Alternative Red Alder Diameter Growth

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Data

We extracted and processed Forest Inventory and Analysis (FIA) data from 4 states listed in the native range of Red Alder in the Silvics of North America.[1]

After subsetting the data to censor observations with missing data, limiting the species to Red Alder (FIA species code 351), and remeasurement intervals ≥ 5 years we get the observations in Table 1.

Table 1: Red Alder Growth Observations by State

State	Observations
AK	224
CA	518
OR	4087
WA	4538

Alternative Model Formulation

An alternative to the ORGANON diameter growth equation[2] which reduces parameter count while retaining key features of the original model is shown below. The key change is the term with a ratio of a transformation of diameter at breast height (`dbh`) squared to crown length. Since β_1 is expected to be negative, this tends to slow growth as more basal area

[1] Burns, Russell M., and Barbara H. Honkala, tech. coords. 1990. Silvics of North America: 1. Conifers; 2. Hardwoods. Agriculture Handbook 654. U.S. Department of Agriculture, Forest Service, Washington, DC. vol.2, 877 p.

[2] Hann, D.W., Marshall, D.D., and Hanus, M.L. 2006. Reanalysis of the SMC-ORGANON equations for diameter-growth rate, height-growth rate, and mortality rate of Douglas-fir. Forest Research Laboratory Research Contribution 49.

accumulates in the tree while moderating that decline by the amount of productive crown capacity as measured by crown length. Basal area in larger trees (`bal`) serves as the inter-tree competition factor, and site index (`si`) as the inherent productivity scaling factor.

Site index is flawed for a number of reasons:

1. It is not consistently obtained for each plot due to missing Red Alder site trees,
2. It is estimated using a number of different and not necessarily compatible `si` equations, and
3. The available `si` equations do not all use the same base age.

In the data set `si` is derived from 18 different site index equations for 17 species. Red Alder site index comprises 12% of the observations. There are 2 base ages used. Preliminary graphical analysis revealed that base age was most correlated with residual bias. Thus in the following, we fit two equations: one where `SIBASE` and `SISP` are treated as a random effects in a mixed model framework, and a second leaving site index out.

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7}+\beta_3 log(si_{s,b}+4.5))} \tag{1}$$

and

$$\Delta dbh = e^{(\beta_0+\beta_1 log(\frac{(dbh+1)^2}{(cr*ht+1)^{\beta_4}})+\beta_2 \frac{bal^{\beta_5}}{dbh+2.7})} \tag{2}$$

where:

- `dbh` = diameter at breast height (inches),
- `bal` = basal area per acre in larger trees ($feet^2/ac$),
- `cr` = crown ratio (fraction of total height),
- `ht` = total height (feet), and
- $si_{s,b}$ = site index (feet) for species `s` and base age `b`.
- $\beta_0 - \beta_5$ are parameters to be estimated.

Nonlinear regression was used with an integrated fitting approach such that individual observations can have differing remeasurement intervals. The error to be minimized is ending `dbh`. Since this effectively minimizes diameter growth it can weight observations with longer remeasurement intervals more heavily. The effect of this needs to be evaluated, but putting more emphasis on longer periods may be beneficial.

The fit statistics for Equation 1 are:

```
Nonlinear mixed-effects model fit by maximum likelihood
  Model: endDIA ~ est_dg(B0, B1, B2, B3, B4, B5, startDIA, startBAL, endBAL,      startCR, en
  Data: tree_subset %>% mutate(SIINT = interaction(as.factor(tree_subset$SIBASE),      as.fac
```

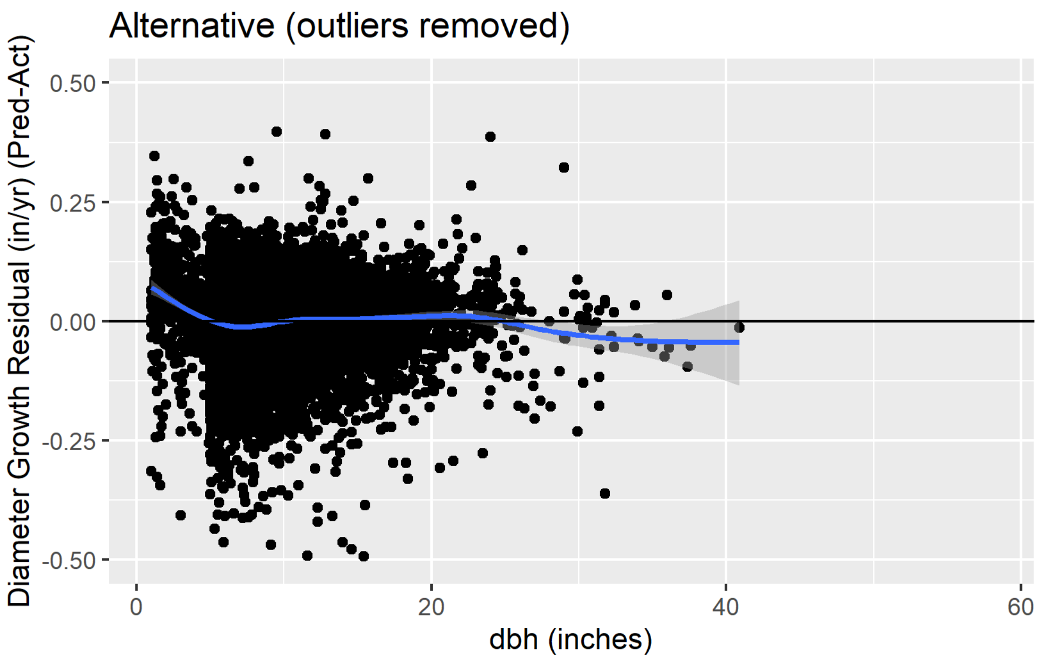
```
       AIC      BIC    logLik
  27971.03 28028.19 -13977.51

Random effects:
 Formula: B3 ~ 1 | SIINT
                B3 Residual
StdDev: 0.03038855 1.075282

Fixed effects:  B0 + B1 + B2 + B3 + B4 + B5 ~ 1
        Value  Std.Error   DF   t-value p-value
B0 -2.2359522 0.24498524 9358  -9.12689   0e+00
B1 -0.7709535 0.01979908 9358 -38.93886   0e+00
B2 -0.1865872 0.04750169 9358  -3.92801   1e-04
B3  0.3073183 0.04802854 9358   6.39866   0e+00
B4  1.2890620 0.03427489 9358  37.60952   0e+00
B5  0.5231778 0.04211535 9358  12.42250   0e+00
 Correlation:
   B0     B1     B2     B3     B4
B1 -0.233
B2 -0.407  0.494
B3 -0.808 -0.017  0.015
B4 -0.437  0.320  0.515 -0.043
B5 -0.388  0.476  0.994  0.019  0.476

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-48.1407968  -0.5045622  -0.1092614   0.4369118   8.7481740

Number of Observations: 9367
Number of Groups: 4

$SIINT
                   B3
50.FALSE   0.01659631
100.FALSE  0.01990841
50.TRUE    0.01227407
100.TRUE  -0.04877880

Residual Standard Error: 1.07528165460809 on 9358 degrees of freedom, AIC: 27971
```

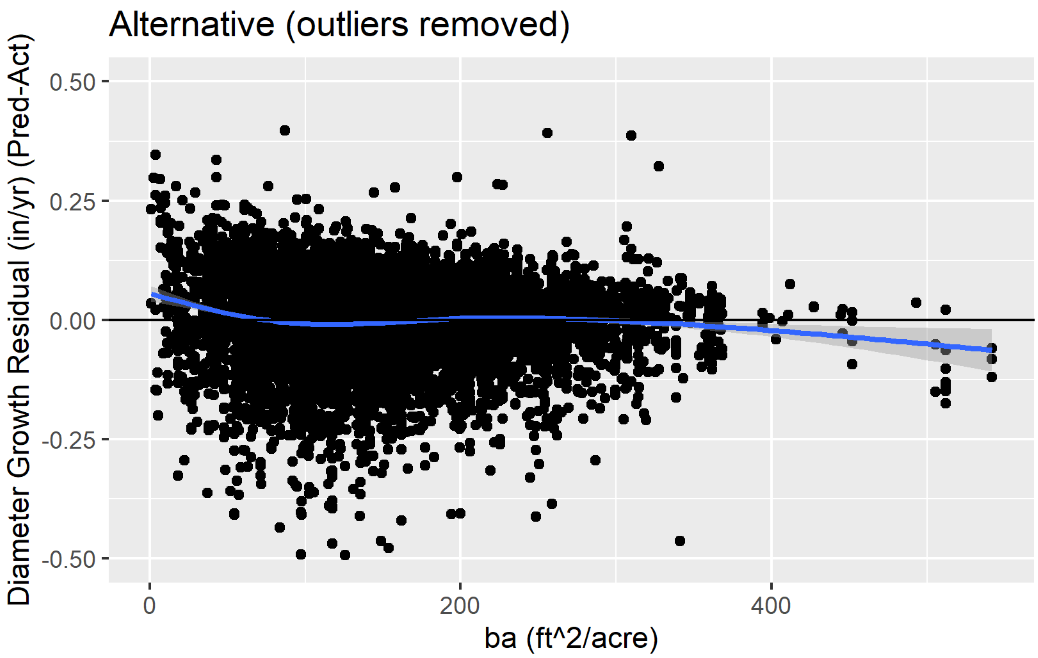
and for Equation 2:

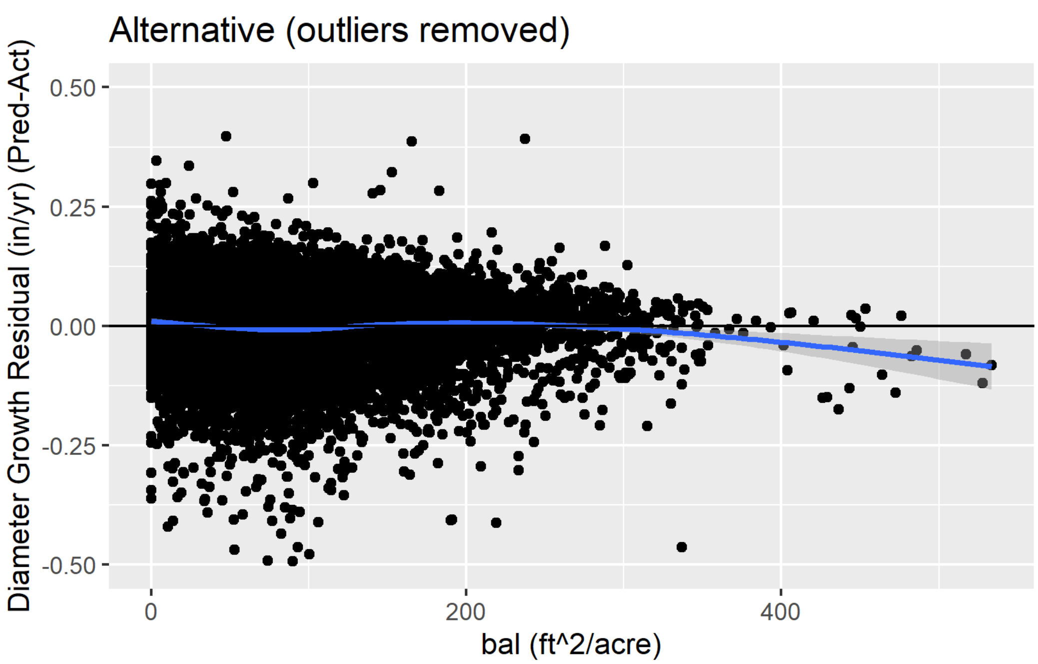
	Coef.	Std. error	t-stat.	p
B0	-0.7566293	0.1285411	-5.886282	0.0e+00
B1	-0.7737909	0.0199156	-38.853574	0.0e+00
B2	-0.1955751	0.0498497	-3.923297	8.8e-05
B4	1.3037868	0.0344134	37.886075	0.0e+00
B5	0.5131623	0.0419910	12.220777	0.0e+00

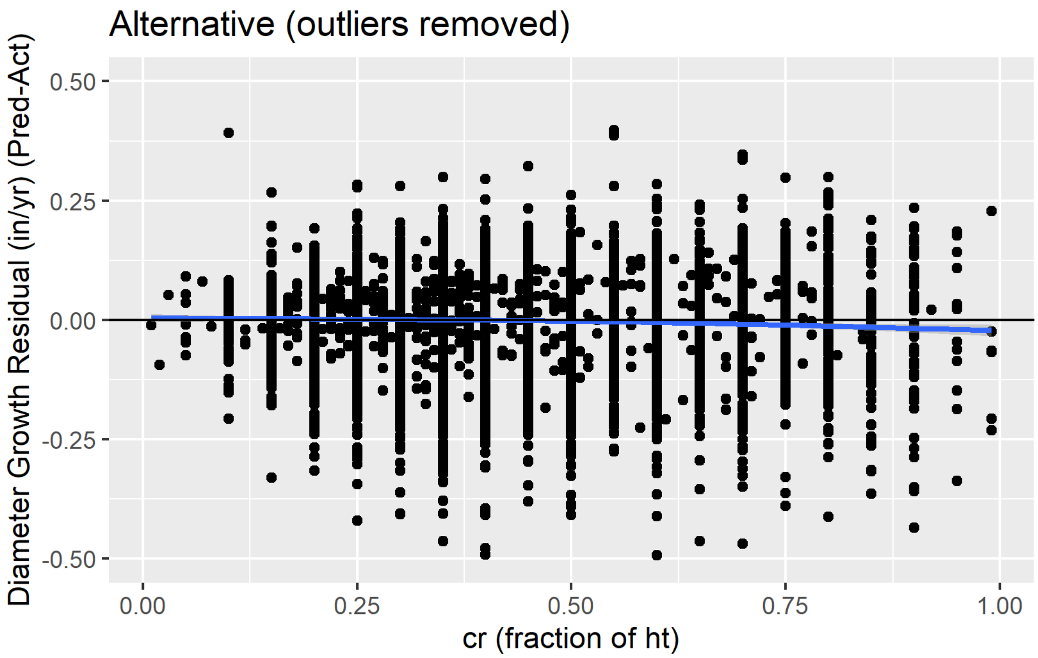
```
Residual Standard Error: 1.07903751116923 on 9362 degrees of freedom, AIC: 28014.5
```

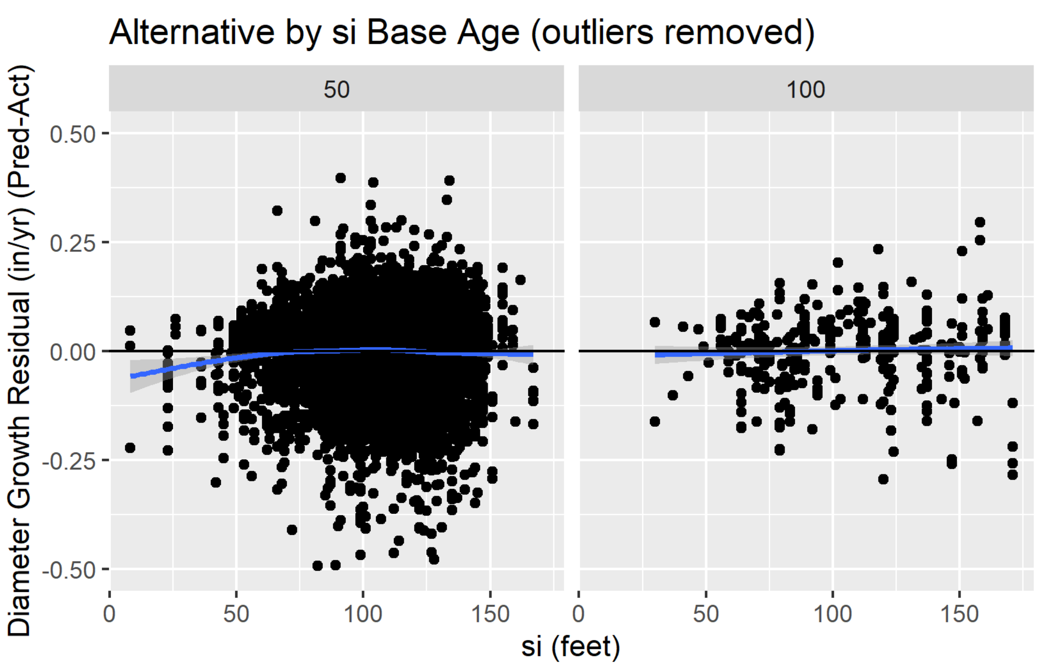
Residual Analysis for Equation 1

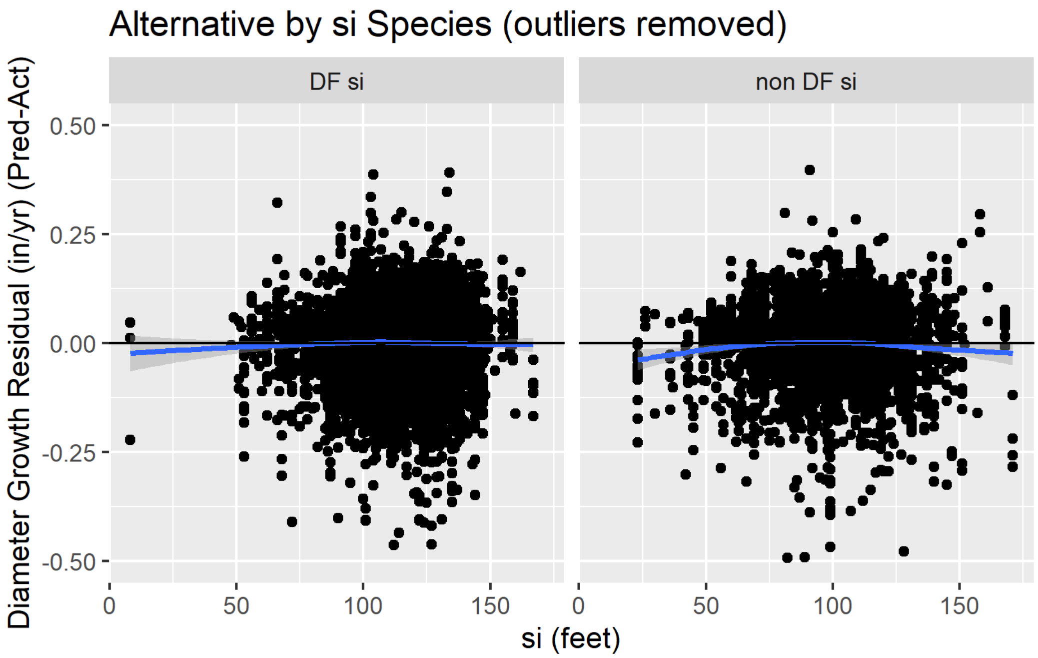



Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
ba (ft^2/acre)

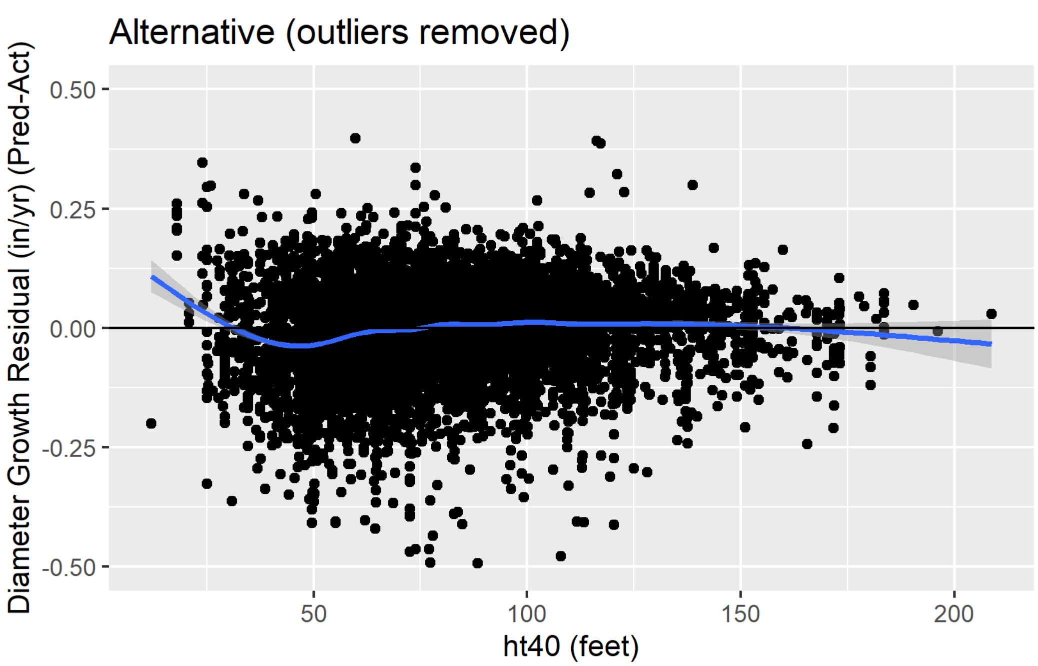

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
bal (ft^2/acre)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

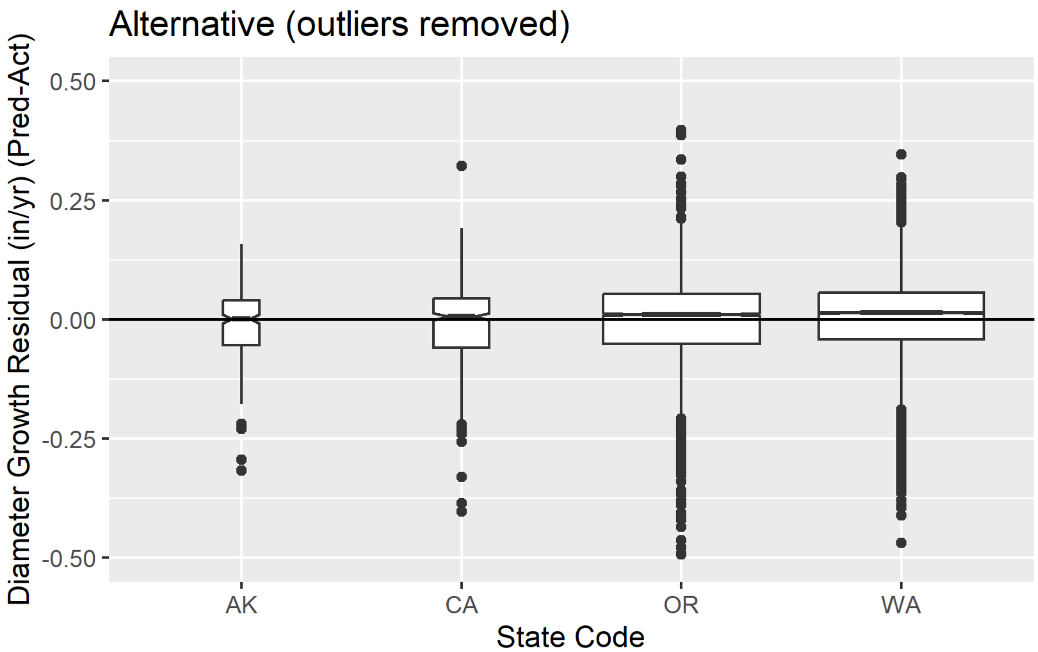

Alternative by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

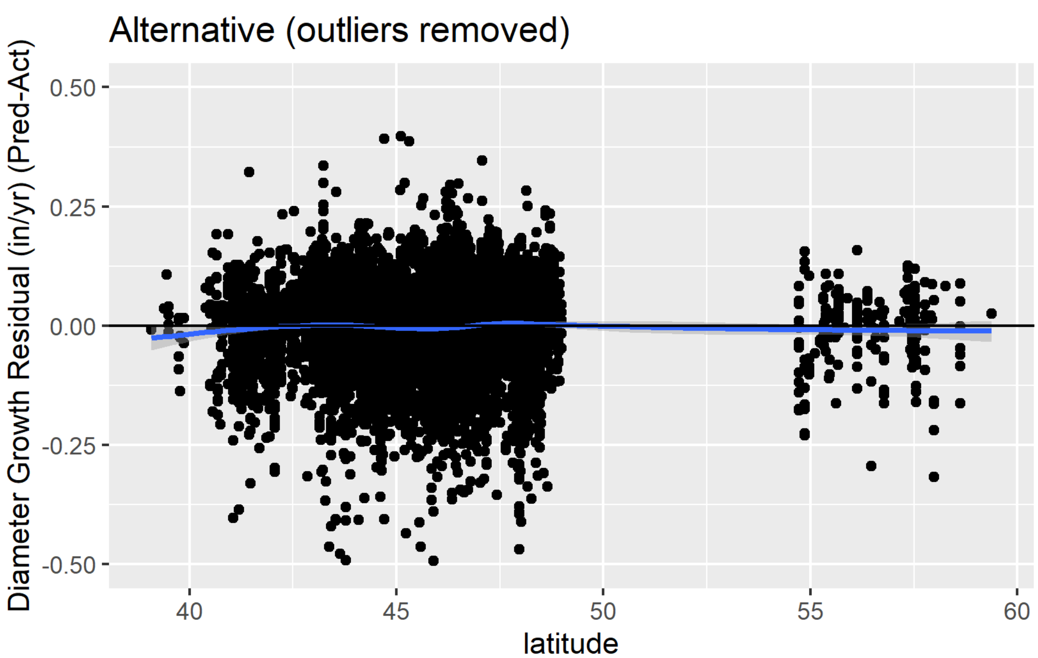


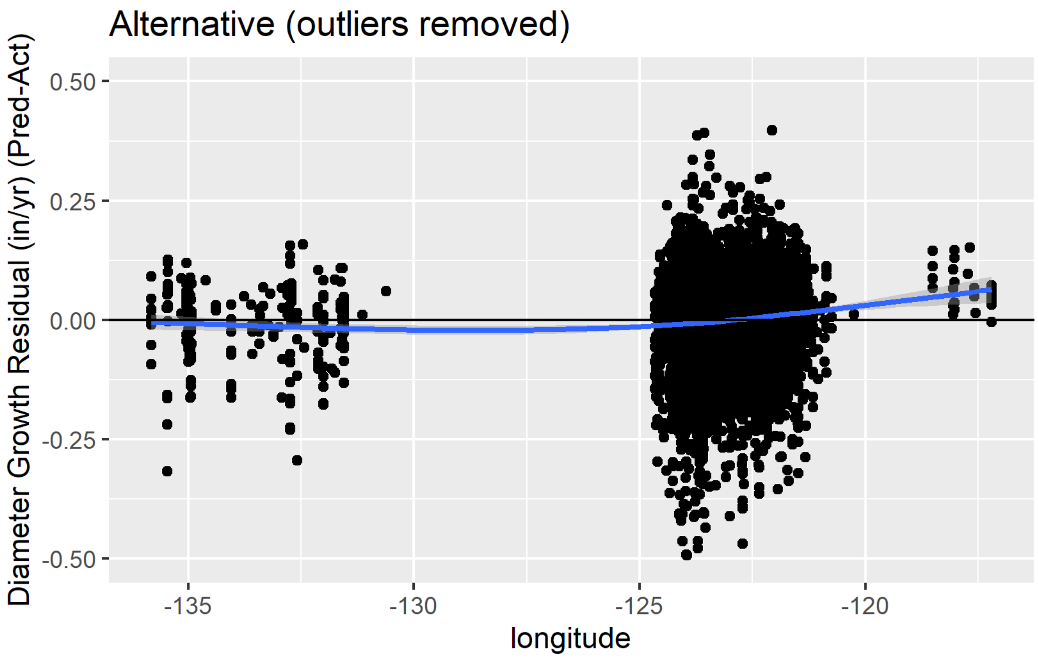
Alternative by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

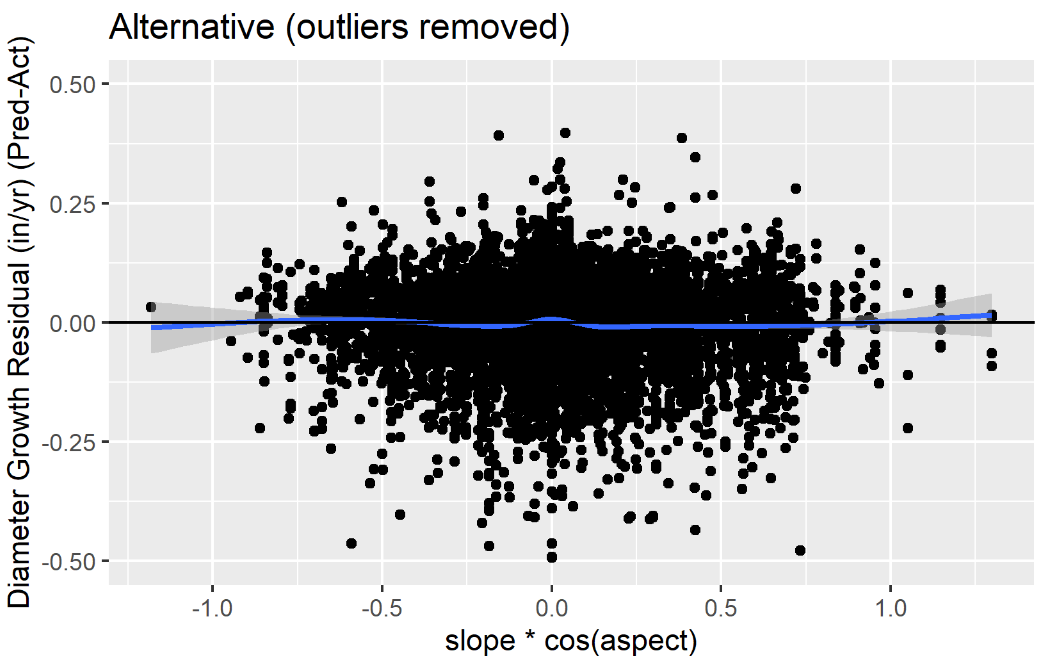


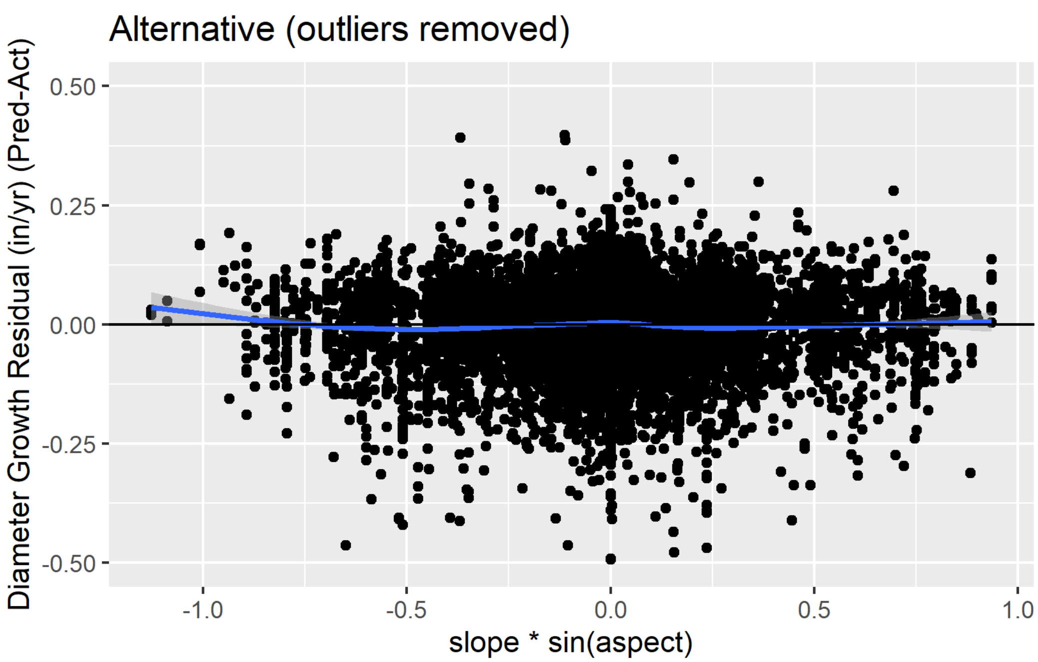
Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
200
ht40 (feet)


Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
AK
CA
OR
WA
State Code

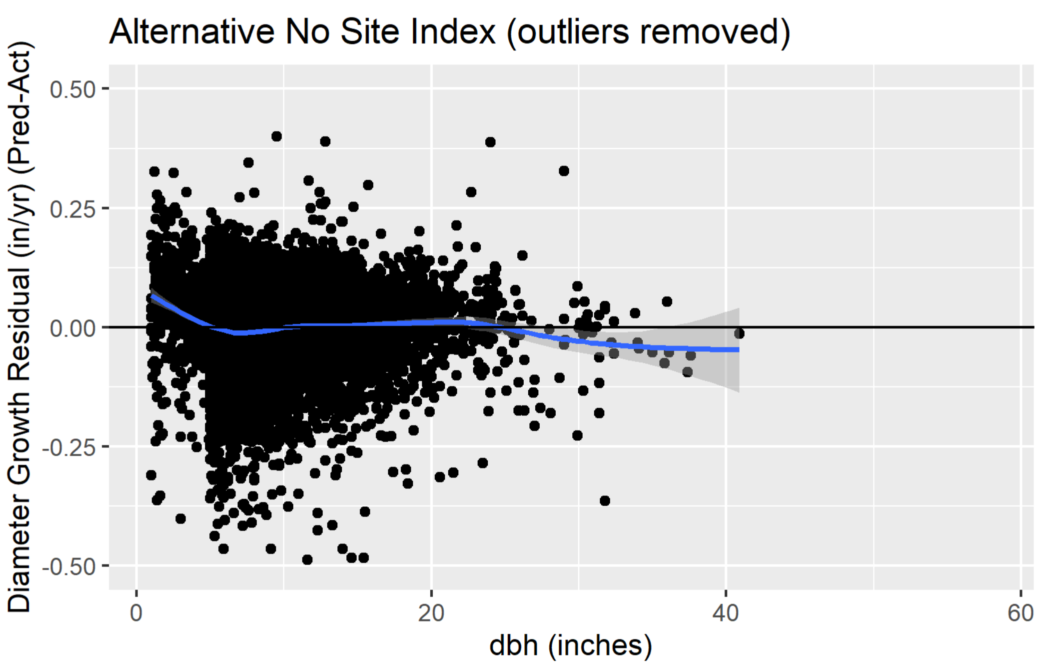

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
45
50
55
60
latitude

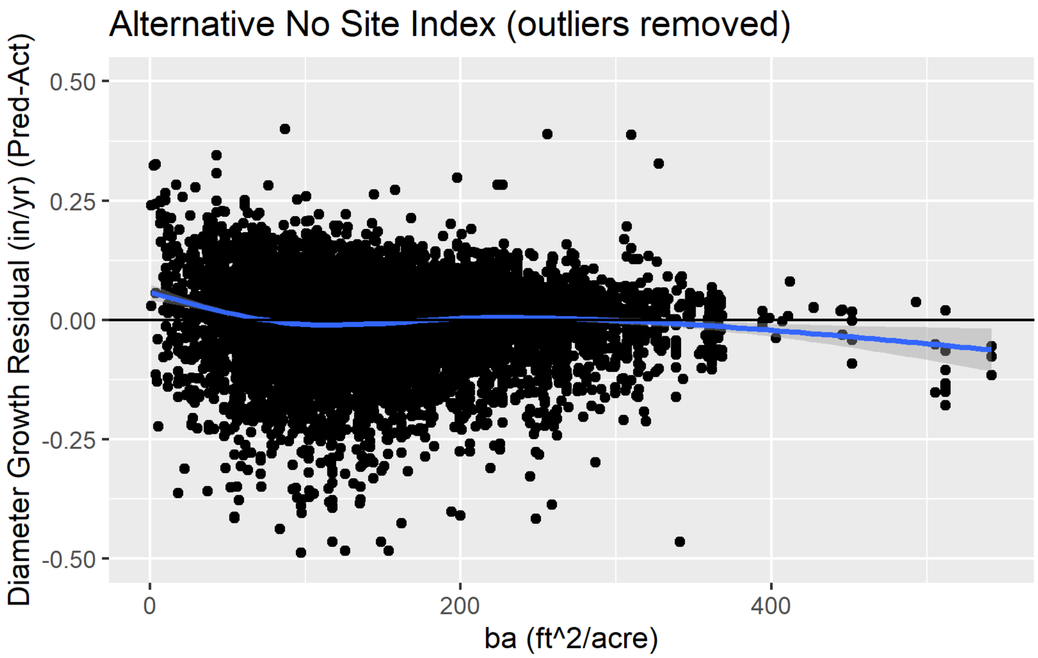

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-135
-130
-125
-120
longitude

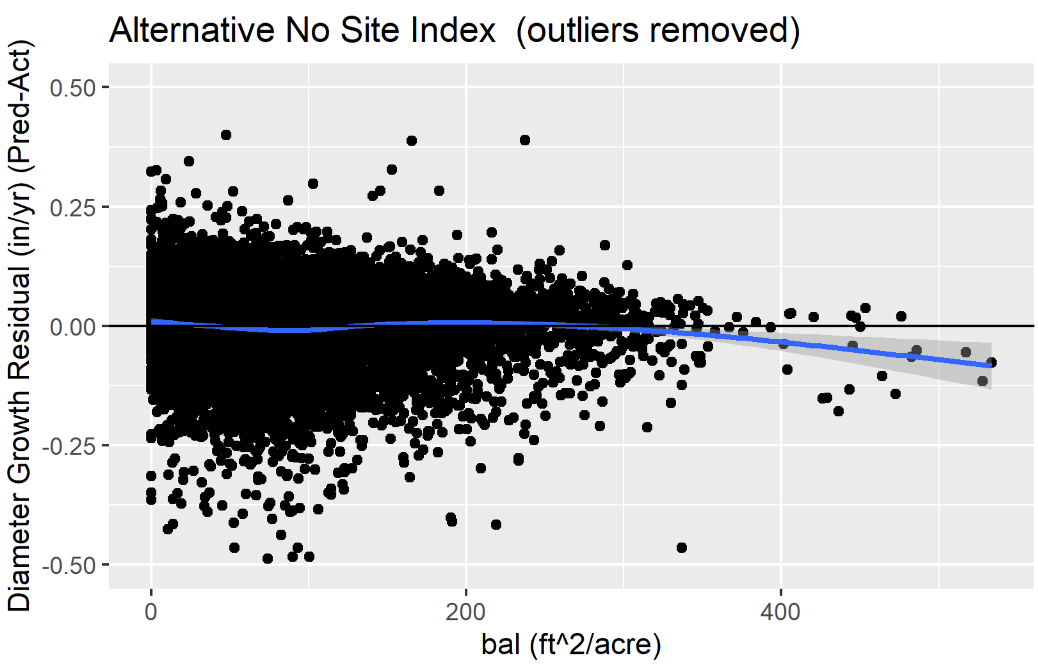

Alternative (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)

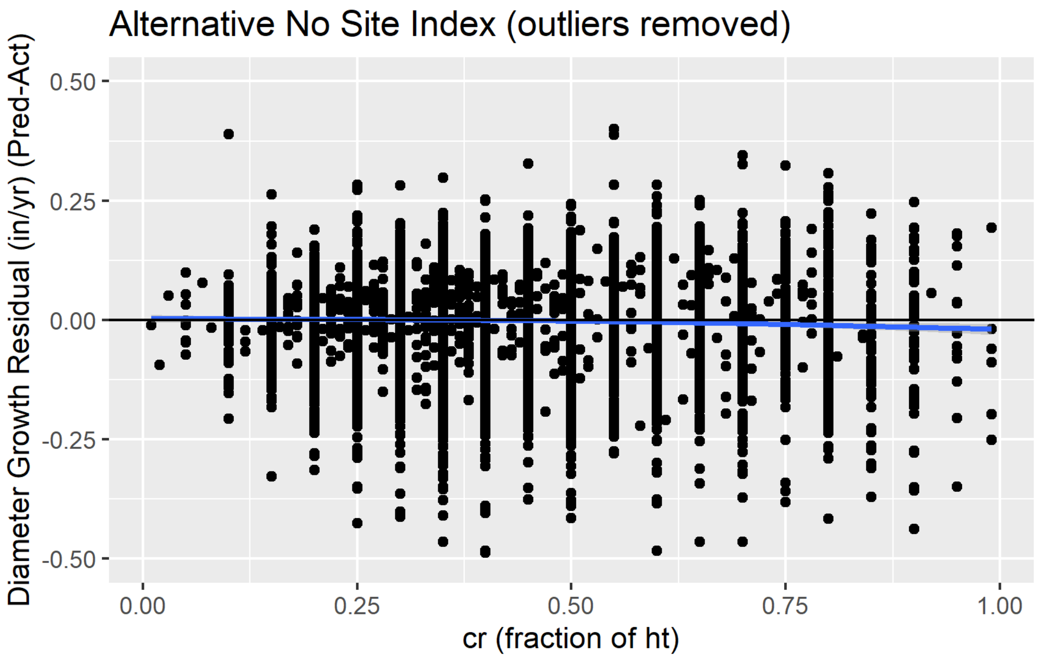


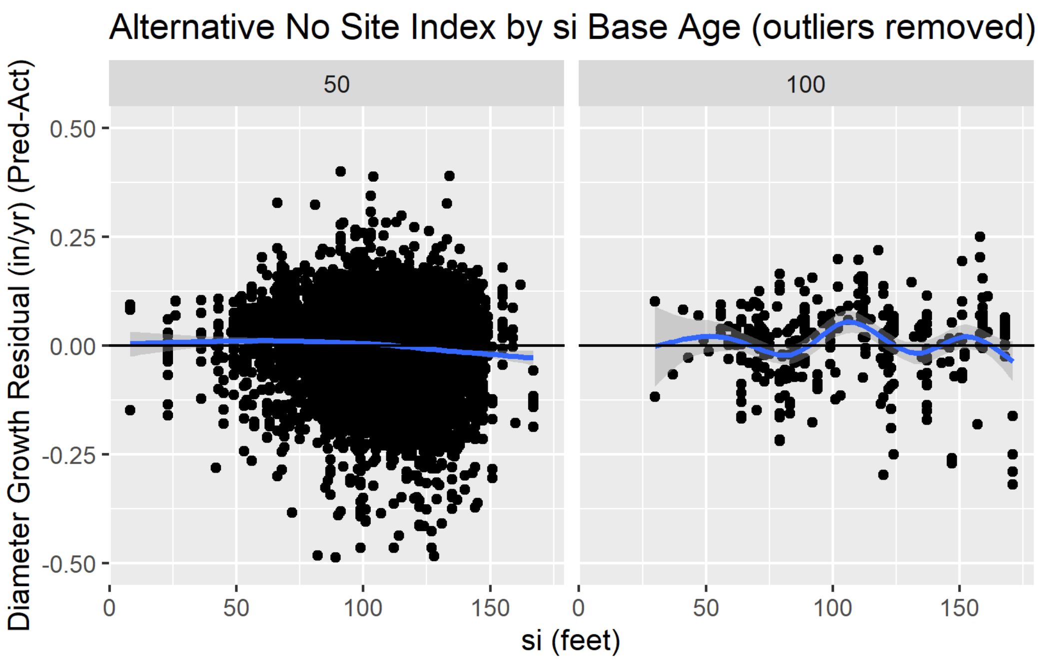
Residual Analysis for Equation 2

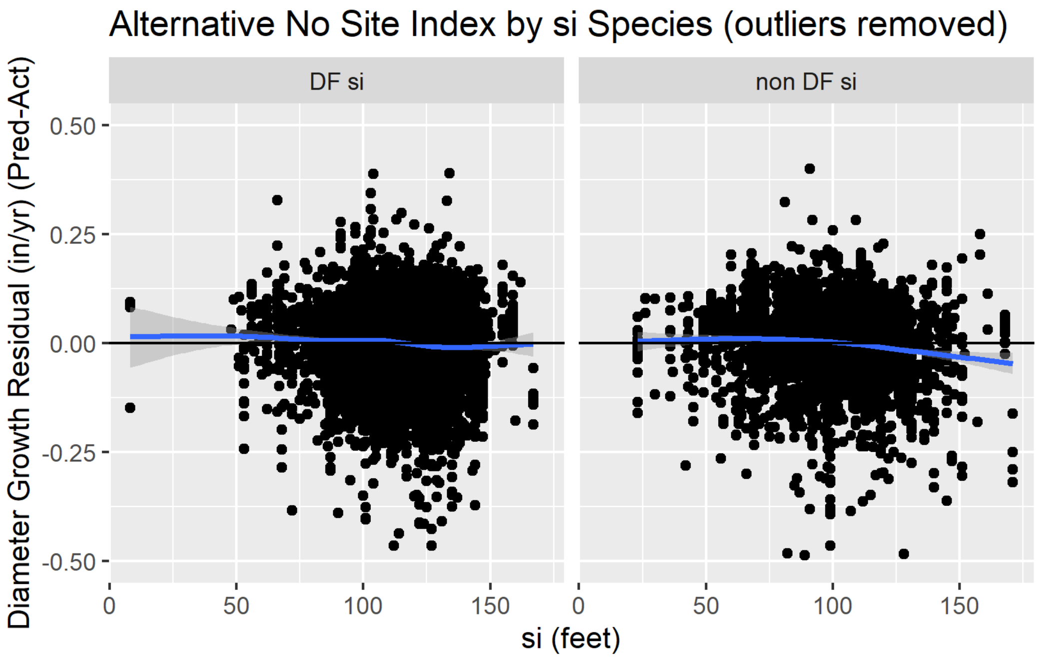



Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
ba (ft^2/acre)

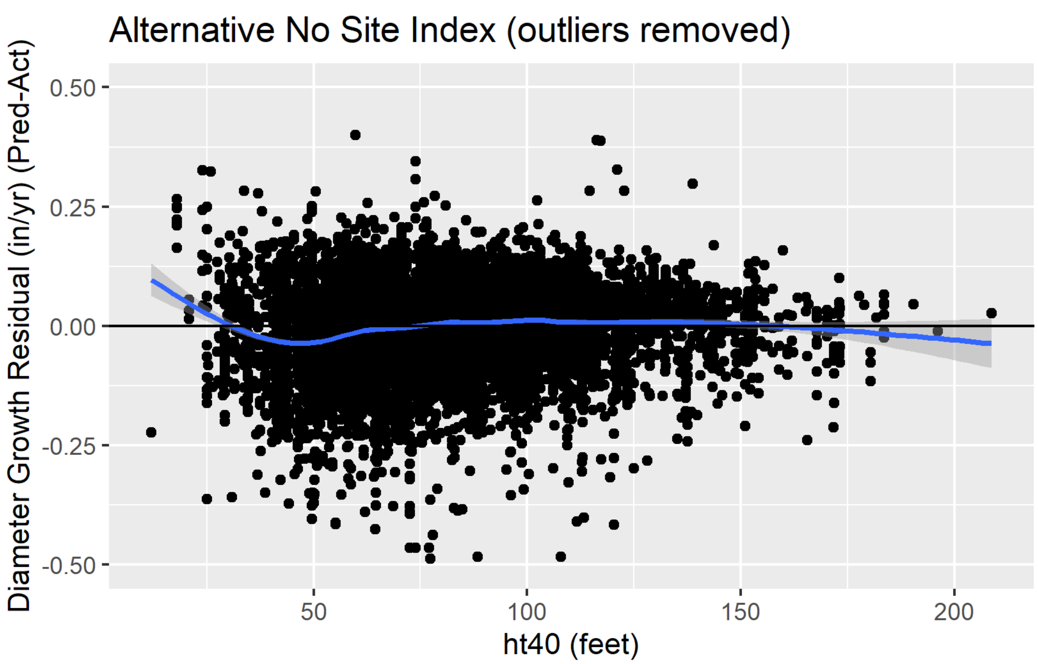

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
200
400
bal (ft^2/acre)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0.00
0.25
0.50
0.75
1.00
cr (fraction of ht)

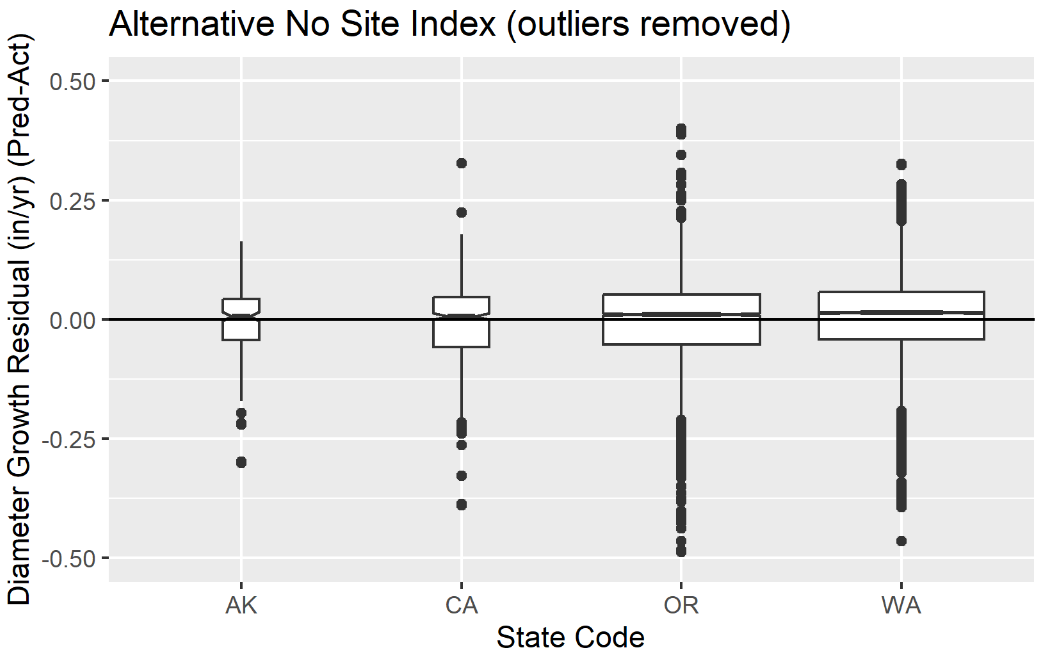

Alternative No Site Index by si Base Age (outliers removed)
50
100
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

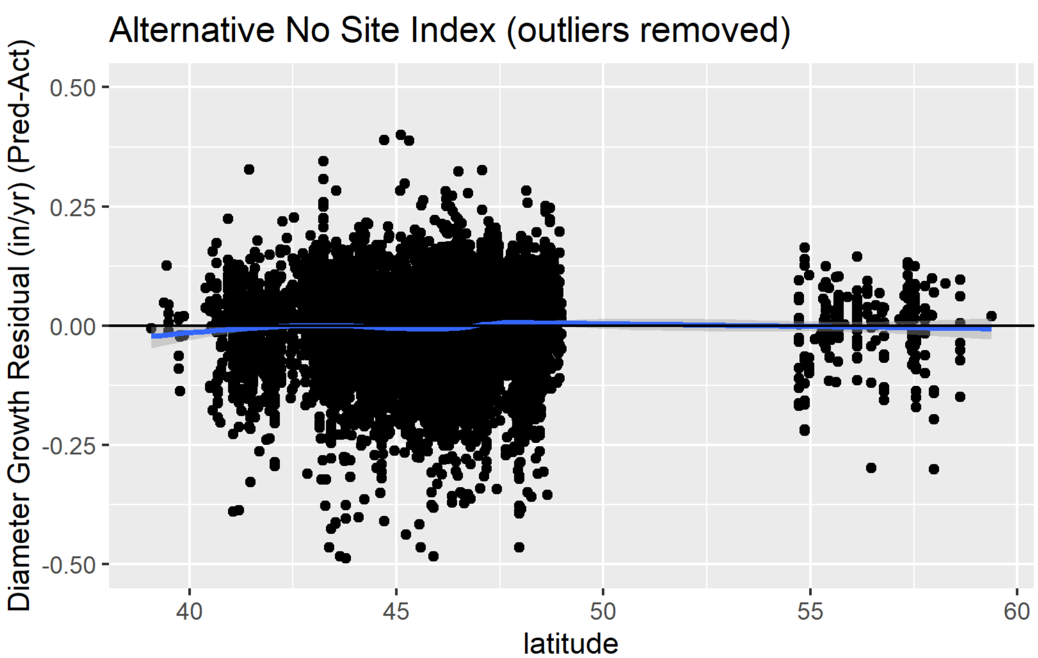


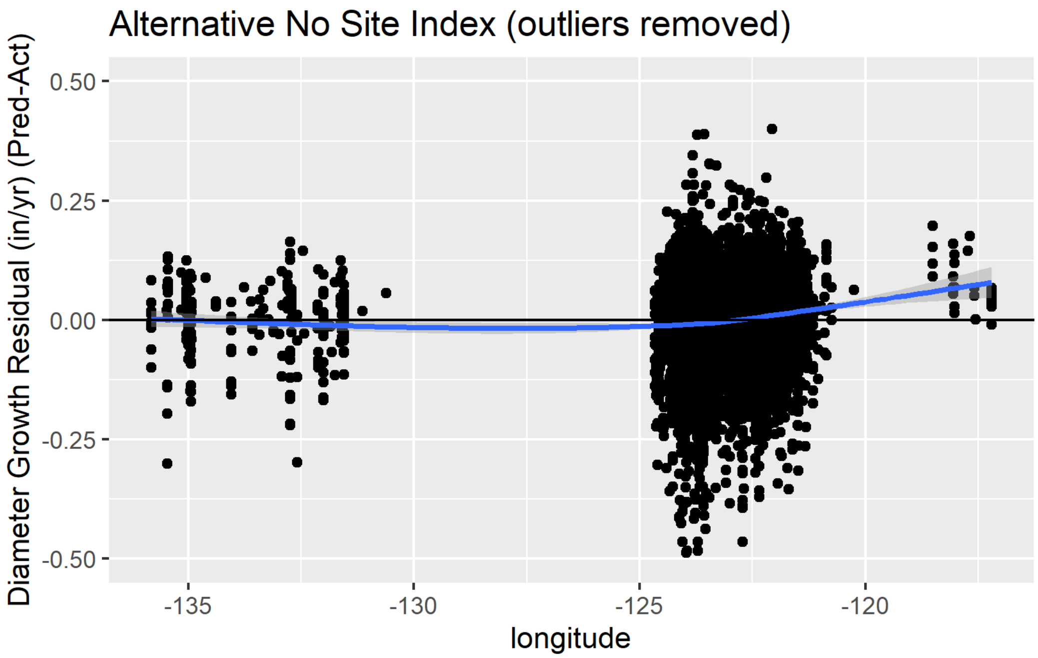
Alternative No Site Index by si Species (outliers removed)
DF si
non DF si
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
0
50
100
150
si (feet)

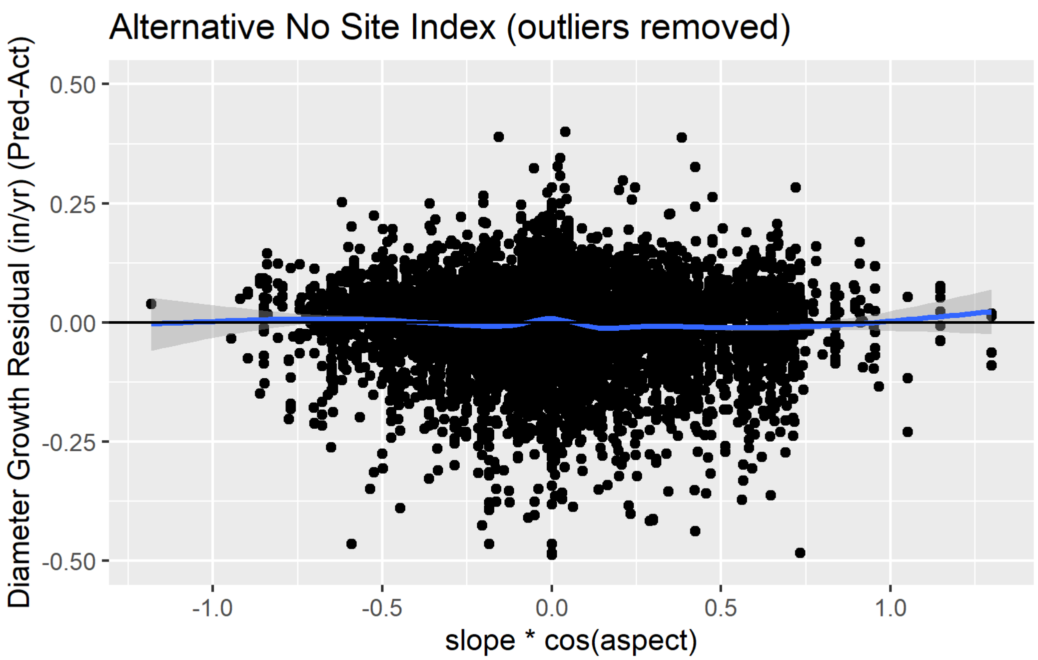


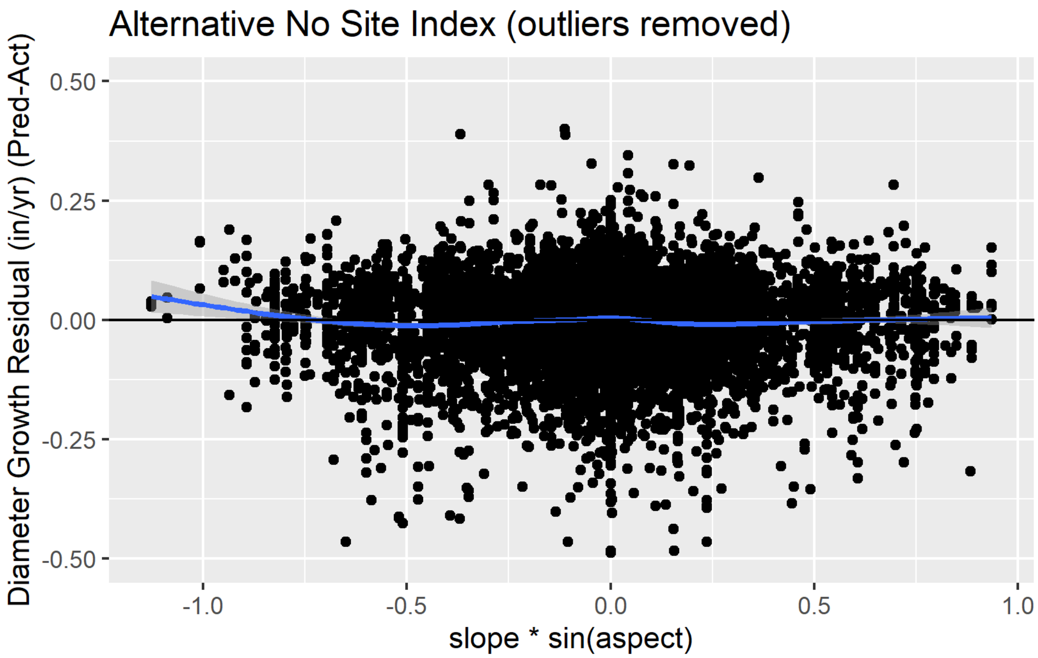
Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
50
100
150
200
ht40 (feet)


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
AK
CA
OR
WA
State Code

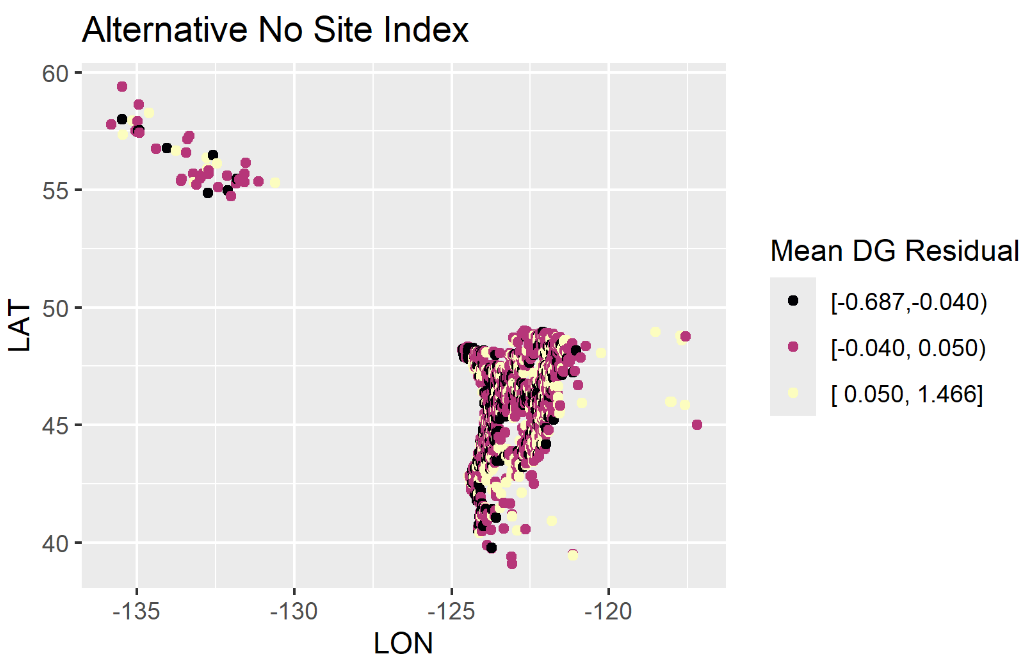

Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
40
45
50
55
60
latitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-135
-130
-125
-120
longitude


Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * cos(aspect)



Alternative No Site Index (outliers removed)
Diameter Growth Residual (in/yr) (Pred-Act)
0.50
0.25
0.00
-0.25
-0.50
-1.0
-0.5
0.0
0.5
1.0
slope * sin(aspect)



Alternative No Site Index
LAT
60
55
50
45
40
-135
-130
-125
-120
LON
Mean DG Residual
[-0.687,-0.040)
[-0.040, 0.050)
[0.050, 1.466]

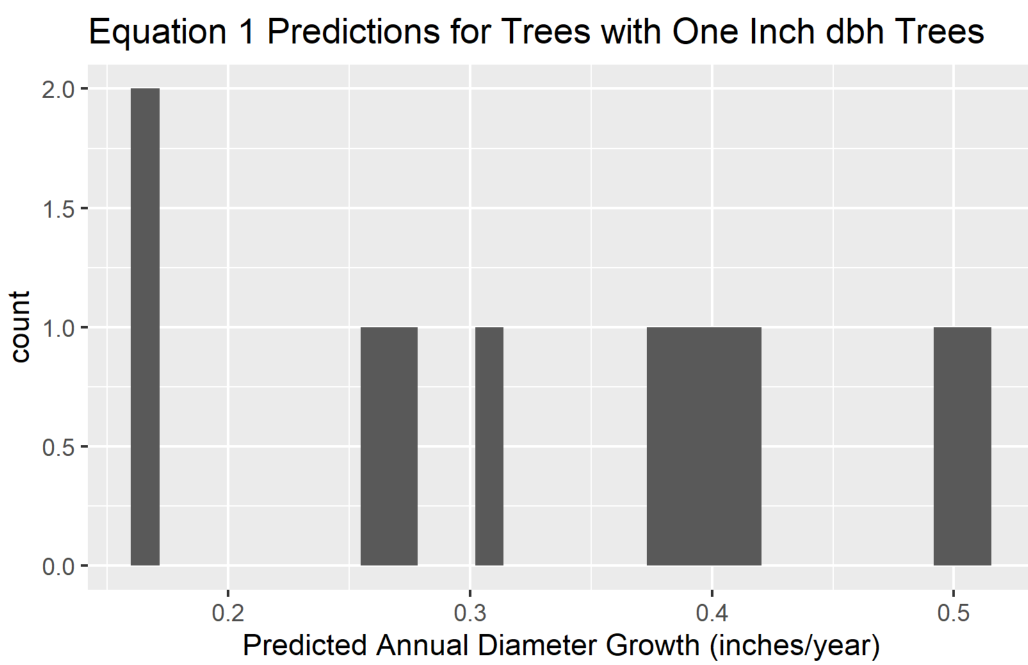
Table 3: Independent Variables for One Inch dbh Trees

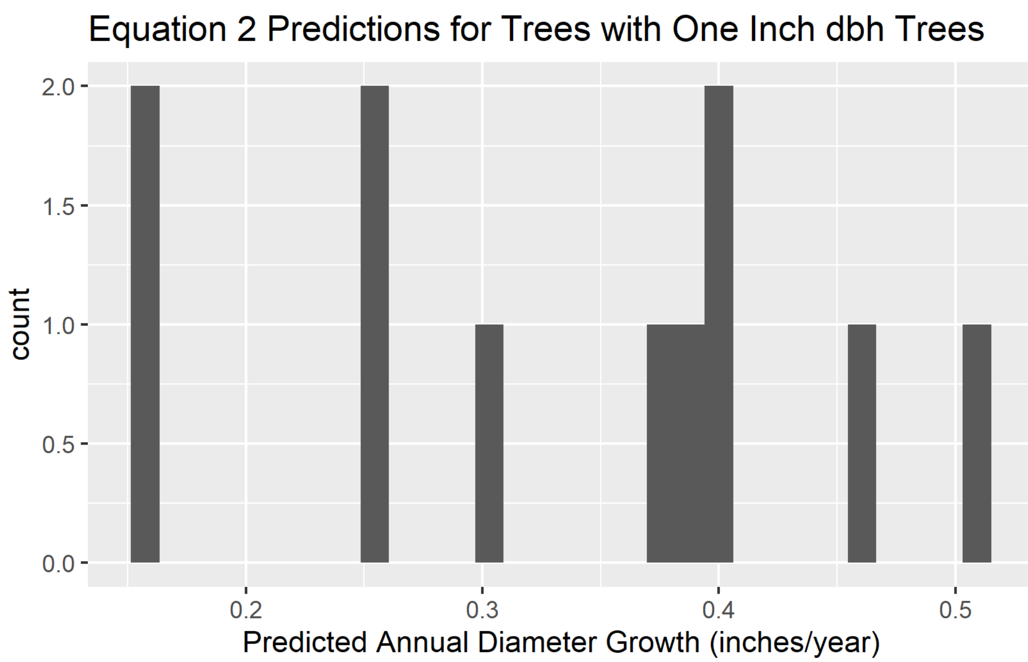
Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
ba	11	101	74	15	57	109	238
bal	11	100	74	15	57	108	237
ht	11	13	2.2	9	12	14	17
cr	11	0.49	0.26	0.2	0.3	0.6	0.99
si	11	123	18	97	114	134	151

Discussion

Removing `si` degrades the fit marginally. There seems to be an issue with red alder with small `dbh` values that will need to be investigated. the effect of this over-prediction is evident in the behavior of small trees shown in the next section. It is entirely possible that the limited data available in small trees is affecting the equation's performance.

Equation Behavior for Very Small Trees




Equation 2 Predictions for Trees with One Inch dbh Trees
2.0
1.5
1.0
0.5
0.0
count
0.2
0.3
0.4
0.5
Predicted Annual Diameter Growth (inches/year)