Alternative Red Maple Diameter Growth

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2026-01-30

Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 32 states listed in the native range of Red Maple in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Red Maple (FIA species code 316), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Red Maple Growth Observations by State

State	Observations
AL	6227
AR	3339
CT	3147
DE	1217
FL	749
GA	10250
IL	602
IN	2751
KY	9919
LA	1295
MA	6537
MD	2550
ME	44626
MI	47733
MN	21472
MO	1538

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

State	Observations
MS	4275
NC	16598
NH	11235
NJ	2368
NY	30891
OH	8505
OK	454
PA	38091
RI	1547
SC	7114
TN	11387
TX	1459
VA	15112
VT	6852
WI	51053
WV	11912

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Red Maple site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 48 different site index equations for 63 species. Red Maple site index comprises 16% of the observations. There are 2 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following,

[2]Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

we fit two equations: one where SIBASE and SISP are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7}+\beta_3 log(si_{s,b}+4.5))} \quad (1)$$

and

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \quad (2)$$

where:

- dbh = diameter at breast height (inches),
- bal = basal area per acre in larger trees ($feet^2/ac$),
- cr = crown ratio (fraction of total height),
- ht = total height (feet), and
- $si_{s,b}$ = site index (feet) for species s and base age b.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending dbh. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, er
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
       AIC      BIC    logLik
  476989.7 477076.5 -238486.8

Random effects:
 Formula: B3 ~ 1 | SIINT
                 B3 Residual
StdDev: 0.002393571 0.451153

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
       Value   Std.Error     DF    t-value p-value
B0 -4.737881 0.029962183 382797 -158.12868       0
B1 -0.379340 0.002780059 382797 -136.45026       0
B2 -0.068424 0.003439002 382797  -19.89633       0
```

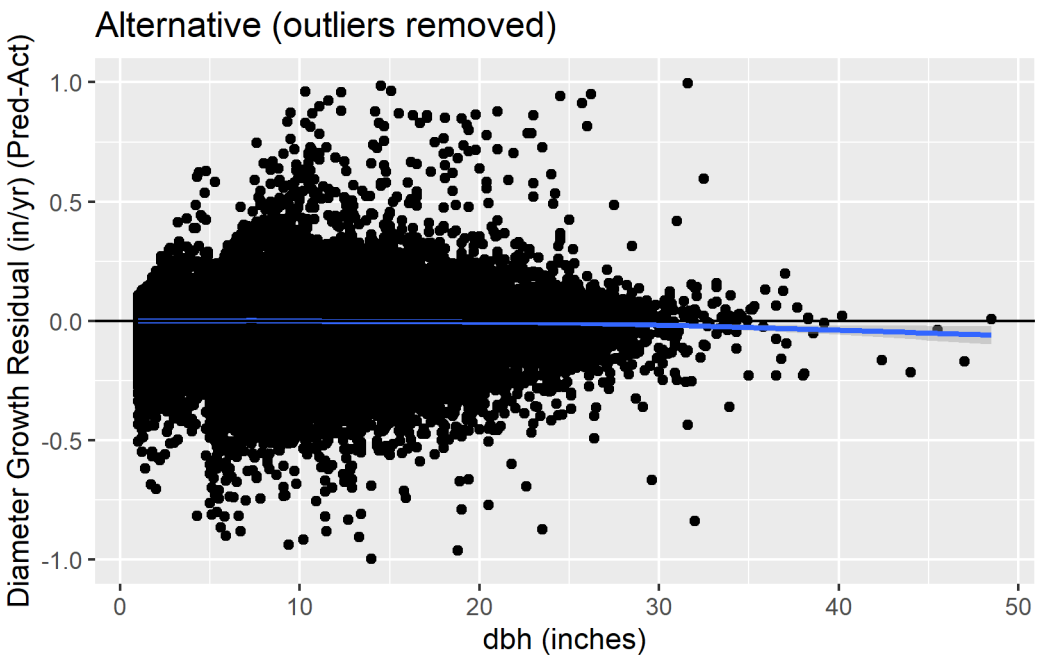
```
B3  0.217616 0.006662818 382797   32.66132       0
B4  3.159693 0.019330045 382797  163.46020       0
B5  0.701246 0.009393673 382797   74.65091       0
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.139
B2 -0.370  0.509
B3 -0.768 -0.073 -0.027
B4 -0.347  0.786  0.623 -0.178
B5 -0.360  0.465  0.993 -0.006  0.566

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-48.7726676  -0.5487345  -0.1384989   0.4064737  21.2114422

Number of Observations: 382805
Number of Groups: 3


$SIINT
                   B3
25.FALSE  0.001426724
50.FALSE -0.002698896
50.TRUE   0.001272173


Residual Standard Error: 0.451152967913115 on 382797 degrees of freedom, AIC: 476989.7
```

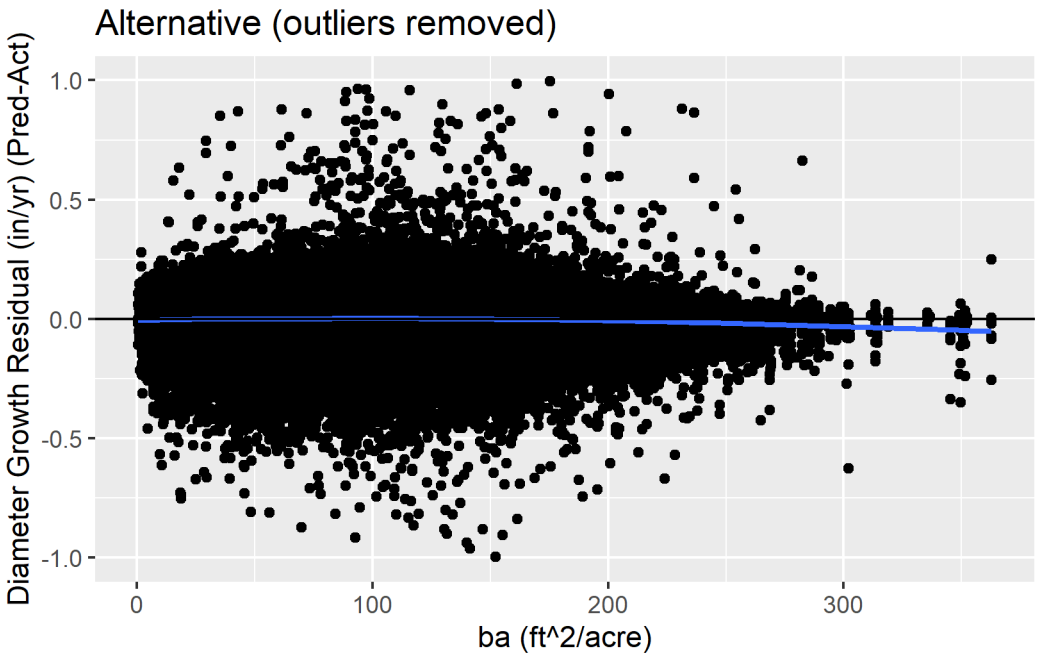
and for Equation 2:

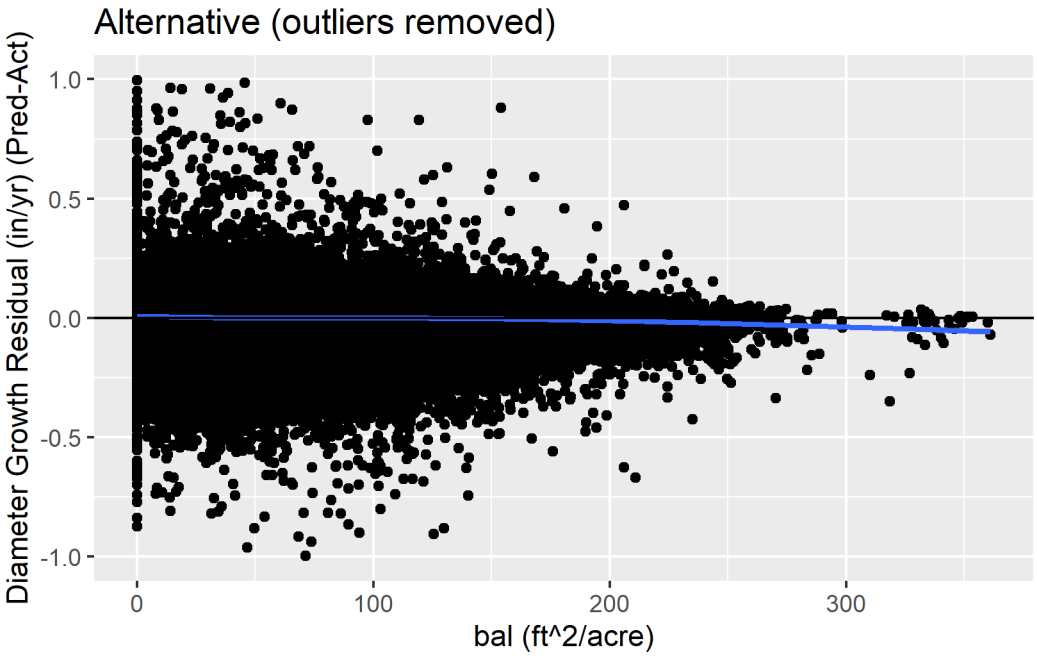
	Coef.	Std. error	t-stat.	p
B0	-3.9480155	0.0182880	-215.88028	0
B1	-0.3724921	0.0027745	-134.25512	0
B2	-0.0653743	0.0034404	-19.00176	0
B4	3.2857389	0.0201899	162.74135	0
B5	0.7015507	0.0098368	71.31866	0

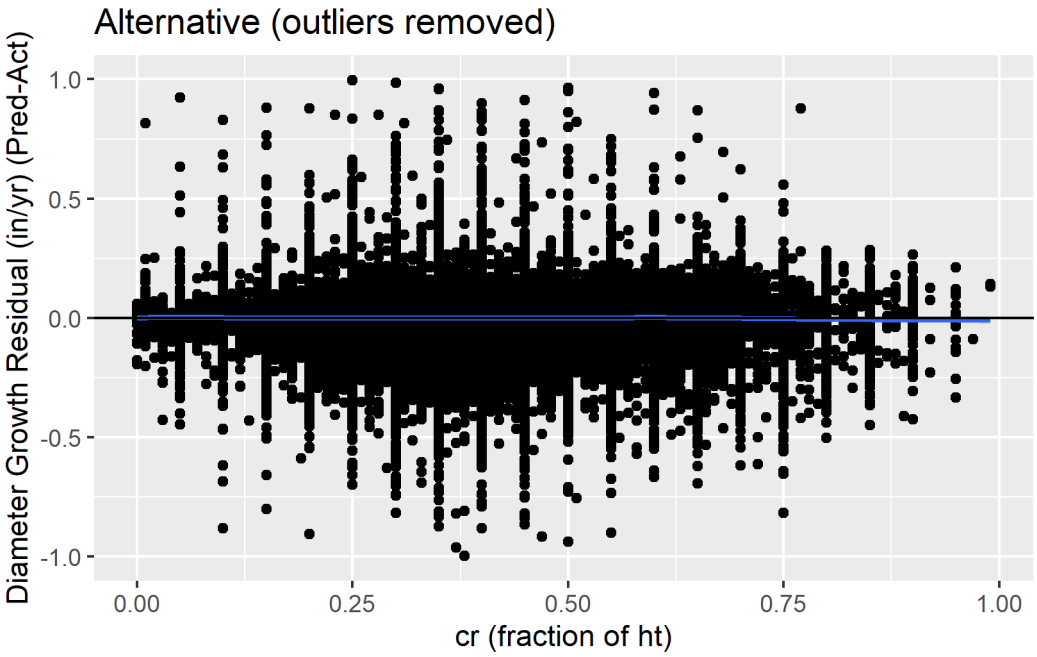
```
Residual Standard Error: 0.451812269309788 on 382800 degrees of freedom, AIC: 478092.2
```

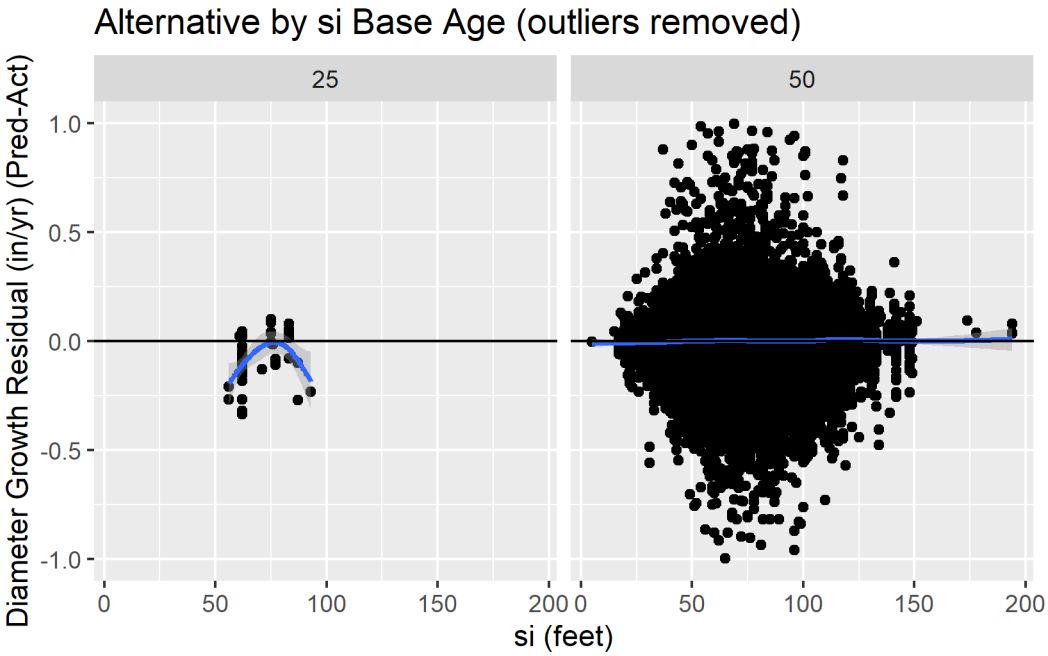
Residual Analysis for Equation 1

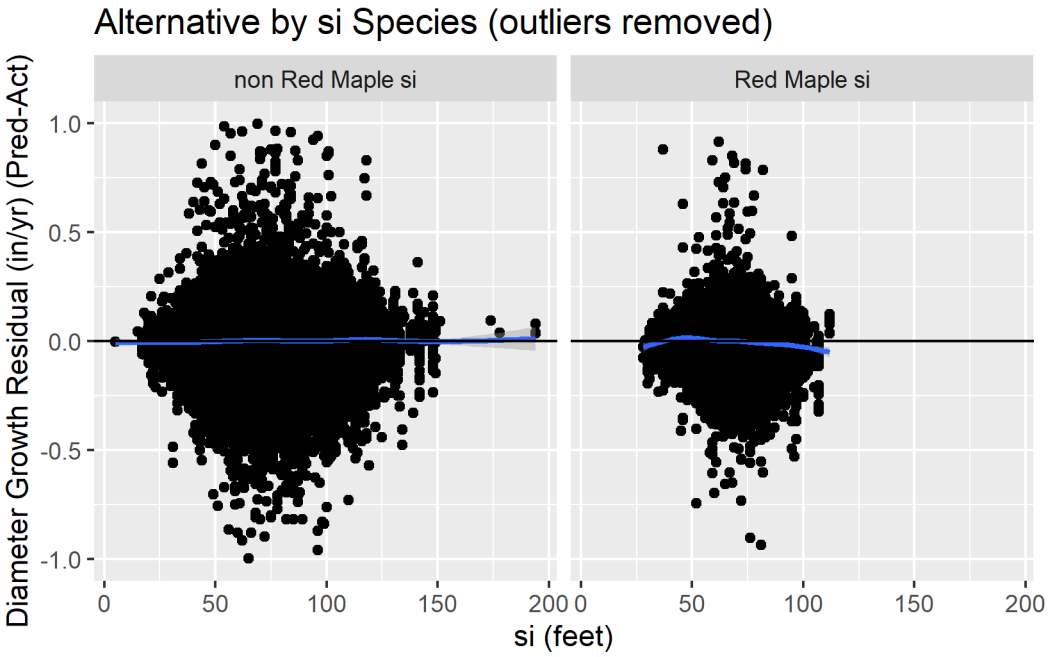


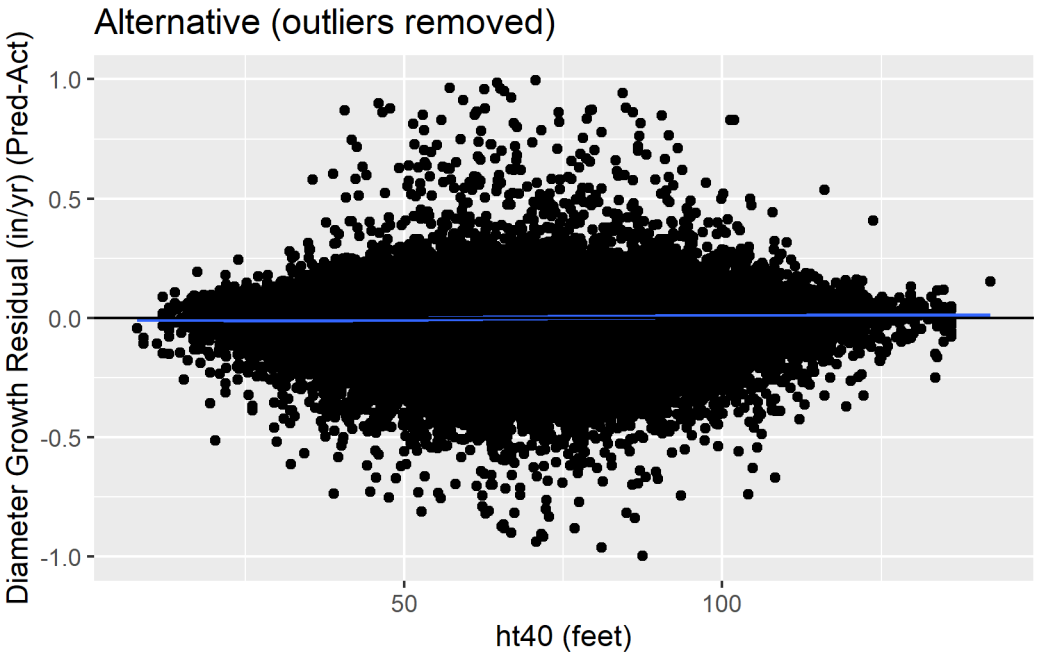


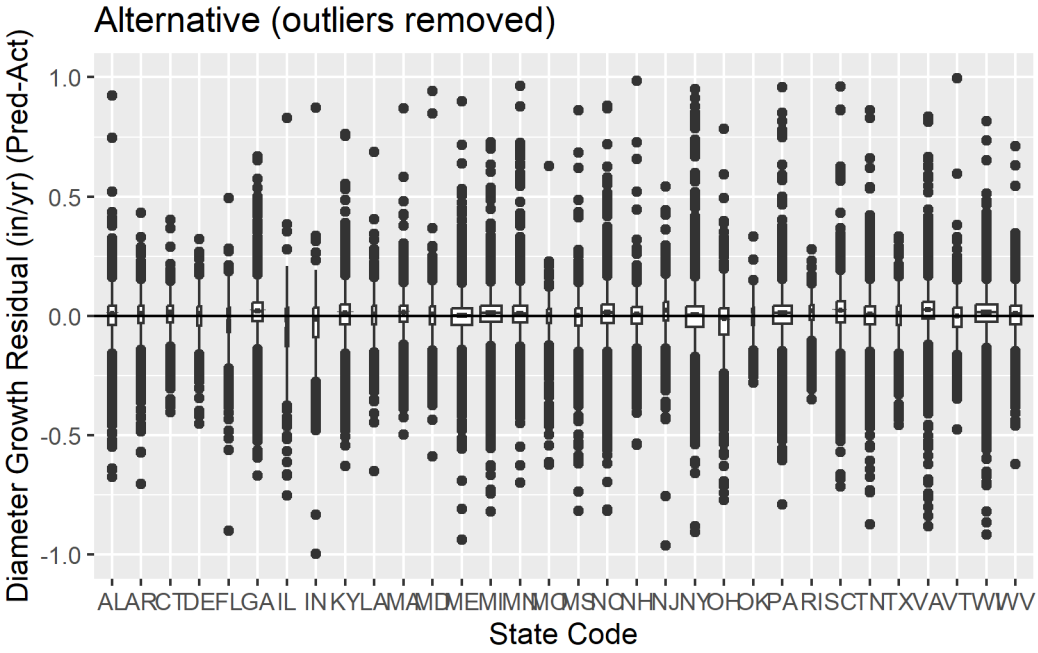

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
bal (ft^2/acre)

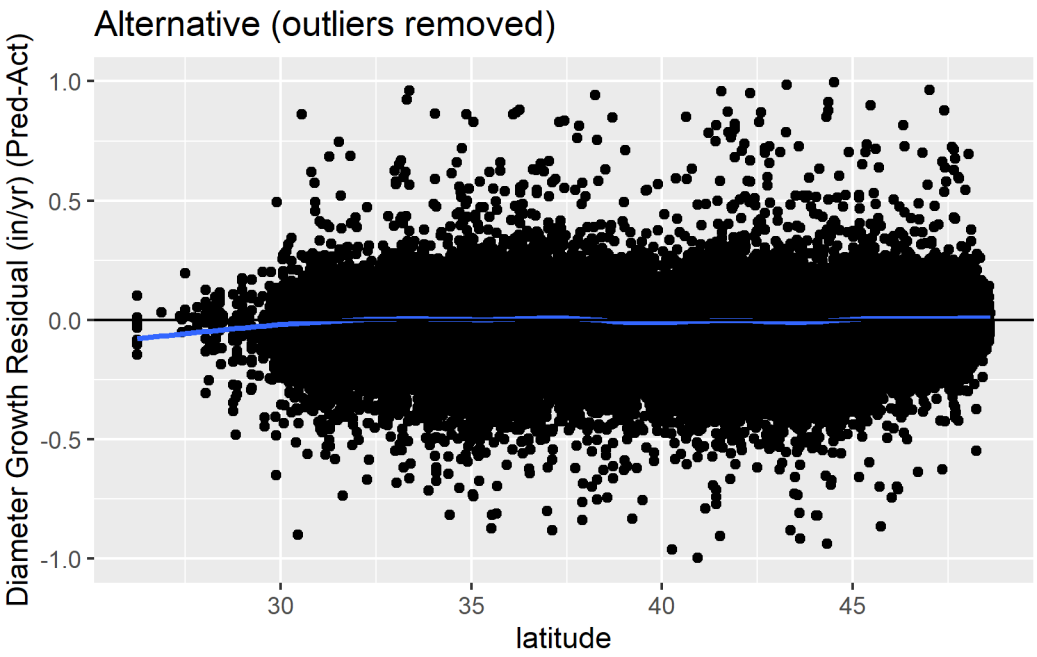

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

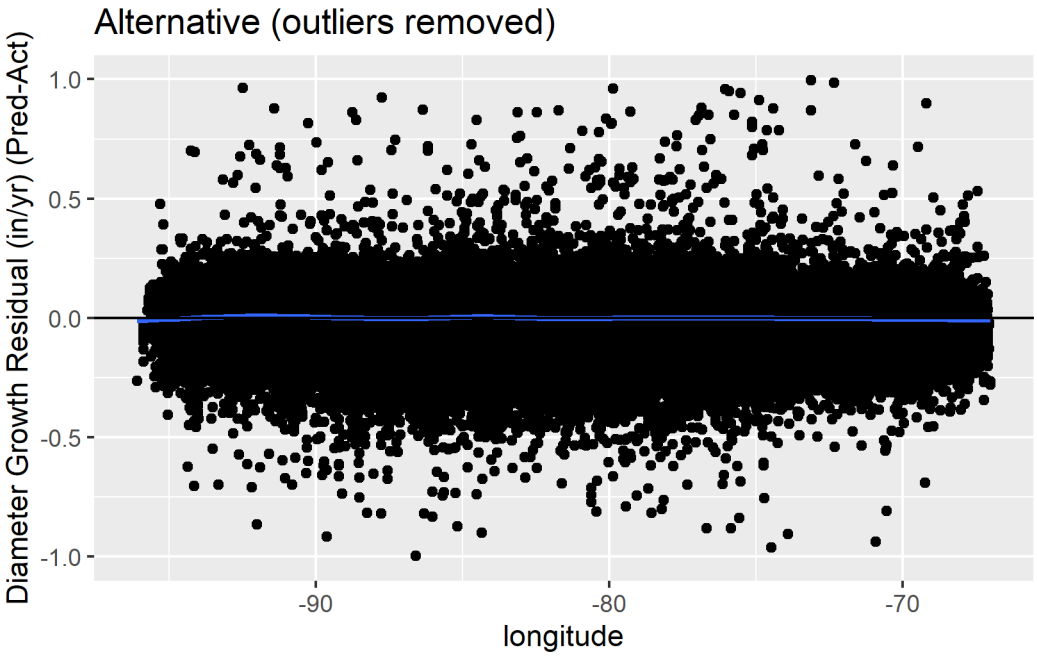

Alternative by si Base Age (outliers removed)
25
50
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
50
100
150
200
si (feet)

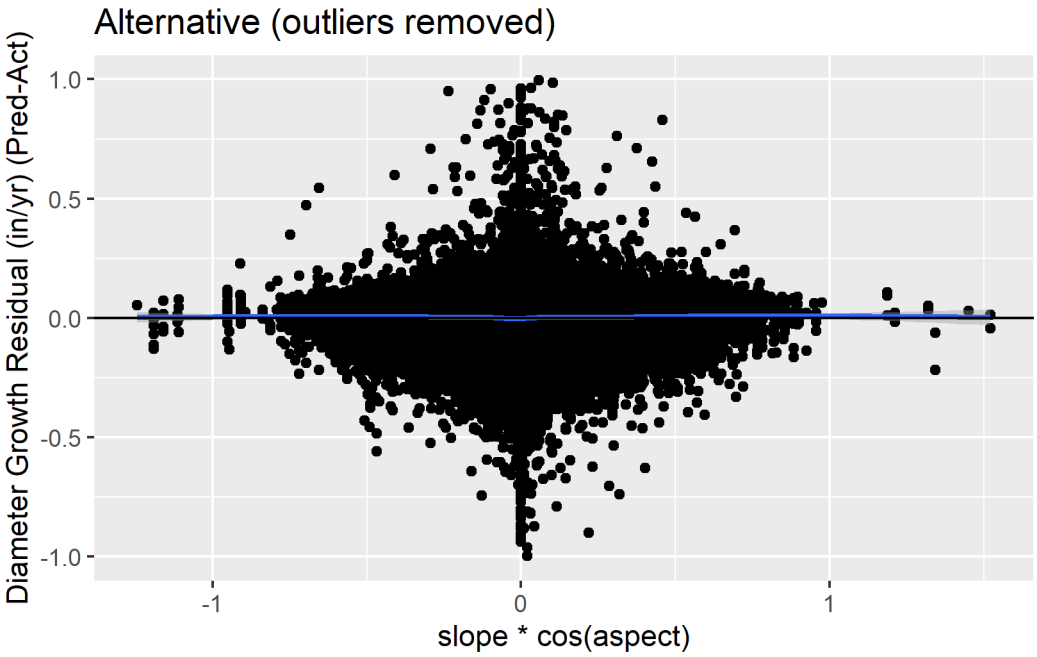

Alternative by si Species (outliers removed)
non Red Maple si
Red Maple si
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
50
100
150
200
si (feet)

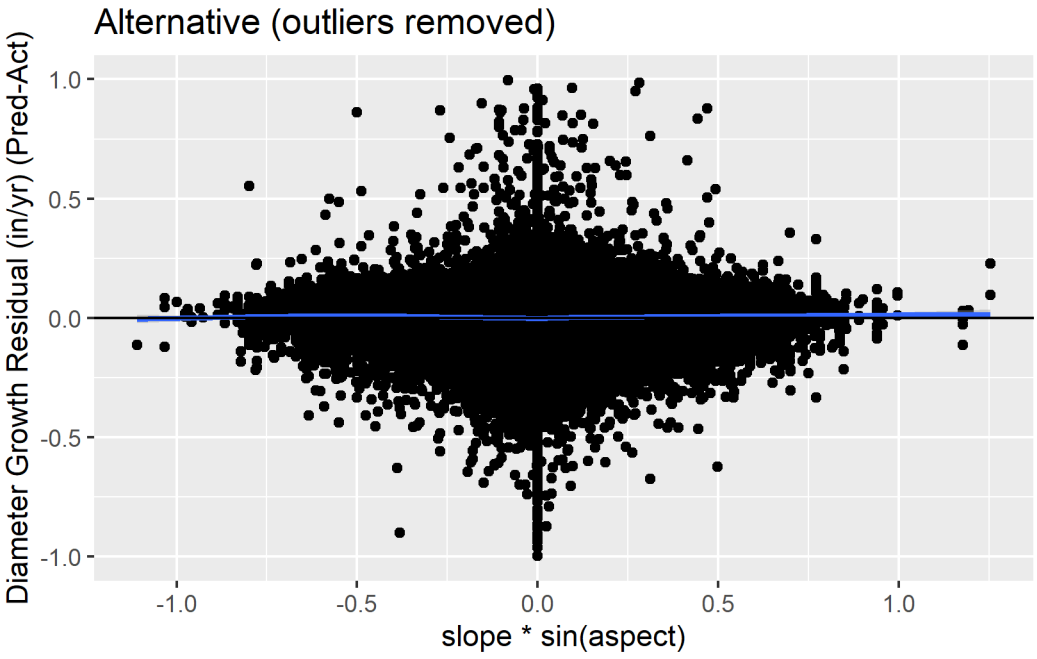

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
50
100
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
ALARCTDEFLGAIL INKYLAMAMDMEMIMNMOMSNCNHNJNYOHOKPARISCTNTXVAVTWIWV
State Code

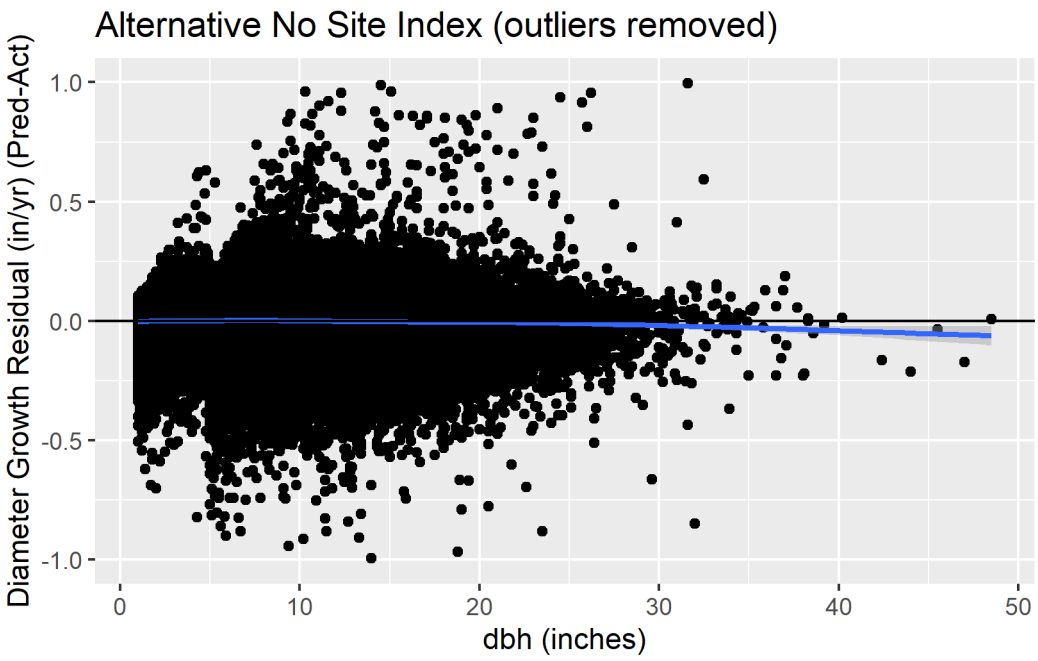

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
30
35
40
45
latitude

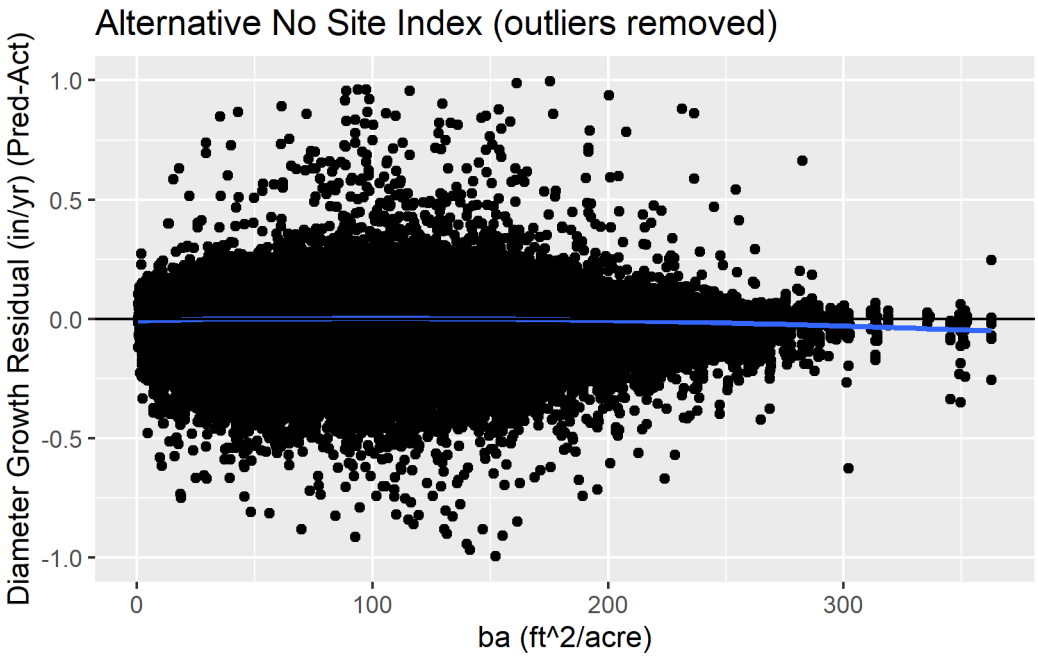

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-90
-80
-70
longitude

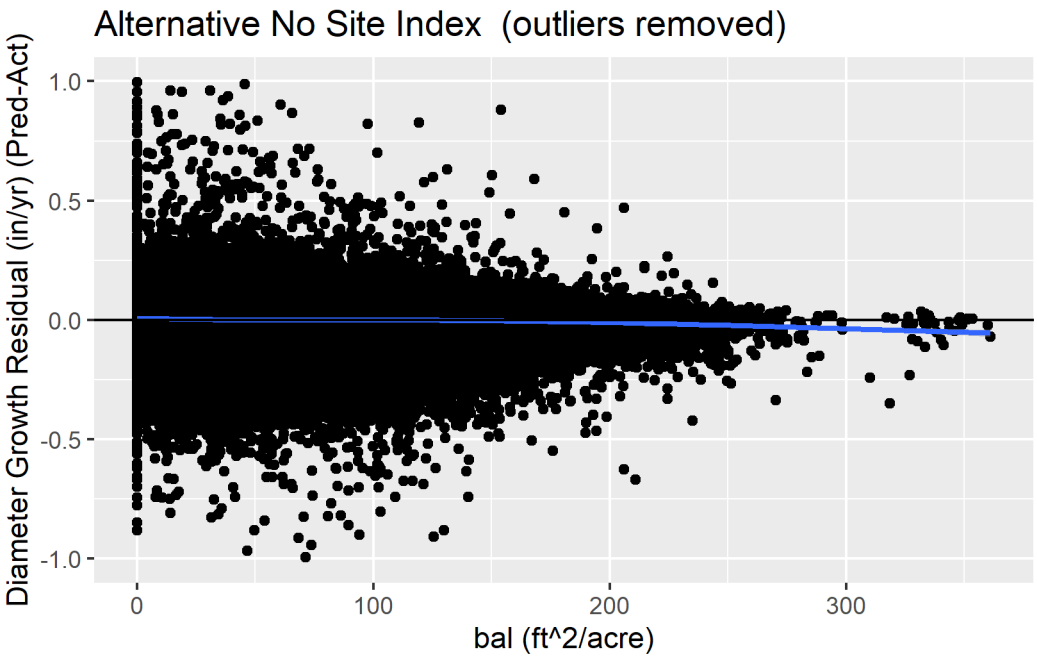

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-1
0
1
slope * cos(aspect)

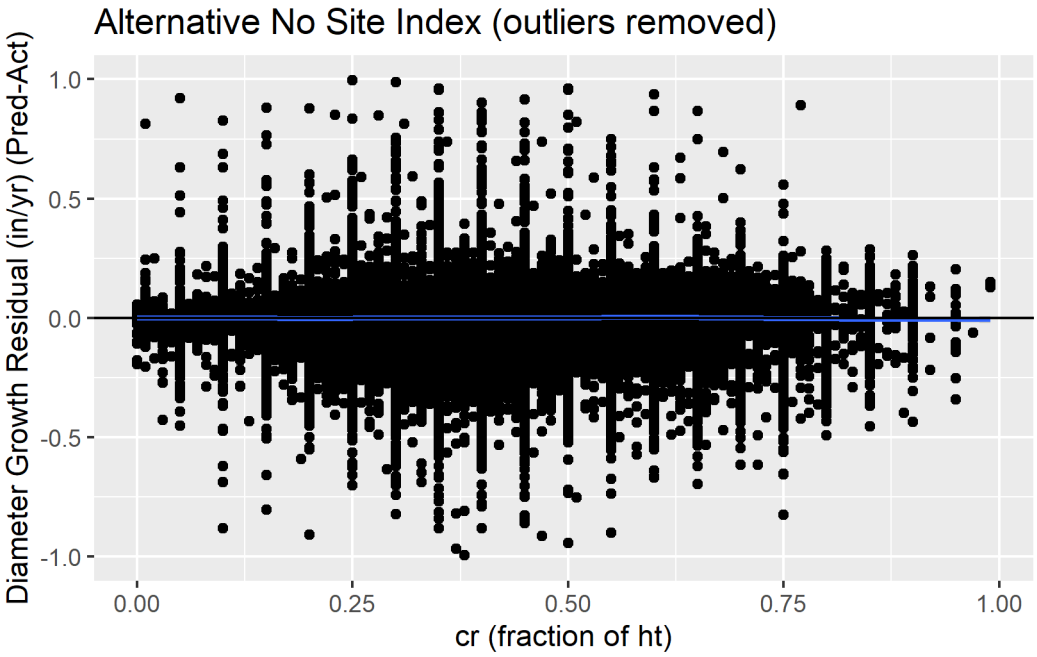

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)

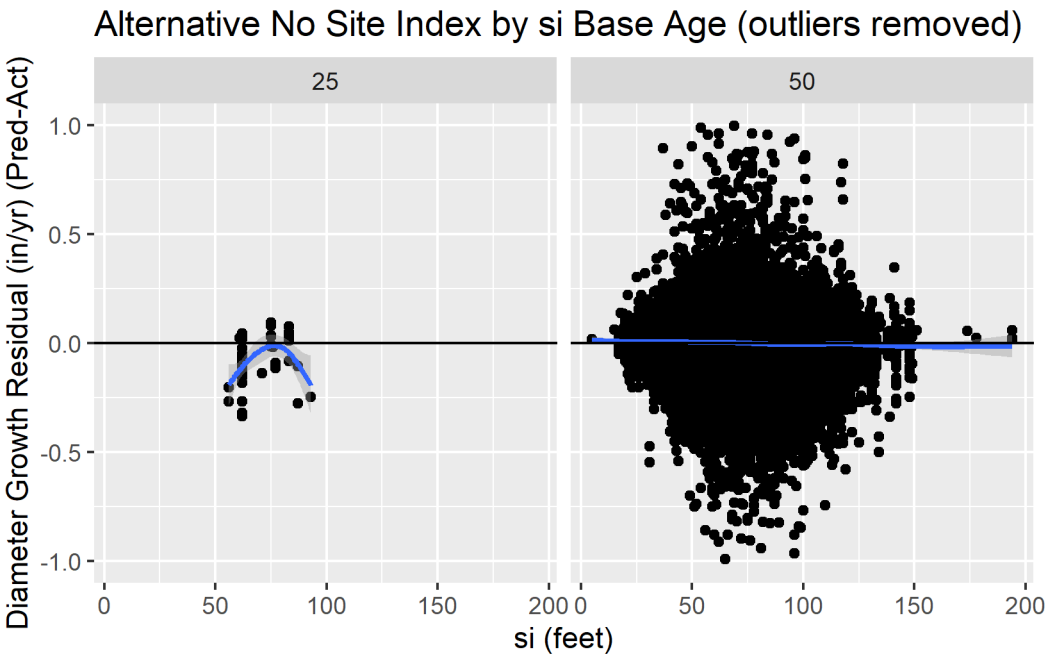
Residual Analysis for Equation 2

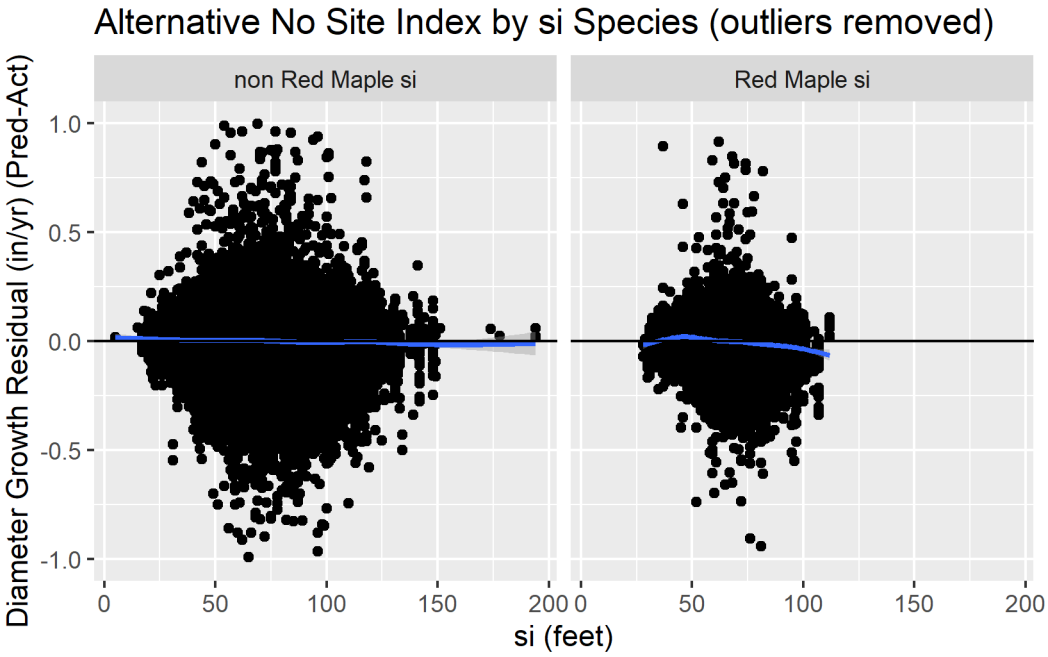


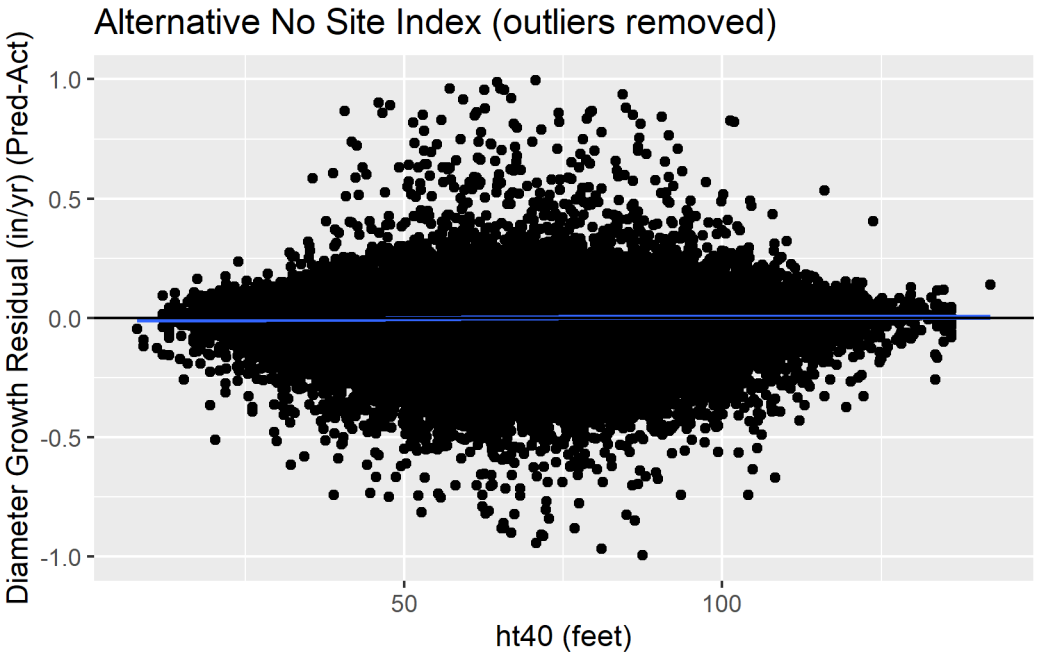


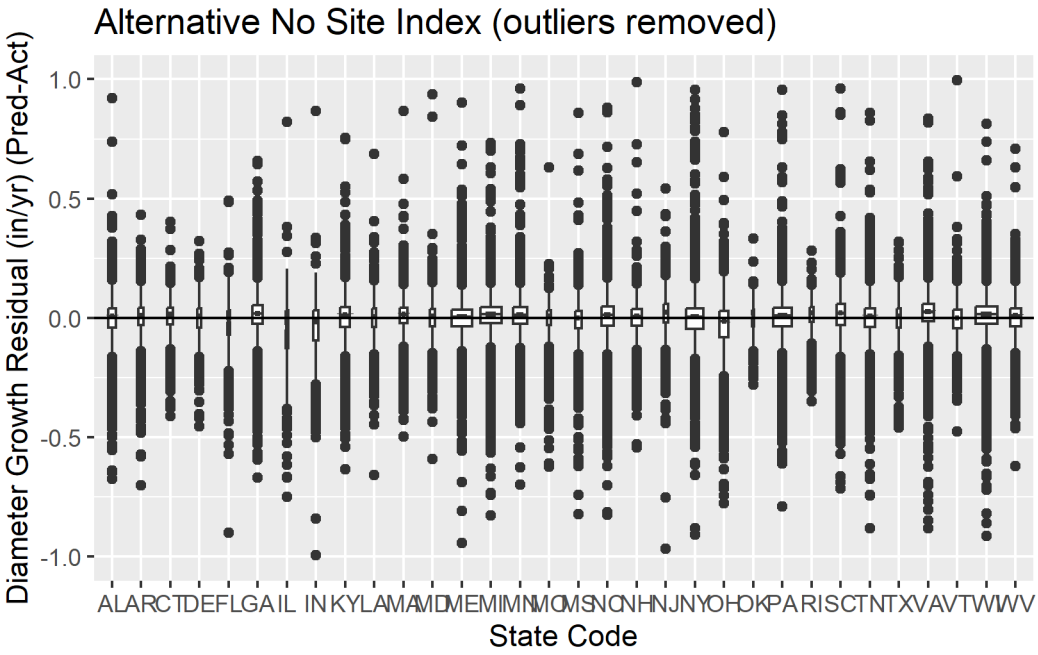

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
100
200
300
bal (ft^2/acre)

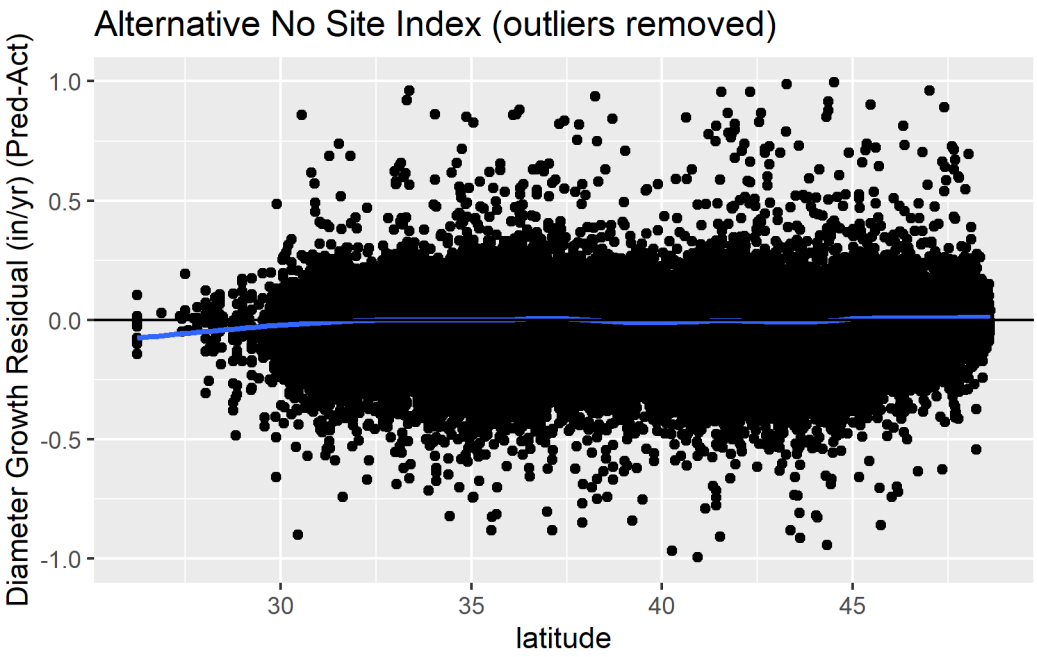

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

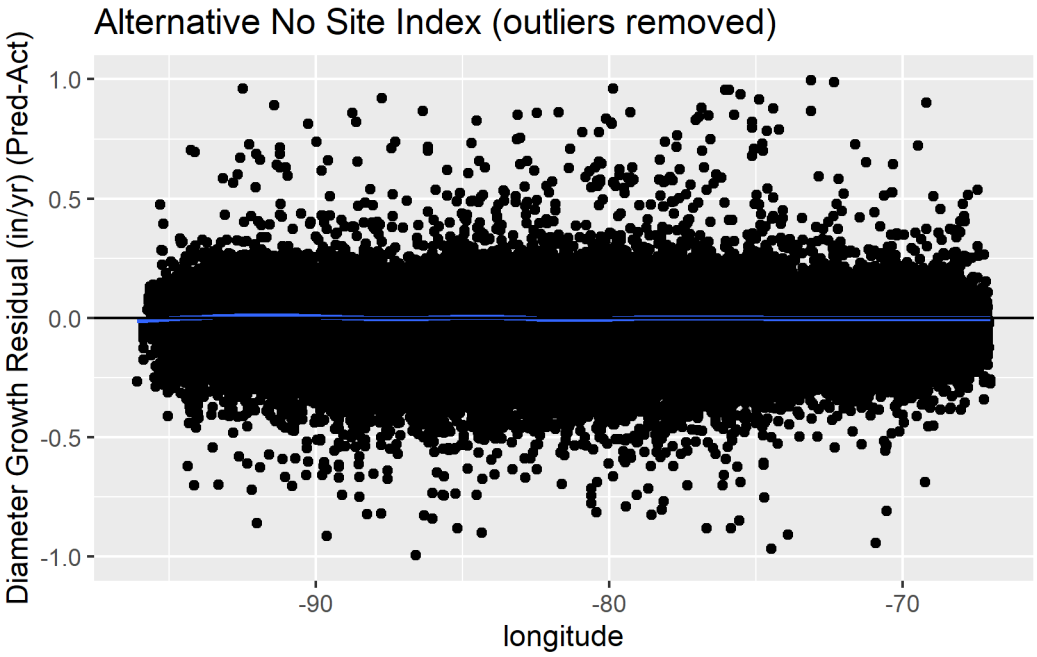

Alternative No Site Index by si Base Age (outliers removed)
25
50
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
50
100
150
200
si (feet)

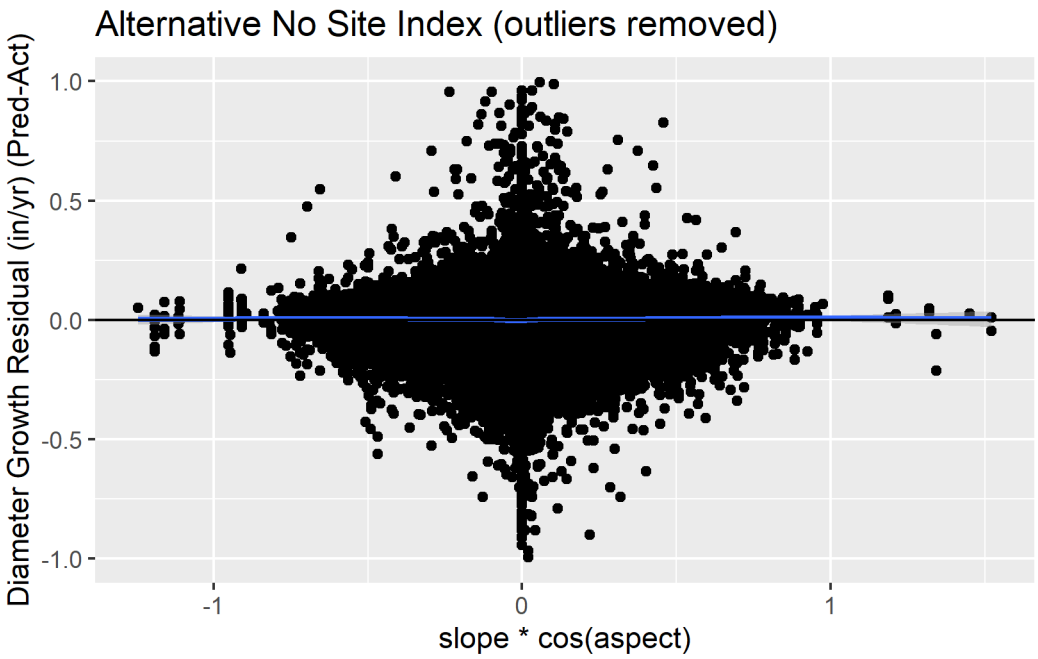

Alternative No Site Index by si Species (outliers removed)
non Red Maple si
Red Maple si
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
0
50
100
150
200
si (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
50
100
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
ALARCTDEFLGAIL INKYLAMAMDMEMIMNMOMSNCNHNJNYOHOKPARISCTNTXVAVTWIWV
State Code


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
30
35
40
45
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-90
-80
-70
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-1
0
1
slope * cos(aspect)

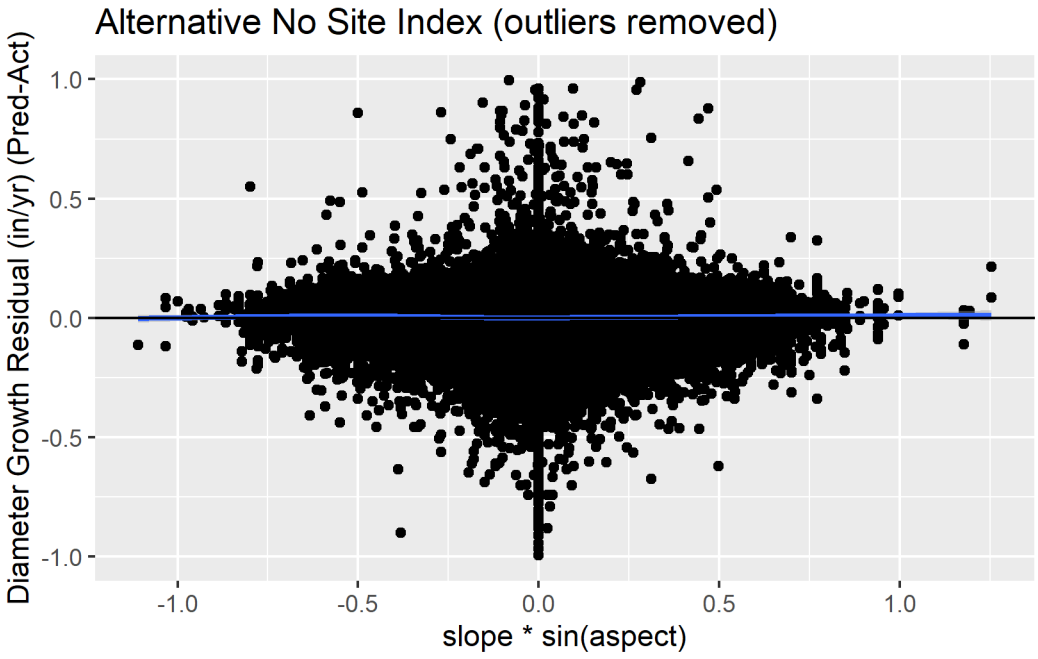
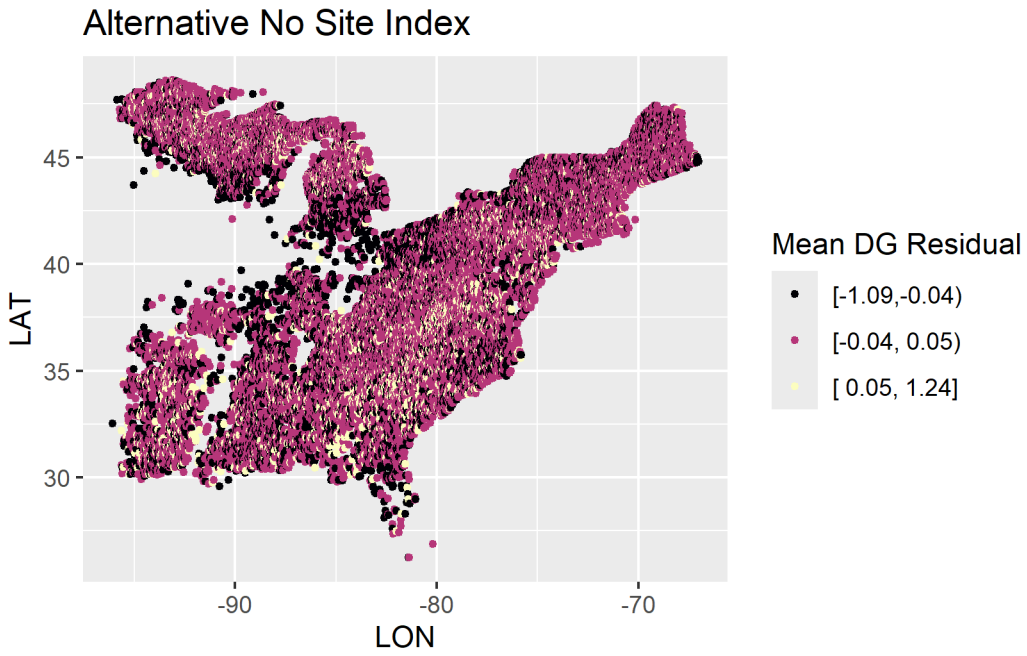

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)

Table 3: Independent Variables for One Inch dbh Trees

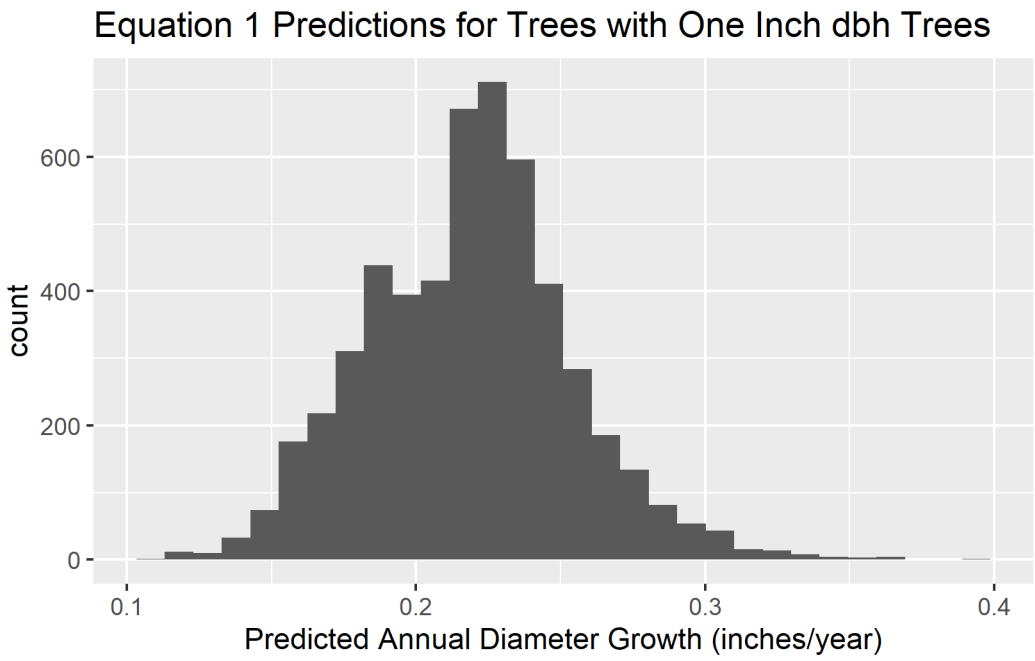
Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	5306	94	43	0.41	63	122	256
bal	5306	93	43	0	62	121	254
ht	5306	15	2.6	8	14	16	32
cr	5306	0.38	0.14	0.03	0.3	0.45	0.9
si	5306	67	16	18	55	77	194

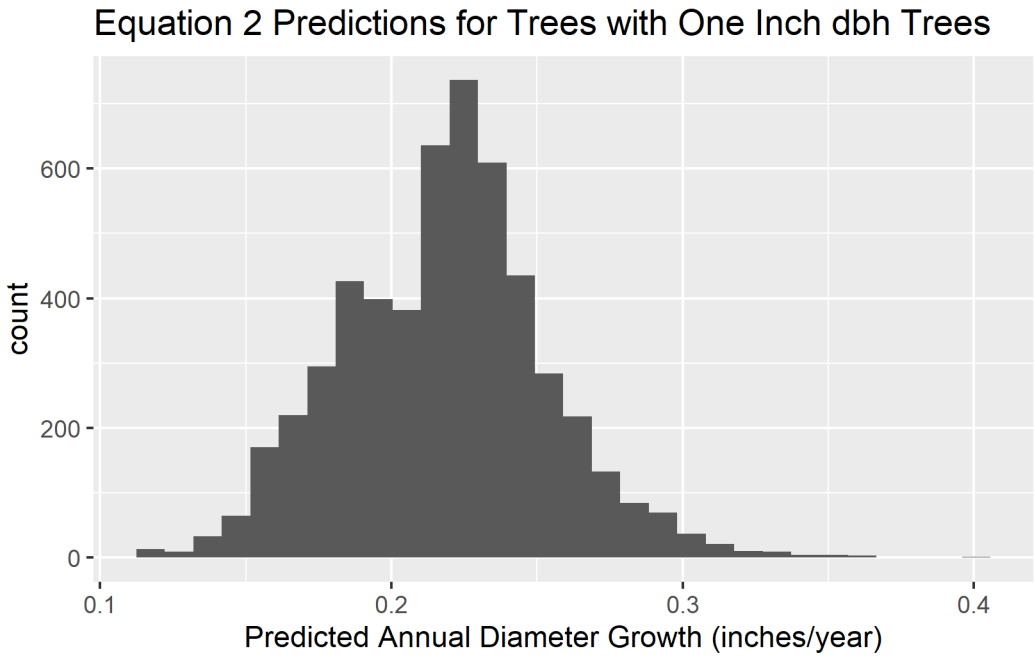


Discussion

Removing `si` does not degrade the fit significantly. Equation 2 residual graphs show that there are some residual trends spatially.

Equation Behavior for Very Small Trees






```
Reading layer `ecoregions' from data source
```

```
  `F:\Projects\FVS Remodel\gis\ecoregions.gpkg' using driver `GPKG'
Simple feature collection with 1632 features and 17 fields
Geometry type: MULTIPOLYGON
Dimension:     XY
Bounding box:  xmin: -2356069 ymin: 272048.5 xmax: 2258225 ymax: 3172577
Projected CRS: USA_Contiguous_Albers_Equal_Area_Conic_USGS_version
```