Intermediate Microeconomics

Greg Leo

Contents

Part I

Bundles & Budget

1 Bundles

Bundles are the fundamental object of study in microeconomics. In our models, when a consumer makes a choice, they choose a **bundle** from the set of bundles available to them (the **budget set**). Bundles can be anything or combination of things you can think of. In this course, however, bundles are *usually* going to be amounts of some things we call **goods** and very often we will just look at two goods.

Bundle: $x = (x_1, x_2)$

Example. Ice Cream Bowls (the bundles) are made of up two goods: scoops of vanilla ice cream and scoops of chocolate ice cream. x_1 is the amount of vanilla. x_2 is the amount of chocolate. $(1, 1)$ represents one scoop of each flavor, $(2, 2)$ two scoops of each flavor, and $(0.28, 100)$ a lot of chocolate (100 scoops) and a little vanilla (0.28 scoops).

Since bundles with two goods are represented by ordered pairs, we can plot bundles on and x_1, x_2 axis. An example of this is shown below.

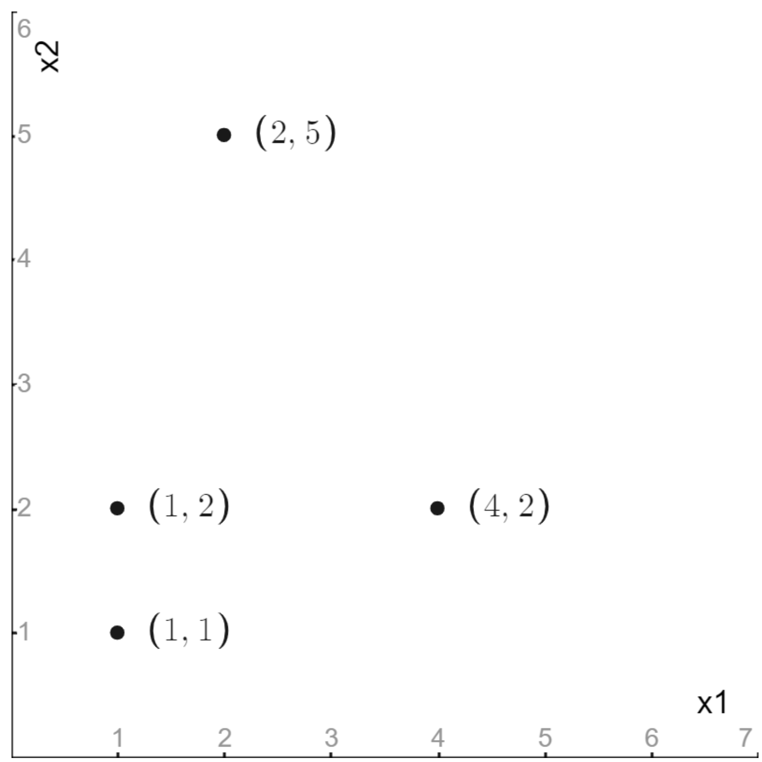

Figure 1.1: A Few Bundles on The Cartesian Plane.

2 Feasible Set

The set of all bundles relevant to a model is called the **Feasible Set**. The feasible set defines the scope of a model.

The Feasible Set: X is the "feasible" set of bundles.

Example. The feasible set for a model about choosing ice cream bowls is the set of all ordered pairs possible ice cream bowls: (x_1, x_2). Of course, it does not make sense to have a negative amount of ice cream, so in this case we might say $X = \mathbb{R}^2_+$. (This notation says that the feasible set is made up of 2 real numbers that are non-negative.)

3 Budget Set

Budget Set: B

The budget set is the set of bundles *available* to a particular consumer. The budget set must be a subset of the feasible set. In set notation we write: $B \subseteq X$

3.1 Budget Sets from Prices and Income

Not everything in the feasible set is going to be achievable for every consumer. Some bundles are affordable and others are not. The set of bundles that a consumer can *actually choose from* is called the **budget set**. Our budget sets will be constructed by assuming consumers have some income and that each good has a price.

Prices: p_1, p_2: Price units of good 1 and good 2.
Income: m.

With these, we can define the cost of a bundle:

Cost of a bundle: $p_1 x_1 + p_2 x_2$

The set of all bundles that a consumer can afford is called the **Budget Set.** We can define if formally this way:

Budget set: $B = \{x | x \in X \,\&\, x_1 p_1 + x_2 p_2 \leq m\}$ **.**[a]

[a]In "normal" language, this says the budget set is the set of bundles such that the price of the bundle is less than income.

Since we are able to plot bundles, we can also plot the budget set. To do this, it is easiest to first, we draw the **Budget Line**. This is the set of bundles that are "just affordable".

Budget Line: $x_1 p_1 + x_2 p_2 = m$

Now we can plot this on an x_1, x_2 plane. Let's put x_2 of the vertical axis. In this case, it is useful to rewrite the budget line into a form we are more familiar with:

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

This is now clearly an equation for a line with intercept $\frac{m}{p_2}$ and slope $-\frac{p_1}{p_2}$. Before we plot it, let's interpret it a little. Notice that if $x_1 = 0$ we get $x_2 = \frac{m}{p_2}$. This says "If I were only to buy x_2, I could afford $\frac{m}{p_2}$ units of x_2. Furthermore, for every unit that we increase x_1 by, x_2 goes down by $-\frac{p_1}{p_2}$. This says "If I am spending all my money, if I want to buy one more unit of x_1, I have to give up $-\frac{p_1}{p_2}$ units of x_2. This is a very important thing to know about the slope of the budget line. **The slope of the budget line represents the trade-off between x_1 and x_2 at the market prices.** We are now ready to plot the budget set. It is the budget line and all of the bundles "below" the budget line.

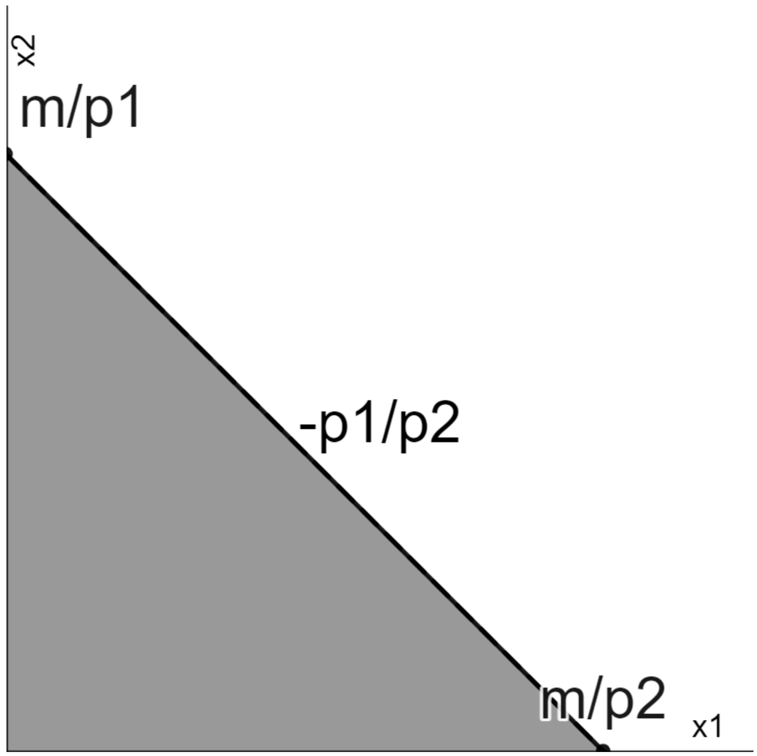

Figure 3.1: Graphical Representation of the Budget Set with slope $-\frac{p_1}{p_2}$ and intercepts $\frac{m}{p_1}$ and $\frac{m}{p_2}$.

3.2 Changing Prices and Income

We are often interested in looking at how the budget set changes when we change on of the parameters of the model: m, p_1, or p_2.

We can work out how the budget set changes by looking at changes in the budget line. There are three key elements to the budget line: the slope $-\frac{p_1}{p_2}$ and the intercepts $\frac{m}{p_1}$ and $\frac{m}{p_2}$.

When income changes, notice that only the intercepts change. If m increases, both intercepts increase. This should be intuitive. Since the intercepts represent how much of a good we can buy if we only buy that good, then if income increases, we can afford more. When income decreases, the opposite happens.

Importantly, when income changes, the slope of the budget line does not change. This is because the trade-off between the goods stays the same regardless of income (as long as the price remain the same).

When a price changes on the other hand, the slop of the budget line changes and **one** of the intercepts changes. For instance, if p_1 goes up, the slope of the budget line becomes steeper (because more x_2 has to be given up to get an extra unit of x_1). Furthermore, the x_1 intercept decreases because less x_1 can be afforded if we only buy x_1.

Some of the possible changes are demonstrated in the graphs below.

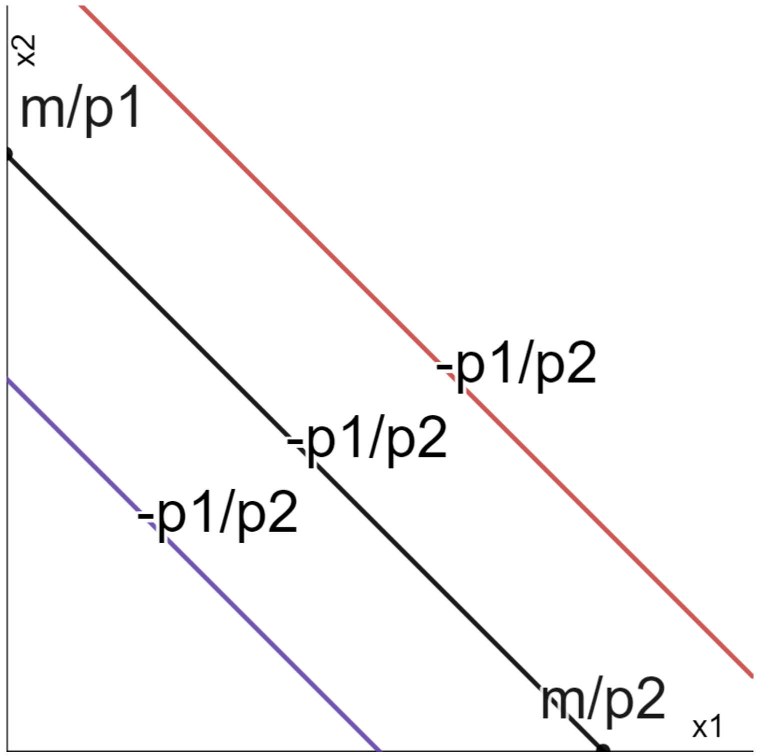

Figure 3.2: How Budget Changes with Income.

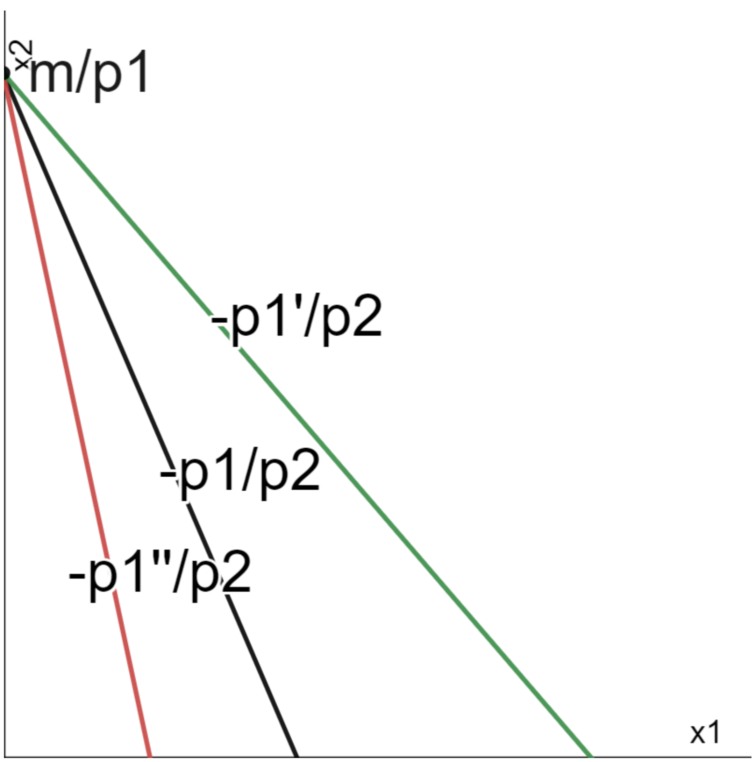

Figure 3.3: How Budget Changes with change in p_1.

In summary:

m **changes:**

Both endpoints change. If m increases, $\frac{m}{p_1}$ (the amount I can buy of good 1 changes) increases and $\frac{m}{p_2}$ (maximum affordable x_2) increases. The slope does not change. If m decreases, the opposite happens.

p_1 **changes:**

p_1. If p_1 goes up, the slope decreases (more negative). If p_1 goes down, the slope increases. The x_2 intercept stays the same.

p_2 **changes:**

p_2. If p_2 goes up, the slope increases and the x_2 intercept decreases. If p_2 goes down the slope decreases (becomes more negative) and the x_2 intercept increases. The x_1 intercept stays the same.

3.3 Taxes

Taxes represent a certain kind of price change. There are two kinds of taxes that are used frequently: *qunatity* and *ad valorem* taxes.

A **quantity tax** is determined by **number of units** (x_i) purchased where an **ad valorem** tax is determined by the **value** of the good purchased ($x_i p_i$).

With a quantity tax of t dollars on good i, the amount paid in tax is tx_i. With an ad valorem tax of percentage τ on good i, the amount pain in tax is $\tau(p_i x_i)$. The key difference is that as price of a good changes, the amount collected by the government does not change with a quantity tax (assuming the amount purchased does not change), but it does with an ad valorem tax. Most sales taxes are ad valorem. However, there are quantity taxes we encounter frequently. Pay close attention next time you are pumping gas, there is usually a sticker showing how much you pay in tax *per gallon*. That's a quantity tax.

Here's what happens to the budget line when we ad a quantity tax and ad valorem tax on good 1.

Quantity tax on good 1:

$$p_1 x_1 + t x_1 + p_2 x_2 = m$$

$$(p_1 + t)\, x_1 + p_2 x_2 = m$$

Ad valorem Tax on good 1:

$$(p_1 x_1) + \tau\,(p_1 x_1) + p_2 x_2 = m$$

$$[(1+\tau)\, p_1] x_1 + p_2 x_2 = m$$

Notice that in both cases, the tax effectively just increases the price of the good. This makes taxes easy to plot, they have the same effect as a price increase. However, there are some complex scenarios you should think about. What if a quantity tax only kicked in after buying a certain amount of some good? What if instead of a tax, a subsidy (a decrease in price) was put on a good? What if that subsidy only held for the first k units of the good? We will talk about many of these scenarios in class and work with them in practice problems.

Part II

Preferences

4 The Preference Relation $\succsim$

4.1 Definitions

Now that we know how to model what a consumer can have, we should talk about what the prefer. We represent preferences with a mathematical tool called a **relation**.

Preference Relation
The preference relation denoted $\succsim$ is a set of statements about **pairs** of bundles. The statement *"bundle x is preferred to bundle x'"* is shortened to:

$$x \succsim x'$$

Example: Ice Cream
Suppose a consumer eats bowls of ice cream. The bundles (bowls) are written with the vanilla scoops first and chocolate second. For example: $(2, 0)$ is two scoops of vanilla and zero of chocolate.
A consumer who likes vanilla more than chocolate might have these preferences:

$$(1, 0) \succsim (0, 1)\,, (2, 0) \succsim (0, 2)$$

A consumer who like more ice cream to less might have these preferences:

$$(2, 0) \succsim (1, 0)\,, (2, 2) \succsim (1, 1)$$

A consumer who gets sick of ice cream: (does anyone want to eat 100 scoops of ice cream?)

$$(1, 0) \succsim (100, 0)$$

A consumer who does not care about flavor might have:

$$(1, 0) \succsim (0, 1)\,, (0, 1) \succsim (1, 0)$$

In the case of the consumer who does not care about flavor above, notice that we have both $(1, 0) \succsim (0, 1)$ and $(0, 1) \succsim (1, 0)$. That is, a scoop of vanilla is just as good as a scoop of chocolate and a scoop of chocolate is just as good as a scoop of vanilla. When this is the case, we say the consumer is **indifferent.**

Indifference Relation: $\sim$
When $x \succsim y$ and $y \succsim x$ we say "x is indifferent to y" and write $x \sim y$.

When a consumer is not indifferent, we say they have strict preference for some bundle.

Strict Preference Relation: $\succ$
When $x \succsim y$ and **not** $y \succsim x$ we say "x is strictly preferred to y" and write $x \succ y$.

4.2 Assumptions on $\succsim$

In economics, we like to make as few assumptions about consumer's preferences as we can. There's a surprising amount we can say about consumer choice with just a few assumptions about the structure of preferences.

The first three assumptions or **axioms** we will look at ensure that for any budget set, consumers will have some favorite or set of favorite bundles. That is, given any set of bundles, they will actually be able to choose *something*. We will talk more about why these assumptions assure that fact in class.

Axiom 1. Reflexive.
For all bundles. The bundle is at least as good as itself.
In set notation:

$$\forall x \in X : x \succsim x$$

This is what we call a *technical* assumption. It does not carry a lot of content for us to talk about, but it helps ensure some minimal structure. After all, if a bundle was not "as least as good as itself", we'd have some trouble since that would imply that either it cannot be compared to itself or that it is both strictly better than itself and at the same time strictly worse than itself.

Axiom 2. Complete.
For every pair of distinct bundles. Either one is at least as good as the other or the consumer is indifferent.
In set notation:

$$\forall x, y \in X \& x \neq y : x \succsim y \, or \, y \succsim x \, or \, both$$

This axiom is a little more interesting. It says that for every pair of bundles, the consumer has *some* preference. The consumer can say "I'm indifferent." but not "I don't know". That is, everything is comparable.

Axiom 3. Transitivity.
If x is at least as good as y and y is at least as good as z then x is at least as good as z.

$$x \succsim y, y \succsim z \,\textbf{implies}\, x \succsim z$$

Transitivity lets us chain together preferences. It is really the **key** and most powerful assumption here. Transitivity ensures (along with the other assumptions) implies we can always put a set of objects into a **ranking** (possible with ties). Once we have a ranking, there's always going to be some things that are at the top of that ranking. Those are the things our consumers will choose.