Raw Class Notes for 3012- Spring 2022

These notes are unedited versions of the notes we typed in class. For more polished notes, please see "Class Notes" on my webpage.

Contents

1 Class 1- 1/19/2022

X is the **feasible set**.

This a set of bundles. x.

We want the feasible set to be all the relevant bundles for a model.

For example, if we are modeling the choice of ice cream bowls. The bowl: "one scoop of vanilla and one of chocolate" is a single bundles in the set of feasible bundles.

A bundle will normally (in this class) consist of two goods.

For example the two goods might be chocolate ice cream and vanilla ice cream.

a bundle is now an amount of each good.

Let's say good 1 is chocolate and good 2 is vanilla.

$x = (1, 1)$ is the bundle "one scoop of chocolate and one scoop of vanilla.

$(0, 2)$ is two scoops of vanilla.

$(5, 1.8)$ five scoops of chocolate and 1.8 scoops of vanilla.

We could add strawberry to the model. now we have 3 goods.

$(1, 1, 1)$ one scoop of each flavor.

Two goods is "enough" to learn about trade-offs so we work with 2.

Let's go back to chocolate and vanilla ice cream.

Let's define the feasible set for this model. We want the feasible set to be all bowls of ice cream with a positive (non-negative) real number of scoops of each flavor.

$$X = \mathbb{R}^2_+$$

$(0, 2), (1000, 5), (1000000, 29)$ all in the feasible set.

$(-1, 2)$ is not.

$$x = (1, 1) = (x_1, x_2)$$

x_1 is the amount of good 1. and x_2 is the amount of good 2.

Budget set is the set of bundles actually available to a particular consumer. B is the **budget set**.

Budget set might be all the bowls of ice cream with no more than two total scoops. The budget set is always a subset of the feasible set.

$$B \subseteq X$$

Let's write formally the set of all bowls of ice cream with no more than two total scoops.

$$B = \left\{x | x \in \mathbb{R}^2_+ \,\&\, x_1 + x_2 \leq 2\right\}$$

$$B = \left\{x|x \in \mathbb{R}^2 \,\&x_1 \geq 0\&x_2 \geq 0\&\, x_1 + x_2 \leq 2\right\}$$

We could a weird budget set:

$$B = \left\{x|x \in \mathbb{R}^2_+\& \left(\sqrt{x_1^2 + x_2^2} \leq 1\right)\right\}$$

This is the the set of all bundles less than distance one from the origin. It is a circle. This is technically possible in our framework.

Normally we think of budgets as coming from income m and prices p_1 and p_2. *Competitive budgets.* The price of any bundle is:

$$p_1 x_1 + p_2 x_2$$

scoops of ice cream cost \$2 each $p_1 = 2$ and $p_2 = 2$. The cost of the bundle $(2, 1)$ is:

$$(2 * 2) + (2 * 1) = 6$$

Suppose I have m dollars. What can I afford.

$$p_1 x_1 + p_2 x_2 \leq m$$

This is the formal version of a *competitive budget*.

$$B = \left\{x|x \in \mathbb{R}^2_+\& p_1 x_1 + p_2 x_2 \leq m\right\}$$

This is the set of all bundles that someone can afford with income m at prices p_1 and p_2.

The **budget line** are all of the bundles that cost exactly m.

$$p_1 x_1 + p_2 x_2 = m$$

We can transform this by isolating x_2.

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

We get the x_2 intercept immediately. $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$. The other intercept can be found by plugging 0 in for x_2. The x_1 intercept is $\frac{m}{p_1}$.

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Budget set is described by:

$$(x_1p_1) + (x_2p_2) \leq m$$

"Spends no more than income"

The important part of this budget is the **Budget Line.**

$$x_1p_1 + x_2p_2 = m$$

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

x_2 intercept is $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$.

We can also get the x_1 intercept by plugging 0 in for x_2.

$$0 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

$$x_1 = \frac{m}{p_1}$$

x_1 intercept is $\frac{m}{p_1}$,x_2 intercept is $\frac{m}{p_2}$. The slope is $-\frac{p_1}{p_2}$.

Interpreting the intercepts:

The x_1 intercept is "how much x_1 can I have if I only buy x_1"

The x_2 intercept is "how much x_2 can I have if I only buy x_2"

These expressions should make sense. $\frac{m}{p_1}$ says "if i spend m on x_1 how many units do I get"

Let's look at another bundle with a similar expression:

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

The slope measures how much good 2 I give up to get one more unit of good 1.

This represents the tradeoff I have to make between the goods given their prices.

$$-\frac{p_1}{p_2}$$

What happens to the budget when one of the parameters of the model p_1, p_2, m changes.

Taxes

Quantity tax:

An amount of money you owe to the government **per unit** of some good you buy. Quantity tax of t on good 1.

$$x_1 t + x_1 p_1 + x_2 p_2 = m$$

$$\left(p_1 + t\right) x_1 + x_2 p_2 = m$$

Ad valorem tax:

A tax on the value of a good purchased:

You owe the government τ times the value of the x_1 you purchase. In the case of Nashville, $\tau \approx 0.09$

$$\tau\left(p_1 x_1\right) + p_1 x_1 + p_2 x_2 = m$$

For example, if $p_1 = 10$ and $x_1 = 10$ I've spent \$100. If $\tau = 0.09$ then the tax is 9% and I owe \$9 in tax. The total cost of x_1 becomes \$109.

$$\left(\left(1+\tau\right) p_1\right) x_1 + p_2 x_2 = m$$

The x_1 intercept when the price of x_1 is p_1 if the amount purchased is less than $\bar{x_1}$ and $p_1 + t$ for any units purchased above $\bar{x_1}$.

Let's calculate the cost of buying $\bar{x_1}$ at price p_1:

$$\bar{x_1} p_1$$

The money I have left over is:

$$m - \bar{x_1} p_1$$

The extra x_1 I can buy with this leftover money at the new price of $p_1 + t$ is:

$$\frac{m - \bar{x_1} p_1}{p_1 + t}$$

The total amount I can afford it $\bar{x_1}$ plus this amount:

$$\bar{x}_1 + \frac{m - \bar{x_1} p_1}{p_1 + t}$$

Preferences:

Now we will try to model "what a consumer wants"

To represent preferences, we use a "relation". A relation is a set of statements about **pairs** of bundles. You are familiar with some relations already like $\geq$ (greater than or equal to).

$$3 \geq 2$$

$$4 \geq 1$$

A another relation on the set of people might be "Is a sibling of". Let's represent this by s. The following statements are true:

$$Greg \, s \, Christina$$

$$Finn \, s \, Remy$$

In economics, we represent preferences as a relation called the "Preference Relation" $\succsim$.

Suppose a consumer doesn't care about flavor, but just likes more ice cream. We represent bundles such as $(1,1)$ is one scoop of vanilla and one scoop of chocolate. The following formal preference statements are true about this consumer:

$$(1,1) \succsim (0,0)$$

$$(2,1) \succsim (0,2)$$

$$(0,2) \succsim (2,0)$$

The term "preferred" is synonymous to "weakly preferred" and will be true if a consumer either strictly prefers the first bundle to the second (as in the first two cases above) or if they are indifferent such as in the third above.

Contrast this to the term "strictly preferred". The strictly preferred relation is represented by $\succ$.

$$(1,1) \succ (0,0)$$

$$(2,1) \succ (0,2)$$

but the following statement is **not true** about the consumer:

$$(0,2) \succ (2,0)$$

However, the consumer is indifferent between these bundles. The indifference relation is represented by $\sim$. THe following is true.

$$(0,2) \sim (2,0)$$

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Preferences

$$\succsim$$

This is the Weak Preference Relation.

Consumer just wants more ice cream: (Scoops of chocolate, scoops of vanilla).

$$(2,0) \succsim (1,0)$$

From the weak preference relation we can infer strict preference and indifference as well.

We will say $x \succ y$ if and only if $x \succsim y$ and not $y \succsim x$

We will say $x \sim y$ if and only if $x \succsim y$ and $y \succsim x$

For instance:

$$(2,0) \succ (1,0)$$

Since $(2,0) \succsim (1,0)$ but not $(1,0) \succsim (2,0)$.

$$(1,0) \sim (0,1)$$

Since $(1,0) \succsim (0,1)$ and $(0,1) \succsim (1,0)$

Example:

Suppose a consumer has preference over the bundles x, y, z:

$x \succsim y$, $y \succsim z$, $z \succsim y$, $x \succsim z$, $x \succsim x$, $y \succsim y$, $z \succsim z$

What is true about their strict preferences?

$x \succ y, x \succ z$

What is true about their indifference relation?

$y \sim z, z \sim y$

The indifference relation is *symmetric*: whenever $x \sim y$ we also have $y \sim x$.

On the other hand, the strict preference relation is *asymmetric*: if we have $x \succ y$ we don't have $y \succ x$.

The fundamental thing we will work with is the weak preference relation.

Three key assumptions we make about $\succsim$.

1. Reflexive: "every bundle is at least as good as itself"

$$\forall x \in X : x \succsim x$$

For all bundles in the feasible set.

2. Completeness: "Every pair of bundles is comparable"

$$\forall x, y \in X \,\&\, x \neq y : \, (x \succsim y) \vee (y \succsim x) \; \textit{or both}$$

Either one bundle is strictly better than the other or they are indifferent, but they can't say "I don't know". This limits the scope of our models. We should only include bundles that are relevant to a consumers. They should be able to form preferences.

3. Transitivity: "For every three bundles x, y, z, if x is at least as good as y and y is at least as good as z, it must be that x is at least as good as z"

$$\forall x, y, z \in X : x \succsim y \& y \succsim z \implies x \succsim z$$

It is possible to have intransitive preferences. Here is an example:

Choosing mates on a dating app:

Three potential mates:

x. Rich, Very Intelligent, Average Looking

y. Financially Constrained, Genius, Good Looking

z. Moderately Wealthy, Average Intelligence, Best Looking

$y \succ x$ since y is smarter and better looking

$z \succ y$ since z is wealthier and better looking

$x \succ z$ since x is wealthier and smarter than z

These are the only things true about $\succsim$ in this case:

$$y \succsim x, z \succsim y, x \succsim z$$

We have $z \succsim y$ and $y \succsim x$. If this was transitive this should imply $z \succsim x$. This isn't true because x **is strictly better than** z. **These preferences are intransitive.**

While this assumption can fail, it will be reasonable for most of what we do, and it is much easier to "do" economics if preferences are transitive since it will ensure the consumer can make a choice from any budget set.

We construct something called a "choice function". The choice function takes a budget set, and returns a bundle or bundles from the budget set such that that bundle or those bundles are better than everything else in the budget set. The bundle chosen meets this condition: x is "chosen" from budget set B if:

$$\forall x' \in B, x \succsim x'$$

That is, the chosen bundle is at least as good as everything else. If this is not true of the chosen bundle, there is something strictly better. From the set of three mates in the previous example, is there one mate that is at least as good as every other one? No! There is not mate meeting this condition. The choice set is empty.

Transitivity (and completeness) ensures there will always be a "best" or set of "best" bundles. It lets us put things in order. Let's look at our example from the beginning of lecture.

$x \succsim y, y \succsim z, z \succsim y, x \succsim z$,$x \succsim x, y \succsim y, z \succsim z$

This is: Reflexive, Complete, Transitive. Let's rank order the objects. We will put one above the other if is strictly preferred, and put them at the same ranking if they are indifferent. From this weak preference relation we have already extract the strict and weak preferences:

$x \succ y, x \succ z$,$y \sim z, z \sim y$. Here's the ranking:

$$1. x$$

$$2. y, z$$

x is first and y and z are second.

Now let's look at the intransitive relation from the dating app example:

With the intransitive relation $x \succ y, y \succ z, z \succ x$

These are cyclic and that means there is no best. We cannot put them in a rank order.

In order to visualize preference, we use "indifference set" "**indifference curves**". An indifference curve is a set of bundles that are all indifferent to eachother.

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Two distinct indifference curves cannot intersect each other if preferences are complete and transitive.

Since they are distinct, we can find x and some x' such that

$$x \succ x'$$

Because they are on distinct indifference curves.

However, since they intersect there is a bundle $\tilde{x}$ on both curves. Thus, $\tilde{x}$ is indifference to both x and x'

$$\tilde{x} \sim x$$

$$\tilde{x} \sim x'$$

By indifference $x' \succsim \tilde{x}$ and $\tilde{x} \succsim x$.

By transitivity we must have:

$$x' \succsim x$$

However by our original assumption:

$$x \succ x'$$

However, these are contraditory since $x \succ x'$ implies that **not** $x' \succsim x$**.**

We have two statements that contradictory:

$x' \succsim x$ and not $x' \succsim x$.

Perfect Substitutes:

Linear indifference curves. Willingness to trade-off between the goods at **any point** is exactly the same.

Perfect Complements:

Always consume the goods in some fixed proportion. *Left/Right Shoes or Baking Pies.* The indifference curves are L-shaped and the kink-points follow a "ray" through the origin of some slope that represents the ratio the goods are consumed in.

Cobb Douglass:

Convex shaped indifference curves. The more of one good you have, you are willing to give up relatively more to get some of the other good.

Bads:

This occurs when you want less of one of the two (or both) of the goods.

With one bad and one good, the indifference curves slope upwards.

With two bads, the indifference curves again slope downward, but prefrence increases as we move towards the origin.

Well-Behaved Preferences

Monotonicity- "everything is a good"

Weakly Monotonic (Monotonic):

$$(x_1, x_2), \left(x_1^{'}, x_2^{'}\right)$$

If $x_1 \geq x_1'$ and $x_2 \geq x_2^{'}$ then $(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$

Furthermore if $x_1 > x_1^{'}$ **and** $x_2 > x_2^{'}$ **then** $(x_1, x_2) \succ \left(x_1^{'}, x_2^{'}\right)$

For instance $(2, 3) \sim (3, 3)$ and $(4, 4) \succ (3, 3)$.

Strictly Monotonic

If $x_1 \geq x_1'$ and $x_2 \geq x_2'$ then $(x_1, x_2) \succsim \left(x_1', x_2'\right)$

Furthermore if $x_1 > x_1'$ **or** $x_2 > x_2'$ **then** $(x_1, x_2) \succ \left(x_1', x_2'\right)$

Convexity