Econ 8100 - Midterm Exam

October 14, 2020

1. A consumer faces prices $p_1 = p_2 = p_3 = 1$. This consumer must consume non-negative amounts of each good and has the utility function:

$$U(x_1, x_2, x_3) = (x_1 - 1)(x_2)(x_3 + 1)$$

A) Prove this consumer's preferences do not meet the following property:

Homotheticity:
$$x \sim x' \Rightarrow tx \sim tx' \, \forall t > 0$$

For parts B-E, assume m > 1 and make sure your answers account for different conditions relating to m.

- B) What are the optimal choices of x_1, x_2, x_3 ?
- C) Write down an expression for the value of the multiplier on the budget constraint in the consumer's Lagrangian function in this problem. Interpret this multiplier and the expression.
- D) Write down the indirect utility function.
- E) What is the elasticity of demand with respect to income for each good? Interpret the values. Under what conditions are they greater than, equal to, or less than one?
- F) What happens in this problem if m < 1?
- **2.** Discuss, in a few paragraphs how the consumer from the previous problem behaves in relation to a consumer with utility function $U(x_1, x_2, x_3) = x_1 x_2 x_3$.

3. A consumer with locally non-satiated and strictly convex preferences has the following expenditure function:

$$e(p_1, p_2, u) = u(p_1^{\alpha} + p_2^{\alpha})^{\frac{1}{\alpha}}$$

- A) Find the consumer's Indirect Utility function.
- B) Find the consumer's Marshallian Demands.
- C) Show that the above expenditure function with $\alpha=2$ is not a valid expenditure function because it violates some property of true expenditure functions.
- D) What does this violation have to do with the substitution effect?