Suppose there is a market with demand $q = 1000 - 5p$. All firms in the market have the same cost function $c(q) = 10q + 100$. The firms compete in Cournot oligopoly.

A) Write down the profit function of firm i in terms of q_i (firm i's quantity) and Q_{-i} (the quantity of all other firms except i).

$$\pi(q_i) = \left(\frac{1000 - Q_{-i} - q_i}{5} \right) q_i - 10q_i - 100$$

B) Find firm i's optimal quantity in terms of Q_{-i}. That is, find i's best response function.

$$\frac{\partial \left(\left(\frac{1000 - Q_{-i} - q_i}{5} \right) q_i - 10q_i - 100 \right)}{\partial q_i} = \frac{1}{5}(-q_i - Q_{-i} + 1000) - \frac{q_i}{5} - 10$$

$$q_i = \frac{1}{2}(950 - Q_{-i})$$

C) Assume all firms produce the same quantity q^*. What is the equilibrium quantity in this market when there are N firms?

$$q^* = \frac{1}{2}(950 - (N-1)q^*)$$

$$q^* = \frac{950}{N+1}$$

D) What is the market quantity, market price, and profit of each firm when there are $N = 18$ firms?

$$Q = (18)\frac{950}{18+1} = 900$$

$$p = 20$$

$$\pi_i = 20 * 50 - 10 * 50 - 100 = 400$$

E) What is the quantity and price under a monopoly?

$$q = \frac{950}{2} = 475$$

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$$p = \frac{1000 - 475}{5} = 105$$

F) How much more consumer welfare is there under the 18 firm oligopoly competition than under monopoly? (Hint: *area under inverse demand but above price*).

$$81000 - \frac{45125}{2} = 58437.5$$