Alternative Ponderosa Pine Diameter Growth

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Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 14 states listed in the native range of Ponderosa Pine in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Ponderosa Pine (FIA species code 122), and remeasurement intervals $\geq$ 5 years we get the observations in Table 1.

Table 1: Ponderosa Pine Growth Observations by State

State	Observations
AZ	9614
CA	8947
CO	4224
ID	1981
MT	4841
ND	36
NE	1505
NM	4997
NV	33
OR	36106
SD	9925
UT	732
WA	6493
WY	1317

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Ponderosa Pine site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 25 different site index equations for 31 species. Ponderosa Pine site index comprises 80% of the observations. There are 3 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7}+\beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, er
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fa
       AIC      BIC    logLik
  290787.3 290862.6 -145385.7

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3 Residual
StdDev: 0.07395863 1.200723

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value  Std.Error    DF   t-value p-value
B0 -5.309932 0.08167771 90741 -65.01079       0
B1 -1.101649 0.01130416 90741 -97.45519       0
B2 -0.174414 0.01550332 90741 -11.25010       0
B3  1.039722 0.03781445 90741  27.49536       0
B4  1.368997 0.01011762 90741 135.30815       0
B5  0.610404 0.01694858 90741  36.01506       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.125
B2 -0.392  0.356
B3 -0.384  0.009  0.004
B4 -0.345 -0.063  0.430 -0.106
B5 -0.367  0.304  0.988  0.013  0.372

Standardized Within-Group Residuals:
         Min           Q1          Med           Q3          Max
-66.30587549  -0.27356123  -0.02141033   0.28769062  19.07167607

Number of Observations: 90751
Number of Groups: 5

$SIINT
```

```
                    B3
50.FALSE    0.07693482
80.FALSE    0.04397161
100.FALSE -0.07392520
50.TRUE     0.05537470
100.TRUE  -0.10235593
```

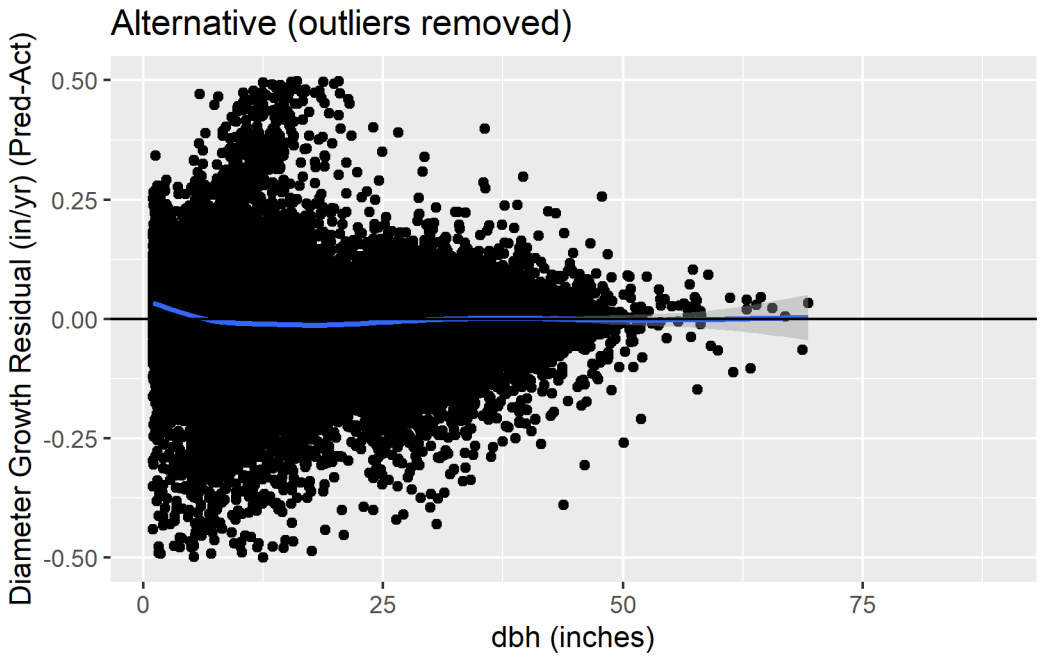
```
Residual Standard Error: 1.20072250094403 on 90741 degrees of freedom, AIC: 290787.3
```

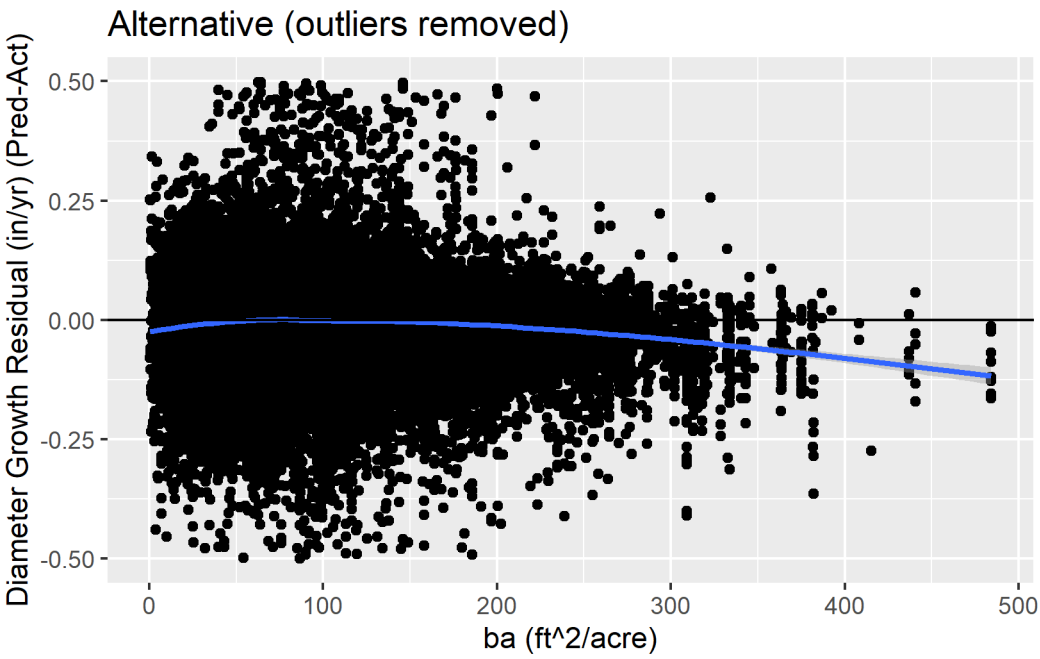
and for Equation 2:

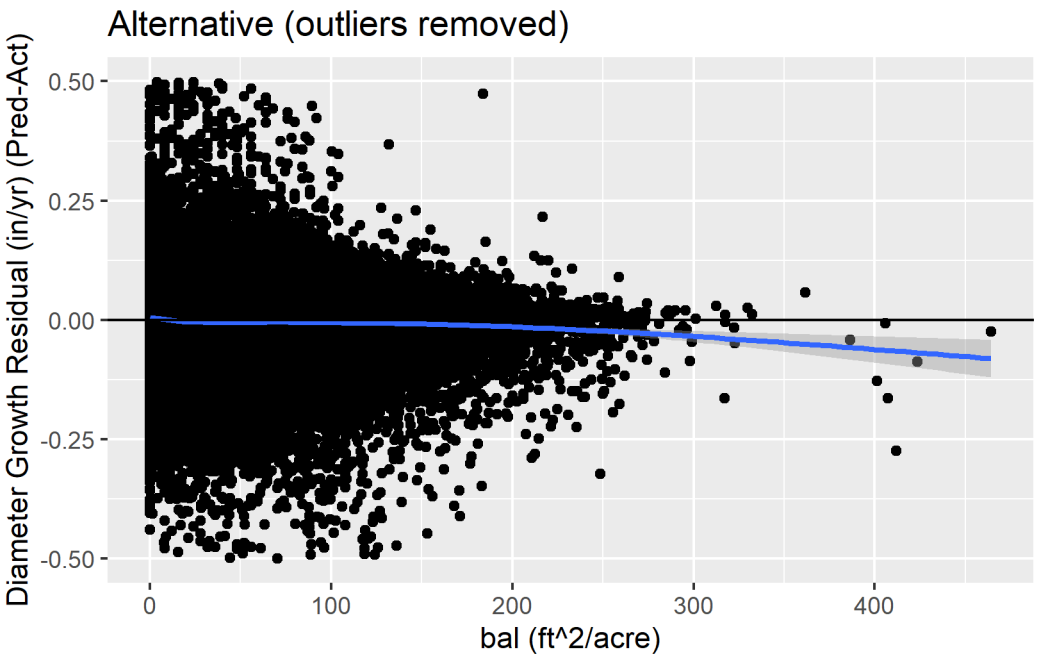
	Coef.	Std. error	t-stat.	p
B0	-1.4841416	0.0508277	-29.19948	0
B1	-1.2470106	0.0123397	-101.05667	0
B2	-0.2017198	0.0187050	-10.78429	0
B4	1.4961869	0.0092050	162.54018	0
B5	0.5721826	0.0175738	32.55892	0

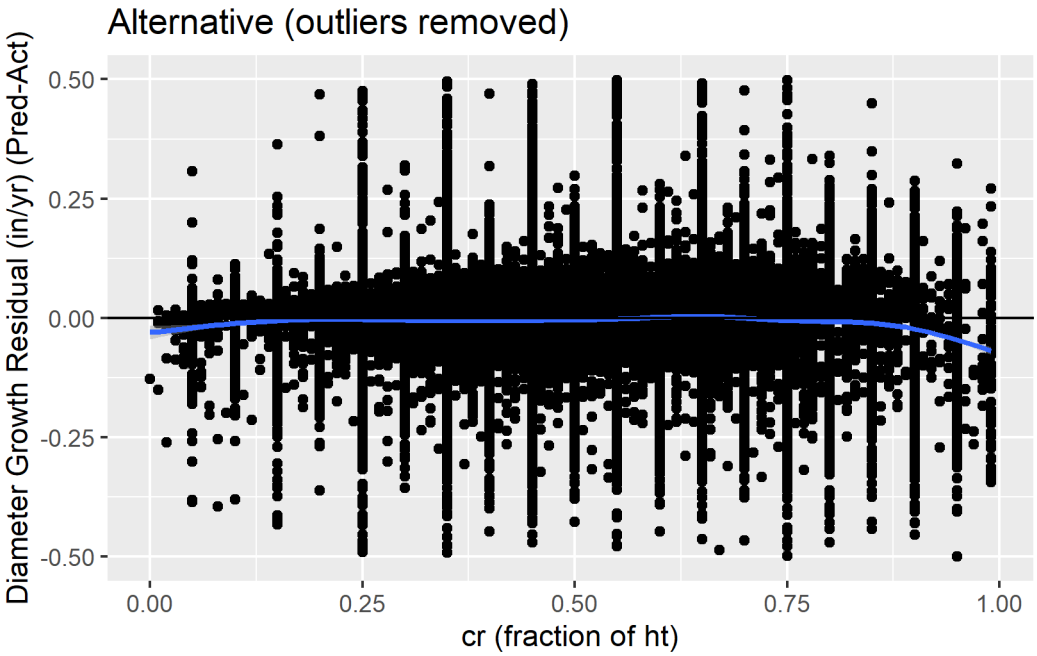
```
Residual Standard Error: 1.2213117267861 on 90746 degrees of freedom, AIC: 293834.1
```

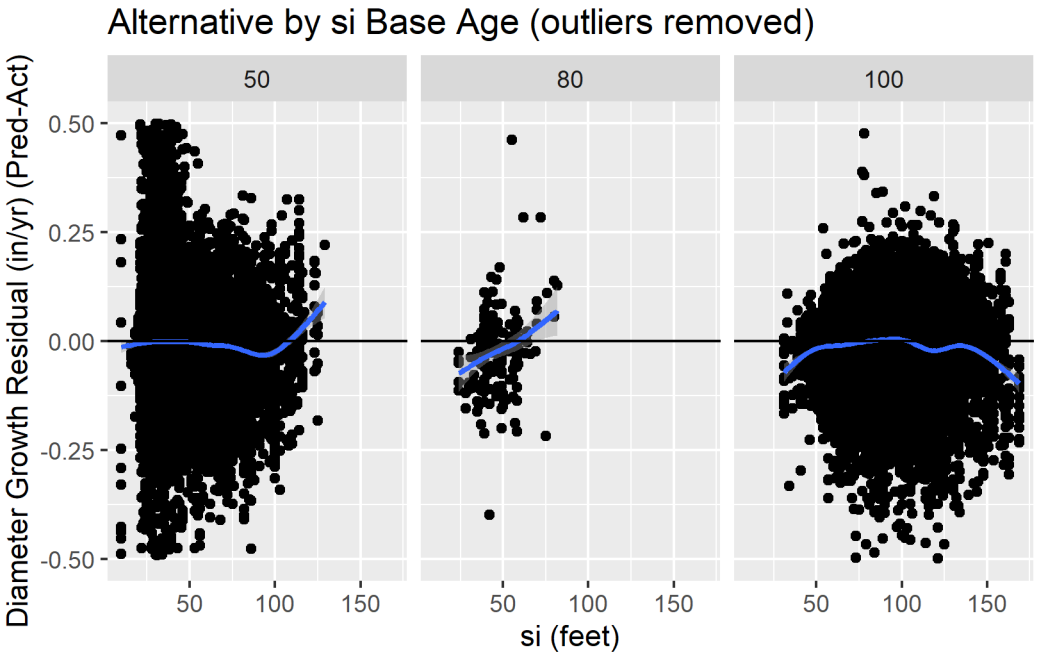
Residual Analysis for Equation 1

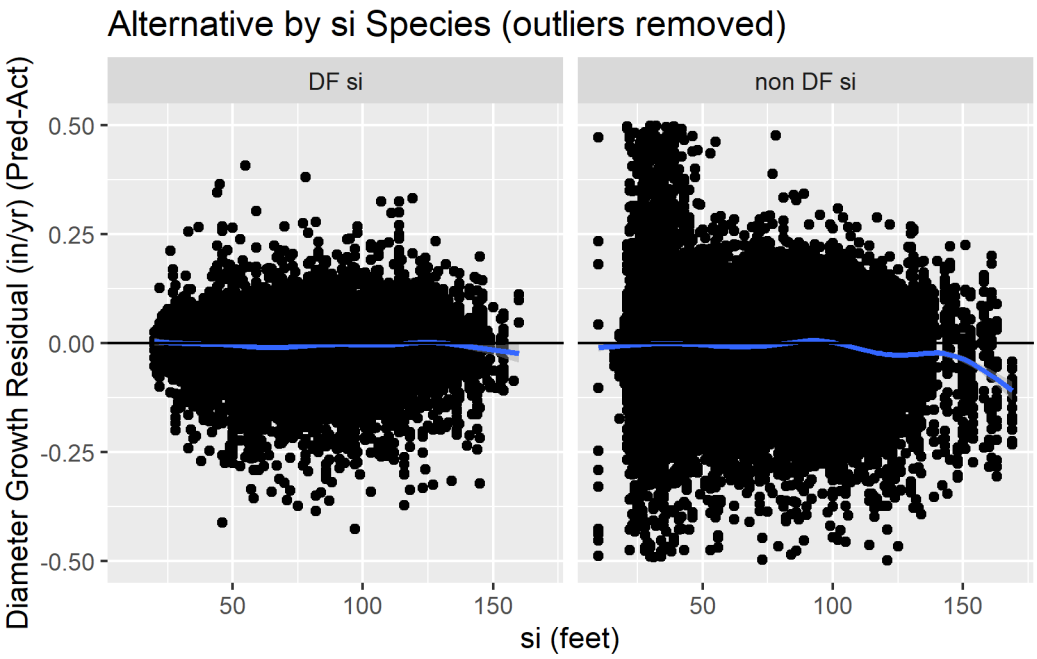


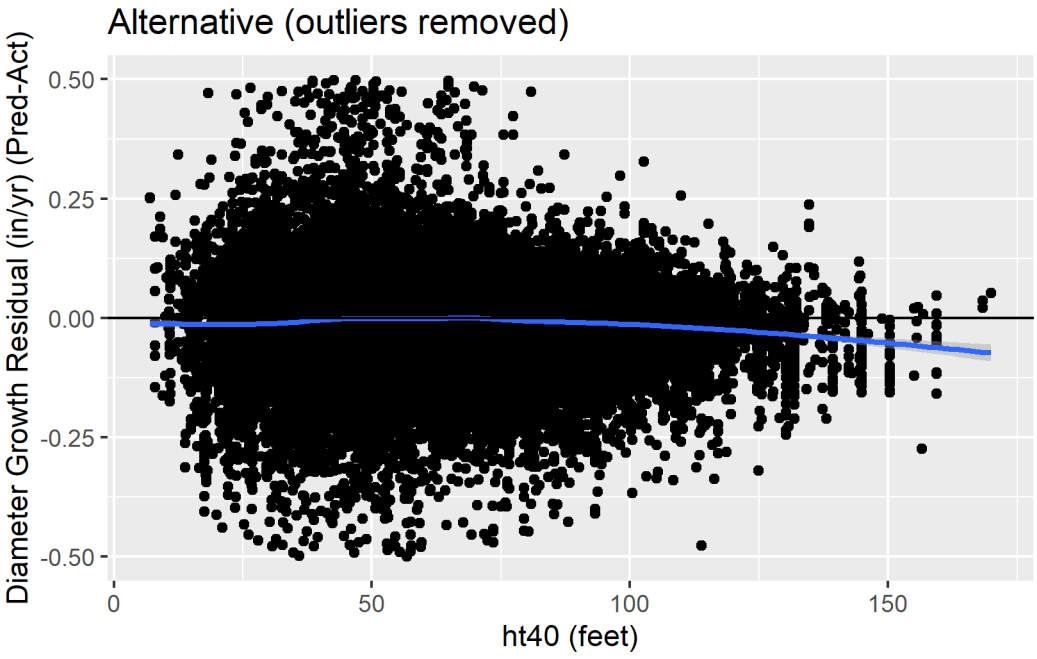

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
500
ba (ft^2/acre)

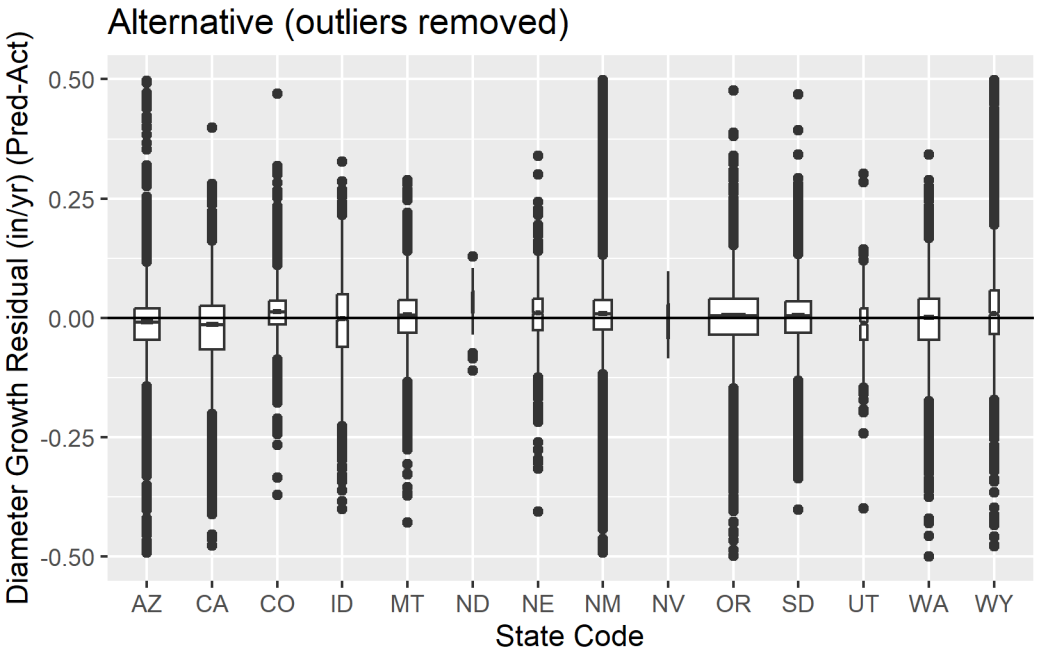

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
bal (ft^2/acre)

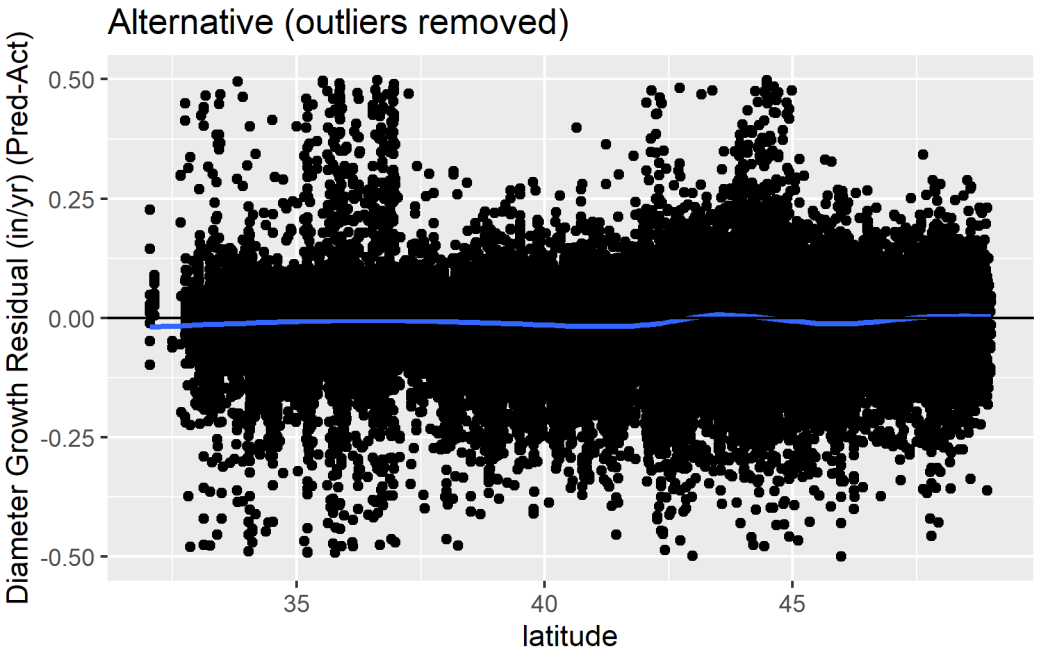

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

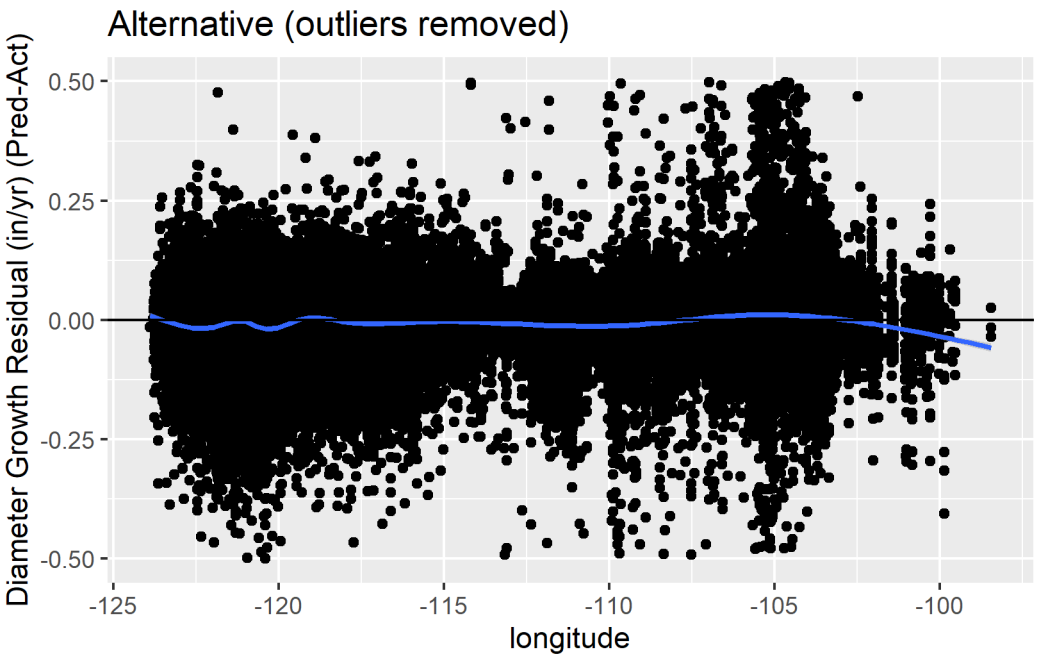

Alternative by si Base Age (outliers removed)
50
80
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

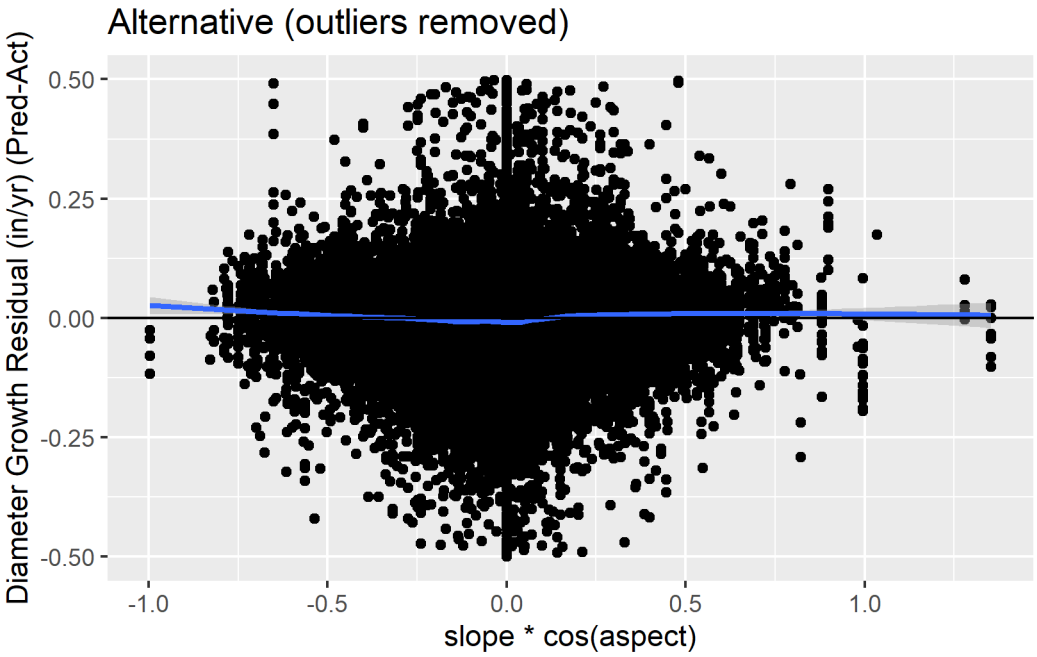

Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

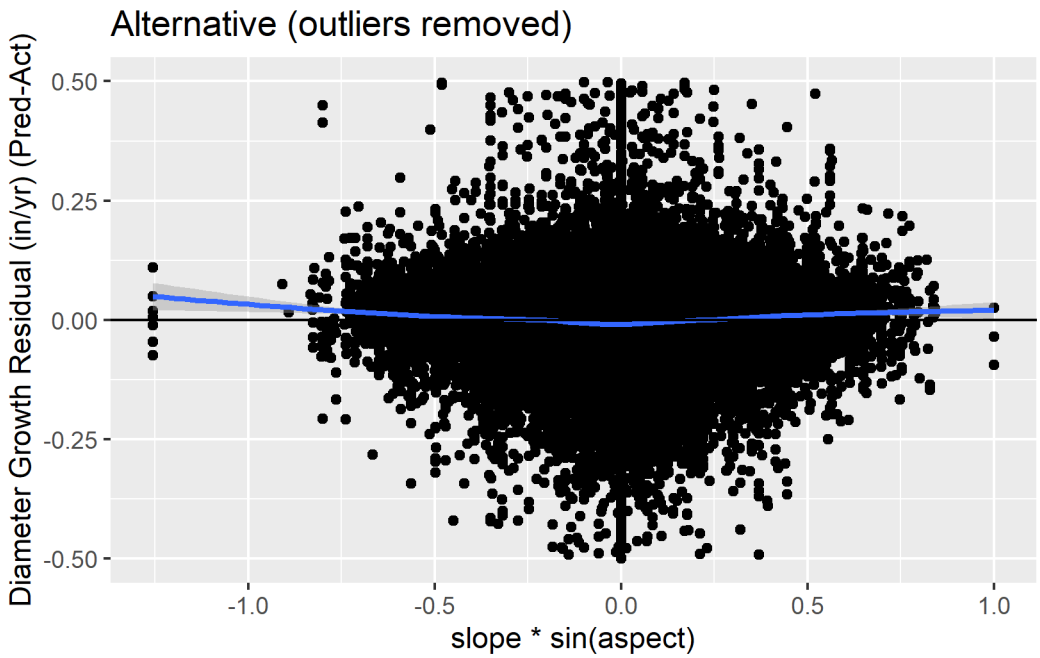

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
AZ
CA
CO
ID
MT
ND
NE
NM
NV
OR
SD
UT
WA
WY
State Code

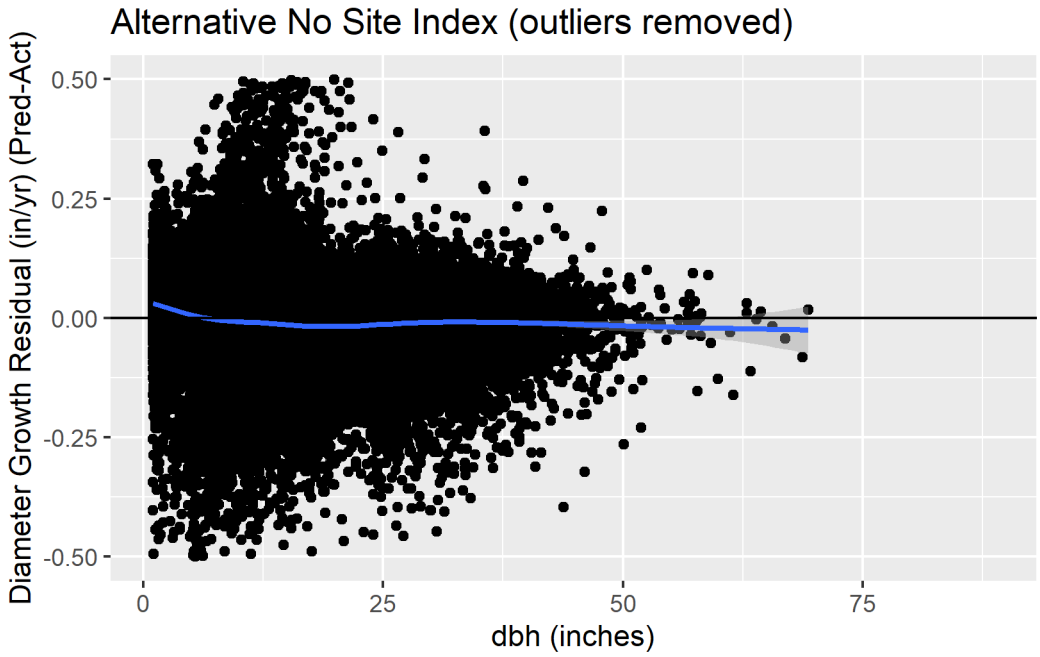

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
35
40
45
latitude

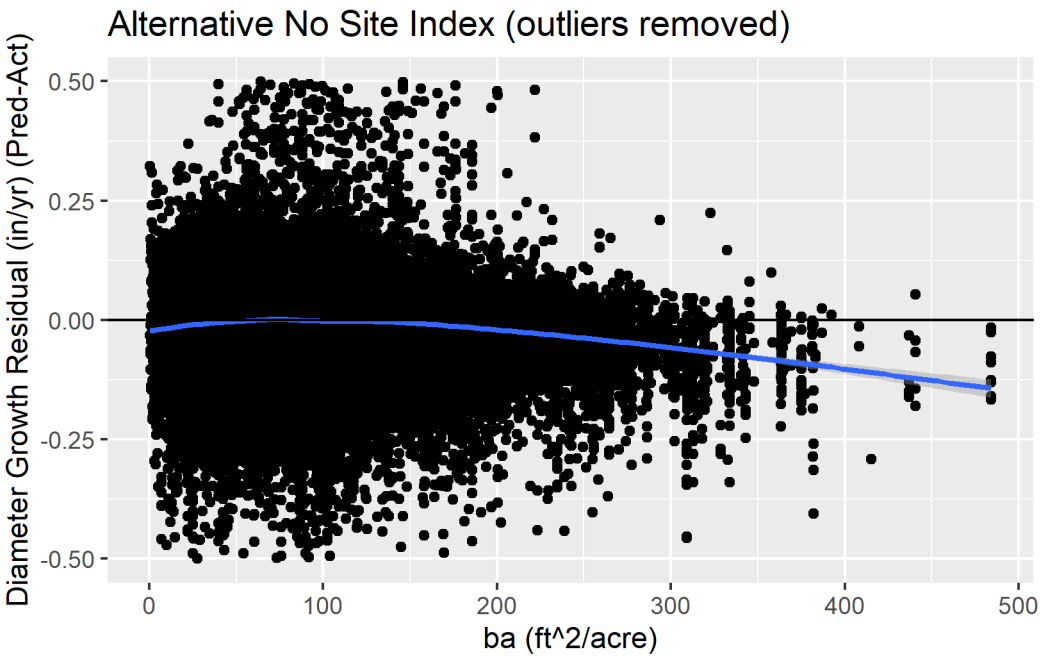

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-125
-120
-115
-110
-105
-100
longitude

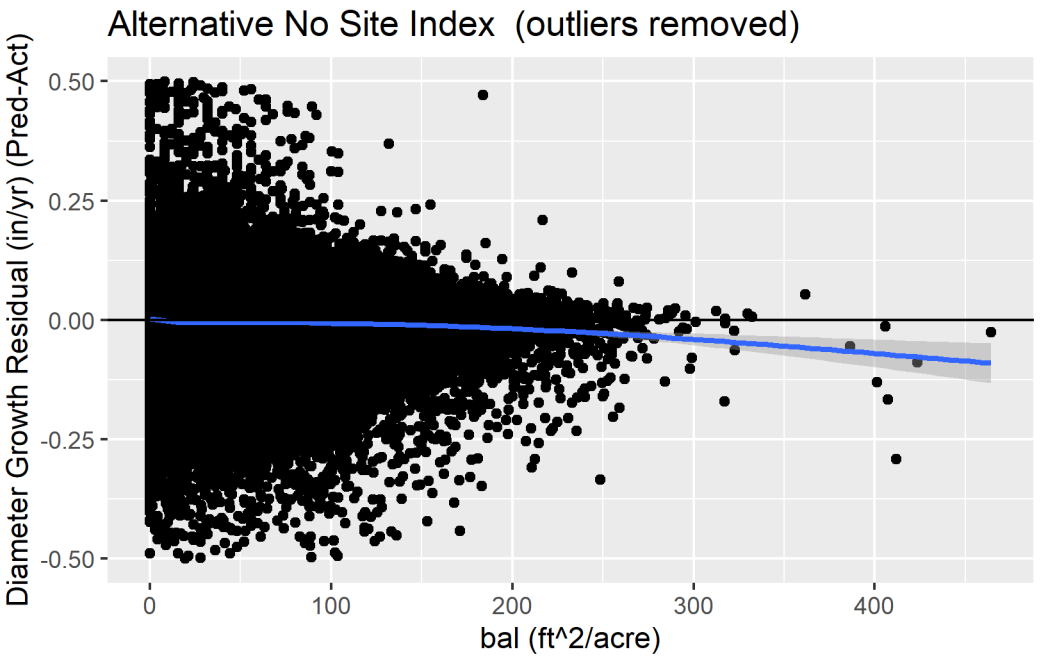

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)

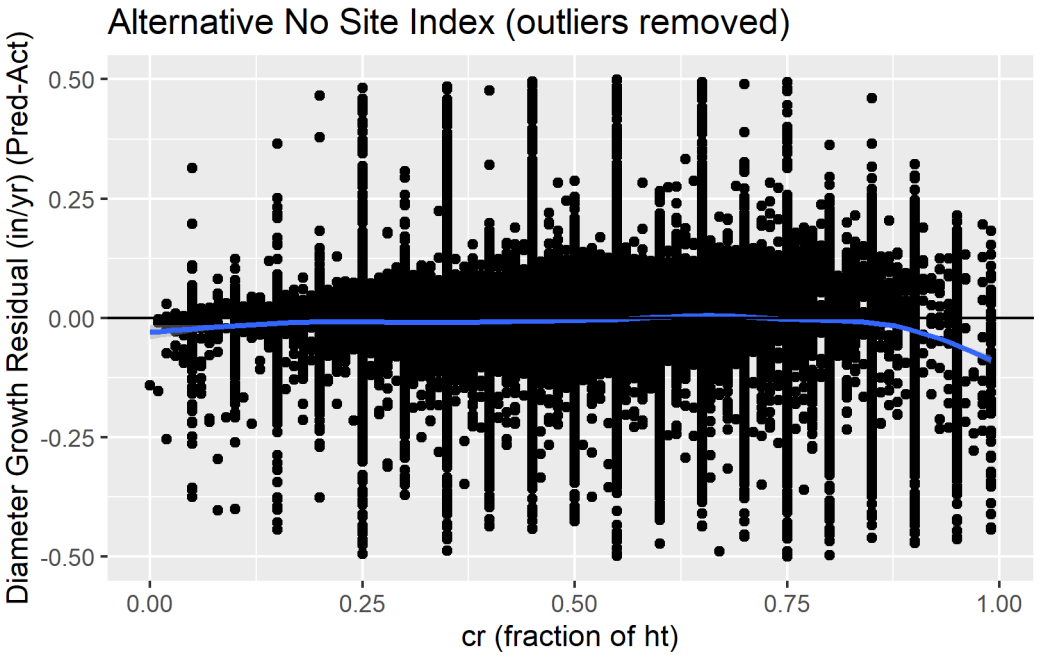


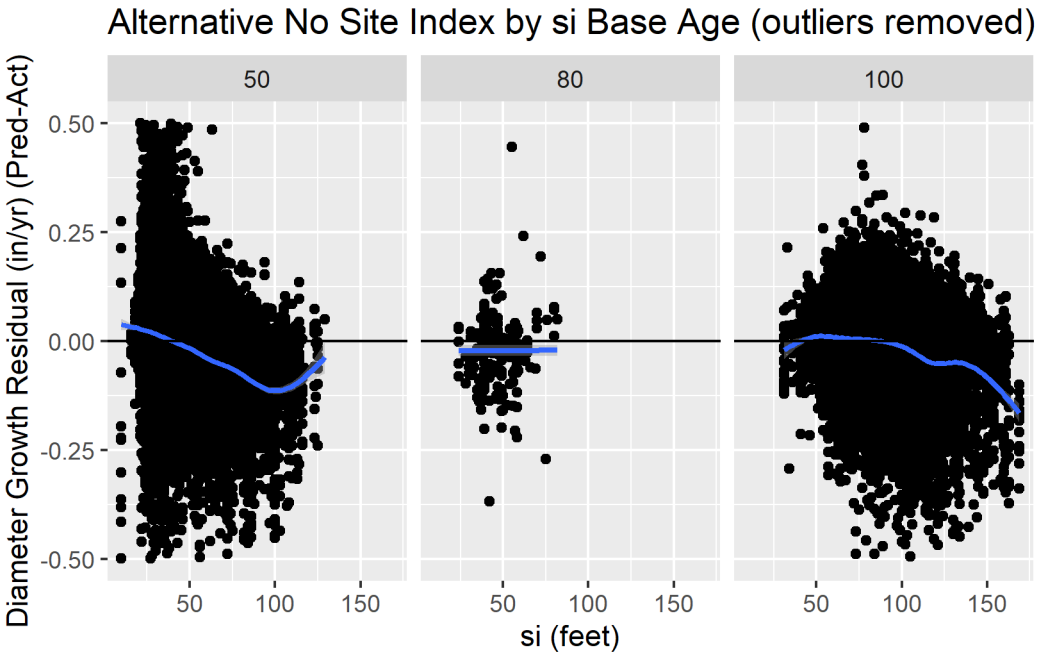
Residual Analysis for Equation 2

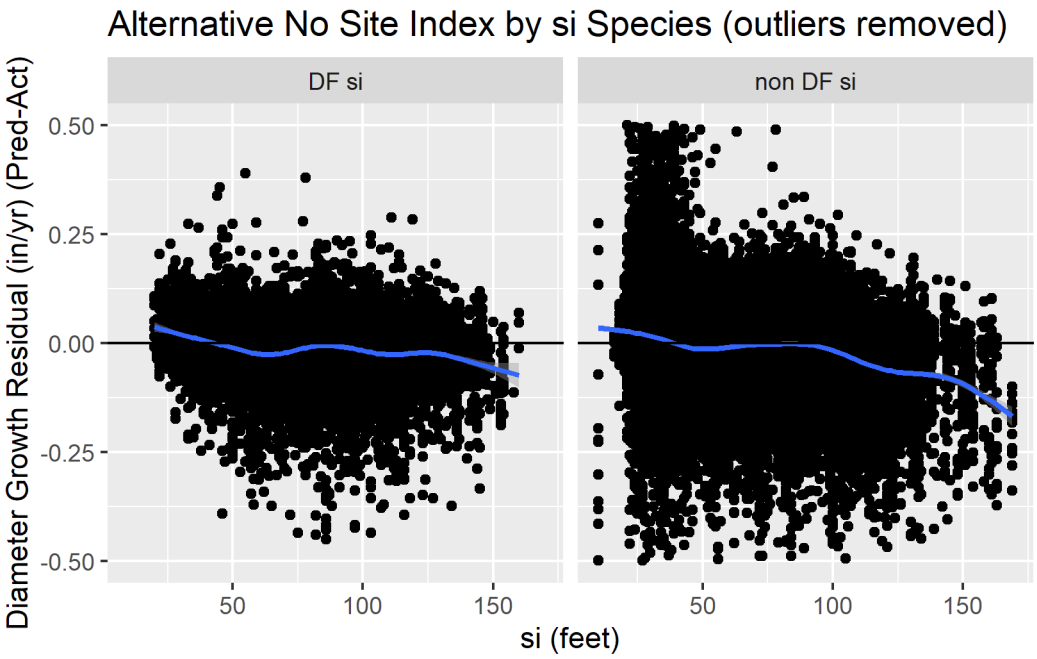


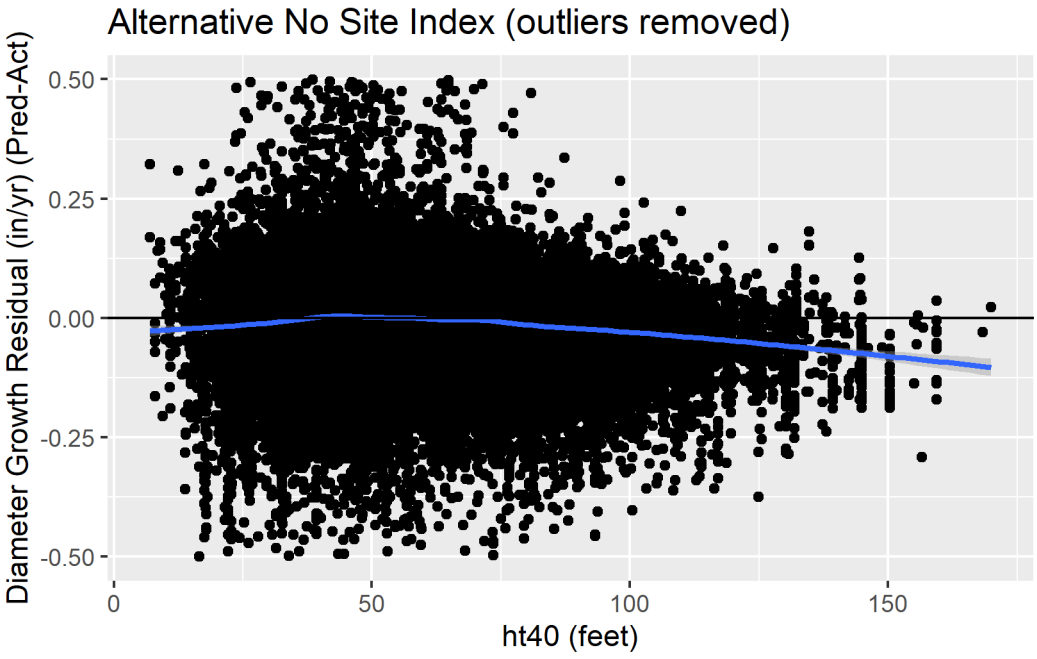

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
500
ba (ft^2/acre)

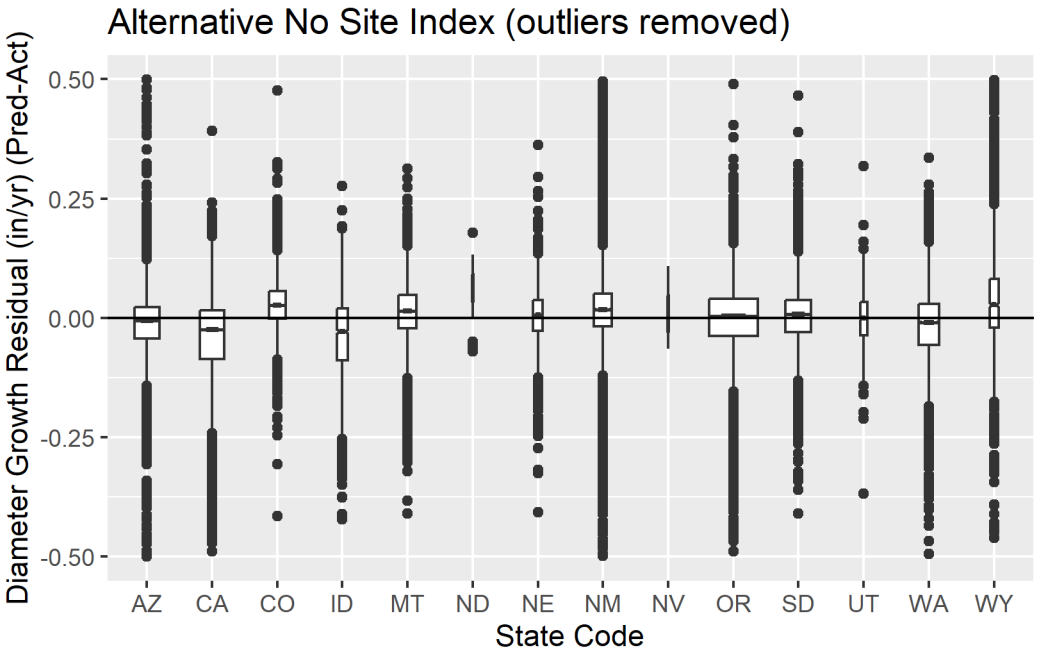

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
100
200
300
400
bal (ft^2/acre)

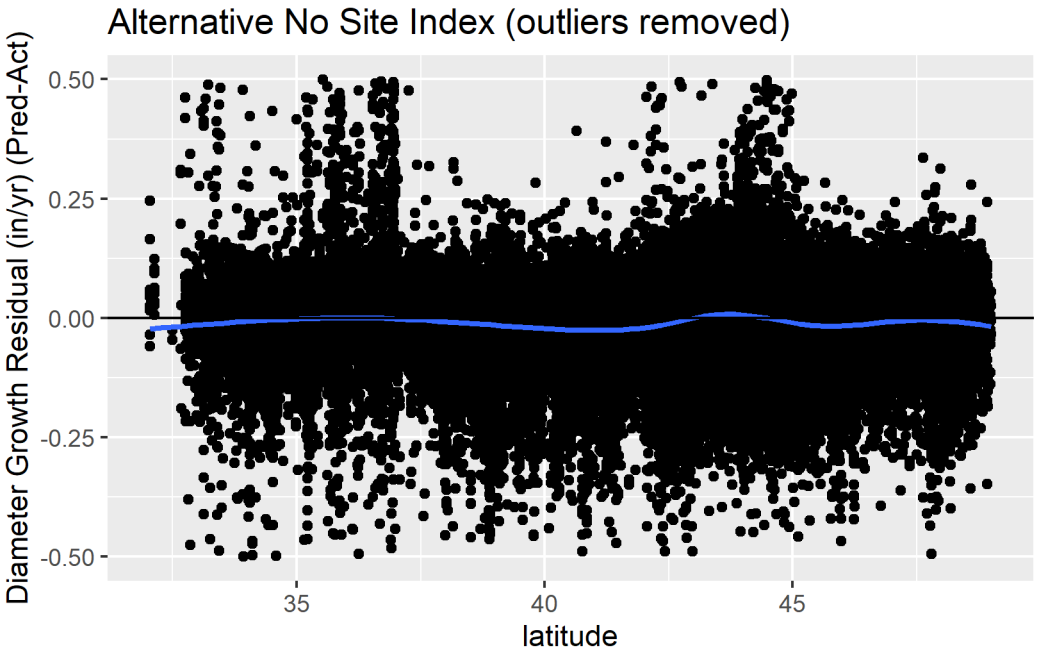

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

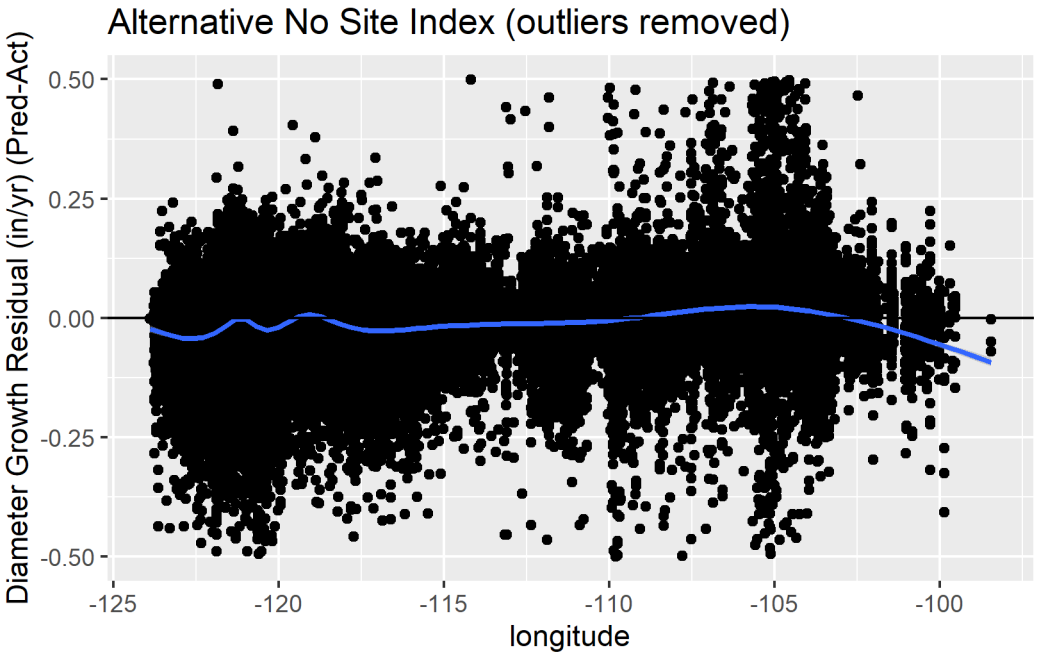

Alternative No Site Index by si Base Age (outliers removed)
50
80
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

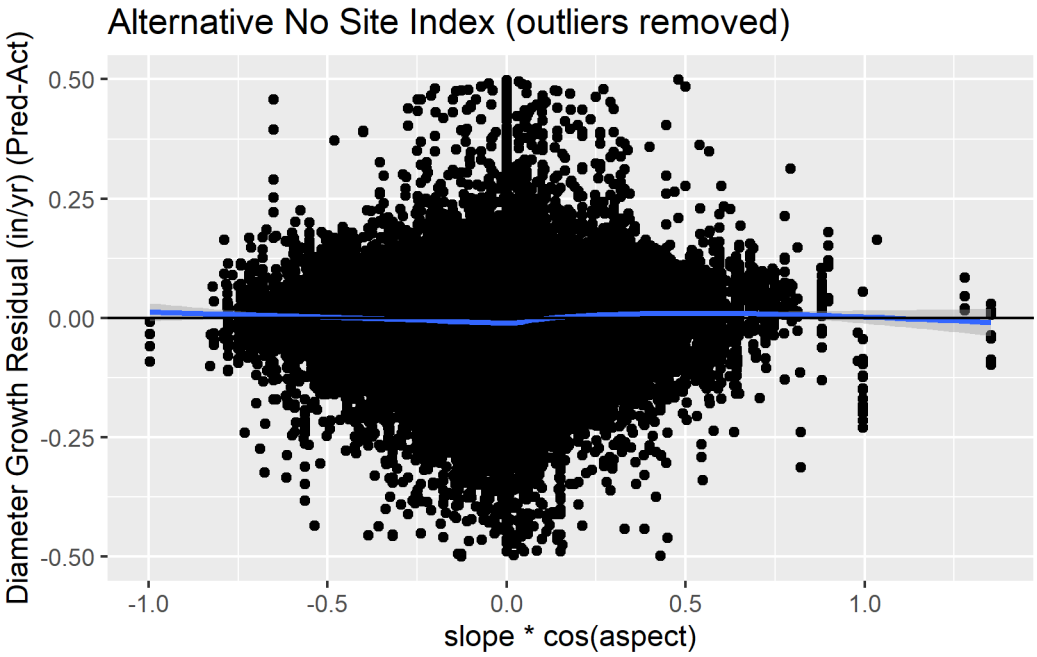

Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

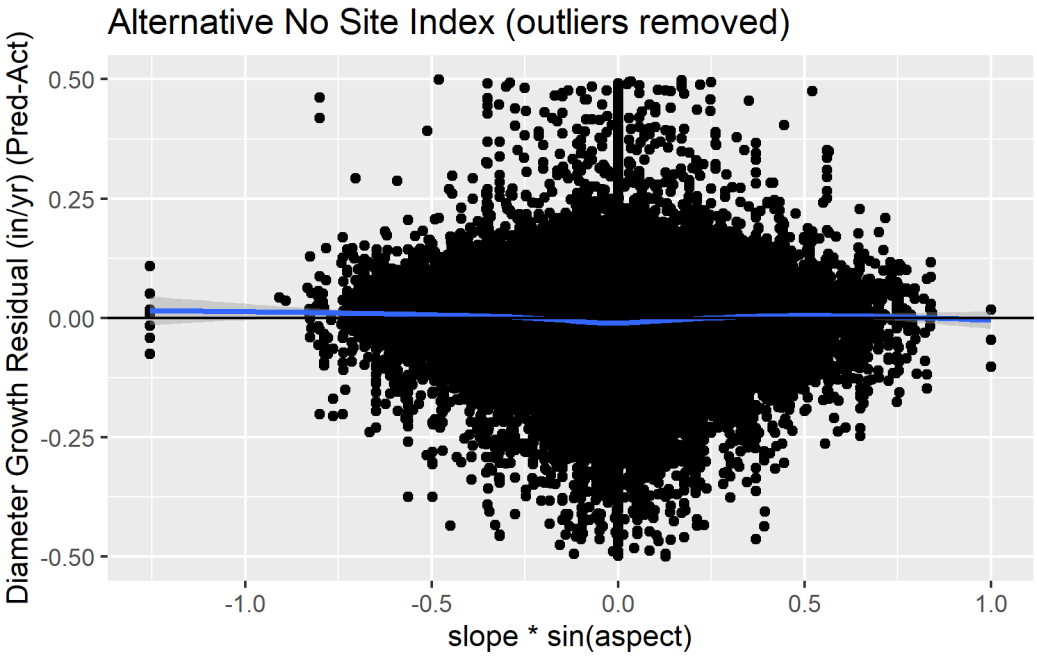

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
AZ
CA
CO
ID
MT
ND
NE
NM
NV
OR
SD
UT
WA
WY
State Code


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
35
40
45
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-125
-120
-115
-110
-105
-100
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)

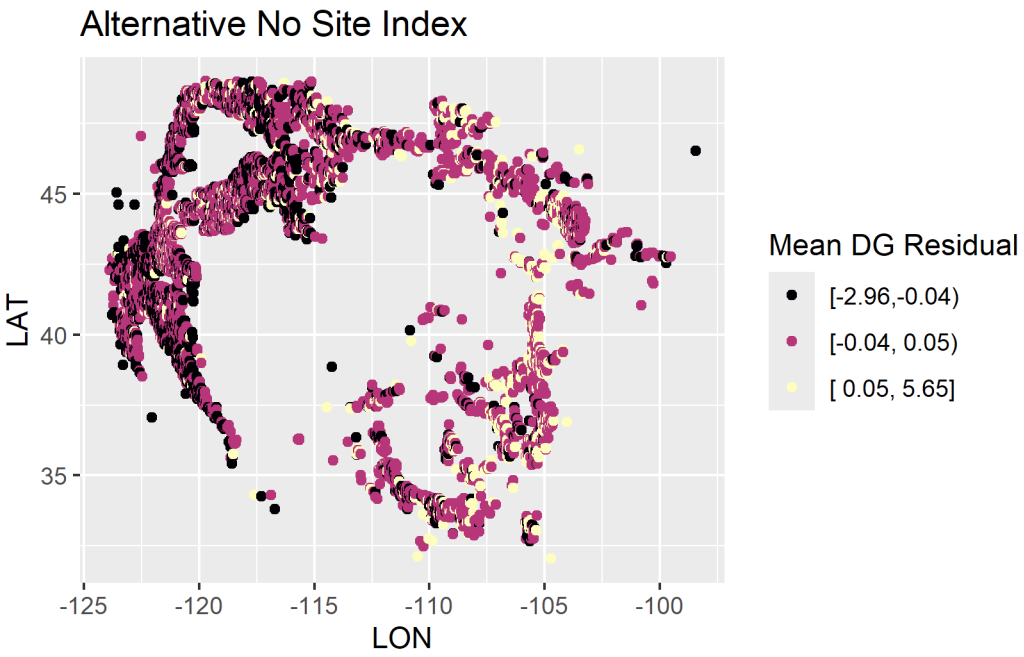

Alternative No Site Index
LAT
45
40
35
-125
-120
-115
-110
-105
-100
LON
Mean DG Residual
[-2.96,-0.04)
[-0.04, 0.05)
[0.05, 5.65]

Table 3: Independent Variables for One Inch dbh Trees

Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	203	95	45	0.41	63	123	251
bal	203	94	45	0	62	122	251
ht	203	8.5	1.8	6	7	10	15
cr	203	0.47	0.22	0.05	0.3	0.64	0.95
si	203	65	28	21	40	89	132

Discussion

Removing `si` degrades the fit significantly. There is a small bias for some states for both equations. The dubious `si` term mitigates some of the bias (expected since it is acting as a localization variable).

There do not appear to be residual trends with latitude and longitude, however, the no site index equations residual map hints at a trend of over prediction for larger longitude (eastward).

Interestingly, for the no `si` equation, there is a trend of under-prediction for higher `ba`. This will need to be investigated in further equation development.

Equation Behavior for Very Small Trees