Alternative Incense Cedar Diameter Growth

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Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 3 states listed in the native range of Incense Cedar in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Incense Cedar (FIA species code 81), and remeasurement intervals $\geq$ 5 years we get the observations in Table 1.

Table 1: Incense Cedar Growth Observations by State

State	Observations
CA	7314
NV	3
OR	2615

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Incense Cedar site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 17 different site index equations for 17 species. Incense Cedar site index comprises 1% of the observations. There are 2 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7}+\beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, en
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
       AIC      BIC   logLik
```

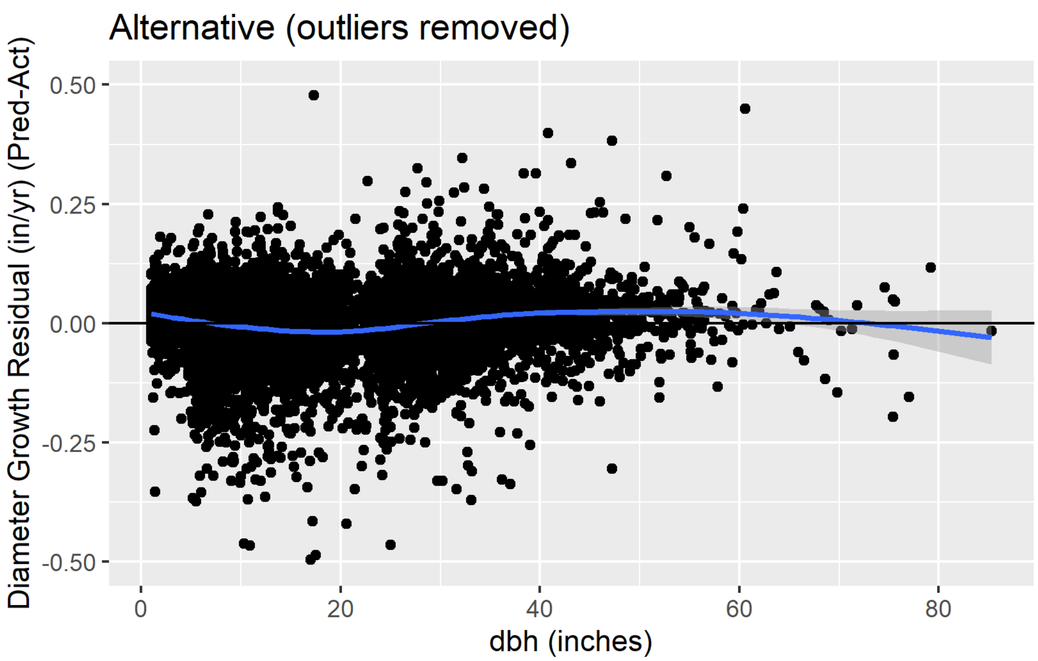
```
  21633.61 21691.24 -10808.8

Random effects:
 Formula: B3 ~ 1 | SIINT
               B3  Residual
StdDev: 0.0632119 0.7176832

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value  Std.Error   DF   t-value p-value
B0 -4.658636 0.13899846 9923 -33.51574       0
B1 -0.694860 0.01438152 9923 -48.31613       0
B2 -0.149961 0.03570928 9923  -4.19950       0
B3  0.601195 0.04117971 9923  14.59930       0
B4  1.783763 0.02826586 9923  63.10665       0
B5  0.521112 0.03820534 9923  13.63976       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.314
B2 -0.527  0.533
B3 -0.475  0.032  0.014
B4 -0.522  0.415  0.626 -0.095
B5 -0.498  0.494  0.991  0.027  0.550

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-9.51836151 -0.50950413 -0.09309793  0.40641442 29.30518247

Number of Observations: 9932
Number of Groups: 4

$SIINT
                   B3
50.FALSE   0.07974169
100.FALSE -0.02274200
50.TRUE    0.03159499
100.TRUE  -0.08859469

Residual Standard Error: 0.717683216123872 on 9923 degrees of freedom, AIC: 21633.6
```

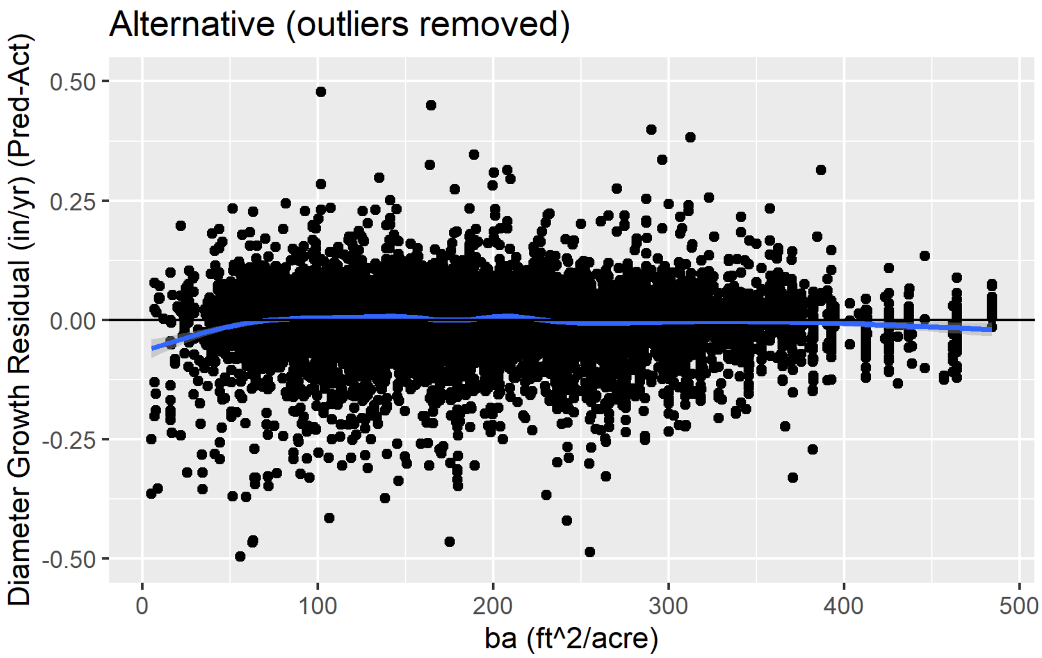
and for Equation 2:

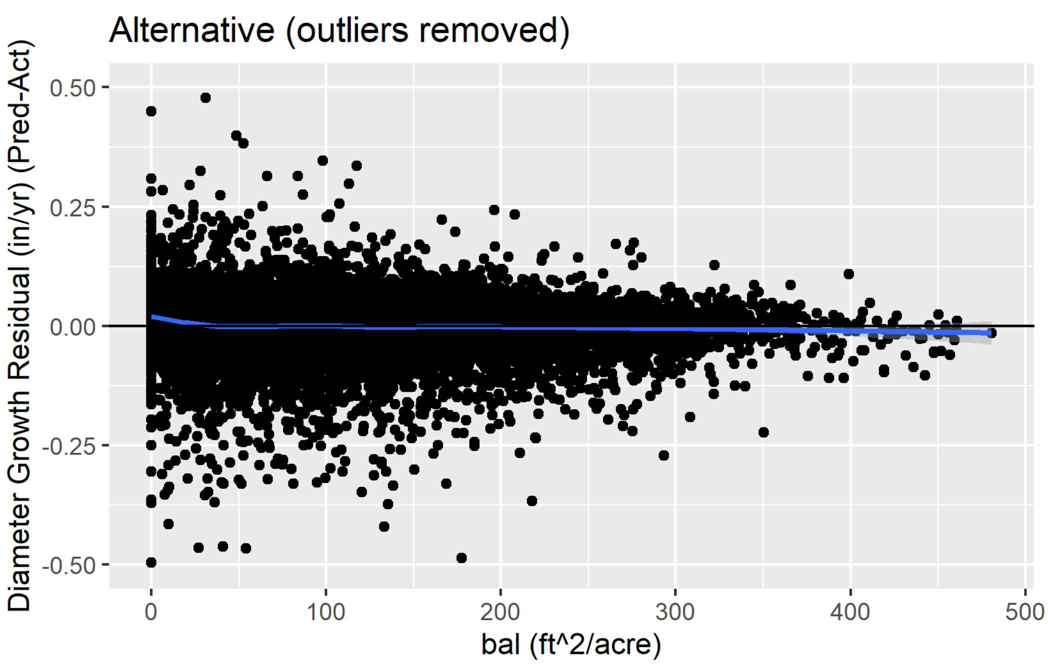
	Coef.	Std. error	t-stat.	p
B0	-2.5198137	0.0992009	-25.401110	0.0000000
B1	-0.6838018	0.0150817	-45.339705	0.0000000
B2	-0.1342665	0.0440084	-3.050933	0.0022873
B4	1.9950859	0.0319159	62.510633	0.0000000
B5	0.4830347	0.0517938	9.326119	0.0000000

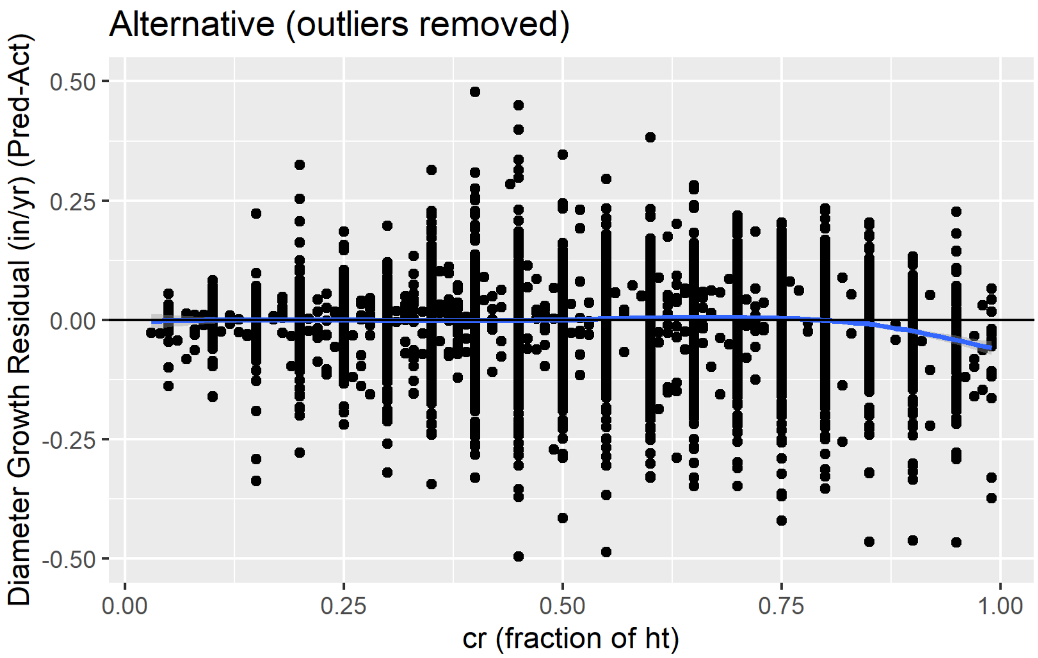
```
Residual Standard Error: 0.744315396278852 on 9927 degrees of freedom, AIC: 22327.1
```

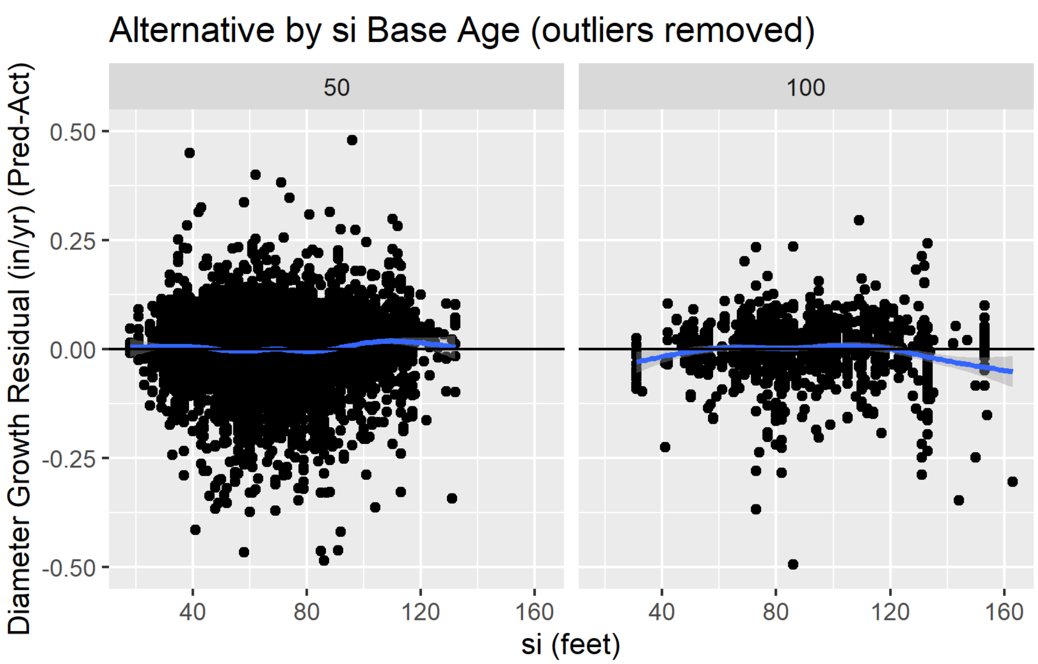
Residual Analysis for Equation 1

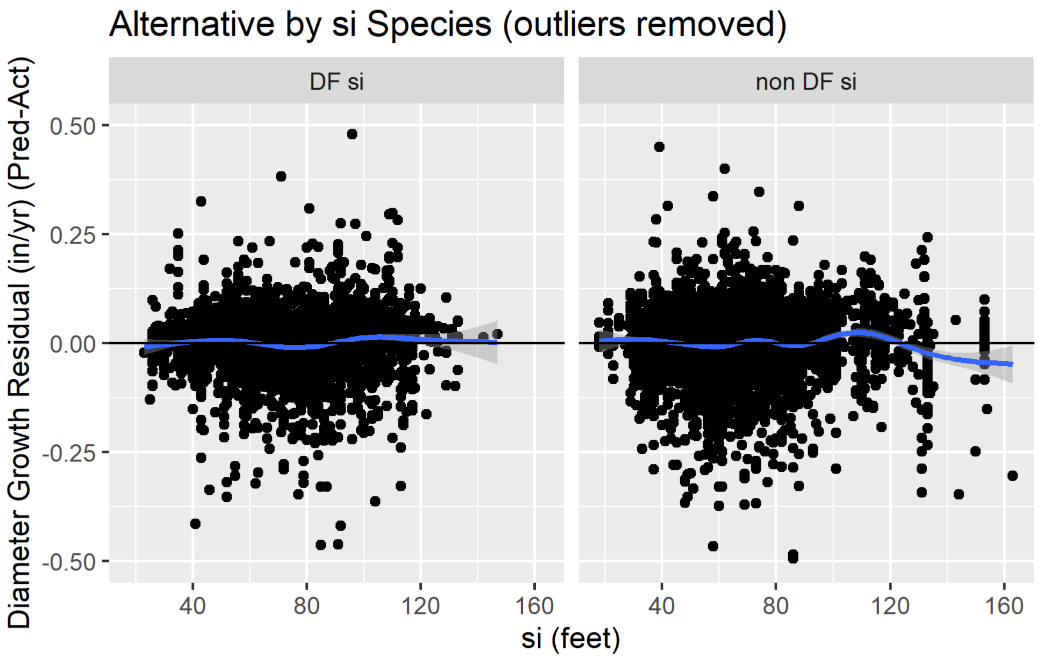



Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
500
ba (ft^2/acre)

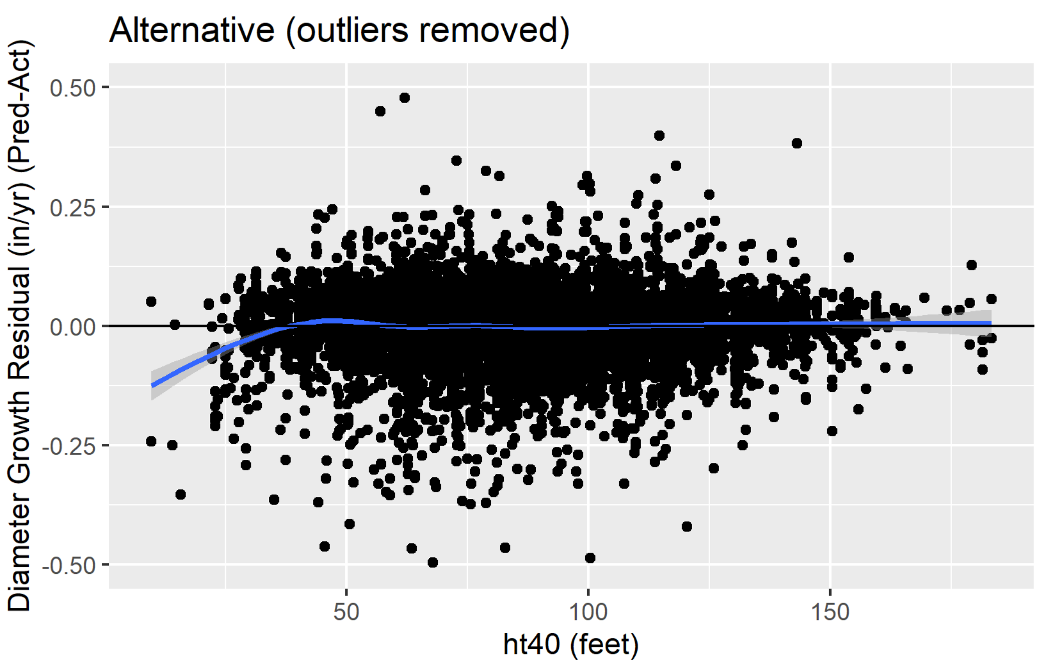

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
500
bal (ft^2/acre)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

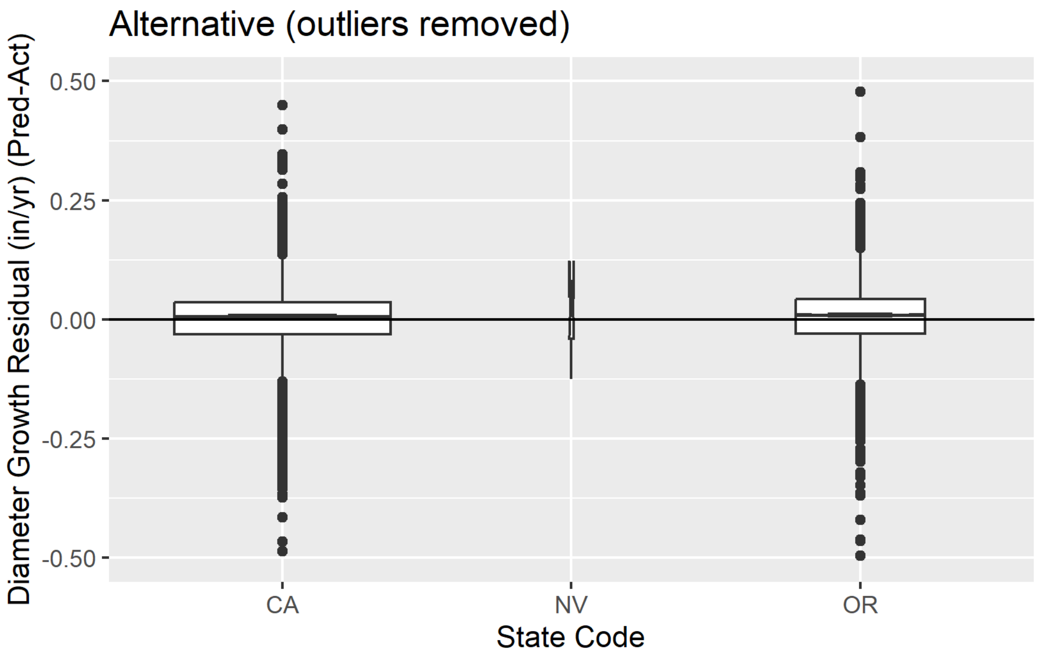

Alternative by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
80
120
160
si (feet)

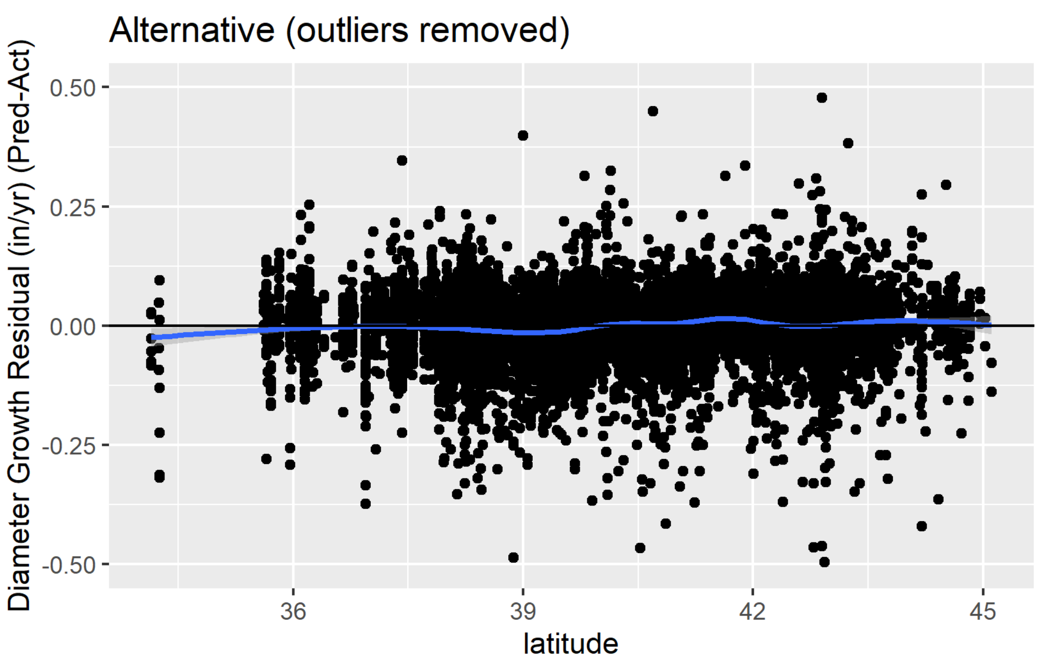


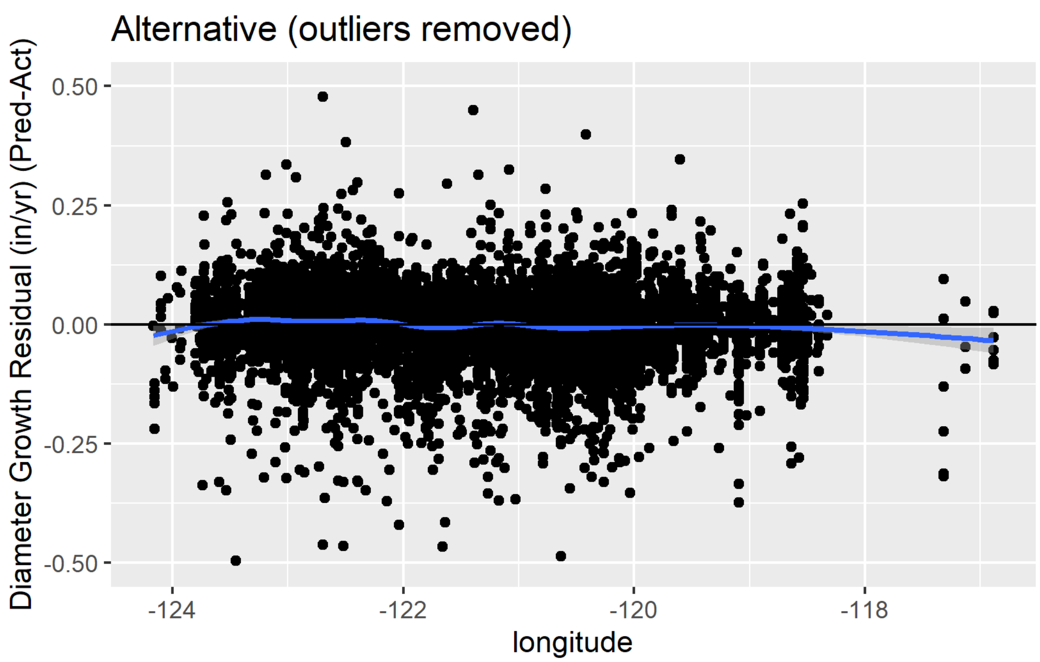
Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
80
120
160
si (feet)



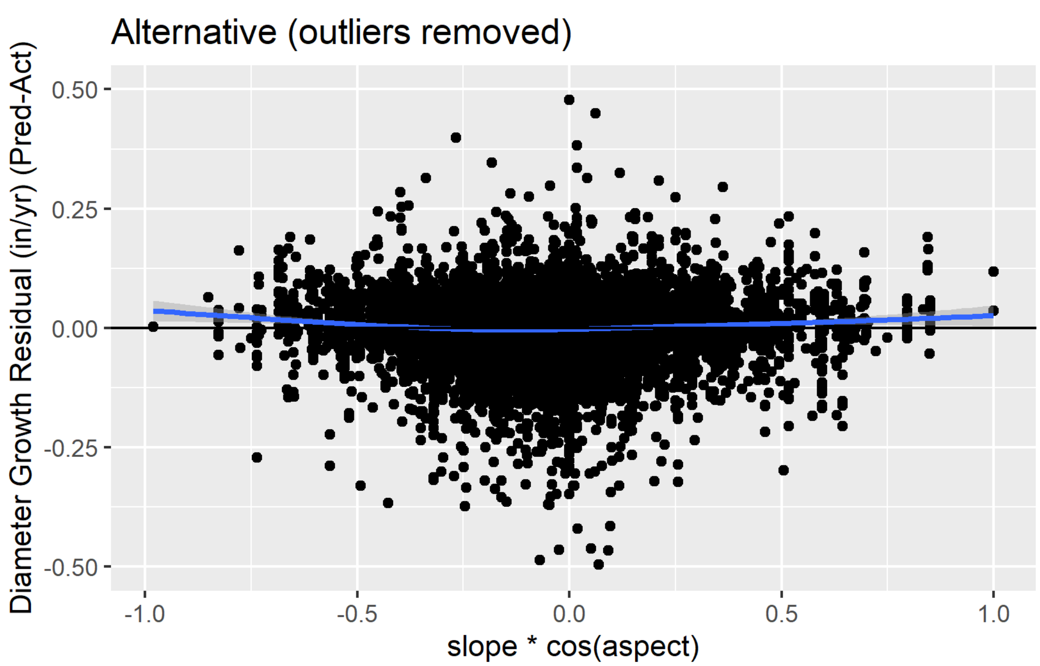
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
CA
NV
OR
State Code

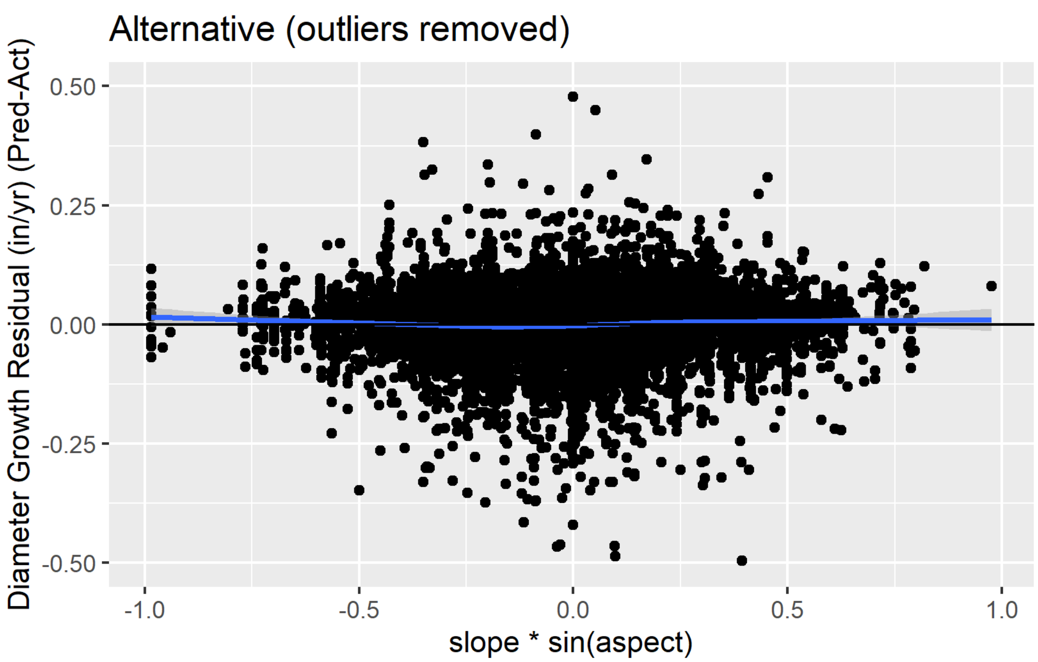

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
36
39
42
45
latitude



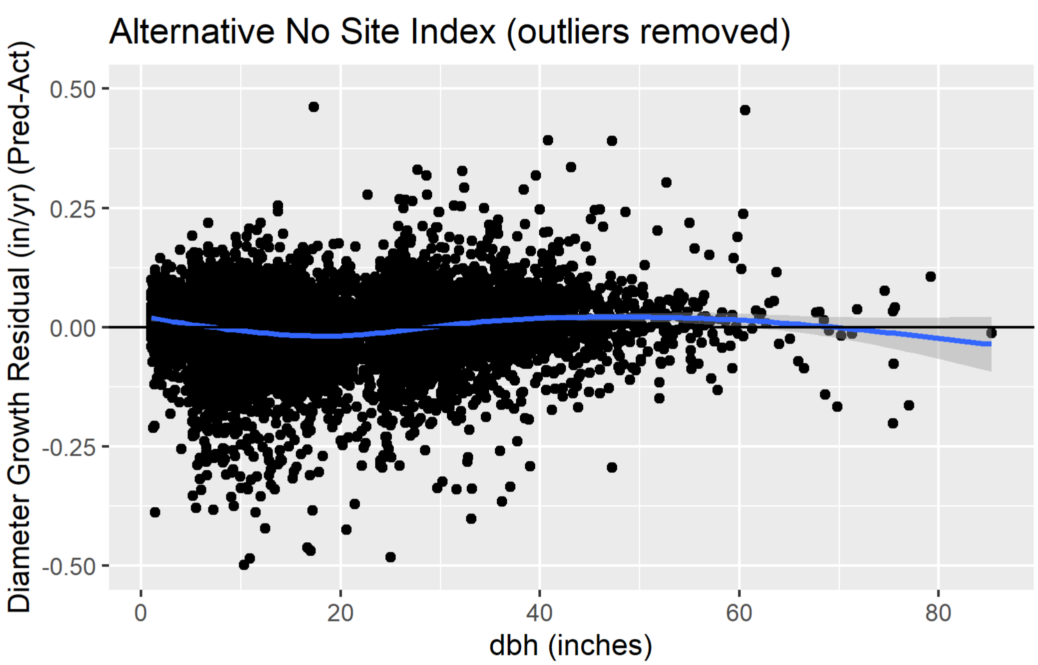
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-124
-122
-120
-118
longitude

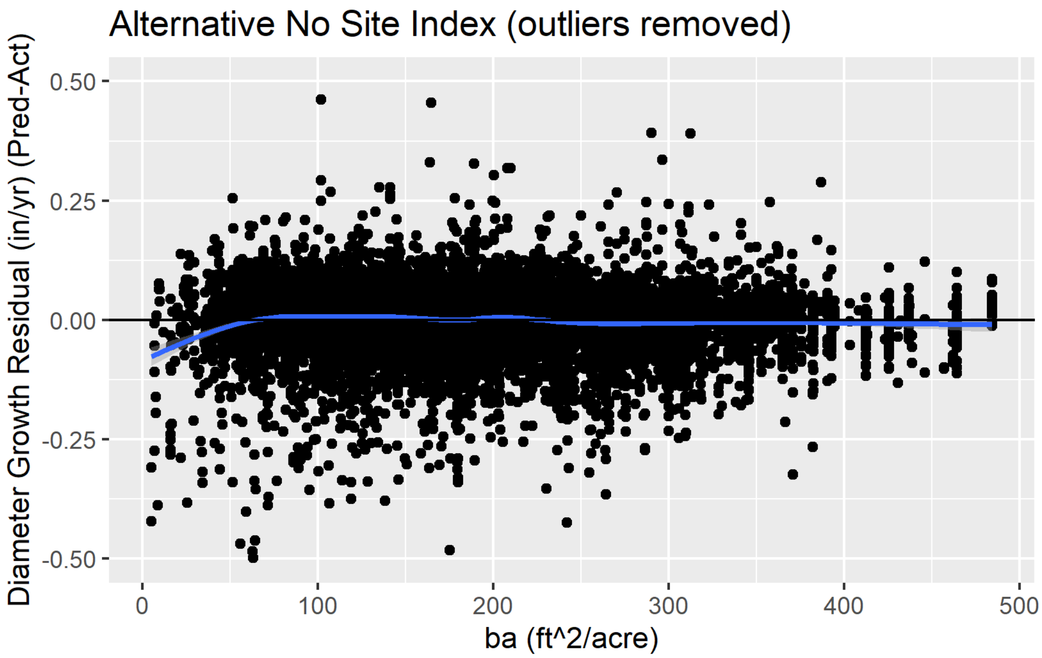


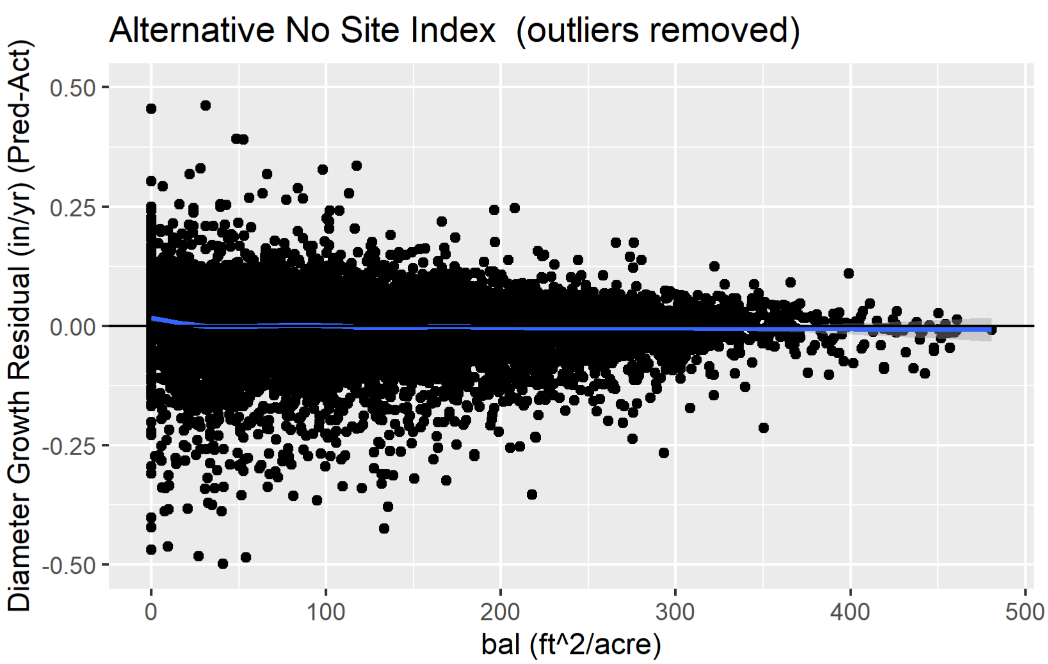
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)

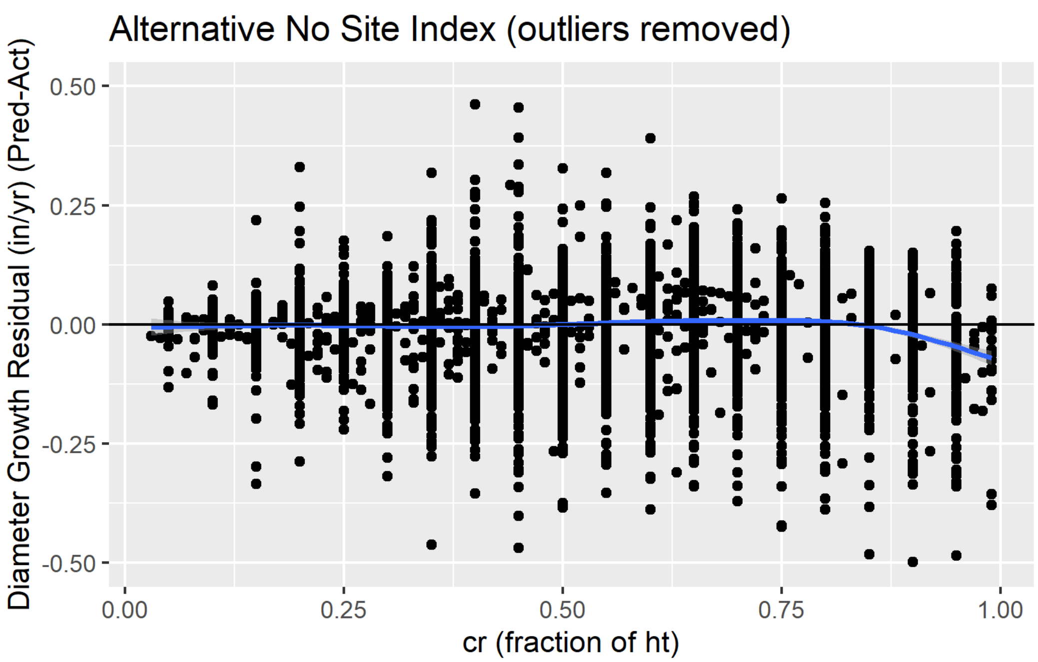


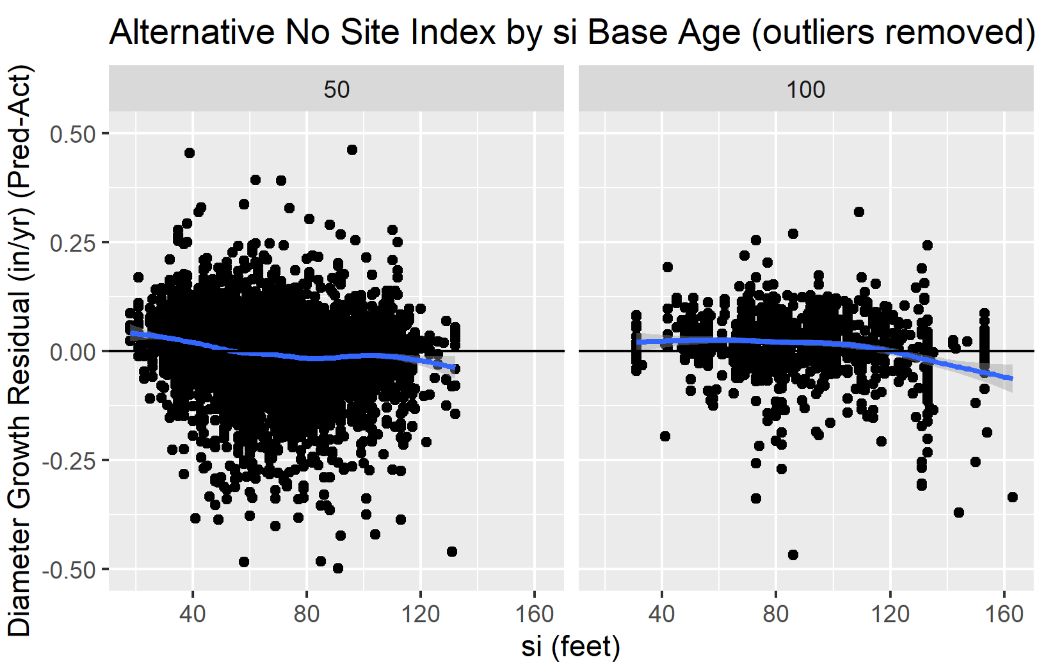
Residual Analysis for Equation 2

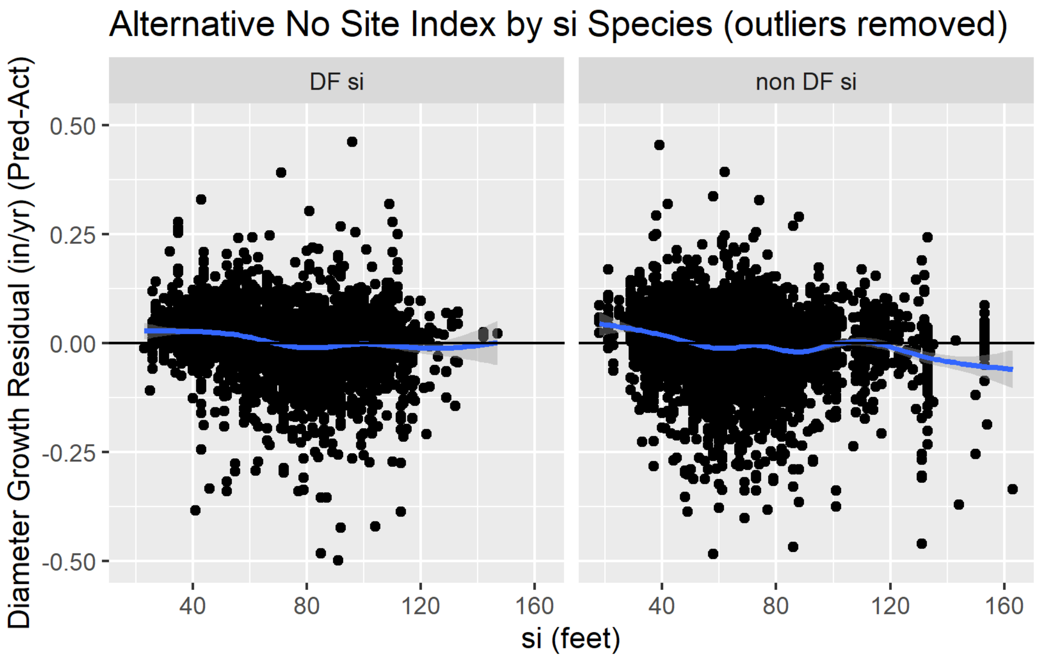


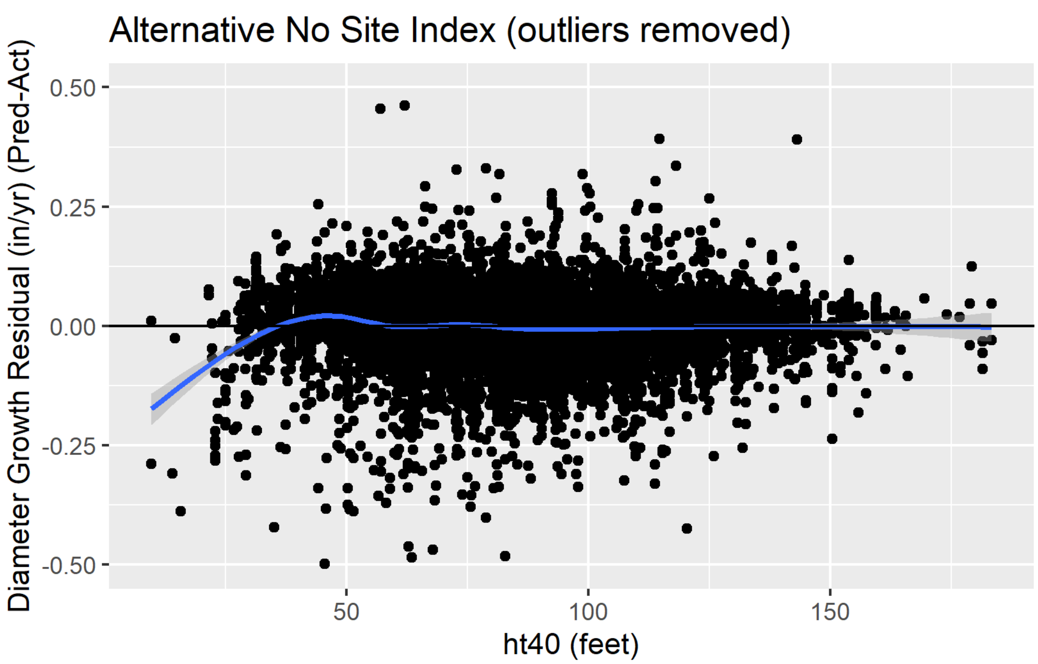

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
500
ba (ft^2/acre)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
500
bal (ft^2/acre)

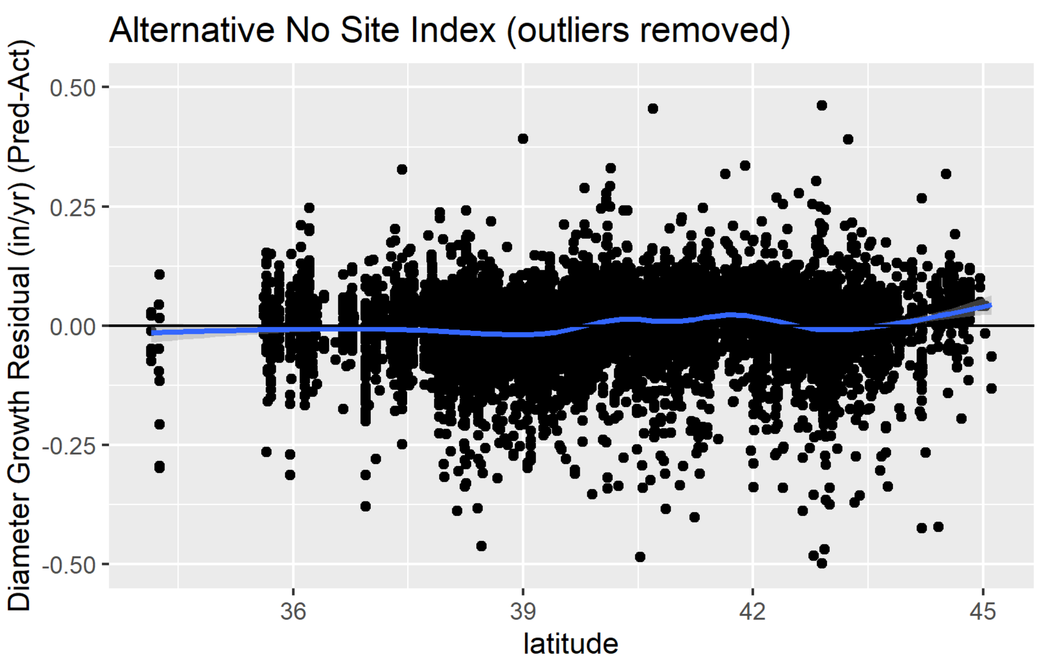

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

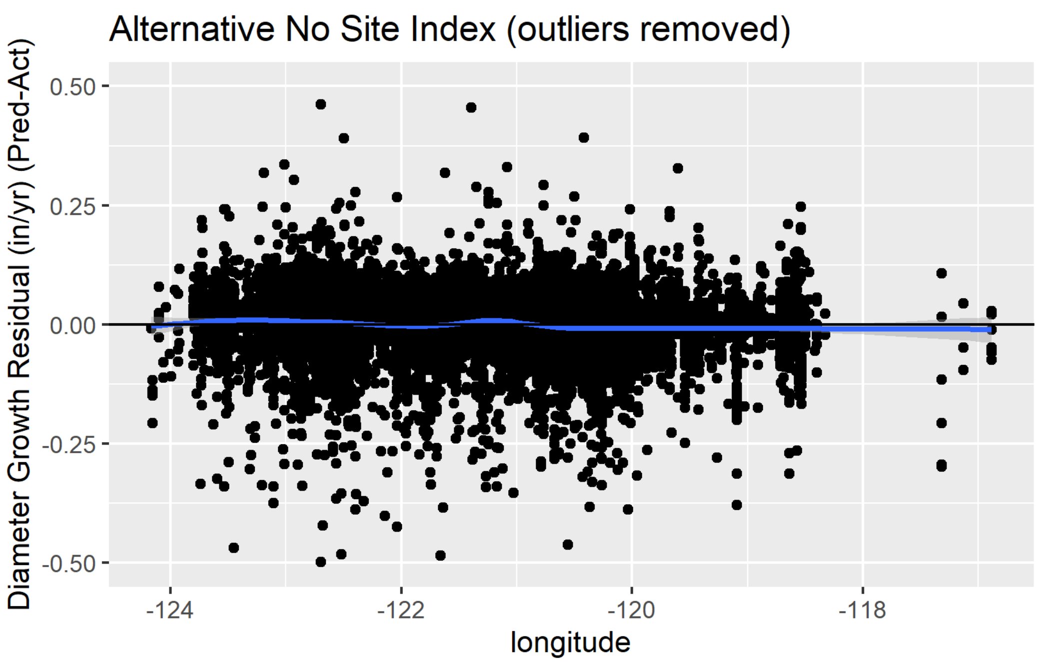

Alternative No Site Index by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
80
120
160
si (feet)

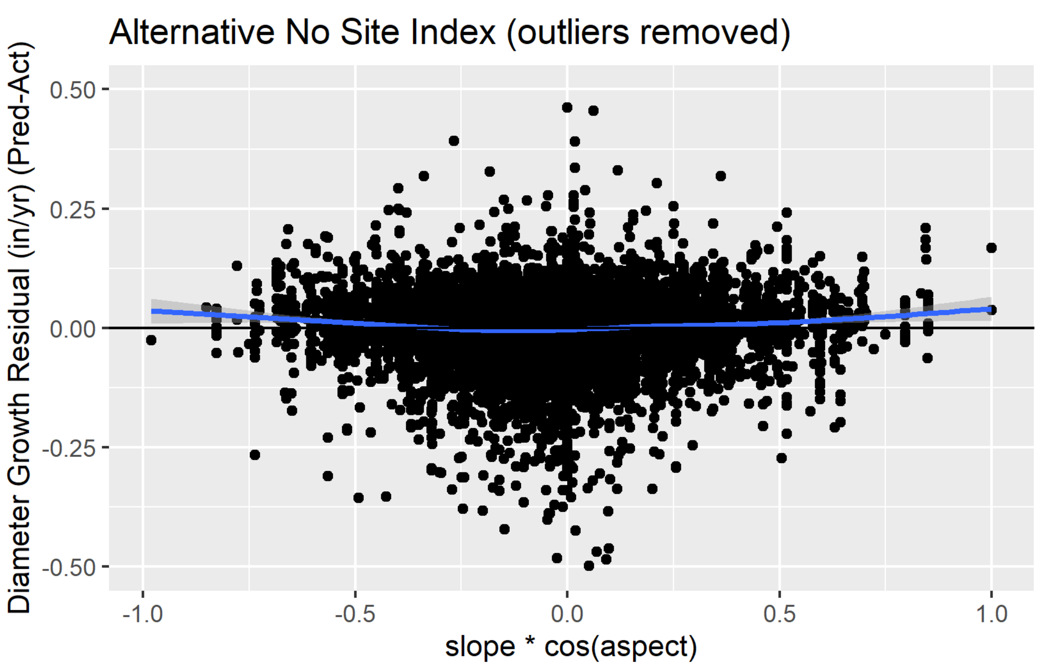

Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
80
120
160
si (feet)

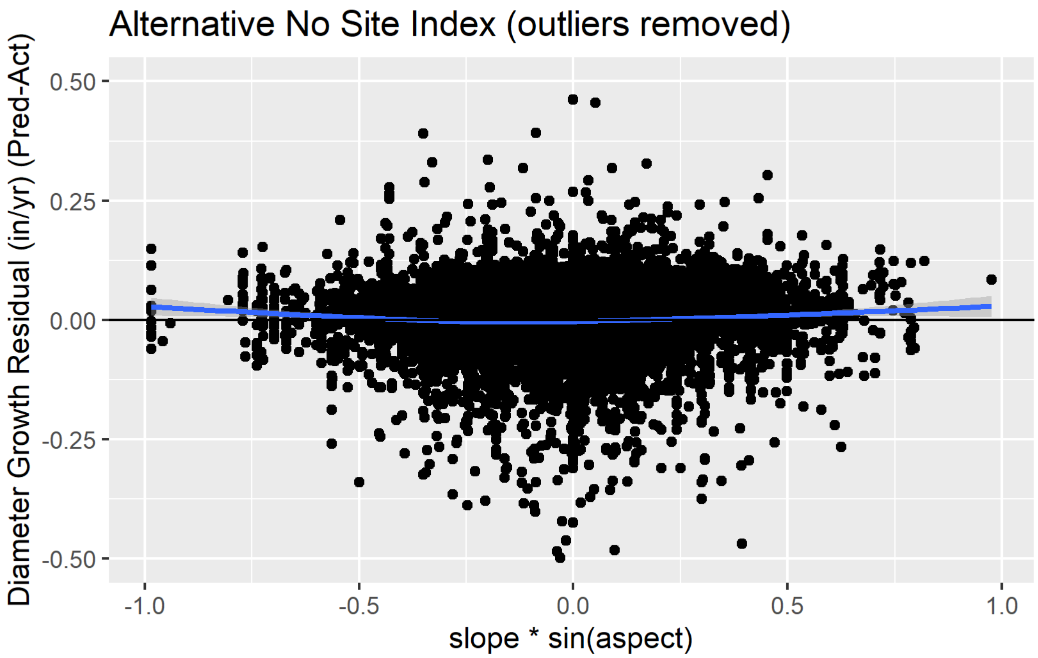

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
CA
NV
OR
State Code

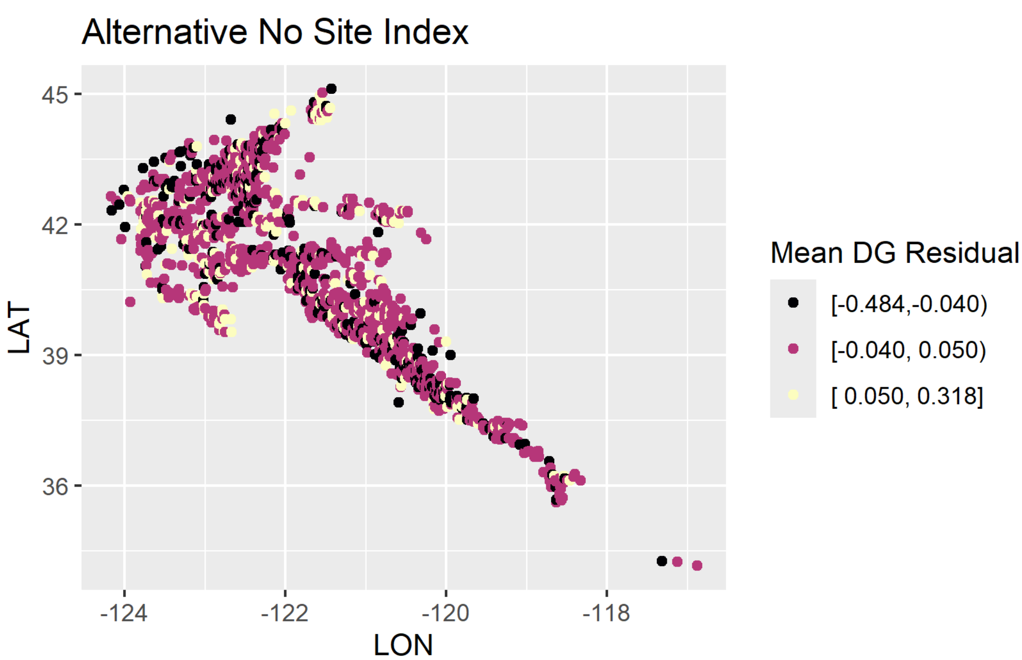

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
36
39
42
45
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-124
-122
-120
-118
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)



Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)



Alternative No Site Index
LAT
45
42
39
36
-124
-122
-120
-118
LON
Mean DG Residual
[-0.484,-0.040)
[-0.040, 0.050)
[0.050, 0.318]

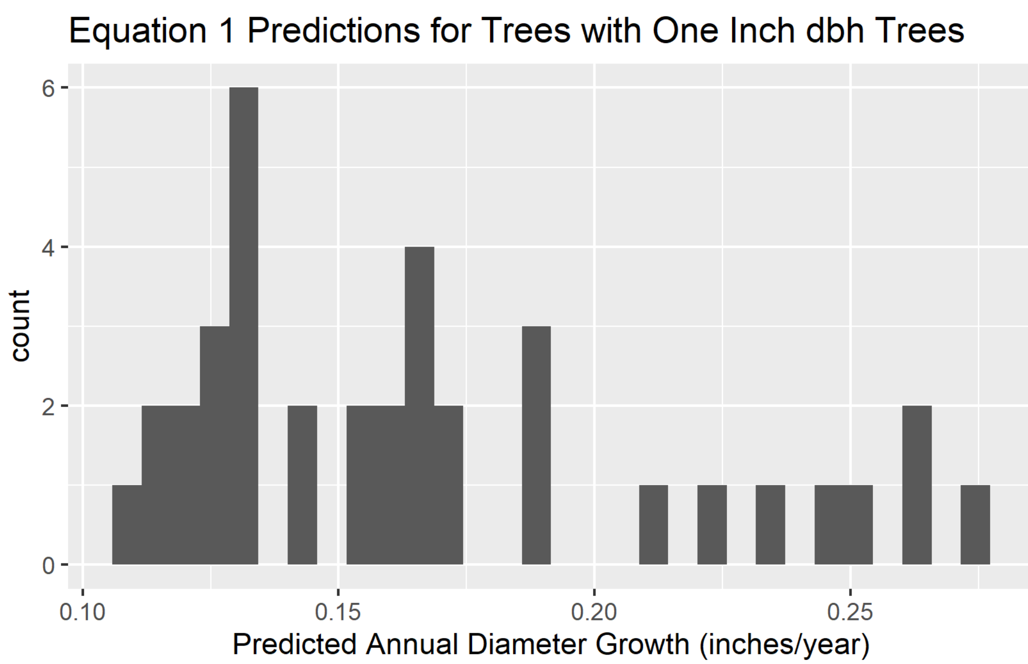
Table 3: Independent Variables for One Inch dbh Trees

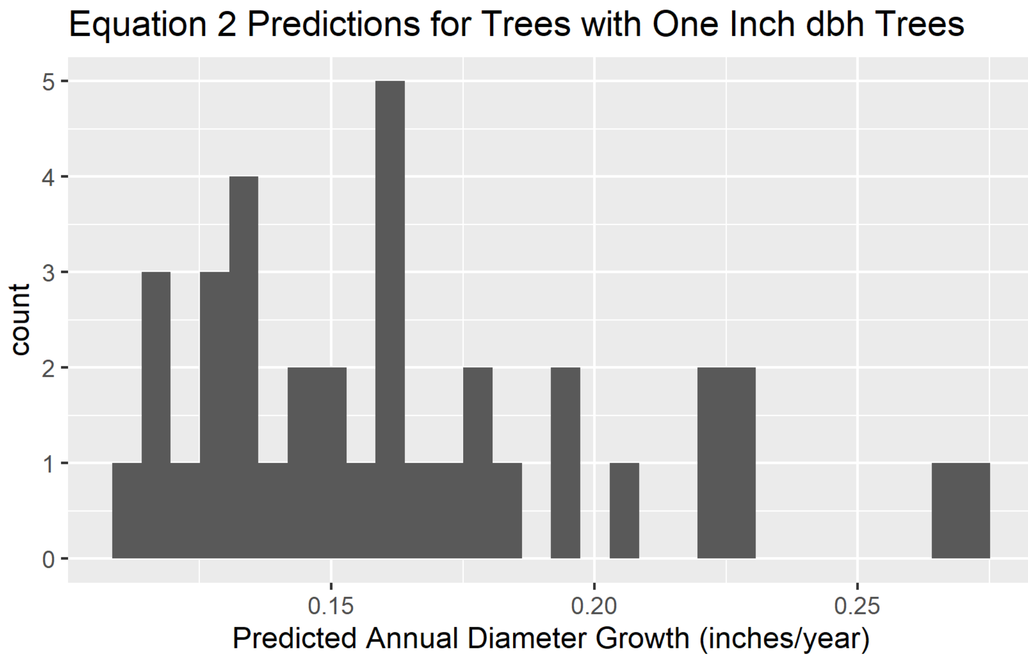
Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	37	179	86	52	117	231	431
bal	37	178	86	52	117	230	430
ht	37	7.3	1.4	5	7	8	12
cr	37	0.42	0.24	0.05	0.2	0.55	0.9
si	37	73	26	30	56	89	129

Discussion

Removing `si` degrades the fit marginally. Otherwise, the alternative model fits the data suprisingly well given the limited data.

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
count
0
1
2
3
4
5
0.15
0.20
0.25
Predicted Annual Diameter Growth (inches/year)