ECON 3012

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Part 1. Budget (2.1-2.7)

1. Bundles

Bundle: $x = (x_1, x_2)$

Example. Ice Cream Bowls. x_1 is the amount of vanilla. x_2 is the amount of chocolate.

$(1, 1)$ one scoop of each flavor.

$(2, 2)$ two scoops of each flavor.

$(0.28, 100)$ a lot of chocolate and a little vanilla.

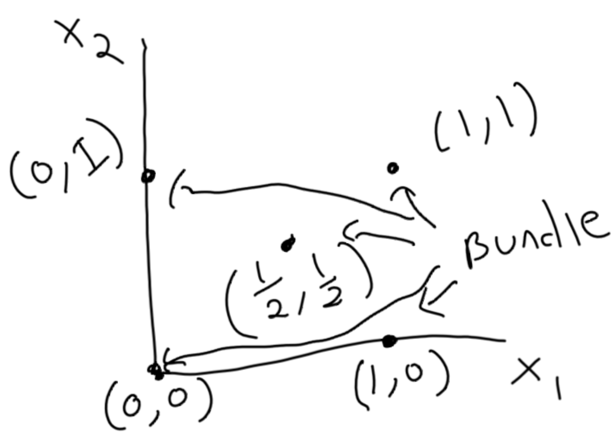

Figure 1.1. Bundles on Cartesian Plane.

2. Feasible Set

The Feasible Set: X is the "feasible" set of bundles.

The feasible set is the universe of bundles that might be relevant in a model. The feasible set defines the scope of a model.

3. Budget Set

Budget Set: B

The budget set is the set of bundles *available* to a particular consumer.

The budget set must be a subset of the feasible set.

In set notation: $B \subseteq X$

3.1. **Budget Sets from Prices and Income. Prices:** p_1, p_2: Price of good 1 and price of good 2.

Cost of a bundle: $p_1x_1 + p_2x_2$.

Income: m.

Budget set: $B = \{x | x \in X \,\&\, x_1p_1 + x_2p_2 \leq m\}$ **.**

In non-math language, this says the budget set is the set of bundles such that the price of the bundle is less than income.

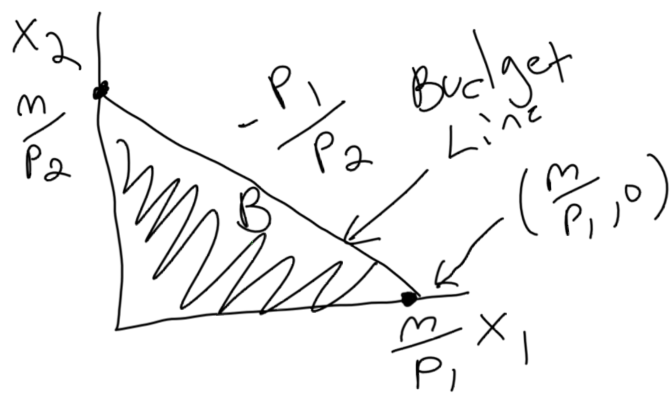

Figure 3.1. Graphical Representation of the Budget Set

3.2. **Changing Prices and Income. Suppose income increases.** m **changes.**

Both endpoints change. If m increaes, $\frac{m}{p_1}$ (the amount I can buy of good 1 changes) increases and $\frac{m}{p_2}$ (maximum affordable x_2) increases. The slope does not change. If m decreases, the opposite happens.

Suppose one of the prices changes.

p_1. If p_1 goes up, the slope decreases (more negative). If p_1 goes down, the slope increases. The x_2 intercept stays the same.

p_2. If p_2 goes up, the slope increases. In p_2 goes down the slope decreases (more negative). The x_1 intercept stays the same.

3.3. **Taxes.** *Quantity tax on good 1:*

$$p_1x_1 + tx_1 + p_2x_2 = m$$

$$(p_1 + t)\,x_1 + p_2x_2 = m$$

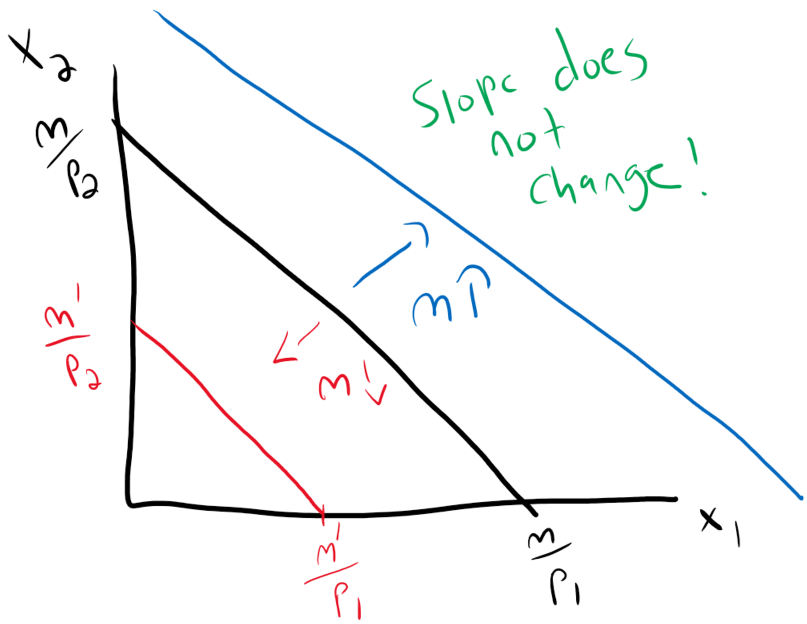

FIGURE 3.2. How Budget Changes with Income

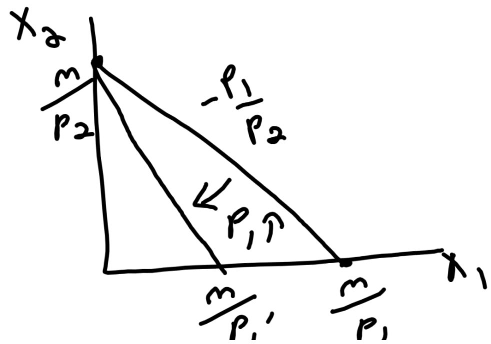

FIGURE 3.3. How Budget Changes with and increase in p_1.

Ad Valorem Tax on good 1:

$$(p_1x_1) + \tau\,(p_1x_1) + p_2x_2 = m$$

$$(1+\tau)\,(p_1x_1) + p_2x_2 = m$$

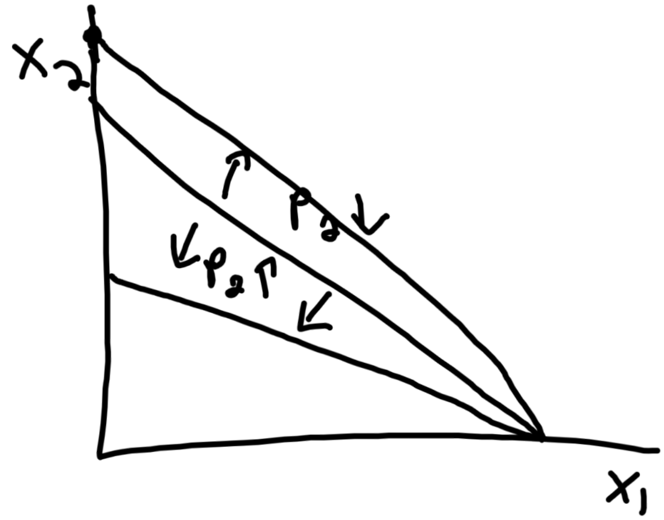

FIGURE 3.4. How Budget Changes with Changes to p_2

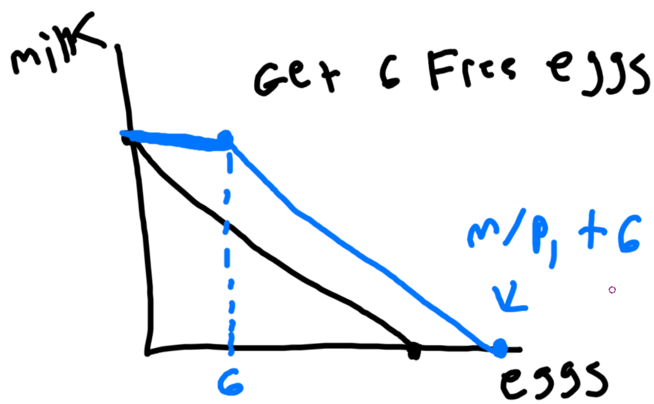

FIGURE 3.5. Six Free Eggs

We will focus on quantity taxes.

3.4. **More Complex Scenarios.**

3.5. **Price Depends on Quantity.**

Part 2. **Preferences (3.1-3.8)**

4. The Preference Relation

4.1. **Definitions.** The preference relation is a set of statements about **pairs** of bundles. The statement x is preferred to bundle x' is shorted to:

$$x \succsim x'$$

Ice Cream Example:

Suppose a consumer eats bowls of ice cream. The bundles (bowls) are written with the vanilla scoops first and chocolate second. For example: $(2, 0)$ is two scoops of vanilla and zero of chocolate.

A consumer who likes vanilla ice cream might have these preferences:

$$(2, 0) \succsim (0, 2)$$

$$(1, 0) \succsim (0, 1)$$

A consumer who like more ice cream to less might have these preferences:

$$(2, 0) \succsim (1, 0)$$

$$(2, 2) \succsim (1, 1)$$

For someone who gets sick of ice cream: (who wants to eat 100 scoops of ice cream?)

$$(1, 0) \succsim (100, 0)$$

For someone who does not care about flavor:

$$(1, 0) \succsim (0, 1) \,\&\, (0, 1) \succsim (1, 0)$$

Indifference Relation: $\sim$

When the following is true: $x \succsim y$ and $y \succsim x$ we say "x is indifferent to y" and write $x \sim y$.

Strict Preference Relation: $\succ$

When the following is true: $x \succsim y$ and **not** $y\backslash succsimx$ we say "x is strictly preferred to y" and write $x \succ y$.

4.2. **Assumptions on $\succsim$.** ***Axiom 1. Reflexive.*** *For all bundles. The bundle is at least as good as itself.*

In set notation:

$$\forall x \in X : x \succsim x$$

Axiom 2. Complete. *For every pair of distinct bundles. Either one is at least as good as the other or the consumer is indifferent.*

In set notation:

$$\forall x, y \in X \& x \neq y : x \succsim y \, or \, y \succsim x \, or \, both$$

A consumer can say "I'm indifferent." but not "I don't know".

Axiom 3. Transitivity.

$$x \succsim y, y \succsim z \,\textbf{implies}\, x \succsim z$$

Transitivity (along with the other assumptions) implies we can always put a set of objects into a ranking.

4.3. **Example of Violating Transitivity.** *Example*: Suppose there are three people on a dating app.

Person 1. Rich, Very Intelligent, Average Looking

Person 2. Financially Constrained, Genius, Good Looking

Person 3. Moderately Well Off, Average Intelligence, Best Looking

Suppose you prefer a person who is better in two aspects than another. We have a cycle:

$$1 \succ 3 \succ 2 \succ 1$$

This kind of multi-dimensional preference can easily cause intransitivity.

4.4. **From Preference to Choice.** Choice Function:

$$C : B \to B$$

$$C\left(B\right) \subseteq B$$

The choice function takes a budget set as input and returns the things the consumer would like to have from that set.

$C\left(B\right)$ is all the objects in B such that those objects are at least as good as everything else in the set.

$$C\left(B\right) = \{x | x \in B : \forall x' \in B, x \succsim x'\}$$

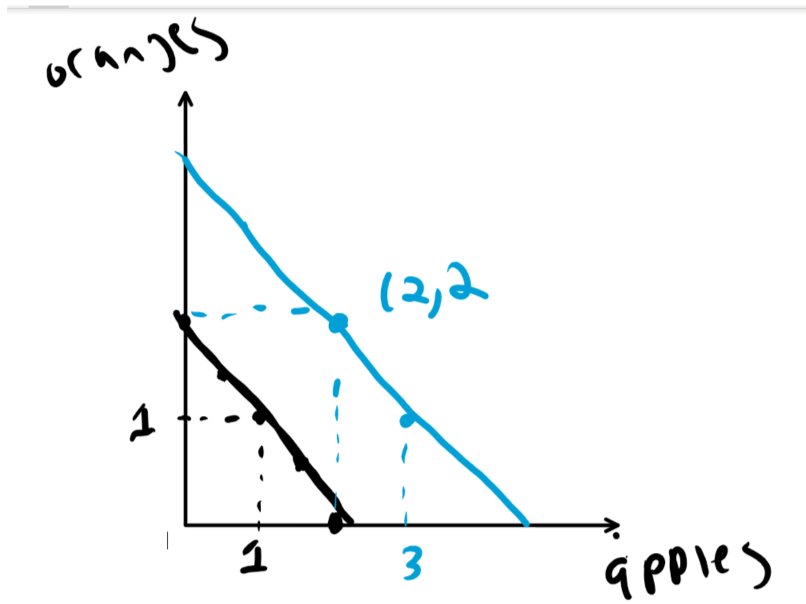

FIGURE 4.1. Indifference curves through the points $(1, 1)$ and $(2, 2)$ for a consumer who will always give up one orange to get one apple.

4.5. **Indifference Curves and the Weakly Preferred Set.** *An indifference curve is a set bundles such that the consumer is indifferent between all of the bundles on the curve.*

Note: There are many indifference curves. We only sketch a few to get an idea of the "shape" of preferences.

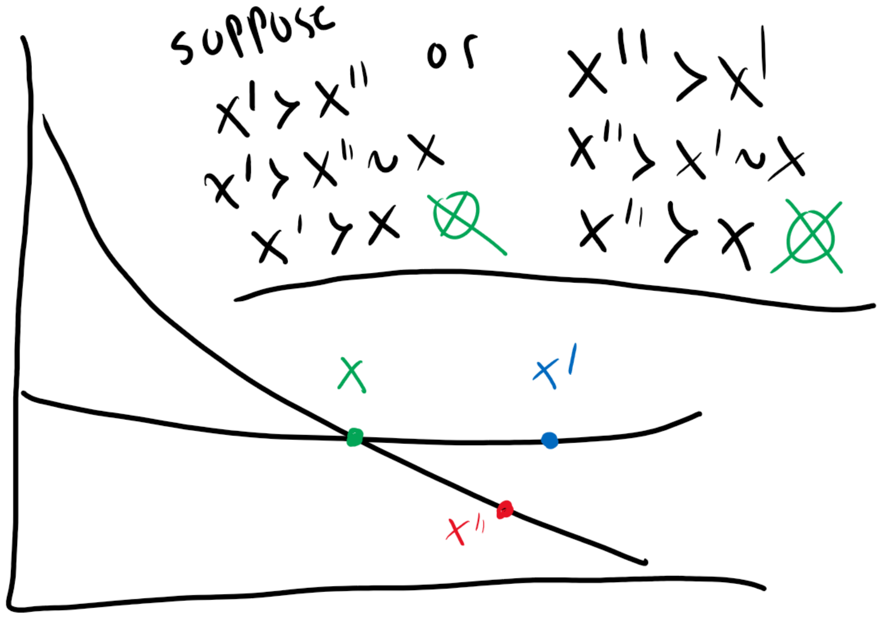

Figure 4.2. Indifference curves cannot cross if preferences are transitive.

4.6. **Indifference Curves Cannot Cross.**

4.7. **Examples of Preferences.**

4.7.1. *Perfect Substitutes.* These preferences are such that my willingness to trade-off between the goods is the same everywhere.

The indifference curves are always downward sloping lines with the same slope. **The slope measures the amount of x_2 you are willing to give up to get 1 more unit of x_1.**

Steep slope: stronger preference for x_1.

Shallow slope: stronger preference for x_2.

Figure 4.3. Indifference curves for perfect substitutes preferences. This consumer would be willing to give up 2 units of x_2 in exchange for 1 unit of x_1.

4.7.2. *Perfect Complements.* Preferences where one good cannot substitute for the other.

Example: Left and right shoes.

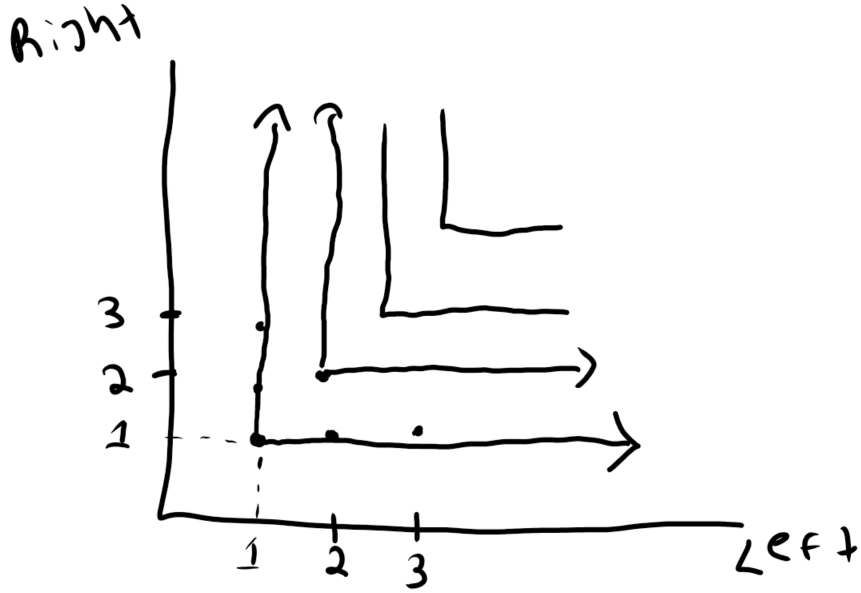

FIGURE 4.4. Indifference curves for perfect complements preferences where Left/Right shoes must be consumed in a 1-to-1 one ratio.

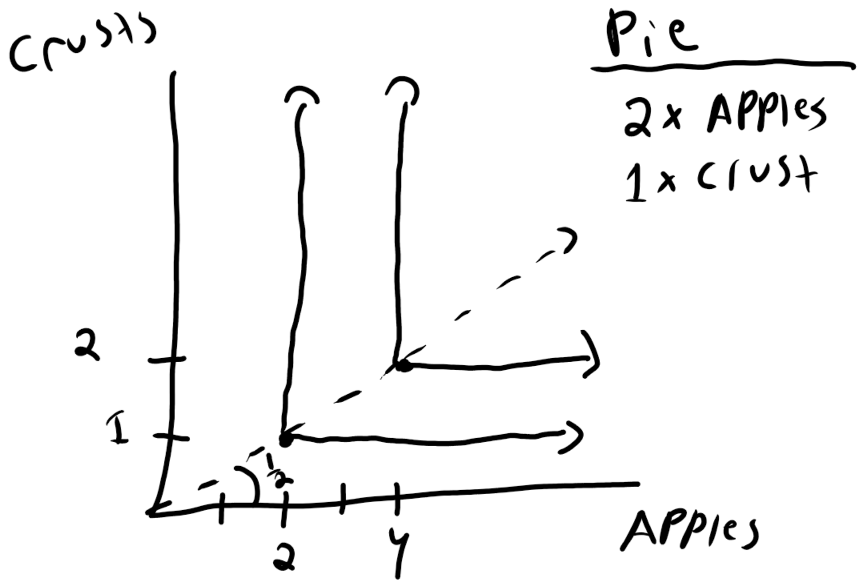

FIGURE 4.5. Indifference curves for perfect complements preferences where the goods are consumed in a 2-to-1 ratio. In this case, 2 apple and 1 crust make a pie.

FIGURE 4.6. When one good is a “bad”, indifference curves slope upward!

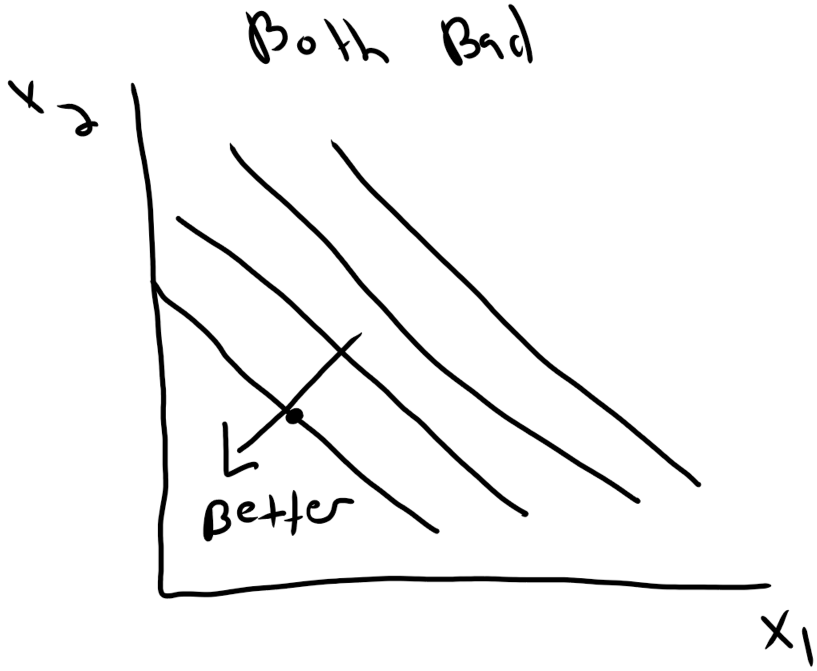

FIGURE 4.7. When both goods are bad, indifference curves slope down, but preference "increases" towards the origin (to the south west).

4.7.3. *Bads.*

4.8. **Further Assumptions: "Well Behaved Preferences".**

4.8.1. *Monotonicity. The assumption that everything is a "good".*

There are two forms of this assumption. *Strict* and *Weak.*

Strict Monotonicity: For two bundles (x_1, x_2) and (y_1, y_2), $(x_1, x_2) \succsim (y_1, y_2)$ if $x_1 \geq y_1$ and $x_2 \geq y_2$. $(x_1, x_2) \succ (y_1, y_2)$ if either $x_1 > y_1$ or $x_2 > y_2$

Perfect substitutes are strictly monotonic.

*Perfect complements are **not** strictly monotonic.*

Weak Monotonicity. (AKA "Monotonic"): For two bundles (x_1, x_2) and (y_1, y_2), $(x_1, x_2) \succsim (y_1, y_2)$ if $x_1 \geq y_1$ and $x_2 \geq y_2$. $(x_1, x_2) \succ (y_1, y_2)$ if **both** $x_1 > y_1$ **and** $x_2 > y_2$

Perfect substitutes are weakly monotonic.

*Perfect complements **are** weakly monotonic.*

4.8.2. *What does monotonicity imply about the indifference curves?* 1. Weakly downward sloping. We cannot have upward sloping indifference curves.

2. Preference increases to the north east (away from the origin).

4.8.3. *Convexity and Strict Convexity. The assumption that mixtures are better than extremes.*

There are two forms of this assumption. *Strict* and *Weak.*

Strictly Convex: For two indifferent bundles $(x_1, x_2) \sim (y_1, y_2)$, for any $t \in (0, 1)$, the mixture given by $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succ (x_1, x_2)$ and $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succ (y_1, y_2)$.

Weakly Convex: For two indifferent bundles $(x_1, x_2) \sim (y_1, y_2)$, for any $t \in [0, 1]$, the mixture given by $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succsim (x_1, x_2)$ and $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succsim (y_1, y_2)$.

The geometry of convex indifference curves. Assume preferences are monotonic:

If preferences are strictly convex, then the indifference curve always lies strictly below a line between any two points on that indifference curve.

If preferences are weakly convex, then the indifference curve always lies weakly below a line between any two points on that indifference curve.

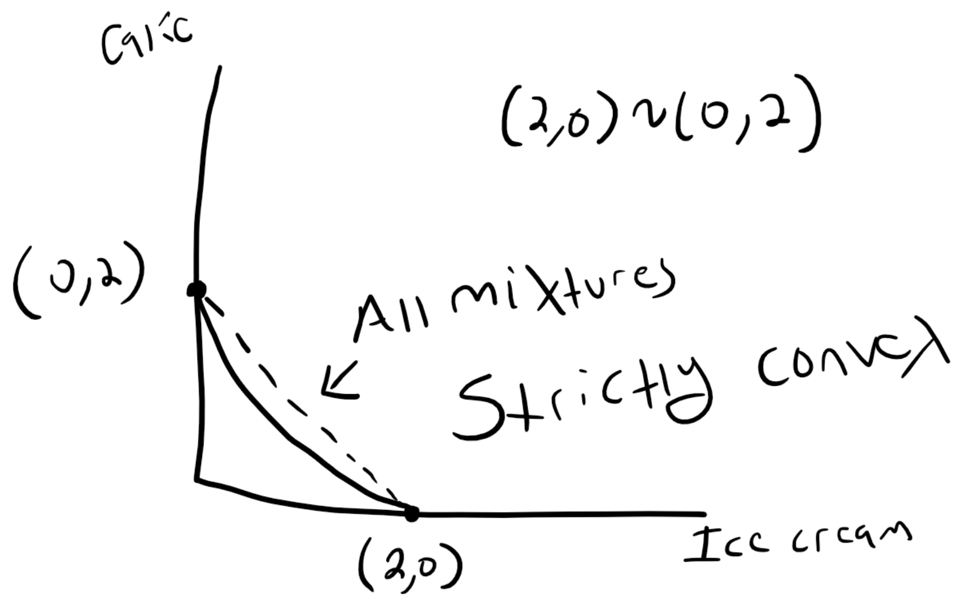

FIGURE 4.8. Example of Convex Indifference Curves

Perfect substitutes, and perfect complements are both **weakly** convex because their indifference curves include "flat" portions.

4.9. **Marginal Rates of Substitution and Slope of the Indifference Curve.** The MRS is defined as the slope of the indifference curve. We need this because optimal bundles will often occur where the slope of the indifference curve is the same as the slope of the budget line.

The MRS measures my willingness to trade off between good 1 and good 2. Specifically, it's how much x_2 I would give up to get one more unit of x_1.

5. Utility (4.1-4.5)

5.1. **Definition.** A utility function is a way of assigning "scores" to bundles, such that better bundles according to $\succsim$ get a higher score. For example, suppose a consumer's preferences are:

$$A \succ B \succ C \sim D$$

Some utility functions that represent these preferences:

$U(A) = 10$, $U(B) = 5$, $U(C) = U(D) = 0$

$U(A) = 12$, $U(B) = 1$, $U(C) = U(D) = -100$

Definition. A utility function $U(x)$ represents preferences $\succsim$ when for every pair of bundles x and y, $U(x) \geq U(y)$ if and only if $x \succsim y$.

That is, if x is better than y according to $\succsim$ it gets a higher utility according to $U()$.

Utility is Ordinal:

Because the magnitude of the numbers are meaningless, and only the relationships matter, we say that these utility functions are "**ordinal**" rather than "**cardinal**". There is no sense in which two times higher utility means that the preference is two times stronger. We can only infer the ranking of things, but not how strong the preferences are from $\succsim$ and a utility function that represents $\succsim$.

5.2. **Monotonic Transformations.** Because utility is ordinal, we are free transform one utility function into another, as long as it maintains the same preferences. Any **strictly increasing** function of a utility function represents the same preferences as the original utility function. For example, suppose:

$$U(x_1, x_2) = x_1 + x_2$$

This represents the preferences of someone who only cares about the total amount of stuff, but not the composition. In fact, this utility function represents *perfect substitutes preferences*. Here are some other utility functions that represent the same preferences:

$$U'(x_1, x_2) = x_1 + x_2 + 100 = U(x_1, x_2) + 100$$

$$U'(x_1, x_2) = (x_1 + x_2)^2 = (U(x_1, x_2))^2$$

Since the functions $f(u) = u + 100$ and $f(u) = u^2$ are strictly increasing for $u \geq 0$ (which is always true for the original utility function). These are monotonic transformations of the original utility function.

5.3. **MRS from Utility Function.** *The* ***Marginal Rate of Substitution*** *(MRS) is the slope of the indifference curve.*

The **Marginal Utility** of good i is $mu_i = \frac{\partial u(x_1,x_2)}{\partial x_i}$.

$$MRS = -\frac{mu_1}{mu_2} = -\frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}}$$

Note that MRS is an **ordinal** property since it represents the slope of indifference curves. Because two preferences that are the same have the same indifference curves, they will also have the same MRS. **Same MRS, same preferences**.

5.4. **Examples of Utility Functions.**

5.4.1. *Perfect Substitutes.*

$$u\left(x_1, x_2\right) = ax_1 + bx_2$$

$$MRS = -\frac{a}{b}$$

Constant MRS implies a constant willingness to trade off between the two goods.

$$u\left(x_1, x_2\right) = \left(ax_1 + bx_2\right)^2$$

$$-\frac{a}{b}$$

5.4.2. *Quasi-Linear. Consumer only gets tired of one of the two goods.*

One common quasi-linear utility function:

$$u\left(x_1, x_2\right) = x_1 + ln\left(x_2\right)$$

$$MRS = -\frac{\frac{\partial(x_1 + ln(x_2))}{\partial x_1}}{\frac{\partial(x_1 + ln(x_2))}{\partial x_2}} = -x_2$$

Another example of a quasi-linear utility function:

$$u\left(x_1, x_2\right) = 10x_1 + \sqrt{x_2}$$

Practice taking the MRS of this function. Notice that it only depends on the amount of x_2!

5.4.3. *Cobb-Douglass. Consumer Gets Tired of Both Goods.*

$$u\left(x_1, x_2\right) = x_1^{\alpha} x_2^{\beta}$$

$$mu_1 = \frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{\beta}$$

$$mu_2 = \frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_2} = \beta x_1^{\alpha} x_2^{\beta-1}$$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}}$$

$$= -\frac{\alpha x_1^{\alpha} x_1^{-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta} x_2^{-1}} = -\frac{\alpha x_1^{-1}}{\beta x_2^{-1}} = -\frac{\alpha}{\beta}\frac{x_2}{x_1}$$

Let's compare two CD Functions:

Increasing the exponent on either good will increase the consumers desire for that good. They will still get tired of it, but between two consumers, one with a larger exponent on a good, that consumer will have a stronger desire for the good at the same bundle.

$$u\left(x_1, x_2\right) = x_1 x_2$$

$$MRS = -\frac{x_2}{x_1}$$

At the point $(1,1)$: $MRS = -1$.

$$\tilde{u}\left(x_1, x_2\right) = x_1^{10} x_2$$

$$MRS = -10\frac{x_2}{x_1}$$

At the point $(1,1)$: $MRS = -10$

Notice that the consumer with $\tilde{u}\left(x_1, x_2\right) = x_1^{10} x_2$ would be willing to give up ten-times more x_2 to get the same amount of x_1 as the consumer with utility function $u\left(x_1, x_2\right) = x_1 x_2$.

5.4.4. *Perfect Complements.*

$$u\left(x_1, x_2\right) = min\left\{ax_1, bx_2\right\}$$

Examples.

Left\Right Shoes

$$u\left(l, r\right) = min\left\{l, r\right\}$$

Apple pies. 2 Apples, 1 Crust makes a pie.

$$u\left(a, c\right) = min\left\{\frac{1}{2}a, c\right\}$$

Notice $\frac{1}{2}a$ is the most pies we could possible make with a apples and c is the most possible pies we could make with c apples.

6. Choice (Chapter 5.1-5.3,5.5)

6.1. **Three Possibilities.** We are going to find the optimal bundles in a budget set. We are going to look for bundles that are all at least as good as everything else in the budget set.

Assume $\succsim$ is reflexive, complete, transitive and $\succsim$ **monotonic**.

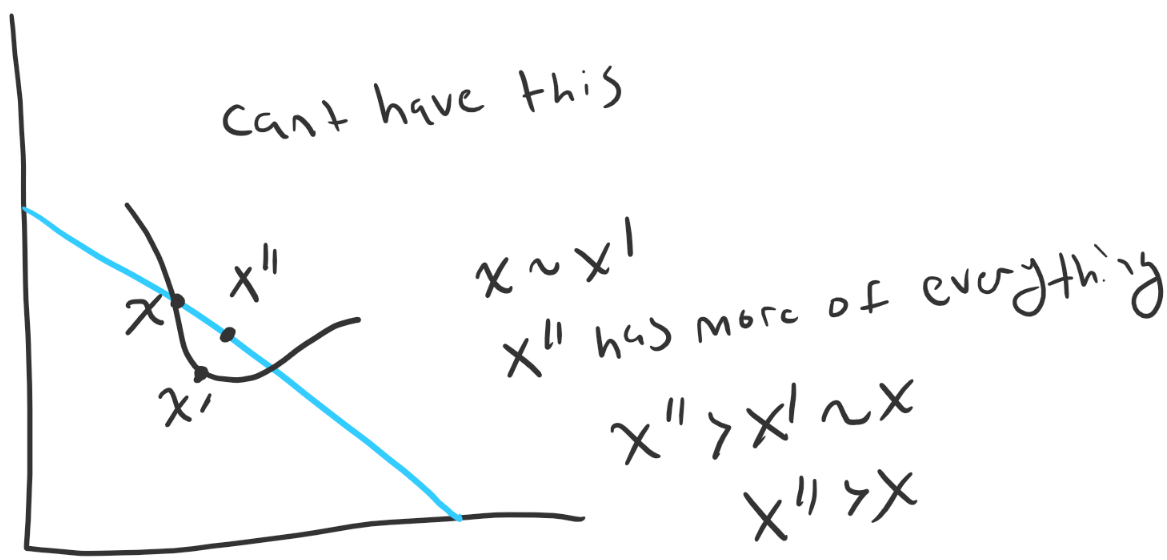

Figure 6.1. An optimal bundle cannot be on an indifference curve that passes "into" the budget set.

There are only three possibilities for an optimal bundle:

1. (Tangent) It is at bundle where the indifference curve at that bundle had the same slope as the budget line.

2. (Touching but not tangent) The bundle is a "non-smooth" point on the indifference curve, but the that point just touches the budget line.

3. (Boundary) We are at one of the boundaries ($x_1 = 0$ or $x_2 = 0$) in this case the slope of the indifference curve and the slope of the budget need not be equal.

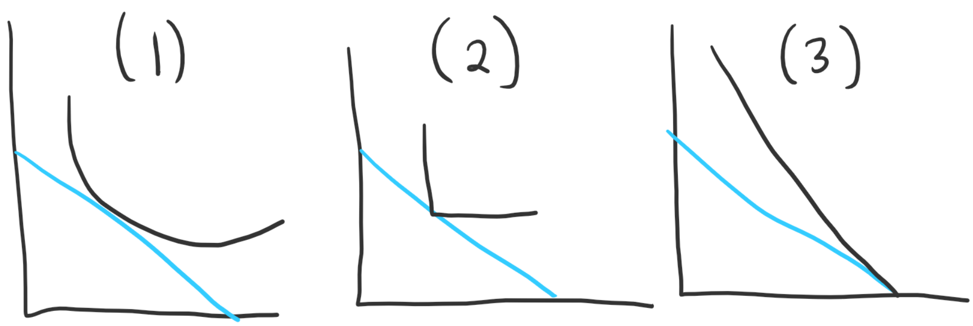

FIGURE 6.2. Graphical Examples of the Three Possibilities

Under some weak conditions (we can take derivatives of the utility function). The tangency condition is necessary for an *interior* optimum (involves consuming some of both things).

That is, if there is an optimal bundle that involves consuming some of both goods, it must have the property that the slope of the indifference curve at that optimal bundle is the same as the slope of the budget line. T

This condition is formalized by the familiar equation:

$$MRS = -\frac{p_1}{p_2}$$

6.2. **Examples.**

6.2.1. *Cobb Douglass:*

$$u(x_1, x_2) = x_1 x_2$$

$$p_1 x_1 + p_2 x_2 = m$$

Tangency condition:

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

Tangency Condition:

$$*\, x_1 p_1 = x_2 p_2$$

Budget Condition:

$$**\ x_1 p_1 + x_2 p_2 = m$$

Plug Tangency Condition into Budget Condition:

$$x_1 p_1 + x_1 p_1 = m$$

$$2 x_1 p_1 = m$$

$$x_1^* = \frac{1}{2} \frac{m}{p_1}$$

Plug this back into one of the two equations:

$$x_1 p_1 = x_2 p_2$$

$$\frac{1}{2} \frac{m}{p_1} p_1 = x_2 p_2$$

$$\frac{1}{2} \frac{m}{p_1} \frac{p_1}{p_2} = x_2$$

$$x_2^* = \frac{1}{2} \frac{m}{p_2}$$

The optimal bundle:

$$\left(\frac{\frac{1}{2} m}{p_1}, \frac{\frac{1}{2} m}{p_2} \right)$$

6.2.2. *Perfect Substitutes.*

$$u(x_1, x_2) = 2x_1 + x_2$$

$p_1 = 1$, $p_2 = 1$, $m = 10$

$$1x_1 + 1x_2 = 10$$

Tangency Condition:

$$-\frac{2}{1} = -\frac{1}{1}$$

$$-2 = -1$$

This is never true. There can't be an interior solution. There has to be a boundary solution. Let's check the utility of both.

$$u(x_1, x_2) = 2x_1 + x_2$$

Only consume x_1:

$$\left(\frac{m}{p_1}, 0\right) = (m, 0)$$

$$u(m, 0) = 2m = 20$$

Only consume x_2:

$$\left(0, \frac{m}{p_2}\right) = (0, m)$$

$$u(0, m) = m = 10$$

Since consuming only x_1 gives me more utility, that is the optimal bundle:

$$(m, 0)$$

6.2.3. *Anything is Optimal.*

$$u(x_1, x_2) = 2x_1 + x_2$$

$p_1 = 2$, $p_2 = 1$, $m = 10$

$$2x_1 + 1x_2 = 10$$

$$-\frac{2}{1} = -\frac{2}{1}$$

$$-2 = -2$$

As long as I spend all of my money, any bundle is optimal.
All of the bundles such that:

$$p_1 x_1 + p_2 x_2 = m$$

6.2.4. *Perfect Complements.*

$$u(x_1, x_2) = min\{x_1, x_2\}$$

$$2x_1 + x_2 = 15$$

We still know the budget condition must be true:

$$** \ 2x_1 + x_2 = 15$$

What is the other condition?
"No Waste Condition". (Equation for the "kink" points).

$$* \, x_1 = x_2$$

Plug one condition into the other:

$$2x_1 + x_2 = 15$$

$$2x_1 + x_1 = 15$$

$$3x_1 = 15$$

$$x_1 = 5$$

Plug back into one of the equations:

$$x_2 = 5$$

6.2.5. *Perfect Complements (2 Apples, 1 Crust).*

$$u\left(x_1, x_2\right) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$2x_1 + x_2 = 15$$

We still know the budget condition must be true:

$$** \; 2x_1 + x_2 = 15$$

What is the other condition?
"No Waste Condition". (Equation for the "kink" points).

$$\frac{1}{2}x_1 = x_2$$

Combine the conditions:

$$2x_1 + x_2 = 15$$

$$2x_1 + \frac{1}{2}x_1 = 15$$

$$\frac{5}{2}x_1 = 15$$

$$x_1 = 6$$

$$x_2 = 3$$

6.2.6. *Max Preferences.*

$$u\left(x_1, x_2\right) = max\left\{x_1, x_2\right\}$$

$$2x_1 + x_2 = 15$$

Try this one at home: what is the optimal bundle?