ECONOMICS 8100

GREG LEO

Part 1. Budget

1. Consumption Set X

Assumptions: (Universe of Choice Objects): X
Bundles: Elements of X. $x \in X$

Assumptions about X.
1. $\emptyset \neq X \subseteq \mathbb{R}^n_+$.
2. X is closed.
3. X is convex.
4. $0 \in X$.

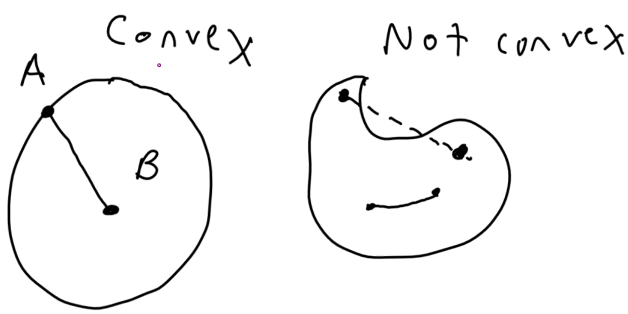

Figure 1.1. Examples of a Convex/Non-Convex Set.

2. Budget Set B

Budget Set: $B \subseteq X$
X defines the scope of the model. B is what an *individual consumer* chooses among.

Example. Budget Set with Prices and Income
$B = \{x | x \in X \,\&\, x_1 p_1 + x_2 p_2 \leq m\}$

Example. Ice Cream Bowls
Every ice cream bowl x has some non-negative number of scoops of Vanilla, Chocolate, Strawberry.

$$X = \mathbb{R}^3_+$$

Budget B is the set of bowls with *no more than one scoop of ice cream.*

$$B = \left\{ x | x \in R^3_+ \& \sum_{i=1}^{3} x_i \leq 1 \right\}$$

This is the unit-simplex in $\mathbb{R}_3$.
$(1, 0, 0) \in B$. (On the boundary.)
$(0.5, 0.5, 0) \in B$. (On the boundary.)
$(0.25, 0.25, 0.25) \in B$. (In the interior.)
$(2, 0, 0) \notin B$

Part 2. **Preference**

3. The Preference Relation

Preference Relation is a **Binary Relation**.
Formally, a binary relation on set X is a subset of the Cartesian product X with itself.

$$\succeq \subseteq X \times X$$

Another way to denote an ordered pair is "in" the relation:
If $(x, y) \in \succeq$ we can also write $x \succeq y$.
Informally we say "x" is at least as good as "y", or "x" preferred "y".

Axioms of $\succeq$.
Axiom 0 *(reflexive):* $\forall x \in X$, $x \succeq x$. This is implied by *axiom 1.*

Axiom 1 *(complete):* $\forall x, x' \in X$, either $x \succeq x'$ or $x' \succeq x$ (or both).
The consumer has "some" preference over every pair of objects.

Axiom 2 *(transitive):* $\forall x, x', x'' \in X$ *if* $x \succeq x'$ *and* $x' \succeq x'' \Rightarrow x \succeq x''$.

$\succeq$ is a "weak order" if it is complete and transitive.

4. Relations and Sets Related to $\succeq$

Subrelations:
$\sim$ is the indifference relation. $x \succeq y$ and $y \succeq x \Leftrightarrow x \sim y$.
$\succ$ is the strict relation. $x \succeq y$ and not $y \succeq x \Leftrightarrow x \succ y$.
Related Sets:
$\succeq (x)$ "upper contour set"

5. From Preferences to Choice

Choice Correspondence.

We will assume that from a budget set B a consumer "chooses" *choice set* C according to their preference $\succeq$. $C = \{x | x \in B \,\&\, \forall x' \in B, x \succeq x'\}$.

Informally, C is the set of objects that are at least as good as anything else in the set.

Example With Transitive Preferences

$X = \{a, b, c\}$. $a \succeq b, c \succeq a, c \succeq b$.

$$C(\{a\}) = a, C(\{b\}) = b, C(\{c\}) = c$$

$$C(\{a, b\}) = a, C(\{a, c\}) = c, C(\{b, c\}) = c$$

$$C(\{a, b, c\}) = c$$

The Problem with Intransitive Preferences

$X = \{a, b, c\}$. $a \succeq b, c \succeq a, b \succeq c$. *This is intransitive!*

Choice correspondence:

$$C : P(X) / \emptyset \to X$$

$$C(\{a\}) = a, C(\{b\}) = b, C(\{c\}) = c$$

$$C(\{a, b\}) = a, C(\{a, c\}) = c, C(\{b, c\}) = b$$

$$C(\{a, b, c\}) = \emptyset$$

This consumer cannot make a choice from the set $\{a, b, c\}$.

Result (Impossibility of Choice): *For every preference relation with an intransitive strict sub-relation $\succ$ over some universe X. There is some budget $B \subseteq X$ with $\#(B) \geq 3$ such that $C(B) = \emptyset$.*