Alternative Western Hemlock Diameter Growth

Greg Johnson and David Marshall

2026-01-20

Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 5 states listed in the native range of Western Hemlock in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Western Hemlock (FIA species code 263), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Western Hemlock Growth Observations by State

State	Observations
CA	164
ID	1538
MT	561
OR	13035
WA	26360

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the

[1]Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

[2]Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

term with a ratio of a transformation of diameter at breast height (dbh) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (bal) serves as the inter-tree competition factor, and site index (si) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Western Hemlock site trees,
2. It is estimated using a number of different and not necessarily compatible si equations, and
3. The available si equations do not all use the same base age.

In the data set si is derived from 22 different site index equations for 19 species. Western Hemlock site index comprises 39% of the observations. There are 3 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where SIBASE and SISP are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7} + \beta_3 log(si_{s,b} + 4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0 + \beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}}) + \beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- dbh = diameter at breast height (inches),
- bal = basal area per acre in larger trees ($feet^2/ac$),
- cr = crown ratio (fraction of total height),
- ht = total height (feet), and
- $si_{s,b}$ = site index (feet) for species s and base age b.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting awhroach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending dbh. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

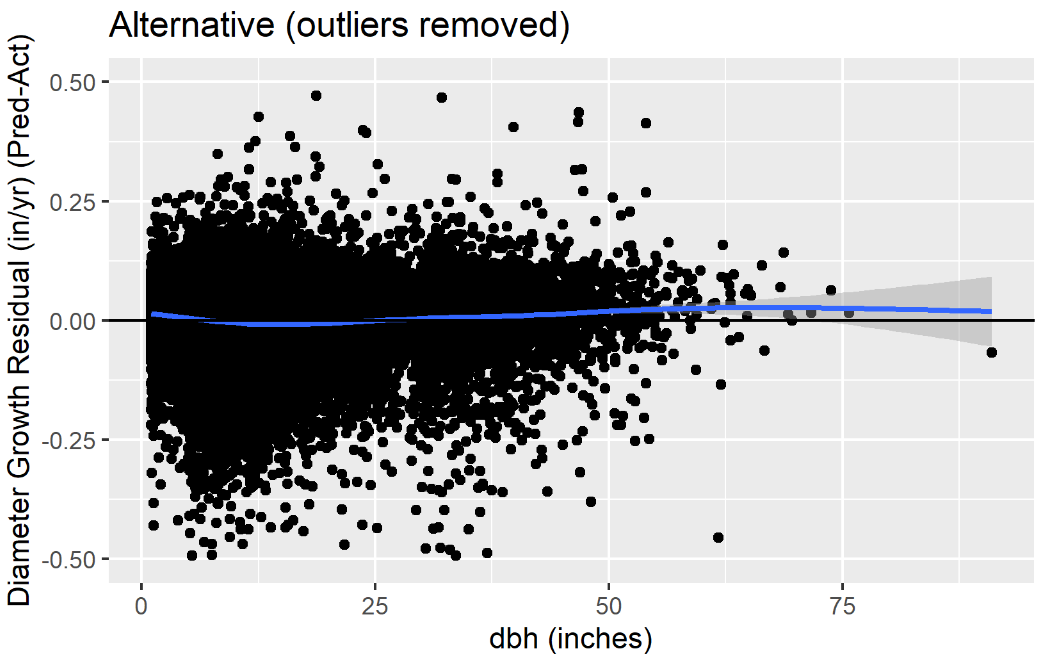
```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, en
  Data: wh_subset %>% mutate(SIINT = interaction(as.factor(wh_subset$SIBASE),      as.factor
       AIC      BIC    logLik
  98490.28 98559.37 -49237.14

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3  Residual
StdDev: 0.03930064 0.7887469

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value  Std.Error    DF   t-value p-value
B0 -4.880500 0.07101778 41648 -68.72223       0
B1 -0.634834 0.00662156 41648 -95.87386       0
B2 -0.221948 0.01597331 41648 -13.89490       0
B3  0.691104 0.02239235 41648  30.86341       0
B4  1.629238 0.01325941 41648 122.87405       0
B5  0.492641 0.01139624 41648  43.22837       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.137
B2 -0.409  0.493
B3 -0.449  0.003  0.002
B4 -0.550  0.239  0.554 -0.004
B5 -0.371  0.465  0.991  0.000  0.500

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-35.1098269  -0.5169631  -0.1211479   0.3806092  29.0121000

Number of Observations: 41658
Number of Groups: 5

$SIINT
                   B3
50.FALSE   0.050530878
80.FALSE   0.003680612
100.FALSE -0.021006131
50.TRUE    0.023764721
100.TRUE  -0.056970080

Residual Standard Error: 0.788746863012293 on 41648 degrees of freedom, AIC: 98490.3
```

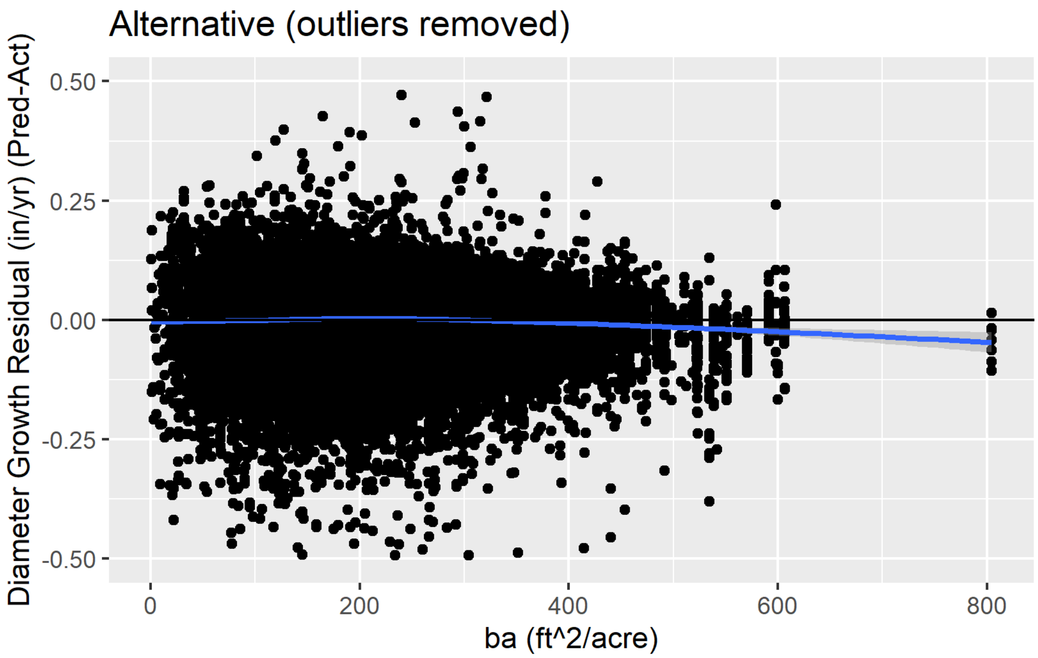
and for Equation 2:

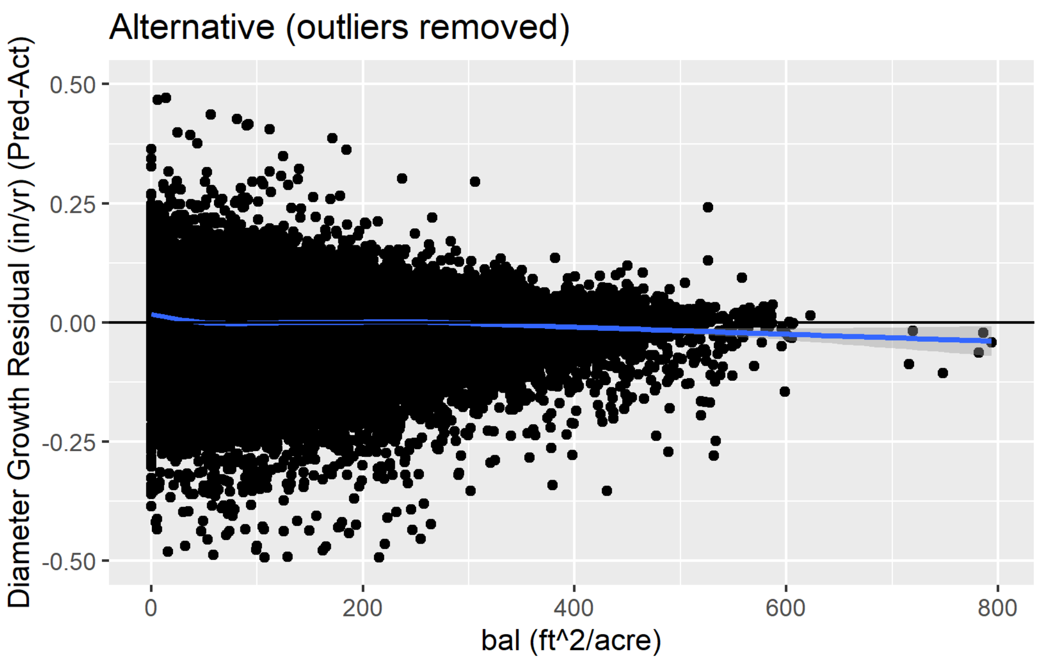
	Coef.	Std. error	t-stat.	p
B0	-1.5940743	0.0438053	-36.38996	0
B1	-0.6664653	0.0070117	-95.05039	0
B2	-0.2506115	0.0182508	-13.73152	0
B4	1.6323658	0.0133854	121.95142	0
B5	0.4743630	0.0114615	41.38752	0

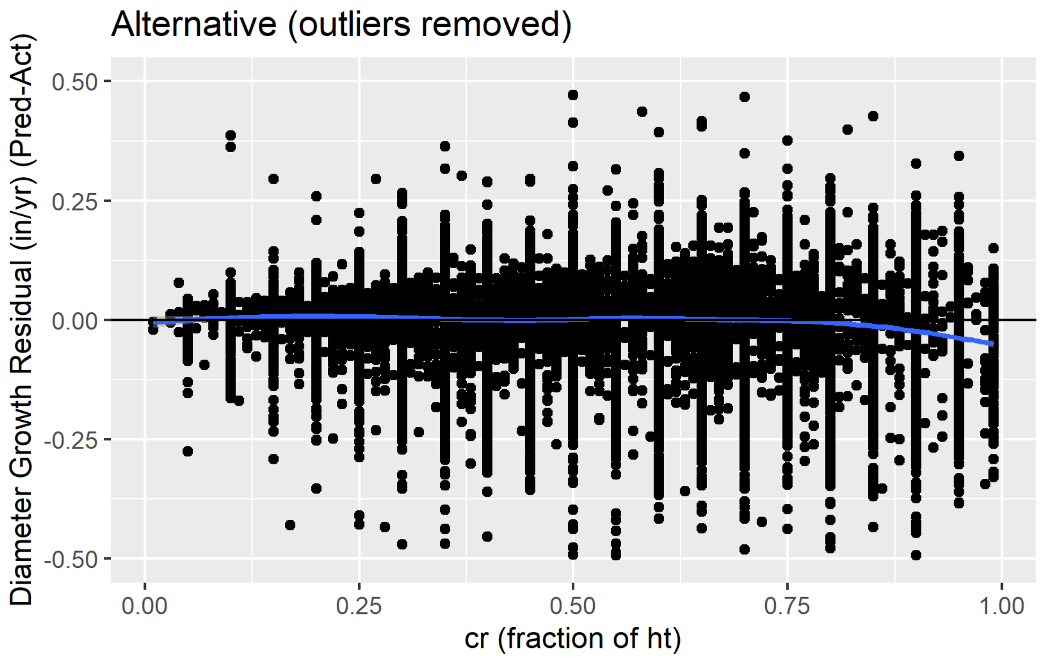
```
Residual Standard Error: 0.82826294159474 on 41653 degrees of freedom, AIC: 102528.5
```

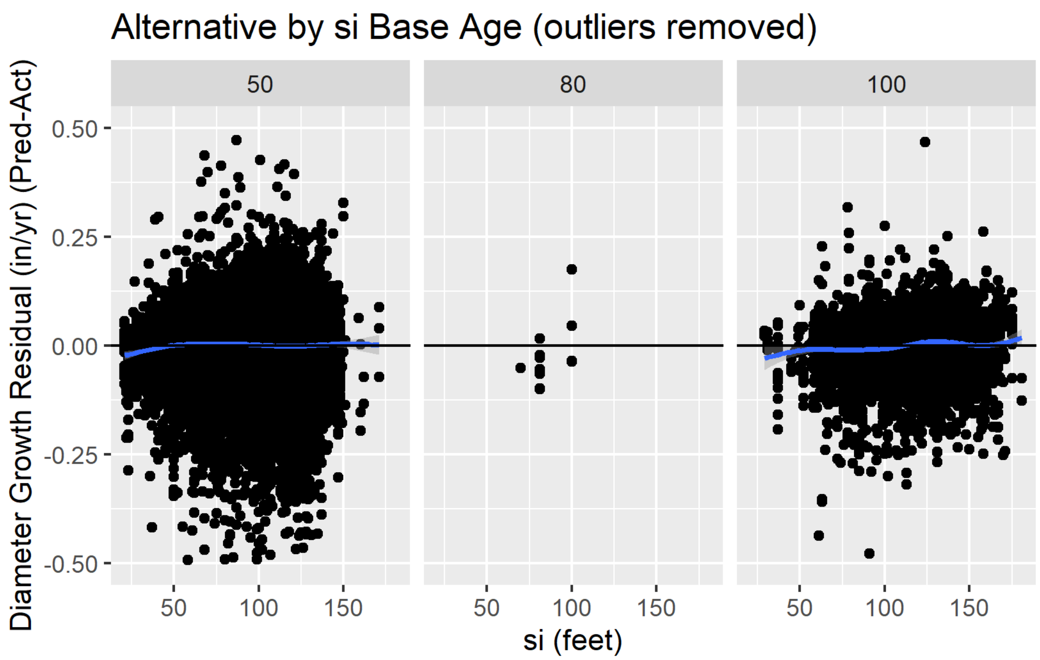
Residual Analysis for Equation 1

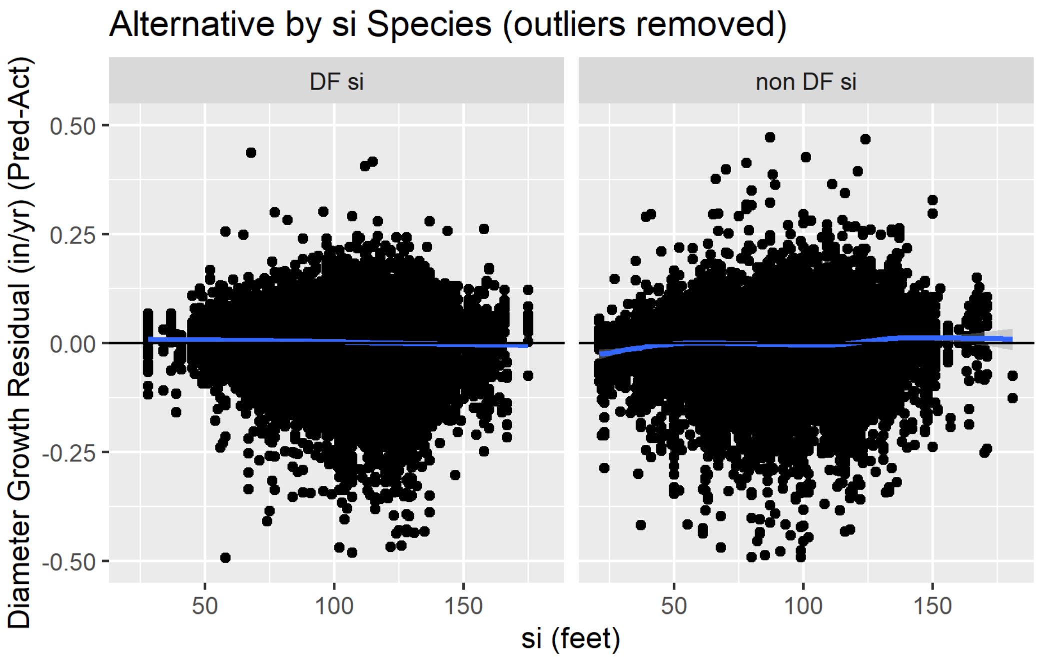


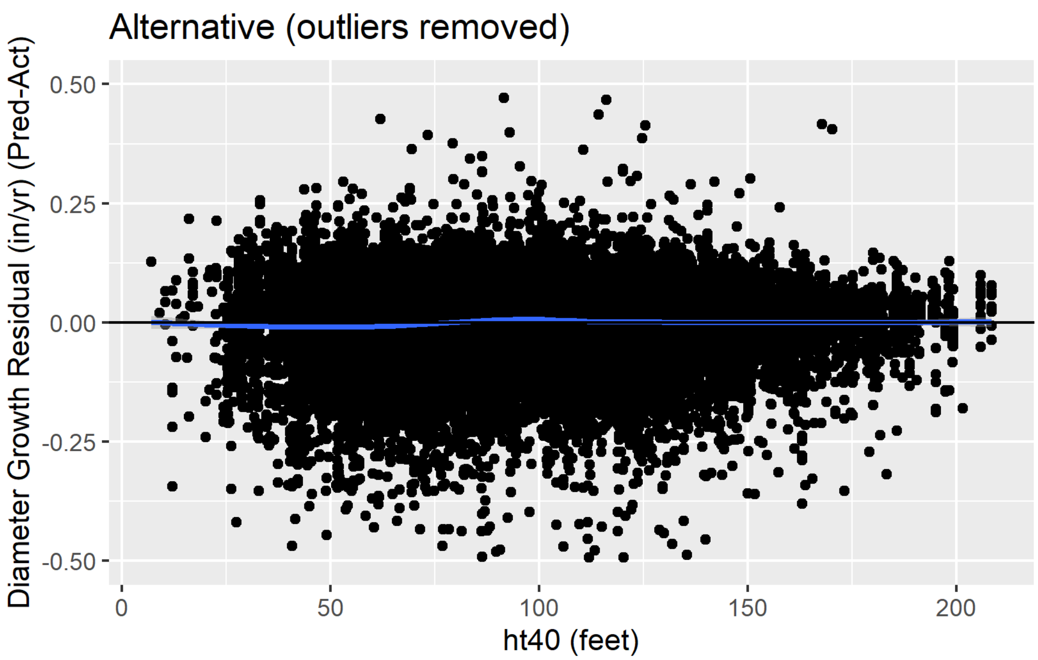

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
800
ba (ft^2/acre)

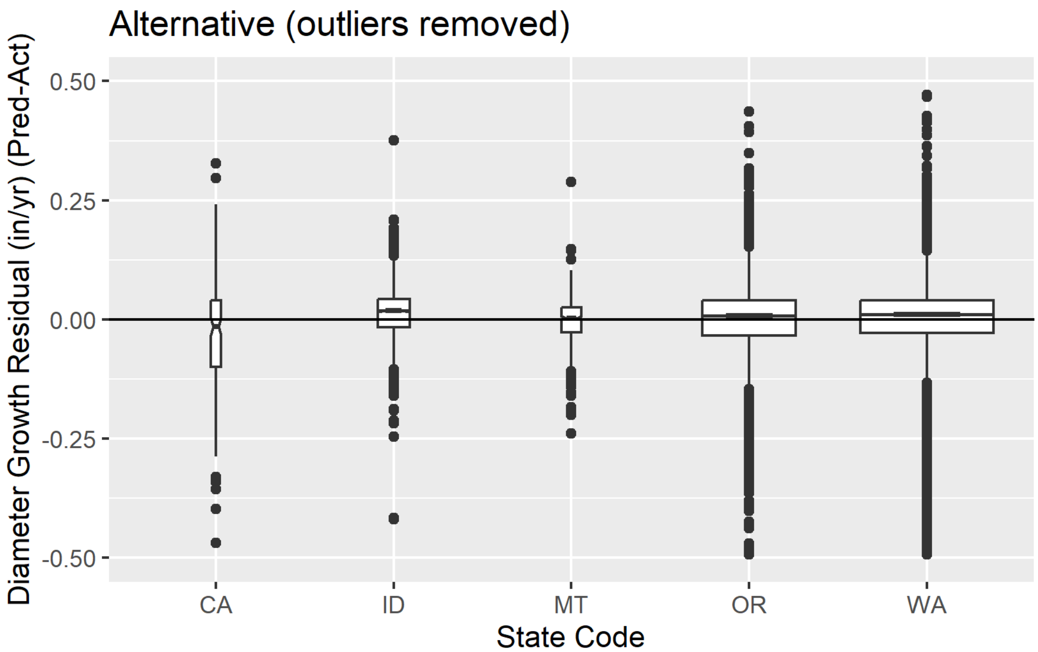

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
800
bal (ft^2/acre)

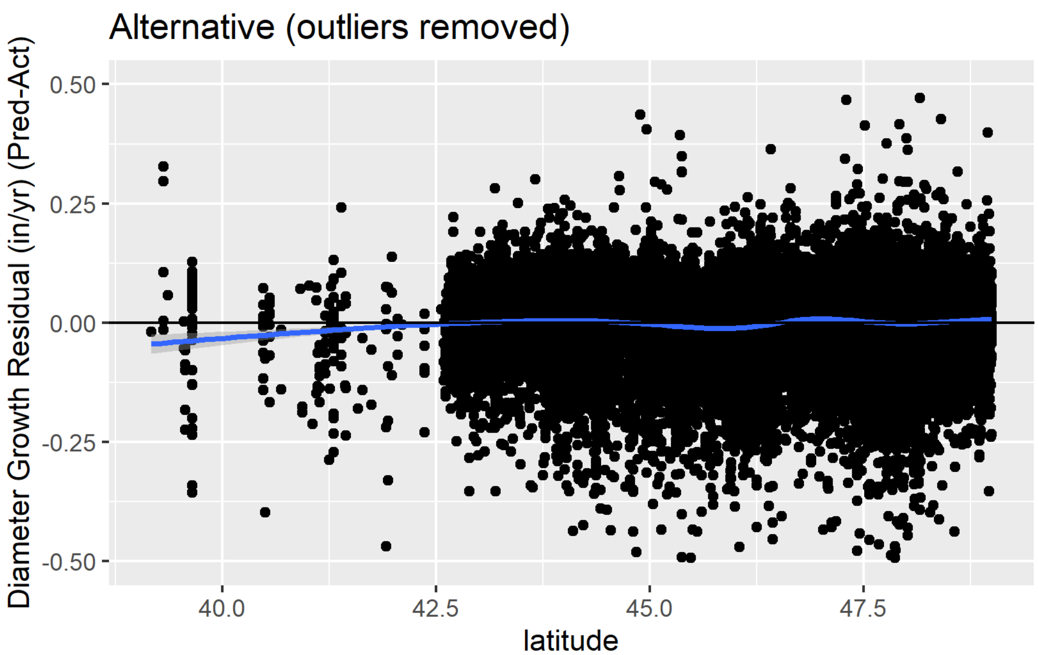

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

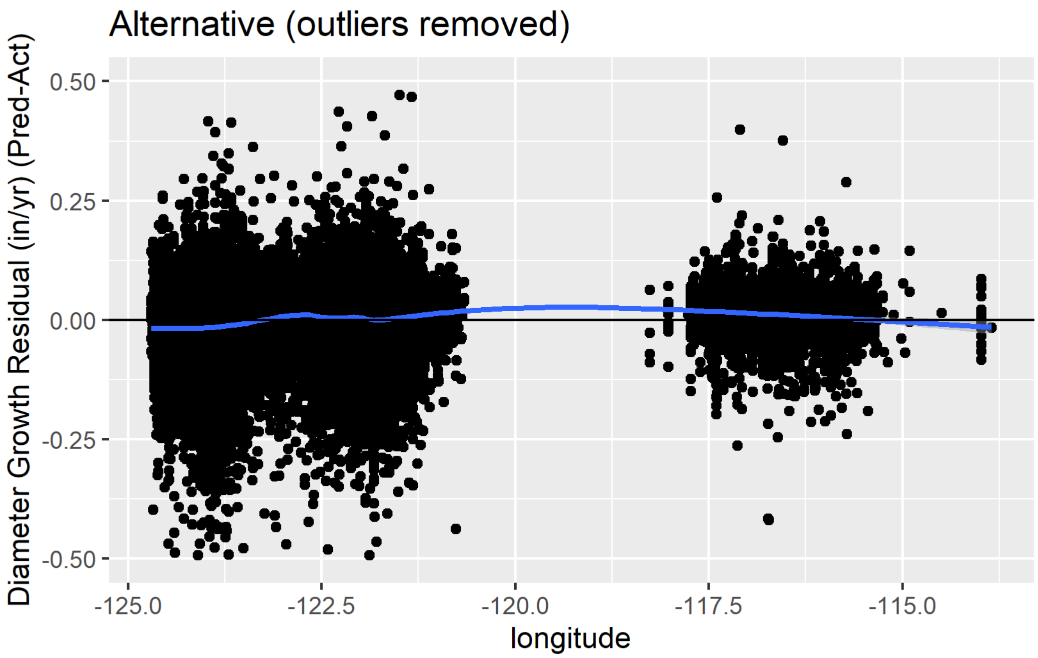

Alternative by si Base Age (outliers removed)
50
80
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

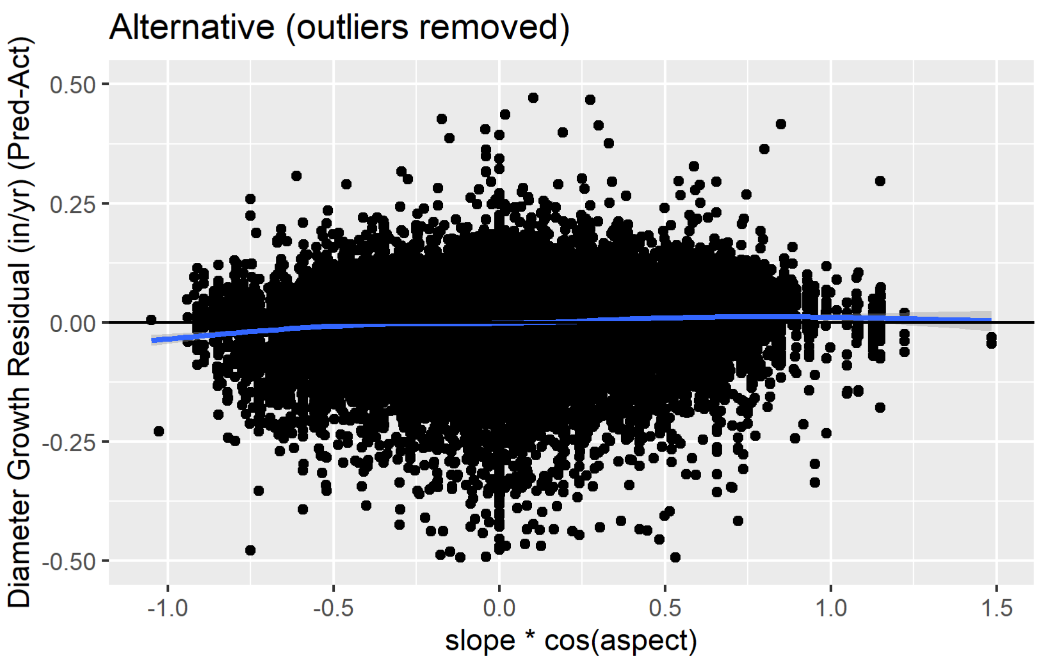

Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

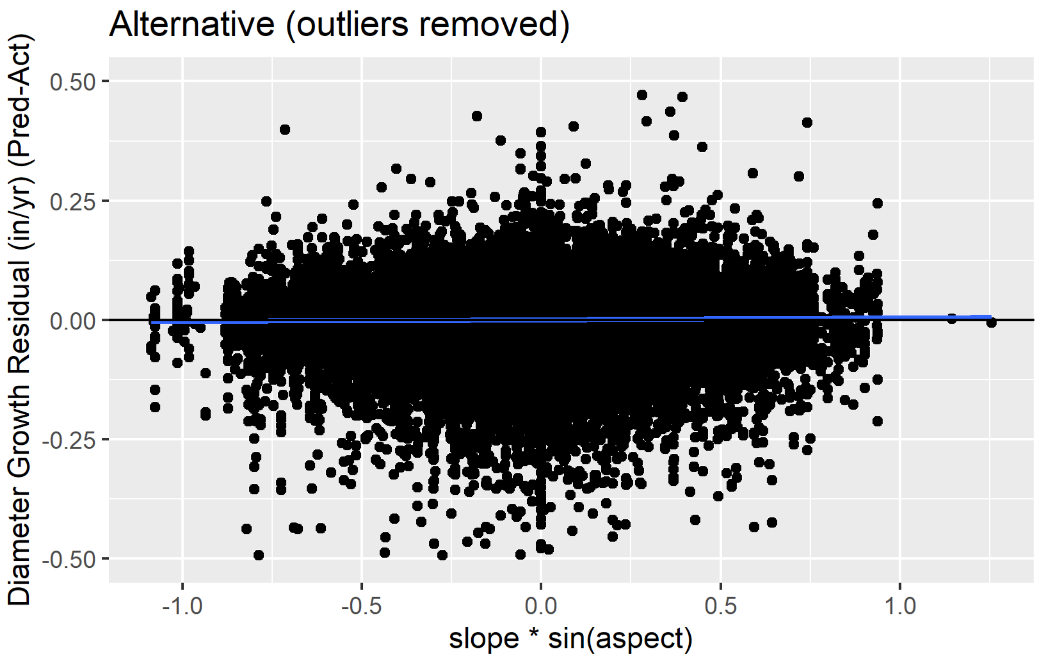

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
200
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
CA
ID
MT
OR
WA
State Code

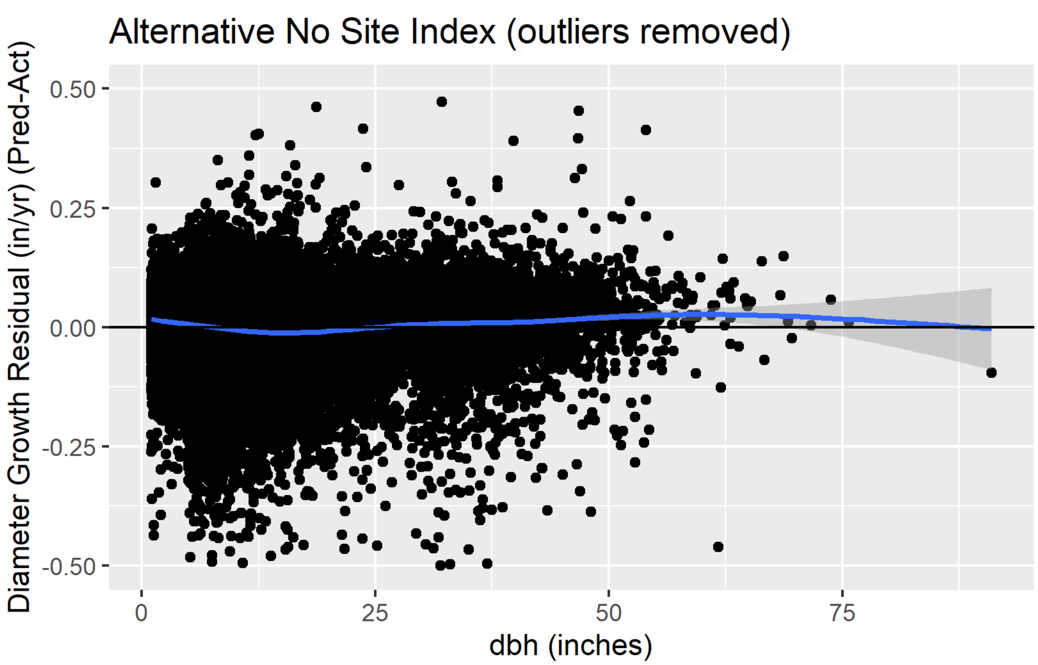

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40.0
42.5
45.0
47.5
latitude

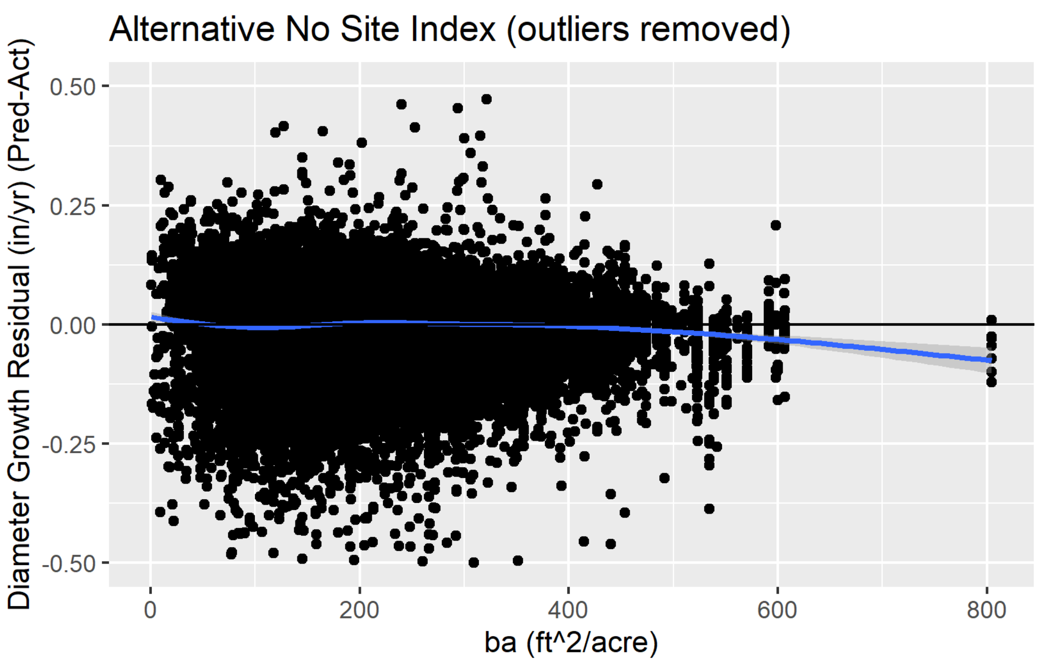

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-125.0
-122.5
-120.0
-117.5
-115.0
longitude

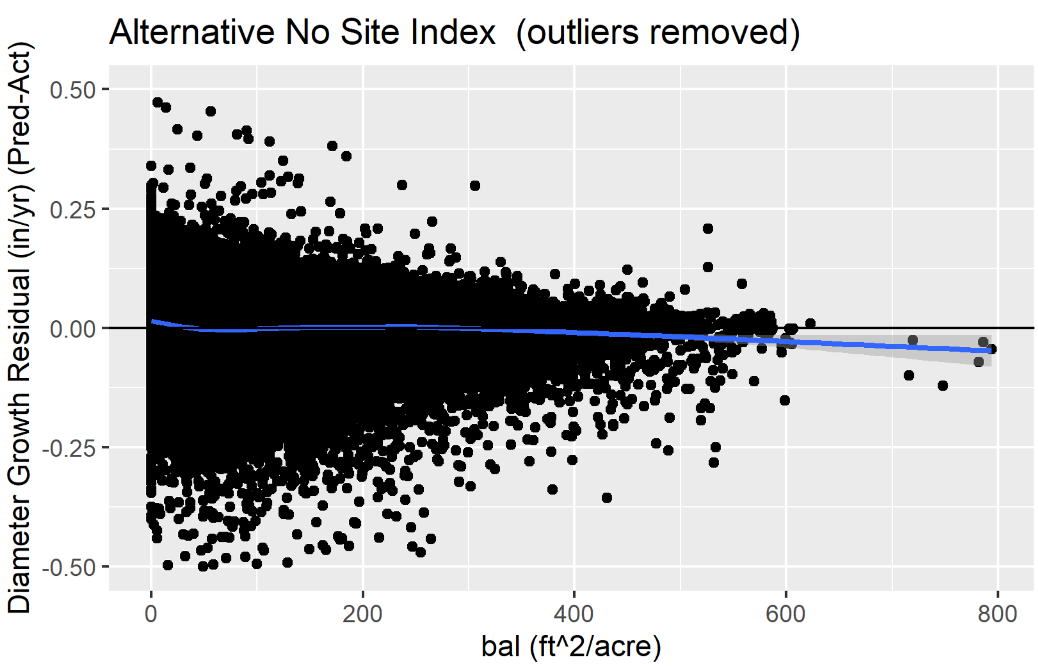

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
1.5
slope * cos(aspect)

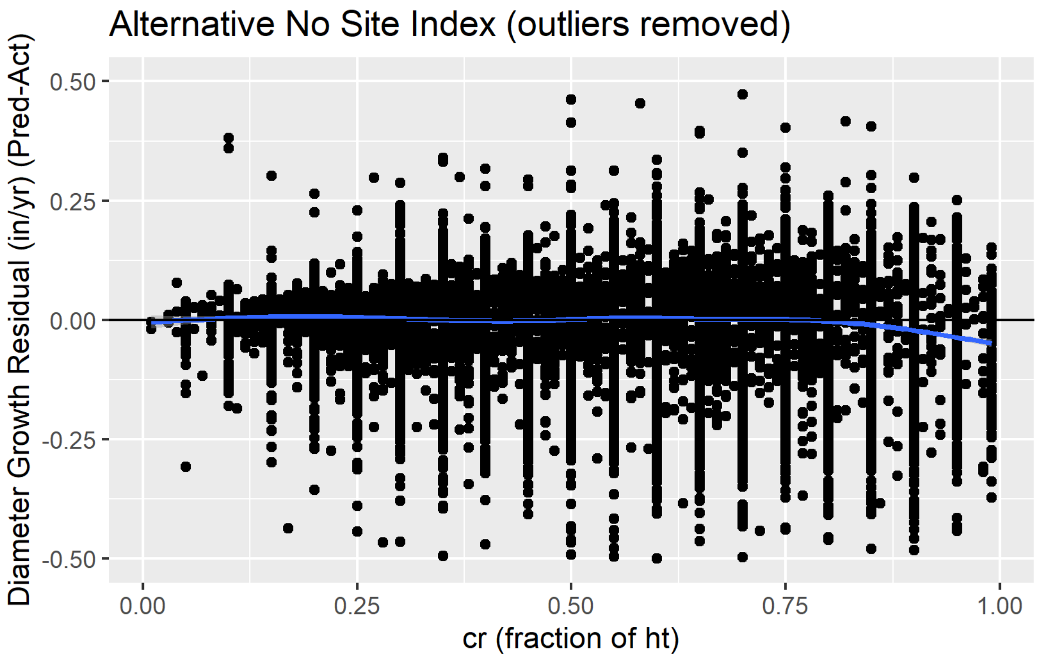


Residual Analysis for Equation 2

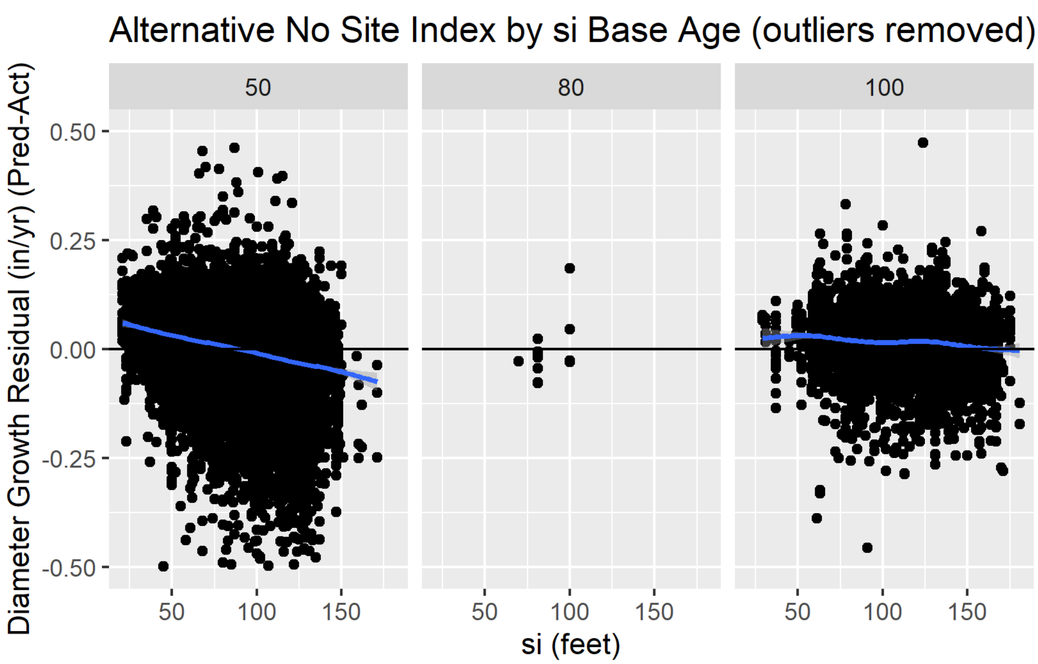



Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
800
ba (ft^2/acre)

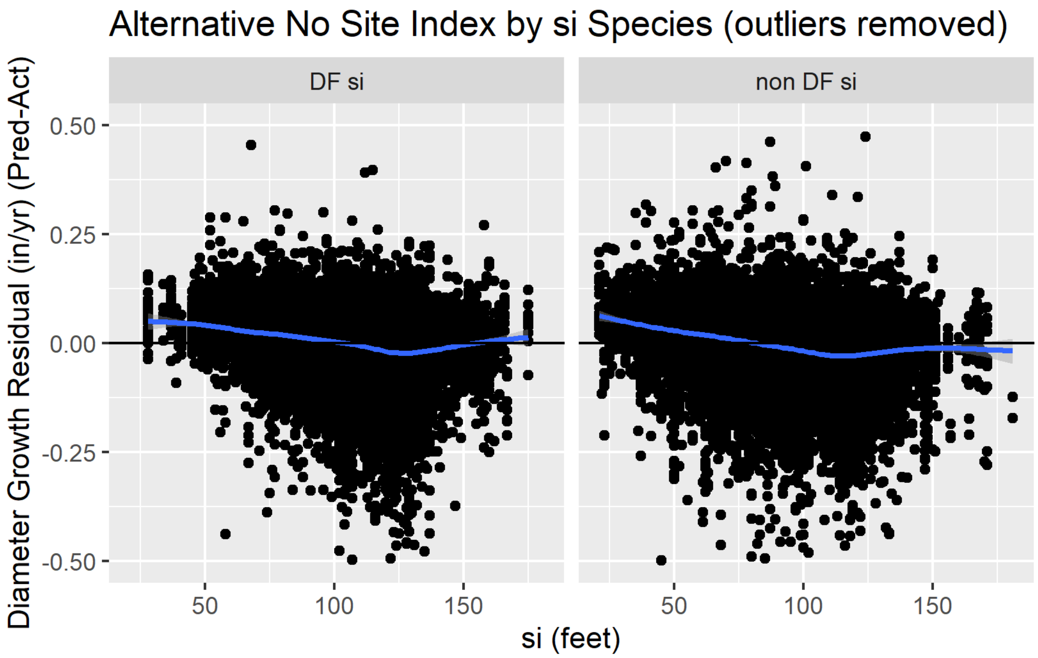

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
600
800
bal (ft^2/acre)



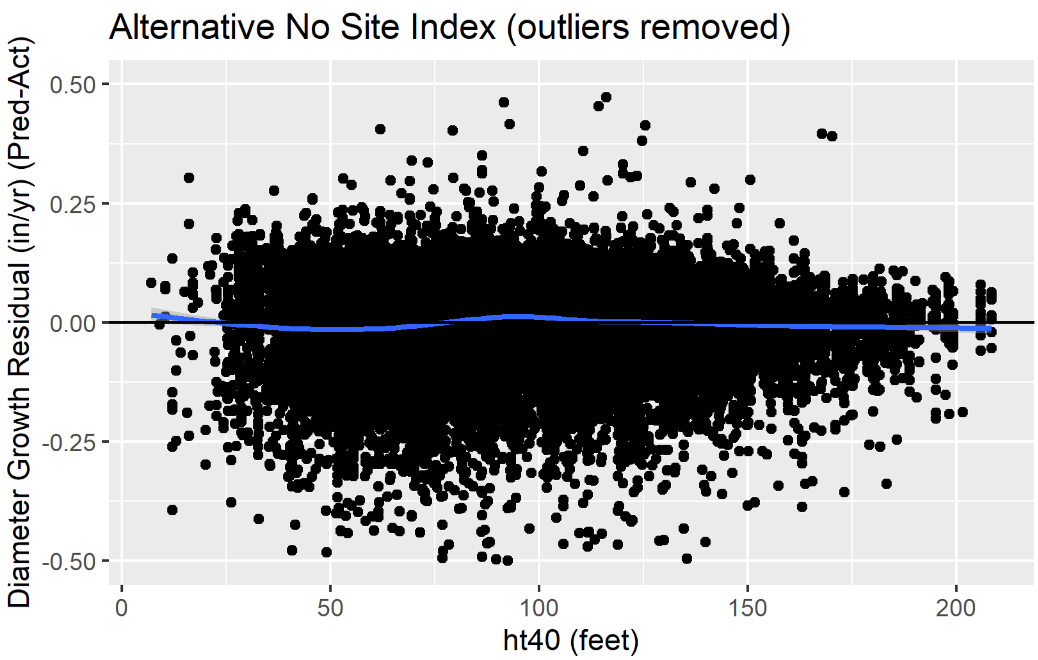
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)



Alternative No Site Index by si Base Age (outliers removed)
50
80
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

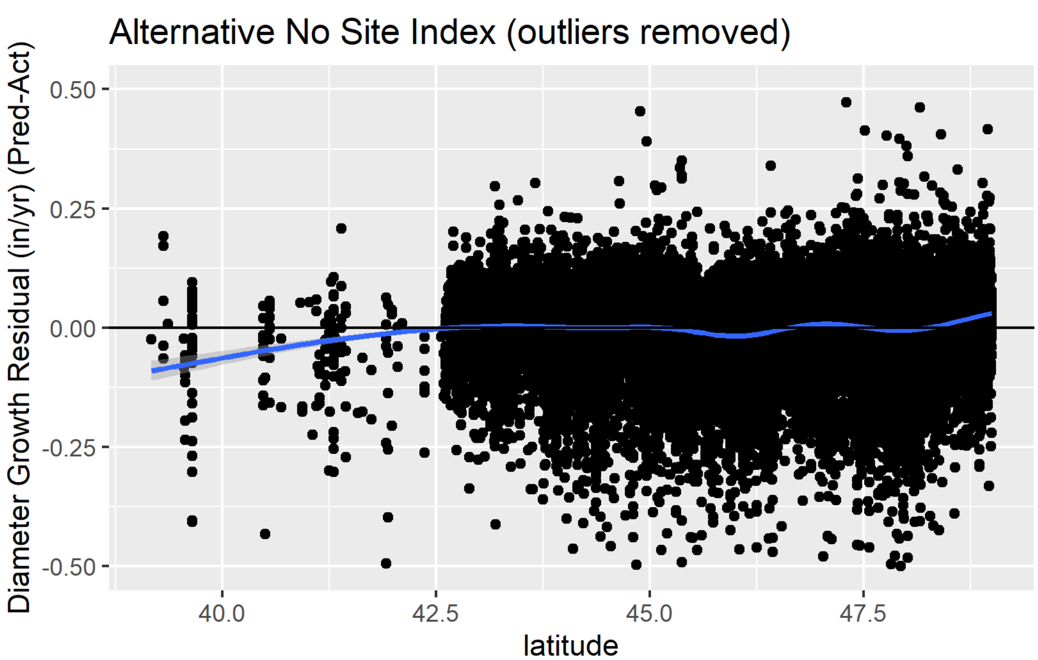


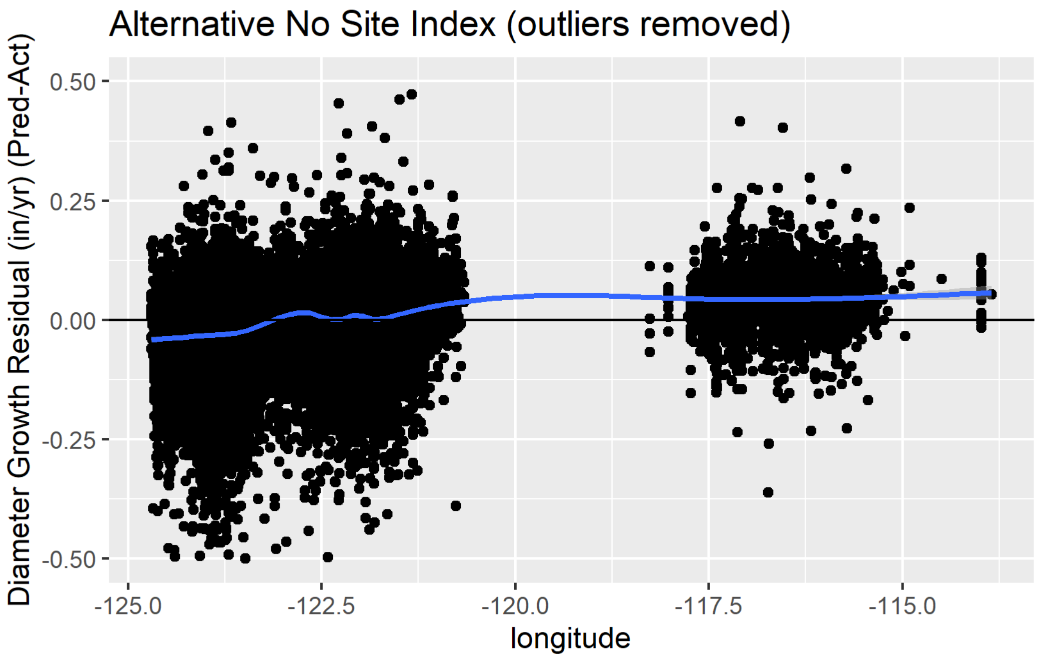
Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
si (feet)

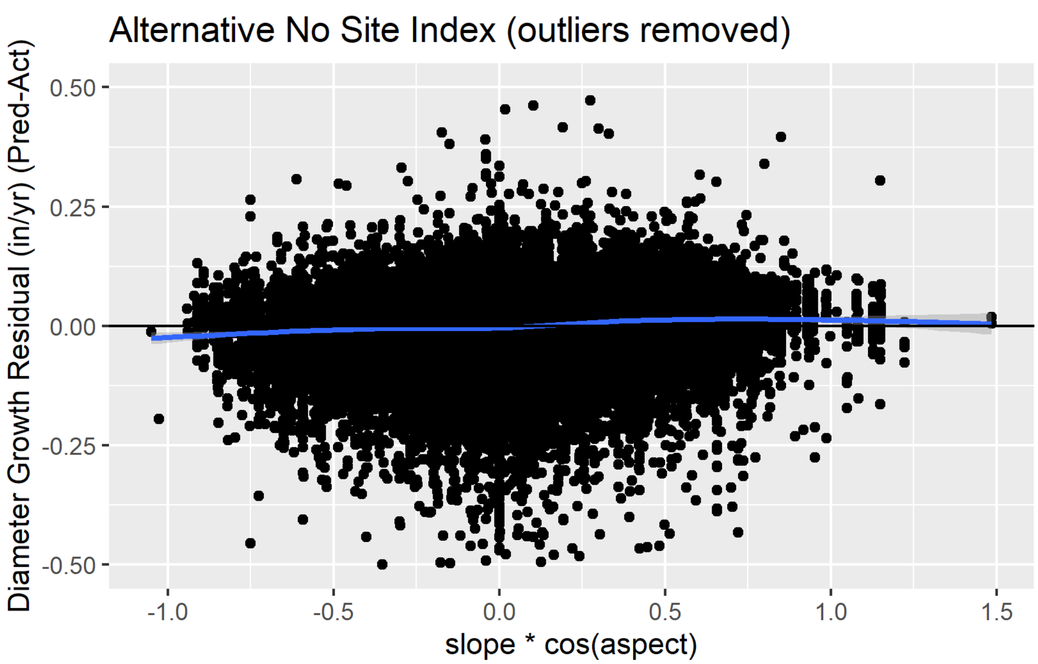


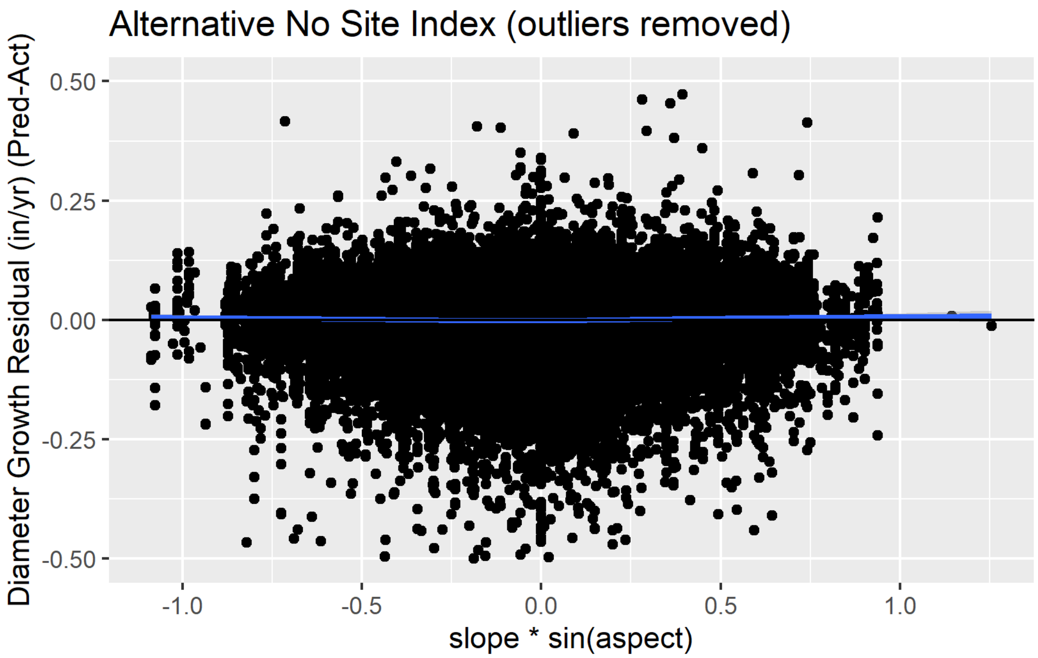
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
200
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
CA
ID
MT
OR
WA
State Code


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40.0
42.5
45.0
47.5
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-125.0
-122.5
-120.0
-117.5
-115.0
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
1.5
slope * cos(aspect)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)

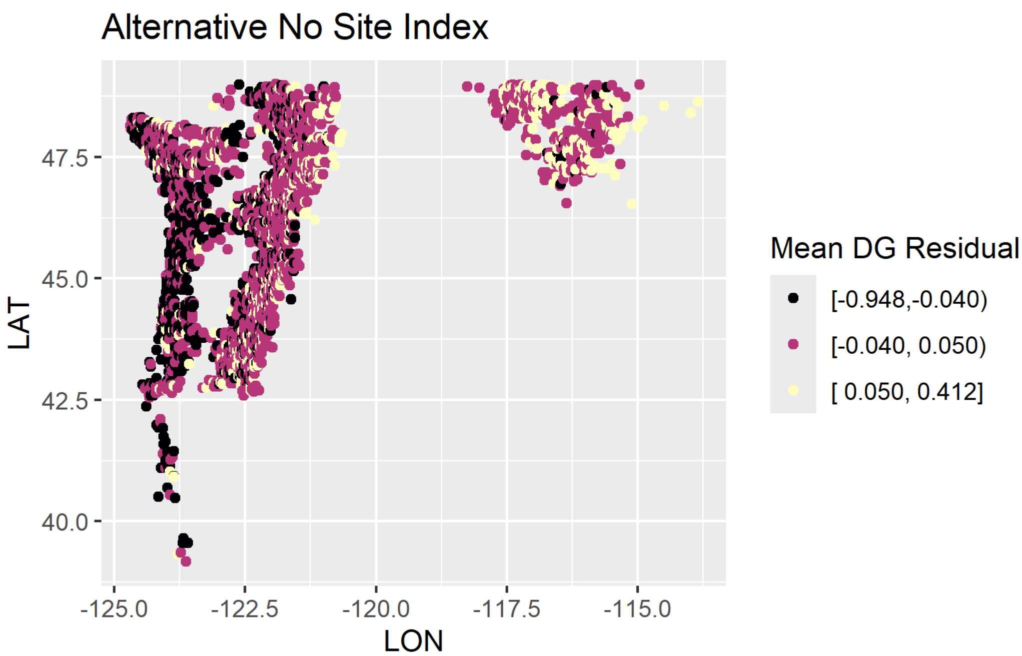

Alternative No Site Index
LAT
47.5
45.0
42.5
40.0
-125.0
-122.5
-120.0
-117.5
-115.0
LON
Mean DG Residual
[-0.948,-0.040)
[-0.040, 0.050)
[0.050, 0.412]

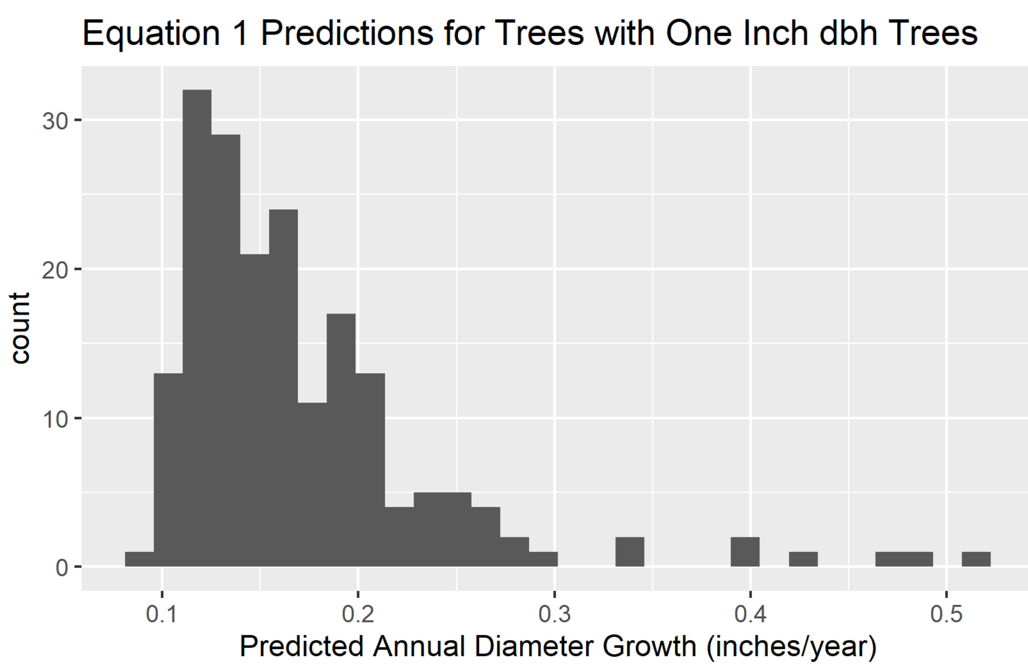
Table 3: Independent Variables for One Inch dbh Trees

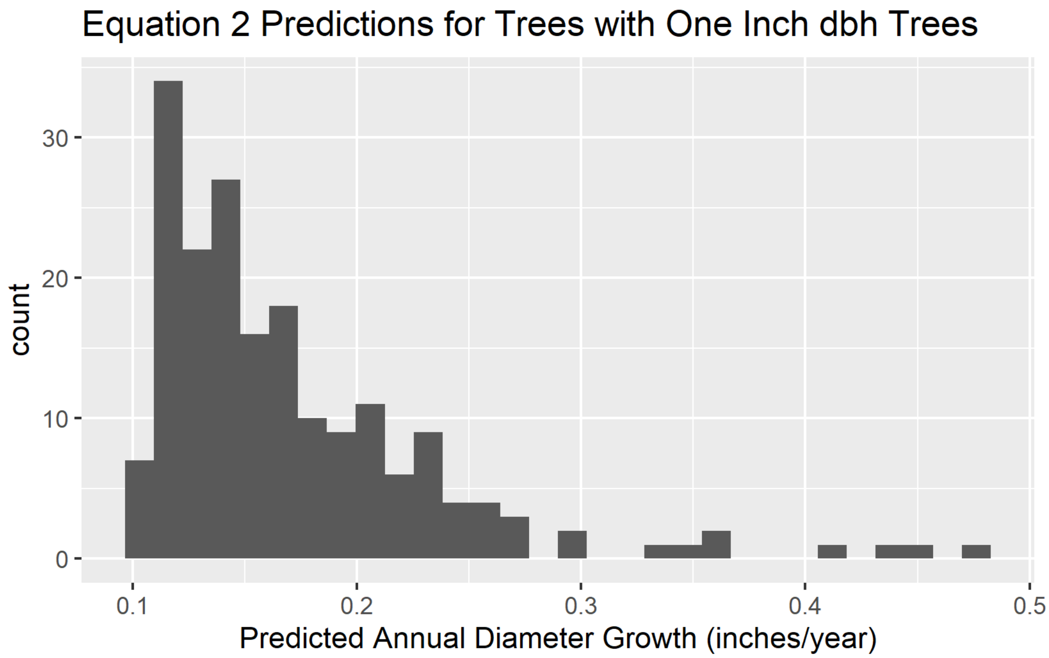
Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	190	179	94	5.4	111	241	523
bal	190	178	94	5	110	240	523
ht	190	9.6	2	6	8	10	20
cr	190	0.5	0.27	0.05	0.25	0.75	0.99
si	190	93	29	24	74	111	171

Discussion

Removing `si` degrades the fit significantly. There is an over-prediction bias for the eastern states for both equations, but is mitigated by the `si` term in Equation 1. The residual trend with longitude confirms this.

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
count
30
20
10
0
0.1
0.2
0.3
0.4
0.5
Predicted Annual Diameter Growth (inches/year)