Raw Class Notes for 3012- Spring 2022

These notes are unedited versions of the notes we typed in class. For more polished notes, please see "Class Notes" on my webpage.

Contents

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X is the **feasible set**.

This a set of bundles. x.

We want the feasible set to be all the relevant bundles for a model.

For example, if we are modeling the choice of ice cream bowls. The bowl: “one scoop of vanilla and one of chocolate” is a single bundles in the set of feasible bundles.

A bundle will normally (in this class) consist of two goods.

For example the two goods might be chocolate ice cream and vanilla ice cream.

a bundle is now an amount of each good.

Let’s say good 1 is chocolate and good 2 is vanilla.

$x = (1, 1)$ is the bundle “one scoop of chocolate and one scoop of vanilla.

$(0, 2)$ is two scoops of vanilla.

$(5, 1.8)$ five scoops of chocolate and 1.8 scoops of vanilla.

We could add strawberry to the model. now we have 3 goods.

$(1, 1, 1)$ one scoop of each flavor.

Two goods is “enough” to learn about trade-offs so we work with 2.

Let’s go back to chocolate and vanilla ice cream.

Let’s define the feasible set for this model. We want the feasible set to be all bowls of ice cream with a positive (non-negative) real number of scoops of each flavor.

$$X = \mathbb{R}^2_+$$

$(0, 2), (1000, 5), (1000000, 29)$ all in the feasible set.

$(-1, 2)$ is not.

$$x = (1, 1) = (x_1, x_2)$$

x_1 is the amount of good 1. and x_2 is the amount of good 2.

Budget set is the set of bundles actually available to a particular consumer. B is the **budget set**.

Budget set might be all the bowls of ice cream with no more than two total scoops. The budget set is always a subset of the feasible set.

$$B \subseteq X$$

Let’s write formally the set of all bowls of ice cream with no more than two total scoops.

$$B = \left\{x | x \in \mathbb{R}^2_+ \,\&\, x_1 + x_2 \leq 2\right\}$$

$$B = \left\{x | x \in \mathbb{R}^2 \, \& x_1 \geq 0 \& x_2 \geq 0 \& \, x_1 + x_2 \leq 2\right\}$$

We could a weird budget set:

$$B = \left\{x | x \in \mathbb{R}^2_+ \& \left(\sqrt{x_1^2 + x_2^2} \leq 1\right)\right\}$$

This is the the set of all bundles less than distance one from the origin. It is a circle. This is technically possible in our framework.

Normally we think of budgets as coming from income m and prices p_1 and p_2. *Competitive budgets.* The price of any bundle is:

$$p_1 x_1 + p_2 x_2$$

scoops of ice cream cost \$2 each $p_1 = 2$ and $p_2 = 2$. The cost of the bundle $(2, 1)$ is:

$$(2 * 2) + (2 * 1) = 6$$

Suppose I have m dollars. What can I afford.

$$p_1 x_1 + p_2 x_2 \leq m$$

This is the formal version of a *competitive budget*.

$$B = \left\{x | x \in \mathbb{R}^2_+ \& p_1 x_1 + p_2 x_2 \leq m\right\}$$

This is the set of all bundles that someone can afford with income m at prices p_1 and p_2.

The **budget line** are all of the bundles that cost exactly m.

$$p_1 x_1 + p_2 x_2 = m$$

We can transform this by isolating x_2.

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

We get the x_2 intercept immediately. $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$. The other intercept can be found by plugging 0 in for x_2. The x_1 intercept is $\frac{m}{p_1}$.

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Budget set is described by:

$$(x_1p_1) + (x_2p_2) \leq m$$

"Spends no more than income"

The important part of this budget is the **Budget Line.**

$$x_1p_1 + x_2p_2 = m$$

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

x_2 intercept is $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$.

We can also get the x_1 intercept by plugging 0 in for x_2.

$$0 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

$$x_1 = \frac{m}{p_1}$$

x_1 intercept is $\frac{m}{p_1}$,x_2 intercept is $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$.

Interpreting the intercepts:

The x_1 intercept is "how much x_1 can I have if I only buy x_1"

The x_2 intercept is "how much x_2 can I have if I only buy x_2"

These expressions should make sense. $\frac{m}{p_1}$ says "if i spend m on x_1 how many units do I get"

Let's look at another bundle with a similar expression:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

The slope measures how much good 2 I give up to get one more unit of good 1.

This represents the tradeoff I have to make between the goods given their prices.

$$-\frac{p_1}{p_2}$$

What happens to the budget when one of the parameters of the model p_1, p_2, m changes.

Taxes

Quantity tax:

An amount of money you owe to the government **per unit** of some good you buy. Quantity tax of t on good 1.

$$x_1 t + x_1 p_1 + x_2 p_2 = m$$

$$(p_1 + t)\, x_1 + x_2 p_2 = m$$

Ad valorem tax:

A tax on the value of a good purchased:

You owe the government τ times the value of the x_1 you purchase. In the case of Nashville, $\tau \approx 0.09$

$$\tau\,(p_1 x_1) + p_1 x_1 + p_2 x_2 = m$$

For example, if $p_1 = 10$ and $x_1 = 10$ I've spent \$100. If $\tau = 0.09$ then the tax is 9% and I owe \$9 in tax. The total cost of x_1 becomes \$109.

$$((1+\tau)\, p_1)\, x_1 + p_2 x_2 = m$$

The x_1 intercept when the price of x_1 is p_1 if the amount purchased is less than $\bar{x_1}$ and $p_1 + t$ for any units purchased above $\bar{x_1}$.

Let's calculate the cost of buying $\bar{x_1}$ at price p_1:

$$\bar{x_1} p_1$$

The money I have left over is:

$$m - \bar{x_1} p_1$$

The extra x_1 I can buy with this leftover money at the new price of $p_1 + t$ is:

$$\frac{m - \bar{x_1} p_1}{p_1 + t}$$

The total amount I can afford it $\bar{x_1}$ plus this amount:

$$\bar{x}_1 + \frac{m - \bar{x_1} p_1}{p_1 + t}$$

Preferences:

Now we will try to model "what a consumer wants"

To represent preferences, we use a "relation". A relation is a set of statements about **pairs** of bundles. You are familiar with some relations already like $\geq$ (greater than or equal to).

$$3 \geq 2$$

$$4 \geq 1$$

A another relation on the set of people might be "Is a sibling of". Let's represent this by s. The following statements are true:

$$Greg\, s\, Christina$$

$$Finn\, s\, Remy$$

In economics, we represent preferences as a relation called the "Preference Relation" $\succsim$.

Suppose a consumer doesn't care about flavor, but just likes more ice cream. We represent bundles such as $(1, 1)$ is one scoop of vanilla and one scoop of chocolate. The following formal preference statements are true about this consumer:

$$(1, 1) \succsim (0, 0)$$

$$(2, 1) \succsim (0, 2)$$

$$(0, 2) \succsim (2, 0)$$

The term "preferred" is synonymous to "weakly preferred" and will be true if a consumer either strictly prefers the first bundle to the second (as in the first two cases above) or if they are indifferent such as in the third above.

Contrast this to the term "strictly preferred". The strictly preferred relation is represented by $\succ$.

$$(1, 1) \succ (0, 0)$$

$$(2, 1) \succ (0, 2)$$

but the following statement is **not true** about the consumer:

$$(0, 2) \succ (2, 0)$$

However, the consumer is indifferent between these bundles. The indifference relation is represented by $\sim$. THe following is true.

$$(0, 2) \sim (2, 0)$$

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Preferences

$$\succsim$$

This is the Weak Preference Relation.

Consumer just wants more ice cream: (Scoops of chocolate, scoops of vanilla).

$$(2,0) \succsim (1,0)$$

From the weak preference relation we can infer strict preference and indifference as well.

We will say $x \succ y$ if and only if $x \succsim y$ and not $y \succsim x$

We will say $x \sim y$ if and only if $x \succsim y$ and $y \succsim x$

For instance:

$$(2,0) \succ (1,0)$$

Since $(2,0) \succsim (1,0)$ but not $(1,0) \succsim (2,0)$.

$$(1,0) \sim (0,1)$$

Since $(1,0) \succsim (0,1)$ and $(0,1) \succsim (1,0)$

Example:

Suppose a consumer has preference over the bundles x, y, z:

$x \succsim y$, $y \succsim z$, $z \succsim y$, $x \succsim z$, $x \succsim x$, $y \succsim y$, $z \succsim z$

What is true about their strict preferences?

$x \succ y, x \succ z$

What is true about their indifference relation?

$y \sim z, z \sim y$

The indifference relation is *symmetric*: whenever $x \sim y$ we also have $y \sim x$.

On the other hand, the strict preference relation is *asymmetric*: if we have $x \succ y$ we don't have $y \succ x$.

The fundamental thing we will work with is the weak preference relation.

Three key assumptions we make about $\succsim$.

1. Reflexive: "every bundle is at least as good as itself"

$$\forall x \in X : x \succsim x$$

For all bundles in the feasible set.

2. Completeness: "Every pair of bundles is comparable"

$$\forall x, y \in X \,\&\, x \neq y :\, (x \succsim y) \vee (y \succsim x) \;\textit{or both}$$

Either one bundle is strictly better than the other or they are indifferent, but they can't say "I don't know". This limits the scope of our models. We should only include bundles that are relevant to a consumers. They should be able to form preferences.

3. Transitivity: "For every three bundles x, y, z, if x is at least as good as y and y is at least as good as z, it must be that x is at least as good as z"

$$\forall x, y, z \in X : x \succsim y \& y \succsim z \implies x \succsim z$$

It is possible to have intransitive preferences. Here is an example:

Choosing mates on a dating app:

Three potential mates:

x. Rich, Very Intelligent, Average Looking

y. Financially Constrained, Genius, Good Looking

z. Moderately Wealthy, Average Intelligence, Best Looking

$y \succ x$ since y is smarter and better looking

$z \succ y$ since z is wealthier and better looking

$x \succ z$ since x is wealthier and smarter than z

These are the only things true about $\succsim$ in this case:

$$y \succsim x, z \succsim y, x \succsim z$$

We have $z \succsim y$ and $y \succsim x$. If this was transitive this should imply $z \succsim x$. This isn't true because x **is strictly better than** z. **These preferences are intransitive.**

While this assumption can fail, it will be reasonable for most of what we do, and it is much easier to "do" economics if preferences are transitive since it will ensure the consumer can make a choice from any budget set.

We construct something called a "choice function". The choice function takes a budget set, and returns a bundle or bundles from the budget set such that that bundle or those bundles are better than everything else in the budget set. The bundle chosen meets this condition: x is "chosen" from budget set B if:

$$\forall x' \in B, x \succsim x'$$

That is, the chosen bundle is at least as good as everything else. If this is not true of the chosen bundle, there is something strictly better. From the set of three mates in the previous example, is there one mate that is at least as good as every other one? No! There is not mate meeting this condition. The choice set is empty.

Transitivity (and completeness) ensures there will always be a "best" or set of "best" bundles. It lets us put things in order. Let's look at our example from the beginning of lecture.

$x \succsim y, y \succsim z, z \succsim y, x \succsim z$,$x \succsim x, y \succsim y, z \succsim z$

This is: Reflexive, Complete, Transitive. Let's rank order the objects. We will put one above the other if is strictly preferred, and put them at the same ranking if they are indifferent. From this weak preference relation we have already extract the strict and weak preferences:

$x \succ y, x \succ z$,$y \sim z, z \sim y$. Here's the ranking:

$$1. x$$

$$2. y, z$$

x is first and y and z are second.

Now let's look at the intransitive relation from the dating app example:

With the intransitive relation $x \succ y, y \succ z, z \succ x$

These are cyclic and that means there is no best. We cannot put them in a rank order.

In order to visualize preference, we use "indifference set" "**indifference curves**". An indifference curve is a set of bundles that are all indifferent to eachother.

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Two distinct indifference curves cannot intersect each other if preferences are complete and transitive.

Since they are distinct, we can find x and some x' such that

$$x \succ x'$$

Because they are on distinct indifference curves.

However, since they intersect there is a bundle $\tilde{x}$ on both curves. Thus, $\tilde{x}$ is indifference to both x and x'

$$\tilde{x} \sim x$$

$$\tilde{x} \sim x'$$

By indifference $x' \succsim \tilde{x}$ and $\tilde{x} \succsim x$.
By transitivity we must have:

$$x' \succsim x$$

However by our original assumption:

$$x \succ x'$$

However, these are contraditory since $x \succ x'$ implies that **not** $x' \succsim x$**.**
We have two statements that contradictory:
$x' \succsim x$ and not $x' \succsim x$.

Perfect Substitutes:

Linear indifference curves. Willingness to trade-off between the goods at **any point** is exactly the same.

Perfect Complements:

Always consume the goods in some fixed proportion. *Left/Right Shoes or Baking Pies.* The indifference curves are L-shaped and the kink-points follow a "ray" through the origin of some slope that represents the ratio the goods are consumed in.

Cobb Douglass:

Convex shaped indifference curves. The more of one good you have, you are willing to give up relatively more to get some of the other good.

Bads:

This occurs when you want less of one of the two (or both) of the goods.

With one bad and one good, the indifference curves slope upwards.

With two bads, the indifference curves again slope downward, but prefrence increases as we move towards the origin.

Well-Behaved Preferences

Monotonicity- "everything is a good"

Weakly Monotonic (Monotonic):

$$(x_1, x_2), \left(x_1^{'}, x_2^{'}\right)$$

If $x_1 \geq x_1'$ and $x_2 \geq x_2^{'}$ then $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Furthermore if $x_1 > x_1^{'}$ **and** $x_2 > x_2^{'}$ **then** $(x_1, x_2) \succ \left(x_1^{'}, x_2^{'}\right)$

For instance $(2, 3) \sim (3, 3)$ and $(4, 4) \succ (3, 3)$.

Strictly Monotonic

If $x_1 \geq x_1'$ and $x_2 \geq x_2^{'}$ then $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Furthermore if $x_1 > x_1^{'}$ **or** $x_2 > x_2^{'}$ **then** $(x_1, x_2) \succ \left(x_1^{'}, x_2^{'}\right)$

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Monotonicity:

"More is better".

Weakly monotonic (monotonic):

If I have at least as much of everything, I am at least as well off and if I have strictly more of everything I am strictly better off.

Strictly monotonic:

If I have at least as much of everything, I am at least as well off and if I have strictly more of *anything* I am strictly better off.

Under both assumptions we know:

$$(2, 2) \succ (1, 1)$$

Under both assumptions we know:

$$(2, 1) \succsim (1, 1)$$

We only know this for sure under **strictly monotonic preferences**:

$$(2, 1) \succ (1, 1)$$

If preferences are only weakly monotonic it is possible:

$$(2, 1) \sim (1, 1)$$

Perfect complements are an example of a weakly but not strictly monotonic preference.

Perfect substitutes are an example of strictly monotonic preferences.

The indifference curves of monotonic preferences are never upward sloping, but if they are only weakly monotonic they may have points of zero or undefined slope.

Convexity:

Intermediate bundles are better than extreme bundles.

Pick two points that are indifferent to eachother.

$$(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$$

Take a convex combination (mixture) of the two bundles. Pick a t between 0 and 1.

$$\left(tx_1 + (1-t)\,x_1^{'}, tx_2 + (1-t)\,x_2^{'}\right)$$

If we pick $t = 0$ we get

$$\left(0x_1 + (1)\,x_1^{'}, 0x_2 + (1)\,x_2^{'}\right) = \left(x_1^{'}, x_2^{'}\right)$$

Pick $t = 1$ we get:

$$(x_1, x_2)$$

If we pick $t = \frac{1}{2}$

$$\left(\frac{1}{2}x_1 + \frac{1}{2}x_1^{'}, \frac{1}{2}x_2 + \frac{1}{2}x_2^{'}\right)$$

This is half-way between the two on a straight line between them.

For t from 0 to 1 we all the bundles on a stright line between them.

Weakly Convex preferences (Convex Prefrences). For any two indifference bundles $(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$ and for all $t \in [0, 1]$

$$\left(tx_1 + (1-t)\,x_1^{'}, tx_2 + (1-t)\,x_2^{'}\right) \succsim (x_1, x_2)\, or \left(x_1^{'}, x_2^{'}\right)$$

Strictly Convex preferences (Convex Prefrences). For any two indifference bundles $(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$ and for all $t \in (0, 1)$

$$\left(tx_1 + (1-t)\,x_1^{'}, tx_2 + (1-t)\,x_2^{'}\right) \succ (x_1, x_2)\, or \left(x_1^{'}, x_2^{'}\right)$$

Strictly convex preferences have indifference curves that bend outward strictly. Weakly convex preference may have flat spots.

Cobb Douglass is strictly convex. Perfect substitutes are weakly convex. Perfect complements are only weakly convex because if we choose two bundles on the same "leg" of the indifference curve, a line between them lies on the indifference curve.

If preferences are complete, transitive, and monotonic, an optimal bundle must occur at a point on the budget line where the indifference through that point "just" touches the budget line.

Now we know where to look for optimal bundles.

How do we represent preferences in a way that will allow us to find the slopes of the indifference curves to find optimal bundles?

Utility Function.

A mathematical representation of preferences.

$u(2,2) = 20$

$u(1,1) = 10$

$u(0,1) = 5$

$u(1,0) = 5$

$u(0,0) = 0$

$u(2,2) = 4$

$u(1,1) = 2$

$u(0,1) = 1$

$u(1,0) = 1$

$u(0,0) = 0$

A utility function is a function that assigns every bundle a number with the property that better bundles get higher numbers.

The function u which maps bundles into real number is a **Utility function** if:

$u(x_1, x_2) \geq u\left(x_1^{'}, x_2^{'}\right)$ if and only if $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Many different utility functions can represent the same preferences.

Let's write some utility functions that represent our families of preferences.

Perfect substitutes:

$$u(x_1, x_2) = 2x_1 + x_2$$

We can find equations for particular indifference curves by plugging some number in for u

$$10 = 2x_1 + x_2$$

$$x_2 = 10 - 2x_1$$

We can find the slope of an indifference curve at any point by taking the ratio of the partial derivatives of the utility function.

$$MRS = -\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}}$$

$$MRS = -\frac{\frac{\partial(2x_1+x_2)}{\partial x_1}}{\frac{\partial(2x_1+x_2)}{\partial x_2}} = -2$$

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$$MRS = -\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}}$$

Slope of an indifference curve at any point.

This is interpreted as the rate a consumer is willing to trade off x_2 to get more x_1.

Example:

$u(x_1, x_2) = x_1 + x_2$. Perfect substitutes.

$$MRS = -\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}} = -1$$

Common Utility Representations.

Perfect Substitutes- $u(x_1, x_2) = ax_1 + bx_2$

$$MRS = -\frac{\frac{\partial(ax_1+bx_2)}{\partial x_1}}{\frac{\partial(ax_1+bx_2)}{\partial x_2}} \quad = \quad -\frac{a}{b}$$

Cobb Douglass- $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$

$$MRS = -\frac{\frac{\partial\left(x_1^{\alpha}x_2^{\beta}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\alpha}x_2^{\beta}\right)}{\partial x_2}} = -\frac{\alpha x_1^{\alpha-1}x_2^{\beta}}{\beta x_1^{\alpha}x_2^{\beta-1}} = -\frac{\alpha x_1^{\alpha}\frac{1}{x_1}x_2^{\beta}}{\beta x_1^{\alpha}x_2^{\beta}\frac{1}{x_2}} = -\frac{\alpha\frac{1}{x_1}}{\beta\frac{1}{x_2}} = -\frac{\alpha}{\beta}\frac{x_2}{x_1}$$

$$\alpha = \beta = 1, MRS = -\frac{x_2}{x_1}$$

$(10, 10), MRS = -\frac{10}{10} = -1.$

$(100, 10), MRS = -\frac{10}{100} = -0.1$ *not willing to give up much* x_2 *to get more* x_1.

$(10, 100), MRS = -\frac{100}{10} = -10$ *willing to give up a lot* x_2 *to get more* x_1.

Quasi-Linear- $u(x_1, x_2) = f(x_1) + x_2$

Common Example:

$ln(x_1) + x_2$

$$MRS = -\frac{\frac{\partial(ln(x_1)+x_2)}{\partial x_1}}{\frac{\partial(ln(x_1)+x_2)}{\partial x_2}} = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

How much x_2 I will give up to get more x_1 is decreasing in the amount of x_1 I have, but **does not depend on how much x_2 I have.**

Another Example:

$\sqrt{x_1} + x_2$

Perfect Complements- $u(x_1, x_2) = min\{ax_1, bx_2\}$

Left\Right Shoes. $min\{x_1, x_2\}$

$u(3,2) = min\{3,2\} = 2$

$u(2,2) = min\{2,2\} = 2$

Pies (2 apples 1 crust).

This is the wrong way to represent this: $u(x_1, x_2) = min\{2x_1, x_2\}$

$(2,1), (2,2)$ both giving me 1 pies.

$$min\{4,1\} = 1$$

$$min\{4,2\} = 2$$

This is the right way. $u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$

With 4 apples, the most pies I could make is 2. # I have divided my number I need per pie. $\frac{4}{2} = 2$.

With 2 crusts, the most pies I could make is 2.

Multiple Utility Functions can Represent the Same Preferences.

Monotonic Transformations. *If we take a strictly increasing function of any utility function, the resulting function represents the same preferences.*

Suppose we have preferences over ice cream where the flavor does not matter.

$u(x_1, x_2) = x_1 + x_2$

$\tilde{u}(x_1, x_2) = x_1 + x_2 + 10$

$\hat{u}(x_1, x_2) = 10(x_1 + x_2)$

$$(1,2) \succ (1,1)$$

$$u(1,2) = 3, u(1,1) = 2$$

$$\tilde{u}(1,2) = 13, u(1,1) = 12$$

$$\hat{u}(1,2) = 30, u(1,1) = 20$$

All of these utility functions, represent the same tradeoff. **They are monotonic transformations of each other.**

$$u'(x_1, x_2) = \log\left(14\sqrt{10(x_1 + x_2) - 1} + 8\right)$$

This represents the same preferences as $u(x_1, x_2) = x_1 + x_2$

$$MRS = -\frac{\frac{\partial\left(\log\left(14\sqrt{10(x_1+x_2)-1}+8\right)\right)}{\partial x_1}}{\frac{\partial\left(\log\left(14\sqrt{10(x_1+x_2)-1}+8\right)\right)}{\partial x_2}} \quad = \quad -1$$

Same MRS, Same Preferences.

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If a consumer if doing something optimal, they must choose a bundle that is on an indifference curve that never passes into the budget set.

Tangency. The slope of the indifference curve is equal to the slope of the budget equation.

If the slope of the indifference curve was not equal to the slope of the budget equation then either I'm willing to give up more x_2 than I have to to get more x_1 or the opposite. Either way, the choice isn't optimal.

Unless I can't give up any more x_2 (or x_1). This condition **has to be true at any optimum.**

We have two conditions to check to find optimal bundles.

1) Find all the points where $MRS = -\frac{p_1}{p_2}$

2) Find all the points on the budget equation. (spent all money)

Any point meeting both conditions is a candidate for being optimal. Often, there's only one point meeting these conditions.

Example:

$$u(x_1, x_2) = x_1 x_2$$

Prices are $p_1 = 1$ and $p_2 = 2$, and $m = 100$.

1) If a bundle is optimal and involves consuming some of both goods, it must be that:

$$MRS = -\frac{p_1}{p_2}$$

Tangency Condition:

$$-\frac{x_2}{x_1} = -\frac{1}{2}$$

$$2x_2 = x_1$$

Budget Condition:

$$1x_1 + 2x_2 = 100$$

Plug the tangency condition into the budget condition:

$$x_1 + x_1 = 100$$

Consumer demand is:

$$x^* = (50, 25)$$

Example:

$$u(x_1, x_2) = x_1 x_2$$

Prices are p_1 and p_2, and m.

$$MRS = -\frac{p_1}{p_2}$$

Tangency Condition:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$x_2 p_2 = x_1 p_1$$

$x_1 p_1$ is the amount of money spent on good 1. $x_2 p_2$ is the amount of money spent on good 2.

If the consumer is doing something optimal, they spend the same amount of money on both goods.

$$p_1 x_1 + p_2 x_2 = m$$

Plugging in the tangency condition:

$$p_2 x_2 + p_2 x_2 = m$$

$$2(p_2 x_2) = m$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Marshallian Demand:

$$x^*\left(p_1,p_2,m\right)=\left(\frac{\frac{1}{2}m}{p_1},\frac{\frac{1}{2}m}{p_2}\right)$$

In this particular case, they spend half their income $\frac{1}{2}m$ on both goods.

What if we had a difference cobb douglass utility function?

$$u\left(x_1,x_2\right)=x_1^{\alpha}x_2^{\beta}$$

$$x^*\left(p_1,p_2,m\right)=\left(\frac{\frac{\alpha}{\alpha+\beta}m}{p_1},\frac{\frac{\beta}{\alpha+\beta}m}{p_2}\right)$$

If we had:

$$u\left(x_1,x_2\right)=x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

$$x^*\left(p_1,p_2,m\right)=\left(\frac{\frac{2}{3}m}{p_1},\frac{\frac{1}{3}m}{p_2}\right)$$

Example of perfect substitutes.

$$u\left(x_1,x_2\right)=x_1+x_2$$

$$p_1=1,p_2=2,m=100$$

Our expectation is that this consumer should only buy x_1.

$$\left(\frac{m}{p_1},0\right)$$

Let's suppose we did not know this up-front and looked at the tangency condition.

$$MRS=-\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}}=-1$$

$$-1=-\frac{1}{2}$$

The tangency condition **can never be met**. Yet, the tangency condition is required to be true for an *interior* optimum. The only thing left is a boundary solution. That is: consume all x_1 or all x_2.

Check the utility of the two intercepts.

$$u\left(\frac{m}{p_1},0\right)=\frac{100}{1}=100$$

$$u\left(0,\frac{m}{p_2}\right)=\frac{100}{2}=50$$

Since I get more utility from buying only x_1, that is the optimal solution.

Let's try another one:

$$u\left(x_1,x_2\right)=2x_1+3x_2$$

$$p_1=2,p_2=4,m=100$$

$$u\left(50,0\right)=2*\left(50\right)=100$$

$$u\left(0,25\right)=3*\left(25\right)=75$$

Buy only x_1 $\left(50,0\right)$.

$$u\left(x_1,x_2\right)=2x_1+4x_2$$

$$p_1=2,p_2=4,m=100$$

$$-\frac{1}{2}=-\frac{1}{2}$$

Slope of the indifference curve is equal to the slope of the budget equation. The consume is willing to buy any bundle that costs m.

$$u\left(50,0\right)=2\left(50\right)=100$$

$$u\left(0,25\right)=4\left(25\right)=100$$

The utility of the endpoints (buying only x_1 or only x_2) is the same, and so **any bundle that costs m is optimal.**

Perfect Complements:

The only case we will run into where this procedure will fail is for perfect complements. Because the MRS is not defined.

For left and right shoes, there is not reason to buy more left shoes than right shoes, or more right shoes than left shoes. Any optimal bundle must have $x_1=x_2$.

For pies, there's no reason to buy more than 2 times the number of apples than I have crusts or more than one half the number of crusts than I have apples.

In either condition, we've wasted money on goods that don't contribute to my utility.

No Waste Condition.

Always consume at the kink of the indifference curve.

Left right shoes:

$$min\left\{x_1, x_2\right\}$$

$$x_1 = x_2$$

2 apples, 1 crust

$$min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$\frac{1}{2}x_1 = x_2$$

The no-waste condition replaces the tangency condition in solving for the optimal bundle.

Example:

$$u\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$p_1 = 1, p_2 = 2, m = 30.$

No waste condition:

$$\frac{1}{2}x_1 = x_2$$

$$x_1 = 2x_2$$

Budget condition:

$$x_1 + 2x_2 = 30$$

Plug the first into the second:

$$2x_2 + 2x_2 = 30$$

$$4x_2 = 30$$

$$x_2 = \frac{30}{4}$$

$$x_1 = \frac{30}{2}$$

$$\left(\frac{30}{2}, \frac{30}{4}\right)$$

Try this one:
Find the **marshallian demand:**

$$u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$p_1, p_2, m.$
Example Quasi-Linear

$$u(x_1, x_2) = \log(x_1) + x_2$$

p_1, p_2, m
Tangency Condition:

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

$$x_1 p_1 = p_2$$

Budget Equation:

$$x_1 p_1 + x_2 p_2 = m$$

Plug one into the other:

$$p_2 + x_2 p_2 = m$$

$$p_2(1 + x_2) = m$$

$$x_2 = \frac{m}{p_2} - 1$$

$$x_1 = \frac{p_2}{p_1}$$

Marshallian Demand:

$$x^*(p_1,p_2,m)=\left(\frac{p_2}{p_1},\frac{m}{p_2}-1\right)$$

Suppose we have $p_1=1,p_2=1,m=100$

$$(1,99)$$

There's a problem here: Suppose we have $p_1=1,p_2=200,m=100$

$$\left(200,-\frac{1}{2}\right)$$

This can't be right. This is impossible.
Instead the consumer will choose the boundary point $\left(\frac{m}{p_1},0\right)$ when the bundle the would like involves a negative amount of x_2.
There is no bundle that meets the tangency condition and is on the budget equation. Both of these conditions **must be true** at an interior optimum.
There is no interior optimum at these prices and income.
There must be a boundary solution.

$$u(100,0)=\log(100)\approx 4.60517$$

$$u\left(0,\frac{100}{200}\right)=u\left(0,\frac{1}{2}\right)=\frac{1}{2}$$

Since no interior solution can be optimal, the best boundary solution must be optimal. The best boundary solution is

$$(100,0)$$

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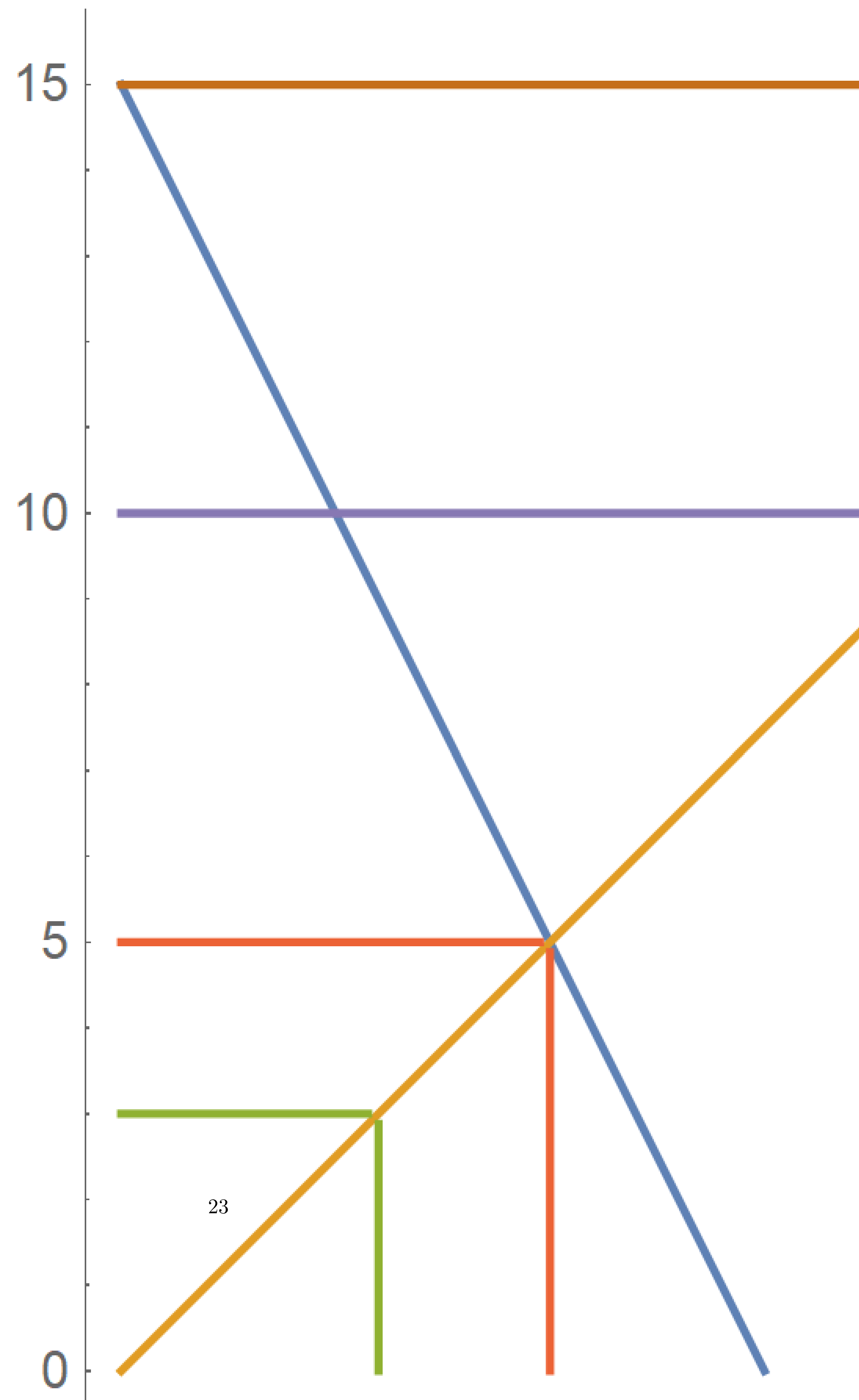
The utility and the budget are:

$$u(x_1,x_2)=min\{x_1,x_2\}$$

Budget: $p_1=2$, $p_2=1$, $m=15$

$$2x_1+1x_2=15$$

Let's plot this problem:


15
10
5
0

No waste condition:

$$x_1 = x_2$$

Budget Equation:

$$2x_1 + 1x_2 = 15$$

Let's solve these together:

$$3x_1 = 15$$

$$x_1 = 5, x_2 = 5$$

Let's do the **max** version of this problem.
The utility and the budget are:

$$u(x_1, x_2) = max\{x_1, x_2\}$$

Budget: $p_1 = 2$, $p_2 = 1$, $m = 15$

$$2x_1 + 1x_2 = 15$$

In this case, plotting indifference shows us that the optimal bundle is $(0, 15)$.

Demand
Marshallian:

$$x_1(p_1, p_2, m), x_2(p_1, p_2, m)$$

For instance for the utility function: $u = x_1x_2$ we've seen the marshallian demands are:

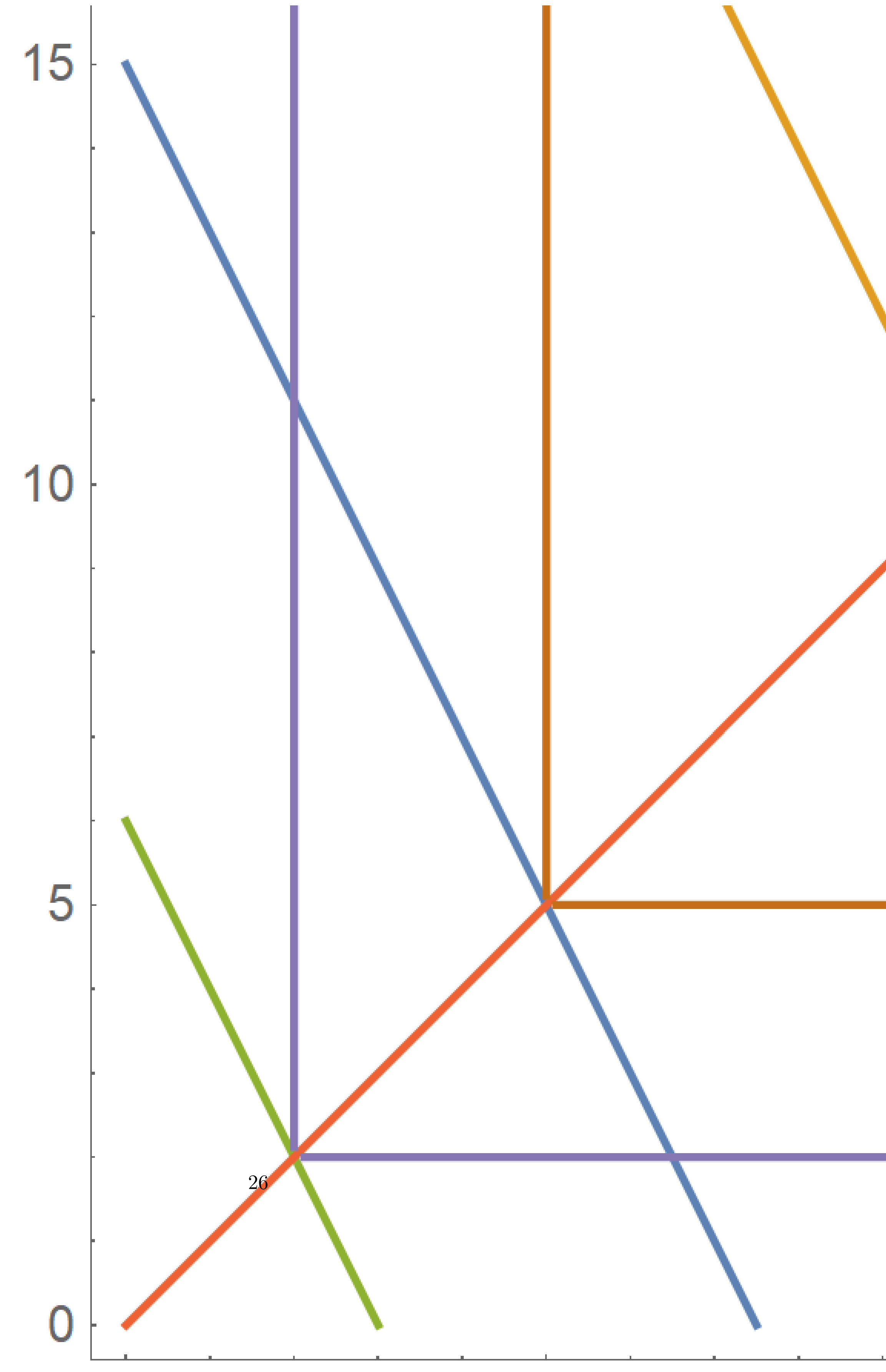
$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

How does demand for a good chance when prices or income changes.
Changes in income.

$$\frac{\frac{1}{2}m}{p_1}$$

Since income is only in the numerator of this function, as income increases, demand increases as well for any fixed prices. We can also do this formally, by taking a derivative.

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m} = \frac{1}{2p_1} > 0$$


15
10
5
0

Inferior good. *When income increases, demand decreases.*

A can't always be inferior. The reason is, to decrease, it has to have increased at some point. Furthermore, when $m = 0$ demand has to be 0.

For instance it might be normal for low levels of income and inferior for high levels of income.

Two graphs associated with the behavior of goods with respect to income.

Income Offer Curve: A plot of demand **bundles** of both goods for fixed prices as income changes.

Example:

$$u = min\{x_1, x_2\}$$

Budget:

$$2x_1 + 1x_2 = m$$

Example of Cobb Douglass Demand:

$$u = x_1 x_2$$

$$2x_1 + 1x_2 = m$$

$$\left(\frac{\frac{1}{2}m}{2}, \frac{\frac{1}{2}m}{1}\right)$$

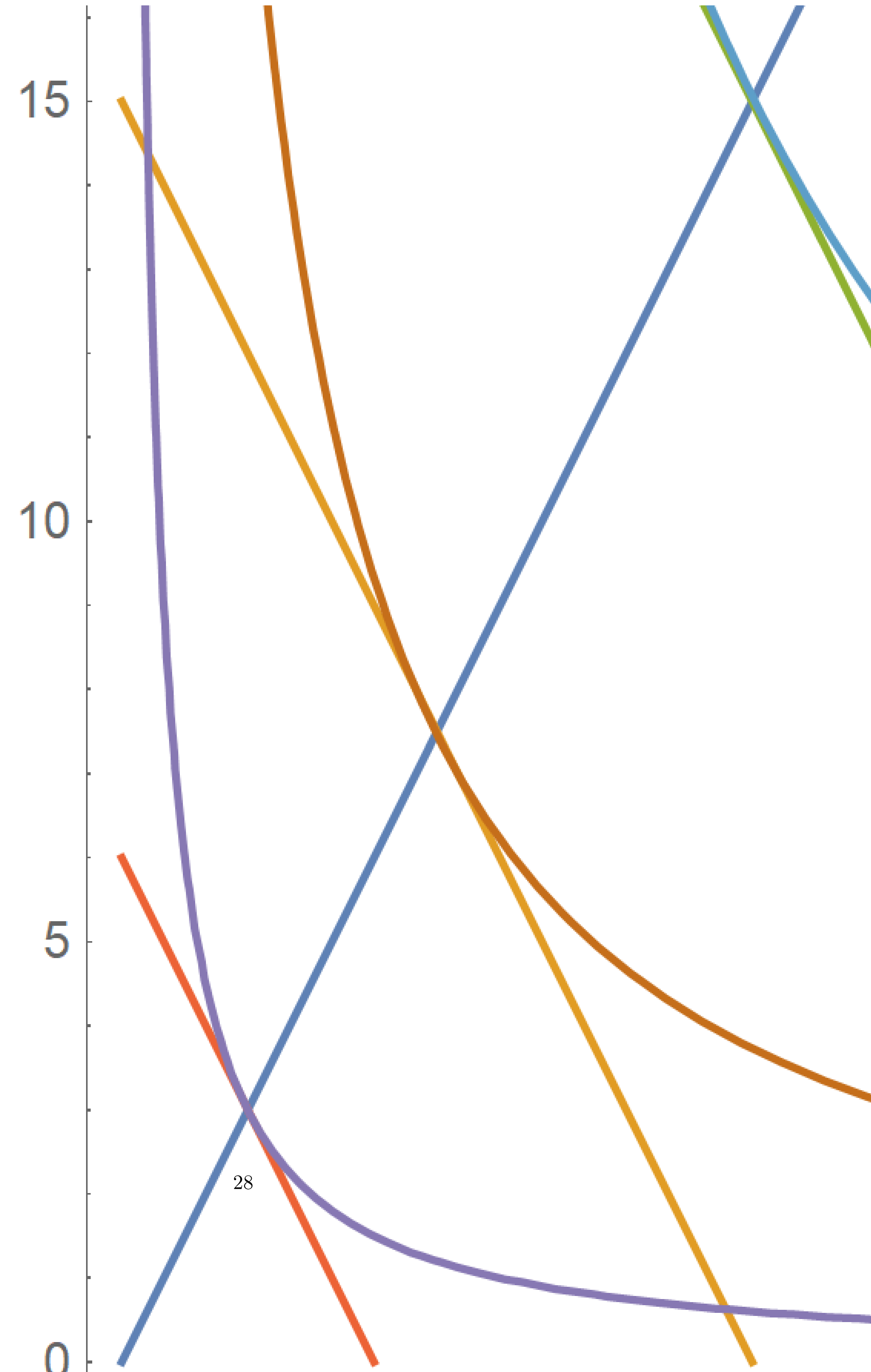
$$x_1 = \frac{\frac{1}{2}m}{2}$$

$$x_2 = \frac{\frac{1}{2}m}{1}$$

$$4x_1 = m$$

$$x_2 = 2x_1$$

In this case the income offer curve is a line with slope 2 coming from the origin.


15
10
5
0

Engle Curve: A plot of the demand for one good for fixed prices as income changes.

Put income on the vertical axis and demand on the horizontal axis. This really answers the question: "for a particular level of demand for some good, what income would be responsible for that amount of demand?"

For example:

$$x_1(2, 1, m) = \frac{\frac{1}{2}m}{2}$$

$$m = 4x_1$$

This is a line with slope of positive 4.

$$m = 2x_2$$

Line with slope of positive 2.

When engle curves are linear, every extra dollar of income will get spent the same way.

$$\left(\frac{\frac{1}{2}m}{2}, \frac{\frac{1}{2}m}{1}\right)$$

Give them an extra dollar, they will spend 50 cents on good 1 and 50 cents on good 2. This buys them an extra $\frac{1}{4}$ unit of good 1 and $\frac{1}{2}$ unit of good 2.

Changes in Own Price

Ordinary- When the price of a good goes up, the demand for that good goes down.

Giffen- When the price of a good goes up, the demand for that good goes up.

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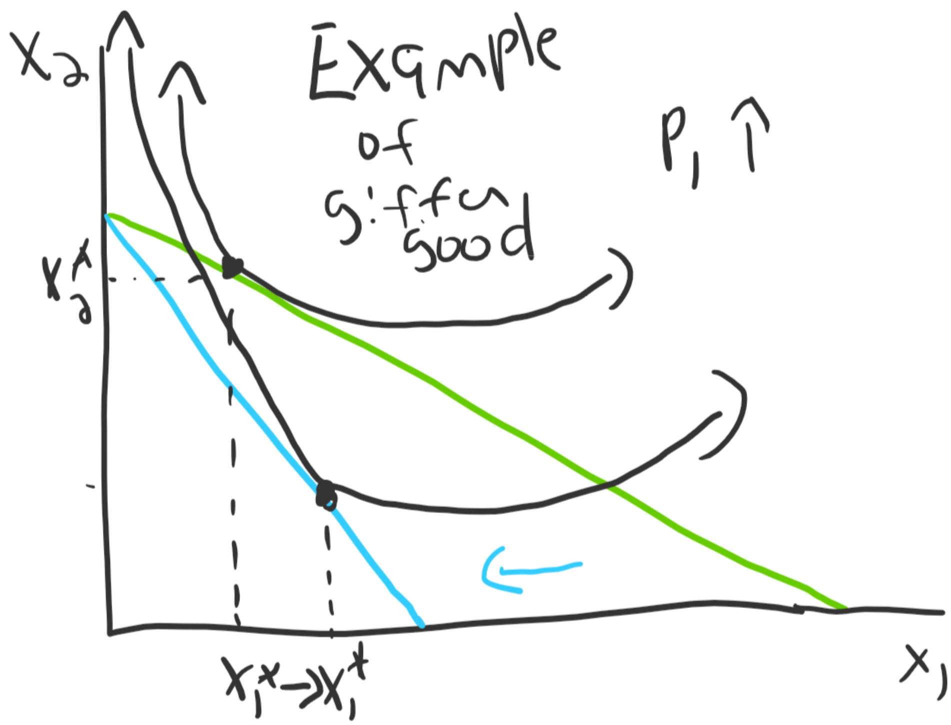


Figure 1: Indifference curves and budget for a “giffen” good. Note that as the price p_1 increases from the green to blue budget, the optimal amount of x_1 increases.

Price Offer Curve. A plot of the optimal bundles as one price changes and the other price and income stay fixed.

Suppose we have utility x_1x_2. The demand is:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

So let's fix $m = 20$, $p_2 = 2$

$$\left(\frac{10}{p_1}, 5\right)$$

Suppose we have perfect complements preferences. Utility function $min\left\{x_1, x_2\right\}$.

$$\left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2}\right)$$

So let's fix $m = 20$, $p_2 = 2$

$$\left(\frac{20}{p_1 + 2}, \frac{20}{p_1 + 2}\right)$$

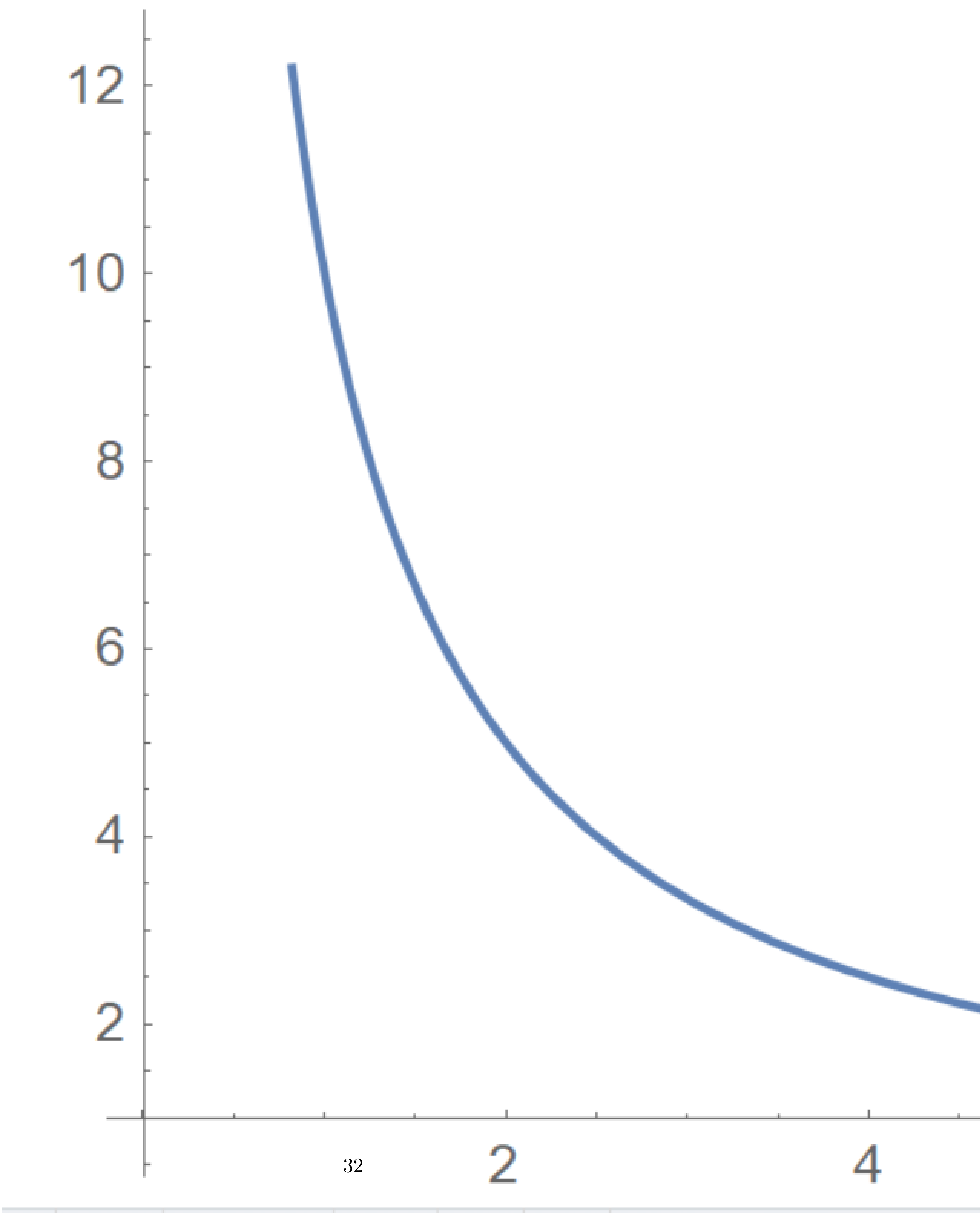
Demand Curve. A plot of the optimal amount of a good as that good's price changes. Plot p on the vertical axis and demand on the horizontal axis.

In the previous cobb douglass example, demand for good 1 is:

$$x_1 = \frac{10}{p_1}$$

To plot this, we need to isolate p_1. This actually called the "inverse demand". At what price is x_1 amount of good 1 demanded.

$$p_1 = \frac{10}{x_1}$$

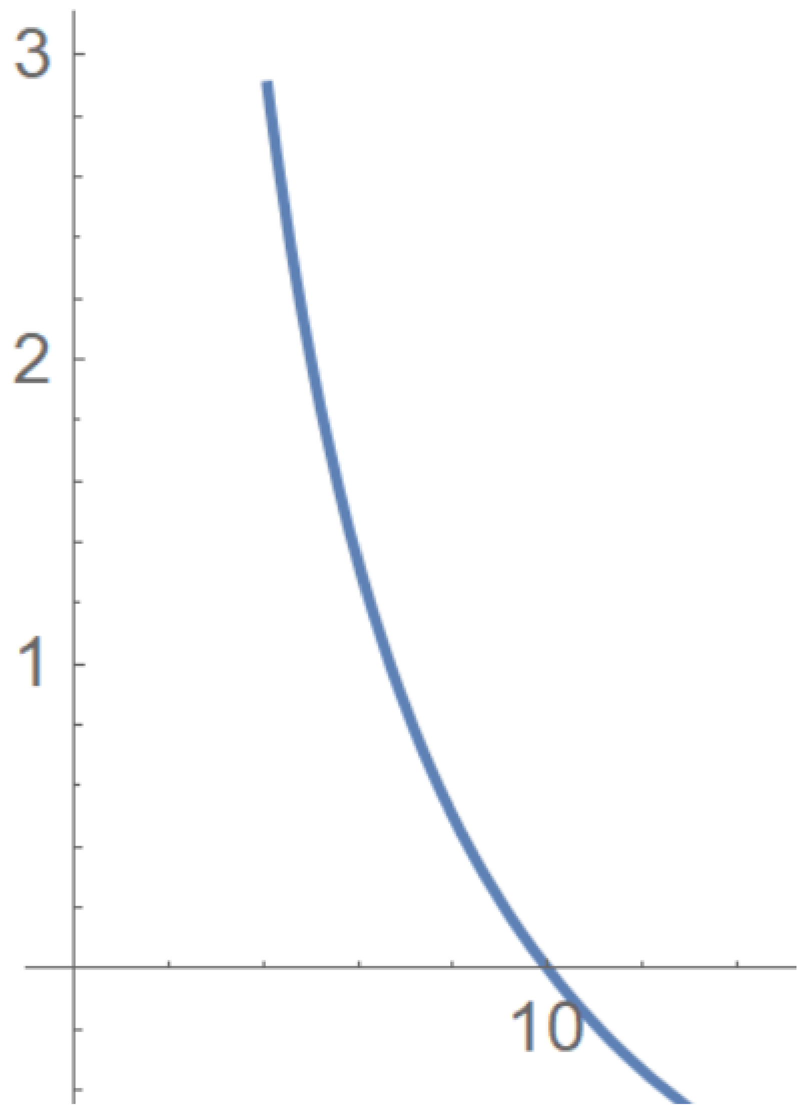

12
10
8
6
4
2
2
4

For the example of perfect complements. Demand for good 1 is:

$$x_1 = \frac{20}{p_1 + 2}$$

The inverse demand is:

$$p_1 = \frac{20}{x_1} - 2$$



Changes in the demand for one good when we change the price of the **other good**.

Two possibilities:

Substitutes: When the price of the other good goes up, demand goes up.

Complements: When the price of the other good goes up, demand goes down.

If when the price of the the other good goes up, demand does not change, we say they are neither.

For example: For cobb douglass $u = x_1 x_2$

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2} \right)$$

Suppose we have perfect complements preferences. Utility function $min\left\{x_1, x_2\right\}$.

$$\left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2} \right)$$

These are complements (as expected) because as the price of the other good goes up, demand goes down.

Let's take a derivative of x_1 with respect to p_2 to find this formally.

$$\frac{\partial \left(\frac{m}{p_1+p_2} \right)}{\partial \left(p_2 \right)} \quad = \quad -\frac{m}{\left(p_1 + p_2\right)^2}$$

m, p_1, p_2 are always positive.

$$-\frac{m}{\left(p_1 + p_2\right)^2} < 0$$

Formally, this shows that the goods are complements because as p_2 goes up, x_1 goes down.