Chapter 19 NAME________________

Technology

Introduction. In this chapter you work with production functions, relating output of a firm to the inputs it uses. This theory will look familiar to you, because it closely parallels the theory of utility functions. In utility theory, an *indifference curve* is a locus of commodity bundles, all of which give a consumer the same utility. In production theory, an *isoquant* is a locus of input combinations, all of which give the same output. In consumer theory, you found that the slope of an indifference curve at the bundle (x_1, x_2) is the ratio of marginal utilities, $MU_1(x_1, x_2)/MU_2(x_1, x_2)$. In production theory, the slope of an isoquant at the input combination (x_1, x_2) is the ratio of the marginal products, $MP_1(x_1, x_2)/MP_2(x_1, x_2)$. Most of the functions that we gave as examples of utility functions can also be used as examples of production functions.

There is one important difference between production functions and utility functions. Remember that utility functions were only "unique up to monotonic transformations." In contrast, two different production functions that are monotonic transformations of each other describe different technologies.

Example: If the utility function $U(x_1, x_2) = x_1 + x_2$ represents a person's preferences, then so would the utility function $U^*(x_1, x_2) = (x_1 + x_2)^2$. A person who had the utility function $U^*(x_1, x_2)$ would have the same indifference curves as a person with the utility function $U(x_1, x_2)$ and would make the same choices from every budget. But suppose that one firm has the production function $f(x_1, x_2) = x_1 + x_2$, and another has the production function $f^*(x_1, x_2) = (x_1 + x_2)^2$. It is true that the two firms will have the same isoquants, but they certainly do not have the same technology. If both firms have the input combination $(x_1, x_2) = (1, 1)$, then the first firm will have an output of 2 and the second firm will have an output of 4.

Now we investigate "returns to scale." Here we are concerned with the change in output if the amount of every input is multiplied by a number $t > 1$. If multiplying inputs by t multiplies output by more than t, then there are increasing returns to scale. If output is multiplied by exactly t, there are constant returns to scale. If output is multiplied by less than t, then there are decreasing returns to scale.

Example: Consider the production function $f(x_1, x_2) = x_1^{1/2} x_2^{3/4}$. If we multiply the amount of each input by t, then output will be $f(tx_1, tx_2) = (tx_1)^{1/2}(tx_2)^{3/4}$. To compare $f(tx_1, tx_2)$ to $f(x_1, x_2)$, factor out the expressions involving t from the last equation. You get $f(tx_1, tx_2) = t^{5/4} x_1^{1/2} x_2^{3/4} = t^{5/4} f(x_1, x_2)$. Therefore when you multiply the amounts of all inputs by t, you multiply the amount of output by $t^{5/4}$. This means there are *increasing* returns to scale.

Example: Let the production function be $f(x_1, x_2) = \min\{x_1, x_2\}$. Then

$$f(tx_1, tx_2) = \min\{tx_1, tx_2\} = \min t\{x_1, x_2\} = t\min\{x_1, x_2\} = tf(x_1, x_2).$$

Therefore when all inputs are multiplied by t, output is also multiplied by t. It follows that this production function has *constant* returns to scale.

You will also be asked to determine whether the marginal product of each single factor of production increases or decreases as you increase the amount of that factor without changing the amount of other factors. Those of you who know calculus will recognize that the marginal product of a factor is the first derivative of output with respect to the amount of that factor. Therefore the marginal product of a factor will decrease, increase, or stay constant as the amount of the factor increases depending on whether the *second* derivative of the production function with respect to the amount of that factor is negative, positive, or zero.

Example: Consider the production function $f(x_1, x_2) = x_1^{1/2}x_2^{3/4}$. The marginal product of factor 1 is $\frac{1}{2}x_1^{-1/2}x_2^{3/4}$. This is a decreasing function of x_1, as you can verify by taking the derivative of the marginal product with respect to x_1. Similarly, you can show that the marginal product of x_2 decreases as x_2 increases.

19.0 Warm Up Exercise. The first part of this exercise is to calculate marginal products and technical rates of substitution for several frequently encountered production functions. As an example, consider the production function $f(x_1, x_2) = 2x_1 + \sqrt{x_2}$. The marginal product of x_1 is the derivative of $f(x_1, x_2)$ with respect to x_1, holding x_2 fixed. This is just 2. The marginal product of x_2 is the derivative of $f(x_1, x_2)$ with respect to x_2, holding x_1 fixed, which in this case is $\frac{1}{2\sqrt{x_2}}$. The TRS is $-MP_1/MP_2 = -4\sqrt{x_2}$. Those of you who do not know calculus should fill in this table from the answers in the back. The table will be a useful reference for later problems.

Marginal Products and Technical Rates of Substitution

$f(x_1, x_2)$	$MP_1(x_1, x_2)$	$MP_2(x_1, x_2)$	$TRS(x_1, x_2)$
$x_1 + 2x_2$			
$ax_1 + bx_2$			
$50x_1x_2$			
$x_1^{1/4}x_2^{3/4}$	$\frac{1}{4}x_1^{-3/4}x_2^{3/4}$		
$Cx_1^a x_2^b$	$Cax_1^{a-1}x_2^b$		
$(x_1 + 2)(x_2 + 1)$	$x_2 + 1$		
$(x_1 + a)(x_2 + b)$			
$ax_1 + b\sqrt{x_2}$			
$x_1^a + x_2^a$			
$(x_1^a + x_2^a)^b$	$bax_1^{a-1}(x_1^a + x_2^a)^{b-1}$	$bax_2^{a-1}(x_1^a + x_2^a)^{b-1}$	

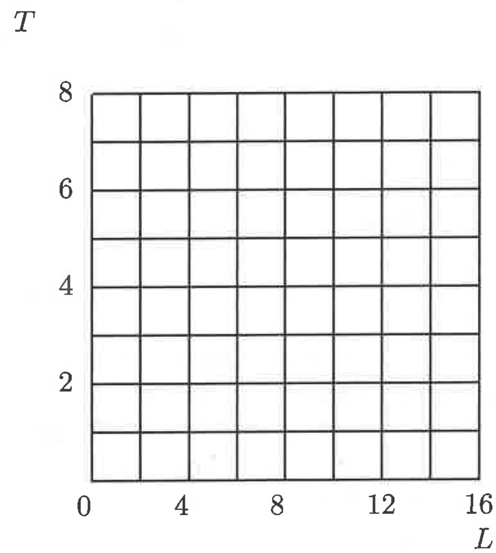
Returns to Scale and Changes in Marginal Products

For each production function in the table below, put an I, C, or D in the first column if the production function has increasing, constant, or decreasing returns to scale. Put an I, C, or D in the second (third) column, depending on whether the marginal product of factor 1 (factor 2) is increasing, constant, or decreasing, as the amount of that factor alone is varied.

$f(x_1, x_2)$	Scale	MP_1	MP_2
$x_1 + 2x_2$			
$\sqrt{x_1 + 2x_2}$			
$.2x_1x_2^2$			
$x_1^{1/4}x_2^{3/4}$			
$x_1 + \sqrt{x_2}$			
$(x_1 + 1)^{.5}(x_2)^{.5}$			
$\left(x_1^{1/3} + x_2^{1/3}\right)^3$			

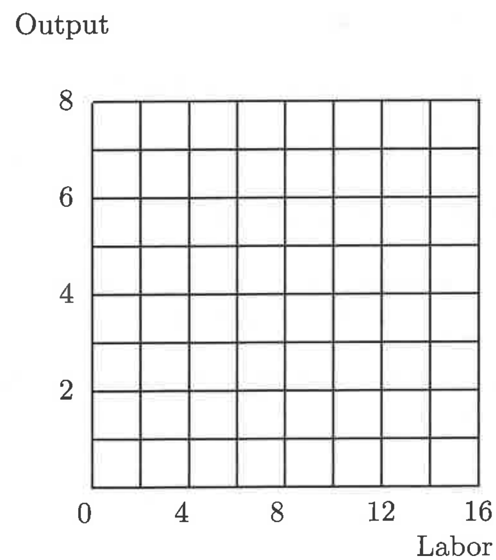
19.1 (0) Prunella raises peaches. Where L is the number of units of labor she uses and T is the number of units of land she uses, her output is $f(L, T) = L^{\frac{1}{2}}T^{\frac{1}{2}}$ bushels of peaches.

(a) On the graph below, plot some input combinations that give her an output of 4 bushels. Sketch a production isoquant that runs through these points. The points on the isoquant that gives her an output of 4 bushels all satisfy the equation $T =$______________________________.



(b) This production function exhibits (constant, increasing, decreasing) returns to scale.________________________.

(c) In the short run, Prunella cannot vary the amount of land she uses. On the graph below, use blue ink to draw a curve showing Prunella's output as a function of labor input if she has 1 unit of land. Locate the points on your graph at which the amount of labor is 0, 1, 4, 9, and 16 and label them. The slope of this curve is known as the marginal ________ of ________ Is this curve getting steeper or flatter as the amount of labor increase?________________________.



(d) Assuming she has 1 unit of land, how much extra output does she get from adding an extra unit of labor when she previously used 1 unit of labor? ______________ 4 units of labor? ______________ If you know calculus, compute the marginal product of labor at the input combination $(1, 1)$ and compare it with the result from the unit increase in labor output found above.______________________________

____.

(e) In the long run, Prunella can change her input of land as well as of labor. Suppose that she increases the size of her orchard to 4 units of land. Use red ink to draw a new curve on the graph above showing output as a function of labor input. Also use red ink to draw a curve showing marginal product of labor as a function of labor input when the amount of land is fixed at 4.

19.2 (0) Suppose x_1 and x_2 are used in fixed proportions and $f(x_1, x_2) = \min\{x_1, x_2\}$.

(a) Suppose that $x_1 < x_2$. The marginal product for x_1 is ________ and (increases, remains constant, decreases)________________ for small increases in x_1. For x_2 the marginal product is ________, and (increases, remains constant, decreases)________________ for small increases in x_2. The technical rate of substitution between x_2 and x_1 is __________ ______ This technology demonstrates (increasing, constant, decreasing) ____________ returns to scale.

(b) Suppose that $f(x_1, x_2) = \min\{x_1, x_2\}$ and $x_1 = x_2 = 20$. What is the marginal product of a small increase in x_1? ________ What is the marginal product of a small increase in x_2? ________ The marginal product of x_1 will (increase, decrease, stay constant)____________ if the amount of x_2 is increased by a little bit.

Calculus **19.3 (0)** Suppose the production function is Cobb-Douglas and $f(x_1, x_2) = x_1^{1/2} x_2^{3/2}$.

(a) Write an expression for the marginal product of x_1 at the point (x_1, x_2).____.

(b) The marginal product of x_1 (increases, decreases, remains constant) ______________ for small increases in x_1, holding x_2 fixed.

(c) The marginal product of factor 2 is ______________, and it (increases, remains constant, decreases) ______________ for small increases in x_2.

(d) An increase in the amount of x_2 (increases, leaves unchanged, decreases) ______________ the marginal product of x_1.

(e) The technical rate of substitution between x_2 and x_1 is ______________.

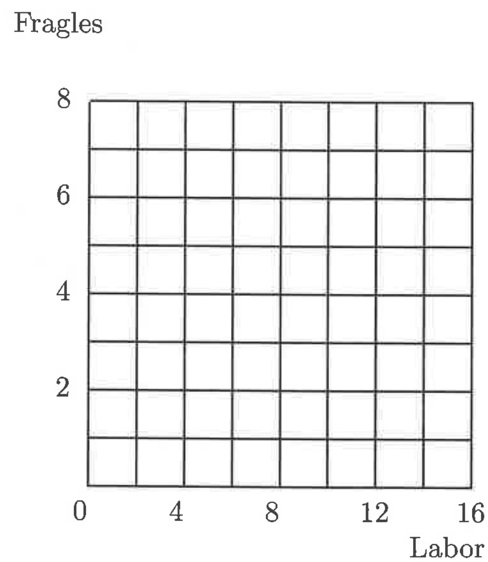
(f) Does this technology have diminishing technical rate of substitution? ____.

(g) This technology demonstrates (increasing, constant, decreasing) ______________ returns to scale.

19.4 (0) The production function for fragles is $f(K, L) = L/2 + \sqrt{K}$, where L is the amount of labor used and K the amount of capital used.

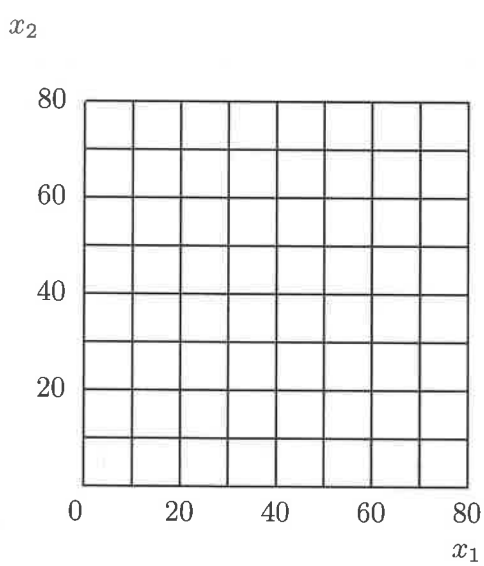
(a) There are (constant, increasing, decreasing) ______________ returns to scale. The marginal product of labor is ______________ (constant, increasing, decreasing).

(b) In the short run, capital is fixed at 4 units. Labor is variable. On the graph below, use blue ink to draw output as a function of labor input in the short run. Use red ink to draw the marginal product of labor as a function of labor input in the short run. The average product of labor is defined as total output divided by the amount of labor input. Use black ink to draw the average product of labor as a function of labor input in the short run.



19.5 (0) General Monsters Corporation has two plants for producing juggernauts, one in Flint and one in Inkster. The Flint plant produces according to $f_F(x_1, x_2) = \min\{x_1, 2x_2\}$ and the Inkster plant produces according to $f_I(x_1, x_2) = \min\{2x_1, x_2\}$, where x_1 and x_2 are the inputs.

(a) On the graph below, use blue ink to draw the isoquant for 40 juggernauts at the Flint plant. Use red ink to draw the isoquant for producing 40 juggernauts at the Inkster plant.

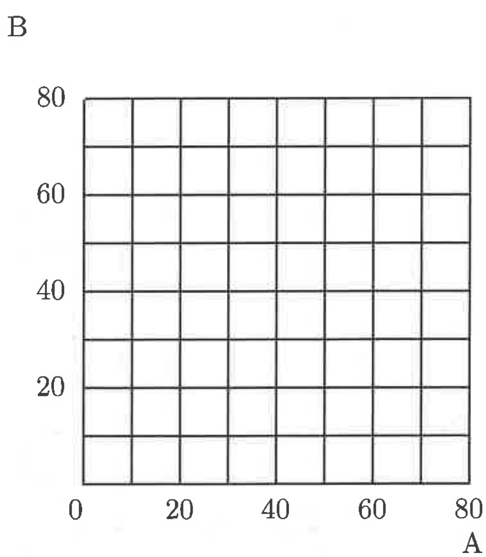


(b) Suppose that the firm wishes to produce 20 juggernauts at each plant. How much of each input will the firm need to produce 20 juggernauts at the Flint plant? ______________ How much of each input will the firm need to produce 20 juggernauts at the Inkster plant? ______________ Label with an *a* on the graph, the point representing the total amount of each of the two inputs that the firm needs to produce a total of 40 juggernauts, 20 at the Flint plant and 20 at the Inkster plant.

(c) Label with a *b* on your graph the point that shows how much of each of the two inputs is needed in toto if the firm is to produce 10 juggernauts in the Flint plant and 30 juggernauts in the Inkster plant. Label with a *c* the point that shows how much of each of the two inputs that the firm needs in toto if it is to produce 30 juggernauts in the Flint plant and 10 juggernauts in the Inkster plant. Use a black pen to draw the firm's isoquant for producing 40 units of output if it can split production in any manner between the two plants. Is the technology available to this firm convex?____.

19.6 (0) You manage a crew of 160 workers who could be assigned to make either of two products. Product A requires 2 workers per unit of output. Product B requires 4 workers per unit of output.

(a) Write an equation to express the combinations of products A and B that could be produced using exactly 160 workers. ______________ On the diagram below, use blue ink to shade in the area depicting the combinations of A and B that could be produced with 160 workers. (Assume that it is also possible for some workers to do nothing at all.)



(b) Suppose now that every unit of product A that is produced requires the use of 4 shovels as well as 2 workers and that every unit of product B produced requires 2 shovels and 4 workers. On the graph you have just drawn, use red ink to shade in the area depicting combinations of A and B that could be produced with 180 shovels if there were no worries about the labor supply. Write down an equation for the set of combinations of A and B that require exactly 180 shovels.____________________.

(c) On the same diagram, use black ink to shade the area that represents possible output combinations when one takes into account both the limited supply of labor and the limited supply of shovels.

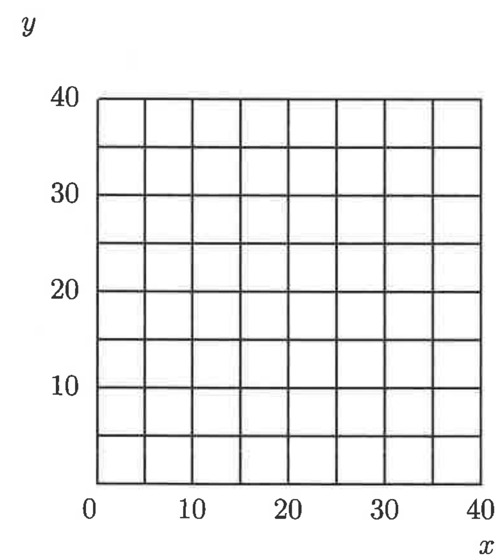
(d) On your diagram locate the feasible combination of inputs that use up all of the labor and all of the shovels. If you didn't have the graph, what equations would you solve to determine this point?____________

____.

(e) If you have 160 workers and 180 shovels, what is the largest amount of product A that you could produce? ____________ If you produce this amount, you will not use your entire supply of one of the inputs. Which one? ____________ How many will be left unused?____________.

19.7 (0) A firm has the production function $f(x, y) = \min\{2x, x + y\}$. On the graph below, use red ink to sketch a couple of production isoquants for this firm. A second firm has the production function $f(x, y) = x + \min\{x, y\}$. Do either or both of these firms have constant returns to scale?

____________ On the same graph, use black ink to draw a couple of isoquants for the second firm.



19.8 (0) Suppose the production function has the form

$$f(x_1, x_2, x_3) = Ax_1^a x_2^b x_3^c,$$

where $a + b + c > 1$. Prove that there are increasing returns to scale.

____.

19.9 (0) Suppose that the production function is $f(x_1, x_2) = Cx_1^a x_2^b$, where a, b, and C are positive constants.

(a) For what positive values of a, b, and C are there decreasing returns to scale?______________ constant returns to scale? ______________ increasing returns to scale?______________.

(b) For what positive values of a, b, and C is there decreasing marginal product for factor 1?______________.

(c) For what positive values of a, b, and C is there diminishing technical rate of substitution?______________.

19.10 (0) Suppose that the production function is $f(x_1, x_2) = (x_1^a + x_2^a)^b$, where a and b are positive constants.

(a) For what positive values of a and b are there decreasing returns to scale?__________ Constant returns to scale? __________ Increasing returns to scale?______________.

19.11 (0) Suppose that a firm has the production function $f(x_1, x_2) = \sqrt{x_1} + x_2^2$.

(a) The marginal product of factor 1 (increases, decreases, stays constant) ______________ as the amount of factor 1 increases. The marginal product of factor 2 (increases, decreases, stays constant) ______________ as the amount of factor 2 increases.

(b) This production function does not satisfy the definition of increasing returns to scale, constant returns to scale, or decreasing returns to scale.

How can this be? ______________________________

______________________________ Find a combination of inputs such that doubling the amount of both inputs will more than double the amount of output. ______________________________ Find a combination of inputs such that doubling the amount of both inputs will less than double output.

______________________________.

Chapter 20

NAME______________________

Profit Maximization

Introduction. A firm in a competitive industry cannot charge more than the market price for its output. If it also must compete for its inputs, then it has to pay the market price for inputs as well. Suppose that a profit-maximizing competitive firm can vary the amount of only one factor and that the marginal product of this factor decreases as its quantity increases. Then the firm will maximize its profits by hiring enough of the variable factor so that the value of its marginal product is equal to the wage. Even if a firm uses several factors, only some of them may be variable in the short run.

Example: A firm has the production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Suppose that this firm is using 16 units of factor 2 and is unable to vary this quantity in the short run. In the short run, the only thing that is left for the firm to choose is the amount of factor 1. Let the price of the firm's output be p, and let the price it pays per unit of factor 1 be w_1. We want to find the amount of x_1 that the firm will use and the amount of output it will produce. Since the amount of factor 2 used in the short run must be 16, we have output equal to $f(x_1, 16) = 4x_1^{1/2}$. The marginal product of x_1 is calculated by taking the derivative of output with respect to x_1. This marginal product is equal to $2x_1^{-1/2}$. Setting the value of the marginal product of factor 1 equal to its wage, we have $p2x_1^{-1/2} = w_1$. Now we can solve this for x_1. We find $x_1 = (2p/w_1)^2$. Plugging this into the production function, we see that the firm will choose to produce $4x_1^{1/2} = 8p/w_1$ units of output.

In the long run, a firm is able to vary all of its inputs. Consider the case of a competitive firm that uses two inputs. Then if the firm is maximizing its profits, it must be that the value of the marginal product of each of the two factors is equal to its wage. This gives two equations in the two unknown factor quantities. If there are decreasing returns to scale, these two equations are enough to determine the two factor quantities. If there are constant returns to scale, it turns out that these two equations are only sufficient to determine the *ratio* in which the factors are used.

In the problems on the weak axiom of profit maximization, you are asked to determine whether the observed behavior of firms is consistent with profit-maximizing behavior. To do this you will need to plot some of the firm's isoprofit lines. An isoprofit line relates all of the input-output combinations that yield the same amount of profit for some given input and output prices. To get the equation for an isoprofit line, just write down an equation for the firm's profits at the given input and output prices. Then solve it for the amount of output produced as a function of the amount of the input chosen. Graphically, you know that a firm's behavior is consistent with profit maximization if its input-output choice in each period lies below the isoprofit lines of the other periods.

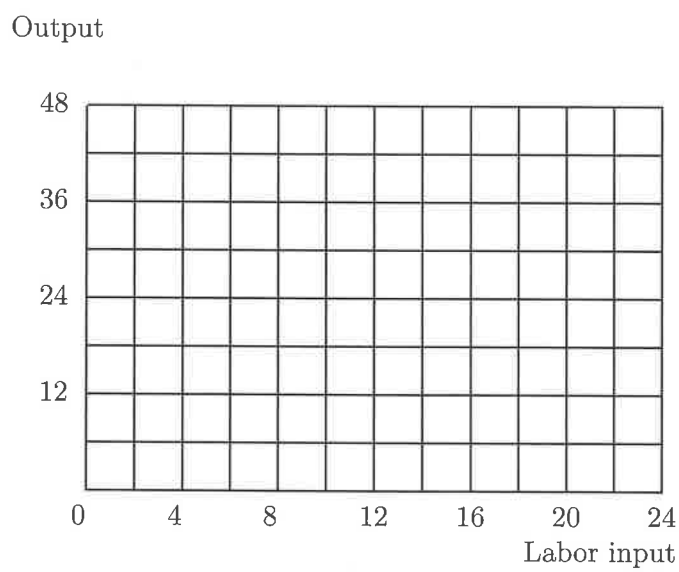
20.1 (0) The short-run production function of a competitive firm is given by $f(L) = 6L^{2/3}$, where L is the amount of labor it uses. (For those who do not know calculus—if total output is aL^b, where a and b are constants, and where L is the amount of some factor of production, then the marginal product of L is given by the formula abL^{b-1}.) The cost per unit of labor is $w = 6$ and the price per unit of output is $p = 3$.

(a) Plot a few points on the graph of this firm's production function and sketch the graph of the production function, using blue ink. Use black ink to draw the isoprofit line that passes through the point $(0, 12)$, the isoprofit line that passes through $(0, 8)$, and the isoprofit line that passes through the point $(0, 4)$. What is the slope of each of the isoprofit lines?

________________ How many points on the isoprofit line through $(0, 12)$ consist of input-output points that are actually possible?________

________ Make a squiggly line over the part of the isoprofit line through $(0, 4)$ that consists of outputs that are actually possible.

(b) How many units of labor will the firm hire? ________ How much output will it produce? ________ If the firm has no other costs, how much will its total profits be?________________________.



(c) Suppose that the wage of labor falls to 4, and the price of output remains at p. On the graph, use red ink to draw the new isoprofit line for the firm that passes through its old choice of input and output. Will the firm increase its output at the new price?________ Explain why,

referring to your diagram.______

______.

Calculus **20.2 (0)** A Los Angeles firm uses a single input to produce a recreational commodity according to a production function $f(x) = 4\sqrt{x}$, where x is the number of units of input. The commodity sells for \$100 per unit. The input costs \$50 per unit.

(a) Write down a function that states the firm's profit as a function of the amount of input.______.

(b) What is the profit-maximizing amount of input? ______ of output? ______ How much profits does it make when it maximizes profits? ______.

(c) Suppose that the firm is taxed \$20 per unit of its output and the price of its input is subsidized by \$10. What is its new input level? ______ What is its new output level? ______ How much profit does it make now?______ (Hint: A good way to solve this is to write an expression for the firm's profit as a function of its input and solve for the profit-maximizing amount of input.)

(d) Suppose that instead of these taxes and subsidies, the firm is taxed at 50% of its profits. Write down its after-tax profits as a function of the amount of input.______ What is the profit-maximizing amount of output? ______ How much profit does it make after taxes?______.

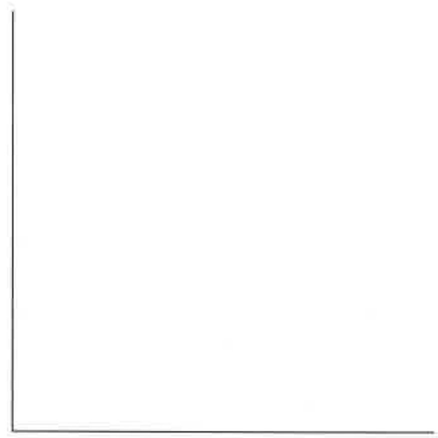
20.3 (0) Brother Jed takes heathens and reforms them into righteous individuals. There are two inputs needed in this process: heathens (who are widely available) and preaching. The production function has the following form: $r_p = \min\{h, p\}$, where r_p is the number of righteous persons produced, h is the number of heathens who attend Jed's sermons, and p is the number of hours of preaching. For every person converted, Jed receives a payment of s from the grateful convert. Sad to say, heathens do not flock to Jed's sermons of their own accord. Jed must offer heathens a payment of w to attract them to his sermons. Suppose the amount of preaching is fixed at $\bar{p}$ and that Jed is a profit-maximizing prophet.

(a) If $h < \bar{p}$, what is the marginal product of heathens? ________ What is the value of the marginal product of an additional heathen?________.

(b) If $h > \bar{p}$, what is the marginal product of heathens? ________ What is the value of the marginal product of an additional heathen in this case?

____.

(c) Sketch the shape of this production function in the graph below. Label the axes, and indicate the amount of the input where $h = \bar{p}$.



(d) If $w < s$, how many heathens will be converted? ________ If $w > s$, how many heathens will be converted?________________________.

20.4 (0) Allie's Apples, Inc. purchases apples in bulk and sells two products, boxes of apples and jugs of cider. Allie's has capacity limitations of three kinds: warehouse space, crating facilities, and pressing facilities. A box of apples requires 6 units of warehouse space, 2 units of crating facilities, and no pressing facilities. A jug of cider requires 3 units of warehouse space, 2 units of crating facilities, and 1 unit of pressing facilities. The total amounts available each day are: 1,200 units of warehouse space, 600 units of crating facilities, and 250 units of pressing facilities.

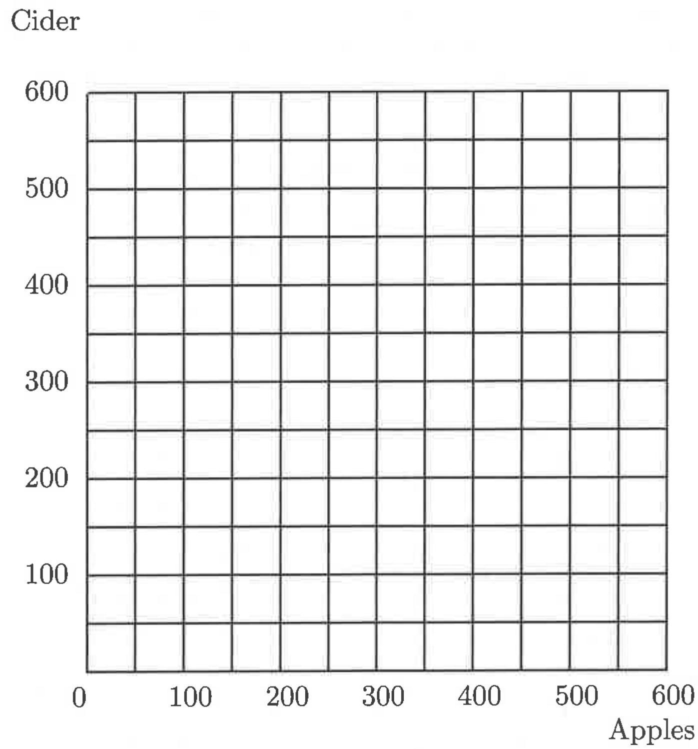
(a) If the only capacity limitations were on warehouse facilities, and if all warehouse space were used for the production of apples, how many boxes of apples could be produced in one day? ________ How many jugs of cider could be produced each day if, instead, all warehouse space were used in the production of cider and there were no other capacity constraints? ________ Draw a blue line in the following graph to represent the warehouse space constraint on production combinations.

(b) Following the same reasoning, draw a red line to represent the constraints on output to limitations on crating capacity. How many boxes of apples could Allie produce if he only had to worry about crating capacity?

________ How many jugs of cider?________________.

(c) Finally draw a black line to represent constraints on output combinations due to limitations on pressing facilities. How many boxes of apples could Allie produce if he only had to worry about the pressing capacity and no other constraints? ______________ How many jugs of cider?________________________________.

(d) Now shade the area that represents feasible combinations of daily production of apples and cider for Allie's Apples.



(e) Allie's can sell apples for \$5 per box of apples and cider for \$2 per jug. Draw a black line to show the combinations of sales of apples and cider that would generate a revenue of \$1,000 per day. At the profit-maximizing production plan, Allie's is producing ______ boxes of apples and ______ jugs of cider. Total revenues are________________.

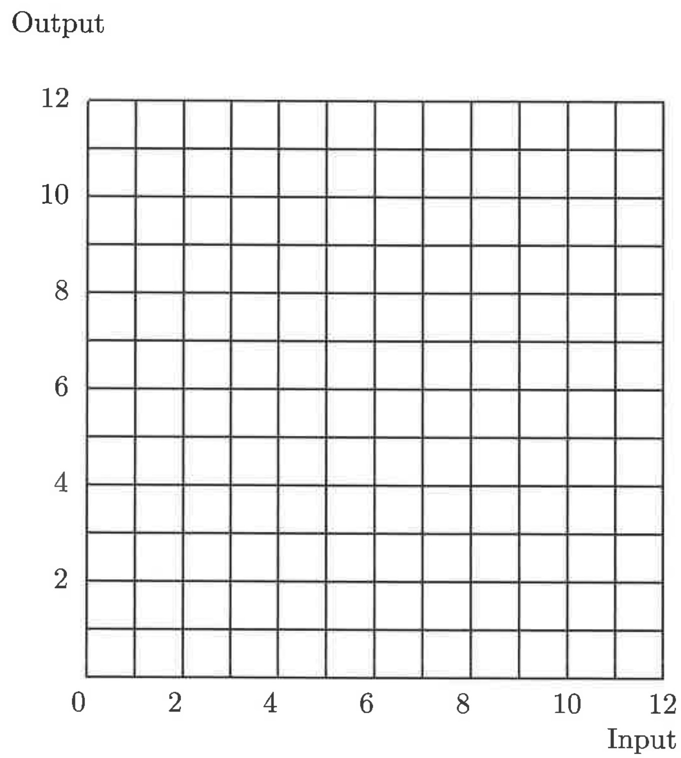
20.5 (0) A profit-maximizing firm produces one output, y, and uses one input, x, to produce it. The price per unit of the factor is denoted by

w and the price of the output is denoted by p. You observe the firm's behavior over three periods and find the following:

Period	y	x	w	p
1	1	1	1	1
2	2.5	3	.5	1
3	4	8	.25	1

(a) Write an equation that gives the firm's profits, π, as a function of the amount of input x it uses, the amount of output y it produces, the per-unit cost of the input w, and the price of output p.__________________.

(b) In the diagram below, draw an isoprofit line for each of the three periods, showing combinations of input and output that would yield the same profits that period as the combination actually chosen. What are the equations for these three lines?____________________________ Using the theory of revealed profitability, shade in the region on the graph that represents input-output combinations that could be feasible as far as one can tell from the evidence that is available. How would you describe this region in words?____.

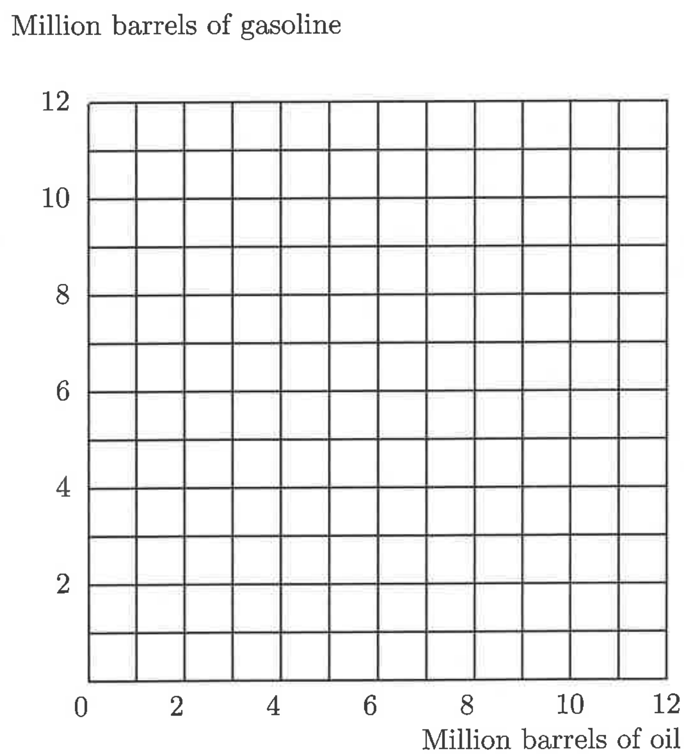


20.6 (0) T-bone Pickens is a corporate raider. This means that he looks for companies that are not maximizing profits, buys them, and then tries to operate them at higher profits. T-bone is examining the financial records of two refineries that he might buy, the Shill Oil Company and the Golf Oil Company. Each of these companies buys oil and produces gasoline. During the time period covered by these records, the price of gasoline fluctuated significantly, while the cost of oil remained constant at \$10 a barrel. For simplicity, we assume that oil is the only input to gasoline production.

Shill Oil produced 1 million barrels of gasoline using 1 million barrels of oil when the price of gasoline was \$10 a barrel. When the price of gasoline was \$20 a barrel, Shill produced 3 million barrels of gasoline using 4 million barrels of oil. Finally, when the price of gasoline was \$40 a barrel, Shill used 10 million barrels of oil to produce 5 million barrels of gasoline.

Golf Oil (which is managed by Martin E. Lunch III) did exactly the same when the price of gasoline was \$10 and \$20, but when the price of gasoline hit \$40, Golf produced 3.5 million barrels of gasoline using 8 million barrels of oil.

(a) Using black ink, plot Shill Oil's isoprofit lines and choices for the three different periods. Label them 10, 20, and 40. Using red ink draw Golf Oil's isoprofit line and production choice. Label it with a 40 in red ink.



(b) How much profits could Golf Oil have made when the price of gasoline was \$40 a barrel if it had chosen to produce the same amount that it did when the price was \$20 a barrel? ______ What profits did Golf actually make when the price of gasoline was \$40? ______.

(c) Is there any evidence that Shill Oil is not maximizing profits? Explain.

______.

(d) Is there any evidence that Golf Oil is not maximizing profits? Explain.

______

______.

20.7 (0) After carefully studying Shill Oil, T-bone Pickens decides that it has probably been maximizing its profits. But he still is very interested in buying Shill Oil. He wants to use the gasoline they produce to fuel his delivery fleet for his chicken farms, Capon Truckin'. In order to do this Shill Oil would have to be able to produce 5 million barrels of gasoline from 8 million barrels of oil. Mark this point on your graph. Assuming that Shill always maximizes profits, would it be technologically feasible for it to produce this input-output combination? Why or why not?

______

______.

20.8 (0) Suppose that firms operate in a competitive market, attempt to maximize profits, and only use one factor of production. Then we know that for any changes in the input and output price, the input choice and the output choice must obey the Weak Axiom of Profit Maximization, $\Delta p \Delta y - \Delta w \Delta x \geq 0$.

Which of the following propositions can be proven by the Weak Axiom of Profit Maximizing Behavior (WAPM)? Respond yes or no, and give a short argument.

(a) If the price of the input does not change, then a decrease in the price of the output will imply that the firm will produce the same amount or less output. ______

______.

(b) If the price of the output remains constant, then a decrease in the input price will imply that the firm will use the same amount or more of the input. ______

______.

(c) If both the price of the output and the input increase and the firm produces less output, then the firm will use more of the input.______

______.

20.9 (1) Farmer Hoglund has discovered that on his farm, he can get 30 bushels of corn per acre if he applies no fertilizer. When he applies N pounds of fertilizer to an acre of land, the *marginal product* of fertilizer is $1 - N/200$ bushels of corn per pound of fertilizer.

(a) If the price of corn is \$3 a bushel and the price of fertilizer is \$$p$ per pound (where $p < 3$), how many pounds of fertilizer should he use per acre in order to maximize profits?______.

(b) (Only for those who remember a bit of easy integral calculus.) Write down a function that states Farmer Hoglund's yield per acre as a function of the amount of fertilizer he uses.______.

(c) Hoglund's neighbor, Skoglund, has better land than Hoglund. In fact, for any amount of fertilizer that he applies, he gets exactly twice as much corn per acre as Hoglund would get with the same amount of fertilizer. How much fertilizer will Skoglund use per acre when the price of corn is \$3 a bushel and the price of fertilizer is \$$p$ a pound? ______ (Hint: Start by writing down Skoglund's marginal product of fertilizer as a function of N.)

(d) When Hoglund and Skoglund are both maximizing profits, will Skoglund's output be more than twice as much, less than twice as much or exactly twice as much as Hoglund's? Explain.______

______

______.

(e) Explain how someone who looked at Hoglund's and Skoglund's corn yields and their fertilizer inputs but couldn't observe the quality of their land, would get a misleading idea of the productivity of fertilizer.______

______.

20.10 (0) A firm has two variable factors and a production function, $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$. The price of its output is 4. Factor 1 receives a wage of w_1 and factor 2 receives a wage of w_2.

(a) Write an equation that says that the value of the marginal product of factor 1 is equal to the wage of factor 1 ________________ and an equation that says that the value of the marginal product of factor 2 is equal to the wage of factor 2.________________ Solve two equations in the two unknowns, x_1 and x_2, to give the amounts of factors 1 and 2 that maximize the firm's profits as a function of w_1 and w_2. This gives $x_1 =$________________ and $x_2 =$________________ (Hint: You could use the first equation to solve for x_1 as a function of x_2 and of the factor wages. Then substitute the answer into the second equation and solve for x_2 as a function of the two wage rates. Finally use your solution for x_2 to find the solution for x_1.)

(b) If the wage of factor 1 is 2, and the wage of factor 2 is 1, how many units of factor 1 will the firm demand?________ How many units of factor 2 will it demand?________ How much output will it produce? ________ How much profit will it make?________________.

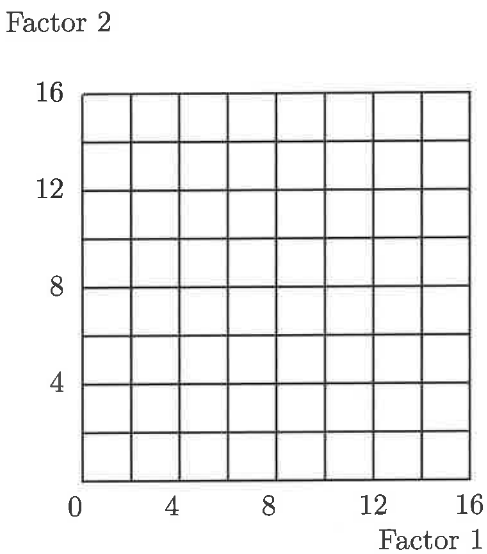
20.11 (0) A firm has two variable factors and a production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. The price of its output is 4, the price of factor 1 is w_1, and the price of factor 2 is w_2.

(a) Write the two equations that say that the value of the marginal product of each factor is equal to its wage. ________________ ________ If $w_1 = 2w_2$, these two equations imply that $x_1/x_2 =$________.

(b) For this production function, is it possible to solve the two marginal productivity equations uniquely for x_1 and x_2?________________.

20.12 (1) A firm has two variable factors and a production function $f(x_1, x_2) = \sqrt{2x_1 + 4x_2}$. On the graph below, draw production isoquants corresponding to an ouput of 3 and to an output of 4.

(a) If the price of the output good is 4, the price of factor 1 is 2, and the price of factor 2 is 3, find the profit-maximizing amount of factor 1 ________, the profit-maximizing amount of factor 2________, and the profit-maximizing output________________.

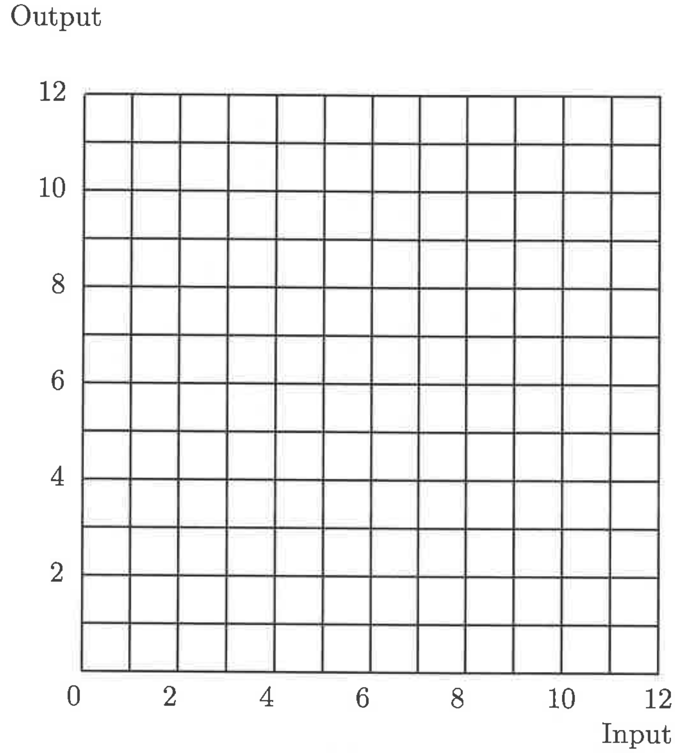


20.13 (0) A profit-maximizing firm produces one output, y, and uses one input, x, to produce it. The price per unit of the factor is denoted by w and the price of the output is denoted by p. You observe the firm's behavior over three periods and find the following:

Period	y	x	w	p
1	1	1	1	1
2	2.5	3	.5	1
3	4	8	.25	1

(a) Write an equation that gives the firm's profits, π, as a function of the amount of input x it uses, the amount of output y it produces, the per-unit cost of the input w, and the price of output p. ______________.

(b) In the diagram below, draw an isoprofit line for each of the three periods, showing combinations of input and output that would yield the same profits that period as the combination actually chosen. What are the equations for these three lines? ______________ Using the theory of revealed profitability, shade in the region on the graph that represents input-output combinations that could be feasible as far as one can tell from the evidence that is available. How would you describe this region in words? ______________.

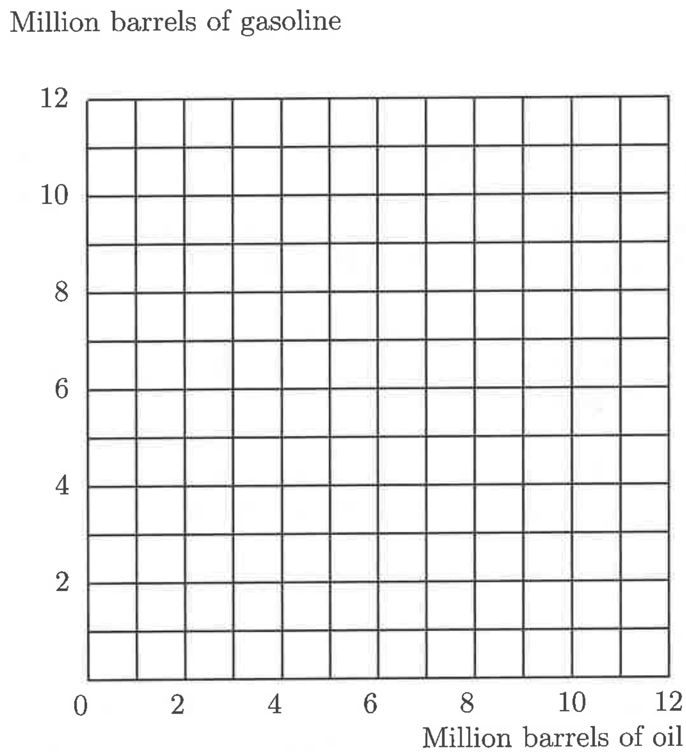


20.14 (0) T-bone Pickens is a corporate raider. This means that he looks for companies that are not maximizing profits, buys them, and then tries to operate them at higher profits. T-bone is examining the financial records of two refineries that he might buy, the Shill Oil Company and the Golf Oil Company. Each of these companies buys oil and produces gasoline. During the time period covered by these records, the price of gasoline fluctuated significantly, while the cost of oil remained constant at \$10 a barrel. For simplicity, we assume that oil is the only input to gasoline production.

Shill Oil produced 1 million barrels of gasoline using 1 million barrels of oil when the price of gasoline was \$10 a barrel. When the price of gasoline was \$20 a barrel, Shill produced 3 million barrels of gasoline using 4 million barrels of oil. Finally, when the price of gasoline was \$40 a barrel, Shill used 10 million barrels of oil to produce 5 million barrels of gasoline.

Golf Oil (which is managed by Martin E. Lunch III) did exactly the same when the price of gasoline was \$10 and \$20, but when the price of gasoline hit \$40, Golf produced 3.5 million barrels of gasoline using 8 million barrels of oil.

(a) Using black ink, plot Shill Oil's isoprofit lines and choices for the three different periods. Label them 10, 20, and 40. Using red ink draw Golf Oil's isoprofit line and production choice. Label it with a 40 in red ink.



(b) How much profits could Golf Oil have made when the price of gasoline was $40 a barrel if it had chosen to produce the same amount that it did when the price was $20 a barrel? ________________ What profits did Golf actually make when the price of gasoline was $40?________________.

(c) Is there any evidence that Shill Oil is not maximizing profits? Explain.

____.

(d) Is there any evidence that Golf Oil is not maximizing profits? Explain.

____

____.

20.15 (0) After carefully studying Shill Oil, T-bone Pickens decides that it has probably been maximizing its profits. But he still is very interested in buying Shill Oil. He wants to use the gasoline they produce to fuel his delivery fleet for his chicken farms, Capon Truckin'. In order to do this Shill Oil would have to be able to produce 5 million barrels of gasoline from 8 million barrels of oil. Mark this point on your graph. Assuming that Shill always maximizes profits, would it be technologically feasible for it to produce this input-output combination? Why or why not?

____

____.

20.16 (0) Suppose that firms operate in a competitive market, attempt to maximize profits, and only use one factor of production. Then we know that for any changes in the input and output price, the input choice and the output choice must obey the Weak Axiom of Profit Maximization, $\Delta p \Delta y - \Delta w \Delta x \geq 0$.

Which of the following propositions can be proven by the Weak Axiom of Profit Maximizing Behavior (WAPM)? Respond yes or no, and give a short argument.

(a) If the price of the input does not change, then a decrease in the price of the output will imply that the firm will produce the same amount or less output.____

____.

(b) If the price of the output remains constant, then a decrease in the input price will imply that the firm will use the same amount or more of the input.____

____.

(c) If both the price of the output and the input increase and the firm produces less output, then the firm will use more of the input.__________

____.

Chapter 21

NAME________________________

Cost Minimization

Introduction. In the chapter on consumer choice, you studied a consumer who tries to maximize his utility subject to the constraint that he has a fixed amount of money to spend. In this chapter you study the behavior of a firm that is trying to produce a fixed amount of output in the cheapest possible way. In both theories, you look for a point of tangency between a curved line and a straight line. In consumer theory, there is an "indifference curve" and a "budget line." In producer theory, there is a "production isoquant" and an "isocost line." As you recall, in consumer theory, finding a tangency gives you only one of the two equations you need to locate the consumer's chosen point. The second equation you used was the budget equation. In cost-minimization theory, again the tangency condition gives you one equation. This time you don't know in advance how much the producer is spending; instead you are told how much output he wants to produce and must find the cheapest way to produce it. So your second equation is the equation that tells you that the desired amount is being produced.

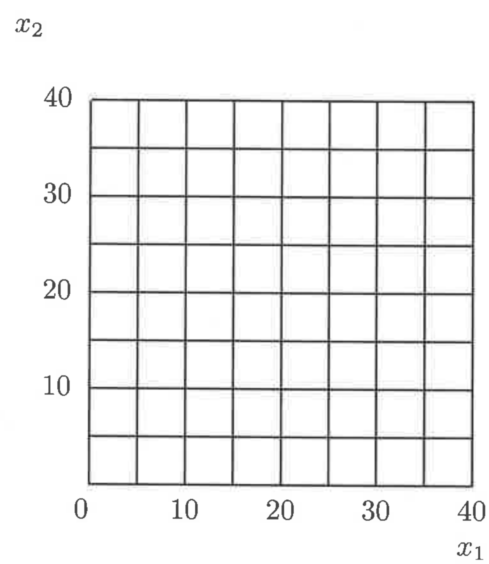
Example. A firm has the production function $f(x_1, x_2) = (\sqrt{x_1} + 3\sqrt{x_2})^2$. The price of factor 1 is $w_1 = 1$ and the price of factor 2 is $w_2 = 1$. Let us find the cheapest way to produce 16 units of output. We will be looking for a point where the technical rate of substitution equals $-w_1/w_2$. If you calculate the technical rate of substitution (or look it up from the warm up exercise in Chapter 18), you find $TRS(x_1, x_2) = -(1/3)(x_2/x_1)^{1/2}$. Therefore we must have $-(1/3)(x_2/x_1)^{1/2} = -w_1/w_2 = -1$. This equation can be simplified to $x_2 = 9x_1$. So we know that the combination of inputs chosen has to lie somewhere on the line $x_2 = 9x_1$. We are looking for the cheapest way to produce 16 units of output. So the point we are looking for must satisfy the equation $(\sqrt{x_1} + 3\sqrt{x_2})^2 = 16$, or equivalently $\sqrt{x_1} + 3\sqrt{x_2} = 4$. Since $x_2 = 9x_1$, we can substitute for x_2 in the previous equation to get $\sqrt{x_1} + 3\sqrt{9x_1} = 4$. This equation simplifies further to $10\sqrt{x_1} = 4$. Solving this for x_1, we have $x_1 = 16/100$. Then $x_2 = 9x_1 = 144/100$.

The amounts x_1 and x_2 that we solved for in the previous paragraph are known as the *conditional factor demands for factors 1 and 2*, conditional on the wages $w_1 = 1$, $w_2 = 1$, and output $y = 16$. We express this by saying $x_1(1, 1, 16) = 16/100$ and $x_2(1, 1, 16) = 144/100$. Since we know the amount of each factor that will be used to produce 16 units of output and since we know the price of each factor, we can now calculate the cost of producing 16 units. This cost is $c(w_1, w_2, 16) = w_1x_1(w_1, w_2, 16) + w_2x_2(w_1, w_2, 16)$. In this instance since $w_1 = w_2 = 1$, we have $c(1, 1, 16) = x_1(1, 1, 16) + x_2(1, 1, 16) = 160/100$.

In consumer theory, you also dealt with cases where the consumer's indifference "curves" were straight lines and with cases where there were kinks in the indifference curves. Then you found that the consumer's choice might occur at a boundary or at a kink. Usually a careful look at the diagram would tell you what is going on. The story with kinks and boundary solutions is almost exactly the same in the case of cost-minimizing firms. You will find some exercises that show how this works.

21.1 (0) Nadine sells user-friendly software. Her firm's production function is $f(x_1, x_2) = x_1 + 2x_2$, where x_1 is the amount of unskilled labor and x_2 is the amount of skilled labor that she employs.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 20 units of output. Draw another isoquant representing input combinations that will produce 40 units of output.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?____________________.

(c) If Nadine uses only unskilled labor, how much unskilled labor would she need in order to produce y units of output?____________________.

(d) If Nadine uses only skilled labor to produce output, how much skilled labor would she need in order to produce y units of output?____________________.

(e) If Nadine faces factor prices $(1, 1)$, what is the cheapest way for her to produce 20 units of output? $x_1 =$ __________, $x_2 =$____________________.

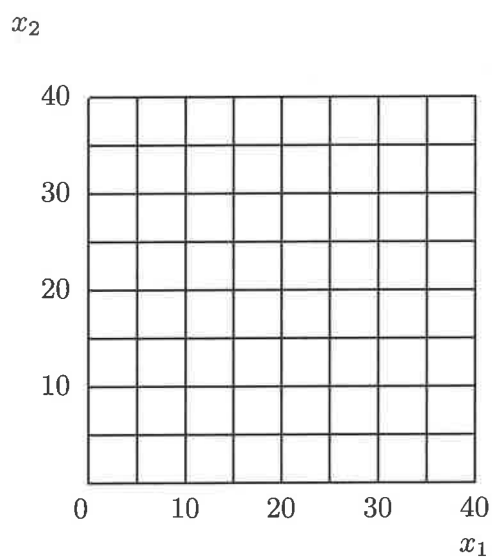
(f) If Nadine faces factor prices $(1, 3)$, what is the cheapest way for her to produce 20 units of output? $x_1 =$ ________ , $x_2 =$ ________.

(g) If Nadine faces factor prices (w_1, w_2), what will be the minimal cost of producing 20 units of output? ________________.

(h) If Nadine faces factor prices (w_1, w_2), what will be the minimal cost of producing y units of output? ________________.

21.2 (0) The Ontario Brassworks produces brazen effronteries. As you know brass is an alloy of copper and zinc, used in fixed proportions. The production function is given by: $f(x_1, x_2) = \min\{x_1, 2x_2\}$, where x_1 is the amount of copper it uses and x_2 is the amount of zinc that it uses in production.

(a) Illustrate a typical isoquant for this production function in the graph below.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale? ________________.

(c) If the firm wanted to produce 10 effronteries, how much copper would it need? ________ How much zinc would it need? ________.

(d) If the firm faces factor prices $(1, 1)$, what is the cheapest way for it to produce 10 effronteries? How much will this cost?________________

____

____.

(e) If the firm faces factor prices (w_1, w_2), what is the cheapest cost to produce 10 effronteries?________________________.

(f) If the firm faces factor prices (w_1, w_2), what will be the minimal cost of producing y effronteries?________________________.

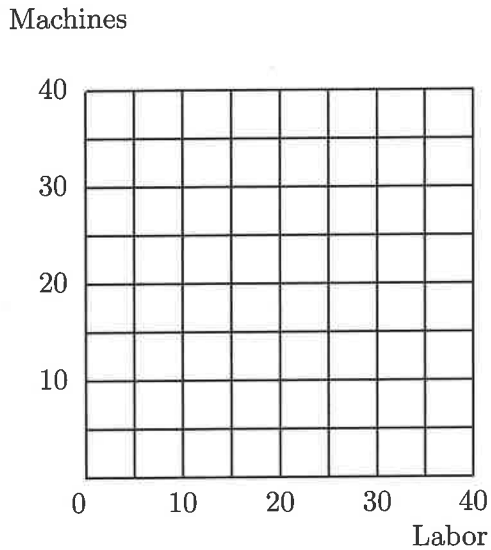
Calculus **21.3 (0)** A firm uses labor and machines to produce output according to the production function $f(L, M) = 4L^{1/2}M^{1/2}$, where L is the number of units of labor used and M is the number of machines. The cost of labor is \$40 per unit and the cost of using a machine is \$10.

(a) On the graph below, draw an isocost line for this firm, showing combinations of machines and labor that cost \$400 and another isocost line showing combinations that cost \$200. What is the slope of these isocost lines?________________________.

(b) Suppose that the firm wants to produce its output in the cheapest possible way. Find the number of machines it would use per worker. (Hint: The firm will produce at a point where the slope of the production isoquant equals the slope of the isocost line.)________________.

(c) On the graph, sketch the production isoquant corresponding to an output of 40. Calculate the amount of labor ________ and the number of machines ________ that are used to produce 40 units of output in the cheapest possible way, given the above factor prices. Calculate the cost of producing 40 units at these factor prices: $c(40, 10, 40) =$____________.

(d) How many units of labor ______ and how many machines ______ would the firm use to produce y units in the cheapest possible way? How much would this cost? ________ (Hint: Notice that there are constant returns to scale.)



21.4 (0) Earl sells lemonade in a competitive market on a busy street corner in Philadelphia. His production function is $f(x_1, x_2) = x_1^{1/3} x_2^{1/3}$, where output is measured in gallons, x_1 is the number of pounds of lemons he uses, and x_2 is the number of labor-hours spent squeezing them.

(a) Does Earl have constant returns to scale, decreasing returns to scale, or increasing returns to scale?________________.

(b) Where w_1 is the cost of a pound of lemons and w_2 is the wage rate for lemon-squeezers, the cheapest way for Earl to produce lemonade is to use ________ hours of labor per pound of lemons. (Hint: Set the slope of his isoquant equal to the slope of his isocost line.)

(c) If he is going to produce y units in the cheapest way possible, then the number of pounds of lemons he will use is $x_1(w_1, w_2, y) =$________ and the number of hours of labor that he will use is $x_2(w_1, w_2, y) =$________ ________ (Hint: Use the production function and the equation you found in the last part of the answer to solve for the input quantities.)

(d) The cost to Earl of producing y units at factor prices w_1 and w_2 is $c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y) =$________________.

21.5 (0) The prices of inputs (x_1, x_2, x_3, x_4) are $(4, 1, 3, 2)$.

(a) If the production function is given by $f(x_1, x_2) = \min\{x_1, x_2\}$, what is the minimum cost of producing one unit of output?________________.

(b) If the production function is given by $f(x_3, x_4) = x_3 + x_4$, what is the minimum cost of producing one unit of output?________________.

(c) If the production function is given by $f(x_1, x_2, x_3, x_4) = \min\{x_1 + x_2, x_3 + x_4\}$, what is the minimum cost of producing one unit of output?

________________________________.

(d) If the production function is given by $f(x_1, x_2) = \min\{x_1, x_2\} + \min\{x_3, x_4\}$, what is the minimum cost of producing one unit of output?

________________________________.

21.6 (0) Joe Grow, an avid indoor gardener, has found that the number of happy plants, h, depends on the amount of light, l, and water, w. In fact, Joe noticed that plants require twice as much light as water, and any more or less is wasted. Thus, Joe's production function is $h = \min\{l, 2w\}$.

(a) Suppose Joe is using 1 unit of light, what is the least amount of water he can use and still produce a happy plant?________________.

(b) If Suppose Joe wants to produce 4 happy plants, what are the minimum amounts of light and water required?________________.

(c) Joe's conditional factor demand function for light is $l(w_1, w_2, h) =$ ________ and his conditional factor demand function for water is $w(w_1, w_2, h) =$________________.

(d) If each unit of light costs w_1 and each unit of water costs w_2, Joe's cost function is $c(w_1, w_2, h) =$________________.

21.7 (1) Joe's sister, Flo Grow, is a university administrator. She uses an alternative method of gardening. Flo has found that happy plants only need fertilizer and talk. (*Warning:* Frivolous observations about university administrators' talk being a perfect substitute for fertilizer is in extremely poor taste.) Where f is the number of bags of fertilizer used and t is the number of hours she talks to her plants, the number of happy plants produced is exactly $h = t + 2f$. Suppose fertilizer costs w_f per bag and talk costs w_t per hour.

(a) If Flo uses no fertilizer, how many hours of talk must she devote if she wants one happy plant? __________ If she doesn't talk to her plants at all, how many bags of fertilizer will she need for one happy plant?

____.

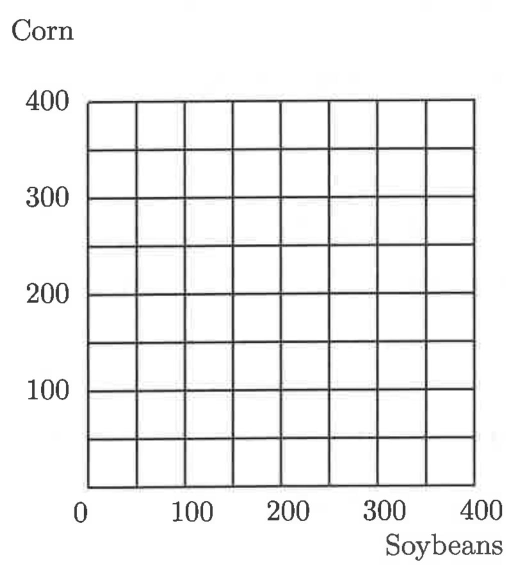
(b) If $w_t < w_f/2$, would it be cheaper for Flo to use fertilizer or talk to raise one happy plant? ____.

(c) Flo's cost function is $c(w_f, w_t, h) =$ ____.

(d) Her conditional factor demand for talk is $t(w_f, w_t, h) =$ __________ if $w_t < w_f/2$ and __________ if $w_t > w_f/2$.

21.8 (0) Remember T-bone Pickens, the corporate raider? Now he's concerned about his chicken farms, Pickens's Chickens. He feeds his chickens on a mixture of soybeans and corn, depending on the prices of each. According to the data submitted by his managers, when the price of soybeans was \$10 a bushel and the price of corn was \$10 a bushel, they used 50 bushels of corn and 150 bushels of soybeans for each coop of chickens. When the price of soybeans was \$20 a bushel and the price of corn was \$10 a bushel, they used 300 bushels of corn and no soybeans per coop of chickens. When the price of corn was \$20 a bushel and the price of soybeans was \$10 a bushel, they used 250 bushels of soybeans and no corn for each coop of chickens.

(a) Graph these three input combinations and isocost lines in the following diagram.



(b) How much money did Pickens' managers spend per coop of chickens when the prices were $(10, 10)$? ________ When the prices were $(10, 20)$? ________ When the prices were $(20, 10)$?________.

(c) Is there any evidence that Pickens's managers were not minimizing costs? Why or why not?

________.

(d) Pickens wonders whether there are any prices of corn and soybeans at which his managers will use 150 bushels of corn and 50 bushels of soybeans to produce a coop of chickens. How much would this production method cost per coop of chickens if the prices were $p_s = 10$ and $p_c = 10$? ________ if the prices were $p_s = 10$, $p_c = 20$? ________ if the prices were $p_s = 20$, $p_c = 10$?________.

(e) If Pickens's managers were always minimizing costs, can it be possible to produce a coop of chickens using 150 bushels and 50 bushels of soybeans?________

____

________.

21.9 (0) A genealogical firm called Roots produces its output using only one input. Its production function is $f(x) = \sqrt{x}$.

(a) Does the firm have increasing, constant, or decreasing returns to scale?

________.

(b) How many units of input does it take to produce 10 units of output? ________ If the input costs w per unit, what does it cost to produce 10 units of output?________.

(c) How many units of input does it take to produce y units of output? ________ If the input costs w per unit, what does it cost to produce y units of output?________.

(d) If the input costs w per unit, what is the average cost of producing y units? $AC(w, y) =$ ________________.

21.10 (0) A university cafeteria produces square meals, using only one input and a rather remarkable production process. We are not allowed to say what that ingredient is, but an authoritative kitchen source says that "fungus is involved." The cafeteria's production function is $f(x) = x^2$, where x is the amount of input and $f(x)$ is the number of square meals produced.

(a) Does the cafeteria have increasing, constant, or decreasing returns to scale? ________________.

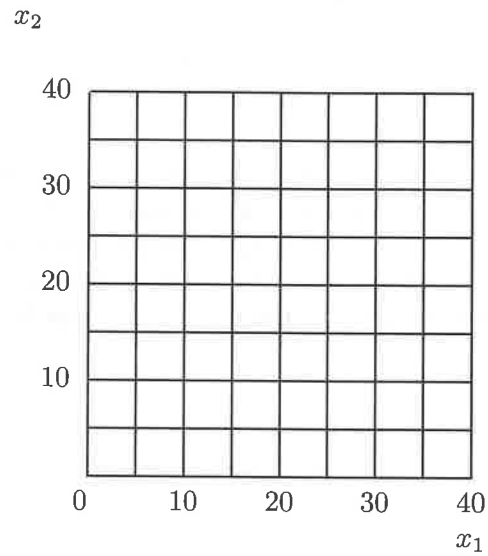
(b) How many units of input does it take to produce 144 square meals? ______ If the input costs w per unit, what does it cost to produce 144 square meals? ________________.

(c) How many units of input does it take to produce y square meals? ________ If the input costs w per unit, what does it cost to produce y square meals? ________________.

(d) If the input costs w per unit, what is the average cost of producing y square meals? $AC(w, y) =$ ________________.

21.11 (0) Irma's Handicrafts produces plastic deer for lawn ornaments. "It's hard work," says Irma, "but anything to make a buck." Her production function is given by $f(x_1, x_2) = (\min\{x_1, 2x_2\})^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of labor used, and $f(x_1, x_2)$ is the number of deer produced.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 4 deer. Draw another production isoquant representing input combinations that will produce 5 deer.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?____________________.

(c) If Irma faces factor prices $(1, 1)$, what is the cheapest way for her to produce 4 deer? __________ How much does this cost?__________.

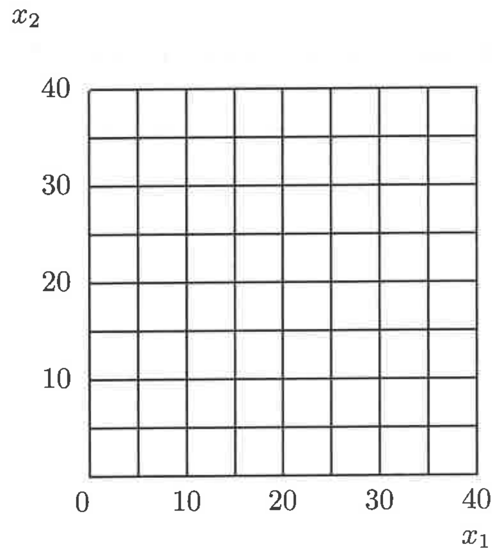
(d) At the factor prices $(1, 1)$, what is the cheapest way to produce 5 deer? __________ How much does this cost?__________.

(e) At the factor prices $(1, 1)$, the cost of producing y deer with this technology is $c(1, 1, y) =$____________________.

(f) At the factor prices (w_1, w_2), the cost of producing y deer with this technology is $c(w_1, w_2, y) =$____________________.

21.12 (0) Al Deardwarf also makes plastic deer for lawn ornaments. Al has found a way to automate the production process completely. He doesn't use any labor–only wood and plastic. Al says he likes the business "because I need the doe." Al's production function is given by $f(x_1, x_2) = (2x_1 + x_2)^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of wood used, and $f(x_1, x_2)$ is the number of deer produced.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 4 deer. Draw another production isoquant representing input combinations that will produce 6 deer.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?______________.

(c) If Al faces factor prices $(1, 1)$, what is the cheapest way for him to produce 4 deer? ________ How much does this cost?__________.

(d) At the factor prices $(1, 1)$, what is the cheapest way to produce 6 deer? ________ How much does this cost?__________.

(e) At the factor prices $(1, 1)$, the cost of producing y deer with this technology is $c(1, 1, y) =$______________.

(f) At the factor prices $(3, 1)$, the cost of producing y deer with this technology is $c(3, 1, y) =$______________.

21.13 (0) Suppose that Al Deardwarf from the last problem cannot vary the amount of wood that he uses in the short run and is stuck with using 20 units of wood. Suppose that he can change the amount of plastic that he uses, even in the short run.

(a) How much plastic would Al need in order to make 100 deer?______

______________.

(b) If the cost of plastic is \$1 per unit and the cost of wood is \$1 per unit, how much would it cost Al to make 100 deer?______________________.

(c) Write down Al's short-run cost function at these factor prices.______

____.

Chapter 22

NAME________________________

Cost Curves

Introduction. Here you continue to work on cost functions. Total cost can be divided into fixed cost, the part that doesn't change as output changes, and variable cost. To get the average (total) cost, average fixed cost, and average variable cost, just divide the appropriate cost function by y, the level of output. The marginal cost function is the derivative of the total cost function with respect to output—or the rate of increase in cost as output increases, if you don't know calculus.

Remember that the marginal cost curve intersects both the average cost curve and the average variable cost curve at their minimum points. So to find the minimum point on the average cost curve, you simply set marginal cost equal to average cost and similarly for the minimum of average variable cost.

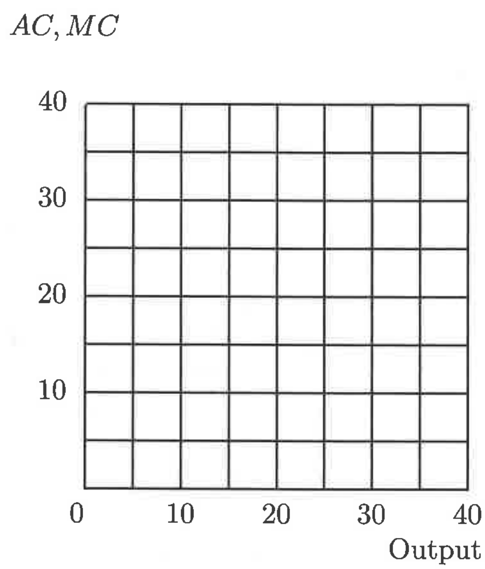
Example: A firm has the total cost function $C(y) = 100 + 10y$. Let us find the equations for its various cost curves. Total fixed costs are 100, so the equation of the average fixed cost curve is $100/y$. Total variable costs are $10y$, so average variable costs are $10y/y = 10$ for all y. Marginal cost is 10 for all y. Average total costs are $(100 + 10y)/y = 10 + 10/y$. Notice that for this firm, average total cost decreases as y increases. Notice also that marginal cost is less than average total cost for all y.

22.1 (0) Mr. Otto Carr, owner of Otto's Autos, sells cars. Otto buys autos for $\$c$ each and has no other costs.

(a) What is his total cost if he sells 10 cars? ________ What if he sells 20 cars? ________ Write down the equation for Otto's total costs assuming he sells y cars: $TC(y) =$________________________.

(b) What is Otto's average cost function? $AC(y) =$ ________ For every additional auto Otto sells, by how much do his costs increase? ________ Write down Otto's marginal cost function: $MC(y) =$________________.

(c) In the graph below draw Otto's average and marginal cost curves if $c = 20$.



(d) Suppose Otto has to pay \$$b$ a year to produce obnoxious television commercials. Otto's total cost curve is now $TC(y) =$ ________ , his average cost curve is now $AC(y) =$ ________ , and his marginal cost curve is $MC(y) =$________________________.

(e) If $b =$ \$100, use red ink to draw Otto's average cost curve on the graph above.

22.2 (0) Otto's brother, Dent Carr, is in the auto repair business. Dent recently had little else to do and decided to calculate his cost conditions. He found that the total cost of repairing s cars is $TC(s) = 2s^2 + 10$. But Dent's attention was diverted to other things ... and that's where you come in. Please complete the following:

Dent's Total Variable Costs:________________________.

Total Fixed Costs:________________________.

Average Variable Costs:________________________.

Average Fixed Costs:________________________.

Average Total Costs:________________________.

Marginal Costs:________________________.

22.3 (0) A third brother, Rex Carr, owns a junk yard. Rex can use one of two methods to destroy cars. The first involves purchasing a hydraulic car smasher that costs \$200 a year to own and then spending \$1 for every car smashed into oblivion; the second method involves purchasing a shovel that will last one year and costs \$10 and paying the last Carr brother, Scoop, to bury the cars at a cost of \$5 each.

(a) Write down the total cost functions for the two methods, where y is output per year: $TC_1(y) =$ ________ , $TC_2(y) =$________________.

(b) The first method has an average cost function ____________ and a marginal cost function ________ For the second method these costs are ________ and________________________________.

(c) If Rex wrecks 40 cars per year, which method should he use? ______ ________ If Rex wrecks 50 cars per year, which method should he use? ____________ What is the smallest number of cars per year for which it would pay him to buy the hydraulic smasher?________________.

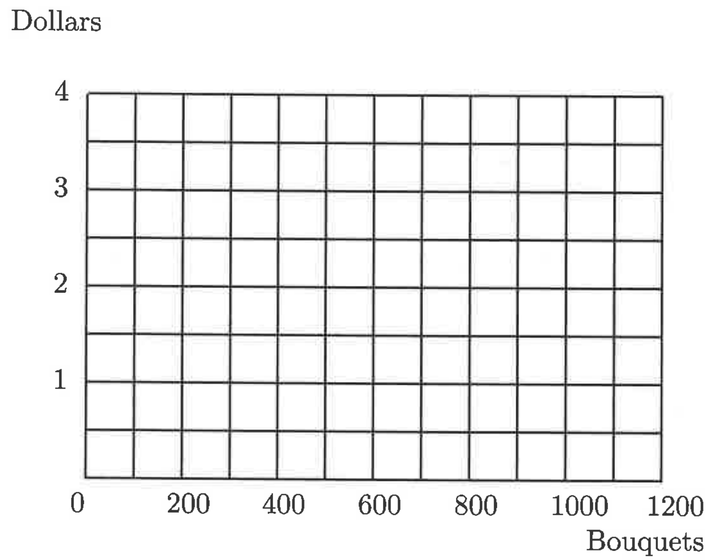
22.4 (0) Mary Magnolia wants to open a flower shop, the Petal Pusher, in a new mall. She has her choice of three different floor sizes, 200 square feet, 500 square feet, or 1,000 square feet. The monthly rent will be \$1 a square foot. Mary estimates that if she has F square feet of floor space and sells y bouquets a month, her variable costs will be $c_v(y) = y^2/F$ per month.

(a) If she has 200 square feet of floor space, write down her marginal cost function: ____________ and her average cost function: ____________ ______ At what amount of output is average cost minimized?__________ At this level of output, how much is average cost?________________.

(b) If she has 500 square feet, write down her marginal cost function: ____________ and her average cost function: ________________ At what amount of output is average cost minimized? __________ At this level of output, how much is average cost?________________.

(c) If she has 1,000 square feet of floor space, write down her marginal cost function: ________ and her average cost function: ________ ________ At what amount of output is average cost minimized? ________ At this level of output, how much is average cost?________.

(d) Use red ink to show Mary's average cost curve and her marginal cost curves if she has 200 square feet. Use blue ink to show her average cost curve and her marginal cost curve if she has 500 square feet. Use black ink to show her average cost curve and her marginal cost curve if she has 1,000 square feet. Label the average cost curves AC and the marginal cost curves MC.



(e) Use yellow marker to show Mary's long-run average cost curve and her long-run marginal cost curve in your graph. Label them LRAC and LRMC.

22.5 (0) Touchie MacFeelie publishes comic books. The only inputs he needs are old jokes and cartoonists. His production function is

$$Q = .1J^{\frac{1}{2}}L^{3/4},$$

where J is the number of old jokes used, L the number of hours of cartoonists' labor used as inputs, and Q is the number of comic books produced.

(a) Does this production process exhibit increasing, decreasing, or constant returns to scale? Explain your answer.________
________.

(b) If the number of old jokes used is 100, write an expression for the marginal product of cartoonists' labor as a function of L. ______
Is the marginal product of labor decreasing or increasing as the amount of labor increases? ______.

22.6 (0) Touchie MacFeelie's irascible business manager, Gander MacGrope, announces that old jokes can be purchased for \$1 each and that the wage rate of cartoonists' labor is \$2.

(a) Suppose that in the short run, Touchie is stuck with exactly 100 old jokes (for which he paid \$1 each) but is able to hire as much labor as he wishes. How much labor would he have to hire in order produce Q comic books? ______.

(b) Write down Touchie's short-run total cost as a function of his output

______.

(c) His short-run marginal cost function is ______.

(d) His short-run average cost function is ______.

Calculus **22.7 (1)** Touchie asks his brother, Sir Francis MacFeelie, to study the long-run picture. Sir Francis, who has carefully studied the appendix to Chapter 19 in your text, prepared the following report.

(a) If all inputs are variable, and if old jokes cost \$1 each and cartoonist labor costs \$2 per hour, the cheapest way to produce exactly one comic book is to use ______ jokes and ______ hours of labor. (Fractional jokes are certainly allowable.)

(b) This would cost ______ dollars.

(c) Given our production function, the cheapest proportions in which to use jokes and labor are the same no matter how many comic books we print. But when we double the amount of both inputs, the number of comic books produced is multiplied by ______.

22.8 (0) Consider the cost function $c(y) = 4y^2 + 16$.

(a) The average cost function is ______.

(b) The marginal cost function is____________________.

(c) The level of output that yields the minimum average cost of production is____________________.

(d) The average variable cost function is____________________.

(e) At what level of output does average variable cost equal marginal cost?____________________.

22.9 (0) A competitive firm has a production function of the form $Y = 2L + 5K$. If $w = \$2$ and $r = \$3$, what will be the minimum cost of producing 10 units of output?____________________.

Chapter 23 NAME____________________

Firm Supply

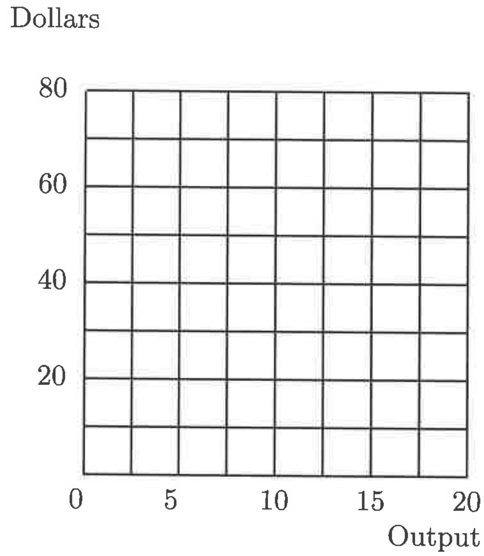
Introduction. The short-run supply curve of a competitive firm is the portion of its short-run marginal cost curve that is upward sloping and lies above its average variable cost curve. The long-run supply curve of a competitive firm is the portion of its short-run marginal cost curve that is upward-sloping and lies above its long-run average cost curve.

Example: A firm has the long-run cost function $c(y) = 2y^2 + 200$ for $y > 0$ and $c(0) = 0$. Let us find its long-run supply curve. The firm's marginal cost when its output is y is $MC(y) = 4y$. If we graph output on the horizontal axis and dollars on the vertical axis, then we find that the long-run marginal cost curve is an upward-sloping straight line through the origin with slope 4. The long-run supply curve is the portion of this curve that lies above the long-run average cost curve. When output is y, long-run average costs of this firm are $AC(y) = 2y + 200/y$. This is a *U-shaped* curve. As y gets close to zero, $AC(y)$ becomes very large because $200/y$ becomes very large. When y is very large, $AC(y)$ becomes very large because $2y$ is very large. When is it true that $AC(y) < MC(y)$? This happens when $2y+200/y < 4y$. Simplify this inequality to find that $AC(y) < MC(y)$ when $y > 10$. Therefore the long-run supply curve is the piece of the long-run marginal cost curve for which $y > 10$. So the long-run supply curve has the equation $p = 4y$ for $y > 10$. If we want to find quantity supplied as a function of price, we just solve this expression for y as a function of p. Then we have $y = p/4$ whenever $p > 40$.

Suppose that $p < 40$. For example, what if $p = 20$, how much will the firm supply? At a price of 20, if the firm produces where price equals long-run marginal cost, it will produce $5 = 20/4$ units of output. When the firm produces only 5 units, its average costs are $2 \times 5 + 200/5 = 50$. Therefore when the price is 20, the best the firm can do if it produces a positive amount is to produce 5 units. But then it will have total costs of $5 \times 50 = 250$ and total revenue of $5 \times 20 = 100$. It will be losing money. It would be better off producing nothing at all. In fact, for any price $p < 40$, the firm will choose to produce zero output.

23.1 (0) Remember Otto's brother Dent Carr, who is in the auto repair business? Dent found that the total cost of repairing s cars is $c(s) = 2s^2 + 100$.

(a) This implies that Dent's average cost is equal to ____________, his average variable cost is equal to ________, and his marginal cost is equal to ______ On the graph below, plot the above curves, and also plot Dent's supply curve.



(b) If the market price is \$20, how many cars will Dent be willing to repair? ________ If the market price is \$40, how many cars will Dent repair?________.

(c) Suppose the market price is \$40 and Dent maximizes his profits. On the above graph, shade in and label the following areas: total costs, total revenue, and total profits.

Calculus **23.2 (0)** A competitive firm has the following short-run cost function: $c(y) = y^3 - 8y^2 + 30y + 5$.

(a) The firm's marginal cost function is $MC(y) =$________.

(b) The firm's average variable cost function is $AVC(y) =$________ (Hint: Notice that total variable costs equal $c(y) - c(0)$.)

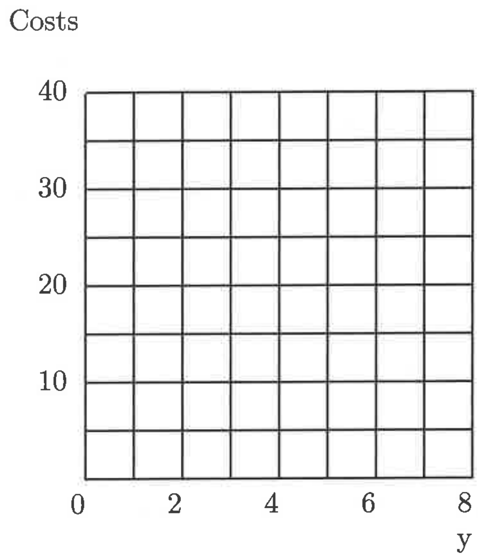
(c) On the axes below, sketch and label a graph of the marginal cost function and of the average variable cost function.

(d) Average variable cost is falling as output rises if output is less than ________ and rising as output rises if output is greater than________.

(e) Marginal cost equals average variable cost when output is________.

(f) The firm will supply zero output if the price is less than ________.

(g) The smallest positive amount that the firm will ever supply at any price is ________ At what price would the firm supply exactly 6 units of output? ________________.



Calculus **23.3 (0)** Mr. McGregor owns a 5-acre cabbage patch. He forces his wife, Flopsy, and his son, Peter, to work in the cabbage patch without wages. Assume for the time being that the land can be used for nothing other than cabbages and that Flopsy and Peter can find no alternative employment. The only input that Mr. McGregor pays for is fertilizer. If he uses x sacks of fertilizer, the amount of cabbages that he gets is $10\sqrt{x}$. Fertilizer costs \$1 per sack.

(a) What is the total cost of the fertilizer needed to produce 100 cabbages? ________ What is the total cost of the amount of fertilizer needed to produce y cabbages? ________________.

(b) If the only way that Mr. McGregor can vary his output is by varying the amount of fertilizer applied to his cabbage patch, write an expression for his marginal cost, as a function of y. $MC(y) =$ ________.

(c) If the price of cabbages is \$2 each, how many cabbages will Mr. McGregor produce? ________ How many sacks of fertilizer will he buy? ________ How much profit will he make? ________.

(d) The price of fertilizer and of cabbages remain as before, but Mr. McGregor learns that he could find summer jobs for Flopsy and Peter in a local sweatshop. Flopsy and Peter would together earn \$300 for the summer, which Mr. McGregor could pocket, but they would have no time to work in the cabbage patch. Without their labor, he would get no cabbages. Now what is Mr. McGregor's total cost of producing y cabbages?

________________________________.

(e) Should he continue to grow cabbages or should he put Flopsy and Peter to work in the sweatshop?________________.

23.4 (0) Severin, the herbalist, is famous for his hepatica. His total cost function is $c(y) = y^2 + 10$ for $y > 0$ and $c(0) = 0$. (That is, his cost of producing zero units of output is zero.)

(a) What is his marginal cost function? ______ What is his average cost function?________________.

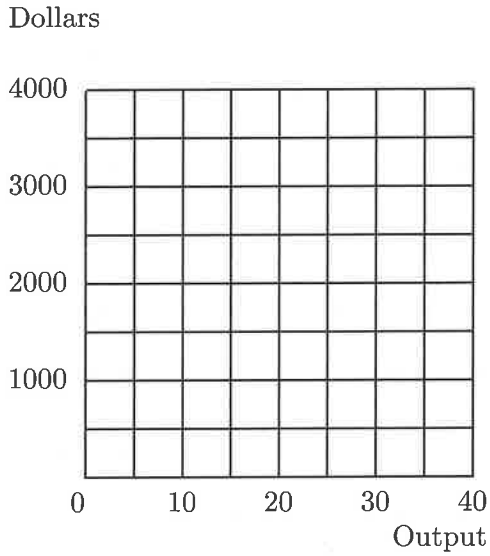
(b) At what quantity is his marginal cost equal to his average cost? ________ At what quantity is his average cost minimized?________.

(c) In a competitive market, what is the lowest price at which he will supply a positive quantity in long-run equilibrium? ________ How much would he supply at that price?________________.

23.5 (1) Stanley Ford makes mountains out of molehills. He can do this with almost no effort, so for the purposes of this problem, let us assume that molehills are the only input used in the production of mountains. Suppose mountains are produced at constant returns to scale and that it takes 100 molehills to make 1 mountain. The current market price of molehills is \$20 each. A few years ago, Stan bought an "option" that permits him to buy up to 2,000 molehills at \$10 each. His option contract explicitly says that he can buy fewer than 2,000 molehills if he wishes, but he can not resell the molehills that he buys under this contract. In order to get governmental permission to produce mountains from molehills, Stanley would have to pay \$10,000 for a molehill-masher's license.

(a) The marginal cost of producing a mountain for Stanley is ________ if he produces fewer than 20 mountains. The marginal cost of producing a mountain is ________ if he produces more than 20 mountains.

(b) On the graph below, show Stanley Ford's marginal cost curve (in blue ink) and his average cost curve (in red ink).



(c) If the price of mountains is \$1,600, how many mountains will Stanley produce?____.

(d) The government is considering raising the price of a molehill-masher's license to \$11,000. Stanley claims that if it does so he will have to go out of business. Is Stanley telling the truth? ______ What is the highest fee for a license that the government could charge without driving him out of business?____

____.

(e) Stanley's lawyer, Eliot Sleaze, has discovered a clause in Stanley's option contract that allows him to resell the molehills that he purchased under the option contract at the market price. On the graph above, use a pencil to draw Stanley's new marginal cost curve. If the price of mountains remains \$1,600, how many mountains will Stanley produce now?____.

23.6 (1) Lady Wellesleigh makes silk purses out of sows' ears. She is the only person in the world who knows how to do so. It takes one sow's ear and 1 hour of her labor to make a silk purse. She can buy as many sows' ears as she likes for \$1 each. Lady Wellesleigh has no other source of income than her labor. Her utility function is a Cobb-Douglas function $U(c, r) = c^{1/3}r^{2/3}$, where c is the amount of money per day that she has to spend on consumption goods and r is the amount of leisure that she has. Lady Wellesleigh has 24 hours a day that she can devote either to leisure or to working.

(a) Lady Wellesleigh can either make silk purses or she can earn \$5 an hour as a seamstress in a sweatshop. If she worked in the sweat shop, how many hours would she work? ________ (Hint: To solve for this amount, write down Lady Wellesleigh's budget constraint and recall how to find the demand function for someone with a Cobb-Douglas utility function.)

(b) If she could earn a wage of \$$w$ an hour as a seamstress, how much would she work?________________________________.

(c) If the price of silk purses is \$$p$, how much money will Lady Wellesleigh earn per purse after she pays for the sows' ears that she uses?__________.

(d) If she can earn \$5 an hour as a seamstress, what is the lowest price at which she will make any silk purses?________________________.

(e) What is the supply function for silk purses? (Hint: The price of silk purses determines the "wage rate" that Lady W. can earn by making silk purses. This determines the number of hours she will choose to work and hence the supply of silk purses.)________________________.

Calculus **23.7 (0)** Remember Earl, who sells lemonade in Philadelphia? You met him in the chapter on cost functions. Earl's production function is $f(x_1, x_2) = x_1^{1/3} x_2^{1/3}$, where x_1 is the number of pounds of lemons he uses and x_2 is the number of hours he spends squeezing them. As you found out, his cost function is $c(w_1, w_2, y) = 2w_1^{1/2} w_2^{1/2} y^{3/2}$, where y is the number of units of lemonade produced.

(a) If lemons cost \$1 per pound, the wage rate is \$1 per hour, and the price of lemonade is p, Earl's marginal cost function is $MC(y) =$ ____________ and his supply function is $S(p) =$__________ If lemons cost \$4 per pound and the wage rate is \$9 per hour, his supply function will be $S(p) =$________________________________.

(b) In general, Earl's marginal cost depends on the price of lemons and the wage rate. At prices w_1 for lemons and w_2 for labor, his marginal cost when he is producing y units of lemonade is $MC(w_1, w_2, y) =$____________________ The amount that Earl will supply depends on the three variables, p, w_1, w_2. As a function of these three variables, Earl's supply is $S(p, w_1, w_2) =$________________________________.

Calculus **23.8 (0)** As you may recall from the chapter on cost functions, Irma's handicrafts has the production function $f(x_1, x_2) = (\min\{x_1, 2x_2\})^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of labor used, and $f(x_1, x_2)$ is the number of lawn ornaments produced. Let w_1 be the price per unit of plastic and w_2 be the wage per unit of labor.

(a) Irma's cost function is $c(w_1, w_2, y) =$ ______________.

(b) If $w_1 = w_2 = 1$, then Irma's marginal cost of producing y units of output is $MC(y) =$ ______ The number of units of output that she would supply at price p is $S(p) =$ ________ At these factor prices, her average cost per unit of output would be $AC(y) =$ ______________.

(c) If the competitive price of the lawn ornaments she sells is $p = 48$, and $w_1 = w_2 = 1$, how many will she produce? ______ How much profit will she make? ______________.

(d) More generally, at factor prices w_1 and w_2, her marginal cost is a function $MC(w_1, w_2, y) =$ ________ At these factor prices and an output price of p, the number of units she will choose to supply is $S(p, w_1, w_2) =$ ______________.

23.9 (0) Jack Benny can get blood from a stone. If he has x stones, the number of pints of blood he can extract from them is $f(x) = 2x^{\frac{1}{3}}$. Stones cost Jack \$$w$ each. Jack can sell each pint of blood for \$$p$.

(a) How many stones does Jack need to extract y pints of blood? ______________.

(b) What is the cost of extracting y pints of blood? ______________.

(c) What is Jack's supply function when stones cost \$8 each? ______________ When stones cost \$$w$ each? ______________.

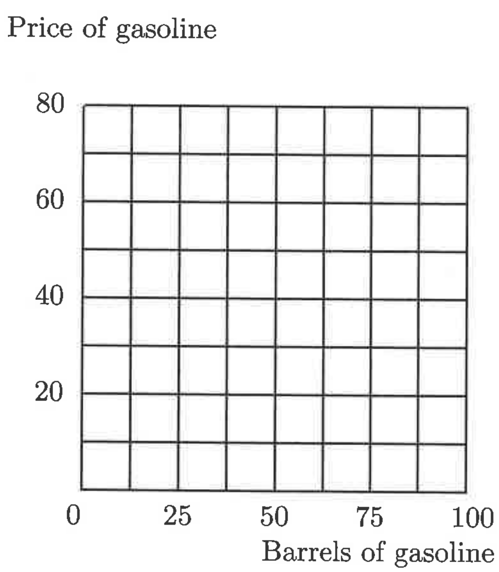
(d) If Jack has 19 relatives who can also get blood from a stone in the same way, what is the aggregate supply function for blood when stones cost $\$w$ each?____.

23.10 (1) The Miss Manners Refinery in Dry Rock, Oklahoma, converts crude oil into gasoline. It takes 1 barrel of crude oil to produce 1 barrel of gasoline. In addition to the cost of oil there are some other costs involved in refining gasoline. Total costs of producing y barrels of gasoline are described by the cost function $c(y) = y^2/2 + p_o y$, where p_o is the price of a barrel of crude oil.

(a) Express the marginal cost of producing gasoline as a function of p_o and y.____.

(b) Suppose that the refinery can buy 50 barrels of crude oil for \$5 a barrel but must pay \$15 a barrel for any more that it buys beyond 50 barrels. The marginal cost curve for gasoline will be __________ up to 50 barrels of gasoline and __________ thereafter.

(c) Plot Miss Manners' supply curve in the diagram below using blue ink.



(d) Suppose that Miss Manners faces a horizontal demand curve for gasoline at a price of \$30 per barrel. Plot this demand curve on the graph above using red ink. How much gasoline will she supply?________________.

(e) If Miss Manners could no longer get the first 50 barrels of crude for $5, but had to pay $15 a barrel for all crude oil, how would her output change?____.

(f) Now suppose that an entitlement program is introduced that permits refineries to buy one barrel of oil at $5 for each barrel of oil that they buy for $15. What will Miss Manners' supply curve be now? ____________

______ Assume that it can buy fractions of a barrel in the same manner. Plot this supply curve on the graph above using black ink. If the demand curve is horizontal at $30 a barrel, how much gasoline will Miss Manners supply now?____.

Chapter 24

NAME______________________

Industry Supply

Introduction. To find the industry supply of output, just add up the supply of output coming from each individual firm. Remember to add quantities, not prices. The industry supply curve will have a kink in it where the market price becomes low enough that some firm reduces its quantity supplied to zero.

The series of problems about the garden gnome industry are designed to help you to understand the distinction between the long run and the short run. To solve these problems, you need to pay careful attention to the timing of decisions. In particular, in this problem, units of capital (gnome molds) can be produced and delivered only one year after they are ordered.

The last three questions of this chapter apply supply and demand analysis to some problems in the economics of illegal activities. In these examples, you will make use of your knowledge of where supply functions come from.

24.0 Warm Up Exercise. Here are some drills for you on finding market supply functions from linear firm supply functions. The trick here is to remember that the market supply function may have kinks in it. For example, if the firm supply functions are $s_1(p) = p$ and $s_2(p) = p - 2$, then the market supply function is $S(p) = p$ for $p \leq 2$ and $S(p) = 2p - 2$ for $p > 2$; that is, only the first firm supplies a positive output at prices below \$2, and both firms supply output at prices above \$2. Now try to construct the market supply function in each of the following cases.

(a) $s_1(p) = p, s_2(p) = 2p, s_3(p) = 3p$.____________________________.

(b) $s_1(p) = 2p, s_2(p) = p - 1$.______________________________.

(c) 200 firms each have a supply function $s_1(p) = 2p - 8$ and 100 firms each have a supply function $s_2(p) = p - 3$.______________________

____.

(d) $s_1(p) = 3p - 12, s_2(p) = 2p - 8, s_3(p) = p - 4$.______________________.

24.1 (1) Al Deardwarf's cousin, Zwerg, makes plaster garden gnomes. The technology in the garden gnome business is as follows. You need a gnome mold, plaster, and labor. A gnome mold is a piece of equipment that costs \$1,000 and will last exactly one year. After a year, a gnome mold is completely worn out and has no scrap value. With a gnome mold, you can make 500 gnomes per year. For every gnome that you make, you also have to use a total of \$7 worth of plaster and labor. The total amounts of plaster and labor used are variable in the short run. If you want to produce only 100 gnomes a year with a gnome mold, you spend only \$700 a year on plaster and labor, and so on. The number of gnome molds in the industry cannot be changed in the short run. To get a newly built one, you have to special-order it from the gnome-mold factory. The gnome-mold factory only takes orders on January 1 of any given year, and it takes one whole year from the time a gnome mold is ordered until it is delivered on the next January 1. When a gnome mold is installed in your plant, it is stuck there. To move it would destroy it. Gnome molds are useless for anything other than making garden gnomes.

For many years, the demand function facing the garden-gnome industry has been $D(p) = 60,000 - 5,000p$, where $D(p)$ is the total number of garden gnomes sold per year and p is the price. Prices of inputs have been constant for many years and the technology has not changed. Nobody expects any changes in the future, and the industry is in long-run equilibrium. The interest rate is 10%. When you buy a new gnome mold, you have to pay for it when it is delivered. For simplicity of calculations, we will assume that all of the gnomes that you build during the one-year life of the gnome mold are sold at Christmas and that the employees and plaster suppliers are paid only at Christmas for the work they have done during the past year. Also for simplicity of calculations, let us approximate the date of Christmas by December 31.

(a) If you invested \$1,000 in the bank on January 1, how much money could you expect to get out of the bank one year later? __________ If you received delivery of a gnome mold on January 1 and paid for it at that time, by how much would your revenue have to exceed the costs of plaster and labor if it is to be worthwhile to buy the machine? (Remember that the machine will be worn out and worthless at the end of the year.)

____.

(b) Suppose that you have exactly one newly installed gnome mold in your plant; what is your short-run marginal cost of production if you produce up to 500 gnomes? ______ What is your average *variable* cost for producing up to 500 gnomes? ______ If you have only one gnome mold, is it possible in the short run to produce more than 500 gnomes?

____.

(c) If you have exactly one newly installed gnome mold, you would produce 500 gnomes if the price of gnomes is above ________ dollars. You would produce no gnomes if the price of gnomes is below ________ dol-

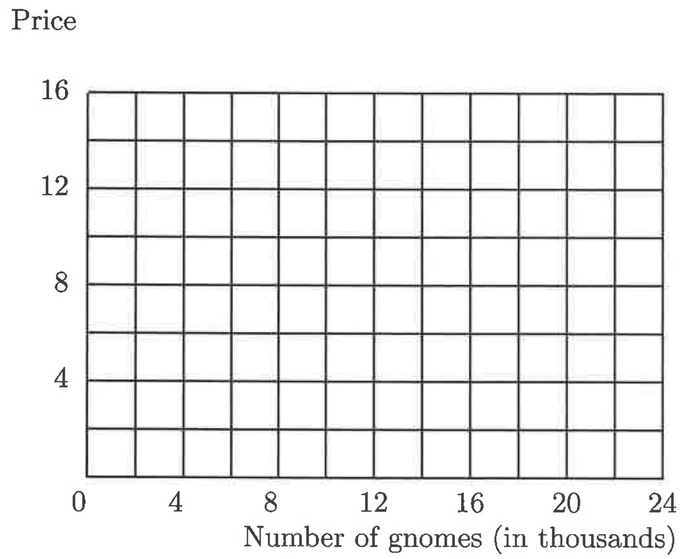
lars. You would be indifferent between producing any number of gnomes between 0 and 500 if the price of gnomes is ________ dollars.

(d) If you could sell as many gnomes as you liked for \$10 each and none at a higher price, what rate of return would you make on your \$1,000 by investing in a gnome mold? __________ Is this higher than the return from putting your money in the bank? __________ What is the lowest price for gnomes such that the rate of return you get from ivesting \$1000 in a gnome mold is as at least 10%? __________ Could the long-run equilibrium price of gnomes be lower than this?______________________.

(e) At the price you found in the last section, how many gnomes would be demanded each year? __________ How many molds would be purchased each year? _______ Is this a long-run equilibrium price?______________.

24.2 (1) We continue our study of the garden-gnome industry. Suppose that initially everything was as described in the previous problem. To the complete surprise of everyone in the industry, on January 1, 2001, the invention of a new kind of plaster was announced. This new plaster made it possible to produce garden gnomes using the same molds, but it reduced the cost of the plaster and labor needed to produce a gnome from \$7 to \$5 per gnome. Assume that consumers' demand function for gnomes in 2001 was not changed by this news. The announcement came early enough in the day for everybody to change his order for gnome molds to be delivered on January 1, 2002, but of course, but the total number of molds available to be used in 2001 was just the 28 molds that had been ordered the previous year. The manufacturer of garden gnome molds contracted to sell them for \$1,000 when they were ordered, so it can't change the price it charges on delivery.

(a) On the graph below, draw the short run industry supply curve and teh demand curve for garden gnomes that applies in the year 2001, after the discovery of the new plaster is announced.



(b) In 2001, what is the short run equilibrium total output of garden gnomes, _________ and what is the short run equilibrium price of garden gnomes? _________ (Hint: Look at the intersection of the supply and demand curves you just drew. Cousin Zwerg bought a gnome mold that was delivered on January 1, 2001, and, as had been agreed, he paid $1,000 for it on that day. On January 1, 2002, when he sold the gnomes he had made during the year and when he paid the workers and the suppliers of plaster, he received a net cash flow of $_________ Did he make more than a 10% rate of return on his investment in the gnome mold?____________ What rate of return did he make?____________________.

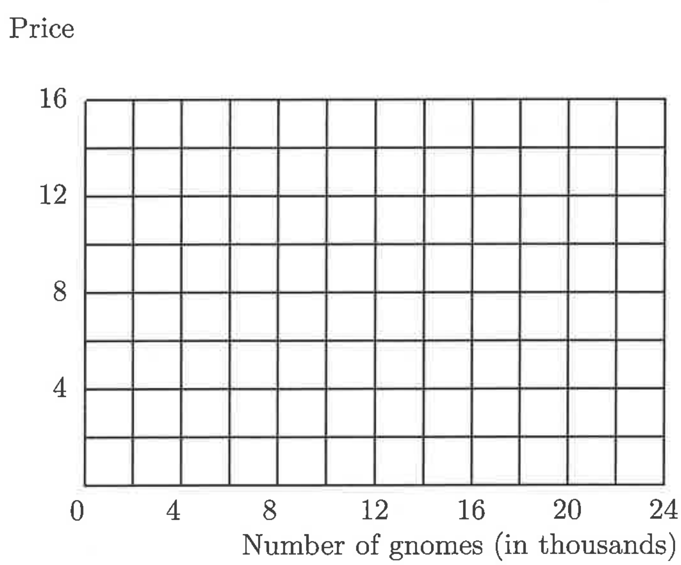
(c) Zwerg's neighbor, Munchkin, also makes garden gnomes, and he has a gnome mold that is to be delivered on January 1, 2001. On this day, Zwerg, who is looking for a way to invest some more money, is considering buying Munchkin's new mold from Munchkin and installing it in his own plant. If Munchkin keeps his mold, he will get a net cash flow of $____________ in one year. If the interest rate that Munchkin faces, both for borrowing and lending is 10%, then should he be willing to sell his mold for $1,000? ______ What is the lowest price that he would be willing to sell it for?_________ If the best rate of return that Zwerg can make on alternative investments of additional funds is 10%, what is the most that Zwerg would be willing to pay for Munchkin's new mold?____________.

(d) What do you think will happen to the number of garden gnomes ordered for delivery on January 1, 2002? Will it be larger, smaller, or the same as the number ordered the previous year? __________ After the passage of sufficient time, the industry will reach a new long-run equilibrium. What will be the new equilibrium price of gnomes? ________

____.

24.3 (1) In the previous problem, we studied the effects of a cost-saving invention. For this problem, we suppose that there was no such invention, but that a tax is introduced. Suppose that on January 1, 2001, the industry was as described in the previous problem (without the invention of the new kind of plaster). On this day, the government surprised the garden gnome industry by introducing a tax on the production of garden gnomes. For every garden gnome produced, the manufacturer must pay a \$1 tax. The announcement came early enough in the day so that there was time for gnome producers to change their orders of gnome molds for 2002. But the gnome molds available to be used in 2001 are those that had been ordered a year previously. Gnome makers had signed contracts promising to pay \$1,000 for each gnome mold that they ordered, and they couldn't back out of these promises. Thus in the short run, during the year 2001, the number of gnome molds is stuck at 28.

(a) On the graph below, draw the short run industry supply curve for garden gnomes that applies in the year 2001, after the new tax is introduced. On the same graph, show the demand curve for garden gnomes.



(b) In 2001, after the tax is introduced, what is the short run equilibrium total output of garden gnomes, __________ and what is the short run

equilibrium price of garden gnomes? ________ (Hint: Look at the intersection of the supply and demand curves you just drew.)

(c) If you have a garden gnome mold, the marginal cost of producing a garden gnome, including the tax, is ______ Therefore all gnome molds (will, will not) ________ be used up to capacity in 2001.

(d) In 2001, what will be the total output of garden gnomes? ________

What will be the price of garden gnomes? ________ What rate of return will Deardwarf's cousin Zwerg make on his investment in a garden gnome mold that he ordered a year ago and paid \$1,000 for at that time?

____.

(e) Remember that Zwerg's neighbor, Munchkin, also has a gnome mold that is to be delivered on January 1, 2001. Knowing about the tax makes Munchkin's mold a less attractive investment than it was without the tax, but still Zwerg would buy it if he can get it cheap enough so that he makes a 10% rate of return on his investment. How much should he be willing to pay for Munchkin's new mold?________________.

(f) What do you think will happen to the number of gnome molds ordered for delivery on January 1, 2002? Will it be larger, smaller, or the same as the number ordered the previous year?________________.

(g) The tax on garden gnomes was left in place for many years, and nobody expected any further changes in the tax or in demand or supply conditions. After the passage of sufficient time, the industry reached a new long-run equilibrium. What was the new equilibrium price of gnomes?

____.

(h) In the short run, who would end up paying the tax on garden gnomes, the producers or the consumers? __________ In the long run, did the price of gnomes go up by more, less, or the same amount as the tax per gnome?________________________________.

(i) Suppose that early in the morning of January 1, 2001, the government had announced that there would be a \$1 tax on garden gnomes, but that the tax would not go into effect until January 1, 2002. Would the producers of garden gnomes necessarily be worse off than if there were no tax? Why or why not?________________________

____.

(j) Is it reasonable to suppose that the government could introduce "surprise" taxes without making firms suspicious that there would be similar "surprises" in the future? Suppose that the introduction of the tax in January 2001 makes gnome makers suspicious that there will be more taxes introduced in later years. Will this affect equilibrium prices and supplies? How?____________

____

____________.

24.4 (0) Consider a competitive industry with a large number of firms, all of which have identical cost functions $c(y) = y^2 + 1$ for $y > 0$ and $c(0) = 0$. Suppose that initially the demand curve for this industry is given by $D(p) = 52 - p$. (The output of a firm does not have to be an integer number, but the number of firms does have to be an integer.)

(a) What is the supply curve of an individual firm? $S(p) =$____________ If there are n firms in the industry, what will be the industry supply curve?

____________.

(b) What is the smallest price at which the product can be sold?______

____________.

(c) What will be the equilibrium number of firms in the industry? (Hint: Take a guess at what the industry price will be and see if it works.)

____________.

(d) What will be the equilibrium price? ____________ What will be the equilibrium output of each firm?____________.

(e) What will be the equilibrium output of the industry?____________.

(f) Now suppose that the demand curve shifts to $D(p) = 52.5 - p$. What will be the equilibrium number of firms? (Hint: Can a new firm enter the market and make nonnegative profits?)____________

____

____

____________.

(g) What will be the equilibrium price? ______

______ What will be the equilibrium output of each firm? ______

What will be the equilibrium profits of each firm?______.

(h) Now suppose that the demand curve shifts to $D(p) = 53 - p$. What will be the equilibrium number of firms? ______ What will be the equilibrium price?______.

(i) What will be the equilibrium output of each firm? ______ What will be the equilibrium profits of each firm?______.

24.5 (3) In 1990, the town of Ham Harbor had a more-or-less free market in taxi services. Any respectable firm could provide taxi service as long as the drivers and cabs satisfied certain safety standards.

Let us suppose that the constant marginal cost per trip of a taxi ride is \$5, and that the average taxi has a capacity of 20 trips per day. Let the demand function for taxi rides be given by $D(p) = 1,200 - 20p$, where demand is measured in rides per day, and price is measured in dollars. Assume that the industry is perfectly competitive.

(a) What is the competitive equilibrium price per ride? (Hint: In competitive equilibrium, price must equal marginal cost.) ______ What is the equilibrium number of rides per day? ______ How many taxicabs will there be in equilibrium?______.

(b) In 1990 the city council of Ham Harbor created a taxicab licensing board and issued a license to each of the existing cabs. The board stated that it would continue to adjust the taxicab fares so that the demand for rides equals the supply of rides, but no new licenses will be issued in the future. In 1995 costs had not changed, but the demand curve for taxicab rides had become $D(p) = 1,220 - 20p$. What was the equilibrium price of a ride in 1995?______.

(c) What was the profit per ride in 1995, neglecting any costs associated with acquiring a taxicab license? ______ What was the profit per taxicab license per day? ______ If the taxi operated every day, what was the profit per taxicab license per year?______.

(d) If the interest rate was 10% and costs, demand, and the number of licenses were expected to remain constant forever, what would be the market price of a taxicab license?____________________.

(e) Suppose that the commission decided in 1995 to issue enough new licenses to reduce the taxicab price per ride to \$5. How many more licenses would this take?____________________.

(f) Assuming that demand in Ham Harbor is not going to grow any more, how much would a taxicab license be worth at this new fare?__________

____________________.

(g) How much money would each current taxicab owner be willing to pay to prevent any new licenses from being issued? ______________ What is the total amount that all taxicab owners together would be willing to pay to prevent any new licences from ever being issued? __________

______ The total amount that consumers would be willing to pay to have another taxicab license issued would be (more than, less than, the same as) ____________ this amount.

24.6 (2) In this problem, we will determine the equilibrium pattern of agricultural land use surrounding a city. Think of the city as being located in the middle of a large featureless plain. The price of wheat at the market at the center of town is \$10 a bushel, and it only costs \$5 a bushel to grow wheat. However, it costs 10 cents a mile to transport a bushel of wheat to the center of town.

(a) If a farm is located t miles from the center of town, write down a formula for its profit per bushel of wheat transported to market.______

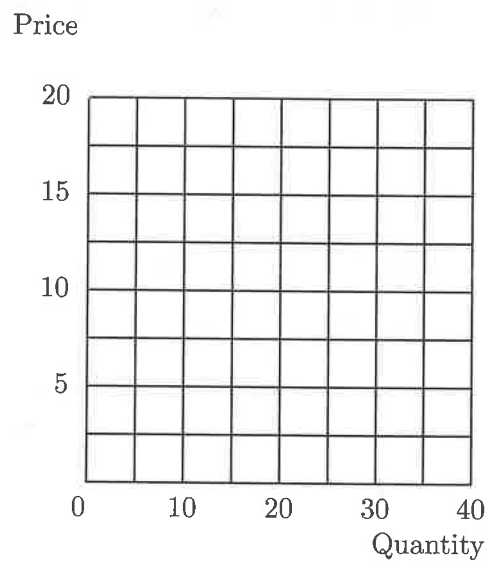
____________________.

(b) Suppose you can grow 1,000 bushels on an acre of land. How much will an acre of land located t miles from the market rent for?__________

____________________.

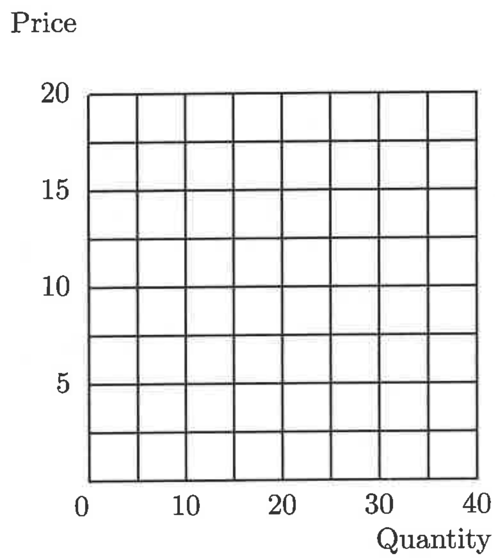
(c) How far away from the market do you have to be for land to be worth zero?____________________.

24.7 (1) Consider an industry with three firms. Suppose the firms have the following supply functions: $S_1(p) = p$, $S_2(p) = p - 5$, and $S_3(p) = 2p$ respectively. On the graph below plot each of the three supply curves and the resulting industry supply curve.



(a) If the market demand curve has the form $D(p) = 15$, what is the resulting market price? ________ Output? ________ What is the output level for firm 1 at this price? ________ Firm 2? ________ Firm 3? ____________________.

24.8 (0) Suppose all firms in a given industry have the same supply curve given by $S_i(p) = p/2$. Plot and label the four industry supply curves generated by these firms if there are 1, 2, 3, or 4 firms operating in the industry.



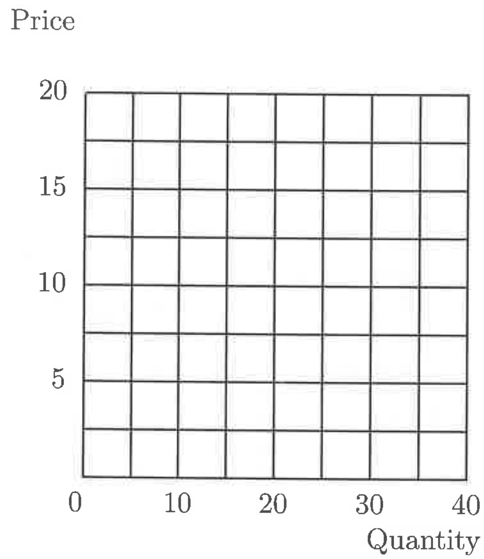
(a) If all of the firms had a cost structure such that if the price was below \$3, they would be losing money, what would be the equilibrium price and output in the industry if the market demand was equal to $D(p) = 3.5$?

Answer: price = ______, quantity = ______ How many firms would exist in such a market?______.

(b) What if the identical conditions as above held except that the market demand was equal to $D(p) = 8 - p$? Now, what would be the equilibrium price and output? ______ How many firms would operate in such a market?______.

24.9 (0) There is free entry into the pollicle industry. Anybody can enter this industry and have the same U-shaped average cost curve as all of the other firms in the industry.

(a) On the diagram below, draw a representative firm's average and marginal cost curves using blue ink. Also, indicate the long-run equilibrium level of the market price.



(b) Suppose the government imposes a tax, t, on every unit of output sold by the industry. Use red ink to draw the new conditions on the above graph. After the industry has adjusted to the imposition of the tax, the competitive model would predict the following: the market price would (increase, decrease) ____________ by amount ________ , there would be (more, the same, fewer) ________ firms operating in the industry, and the output level for each firm operating in the industry would ________________ (increase, stay the same, decrease).

(c) What if the government imposes a tax, l, on every *firm* in the industry. Draw the new cost conditions on the above graph using black ink. After the industry has adjusted to the imposition of the tax the competitive model would predict the following: the market price would (increase, decrease) ____________ , there would be (more, the same, fewer) ____________ firms operating in the industry, and the output level for each firm operating in the industry would ____________ (increase, stay the same, decrease).

24.10 (0) In many communities, a restaurant that sells alcoholic beverages is required to have a license. Suppose that the number of licenses is limited and that they may be easily transferred to other restaurant owners. Suppose that the conditions of this industry closely approximate perfect competition. If the average restaurant's revenue is \$100,000 a year, and if a liquor license can be leased for a year for \$85,000 from an existing restaurant, what is the average variable cost in the industry?

________.

24.11 (2) In order to protect the wild populations of cockatoos, the Australian authorities have outlawed the export of these large parrots. An illegal market in cockatoos has developed. The cost of capturing an Australian cockatoo and shipping him to the United States is about \$40 per bird. Smuggled parrots are drugged and shipped in suitcases. This is extremely traumatic for the birds and about 50% of the cockatoos shipped die in transit. Each smuggled cockatoo has a 10% chance of being discovered, in which case the bird is confiscated and a fine of \$500 is charged. Confiscated cockatoos that are alive are returned to the wild. Confiscated cockatoos that are found dead are donated to university cafeterias.*

(a) The probability that a smuggled parrot will reach the buyer alive and unconfiscated is ________ Therefore when the price of smuggled parrots is p, what is the expected gross revenue to a parrot-smuggler from shipping a parrot? ________.

(b) What is the expected cost, including expected fines and the cost of capturing and shipping, per parrot? ________.

(c) The supply schedule for smuggled parrots will be a horizontal line at the market price ________ (Hint: At what price does a parrot-smuggler just break even?)

(d) The demand function for smuggled cockatoos in the United States is $D(p) = 7,200 - 20p$ per year. How many smuggled cockatoos will be sold in the United States per year at the equilibrium price? ________ How many cockatoos must be caught in Australia in order that this number of live birds reaches U.S. buyers? ________.

(e) Suppose that instead of returning live confiscated cockatoos to the wild, the customs authorities sold them in the American market. The profits from smuggling a cockatoo do not change from this policy change. Since the supply curve is horizontal, it must be that the equilibrium price of smuggled cockatoos will have to be the same as the equilibrium price when the confiscated cockatoos were returned to nature. How many live cockatoos will be sold in the United States in equilibrium? ________

* The story behind this problem is based on actual fact, but the numbers we use are just made up for illustration. It would be very interesting to have some good estimates of the actual demand functions and cost functions.

How many cockatoos will be permanently removed from the Australian wild?____________________.

(f) Suppose that the trade in cockatoos is legalized. Suppose that it costs about \$40 to capture and ship a cockatoo to the United States in a comfortable cage and that the number of deaths in transit by this method is negligible. What would be the equilibrium price of cockatoos in the United States? ________ How many cockatoos would be sold in the United States? ________ How many cockatoos would have to be caught in Australia for the U.S. market?____________________.

24.12 (0) The horn of the rhinoceros is prized in Japan and China for its alleged aphrodisiac properties. This has proved to be most unfortunate for the rhinoceroses of East Africa. Although it is illegal to kill rhinoceroses in the game parks of Kenya, the rhinoceros population of these parks has been almost totally depleted by poachers. The price of rhinoceros horns in recent years has risen so high that a poacher can earn half a year's wages by simply killing one rhinoceros. Such high rewards for poaching have made laws against poaching almost impossible to enforce in East Africa. There are also large game parks with rhinoceros populations in South Africa. Game wardens there were able to prevent poaching almost completely and the rhinoceros population of South Africa has prospered. In a recent program from the television series *Nova*, a South African game warden explained that some rhinoceroses even have to be "harvested" in order to prevent overpopulation of rhinoceroses. "What then," asked the interviewer, "do you do with the horns from the animals that are harvested or that die of natural causes?" The South African game warden proudly explained that since international trade in rhinoceros horns was illegal, South Africa did not contribute to international crime by selling these horns. Instead the horns were either destroyed or stored in a warehouse.

(a) Suppose that all of the rhinoceros horns produced in South Africa are destroyed. Label the axes below and draw world supply and demand curves for rhinoceros horns with blue ink. Label the equilibrium price and quantity.

(b) If South Africa were to sell its rhinoceros horns on the world market, which of the curves in your diagram would shift and in what direction?

________________________ Use red ink to illustrate the shifted curve or curves. If South Africa were to do this, would world consumption of rhinoceros horns be increased or decreased? ____________ Would the world price of rhinoceros horns be increased or decreased? ______ ________ Would the amount of rhinoceros poaching be increased or decreased? ____.

24.13 (1) The sale of rhinoceros horns is not prohibited because of concern about the wicked pleasures of aphrodisiac imbibers, but because the supply activity is bad for rhinoceroses. Similarly, the Australian reason for restricting the exportation of cockatoos to the United States is not because having a cockatoo is bad for you. Indeed it is legal for Australians to have cockatoos as pets. The motive for the restriction is simply to protect the wild populations from being overexploited. In the case of other commodities, it appears that society has no particular interest in restricting the supply activities but wishes to restrict consumption. A good example is illicit drugs. The growing of marijuana, for example, is a simple pastoral activity, which in itself is no more harmful than growing sweet corn or brussels sprouts. It is the consumption of marijuana to which society objects.

Suppose that there is a constant marginal cost of $5 per ounce for growing marijuana and delivering it to buyers. But whenever the marijuana authorities find marijuana growing or in the hands of dealers, they seize the marijuana and fine the supplier. Suppose that the probability that marijuana is seized is .3 and that the fine if you are caught is $10 per ounce.

(a) If the "street price" is $\$p$ per ounce, what is the expected revenue net of fines to a dealer from selling an ounce of marijuana? ________

What then would be the equilibrium price of marijuana?________.

(b) Suppose that the demand function for marijuana has the equation $Q = A - Bp$. If all confiscated marijuana is destroyed, what will be the equilibrium consumption of marijuana? ________ Suppose that confiscated marijuana is not destroyed but sold on the open market. What will be the equilibrium consumption of marijuana?________.

(c) The price of marijuana will (increase, decrease, stay the same)________

____.

(d) If there were increasing rather than constant marginal cost in marijuana production, do you think that consumption would be greater if confiscated marijuana were sold than if it were destroyed? Explain.________

____

____.