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Taking turns

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ABSTRACT

Two individuals face a regular task that requires the effort of only one. They take turns but sometimes arrange to swap obligations. These swaps account for their changing, private costs. While seemingly primitive, flexible turn-taking is surprisingly efficient, even relative to what can be achieved by mechanisms using monetary transfers. I model and experimentally evaluate a simple form of flexible turn-taking and then present a second form that is both consistent with patterns of subject behavior and approximately second-best in a benchmark case.

1. Introduction

Turn-taking is a fundamental social behavior and a major developmental milestone for children (Sheridan et al., 2014). It has also been observed in animal species (Harcourt et al., 2010; Voelkl et al., 2015). Turn-taking is a fair and natural arrangement in settings where one individual's effort is needed to complete a regularly occurring and mutually beneficial task. Examples include a parent waking to calm a crying baby, a monitor keeping watch in a dangerous environment, or a doctor on-call for a late-night emergency.

Under stochastic, private costs, rote turn-taking can result in inefficient assignment. A flexible turn-taking arrangement, with the possibility of swapping turns, overcomes some of this inefficiency. I formalize,[1] a simple model of flexible turn-taking as a dynamic economic mechanism referred to as *recurring rotation*. A benefit of modeling this in the framework of mechanism design is that flexible turn-taking can be compared to alternative solutions. The result of this comparison is surprising. Despite its simplicity, recurring rotation achieves impressive efficiency even relative to what may be achieved in this environment by mechanisms that use monetary transfer. For instance, with uniformly distributed costs, recurring rotation captures about three-quarters of the achievable efficiency.[2] I present recurring rotation in section 2.

Peeling back the gentle exterior of this mechanism reveals a complex set of incentives. The theoretical properties presented here are the properties of this indirect mechanism *in equilibrium*. However, given the complexity, it is natural to ask whether the empirical properties are likely to match the theoretical properties.

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[1] Specifically, I model recurring rotation as a *Perfect Public Equilibrium* (*PPE*) of a repeated, private cost version of the volunteer's dilemma game (Diekmann, 1985).

[2] The robustness of the incentives in recurring rotation, which make it appropriate for an informal interpersonal environment, implies it is part of a particular subclass of *Perfect Public Equilibrium* (*PPE*) known as *Ex-post Incentive Compatible Perfect Public Equilibrium* or *EPPPE*, and first-best is not achievable by an *EPPPE* in the environment studied here (Miller, 2012).

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In section 3, I report the results of a laboratory experiment designed test the empirical properties and to evaluate actual behavior of subjects under the recurring rotation mechanism. Interestingly, although the mechanism achieved efficiency close to the theoretical prediction, subject behavior departed systematically from predictions. These patterns of unexpected behavior appear to arise from how subjects approach the decision problem induced by the incentives of the mechanism and cannot be explained by pro-social interest or strategic concerns. This lends to the possibility that the subjects used heuristics borrowed from their experience in turn-taking arrangements with a different structure.

In section 4, I present an alternative arrangement: *obligation takeover.* Obligation takeover permits a "debt" of turns where recurring rotation allows only delay. While more complex, obligation takeover retains a familiar structure, and the asymmetries in subject behavior that appear anomalous under recurring rotation are part of equilibrium behavior in obligation takeover. Furthermore, in the benchmark case of uniformly distributed costs, obligation takeover achieves second-best efficiency for perfectly patient players. In this case, no improvement is available from any mechanism appropriate for this environment, even those using monetary transfers. This may attest to the durability of flexible turn-taking as a social arrangement and suggests further study of the how these arrangements are commonly applied in real-world applications.

To my knowledge, this paper is the first formal study of flexible turn-taking. It may be understood as both a normative and a positive exposition of these institutions. The mechanisms in this paper may be thought of as abstract models of the kinds of flexible turn-taking arrangements people actually use. The results also permit normative conclusions; flexible turn-taking can provide a large amount of efficiency without the use of money, but the incentives underlying these mechanisms are remarkably complex, despite the familiar exterior.

This work is related to several areas of the mechanism design literature. In the area of robust mechanism design with transfers, Drexl and Kleiner (2015) and Shao and Zhou (2013) consider robust allocation of a valuable good in a one-shot environment with transfers, and construct optimal mechanisms. Athey and Miller (2007) consider a repeated trade setting and demonstrate that first-best efficiency can be achieved by robust mechanisms under relaxed budget balance conditions. Miller (2012) focuses on robust mechanisms for collusion of two firms in repeated settings with transfers. Like these papers, the mechanisms described here do not require a planner to enforce information structure. However, unlike in these, I explicitly analyze mechanisms where players do not use money transfers.

A procedure for constructing the set of payoffs achievable by robust mechanisms without money transfers in repeated environments is discussed in Miller (2012, p. 792) and is based on the tangent hyperplane method developed in Fudenberg et al. (1994). However, this exercise provides little insight into the potential structure of such mechanisms. A primary contribution of this paper is to show that substantial efficiency can be achieved by robust and familiar turn-taking mechanisms.

Several papers also characterize or explicitly construct mechanisms using only continuation transfers in repeated settings without money transfers, but without focusing on robust mechanisms. Athey and Bagwell (2001) consider a repeated Bertrand environment with discrete cost-types and demonstrate that first-best profits can be achieved by impatient firms without money transfers through the use of promises about future market-share. The conclusion of their paper discusses the potential extension to interpersonal relationships that are the focus of this paper. In contrast to Athey and Bagwell (2001) however, I focus on a characterizing simple, *robust* mechanisms, in an environment with a continuous type-space.

Several papers consider collusion in repeated auctions. Aoyagi (2003) considers repeated auctions with a type-space on the unit interval, and constructs highly efficient collusion mechanisms. However, these mechanisms require a coordinating institution. Blume and Heidhues (2008) and Skrzypacz and Hopenhayn (2004) also consider repeated auction environments, but where communication and monitoring are restricted.[3] Although the specifics of the environment are quite different, the mechanism for bid-rotation developed in Skrzypacz and Hopenhayn (2004) is similar to the *obligation takeover* mechanism. Although these papers provide a great deal of insight into the details of using continuation transfers to incentivize mechanisms, the specifics of the environments are quite different from those considered here.[4]

Mobius (2001) considers a model where two players can offer each-other "favors". A favor is an opportunity for one player to offer a fixed benefit of b to another while incurring a fixed cost c. The ability for one player to offer the favor is privately known and arrives at random times. Lau (2011) extends the favor trading environment to random cost and benefits but with one-sided private information and comes closest in the favor-trading literature to the kind of private information in the present environment. In addition to these theoretic papers, Roy (2012) discusses an experimental implementation of the Mobius (2001) environment.

Though the type of private information is different from that considered here, the mechanism (Mobius, 2001) develops, the "chips" mechanism, is similar to the obligation takeover mechanism in terms of integer accounting of obligations. Hauser and Hopenhayn (2008) consider alternative versions with improved efficiency in the favor trading environment, and Abdulkadiroglu and Bagwell (2012) derive optimal chips mechanisms. Chips mechanisms are further studied by Olszewski and Safronov (2016) who demonstrate the efficiency of a class of these mechanisms in a more generalized environment. The results on the optimality of obligation takeover in this paper can be seen as extending this line of research on the efficiency chips-like mechanisms to instances where there is two-sided private information about costs and ex-post incentive compatibility is required.

[3] Blume and Heidhues (2006) study a scenario in which the information environment is limited to an extent that players must rely on private strategies (rather than the public strategies studied here and elsewhere in the repeated auction literature) to achieve improvements over competitive bidding.

[4] Similar to these papers in its use of auctions, Guo et al. (2009) develops a mechanism for repeated allocation without transfers using auctions of a fiat currency in a binary valuation environment.

The concept of "budgeting" also plays a role in the approximately efficient (first-best) mechanisms presented by Jackson and Sonnenschein (2007) for a general class of repeated private information problems. Their mechanisms force players' private information reports to match the true underlying distribution over a long sequence of repeated instances. Ex-post incentive compatibility, which I focus on in this paper, does not permit mechanisms that allow a rich enough set of reports to achieve first best outcomes. Recurring rotation and obligation takeover can be seen as abstracted or simplified versions of this budgeting which meet the ex-post incentive requirement.

Other experiments are also related, especially those on turn-taking style cooperation in games without private information. Cason et al. (2013) study a common-pool resource allocation game in which turn-taking is the efficient cooperative outcome. They find that subjects who successfully engage in turn-taking are able to teach this strategy to future partners. Kuzmics et al. (2014)[5] studies repeated allocation games in the laboratory and also finds that turn-taking is a prevalent form of behavior.[6]

Kaplan and Ruffle (2012) study a repeated entry game with private information but where communication is not permitted. They demonstrate that the form of cooperation depends on the particular distribution of private information. With a low variance in private value, rigid turn-taking emerges. However, with higher variance, cut-off strategies based on private value emerge. In contrast, the mechanisms considered here combine both of these elements in an environment where the parties are permitted to communicate.

In addition to these, the current paper may be seen as a repeated extension of the volunteer's dilemma literature. See for instance Diekmann (1985), Goeree et al. (2017), Weesie (1994), Weesie and Franzen (1998) and a similar model related to the volunteers dilemma (Bliss and Nalebuff, 1984).

An interesting line of research in linguistics, formally starting with Sacks et al. (1974), studies turn-taking in conversation, which has been found to be a culturally universal system for organizing discussion (Stivers et al., 2009). Though, at first glance, this literature seems to be linked only in name to the institutions studied here, the role of turn-taking in conversation may be seen as a set of rules for allocation the scarce resource of opportunities to speak. When its function is considered in this way, it is more clear that conversational turn-taking is likely universal in the same way and for the same reasons that it is an important and universal institution for allocating indivisible goods, opportunities, or tasks over time.

2. Recurring rotation mechanism

Alice and Bob have a household task that must be done every day. It only takes one person to do it. Neither likes the task, but on some days it is more inconvenient than on others. Every day, the cost for each of them is independently drawn from a common distribution. They have agreed on a turn-taking arrangement with possible swaps. It works like this...

The partner who did not complete the task yesterday is obligated to complete it today unless there is a mutual decision to swap. All else equal, neither wants to be the one who has to do it. However, since completing the task in this arrangement results in becoming non-obligated on the next day, a partner with a low enough cost will prefer to complete the task immediately, delaying future obligation. Suppose Alice is obligated and has a high enough cost to prefer putting off the task – remaining obligated tomorrow. Bob has a low cost and prefers to complete the task immediately rather than be obligated tomorrow. They can make a mutually beneficial swap. This is the *recurring rotation* mechanism.

In this section, I derive several results on the incentive properties, equilibrium, and efficiency of recurring rotation for general cost distributions. I also compare it to other mechanisms and provide more detailed numerical results for the uniform distribution.

2.1. Environment

Two players engage in a repeated game. Each has discount factor β. In each period, a task must be completed. The cost of performing the task is given by θ_i for player i. This cost is private information and is drawn independently in each period from identical, and commonly known distribution $F(\theta_i)$ on the (normalized) domain $[0, 1]$. The value of having the task completed is fixed and normalized to 0 for both.

The players use a mechanism which determines who will complete the task in each period. If neither completes the task, they each get a payoff that is less than -1. This ensures that, no matter what their costs, if either knew the other was not going to do they task, he or she would prefer to do the task rather than leave it undone and gives the stage game the structure of a Volunteer's Dilemma (Diekmann, 1985).

The Volunteer's Dilemma has asymmetric Nash equilibria in which any player completes the task with certainty. Thus, the mechanism simply fixes players' beliefs about which Nash equilibrium should be played in a period. Since the repeated game then involves a repetition of Nash play at each stage, no player has incentive to deviate from the proposed assignment.[7]

[5] Kuzmics et al. (2014) provides an interesting interpretation of the Thue–Morse sequence (see Allouche and Shallit, 1999) in terms of an equilibrium of a repeated allocation game. A similar result is given in Cooper and Dutle (2013) for the example of structuring a fair duel.

[6] In a computational model, Neill (2003) shows that turn-taking can be achieved in a noisy environment, even when agents use limited memory strategies.

[7] To be complete, I assume punishment for this deviation involves repetition of the asymmetric Nash equilibrium in which the deviating player completes the task in perpetuity.

		Non-Obligated Action: *Not Offer*	Non-Obligated Action: *Offer*
Obligated Action	*Not Offer*	Obligated	Non-Obligated
Obligated Action	*Offer*	Obligated	Obligated $Pr = q$ Non-Obligated $Pr = 1 - q$

Fig. 1. Partner completing task by joint action profile.

The players can communicate, but there is no formal authority to structure communication, prevent them from learning about each other's private information, or absorb budget imbalance. These assumptions capture the interpersonal nature of their relationship. This assumption does not affect the modeling of recurring rotation; it is robust to these features. Rather, this assumption restricts the class of available alternative mechanisms to those that are suitable for such informal, interpersonal situations. This is discussed further in subsection 2.7.

2.2. Recurring rotation

The mechanism has two states that correspond to which player is obligated. From either player's perspective, the states are labeled o for "I am **O**bligated" and n for "I am **N**ot Obligated". Whoever completes the task becomes non-obligated in the next period. The players communicate by offering to do the task. If only one player offers, that player completes the task. If neither offer, the obligated completes the task. The disagreement that arises when both offer can be handled more generally. Various versions are possible. The *deferment* version settles the disagreement on the side of the obligated, while *option* settles it on the side of the non-obligated. A hybrid version that nests these two deterministic versions allows an arbitrary rule for deciding who completes the task when both offer. q will indicate this rule and corresponds to the probability that the obligated is assigned the task when both offer. $q = 0$ corresponds to *option* and $q = 1$ corresponds to *deferment*.

A summary of the mechanism in terms of which partner completes the task, conditional on their actions, is given in Fig. 1.

2.2.1. Dominant threshold strategy

When should a player offer? A interesting feature of this mechanism is the symmetry of the decision problem faced in either state. A player's best response is a threshold strategy. When cost is below this threshold, the player should offer. When cost is above, the player should not offer. The *same* threshold determines action in both states.

Since the players are ex-ante identical and other elements of the game are stationary, I focus on identical Markov strategies. Here, a strategy maps the state and players type θ_i into the action space $\{Offer, Do\ Not\ Offer\}$. Let $V(o)$ and $V(n)$ be the present values associated with being obligated and non-obligated respectively.[8] Regardless of the state, the present value of the player who executes the task with cost θ_i is $\beta V(n) - \theta_i$. The present value of not executing the task is $\beta V(o)$. These are the only two possible outcomes.

A player will want to do the task if and only if $\theta_i \le \beta(V(n) - V(o))$. Whenever $\beta(V(n) - V(o)) \in [0, 1]$ there is a well-defined threshold cost $\tilde{\theta} = \beta(V(n) - V(o))$ below which a player, in either state, prefers to complete the task. Suppose a player has $\theta_i < \tilde{\theta}$. Whether the player is obligated or non-obligated, it is weakly dominant to offer since it maximizes the probability of completing the task – the best outcome when cost is below $\tilde{\theta}$. Likewise, a player with $\theta_i > \tilde{\theta}$ does not want to complete the task. In this situation it is weakly dominant to not offer since it minimizes the probability of completing the task- the worst outcome when cost is above $\tilde{\theta}$. Thus, when players act according to symmetric Markov strategies, their actions are determined solely by a single threshold $\tilde{\theta}$.

However, the value functions are dependent on the strategies of the players, and thus the threshold $\tilde{\theta}$. In equilibrium, players acting according to threshold $\tilde{\theta}$ must generate a discounted difference in state value $\beta(V(n) - V(o))$ that is equal to the threshold. If this is not the case, players have incentive to adjust their strategies. A consistent equilibrium within recurring rotation involves a threshold $\tilde{\theta}$ that solves this fixed point problem. To determine these consistent equilibria, it is first necessary to derive an explicit form of $\beta(V(n) - V(o))$ in terms of the threshold.

2.2.2. Equilibrium condition

The calculation $\beta(V(n) - V(o))$ is simplified by noting that whenever the two players are on opposite sides of the threshold $\tilde{\theta}$, it does not matter who is obligated. Refer to Fig. 1. Regardless of who is obligated, if one player has a cost

[8] These are stationary and not individual specific due to the identical Markov strategy assumption.

below threshold and the other has a cost above, only the player with the low cost will offer. That player will complete the task and move to the non-obligated state. Thus, the difference in the two state values *only depends on what happens when both are on the same side of the threshold*. With this simplification, $(V(n) - V(o))$ can be determined as follows.

Both Above:

When both players are above the threshold, the obligated always carries out the task at an average cost $E\left(\theta_i|\theta_i \geq \tilde{\theta}\right)$ and becomes non-obligated which carries discounted continuation value $\beta V(n)$. The non-obligated does not carry out the task and becomes obligated, which has discounted value $\beta V(o)$. Thus, the difference in value of being non-obligated and obligated, conditional on both being above the threshold, is given by $\left[E\left(\theta_i|\theta_i \geq \tilde{\theta}\right) + \beta V(o) - \beta V(n)\right]$. This event happens with probability $\left(1 - F\left(\tilde{\theta}\right)\right)^2$.

Both Below:

When both are below the threshold, the player chosen to do the task by the tie-breaking rule completes the task at average cost $E\left(\theta_i|\theta_i \leq \tilde{\theta}\right)$ and becomes non-obligated, which has discounted continuation value $\beta V(n)$. The player who does not complete the task becomes obligated, which has discounted value $\beta V(o)$. Taking in to account the tie-breaking rule q, the difference in the value of being non-obligated and obligated, conditional on both being *below* the threshold, is given by $(2q-1)\left[E\left(\theta_i|\theta_i \leq \tilde{\theta}\right) + \beta V(o) - \beta V(n)\right]$. For instance, when $q = 1$, the obligated completes the task for sure and this term is: $\left[E\left(\theta_i|\theta_i \geq \tilde{\theta}\right) + \beta V(o) - \beta V(n)\right]$. When $q = 0$, the non-obligated completes the task for sure and this term is the negative of the previous. The event that both are below the threshold happens with probability $\left(1 - F\left(\tilde{\theta}\right)\right)^2$.

Taking the sum of these two differences weighted by the probabilities of occurrence gives:

$$V(n) - V(o) = F\left(\tilde{\theta}\right)^2 (2q-1)\left[E\left(\theta_i|\theta_i \leq \tilde{\theta}\right) + \beta V(o) - \beta V(n)\right] + \left(1 - F\left(\tilde{\theta}\right)\right)^2 \left[E\left(\theta_i|\theta_i \geq \tilde{\theta}\right) + \beta V(o) - \beta V(n)\right] \tag{1}$$

In equilibrium, $\theta^* = \beta(V(n) - V(o))$. Imposing this relationship on equation (1) provides the following equilibrium condition:

$$\theta^* = \beta\left[\left(1 - F\left(\theta^*\right)\right)^2 \left[E\left(\theta_i|\theta_i \geq \theta^*\right) - \theta^*\right] - F\left(\theta^*\right)^2 (2q-1)\left[\theta^* - E\left(\theta_i|\theta_i \leq \theta^*\right)\right]\right] \tag{2}$$

2.3. Equilibrium – uniqueness and location

The equilibrium threshold θ^* solves the fixed-point problem in equation (2). Notice the right side of this relates weighted versions of the terms $E(\theta_i|\theta_i \geq \theta^*) - \theta^*$ and $\theta^* - E(\theta_i|\theta_i \leq \theta^*)$. These are, respectively, the difference in the average cost of execution and the continuation transfer associated with a deferment when both have cost above the threshold and the negative of the same difference when both have cost below the threshold.[9]

In solving for the equilibrium, it proves useful to convert the fixed-point problem into a root problem. Define:

$$g\left(\tilde{\theta}\right) = \frac{1}{\beta}\tilde{\theta} + F\left(\tilde{\theta}\right)^2 (2q-1)\left[\tilde{\theta} - E\left(\theta_i|\theta_i \leq \tilde{\theta}\right)\right] - \left(1 - F\left(\tilde{\theta}\right)\right)^2 \left[E\left(\theta_i|\theta_i \geq \tilde{\theta}\right) - \tilde{\theta}\right] \tag{3}$$

A threshold $\tilde{\theta}$ is an equilibrium of recurring rotation if and only if $g\left(\tilde{\theta}\right) = 0$. The function $g\left(\tilde{\theta}\right)$ has three properties (for continuous and strictly increasing cost distributions) that imply the equilibrium threshold θ^* must be unique, below half of the mean type, increasing in β, and decreasing in q. These properties, presented in the lemmas below, are also demonstrated graphically in Fig. 2.

Lemma 1. $g(0) < 0$.

Proof. $g(0) = -E(\theta_i|\theta_i \geq 0) = -E(\theta_i) < 0$. $\square$

Lemma 2. *g increases strictly over the interval* $[0, E(\theta_i)]$.

[9] The term $E(\theta_i|\theta_i \geq \theta^*) - \theta^*$ is familiar in survival analysis and is often referred to as “mean residual lifetime”. $\theta^* - E(\theta_i|\theta_i \leq \theta^*)$ is a related term referred to as “mean advantage over inferiors” by Bagnoli and Bergstrom (2005).

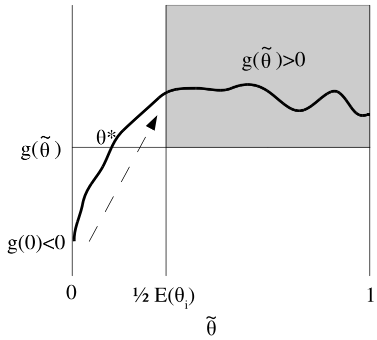

Fig. 2. Properties of $g\left(\tilde{\theta}\right)$.

Proof. See Appendix Section A.1. □

Lemma 3. g is strictly positive over the interval $\left(\frac{1}{2}E\left(\theta_i\right), 1\right]$.

Proof. See Appendix Section A.1. □

Proposition 4. *For any continuous and strictly increasing F and for any $q \in [0, 1]$ and $\beta \in [0, 1)$, the recurring rotation mechanism has a unique equilibrium with a threshold that is below the half of the mean type.*

Proof. This follows from the combination of Lemmas 1, 2, 3. The equilibrium condition is $g\left(\tilde{\theta}\right) = 0$. $g(0) < 0$ and $g\left(E\left(\theta_i\right)\right) > 0$. Since g increases strictly over $[0, E\left(\theta_i\right)]$, it must cross 0 exactly once in the interval $[0, \frac{1}{2}E\left(\theta_i\right)]$, and not again over $\left(\frac{1}{2}E\left(\theta_i\right), 1\right]$ since g remains strictly positive over this interval. □

2.4. Efficiency

Since no transfers are used, the efficiency can be measured by the average cost borne by the partners in executing the task. To simplify the expression of this average cost, notice that, under any version of the mechanism, if at least one offers, then the partner who does the task must have a cost below θ^*. Since the players' costs are independent, the average cost of execution when at least one offers is $E\left(\theta_i | \theta_i \leq \theta^*\right)$. When neither player offers, the obligated player is always chosen to do the task and must have cost above θ^*, executing the task at an average cost $E\left(\theta_i | \theta_i \geq \theta^*\right)$. Further, since the probability that at least one offers is $1 - \left(1 - F\left(\theta^*\right)\right)^2$ and the probability that neither offer is $\left(1 - F\left(\theta^*\right)\right)^2$, the expression for average cost of execution is $\bar{AC} = \left[1 - \left(1 - F\left(\theta^*\right)\right)^2\right] E\left(\theta_i | \theta_i \leq \theta^*\right) + \left(1 - F\left(\theta^*\right)\right)^2 E\left(\theta_i | \theta_i \geq \theta^*\right)$. After simplifying:

$$\bar{AC} = E\left(\theta_i\right) - F\left(\theta^*\right)\left(E\left(\theta_i\right) - E\left(\theta_i | \theta_i \leq \theta^*\right)\right) \tag{4}$$

The derivative of $\bar{AC}$ with respect to θ^* is $-f\left(\theta^*\right)E\left(\theta_i\right) + f\left(\theta^*\right)\theta^*$, which is negative as long as $\theta^* \leq E\left(\theta_i\right)$. Since all equilibria occur below the mean, the average cost is decreasing in the equilibrium threshold. This result is useful for comparative statics.

Lemma 5. *Higher equilibrium threshold implies lower average cost (higher efficiency).*

2.5. Comparative statics

Efficiency of recurring rotation is increasing in patience (β) and decreasing in the tie-breaking rule (q). The option version ($q = 0$) of the mechanism is the most efficient for any cost distribution.

Corollary 6. *θ^* and efficiency are increasing in β.*

Proof.

$$\frac{\delta\theta^*}{\delta\beta} = -\frac{\frac{\delta g(\theta^*, \beta)}{\delta\beta}}{\frac{\delta g(\theta^*, \beta)}{\delta\theta^*}} = \frac{\frac{1}{\beta^2}\theta^*}{\frac{\delta g(\theta^*, \beta)}{\delta\theta^*}} \tag{5}$$

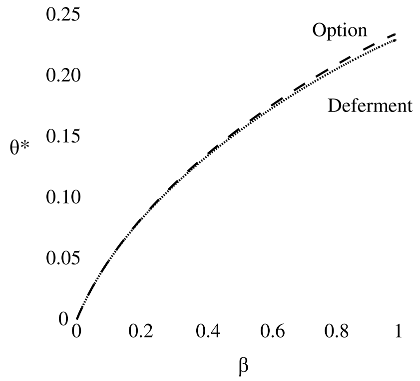

Fig. 3. Equilibria under uniform distribution by discount factor.

This is positive since $\frac{\delta g(\theta^*,\beta)}{\delta \theta^*} \geq 0$ at the equilibrium by Lemma 2 and Proposition 4. Increasing efficiency follows from Lemma 5. $\square$

Corollary 7. *θ^* and efficiency are decreasing in q.*

Proof.

$$\frac{\delta \theta^*}{\delta q} = -\frac{\frac{\delta g(\theta^*,q)}{\delta q}}{\frac{\delta g(\theta^*,q)}{\delta \theta^*}} = -\frac{2F\left(\theta^*\right)^2\left[\theta^* - E\left(\theta_i|\theta_i \leq \theta^*\right)\right]}{\frac{\delta g(\theta^*,q)}{\delta \theta^*}} \tag{6}$$

This is negative since $\theta^* - E\left(\theta_i|\theta_i \leq \theta^*\right) \geq 0$ and $\frac{\delta g(\theta^*,\beta)}{\delta \theta^*} \geq 0$ at the equilibrium, by Lemma 2 and Proposition 4. Increasing efficiency follows from Lemma 5. $\square$

Corollary 6, that efficiency increases with patience, is rather intuitive and due to the fact that players are less willing to take high costs in the current period to change the potential outcomes of future periods when the future is discounted more heavily. This result also implies that patient players swap more often in equilibrium than less patient players. As β increases, turn-taking becomes less rigid.

Corollary 7, that the option version is most efficient, is less intuitive. Recall that the best-response threshold is equal to the discounted difference in the value of being non-obligated and obligated. Further, recall that the choice of q only matters in situations when both players *want* to complete the task. When q is small, the non-obligated is assigned the favorable outcome more often at the expense of the obligated player. This increases the difference between the non-obligated and obligated values for any fixed threshold and results in a higher equilibrium threshold.

It is an interesting result that the seemingly less natural and less 'cooperative' version of the mechanism is more efficient, especially since assignments in the option version depend only by the cost of the non-obligated player. In numerical tests, the difference in efficiency of the option and deferment versions tended to be small. This is because the versions differ only in how they handle the situation that both *want* to do the task – a relatively rare event in equilibrium. It is possible to put an analytical bound on this difference, see online appendix subsection C.1.

These results provide some general insight into the workings of the recurring rotation mechanism and the location of equilibria. Next, I look more deeply at the case of uniform costs.

2.6. Example: uniform costs

When $\theta_i \sim U(0,1)$, θ^* is the solution to the cubic[10] equation: $\theta^* = \frac{1}{2}\beta\left[\left(1-\theta^*\right)^3 - (2q-1)\left(\theta^*\right)^3\right]$. For option $(q=0)$ and deferment $(q=1)$, respectively these are:

$$q = 0: \theta^* = \frac{1}{2}\beta\left[\left(1-\theta^*\right)^3 + \left(\theta^*\right)^3\right] \tag{7}$$

$$q = 1: \theta^* = \frac{1}{2}\beta\left[\left(1-\theta^*\right)^3 - \left(\theta^*\right)^3\right] \tag{8}$$

A graph of the solutions is shown in Fig. 3.

[10] At $q=0$ the equation is quadratic.

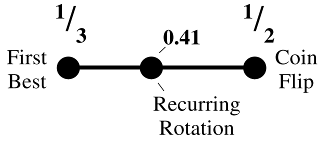

Fig. 4. Efficiency comparison.

Consistent with the analytical results above, both versions have thresholds increasing in β and the equilibrium threshold in the option version is larger than the deferment version. As $\beta \to 1$, the equilibria approach 0.232 for option and 0.226 for deferment. Using equation (4) together with the calculated equilibrium thresholds, the resulting average stage costs are 0.411 for option and 0.413 for deferment as $\beta \to 1$.

To put these costs in perspective, suppose players were to take turns in a rigid way or flip a coin. The average stage cost would be the mean of the distribution: $\frac{1}{2}$. Since they can achieve this stage cost without sharing *any* information about their costs, it represents a of lower bound on the achievable efficiency.[11] Compared to this lower bound, recurring rotation provides a substantial efficiency improvement.

On the other hand, in a perfect information setting, they could always assign the task to the player with lower cost. This would yield an average stage cost equal to the expected value of the minimum of the partners' costs. In the uniform case, this would be $\frac{1}{3}$. Thus, in uniform environment, recurring rotation falls about half-way between the lower-bound and first-best efficiency. This is plotted on the line segment in Fig. 4.

However, first-best is not be the most appropriate comparison. Players do not have perfect information. Because of this, achieving first best would require a mechanism that perfectly reveals costs. In this environment, with no planner to enforce communication rules or absorb budget imbalance, there is not a suitable mechanism that achieves first-best even with monetary transfers. An explanation of this requires some discussion of the mechanism design literature.

2.7. Finding the appropriate benchmark

In a repeated game, players may choose their actions based on the history of play. In recurring rotation, the relevant history is completely summarized by who is currently obligated. Because the only important aspect of the history is one that is commonly known, recurring rotation is known as a *perfect public equilibrium (PPE)* (Fudenberg et al., 1994). There are other aspects of the history that are not used. For instance, a player *could* additionally consider past cost realizations, an aspect of the history that is not publicly known.

In a PPE, various *states* of the equilibrium provide different amounts of expected utility to players. For instance, in recurring rotation, it is less desirable to be obligated than non-obligated. A transition from one state to another in a *PPE is* a transfer – not of money but rather of continuation utility. These transfers can be used to incentivize the revelation of private information just as money transfers provide incentives in many one-shot mechanisms. Because of this, a *PPE* may be considered a type of dynamic mechanism where incentives for revealing information are provided, at least in part, by continuation utility transfers (Miller, 2012).

In some settings, first-best efficiency is achievable by a *PPE*, even without money transfers (Athey and Bagwell, 2001; Fudenberg et al., 1994). However, the *whole* class of *PPE* is not the appropriate benchmark here. In recurring rotation, a player's incentive to report their cost truthfully, relative to the threshold θ^*, is weakly dominant, holding beliefs about future strategies fixed. The partners' *stage beliefs* can be distorted in any way by cheap-talk, spying, etc. Their incentives remain the same. The induced set of dynamic stage mechanisms have what Bergemann and Välimäki (2010) refer to as *periodic ex-post incentive compatibility.* That is, the incentives do not depend on history or beliefs about the current state of the other player, but may depend on future states.

Ex-post incentive compatibility provides robustness appropriate for an interpersonal setting (Chung and Ely, 2002). No planner is required to structure communication or isolate the partners to prevent changes in beliefs. The subclass of *PPE* with similarly robust incentives are what Miller (2012) refers to as *Ex Post Perfect Public Equilibrium (EPPPE).*

Further, Miller (2012) provides an impossibility result for this more restricted class of mechanisms. In an economically interesting class of environments, including the one studied here, first-best is not achievable by an EPPPE even with money transfers (under a no-subsidy condition on the ex-post budget).[12] This means that, even if money transfers are allowed, but no subsidy can be provided for the players, first-best is not achievable by a robust mechanism in this environment. This leaves open the question of what *is* achievable by an EPPPE either with or without money transfers.

[11] They could do even worse, but systematically assigning the task to the player with a higher cost would require some information transfer, which could be put to more beneficial use.

[12] Athey and Miller (2007) consider the repeated trade setting under weaker budget assumptions where first-best can sometimes be achieved, especially by patient players.

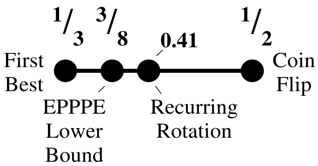

Fig. 5. Efficiency comparison.

The efficiency achievable by a one-shot ex-post incentive compatible mechanism with transfers under the no-subsidy condition is an upper-bound on achievable efficiency in each period of an EPPPE without money transfers (Miller, 2012).[13] This result simplifies the problem of bounding the optimal EPPPE efficiency to the problem of characterizing optimal one-shot ex-post incentive compatible mechanisms. While this is not a well-studied problem[14] due to the overwhelming variety of available mechanisms, Shao and Zhou (2013) provides a useful result for the uniform environment. They prove that, in the one-shot allocation of a valuable good between two players with value independently and uniformly distributed on $[0,1]$, the highest welfare achievable by an ex-post incentive compatible mechanism under no-subsidy is $\frac{5}{8}$. Since the allocation problem may be thought of as the negative reflection of the assignment problem,[15] this implies that second-best optimal average stage cost in an EPPPE with monetary transfers is $\frac{3}{8}$. This can be used as a lower-bound on the achievable average stage costs of an EPPPE without monetary transfers as well.[16]

2.8. Efficiency comparison

The line segment in Fig. 5 plots recurring rotation in relation to the second-best benchmark. The recurring rotation mechanism achieves about $\frac{3}{4}$ of the efficiency of an optimal ex-post[17] incentive compatible mechanism with subsidy-free transfers in the uniform environment.

The efficiency lost over second-best comes from the fact that the equilibrium threshold is too low. For comparison, if players used threshold $\frac{1}{2}$, they would achieve second-best average stage cost of $\frac{3}{8}$. However, the incentives provided by recurring rotation are smaller than necessary to get players to implement this optimal threshold. This results in inefficient sorting of the players costs. This inefficiency is not a problem limited to the uniform case. In fact, the efficiency of recurring rotation is always bounded below second-best optimal. I discuss this further in Appendix subsection A.2.

A mechanism using a single threshold at the mean cost, with appropriate transfers, is always ex-post incentive compatible and optimal among single-threshold mechanisms (see Lemmas 11 and 12 in subsection A.2 of the appendix). By comparing the efficiency of recurring rotation to this mean-threshold mechanism, it is possible to put a lower-bound on the efficiency lost by using recurring rotation instead of an optimal mechanism (see Proposition 13 in the appendix subsection A.2). However, despite the fact that recurring rotation is suboptimal relative to the best threshold mechanism, in the online appendix subsection C.2, I demonstrate that there are some environments where the welfare loss is small.

To complement these results, in the next section I present results from an experiment designed to test the empirical properties of recurring rotation.

3. Experiment

Given the surprising complexity of the incentives of recurring rotation, it is easy to imagine that actual behavior would deviate from theoretical predictions. In this section, I report the results of an experiment designed to evaluate the performance of the recurring rotation mechanism under true behavior.

This experiment put subjects in the scenario of playing an abstracted version of the repeated task game with uniformly distributed costs. The subjects used the deferment version of the recurring rotation mechanism to determine assignments. On average, subjects chose threshold strategies higher than the predicted equilibrium threshold. Further, many subjects choose higher thresholds in the obligated state than the non-obligated state (“gap” strategies).

[13] An EPPPE without money transfers uses instead transfers of utility within the repeated game to provide incentives. In an equilibrium, these transfers must come from within a set of consistent continuation values (Abreu et al., 1986, 1990). Monetary transfers are not restricted in this way. Barring budget issues, players are free to transfer any amount. Consistency is not an issue. Because of this, the set of feasible transfers using money is at least as large as the set of feasible transfers in a dynamic mechanism without money. However, in a repeated setting where money transfers are allowed, all of the incentives can be handled by money without resorting to more complicated utility transfers.

[14] Hagerty and Rogerson (1987) demonstrate that in the static trade setting, a posted price mechanism must be optimal under ex-post incentive compatibility, ex-post individual rationality (participation constraint), and strong ex-post budget balance (transfers must balance exactly).

[15] The problem of assigning a costly duty to do the task is the same as the problem of allocating the valuable right not to do the task.

[16] It is also possible to directly approximate the set of attainable average stage costs without monetary transfers as $\beta \to 1$ using the tangent hyperplane methods of Fudenberg et al. (1994). This procedure is also discussed in Miller (2012, p. 792). This numerical exercise also yields a bound on the average stage cost of $\frac{3}{8}$, and suggests that eschewing monetary transfers is not restrictive in this environment for patient players. In fact, the mechanism in section 4 approximates this efficiency.

[17] Since ex-post and dominant strategy incentive compatibility are equivalent for the private valuation environment, they are used interchangeably in this paper.

Despite variance in subject behavior, efficiency was close to what is expected in equilibrium. However, the pervasive use of gap strategies substantially decreased efficiency from what it would have been otherwise.[18] Further, the gaps remain in a treatment of the experiment designed to control strategic and pro-social variation by pairing subjects with computer partners. This suggests that the gaps may be due to heuristics developed under a turn-taking arrangement with different structure.

3.1. Design details

Data was collected at the University of California, Santa Barbara Experimental and Behavior Economics Laboratory using ZTREE (Fischbacher, 2007). The Online Recruitment System for Economic Experiments was used to recruit subjects (Greiner, 2004). A total of 199 subjects participated in the experiment, including pilot sessions. Equal numbers of men and women were invited to each session to provide rough gender balance. Each session of the experiment lasted about 50 minutes. No subjects participated in more than one session.

The experiment consisted of two key treatments. In the first, subjects were partnered with other subjects in their session. In the second, subjects were paired with a computer playing a known strategy – offering with probability $\frac{1}{2}$. In both treatments, the environment was the same and the subjects were aware of all of the details of the game.

In the person/person treatment there were four sessions. Three session used 16 subjects and one session used 18 subjects for a total of 66. Subjects were paired randomly with another subject from the session. One subject from each pair was randomly selected to start as the obligated partner. For the first 20 rounds, players found out their random cost, distributed uniformly between \$0 and \$3 in penny increments, before choosing an action. Simultaneously, the obligated player chose whether to ask his/her partner to do the task and the non-obligated chose whether to agree to do the task should his/her partner ask. After round 20, subjects instead submitted a threshold strategy before learning about their private costs[19] – the obligated choosing above what cost to ask and the non-obligated below what cost to agree.[20]

Below, I refer to the situation where the obligated asks the non-obligated to complete the task as the obligated partner asking for a "swap".

After each round, subjects found out whether the obligated had asked and whether the non-obligated agreed or would have agreed (if the obligated did not ask).[21] In each case, they also found out how much they paid in the round, if anything, and who would be obligated on the next round. Cumulative payoffs were not shown throughout the experiment, though no effort was made to prevent subjects from recording data with pen and paper (this was rare).

The probability of continuing with a partner in each round was $\frac{9}{10}$ to generate an effective discount factor of $\beta = \frac{9}{10}$. At the end of each round, players found out whether they would continue with the same partner or whether they had been paired with a new partner ($\frac{1}{10}$ probability) for the next round. Whenever a new partnership was formed, the software randomly chose one partner to be obligated. After 48 rounds,[22] the game ended on the next partnership termination.[23]

Subjects were paid for every round of the game. This scheme was chosen for simplicity and to induce far-sighted behavior. The rationale for this choice is based on the experimental comparison of repeated game payment schemes in Sherstyuk et al. (2013). Subjects started the experiment with \$5 and earned an additional \$3.20 every 4 rounds.[24] Subjects were reminded after each round that they would receive \$3.20 every 4 rounds, but they did not know explicitly when this occurred. This was chosen to avoid focal-points and strategy distortion on the paying rounds. On average, players in the person/person experiment earned just over \$15.

[18] Efficiency is lower relative to what would have been achieved if subjects played non-gap strategies equal to the average of what they chose in the obligated and non-obligated states.

[19] This "strategy method" is a common experimental technique for efficiently collecting information about subjects' underlying strategies. Rather than collecting a single data point about the subject's strategy at the interim stage, this method elicits the entire strategy at the ex-ante stage. Importantly, the choice of when to collect this data does not affect the actual game in any way.

[20] The language used in the experiment is slightly different than the way the actions are described in the previous section. The obligated choose whether to ask the non-obligated to do the task and the non-obligated chose whether to agree conditional on being asked. In a pilot study, this language made the mechanism more transparent to subjects over having each player offer to do the task (or not) regardless of state. An obligated partner asking the non-obligated to do the task is equivalent to not offering to do the task in theoretical game. Similarly, the non-obligated partner agreeing to do the task is equivalent to offering in theoretical game.

[21] This design feature was chosen to ensure that the amount of information subjects received about their partner's strategy was not conditional on actions.

[22] The subjects did not know this cut-off, but rather that there was *some* specified cutoff. 48 was chosen based on the average play time of groups in pilot sessions.

[23] Although 58 is the expected number of rounds, actual average duration, in rounds, of the sessions were shorter. The sessions lasted 49, 51, 52, 58 rounds respectively.

[24] Pilot-study data and theoretical efficiencies of the recurring rotation mechanism were used to select these payment parameters along with the range of the cost distribution to target an average subject payment of \$15 for an hour experiment while providing as much curvature on the pay-off function as possible without risking many subjects ending up below the \$5 lab minimum guaranteed payment. Subjects that suspected they might be below the minimum could face distorted incentives. Under these parameters, for 58 rounds, rigid turn-taking would result in an average payoff of \$7.90 for the experiment while second-best play would result in an average of \$18.78 and equilibrium play in roughly \$15.21. The parameter choice was relatively accurate to the intended targets. Under these parameters, only 2 subjects ended up below \$5 in the person/person treatment. Both earned about \$3.50 in the experiment (and had payment rounded to \$5). At this level, even if the two subjects had been aware their cumulative earnings were below the \$5 limit, the round incentives were still meaningful since moving above the threshold might have taken only a few rounds.

In the computer treatment, all details were identical. However, instead of being paired with a new subject after partnership termination, the partnership was instead restarted. Each time this happened, the software chose randomly whether the subject or the computer would start as the obligated partner. Between the person/person and the person/computer treatments, partnership lengths were matched within sessions. That is, session one of the person/person treatment had the same partnership lengths and ultimate session length as session one of the person/computer treatment. A total of 63 subjects participated over 4 sessions. Three sessions included 16 subjects and one session included 15. Earnings in the computer treatment were higher with average of about \$20 per subject.[25]

3.2. Results: person/person experiment

On average, selected thresholds were larger than those expected in a symmetric Markov equilibrium. Over all four sessions, the average chosen threshold was \$0.99 while equilibrium prediction is \$0.64. There was a great deal of variance in threshold choice. Histograms of average threshold choice by subject are available in the online appendix subsection C.5.

Average play did not vary much over the rounds.[26] Changes in play appear to have settled down by round 35 (see online appendix subsection C.6), and I focus on rounds 35–48 for calculating steady-state properties of the mechanism. Over these rounds, the average threshold conditional on being obligated was \$1.11 and \$0.78 when non-obligated.

From a strategic standpoint, one of the most obvious questions to ask about subject behavior is how it compares to best-response. Due to the variance in subject behavior, a full best-response analysis is difficult. However, from an aggregate perspective, it appears subjects' biggest mistake relative to best-response was not asking for deferment often enough when obligated.

The 'average' subject chose thresholds of \$1.11 and \$0.78 when obligated and non-obligated respectively. Suppose two such 'average' subjects meet. With their strategies, the difference in continuation value of being obligated and non-obligated discounted by $\beta = 0.9$ is roughly \$0.63. For them, this is the best response threshold, regardless of state. In comparison, the average subject tended to offer swaps too often when non-obligated and, to a greater extent, reject swaps too often when obligated.

Furthermore, these gaps are pervasive at a subject level. I use the following model to test the significance of the threshold gap for each subject. A player i's threshold $\theta_{i,t}$ in round t is estimated by an individual constant α_i plus individual-specific change δ_i when the player is obligated ($o_{i,t} = 1$ when subject i is obligated in round t).

$$\theta_{i,t} = \alpha_i + \delta_i o_{i,t} + \epsilon_{i,t} \tag{9}$$

59 percent of subjects have a significantly positive difference between obligated and non-obligated threshold at the 5 percent level (against one-sided alternative), only 9 percent have a significantly negative difference. Thus, the bias towards playing larger thresholds when obligated is robust at the subject level. Below, I demonstrate that these gaps were costly.

3.3. Efficiency estimation

Several empirical facts have implications for the efficiency achieved by subjects. On average, subjects chose thresholds higher than equilibrium. There was substantial variation among subjects. Many subjects choose "gap" strategies. In this section, I estimate the empirical efficiency of recurring rotation using subject behavior.

For each subject, I calculate the average obligated and non-obligated thresholds (normalized to $[0, 1]$ for comparison to the previous section) over rounds 35–48. For every possible pair of subjects (with replacement) I calculate the long-run efficiency they would achieve using these strategies. This calculation is detailed in appendix subsection B.1. Averaging over all of these pairs provides the estimated population efficiency. This procedure yields an estimate of 0.421[27] – only slightly worse than the equilibrium prediction: 0.416.

To get a measure of efficiency without gap strategies, I use the same procedure as above but assign each hypothetical player to use, in both states, their unconditional average threshold. This yields an average cost of 0.399, a substantial improvement over 0.416 estimated under gap strategies. This is not surprising since players tended to choose thresholds larger than the predicted equilibrium threshold. By Lemma 5, *symmetric* thresholds closer to $\frac{1}{2}$ provide higher efficiency.[28]

For comparison, the difference between an average cost of 0.399 and 0.421 corresponds to an average of \$3 per pair given up over 50 rounds; about 10 percent of an average pair's experimental earnings. These estimates suggest that gap strategies are not just an unexpected phenomenon, but also an important factor in the empirical efficiency of recurring rotation.

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[25] This is not surprising. The computer's strategy of offering half of the time is quite favorable to the subjects.

[26] CDFs of average thresholds over 7-round blocks by subject and split by obligated and non-obligated states are available in the online appendix subsection C.5.

[27] This also corresponds closely to the actual achieved average cost for subjects in the experiment. Over rounds 35–48 the partner who ended up completing the task in each pair paid an average of \$1.26 or .42 when normalized to $[0, 1]$.

[28] Fig. C.14 in the online appendix demonstrates visually why such a scenario leads to improved efficiency. The figure is a contour plot of the efficiency achieved when players choose arbitrary strategies with no gap. Contour lines are drawn at .416 (equilibrium efficiency for $\beta = .9$), .4, .39, and .38. Most pairs tend to fall in the darker blue area of the figure where the joint strategy profile yields efficiency better than equilibrium.

3.4. Results: person/computer treatment

In this treatment, subjects were paired with a computer known to offer to do the task with probability $\frac{1}{2}$ in both states. From a strategic point of view, this is equivalent to having the computer use a threshold of \$1.50, or $\frac{1}{2}$ when normalized to $[0, 1]$. The online appendix subsection C.5 contains histograms as well as cumulative distribution plots of average thresholds by subject separated by obligated and non-obligated states. As in the person/person treatment, changes in play appear to have settled down by round 35. A formal test of this is provided in the online appendix subsection C.6.

For rounds 35–48 the average threshold chosen by subjects in the obligated state was \$0.83 and \$0.66 in the non-obligated state. Note that a gap remains at the aggregate level.

3.4.1. Best response analysis

As in the person/person treatment, subjects set average thresholds higher than predicted by best-response. In this case, the best response can be calculated from a modified form of equation (2) allowing different threshold values for two players $\tilde{\theta}_1$ and $\tilde{\theta}_2$.

$$\frac{1}{\beta}\tilde{\theta}_1 = F\left(\tilde{\theta}_1\right) F\left(\tilde{\theta}_2\right)\left[E\left(\theta_1|\theta_1 \leq \tilde{\theta}_1\right) - \tilde{\theta}_1\right] + \left(1 - F\left(\tilde{\theta}_1\right)\right)\left(1 - F\left(\tilde{\theta}_2\right)\right)\left[E\left(\theta_1|\theta_1 \geq \tilde{\theta}_1\right) - \theta_1\right] \tag{10}$$

Normalized to $[0, 1]$, the computer implements threshold $\tilde{\theta}_2 = \frac{1}{2}$. Thus, the best response for player 1 is the solution to the following quadratic equation:

$$\tilde{\theta}_1 = \frac{9}{10}\left[\frac{\left(1 - \tilde{\theta}_1\right)^2}{4} - \frac{\tilde{\theta}_1^2}{4}\right] \tag{11}$$

The solution is $\frac{9}{58} \approx 0.156$ in the normalized game or about \$0.47 for the parameters faced by subjects. The aggregate obligated and non-obligated thresholds are significantly higher than the best response at better than the 1 percent level.

3.4.2. Gap strategies

Using the same procedure as the person/person treatment, 49 percent of subjects have a significantly positive gap (at the 5 percent level against one-sided alternative), 3 percent have a significantly negative gap. Here, slightly fewer subjects show a significantly positive gap than in the person/person treatment where the figure was 59 percent. However, this still dwarfs the percentage with a significantly negative gap.

3.5. Discussion

What could account for these gaps in subjects' strategies? Perhaps subjects are responding to incentives that extend beyond the monetary incentives of the game. They may be pro-social. Alternatively, gaps could be a strategic response to subjects' beliefs about their opponents' strategies; perhaps they are non-Markov. I analyze these two possibilities with subject data in online appendix subsection C.7. The evidence for either explanation is limited.

On the other hand, the computer partner treatment explicitly controls for both pro-social and strategic aspects of the game. The computer's strategy is known, and explicitly Markov. The fact that gap strategies are prevalent in the computer treatment suggests that gaps cannot be explained by pro-social behavior or strategic response. The persistence of these gaps suggests these patterns arise from the way subjects approach the decision problem, rather than from unmodeled incentives.

One explanation is that the experience subjects draw from to make choices in the lab comes from their exposure to situations where gaps are a more natural part of the arrangement. It may be that recurring rotation, as a model of flexible turn-taking, is exceptional in terms of this symmetric threshold feature relative to the kinds of arrangements people normally use in these scenarios. This explanation is consistent with the model of decision making in Samuelson (2001).

In the next section, I discuss a slightly different form of flexible turn taking (*obligation takeover*) which is both more efficient than recurring rotation and in which gaps appear as part of equilibria. If the hypothesis that the anomalous gaps in subject behavior appear because subjects recognize recurring rotation as being analogous to situations that are more familiar and where gaps are appropriate, then obligation takeover may be a model that is closer to reality.

The question of what form of flexible turn-taking is most natural is interesting in its own right. This question could be investigated in carefully designed experiments where subjects are not forced to use any particular mechanism but can instead select arrangements on their own. Investigation of these arrangements in the field is likely possible as well. Different arrangements such as recurring rotation and obligation takeover make different predictions about observable outcomes such as the sequence of realized task executions. However, these predictions may be sensitive to the underlying distribution of costs which would be difficult to measure due to the private-information nature of these environments.

4. Obligation takeover mechanism

This simplicity of recurring rotation comes at the cost of efficiency. In recurring rotation, deferring shifts future obligations. However, this shift is not costly enough to get partners to use efficient thresholds. An alternative arrangement which makes deferment more costly would offer improved efficiency. One way to do this is to give the deferring player extra turns of obligation, rather than simply shifting existing obligations. These extra obligations come as "debt". This is a natural extension since non-monetary debt is common in interpersonal relationships, as demonstrated by the prevalence of the idiom "I owe you one".

In this section, I present a mechanism called *obligation takeover* which introduces this "debt of turns."[29] The introduction of debt has two important effects. First, it improves efficiency. In fact, for perfectly patient players, obligation takeover achieves second-best efficiency under the uniform cost benchmark and that of the best threshold mechanism for any symmetric cost distribution. It also performs very well even for moderate patience levels.

Second, it causes asymmetries in the equilibrium that do not appear in recurring rotation. However, these asymmetries have a similar pattern to the "gaps" used by subjects in the recurring rotation experiment. The asymmetries are caused by the fact that the cost of acquiring additional debt for the obligated partner is larger than the associated benefit for the non-obligated.

4.1. Mechanism details

In obligation takeover, a deferment results in one extra period of obligation for the deferring partner. Specifically, the deferring player "takes over" the nearest future obligation of her partner.

Assume the players start with a plan to alternate who is obligated. Let $\mathbf{p}^t \in \prod_{k=t}^{\infty}\{1,2\}$ be a vector that represents the "plan" of obligation. At time t, the player identified by the first element of the vector $\mathbf{p}^t$ is obligated. When the obligated completes the task, the plan continues: $\mathbf{p}^{t+1}$ is equal to $\mathbf{p}^t$ with the first element truncated. If, on the other hand, the non-obligated completes the task, the plan proceeds but with an additional period of obligation added at the nearest position for the obligated player.

For instance if the plan is alternating so that $\mathbf{p}^t = 1, 2, 1, 2, 1, 2, 1, 2...$ and the obligated individual completes the task, then $\mathbf{p}^{t+1} = 2, 1, 2, 1, 2, 1, 2, 1....$ On the other hand, if the non-obligated completes the task then $\mathbf{p}^{t+1} = 1, 1, 2, 1, 2, 1, 2, 1....$ In this example the difference between the two resulting plans is precisely one extra period of obligation for player 1.

Under this arrangement, $\mathbf{p}^t$ will always include some repetition of either 1 or 2 followed by alternation. Because of this, the state of the mechanism can be represented by a single variable $z \in \mathbb{Z} \setminus 0$. $|z|$ represents the number of repetitions of obligation and $\operatorname{sign}(z)$ represents for whom that repetition pertains. For instance, $\mathbf{p}^t = 1, 1, 1, 1, 2, 1, 2, ...$ can be represented by state variable $z = 4$. While $\mathbf{p}^t = 2, 2, 2, 1, 2, 1, 2...$ can be represented by $z = -3$.

In a sense, z represents the debt owed by one partner to the other. Assuming a positive z represents extra obligation for 1, then it is the case that player 1 will have to complete z more executions of the task than player 2 before 2 can begin to accrue debt. Furthermore, in any state, the sequence of planned obligations has the property that, in absence of further deferment, players' total task executions are equalized as quickly as possible before returning to alternation. Because of this, the mechanism has an elegant way of promoting fairness in the history of task-execution, while allowing the efficiency gains associated with deferment.

For computational reasons, I will assume that there is a limit on the number of obligations a player can accrue so that $|z| \le \bar{z}$ – a debt limit. In a state where $|z| = \bar{z}$, the player currently obligated is forced to complete the task without the possible intervention of the non-obligated.[30]

4.2. Equilibria

As in recurring rotation, assume available actions are to 'offer' or 'not offer' to do the task and that players implement symmetric Markov strategies. Let $V_i(z)$ be the discounted ex-ante utility for player i in state z. For any V function, there is a unique Markov strategy that is weakly dominant with respect to stage beliefs in each period. As in recurring rotation, there are only two outcomes in each period. If the obligated completes the task, the state decreases. If the non-obligated completes the task, the state increases.

Consider player 1 in a state $|z| \neq 1$. Regardless of whether player 1 is obligated, completing the task provides utility $-\theta_1 + \beta V_1(z-1)$. Not completing the task provides utility $\beta V_1(z+1)$. Player 1 prefers to complete the task if and only if $\theta_1 < \beta(V_1(z-1) - V_1(z+1))$. Thus, $\tilde{\theta}_1(z) = \beta(V_1(z-1) - V_1(z+1))$ is the cost threshold that determines player 1's

[29] This mechanism can be seen as extending (in the sense of integer accounting of debt) the "chips" mechanisms discussed in the favor trading literature (see: Abdulkadiroglu and Bagwell, 2012; Hauser and Hopenhayn, 2008; Mobius, 2001) to include ex-post incentive compatibility in this repeated volunteering environment with two-sided private information.

[30] In equilibrium, the effect of the limit on efficiency is practically inconsequential as long as the limit is not very small. This is because the equilibrium strategies of the players make the probability of visiting larger states exponentially rare. A computational analysis of efficiency with limited states is provided in appendix subsection C.3.

preference over completing the task in state z. A summary of the thresholds for each state is given below (for $|z|=1$, a slight modification is needed since there is no state 0). Technically, the expressions below should include additional structure to account for the situation that the difference in state values is above 1 or below 0. To avoid unnecessary complication, it should be understood that when $\beta\left(V_1(z-1)-V_1(z+1)\right)\geq 1$, the optimal strategy is $\tilde{\theta}_1(z)=1$ and when $\beta\left(V_1(z-1)-V_1(z+1)\right)\leq 0$ the optimal strategy is $\tilde{\theta}_1(z)=0$. Although the thresholds are written for player 1, by symmetry, player 2's thresholds are simply the reverse $\left(\tilde{\theta}_2(z)=\tilde{\theta}_1(-z)\right)$.

$$|z|\neq 1: \tilde{\theta}_1(z)=\beta\left(V_1(z-1)-V_1(z+1)\right) \tag{12}$$

$$z=1: \tilde{\theta}_1(1)=\beta\left(V_1(-1)-V_1(2)\right) \tag{13}$$

$$z=-1: \tilde{\theta}_1(-1)=\beta\left(V_1(-2)-V_1(1)\right) \tag{14}$$

As in recurring rotation, I focus on symmetric equilibria and drop the player index. For simplicity, the rule that determines task assignment, conditional on actions, is assumed to be identical to the deferment version of the recurring rotation mechanism. Swaps happen only by mutual agreement.

In state z, a player with cost below [above] $\tilde{\theta}(z)$ has a weakly dominant strategy to offer [not offer] since that action maximizes the probability of a partner getting her most favorable outcome. Thus, when players act according to symmetric Markov strategies, their actions are determined solely by the vector of thresholds.

However, just as in recurring rotation, the value function depends on the strategies of players. In equilibrium, the vector of threshold strategies must generate discounted state value differences equal to the thresholds. Expressions for the continuation values for each state as a function of the vector of thresholds is derived in the appendix subsection A.3. Imposing the equilibrium conditions, these can be modified into expression that relate only the thresholds. Again, to avoid complicating the expressions, it should be understood that the right-hand-side of these expressions are bounded between 0 and 1 (as follows from the equilibrium condition).

$$z>1: \tilde{\theta}(z)=\beta\left(\tilde{\theta}(z-1)+C\left(z+1,\tilde{\theta}\right)-C\left(z-1,\tilde{\theta}\right)+T\left(z+1,\tilde{\theta}\right)-T\left(z-1,\tilde{\theta}\right)\right) \tag{15}$$

$$z<-1: \tilde{\theta}(z)=\beta\left(\tilde{\theta}(z+1)+C\left(z+1,\tilde{\theta}\right)-C\left(z-1,\tilde{\theta}\right)+T\left(z-1,\tilde{\theta}\right)-T\left(z+1,\tilde{\theta}\right)\right) \tag{16}$$

$$\tilde{\theta}(1)=\beta\left(C\left(2,\tilde{\theta}\right)-C\left(-1,\tilde{\theta}\right)+T\left(-1,\tilde{\theta}\right)+T\left(2,\tilde{\theta}\right)\right) \tag{17}$$

$$\tilde{\theta}(-1)=\beta\left(C\left(1,\tilde{\theta}\right)-C\left(-2,\tilde{\theta}\right)+T\left(-2,\tilde{\theta}\right)+T\left(1,\tilde{\theta}\right)\right) \tag{18}$$

Each of these fixed point equations involves functions C and T. C represents the average stage cost borne by the player in each state when both use threshold strategies $\tilde{\theta}$.

$$C\left(z,\tilde{\theta}\right)=\begin{cases} E\left(\theta_i|\theta_i\leq\tilde{\theta}(z)\right)F\left(\tilde{\theta}(z)\right)F\left(\tilde{\theta}(-z)\right)+E\left(\theta_i\right)\left(1-F\left(\tilde{\theta}(-z)\right)\right) & z\geq 1\\ E\left(\theta_i|\theta_i\leq\tilde{\theta}(z)\right)F\left(\tilde{\theta}(z)\right)\left(1-F\left(\tilde{\theta}(-z)\right)\right) & z\leq -1\end{cases} \tag{19}$$

T represents the expected transfer of continuation utility associated with a deferment in each state.

$$T\left(z,\tilde{\theta}\right)=\begin{cases} \tilde{\theta}(z)\left(1-F\left(\tilde{\theta}(z)\right)\right)F\left(\tilde{\theta}(-z)\right) & z\geq 1\\ \tilde{\theta}(z)F\left(\tilde{\theta}(z)\right)\left(1-F\left(\tilde{\theta}(-z)\right)\right) & z\leq -1\end{cases} \tag{20}$$

Proposition 8. *An equilibrium exists for obligation takeover.*

Proof. An equilibrium in obligation takeover is a solution to the multi-dimensional fixed-point problem above. For finite $\bar{z}$, this system continuously maps the $2(\bar{z}-1)$-dimensional unit cube into itself. Brouwer's fixed point theorem guarantees an equilibrium. When there is no limit on the number of states, the system maps the infinite dimensional unit-cube $I^\infty=\prod_{i=1}^{\infty}[0,1]$ into itself. I^∞ is a convex and compact (by Tychonoff's theorem) subset of $\mathbb{R}^\infty$. Schauder's fixed point theorem guarantees an equilibrium. □

Unlike in recurring rotation, the dimensionality of the problem makes finding closed form solutions difficult. Below, I present numerical approximations of the efficiency achieved by obligation takeover for the uniform cost benchmark over different discount factors. Online appendix subsection C.4 contains results on the efficiency achieved by the mechanism for a range of symmetric beta distributions.[31]

[31] Unlike in recurring rotation, I do not prove that there is a unique equilibrium for this mechanism. Even if there are multiple equilibria, the efficiencies of the equilibria computed below still represent an approximate lower-bound on the achievable efficiency.

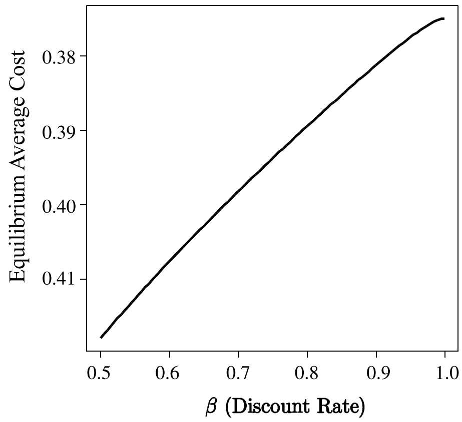

Fig. 6. Average cost in β of obligation takeover with unlimited states.

4.3. *Efficiency*

Fig. 6 plots the approximate efficiency achieved by obligation takeover with an unlimited number of states[32] for discount factors $\beta \in [.5, 1)$. It is striking how much efficiency is generated even for relatively low discount factors. Though this plot is an approximation of the efficiency with unlimited states, computations in the online appendix subsection C.3 suggests the efficiency lost by using finite limits tends to be small, due to the rarity of long chains of obligation. Further, for large β, the approximate efficiency (average cost) is nearly identical to second best: $\frac{3}{8}$. In fact for perfectly patient players, obligation takeover can achieve second best efficiency.

Proposition 9. *For any symmetric distribution and perfectly patient players with overtaking preferences, obligation takeover can achieve efficiency equal to that of the best threshold mechanism.*

Proof. See appendix subsection A.1, an intuitive sketch is provided below. □

By Lemma 12 in appendix subsection A.2, the best threshold mechanism uses a threshold of $E(\theta_i)$. When both players use this threshold in every state, the difference in expected stage-cost of being obligated and non-obligated is:

$$F\left(E\left(\theta_i\right)\right)^2\left[E\left(\theta_i|\theta_i \leq E\left(\theta_i\right)\right)\right] + \left(1 - F\left(E\left(\theta_i\right)\right)\right)^2\left[E\left(\theta_i|\theta_i \leq E\left(\theta_i\right)\right)\right] \tag{21}$$

For symmetric distributions, this simplifies to $\frac{1}{2}E\left(\theta_i\right)$. By deferring, the obligated partner reaches a state in which he has *two* more obligations than if he did not defer. Similarly, by accepting a deferment, the non-obligated partner reaches a state in which he has *two* less obligations than he would have otherwise. Since players continue to use the same thresholds in every state, the difference in value of both of these state changes is exactly two times the difference in stage cost, which in this case is $2\left(\frac{1}{2}E\left(\theta_i\right)\right) = E\left(\theta_i\right)$. Thus, for symmetric distributions, and for perfectly patient players with overtaking preferences,[33] using thresholds equal to the mean cost generates differences in the state values that are consistent with using those thresholds – an equilibrium.

Corollary 10. *For the uniform distribution, obligation takeover can achieve second-best efficiency for perfectly patient players with overtaking preferences.*

Proof. This follows from Proposition 9 and the fact that the threshold mechanism with threshold $E\left(\theta_i\right)$ is optimal among ex-post incentive compatible stage-mechanisms in this environment (see section 2.7). □

[32] In order to determine the efficiency of the mechanism under fixed discount factor β, but without limit on the states, I utilize the following procedure: for each β, the equilibrium is approximated under a limited number of states and the efficiency is calculated for the computed equilibrium. After this, the efficiency is again calculated for the computed equilibrium with the number of states doubled. This process is repeated until the difference in efficiency of the two most recently computed equilibria converges to 0.

[33] The change in state leads to only a finite change in the undiscounted stream of stage payoffs. A perfectly patient player with limit of means preferences is indifferent between these states.

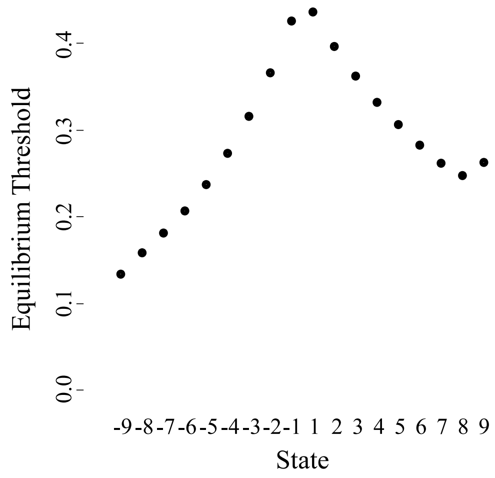

Fig. 7. Equilibrium threshold values for $\beta = .9$ and $\bar{z} = 10$.

4.4. Equilibrium characterization

Here, I return to discounting preferences and give a brief overview and intuitive explanation for some of the most striking characteristics of equilibria of obligation takeover for players with fixed discount factors below 1. In recurring rotation, there is little to say about the equilibrium beyond a single value, since even impatient partners use the same threshold in both states. This is not the case in obligation takeover. In equilibrium, different thresholds may be used in each state. A computed example equilibrium for $\beta = .9$ and $\bar{z} = 10$ is provided in Fig. 7.

The various asymmetries in the equilibrium come from the fact that deferments carry different values to each partner in different states. This was not the case in recurring rotation. There, a deferment always has the same effect on the sequence of obligations. A deferment always leads to a state in which the currently obligated partner is again obligated. In obligation takeover, the sequence of obligations is affected differently by a deferment in each state. For instance, when a long string of obligations has already been accrued, an additional deferment only adds an additional obligation at the end of the already long chain. When the players discount, the effect of this additional obligation is diminished relative to its effect if the chain of obligations had been short.

Because of this, the cost to the obligated player and the value to the non-obligated of having an additional obligation added are diminished in larger states. In equilibrium, the player's thresholds depend on these values or costs. This effect leads to thresholds that diminish as the partners move away from states 1 or -1. This can be seen in the example above in Fig. 7.

A related effect leads to thresholds that are asymmetric between analogous obligated and non-obligated sates. As additional obligations are accrued, the lower thresholds diminish the efficiency of the stage mechanisms. The effect of this weighs more heavily on the obligated player who is more likely to get stuck executing the task at high cost. Because of this, the obligated partner is willing to take higher costs to avoid additional obligations than the non-obligated is to accept them in any particular state, leading to the asymmetry across analogous states, which can also be seen in Fig. 7.[34] Interestingly, these obligated/non-obligated gaps are precisely the kind that appeared anomalous in subject behavior under recurring rotation.

This source of asymmetries can be see directly in the equilibrium conditions. For instance, the following equation is derived from (13) and (14) and gives an expression for the gap between analogous obligated and non-obligated thresholds in state 1:

$$\theta_1^*(1) - \theta_1^*(-1) = \beta\left[(V_1(1) + V_1(-1)) - (V_1(2) + V_1(-2))\right] \tag{22}$$

By symmetry, $V_1(1) + V_1(-1)$ and $V_1(2) + V_1(-2)$ represent the total efficiency of the mechanism in states 1 and 2 respectively. In this case, the gap between obligated and non-obligated thresholds is proportional to the difference in total efficiency of the mechanism in these two states.

[34] Note that in state 9, the obligated player uses a higher threshold than in state 8. This can be attributed to the fact that in state 10, the obligated player is forced to complete the task. The obligated player is willing to accept slightly larger costs to avoid this distortion.

5. Discussion

5.1. Conclusion

This work represents, to my knowledge, the first formal analysis of flexible turn-taking. Whatever the precise form of flexible turn-taking people may use, it is likely to provide substantial efficiency, perhaps nearly optimal. If this is the case, it is not surprising that flexible turn-taking is such a durable institution. Nothing suitable would do much better.

However, the complexity of the underlying incentives in these mechanisms suggests the caution that actual behavior might depart from theoretical predictions in ways that impact the empirical properties. In an experiment, although the efficiency achieved by subjects under recurring rotation was close to that expected in theory, there were robust anomalies in subject behavior which cannot be attributed to pro-social behavior or strategic concerns. This suggests these patterns arise at in the decision process rather than because of unmodeled incentives. One explanation is that subjects may have used heuristics from forms of flexible turn-taking with slightly different structure. For instance, the strategies subjects implement in the experiment are consistent with the kind of "gaps" that appear theoretically in the obligation takeover mechanism.

Work confirming the precise type of flexible turn-taking people naturally implement would allow a more detailed comparison between these social arrangements and other mechanisms. Further, there are several interesting theoretical extensions possible including analysis of arrangements for larger groups, the effect of cost distributions that are asymmetric between players, the potential for players to condition on partially observable information, correlation between players and across time, and the effect of prosociality.

Further, while I have looked at turn-taking in the context of tasks assignment, the results of this paper extend immediately to the repeated allocation problem and should provide insight into analogous environments such as the repeated trade, and collusive environments considered elsewhere.

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Appendix A. Theoretical appendix

A.1. Proofs of results in text

Proof of Lemma 2.

$$g\left(\tilde{\theta}\right)=\frac{1}{\beta}\tilde{\theta}+(2q-1)\,F\left(\tilde{\theta}\right)^{2}\left[\tilde{\theta}-E\left(\theta_i|\theta_i\le\tilde{\theta}\right)\right]-\left(1-F\left(\tilde{\theta}\right)\right)^{2}\left[E\left(\theta_i|\theta_i\ge\tilde{\theta}\right)-\tilde{\theta}\right]\tag{A.1}$$

The following two relationships are useful in re-writing this equation:

$$F\left(\tilde{\theta}\right)E\left(\theta_i|\theta_i\le\tilde{\theta}\right)=F\left(\tilde{\theta}\right)\frac{\int_0^{\tilde{\theta}}\zeta f(\zeta)\,d\zeta}{F\left(\tilde{\theta}\right)}=\int_0^{\tilde{\theta}}\zeta f(\zeta)\,d\zeta\tag{A.2}$$

$$\left(1-F\left(\tilde{\theta}\right)\right)E\left(\theta_i|\theta_i\ge\tilde{\theta}\right)=\left(1-F\left(\tilde{\theta}\right)\right)\frac{\int_{\tilde{\theta}}^{1}\zeta f(\zeta)\,d\zeta}{1-F\left(\tilde{\theta}\right)}=\int_{\tilde{\theta}}^{1}\zeta f(\zeta)\,d\zeta\tag{A.3}$$

Using these relationships, equation (A.1) can be rewritten:

$$\begin{aligned}g\left(\tilde{\theta}\right)=&\ \tilde{\theta}\left(\frac{1}{\beta}+(2q-1)\,F\left(\tilde{\theta}\right)^{2}+\left(1-F\left(\tilde{\theta}\right)\right)^{2}\right)\\&-\left((2q-1)\,F\left(\tilde{\theta}\right)\int_0^{\tilde{\theta}}\zeta f(\zeta)\,d\zeta+\left(1-F\left(\tilde{\theta}\right)\right)\int_{\tilde{\theta}}^{1}\zeta f(\zeta)\,d\zeta\right)\end{aligned}\tag{A.4}$$

The derivative of the term on the first line of equation (A.4) with respect to $\tilde{\theta}$ is:

$$\left(\frac{1}{\beta}+(2q-1)\,F\left(\tilde{\theta}\right)^{2}+\left(1-F\left(\tilde{\theta}\right)\right)^{2}\right)+2\,(2q-1)\,\tilde{\theta}f\left(\tilde{\theta}\right)F\left(\tilde{\theta}\right)-2\tilde{\theta}f\left(\tilde{\theta}\right)\left(1-F\left(\tilde{\theta}\right)\right)\tag{A.5}$$

The derivative of the term on the second line of equation (A.4) with respect to $\tilde{\theta}$ is:

$$-(2q-1)\, f\left(\tilde{\theta}\right)\int_0^{\tilde{\theta}} \zeta f(\zeta)\, d\zeta - (2q-1)\,\tilde{\theta} F\left(\tilde{\theta}\right) f\left(\tilde{\theta}\right) + f\left(\tilde{\theta}\right)\int_{\tilde{\theta}}^{1} \zeta f(\zeta)\, d\zeta + \tilde{\theta}\left(1-F\left(\tilde{\theta}\right)\right) f\left(\tilde{\theta}\right) \tag{A.6}$$

Together:

$$\begin{aligned} g'\left(\tilde{\theta}\right) = {} & \left(\frac{1}{\beta} + (2q-1)\, F\left(\tilde{\theta}\right)^2 + \left(1-F\left(\tilde{\theta}\right)\right)^2\right) + 2\,(2q-1)\,\tilde{\theta} f\left(\tilde{\theta}\right) F\left(\tilde{\theta}\right) - 2\tilde{\theta} f\left(\tilde{\theta}\right)\left(1-F\left(\tilde{\theta}\right)\right) \\ & - (2q-1)\, f\left(\tilde{\theta}\right)\int_0^{\tilde{\theta}} \zeta f(\zeta)\, d\zeta - (2q-1)\,\tilde{\theta} F\left(\tilde{\theta}\right) f\left(\tilde{\theta}\right) + f\left(\tilde{\theta}\right)\int_{\tilde{\theta}}^{1} \zeta f(\zeta)\, d\zeta + \tilde{\theta}\left(1-F\left(\tilde{\theta}\right)\right) f\left(\tilde{\theta}\right) \end{aligned} \tag{A.7}$$

This simplifies to:

$$\begin{aligned} g'\left(\tilde{\theta}\right) = {} & (2q-1)\left[F\left(\tilde{\theta}\right)^2 + f\left(\tilde{\theta}\right) F\left(\tilde{\theta}\right)\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right]\right] \\ & + \frac{1}{\beta} + \left(1-F\left(\tilde{\theta}\right)\right)^2 + f\left(\tilde{\theta}\right)\left(1-F\left(\tilde{\theta}\right)\right)\left[E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) - \tilde{\theta}\right] \end{aligned} \tag{A.8}$$

Since the term multiplying $2q-1$ must be positive, we may bound the derivative below by choosing $\beta = 1$ and $q = 0$.

$$g'\left(\tilde{\theta}\right) \ge 2\left(1-F\left(\tilde{\theta}\right)\right) - f\left(\tilde{\theta}\right) F\left(\tilde{\theta}\right)\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] + f\left(\tilde{\theta}\right)\left(1-F\left(\tilde{\theta}\right)\right)\left[E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) - \tilde{\theta}\right] \tag{A.9}$$

Thus, $g()$ is increasing if:

$$2\left(1-F\left(\tilde{\theta}\right)\right) + f\left(\tilde{\theta}\right)\left(1-F\left(\tilde{\theta}\right)\right)\left[E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) - \tilde{\theta}\right] \ge f\left(\tilde{\theta}\right) F\left(\tilde{\theta}\right)\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] \tag{A.10}$$

Since $2\left(1-F\left(\tilde{\theta}\right)\right) \ge 0$, the following is sufficient:

$$f\left(\tilde{\theta}\right)\left(1-F\left(\tilde{\theta}\right)\right)\left[E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) - \tilde{\theta}\right] \ge f\left(\tilde{\theta}\right) F\left(\tilde{\theta}\right)\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] \tag{A.11}$$

Since, by assumption, density is strictly positive everywhere, this simplifies to:

$$\left(1-F\left(\tilde{\theta}\right)\right)\left[E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) - \tilde{\theta}\right] \ge F\left(\tilde{\theta}\right)\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] \tag{A.12}$$

Noting that $\left(1-F\left(\tilde{\theta}\right)\right) E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) + F\left(\tilde{\theta}\right) E\left(\theta_i|\theta_i \le \tilde{\theta}\right) = E(\theta_i)$ (refer to (A.2) and (A.3)), this simplifies to the following sufficient condition for increasing $g()$.

$$E(\theta_i) \ge \tilde{\theta} \qquad \square \tag{A.13}$$

Proof of Lemma 3.

g is strictly positive when:

$$g\left(\tilde{\theta}\right) = \frac{1}{\beta}\tilde{\theta} + (2q-1)\, F\left(\tilde{\theta}\right)^2\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] - \left(1-F\left(\tilde{\theta}\right)\right)^2\left[E\left(\theta_i|\theta_i \ge \tilde{\theta}\right) - \tilde{\theta}\right] > 0 \tag{A.14}$$

Since $\left[\tilde{\theta} - E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] \ge 0$ and $\frac{1}{\beta}\tilde{\theta}$ is decreasing in β, let $q = 0$ and $\beta = 1$. This results in the following sufficient condition:

$$2\left(1-F\left(\tilde{\theta}\right)\right)\tilde{\theta} + F\left(\tilde{\theta}\right)\left[E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right] > \left(1-F\left(\tilde{\theta}\right)\right)\left[E(\theta_i)\right] \tag{A.15}$$

When $\tilde{\theta} = 1$, this simplifies to $E(\theta_i) > 0$, which is true by assumption that density is strictly positive everywhere. Over the interval $(\frac{1}{2}E(\theta_i), 1)$, drop the positive term $\left(F\left(\tilde{\theta}\right)\right)\left[E\left(\theta_i|\theta_i \le \tilde{\theta}\right)\right]$ to yield yet another sufficient condition:

$$2\left(1-F\left(\tilde{\theta}\right)\right)\tilde{\theta} > \left(1-F\left(\tilde{\theta}\right)\right)\left[E(\theta_i)\right] \tag{A.16}$$

Since density is strictly positive everywhere, $1-F\left(\tilde{\theta}\right) > 0$ over the interval $(\frac{1}{2}E(\theta_i), 1)$. Eliminating this term yields the desired result.

$$\tilde{\theta} > \frac{1}{2} E(\theta_i) \qquad \square \tag{A.17}$$

Proof of Proposition 9. By Lemma 12, the optimal threshold mechanism uses threshold $E(\theta_i)$. Since obligation takeover uses no monetary transfers, it can achieve the efficiency of the best threshold mechanism if using threshold $E(\theta_i)$ in every state is an equilibrium.

Let $\tilde{\theta}(z) = E(\theta_i)$ for all z. For any $|z| \geq 2$, $C\left(z+1, \tilde{\theta}\right) = C\left(z-1, \tilde{\theta}\right)$ and $T\left(z+1, \tilde{\theta}\right) = T\left(z-1, \tilde{\theta}\right)$. Thus, the fixed point conditions are met for any state with $|z| \geq 2$. For $z = 1$ and $z = -1$, the following must be true for this to be an equilibrium:

$$E(\theta_i) = C\left(2, \tilde{\theta}\right) - C\left(-1, \tilde{\theta}\right) + T\left(-1, \tilde{\theta}\right) + T\left(2, \tilde{\theta}\right) \tag{A.18}$$

$$E(\theta_i) = C\left(1, \tilde{\theta}\right) - C\left(-2, \tilde{\theta}\right) + T\left(-2, \tilde{\theta}\right) + T\left(1, \tilde{\theta}\right) \tag{A.19}$$

When $\tilde{\theta}(z) = E(\theta_i)$ for all z and F is symmetric, $T\left(z, \tilde{\theta}\right) = \frac{1}{4} E(\theta_i)$ for all z. While $C\left(2, \tilde{\theta}\right) = C\left(1, \tilde{\theta}\right)$ and $C\left(-2, \tilde{\theta}\right) = C\left(-1, \tilde{\theta}\right)$, C functions in states with opposite signs are not equal. The equilibrium condition simplifies to checking:

$$\begin{aligned} E(\theta_i) &= E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i)) F(E(\theta_i)) \\ &\quad + E(\theta_i)(1 - F(E(\theta_i))) - E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i))(1 - F(E(\theta_i))) + \frac{1}{2} E(\theta_i) \end{aligned} \tag{A.20}$$

Since, $E(\theta_i) = E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i)) + E(\theta_i|\theta_i \geq E(\theta_i))(1 - F(E(\theta_i)))$, this is:

$$\begin{aligned} E(\theta_i) &= E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i)) F(E(\theta_i)) \\ &\quad + E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i))(1 - F(E(\theta_i))) + E(\theta_i|\theta_i \geq E(\theta_i))(1 - F(E(\theta_i)))^2 \\ &\quad - E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i))(1 - F(E(\theta_i))) + \frac{1}{2} E(\theta_i) \end{aligned} \tag{A.21}$$

Which simplifies to:

$$E(\theta_i) = E(\theta_i|\theta_i \leq E(\theta_i)) F(E(\theta_i)) F(E(\theta_i)) + E(\theta_i|\theta_i \geq E(\theta_i))(1 - F(E(\theta_i)))^2 + \frac{1}{2} E(\theta_i) \tag{A.22}$$

For any symmetric distribution, $F(E(\theta_i)) = 1 - F(E(\theta_i)) = \frac{1}{2}$. Thus, this simplifies to:

$$E(\theta_i) = \frac{1}{2} E(\theta_i) + \frac{1}{2} E(\theta_i) = E(\theta_i) \tag{A.23}$$

Thus, $\tilde{\theta}(z) = E(\theta_i)$ for all z is an equilibrium of obligation takeover for perfectly patient players with overtaking preferences under any symmetric F.[35] $\square$

A.2. Suboptimality of recurring rotation

Lemma 11. *Any threshold $\tilde{\theta}$ may be implemented by an ex-post incentive compatible mechanism using budget balanced money transfers.*

Proof. Consider a "threshold mechanism" using transfers of $\frac{\tilde{\theta}}{2}$. The player completing the task will be paid $\frac{\tilde{\theta}}{2}$ by the other. Both players report whether they are above or below the threshold. If both are above or both are below, one is chosen at random to execute the task. Otherwise, the player reporting a low cost completes the task.

Note that a player prefers to complete the task and receive a payment of $\frac{\tilde{\theta}}{2}$ rather than pay $\frac{\tilde{\theta}}{2}$, as long as that player has a cost below $\tilde{\theta}$. Otherwise, the player prefers not to complete the task and pay $\frac{\tilde{\theta}}{2}$. Regardless of the other player's choice, offering to do the task maximizes a player's probability of doing the task. Not-offering maximizes the probability of not doing the task. Thus, a player with $\theta_i \leq \tilde{\theta}$ has a weakly dominant strategy of offering to do the task while a player with $\theta_i \geq \tilde{\theta}$ has a weakly dominant strategy of not-offering. Because of this, reporting costs according to the threshold is dominant strategy (ex-post) incentive compatible. $\square$

Lemma 12. *For any $F()$, the threshold mechanism that maximizes welfare (minimizes average stage cost) uses a threshold equal to the mean type.*

[35] The change in state leads to only a finite change in the undiscounted stream of stage payoffs. A perfectly patient player with limit of means preferences is indifferent between these states.

Proof. By Lemma 11, any threshold mechanism can be implemented by an ex-post incentive compatible mechanism with budget-balanced transfers. These transfers are welfare neutral, and so the mechanism's welfare is given by its average stage cost. The average stage cost under a threshold mechanism is given by:

$$\bar{AC}\left(\tilde{\theta}\right) = F\left(\tilde{\theta}\right) E\left(\theta_i | \theta_i \le \tilde{\theta}\right) + \left(1 - F\left(\tilde{\theta}\right)\right) E\left(\theta_i\right) \tag{A.24}$$

It's derivative is:

$$\frac{\delta \bar{AC}\left(\tilde{\theta}\right)}{\delta \tilde{\theta}} = f\left(\tilde{\theta}\right)\tilde{\theta} - f\left(\tilde{\theta}\right) E\left(\theta_i\right) \tag{A.25}$$

Since $\frac{\delta \bar{AC}\left(\tilde{\theta}\right)}{\delta \tilde{\theta}}$ is negative below the mean and positive above the mean, $\bar{AC}\left(\tilde{\theta}\right)$ is quasi-convex and reaches its global minimum at $\tilde{\theta} = E\left(\theta_i\right)$. □

Proposition 13. *For distributions with positive density everywhere, the average stage cost of recurring rotation is bounded above second best optimal. The stage average welfare gap is at least* $\bar{AC}\left(\frac{E(\theta_i)}{2}\right) - \bar{AC}\left(E\left(\theta_i\right)\right)$.

Proof. By Lemmas 11 and 12, the optimal threshold mechanism incurring average stage cost $\bar{AC}\left(E\left(\theta_i\right)\right)$ can be implemented by a robust mechanism. Thus, the second-best optimal average stage cost must be at least as small as $\bar{AC}\left(E\left(\theta_i\right)\right)$.

By Proposition 4, the equilibrium threshold in recurring rotation is always below half of the mean type. The average stage cost associated with using a threshold $\tilde{\theta}$ is $\bar{AC}\left(\tilde{\theta}\right) = E\left(\theta_i\right) - F\left(\tilde{\theta}\right)\left(E\left(\theta_i\right) - E\left(\theta_i | \theta_i \le \tilde{\theta}\right)\right)$. This is strictly decreasing over $[0, E\left(\theta_i\right)]$ and so the average cost of recurring rotation must be larger than $\bar{AC}\left(\frac{E(\theta_i)}{2}\right)$.

Together, this implies that the gap in welfare (in terms of average stage cost) between recurring rotation and a second-best optimal mechanism is at least $\bar{AC}\left(\frac{E(\theta_i)}{2}\right) - \bar{AC}\left(E\left(\theta_i\right)\right) > 0$. □

Using the result above for the uniform distribution predicts a gap of at least $\frac{1}{32} \approx 0.0278$. In fact, the gap for the option version of the mechanism as $\beta \to 1$ is approximately $0.411 - 0.375 = 0.036$ since $\frac{3}{8} = 0.375$ is known to be the second-best optimal cost.

I do not claim that the mean-threshold mechanism provides a tight bound on the welfare achievable by robust mechanisms for any distribution except uniform. In fact, Miller (2012, example 4) provides a counter-example under which the mean-threshold mechanism is not optimal under a particular asymmetric distribution where density is $\frac{2}{5}$ for $\theta_i \le \frac{1}{2}$ and $\frac{8}{5}$ for $\theta_i > \frac{1}{2}$. However, several results suggest that the best threshold mechanism may still be a good benchmark.

General results showing that threshold mechanisms are optimal among the class of *deterministic* mechanisms for type distributions with monotonic hazard rate are given in Shao and Zhou (2013), Drexl and Kleiner (2015). In addition, Hagerty and Rogerson (1987) prove that a threshold, specifically a *posted price* mechanism, is optimal in a trade setting under dominant strategy incentive compatibility, ex-post individual rationality and strong ex-post budget balance. Further, Athey and Miller (2007) reports that in numerical tests where the threshold mechanism was not optimal, the computed optimal mechanism improved efficiency very little.

Using the best threshold rather than the true optimal for comparison has the benefit that the best threshold mechanism is well defined by Lemma 11 Lemma 12 as the mean threshold mechanism and has an average stage cost that is easy to calculate. This provides the flexibility to make efficiency comparisons in environments with cost distributions that are not uniform. Despite the fact that recurring rotation is suboptimal relative to the best threshold mechanism, in online appendix subsection C.2. I demonstrate that there are some environments where the welfare loss is small.

A.3. Constructing $V_1(z)$ for obligation takeover

The value function can be constructed from a set of threshold strategies. Below I derive expressions for the value function of player 1. The value function for player 2 is simply the reverse. For instance, $V_2(z) = V_1(-z)$ just as $\tilde{\theta}_2(z) = \tilde{\theta}_1(-z)$.

When obligated, $(z \ge 1)$, Player 1 pays an average cost of $E\left(\theta_1\right)$ when player 2 has cost above threshold $\tilde{\theta}(-z)$. The probability of this occurrence is $\left(1 - F\left(\tilde{\theta}(-z)\right)\right)$. On the other hand, player 2 is below this threshold with probability $F\left(\tilde{\theta}(-z)\right)$. In this case, if player 1 has cost below threshold $\tilde{\theta}(z)$, then there is no swap and player 1 pays an average cost of $E\left(\theta_1 | \theta_1 \le \tilde{\theta}(z)\right)$. If player 1 is above threshold $\tilde{\theta}(z)$, there is a swap. Player 1 does not pay. Thus, the average stage cost for player 1 in state z is:

$$-E\left(\theta_i | \theta_i \le \tilde{\theta}(z)\right) F\left(\tilde{\theta}(z)\right) F\left(\tilde{\theta}(-z)\right) - E\left(\theta_i\right)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \tag{A.26}$$

For $z > 1$, The state moves to $z - 1$ unless there is a swap (for $z = 1$, a slight correction is needed to account for the fact that there is no state 0). The discounted continuation value of this is $\beta(V(z-1))$. Only if there is a swap, which happens with probability $\left(1 - F\left(\tilde{\theta}(z)\right)\right) F\left(\tilde{\theta}(-z)\right)$, does the state move to $z + 1$. The discounted continuation value of this is $\beta(V(z+1))$. The continuation portion of the value function can thus be written:

$$\beta(V(z-1)) - \beta(V_1(z-1) - V_1(z+1))\left(1 - F\left(\tilde{\theta}(z)\right)\right) F\left(\tilde{\theta}(-z)\right) \tag{A.27}$$

In equilibrium however, $\beta(V_1(z-1) - V_1(z+1)) = \tilde{\theta}(z)$. Imposing this relationship, (A.27) can be rewritten:

$$\beta(V_1(z-1)) - \tilde{\theta}(z)\left(1 - F\left(\tilde{\theta}(z)\right)\right) F\left(\tilde{\theta}(-z)\right) \tag{A.28}$$

Adding together expressions (A.26) and (A.28), a player's value function in an obligated state can be written:

$$\begin{aligned} &\forall z > 1: V_1(z) = \beta(V_1(z-1)) - \tilde{\theta}(z)\left(1 - F\left(\tilde{\theta}(z)\right)\right) F\left(\tilde{\theta}(-z)\right) \\ &- E\left(\theta_i|\theta_i \le \tilde{\theta}(z)\right) F\left(\tilde{\theta}(z)\right) F\left(\tilde{\theta}(-z)\right) - E(\theta_i)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \end{aligned} \tag{A.29}$$

To simplify this expression, note that the term below represents the average cost paid by the obligated player in a stage with state z under strategy $\tilde{\theta}$. Denote this cost $C\left(z, \tilde{\theta}\right)$.

$$C\left(z, \tilde{\theta}\right) = E\left(\theta_i|\theta_i \le \tilde{\theta}(z)\right) F\left(\tilde{\theta}(z)\right) F\left(\tilde{\theta}(-z)\right) + E(\theta_i)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \tag{A.30}$$

The following term is the expected transfer of continuation utility associated with a deferment. This is denoted $T\left(z, \tilde{\theta}\right)$.

$$T\left(z, \tilde{\theta}\right) = \tilde{\theta}(z)\left(1 - F\left(\tilde{\theta}(z)\right)\right) F\left(\tilde{\theta}(-z)\right) \tag{A.31}$$

Together, for $z > 1$:

$$\forall z > 1: V_1(z) = \beta(V_1(z-1)) - C\left(z, \tilde{\theta}\right) - T\left(z, \tilde{\theta}\right) \tag{A.32}$$

In any state with $z < -1$ such that player 1 is non-obligated, the "status quo" is returning to a higher state, which has discounted value $\beta V_1(z+1)$. Only if player 2 is above threshold $\tilde{\theta}(-z)$ and player 1 is below threshold $\tilde{\theta}(z)$ does player 1 pay an average cost $E\left(\theta_1|\theta_1 \le \tilde{\theta}(z)\right)$ and receive continuation utility transfer $\beta(V_1(z-1) - V_1(z+1)) = \tilde{\theta}(z)$ due to the state-change. In this way, a player's utility in a non-obligated state can be written:

$$\begin{aligned} &\forall z < -1: V_1(z) = \beta(V_1(z+1)) + \tilde{\theta}(z) F\left(\tilde{\theta}(z)\right)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \\ &\quad - E\left(\theta_i|\theta_I \le \tilde{\theta}(z)\right) F\left(\tilde{\theta}(z)\right)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \end{aligned} \tag{A.33}$$

As in the obligated states, the following terms represent the average stage cost paid by the non-obligated player and the expected continuation transfer associated with a deferment:

$$C\left(z, \tilde{\theta}\right) = E\left(\theta_i|\theta_i \le \tilde{\theta}(z)\right) F\left(\tilde{\theta}(z)\right)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \tag{A.34}$$

$$T\left(z, \tilde{\theta}\right) = \tilde{\theta}(z) F\left(\tilde{\theta}(z)\right)\left(1 - F\left(\tilde{\theta}(-z)\right)\right) \tag{A.35}$$

Non-obligated continuation value can be written:

$$\forall z < -1: V_1(z) = \beta(V(z+1)) - C\left(z, \tilde{\theta}\right) + T\left(z, \tilde{\theta}\right) \tag{A.36}$$

For states $z = 1$ and $z = -1$, a slight modification must be made to account for the fact that there is no state 0.

$$V_1(1) = \beta(V(-1)) - C\left(1, \tilde{\theta}\right) - T\left(1, \tilde{\theta}\right) \tag{A.37}$$

$$V_1(-1) = \beta(V(1)) - C\left(-1, \tilde{\theta}\right) + T\left(-1, \tilde{\theta}\right) \tag{A.38}$$

Appendix B. Experimental appendix

B.1. Procedure for empirical efficiency

Letting $\tilde{\theta}_{i,o}$ be player $i's$ threshold strategy when player o is obligated, the equation for average cost under strategy profile $\tilde{\boldsymbol{\theta}}$ in the state that i is obligated is given by:

$$\bar{AC}\left(\tilde{\boldsymbol{\theta}}, i\right) = F\left(\tilde{\theta}_{i,i}\right) E\left(\theta_i | \theta_i \leq \tilde{\theta}_{i,i}\right) + F\left(\tilde{\theta}_{j,i}\right)\left[1 - F\left(\tilde{\theta}_{i,i}\right)\right] E\left(\theta_j | \theta_j \leq \tilde{\theta}_{j,i}\right) + \left[1 - F\left(\tilde{\theta}_{j,i}\right)\right]\left[1 - F\left(\tilde{\theta}_{i,i}\right)\right] E\left(\theta_i | \theta_i \geq \tilde{\theta}_{i,i}\right) \tag{B.1}$$

Since the average stage cost depends on the state (who is obligated), the joint welfare in the long-run depends on the how often each state occurs. This was not the case when both players use the same strategy since the average stage cost is the same in both states and, in the long-run, each state is equally likely. The joint welfare of disequilibrium play with asymmetric strategies can be calculated by weighting the average cost in each state by the long-run visiting probabilities of the states. The sequence of which player is obligated follows a stationary Markov process and the probability player 1 is obligated in the long-run is:

$$\pi_1\left(\tilde{\boldsymbol{\theta}}\right) = \frac{F\left(\tilde{\theta}_{1,2}\right) F\left(\tilde{\theta}_{2,2}\right) - F\left(\tilde{\theta}_{1,2}\right) + 1}{2 + F\left(\tilde{\theta}_{1,1}\right) F\left(\tilde{\theta}_{2,1}\right) - F\left(\tilde{\theta}_{2,1}\right) + F\left(\tilde{\theta}_{1,2}\right) F\left(\tilde{\theta}_{2,2}\right) - F\left(\tilde{\theta}_{1,2}\right)} \tag{B.2}$$

The average stage cost under "gap" strategies $\tilde{\boldsymbol{\theta}}$ is then given by:

$$\pi_1\left(\tilde{\boldsymbol{\theta}}\right) \bar{AC}\left(\tilde{\boldsymbol{\theta}}, 1\right) + \pi_2\left(\tilde{\boldsymbol{\theta}}\right) \bar{AC}\left(\tilde{\boldsymbol{\theta}}, 2\right) \tag{B.3}$$

Appendix C. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.geb.2017.02.003.

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