8100 Problem Set 6.

November 17, 2021

1. Suppose there are two firms with cost functions $c_1(y) = \alpha_1 y$ and $c_2(y) = \alpha_2 y$ with $\alpha_2 > \alpha_1 > 0$. Show that, in equilibrium, firm one will produce more output in equilibrium.

2. Two firms have production function $f(x_1, x_2) = \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^{-1}$. Inverse demand is $p = \frac{400}{y_1+y_2}$.

A) What are the conditional factor demands for x_1 and x_2.

B) What is the cost function for the firms?

C) Show that this cost function is of the form $c(1)\, y^{\delta}$ where δ is the degree of homogeneity of f.

D) Under what conditions would a price-taking firm choose to produce?

Now assume $w_1 = w_2 = 1$**.**

E) What is the profit function of a firm who is not a price-taker, but instead assumes price is given by the inverse demand function?

F) What is firm i's profit maximizing choice of y_i if firm j produces y_j? That is, what are the firms' best response functions?

G) Prove there is no set of $y_1 \neq y_2$ such that y_1 is a best response to y_2 and vise-versa. That is, show there is no assymetric Nash equilibrium for this Cournot game.

H) What is the symmetric Cournot equilibrium (ignore $y_1 = y_2 = 0$)?

I) Suppose firm one can commit to an output y_1. Firm two observes this output before choosing y_2. Firm one can anticipate firm 2 will best respond to y_1. Write down firm 1's profit function only in term of y_1.

J) What is firm 1's optimal choice for y_1?