1. Let c be the cup holders and p be the horsepower of a car.

a) There are lots of things that will work here. A simple example is: $u(c,p) = c + \frac{p}{1+p}$. Note that $\frac{p}{1+p}$ is a "sigmoid" (s-shapes) function ranging from 0 to 1 over the domain $[0,\infty)$. Any similar function would work since no finite increase in the amount of power will overcome the addition of a single cup holder.

b) There are numerous ways to show this. Hopefully this argument will be instructive:

$\succsim(x)$ and $\precsim(x)$ are both unions of closed rays (lines extending from a point to infinity) and in the case of $\succsim(x)$ an additional closed line segment. While the union of an infinite number of closed sets is not nessicarily close. The union of a finite number of them is closed and this is good enough to show that $\precsim(x)$ is closed. $\succsim(x)$ on the other hand has an infinite number of these closed rays. In topology this sort of thing is referred to as the close long ray.

We can prove in a more mundane way that is is closed. In a closed set, every convergent sequence in the set converges to a point in the set. For a convergent sequence in Euclidian space, the distance between points in the sequence must eventually get arbitrarily close together (they are *Cauchy*). Since the rays are a finite distance apart, every convergent sequence in $\succsim(x)$ sets eventually has every point on a single ray. Because each of these rays is closed, the sequence thus converges to a point on that closed ray. Thus, the entire set of them is closed.

c) $\precsim(c,p)$ is open at $(c+1,p)$ since $(c+1,p-\epsilon) \in \precsim(c,p)$ for all $\epsilon > 0$ but $(c+1,p)$ is in $\succ(c,p)$.

2.

a) For every pair (unordered) of objects (there are $\frac{n(n-1)}{2}$ such pairs), there is a $\frac{1}{4}$ chance that $x \succ x'$ and $x' \succ x$. There is a $\frac{3}{4}$ chance that this is not the case. Thus, the probability is $\left(\frac{3}{4}\right)^{\frac{n(n-1)}{2}}$.

b) Now we take out the probability that neither $x \succ x'$ nor $x' \succ x$ (which occurs with probability $\frac{1}{4}$) and we have $\left(\frac{1}{2}\right)^{\frac{n(n-1)}{2}}$.

c) If $\succ$ meets these conditions it is a complete total order and ranks the objects. There are $n!$ such rankings possible. There are $2^{n(n-1)}$ total ways to create $\succ$. Thus the probability is $\frac{n!}{2^{n(n-1)}}$.

3. Since V is strictly increasing $V(U(x)) \geq V(U(x'))$ if and only if $U(x) \geq U(x')$ if and only if $x \succsim x'$ since U represents $\succsim$. Thus, V represents $\succsim$.

4. Let x, x' be in the intersection. Then x and x' are in each A_i. Because each A_i is convex $t(x) + (1-t)x' \in A_i$ for all i, $t(x) + (1-t)x'$ is in the intersection of all A_i. Thus, $\bigcap A_i$ is convex.