GUIDE TO EXAM 2

MATH 4101 — FALL 2025

1. Basic Info

- Exam 2 is October 23, 2025, in class during the section you are enrolled.
- Please if possible bring a phone to scan and upload your exam into gradescope. This saves me a lot of time! If a handful of people cannot do this it is not a big deal.
- The exam covers material from Sections 2.3, 3.1, 3.2, 3.3 of our textbook as well as associated homework assignments.

2. Format

The exam will consist of 2 definition/theorem statements (from the list below), two proof problems from section 3 below, and two additional proof problems.

The definitions/theorems will come from this list:

(1) Cauchy sequence of real numbers.
(2) Limit of a sequence of real numbers.
(3) Completeness of the real numbers.
(4) $\lim_{j\to\infty} x_j = \infty$
(5) Upper/lower bound for a set
(6) Least upper bound/greatest lower bound of a set
(7) Supremum/infimum
(8) Monotone sequence
(9) Limit point of a sequence
(10) Subsequence of a sequence
(11) $\limsup$ and $\liminf$
(12) Open set
(13) Neighborhood of a point
(14) Limit point of a set

(15) Closed set
(16) Interior of a set
(17) Closure of a set
(18) Dense subset of another set
(19) Compact set

Consult the Chapter 2, 3 summaries in the book for authoritative statements of these definitions/theorems. You can also state things the way I have stated them in class.

Proofs will be inspired by homework problems, recommended problems, and proofs from class. You may also be asked to digest a new definition and prove a fact/theorem about it (this tests general fluency as a result of studying, practicing, asking questions, thinking critically, etc.). Unless otherwise stated[1], proofs should be justified using standard results about natural/rational numbers, named or easily nameable theorems we have covered, as well as the some results which will be provided.

Examples of "easily nameable theorems" are:

(1) Convergent sequences are Cauchy
(2) The reals form a field
(3) (trichotomy) Every real is either positive, negative, or zero.
(4) The sums and products of positive reals are positive.
(5) Axiom of Archimedes
(6) Bounded monotonic increasing sequences converge to the supremum of the sequence.
(7) Limits preserve addition, multiplication, and division (with non-zero denominator).
(8) Limits preserve non-strict inequalities.
(9) Every positive real has a unique real positive square root.
(10) x is a limit point of a sequence if and only if a subsequence converges to x
(11) A bounded sequence converges if and only if its limsup and liminf are equal.
(12) Open sets are preserved under arbitrary union and finite intersection.
(13) A set is closed if and only if its complement is open.
(14) Closed sets are preserved under arbitrary intersection and finite union.
(15) The closure of a set is closed.
(16) A set is compact if and only if it is closed and bounded.

[1] You may be asked to prove something straight from a definition.

(17) A set is compact if and only if every open cover has a finite subcover.

(18) A nested sequence of nonempty compact sets has nonempty intersection.

You should not justify something with a homework problem.[2]

3. Specific proofs

In order to make studying more straightforward, two of the following problems will appear verbatim on the exam.

(1) Show that the following *finite intersection property* for a set A is equivalent to compactness.

> (Finite intersection property) If $\mathcal{B} = (B_\alpha)_{\alpha \in I}$ is any collection of closed sets such that the intersection of any finite number of them contains a point of A, then the intersection of all of them contains a point of A.

(2) Given nonempty sets $A, B \subset \mathbb{R}$ define $A + B = \{a + b : a \in A, b \in B\}$. Prove that if A and B are compact then $A + B$ is compact.

(3) If A is compact, prove that $\sup A$ belongs to A.

(4) Prove that if a set $A \subset \mathbb{R}$ contains its supremum, then the set is not open.

There will be two additional proof problems not from this list.

[2] Example of what not to do: "By Homework 2 problem 12..."