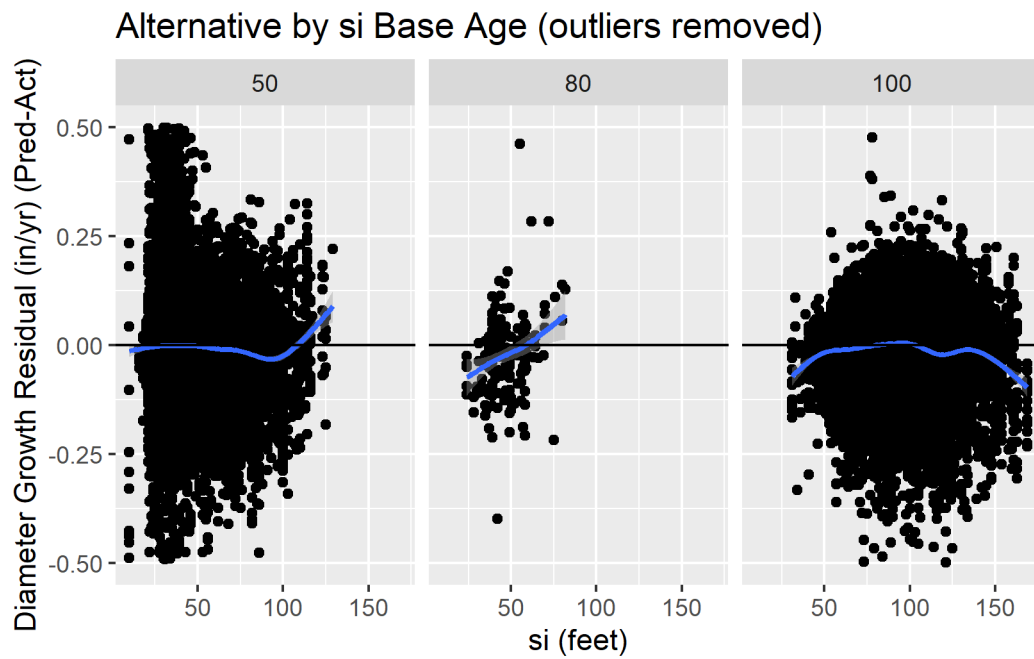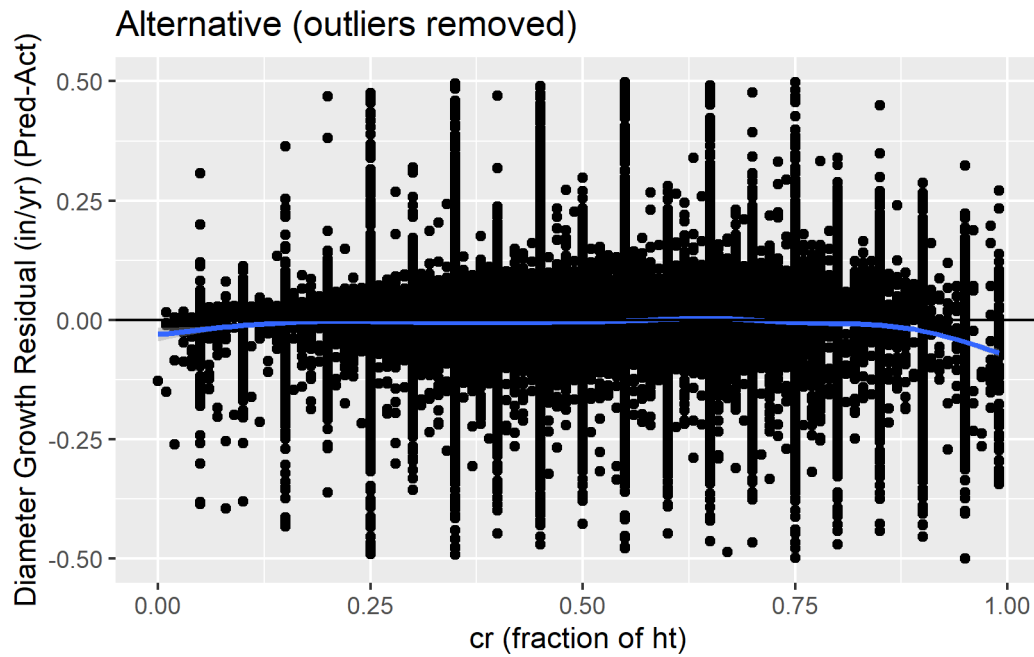|     | Coef.      | Std. error | t-stat.     | p |
| --- | ---------- | ---------- | ----------- | - |
| B0  | -1.4852902 | 0.0508652  | -29.20052   | 0 |
| B1  | -1.2468203 | 0.0123443  | -101.00386  | 0 |
| B2  | -0.2013161 | 0.0187025  | -10.76413   | 0 |
| B4  | 1.4962747  | 0.0092124  | 162.42032   | 0 |
| B5  | 0.5724456  | 0.0176056  | 32.51501    | 0 |

Residual Standard Error: 1.22133686610336 on 90723 degrees of freedom, AIC: 293763.3

**Residual Analysis for Equation 1**



Alternative (outliers removed)

4

Alternative (outliers removed)



Alternative (outliers removed)

## Alternative (outliers removed)



## Alternative by si Base Age (outliers removed)

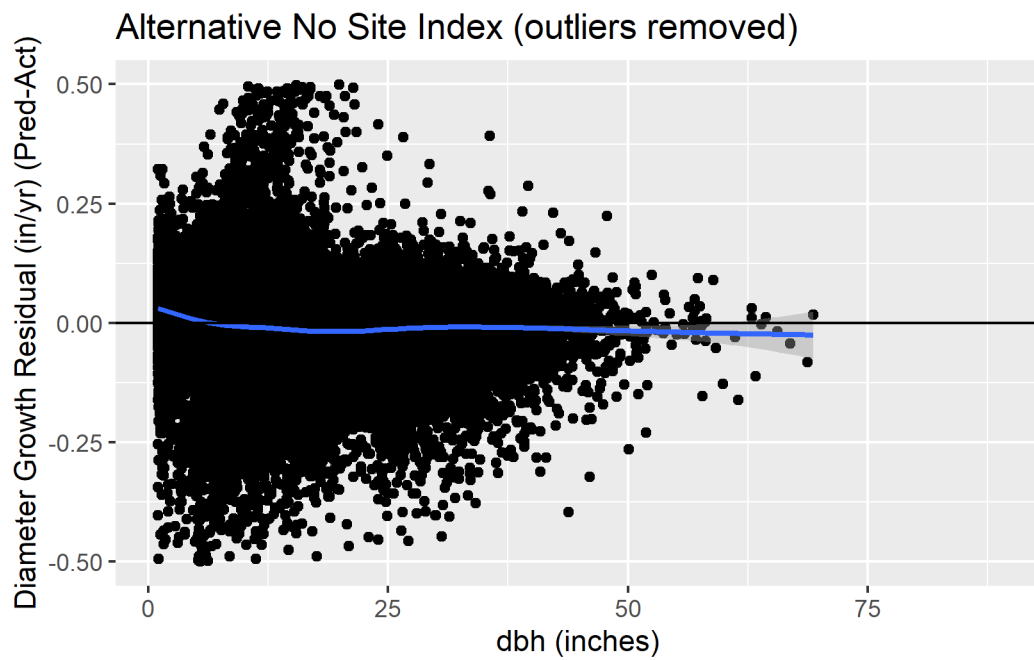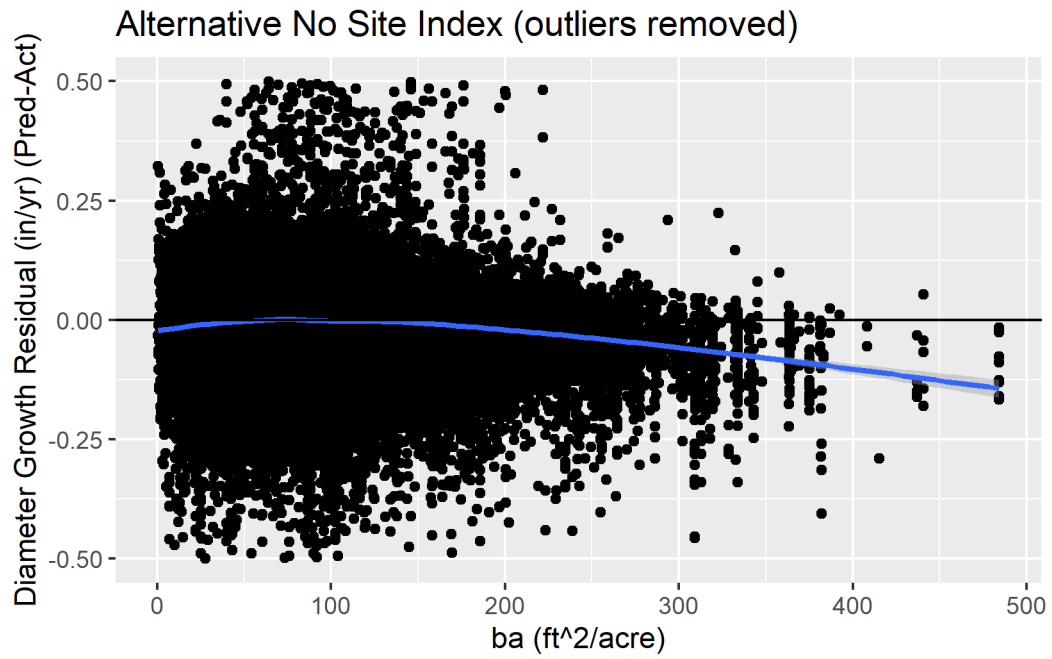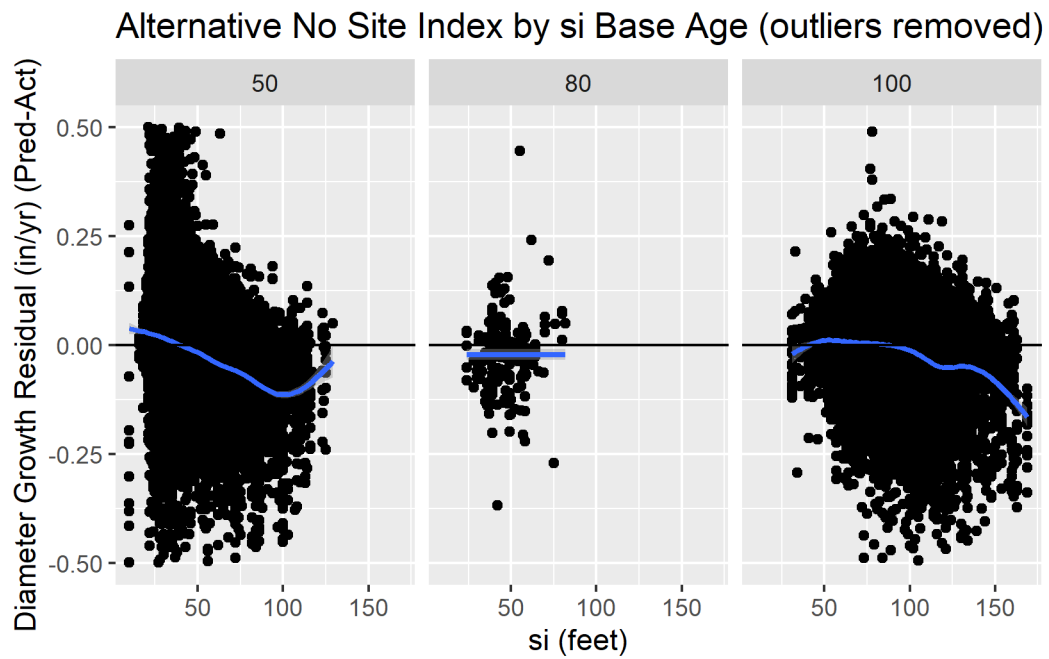Alternative by si Species (outliers removed)



Alternative (outliers removed)

Alternative (outliers removed)



Alternative (outliers removed)
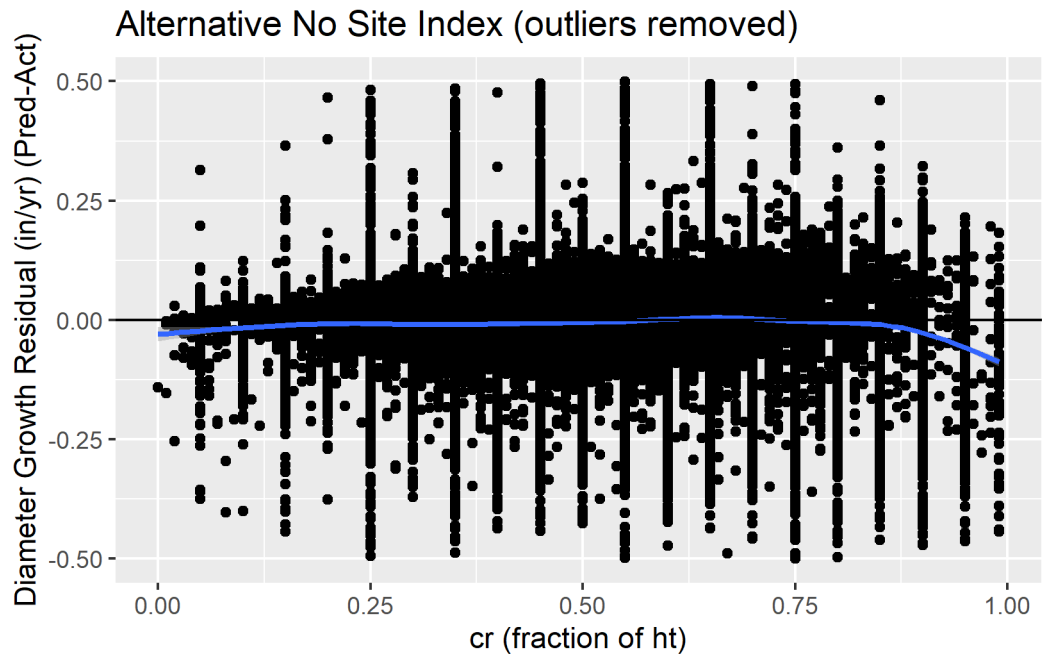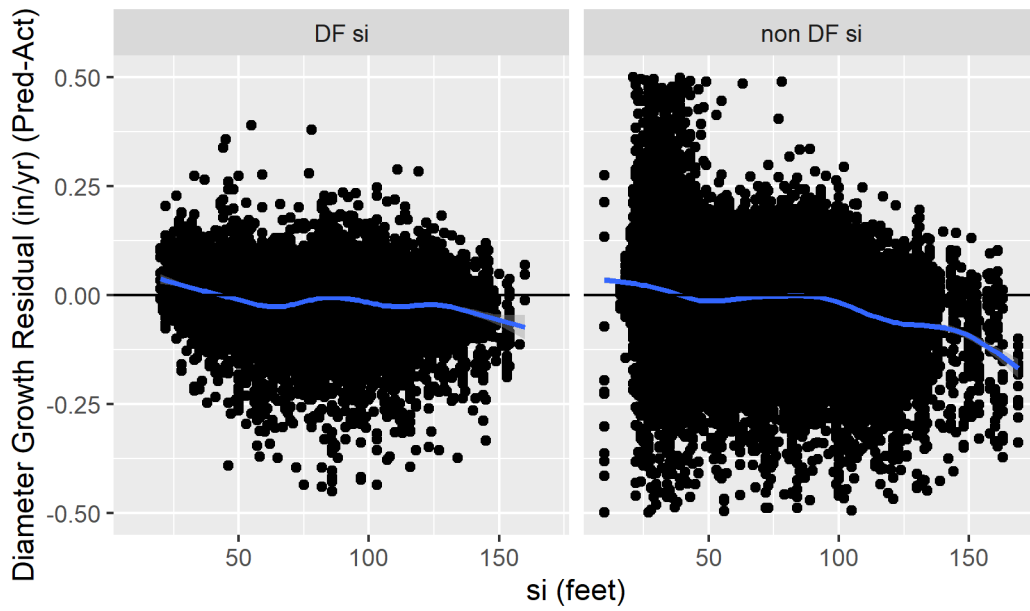
Alternative (outliers removed)



Alternative (outliers removed)
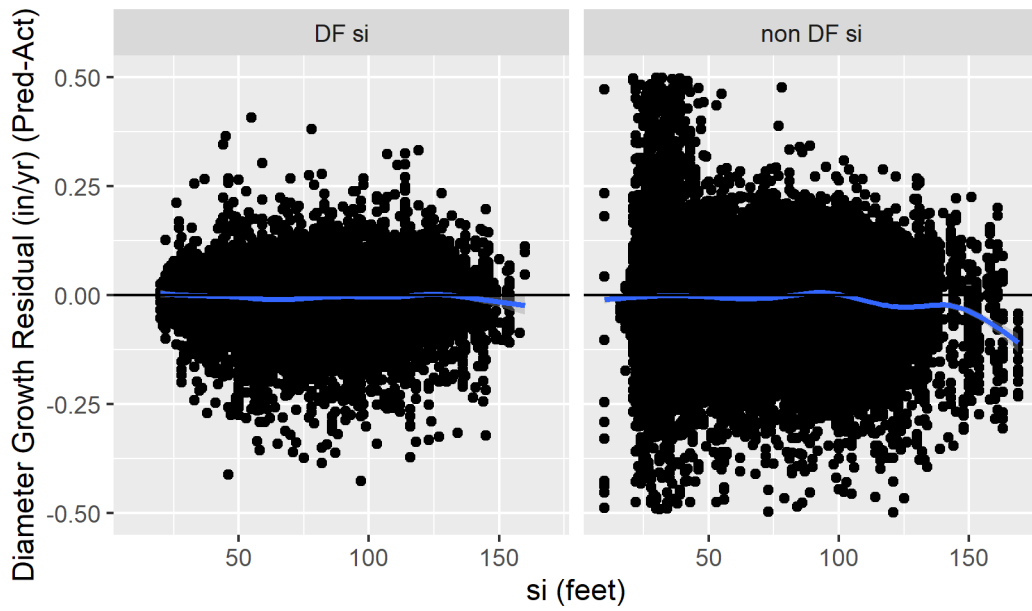
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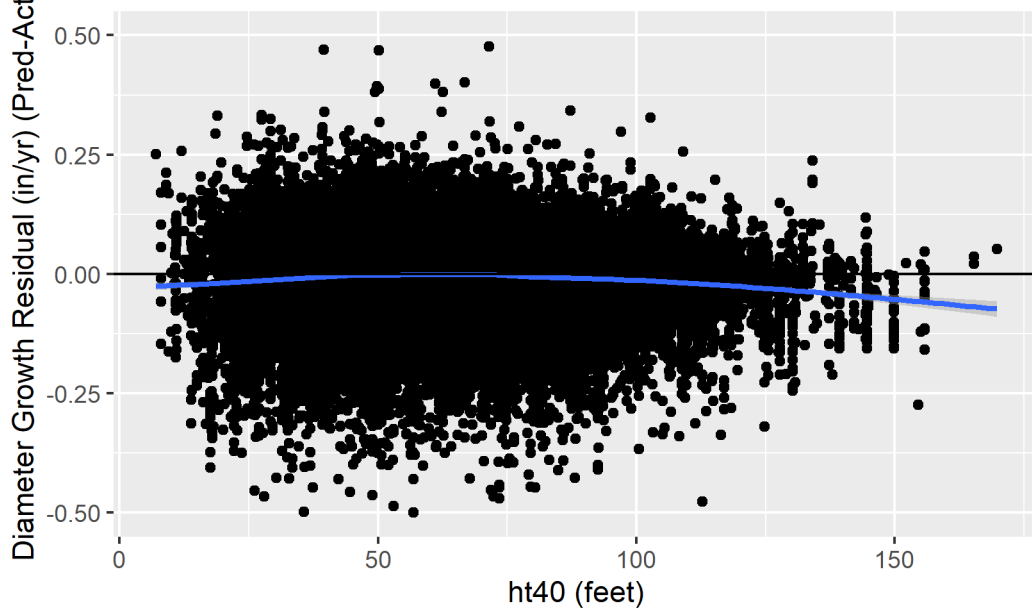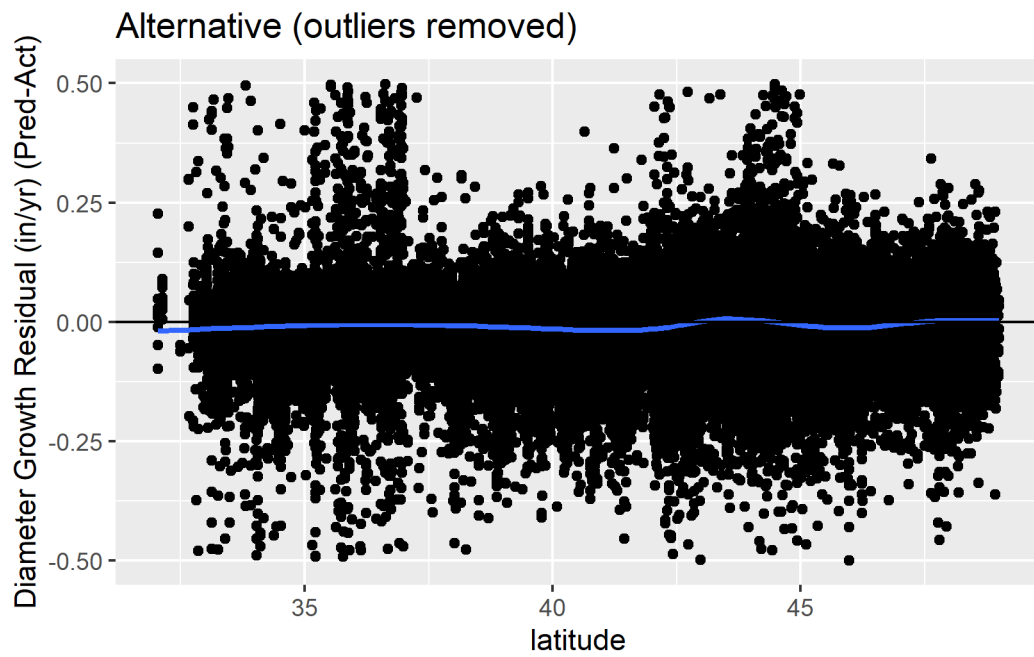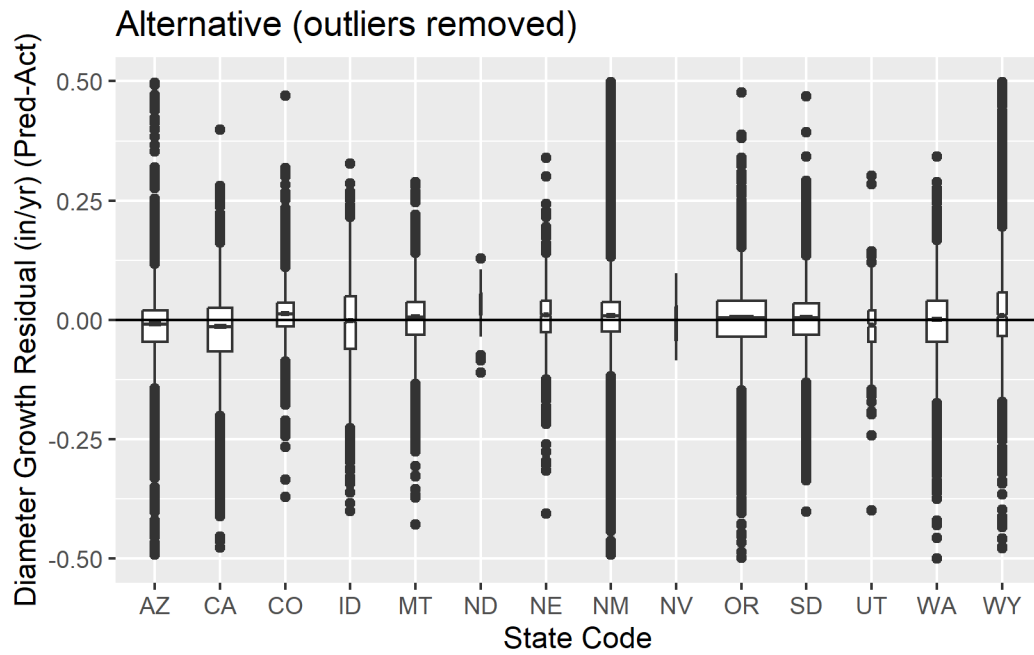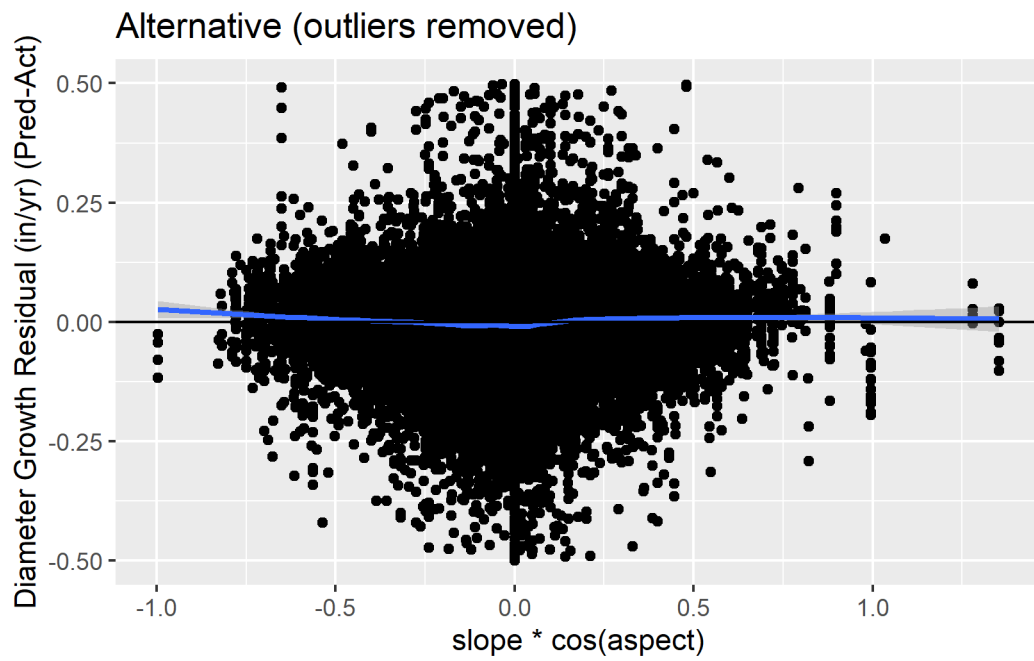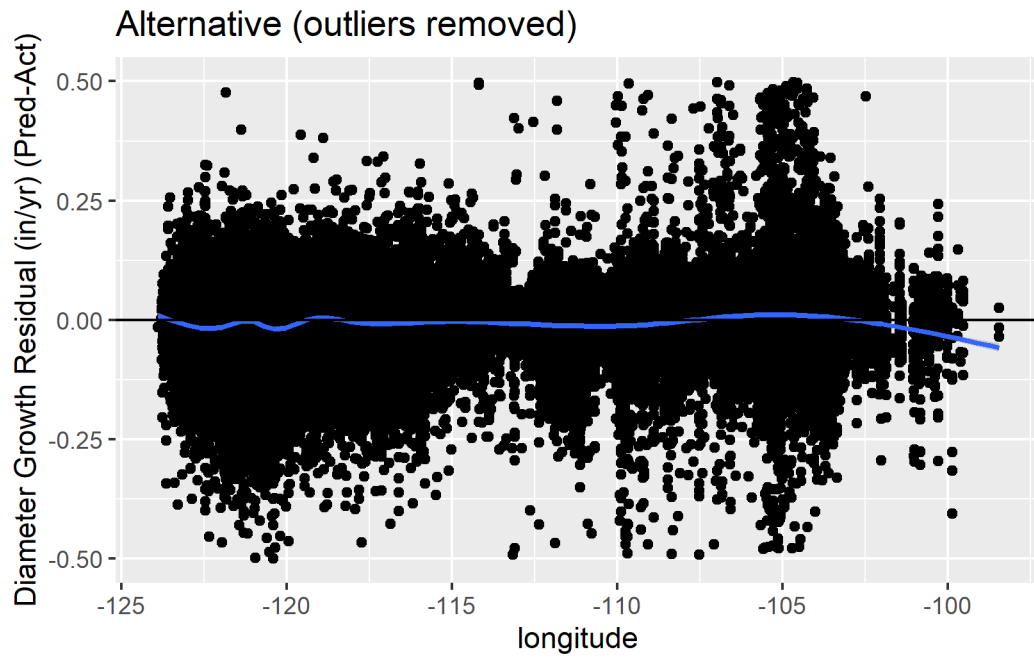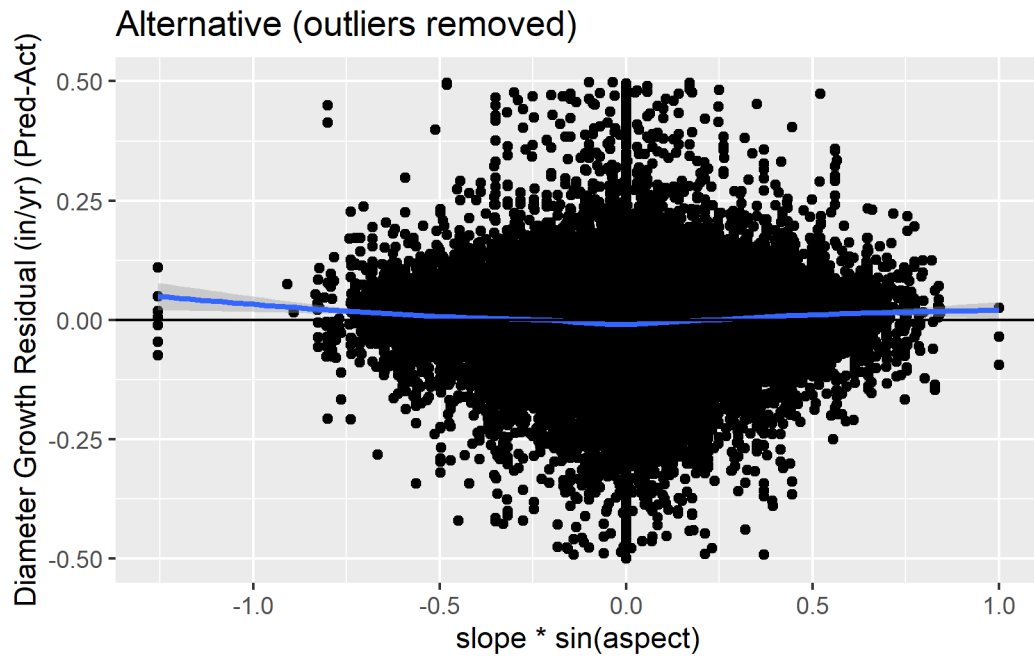
**Residual Analysis for Equation 2**



Alternative No Site Index (outliers removed)

Alternative No Site Index (outliers removed)



Alternative No Site Index (outliers removed)

## Alternative No Site Index (outliers removed)



## Alternative No Site Index by si Base Age (outliers removed)
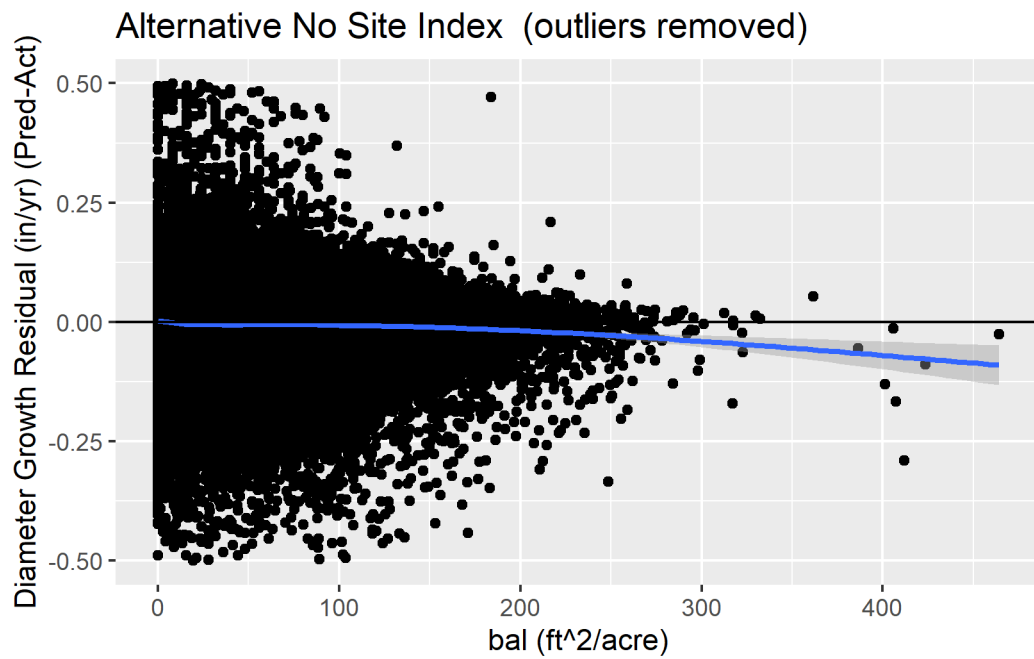
## Alternative No Site Index by si Species (outliers removed)



## Alternative No Site Index (outliers removed)

Alternative No Site Index (outliers removed)



Alternative No Site Index (outliers removed)

Alternative No Site Index (outliers removed)



Alternative No Site Index (outliers removed)
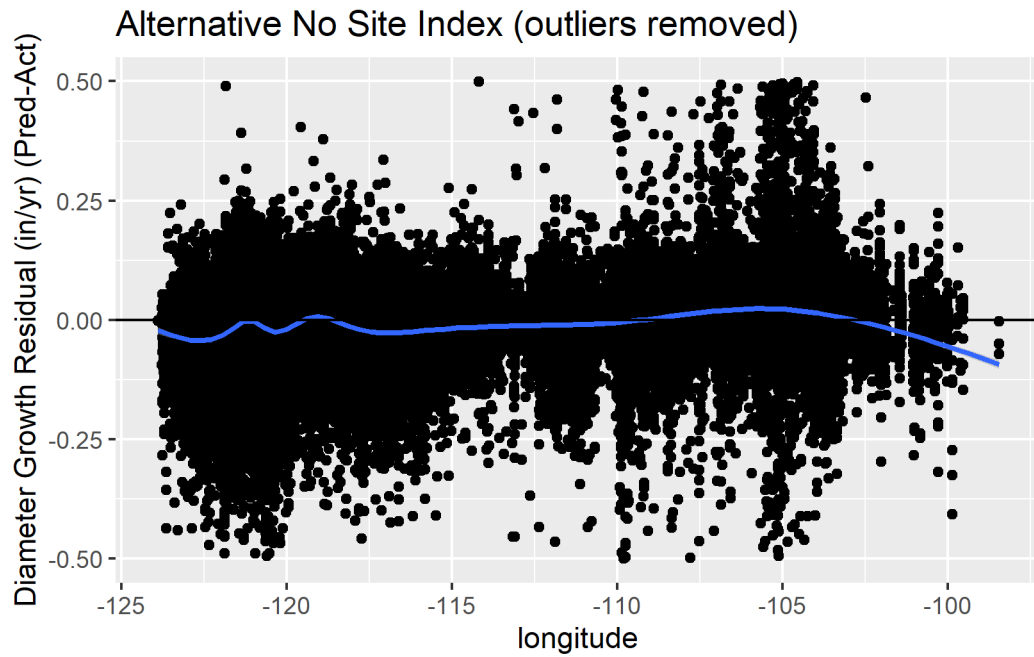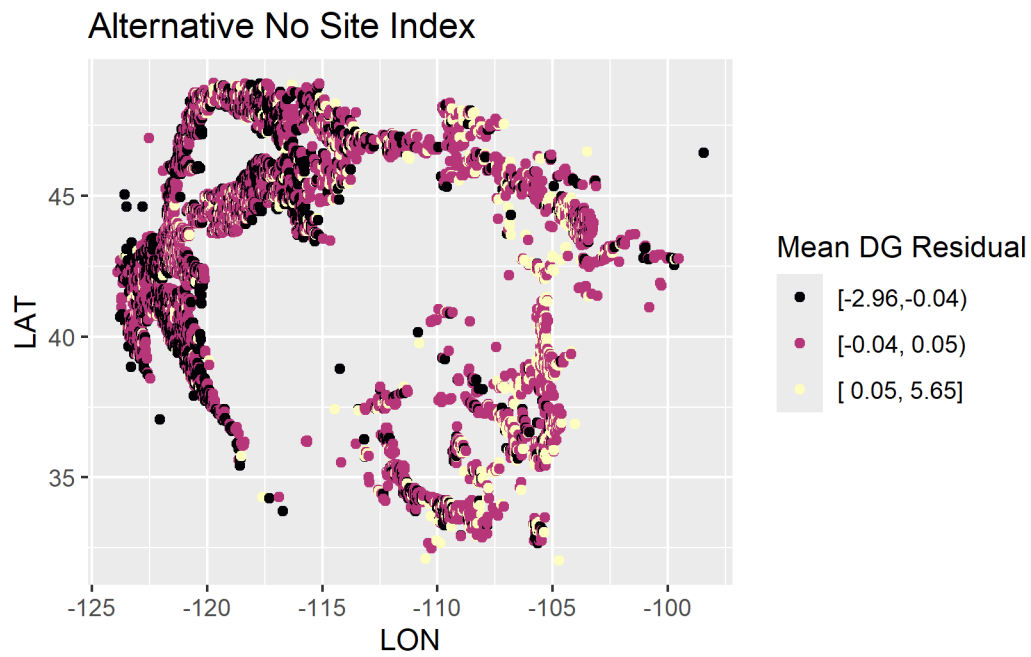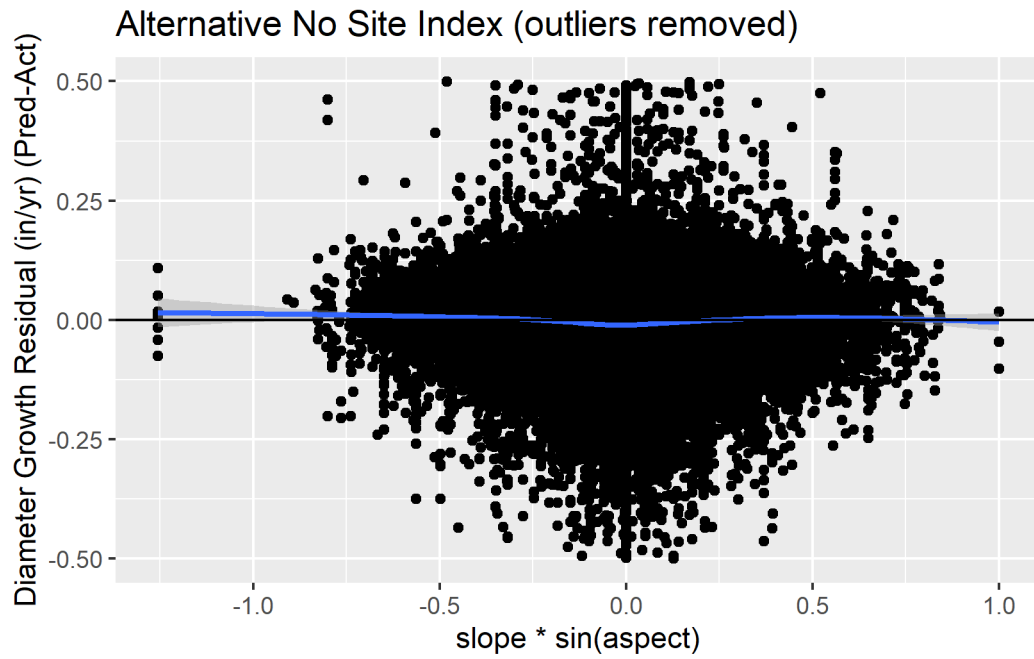
Alternative No Site Index (outliers removed)



Alternative No Site Index
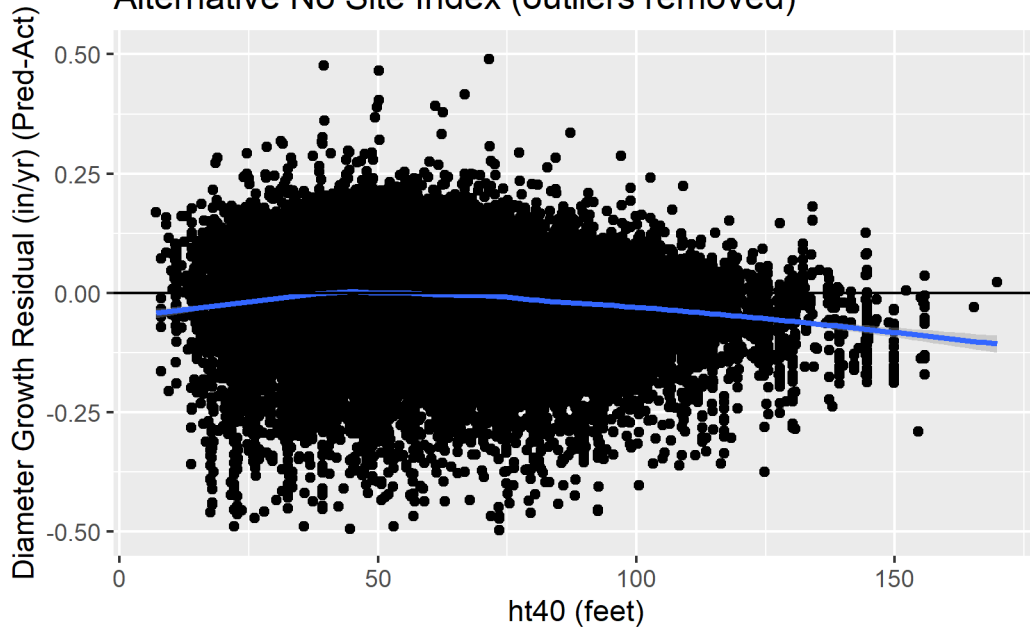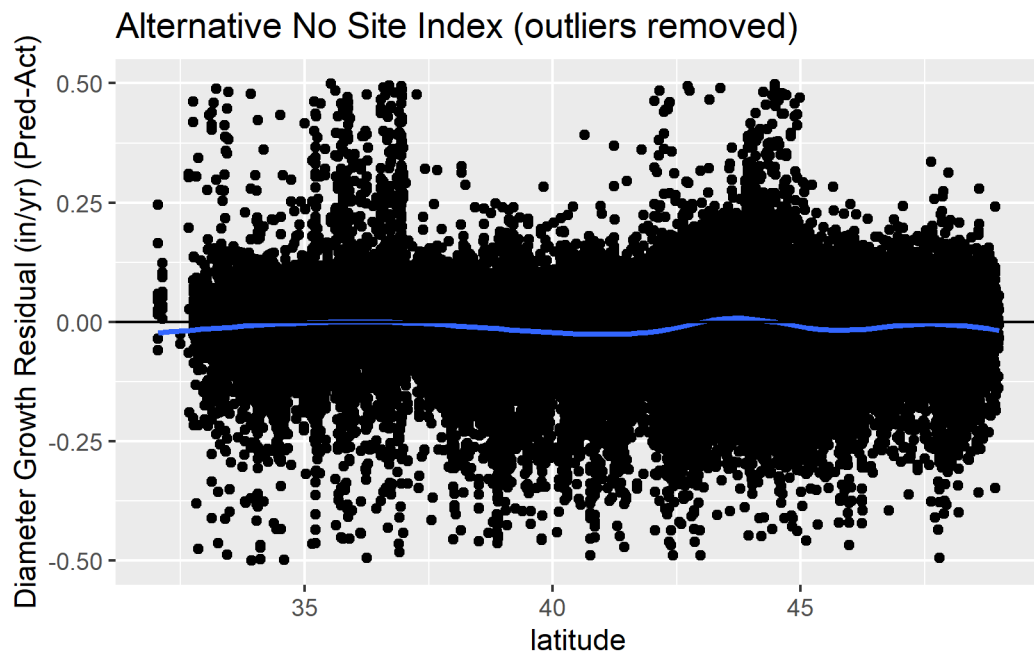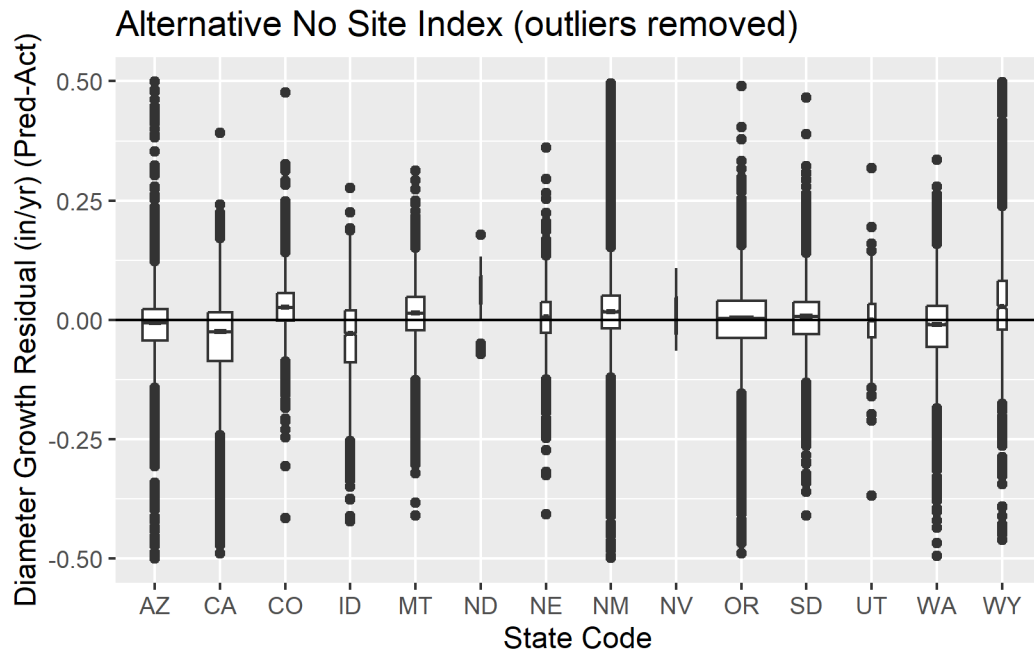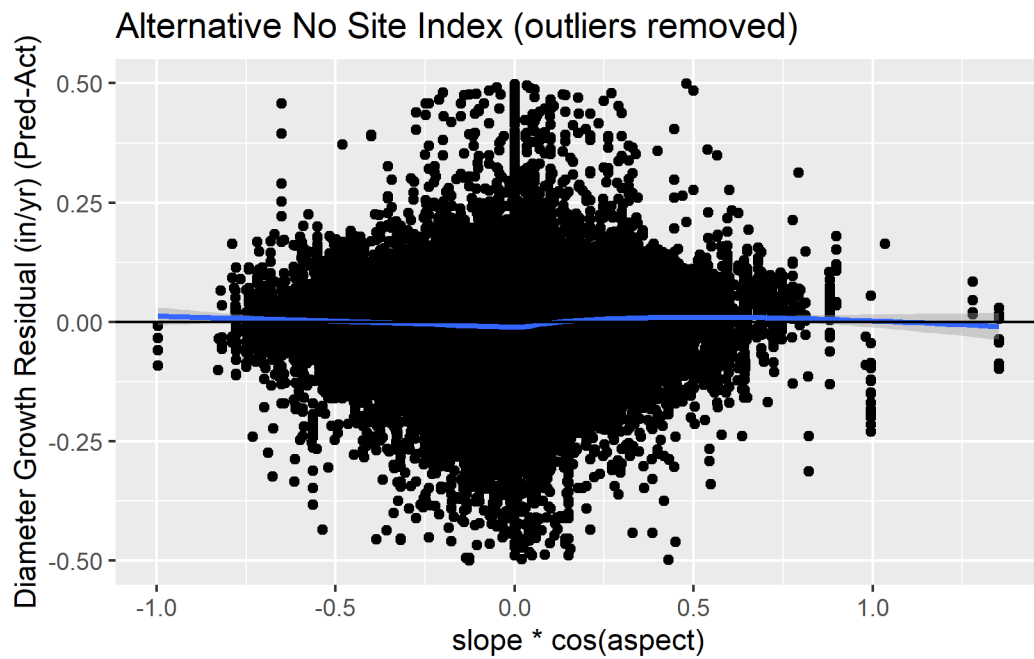
Table 3: Independent Variables for One Inch dbh Trees

| Variable | N | Mean | Std. Dev. | Min | Pctl. 25 | Pctl. 75 | Max |
|----------|-----|------|-----------|------|----------|----------|------|
| ba | 203 | 95 | 45 | 0.41 | 63 | 123 | 251 |
| bal | 203 | 94 | 45 | 0 | 62 | 122 | 251 |
| ht | 203 | 8.5 | 1.8 | 6 | 7 | 10 | 15 |
| cr | 203 | 0.47 | 0.22 | 0.05 | 0.3 | 0.64 | 0.95 |
| si | 203 | 65 | 28 | 21 | 40 | 89 | 132 |

## Discussion

Removing `si` degrades the fit significantly. There is a small bias for some states for both equations. The dubious `si` term mitigates some of the bias (expected since it is acting as a localization variable).

There do not appear to be residual trends with latitude and longitude, however, the no site index equations residual map hints at a trend of over prediction for larger longitude (eastward).

Interestingly, for the no `si` equation, there is a trend of under-prediction for higher `ba`. This will need to be investigated in further equation development.

**Equation Behavior for Very Small Trees**

## Equation 1 Predictions for Trees with One Inch dbh Trees
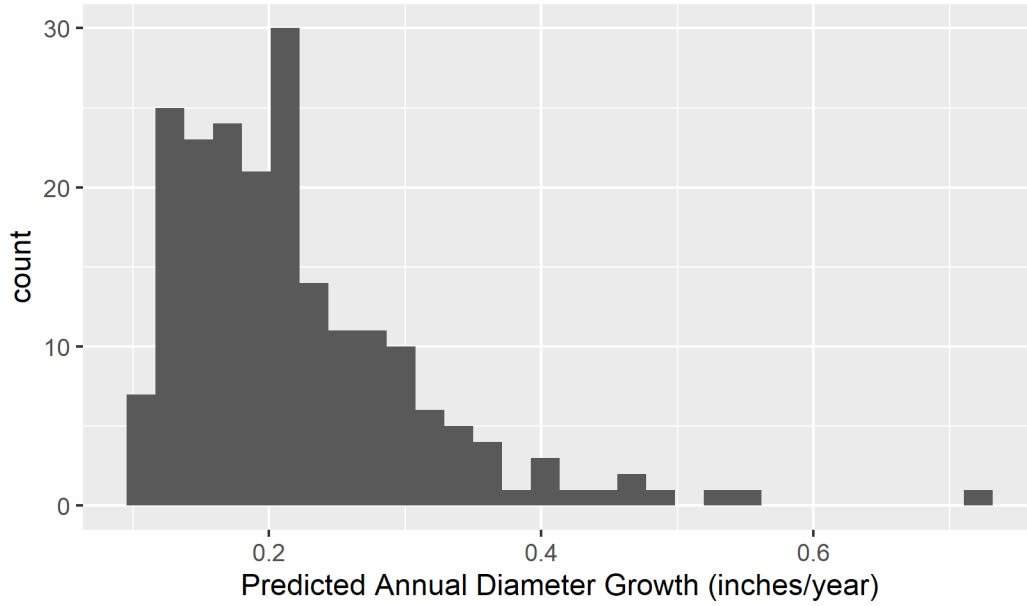


## Equation 2 Predictions for Trees with One Inch dbh Trees