Alternative Balsam Fir Diameter Growth

Greg Johnson, David Marshall, Aaron Weiskittel

2026-01-27

Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 7 states listed in the native range of Balsam Fir in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Balsam Fir (FIA species code 12), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Balsam Fir Growth Observations by State

State	Observations
CT	1
MA	40
ME	83061
MN	40793
NH	11573
VT	4172
WV	6

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

term with a ratio of a transformation of diameter at breast height (dbh) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (bal) serves as the inter-tree competition factor, and site index (si) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Balsam Fir site trees,
2. It is estimated using a number of different and not necessarily compatible si equations, and
3. The available si equations do not all use the same base age.

In the data set si is derived from 25 different site index equations for 27 species. Balsam Fir site index comprises 35% of the observations. There are 1 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where SIBASE and SISP are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7} + \beta_3 log(si_{s,b} + 4.5))} \quad (1)$$

and

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \quad (2)$$

where:

- dbh = diameter at breast height (inches),
- bal = basal area per acre in larger trees ($feet^2/ac$),
- cr = crown ratio (fraction of total height),
- ht = total height (feet), and
- $si_{s,b}$ = site index (feet) for species s and base age b.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending dbh. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

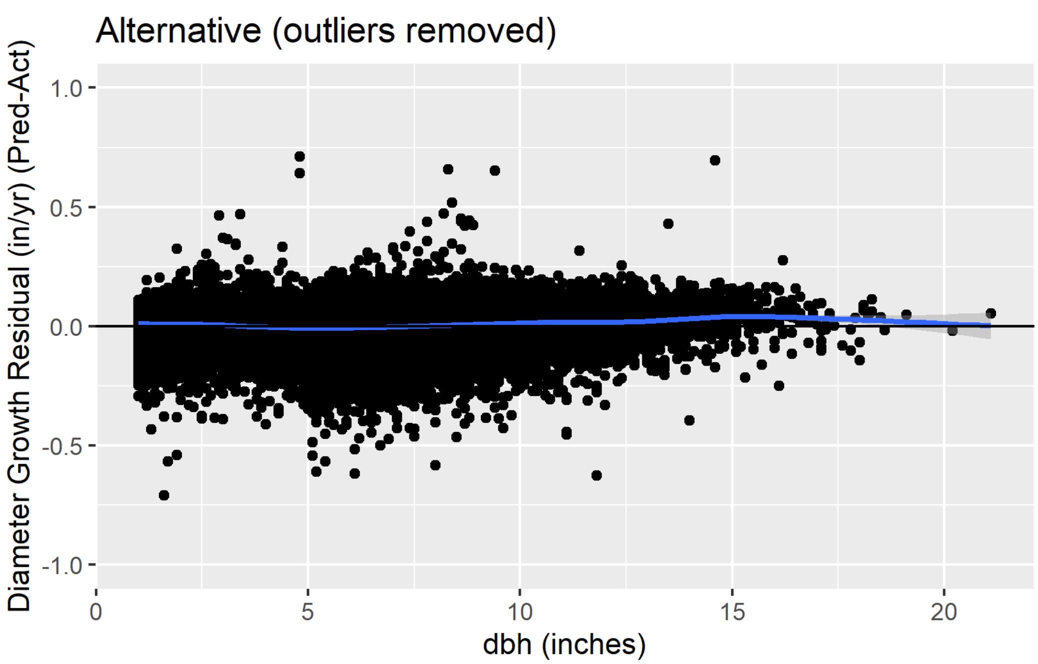
```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, en
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
       AIC      BIC    logLik
  71704.57 71783.35 -35844.29

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3  Residual
StdDev: 0.01237966 0.3127649

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value  Std.Error     DF    t-value p-value
B0 -3.667189 0.03537306 139639 -103.67179       0
B1 -0.367941 0.00351713 139639 -104.61422       0
B2 -0.042757 0.00401374 139639  -10.65266       0
B3  0.129654 0.01194394 139639   10.85522       0
B4  2.527050 0.01916143 139639  131.88214       0
B5  0.698240 0.01727999 139639   40.40744       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.177
B2 -0.338  0.491
B3 -0.573 -0.023 -0.036
B4 -0.342  0.790  0.613 -0.086
B5 -0.316  0.465  0.994 -0.030  0.560

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-32.3852603  -0.5899003  -0.1846415   0.4081703  24.9075325

Number of Observations: 139646
Number of Groups: 2

$SIINT
                  B3
50.FALSE -0.01236728
50.TRUE   0.01236728

Residual Standard Error: 0.312764862293159 on 139639 degrees of freedom, AIC: 71704.6
```

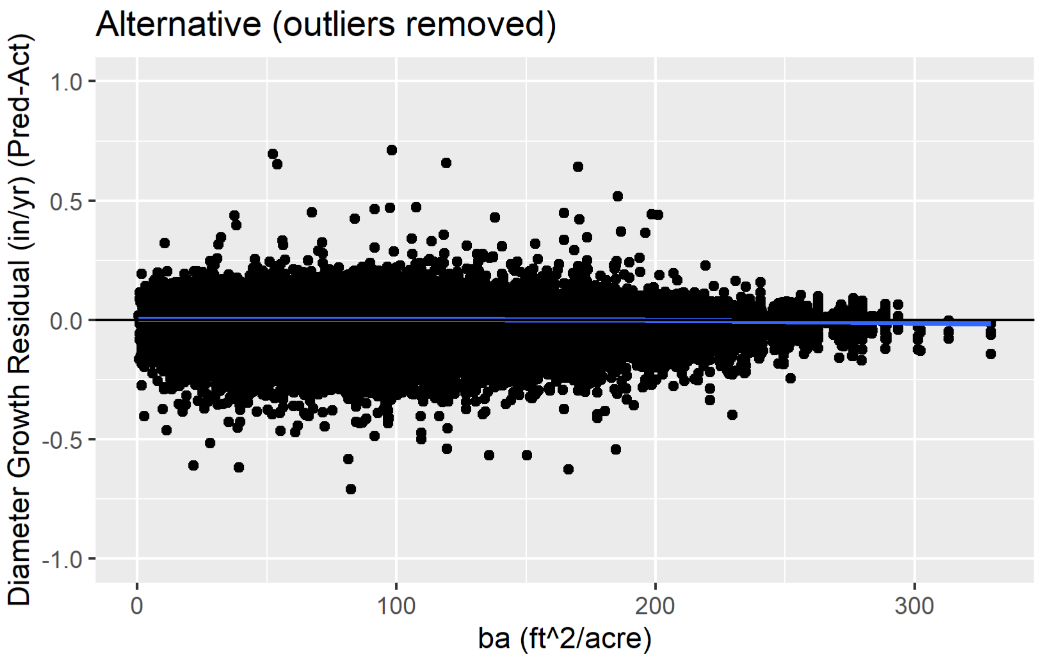
and for Equation 2:

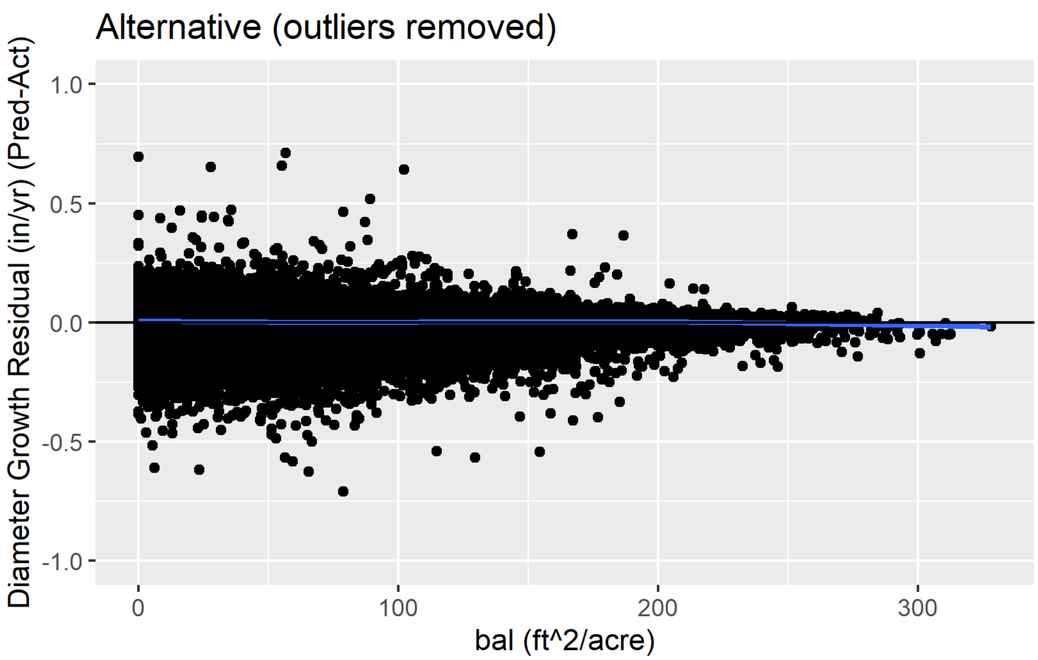
	Coef.	Std. error	t-stat.	p
B0	-3.1801582	0.0188532	-168.68042	0
B1	-0.3525681	0.0034588	-101.93279	0
B2	-0.0387618	0.0036883	-10.50944	0
B4	2.5968033	0.0202114	128.48219	0
B5	0.7168023	0.0175847	40.76286	0

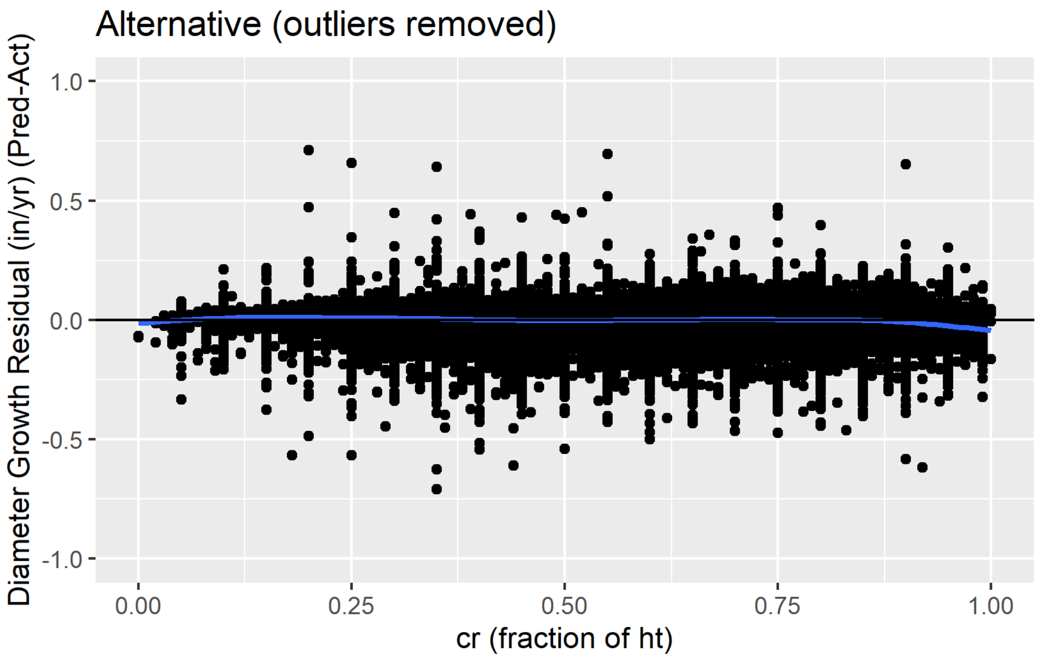
`Residual Standard Error: 0.313789812510913 on 139641 degrees of freedom, AIC: 72596.8`

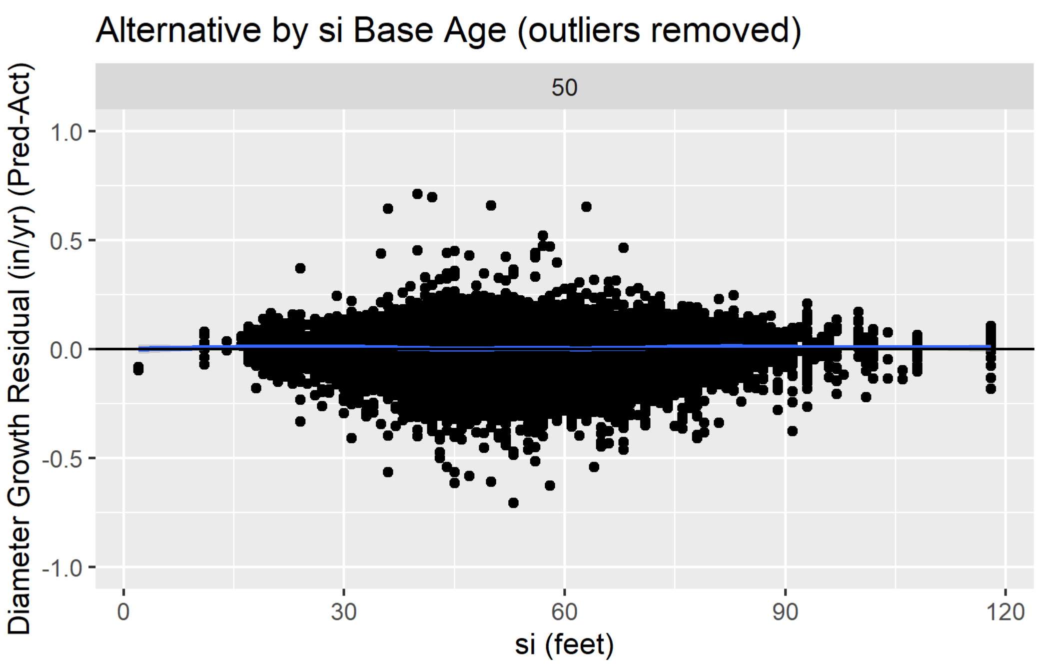
Residual Analysis for Equation 1

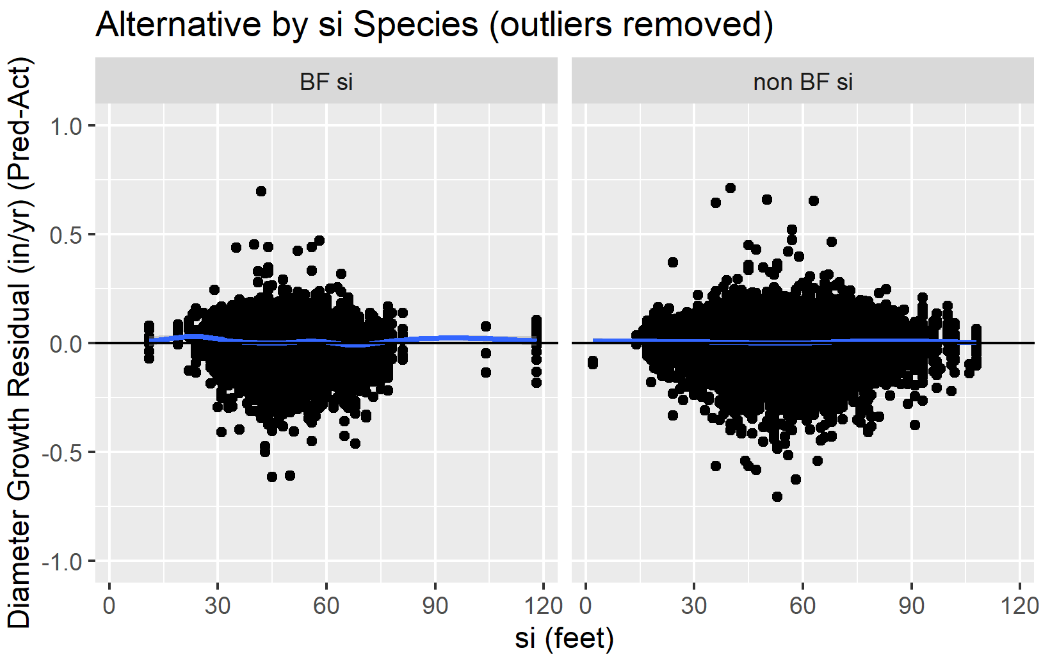


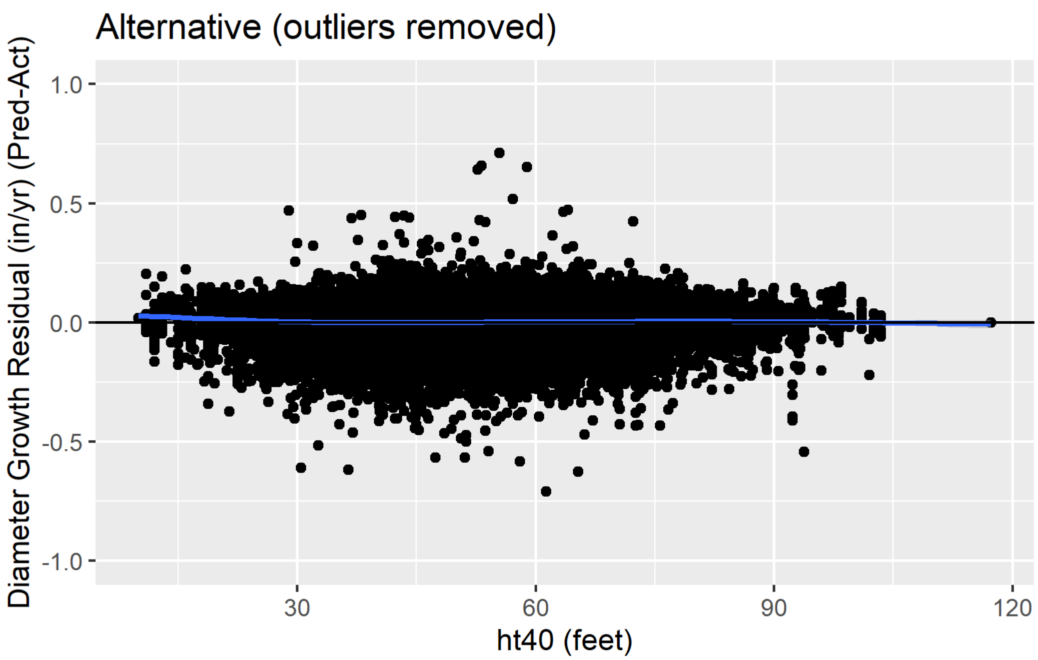

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
ba (ft^2/acre)

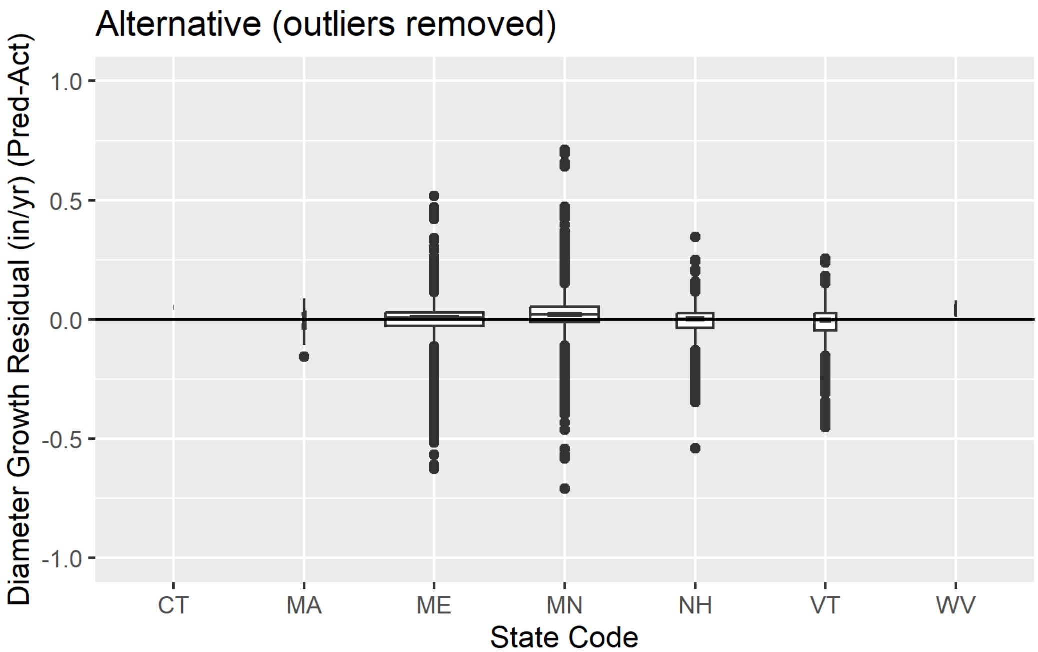

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
bal (ft^2/acre)

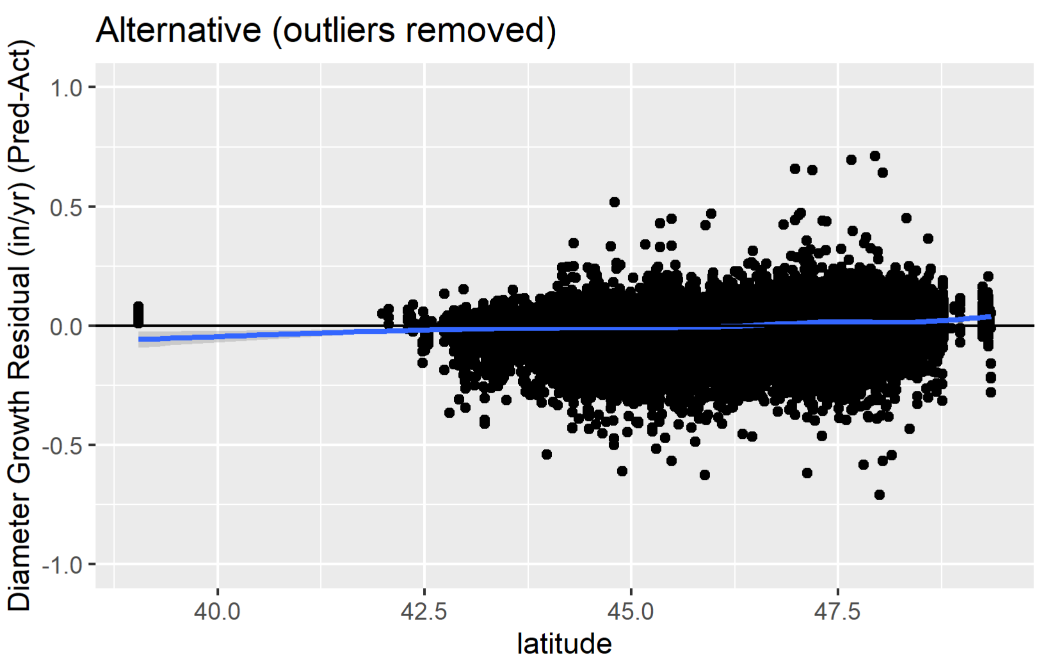

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

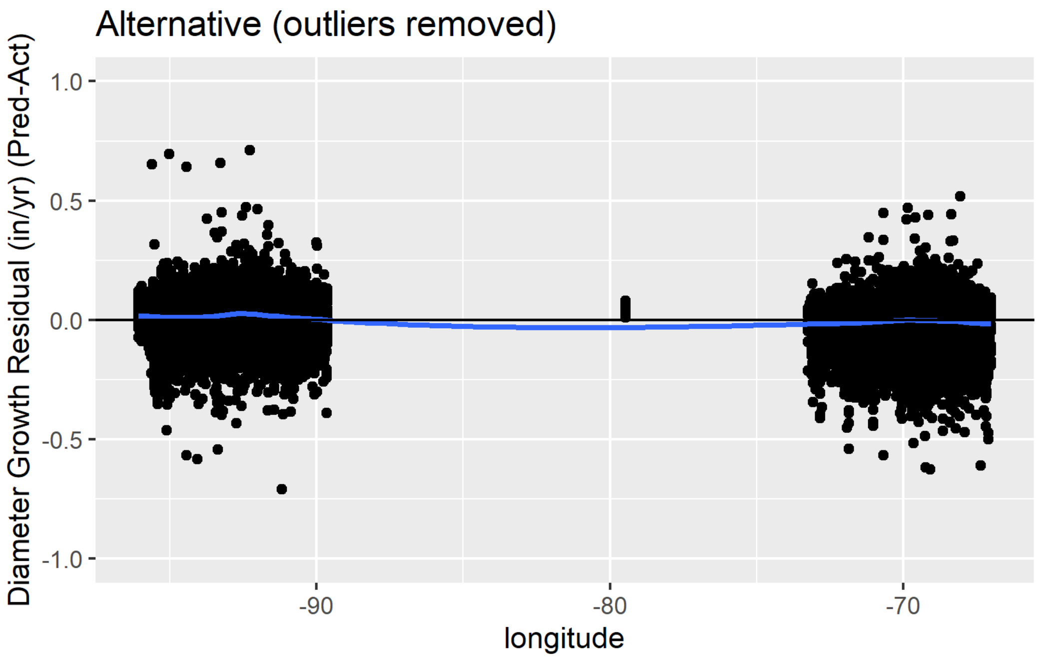

Alternative by si Base Age (outliers removed)
50
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)


Alternative by si Species (outliers removed)
BF si
non BF si
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

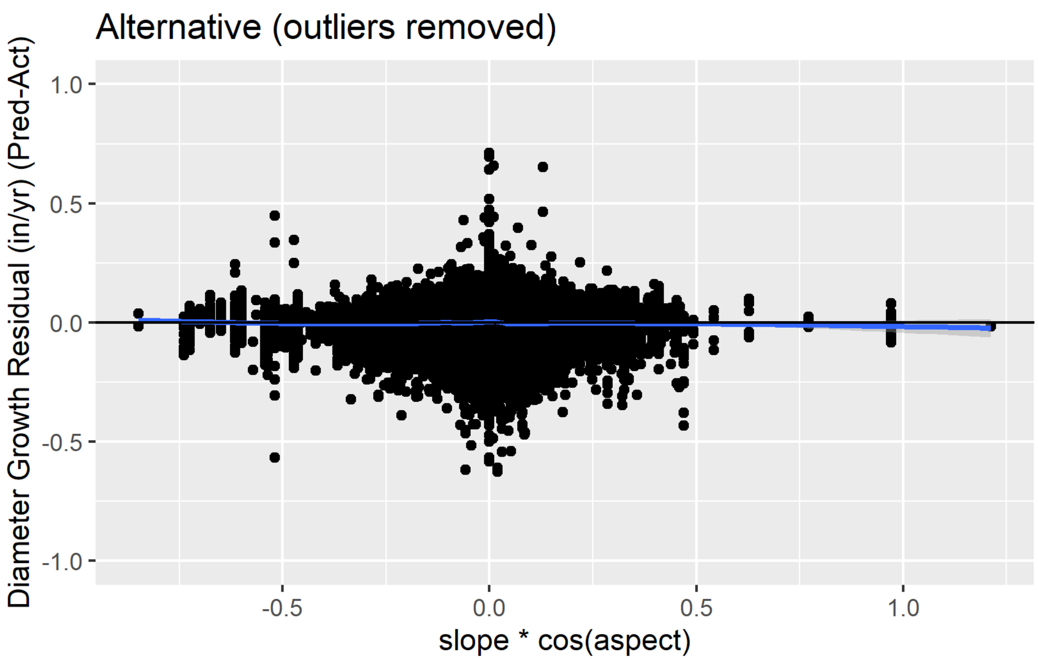

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
30
60
90
120
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
CT
MA
ME
MN
NH
VT
WV
State Code

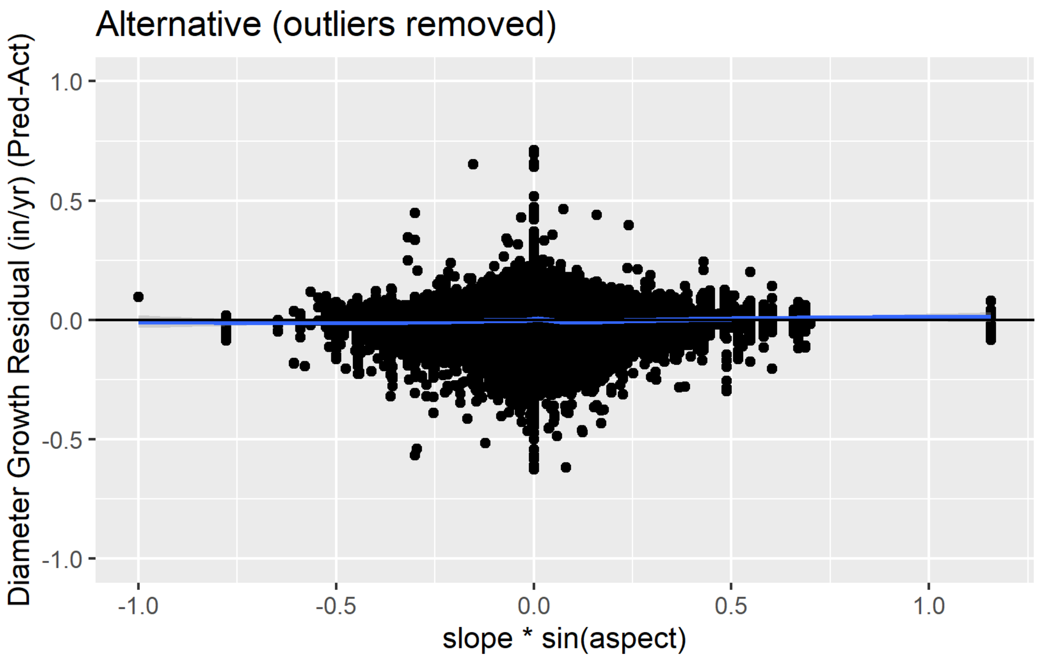

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
40.0
42.5
45.0
47.5
latitude



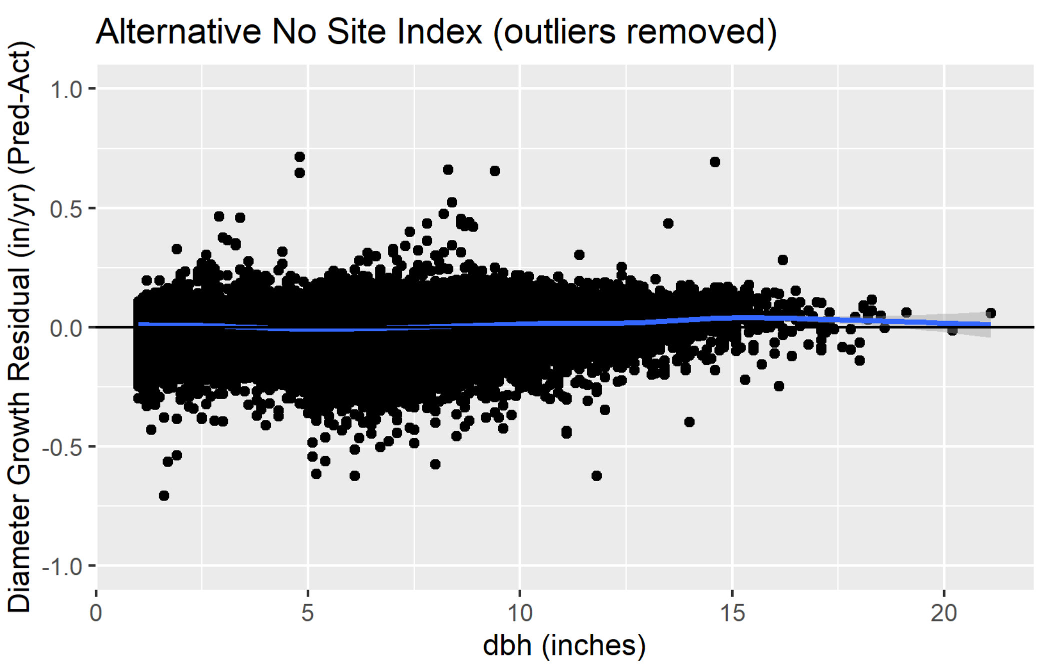
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-90
-80
-70
longitude

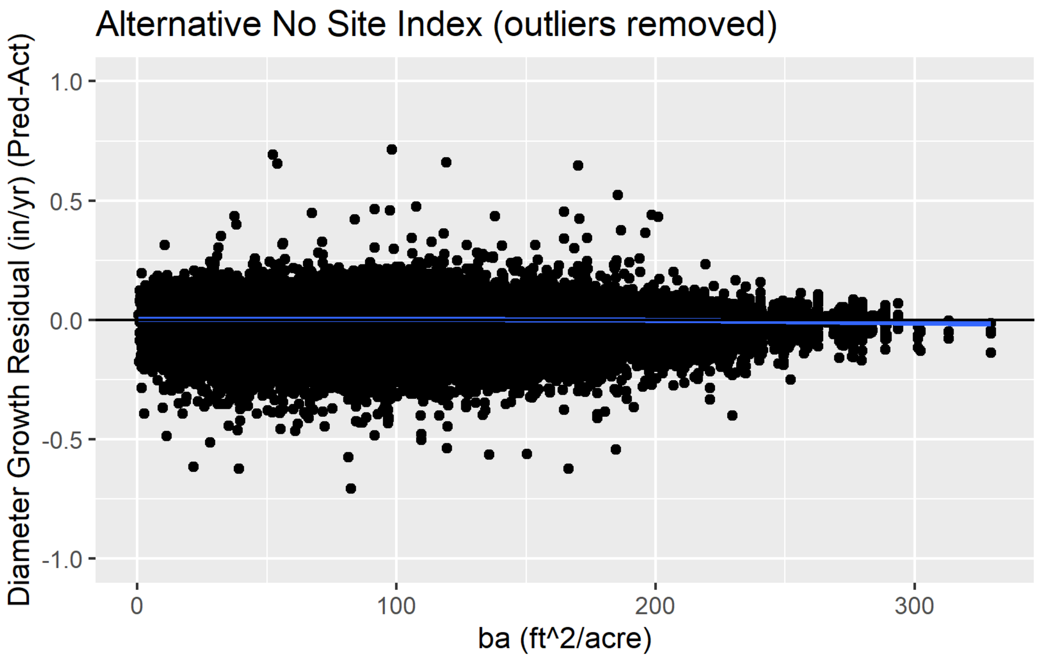


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)

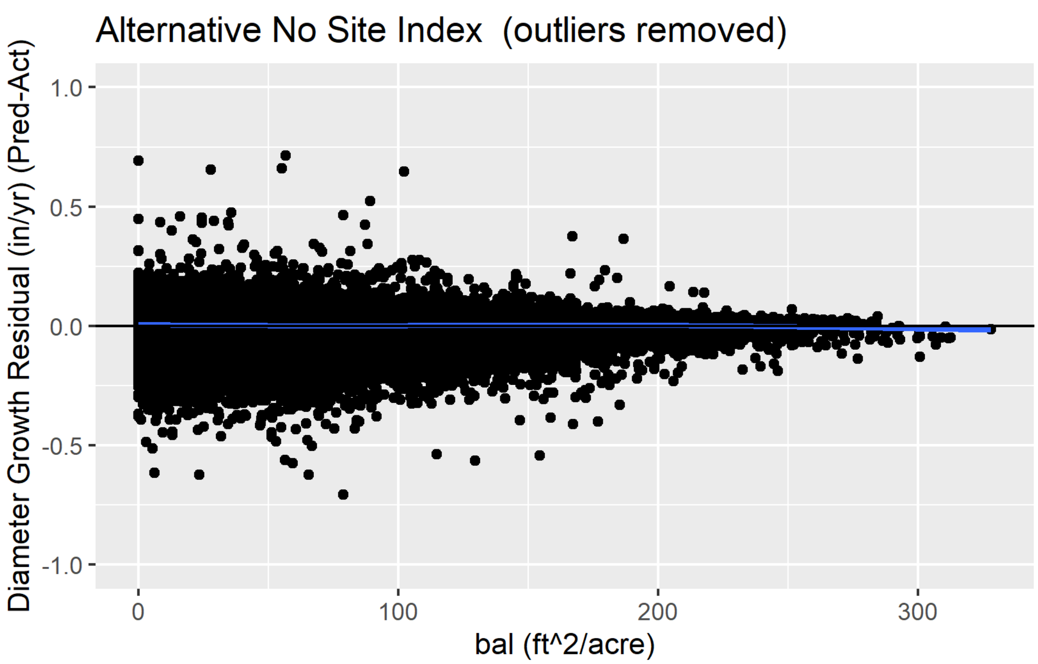


Residual Analysis for Equation 2

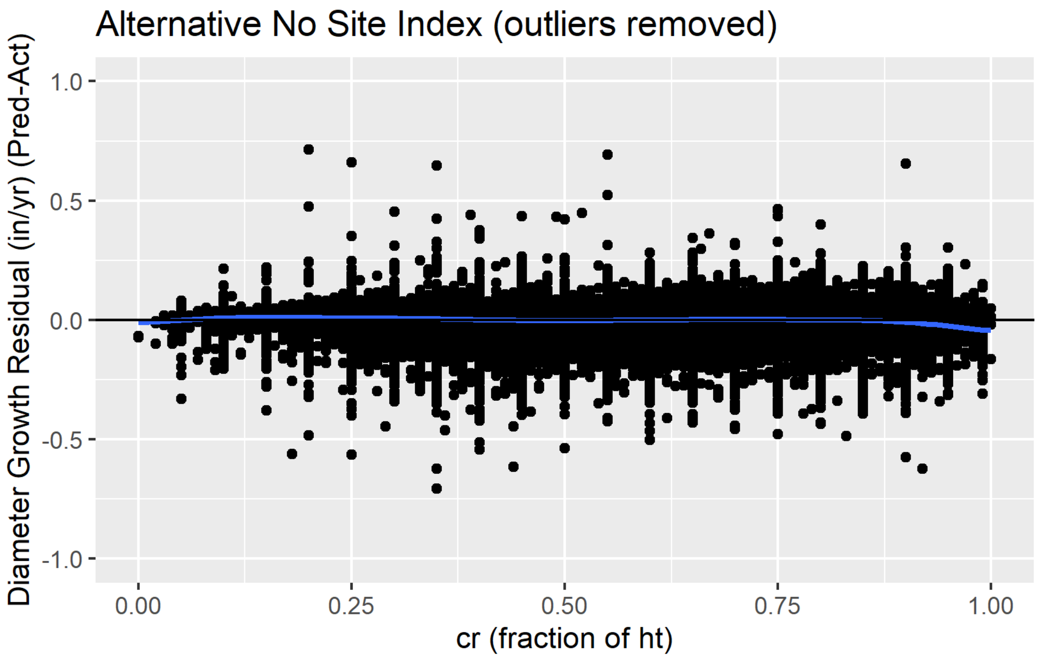


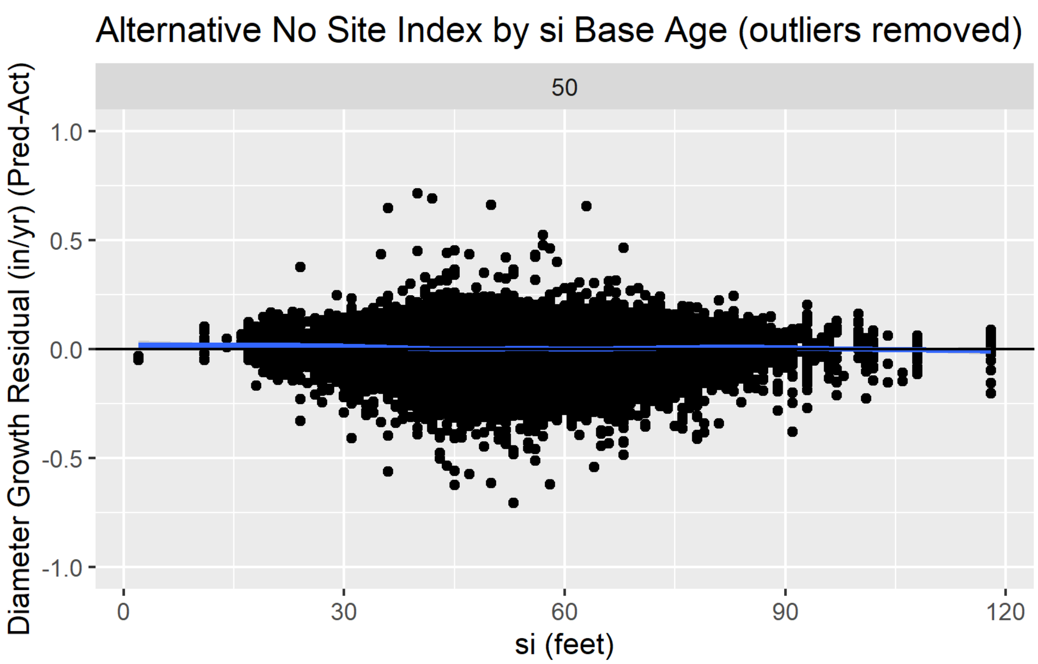


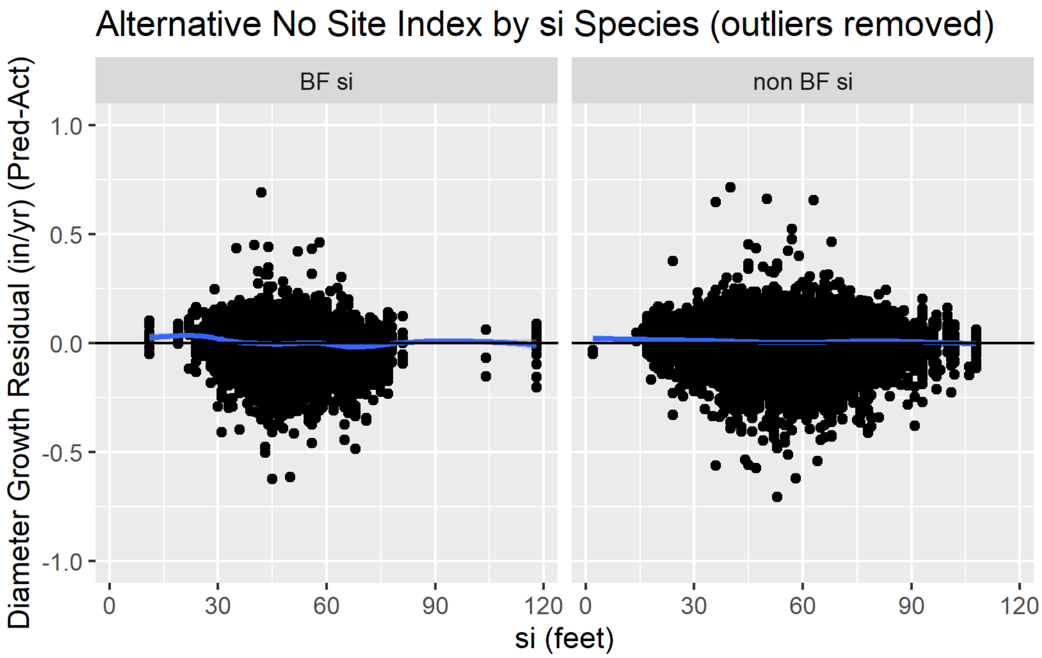
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
ba (ft^2/acre)



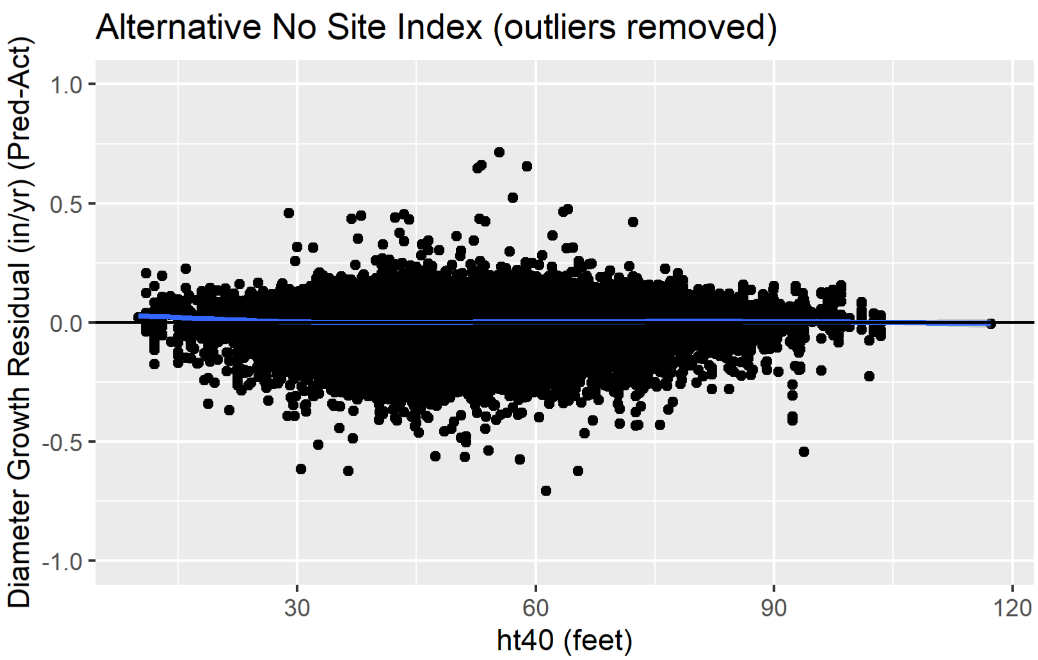
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
bal (ft^2/acre)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)


Alternative No Site Index by si Base Age (outliers removed)
50
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

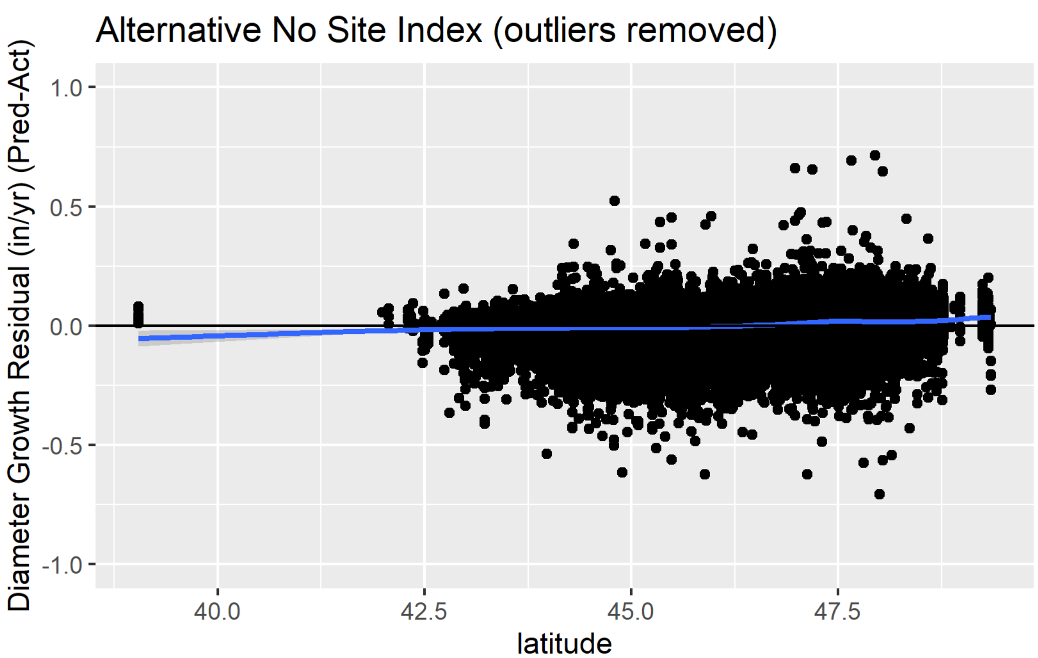


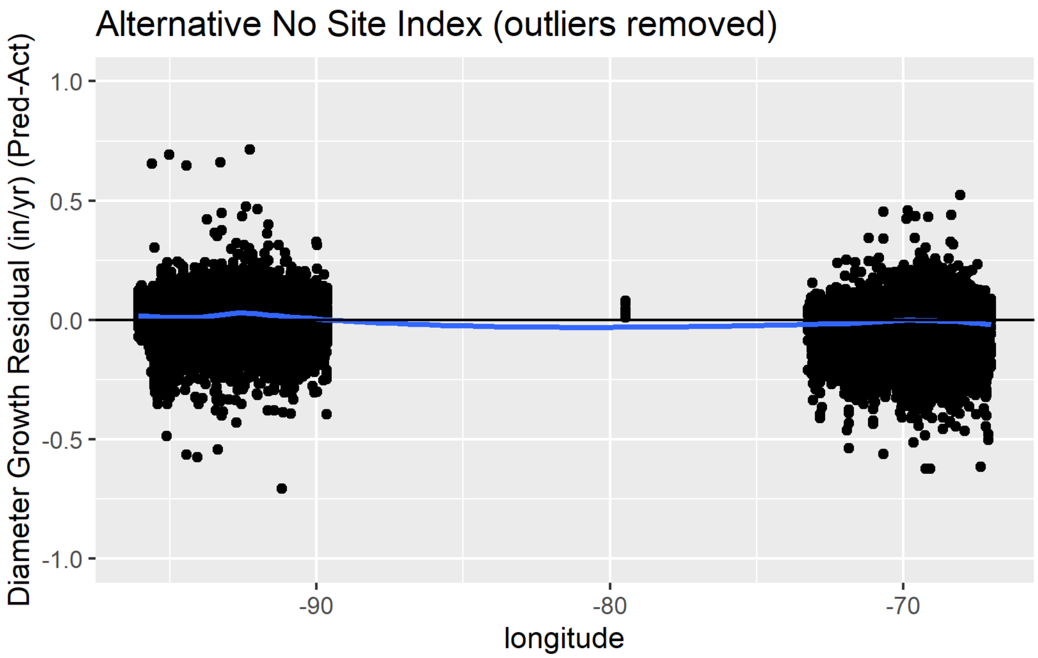
Alternative No Site Index by si Species (outliers removed)
BF si
non BF si
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
30
60
90
120
si (feet)

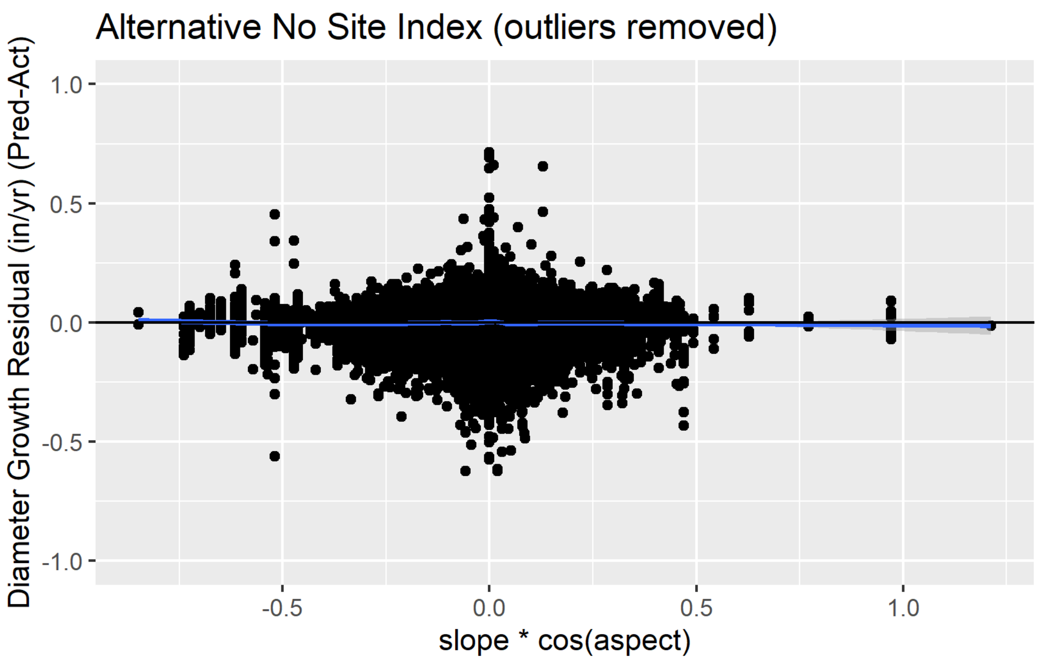


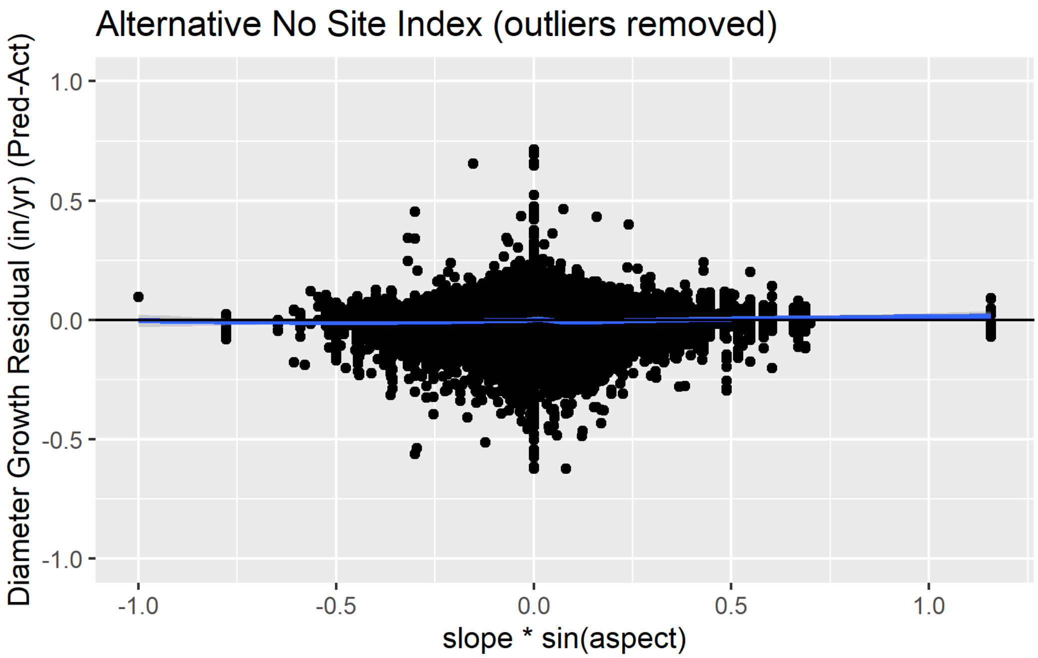
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
30
60
90
120
ht40 (feet)

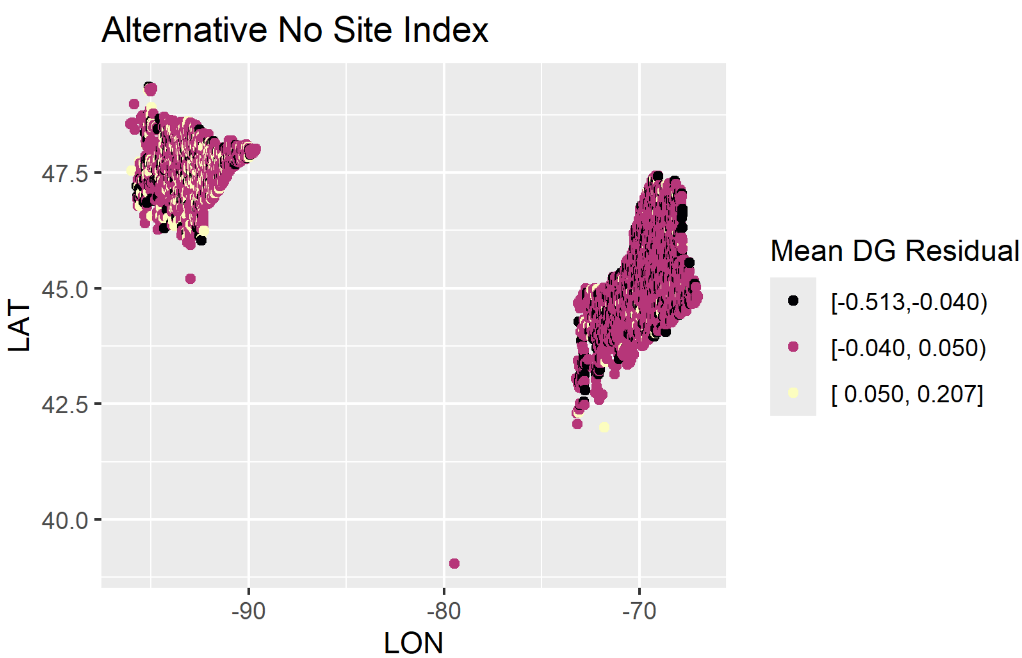

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
CT
MA
ME
MN
NH
VT
WV
State Code


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
40.0
42.5
45.0
47.5
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-90
-80
-70
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)





Discussion

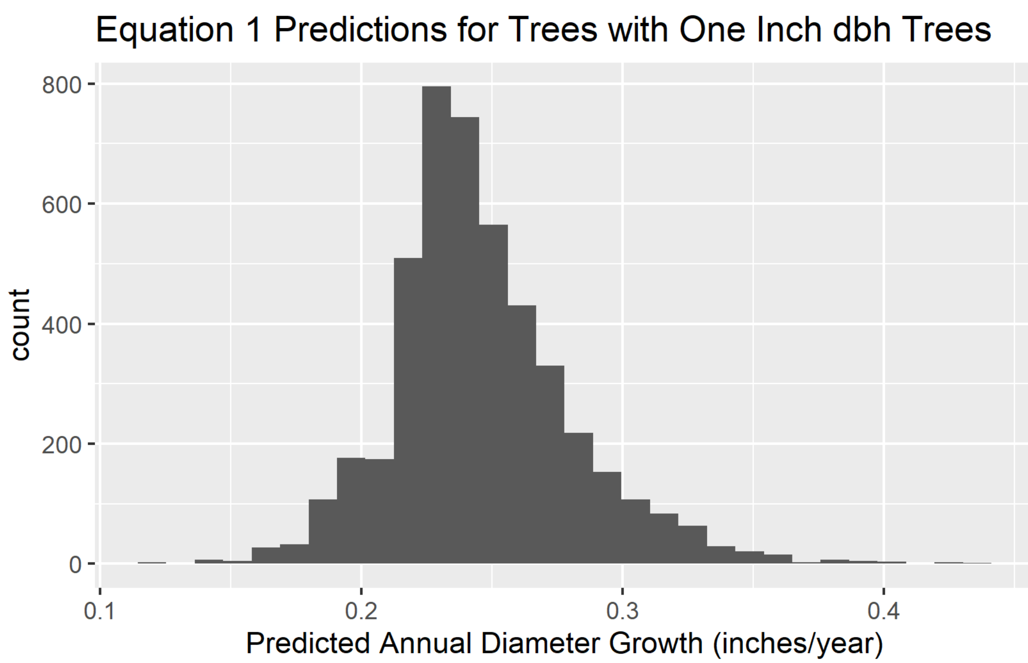
Removing `si` does not degrade the fit significantly. Only MN appears to be over-predicted.

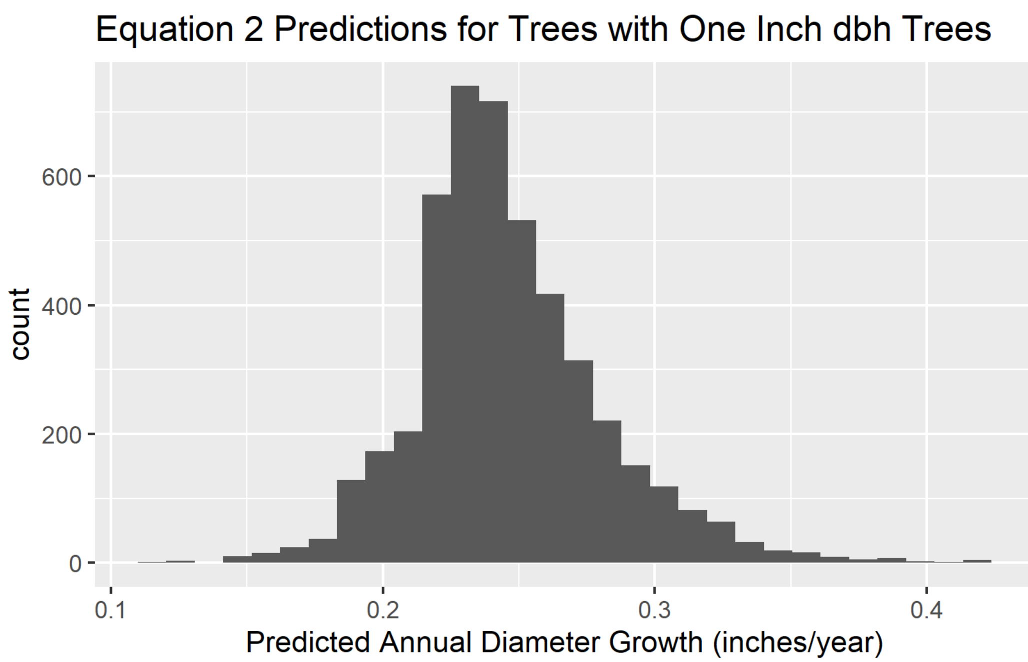
Table 3: Independent Variables for One Inch dbh Trees

Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	4621	111	49	0.41	75	144	313
bal	4621	109	49	0	74	143	313
ht	4621	10	1	5	10	10	25
cr	4621	0.46	0.22	0.03	0.3	0.6	1
si	4621	51	12	17	44	58	108

Equation 1 residual graphs show that there is some indication of residual trends spatially (especially in the western states).

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
count
600
400
200
0
0.1
0.2
0.3
0.4
Predicted Annual Diameter Growth (inches/year)