

## ECON 3012

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### Part 1. Budget (2.1-2.7)

#### 1. BUNDLES

**Bundle:**  $x = (x_1, x_2)$

**Example.** Ice Cream Bowls.  $x_1$  is the amount of vanilla.  $x_2$  is the amount of chocolate.

$(1, 1)$  one scoop of each flavor.

$(2, 2)$  two scoops of each flavor.

$(0.28, 100)$  a lot of chocolate and a little vanilla.

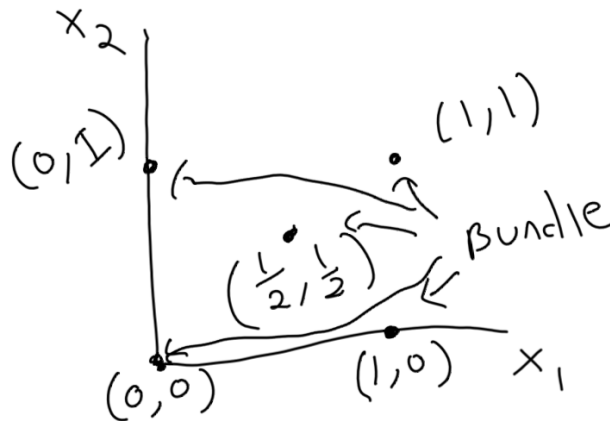


FIGURE 1.1. Bundles on Cartesian Plane.

#### 2. FEASIBLE SET

**The Feasible Set:**  $X$  is the “feasible” set of bundles.

The feasible set is the universe of bundles that might be relevant in a model. The feasible set defines the scope of a model.

## 3. BUDGET SET

**Budget Set:  $B$** 

The budget set is the set of bundles *available* to a particular consumer.

The budget set must be a subset of the feasible set.

In set notation:  $B \subseteq X$

**3.1. Budget Sets from Prices and Income. Prices:**  $p_1, p_2$ : Price of good 1 and price of good 2.

**Cost of a bundle:**  $p_1x_1 + p_2x_2$ .

**Income:**  $m$ .

**Budget set:**  $B = \{x | x \in X \text{ \& } x_1p_1 + x_2p_2 \leq m\}$ .

In non-math language, this says the budget set is the set of bundles such that the price of the bundle is less than income.

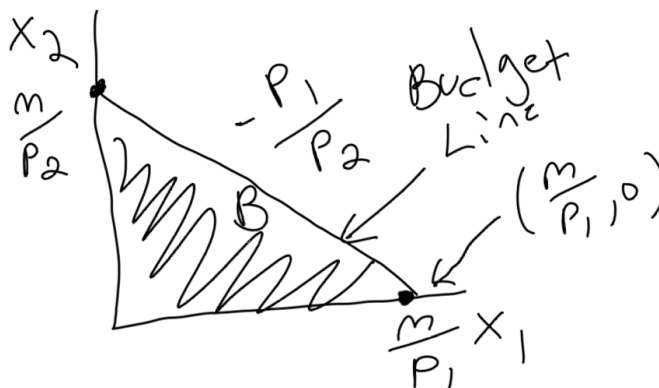


FIGURE 3.1. Graphical Representation of the Budget Set

**3.2. Changing Prices and Income. Suppose income increases.  $m$  changes.**

Both endpoints change. If  $m$  increases,  $\frac{m}{p_1}$  (the amount I can buy of good 1 changes) increases and  $\frac{m}{p_2}$  (maximum affordable  $x_2$ ) increases. The slope does not change. If  $m$  decreases, the opposite happens.

**Suppose one of the prices changes.**

$p_1$ . If  $p_1$  goes up, the slope decreases (more negative). If  $p_1$  goes down, the slope increases. The  $x_2$  intercept stays the same.

$p_2$ . If  $p_2$  goes up, the slope increases. If  $p_2$  goes down the slope decreases (more negative). The  $x_1$  intercept stays the same.

**3.3. Taxes. Quantity tax on good 1:**

$$p_1x_1 + tx_1 + p_2x_2 = m$$

$$(p_1 + t)x_1 + p_2x_2 = m$$

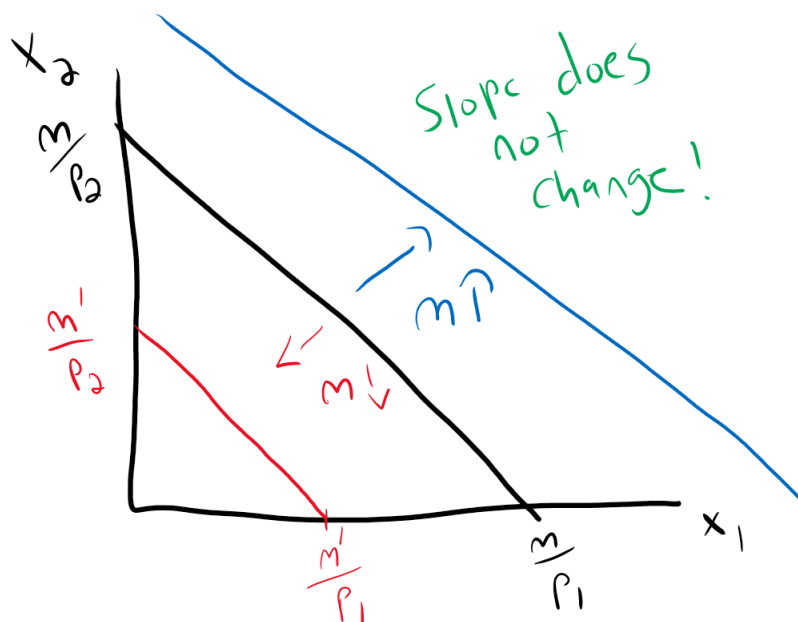
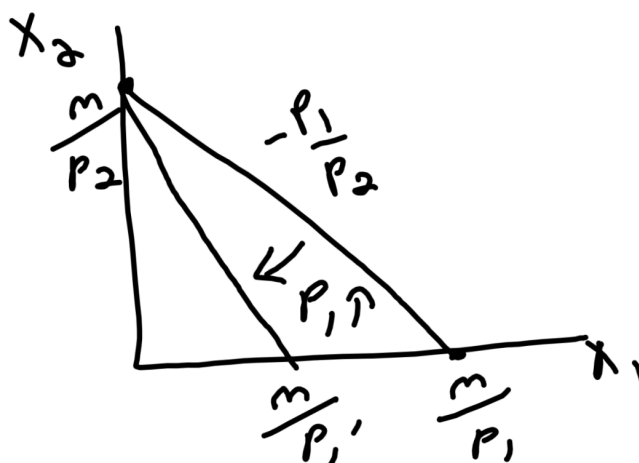


FIGURE 3.2. How Budget Changes with Income

FIGURE 3.3. How Budget Changes with an increase in  $p_1$ .

*Ad Valorem Tax on good 1:*

$$(p_1 x_1) + \tau (p_1 x_1) + p_2 x_2 = m$$

$$(1 + \tau) (p_1 x_1) + p_2 x_2 = m$$

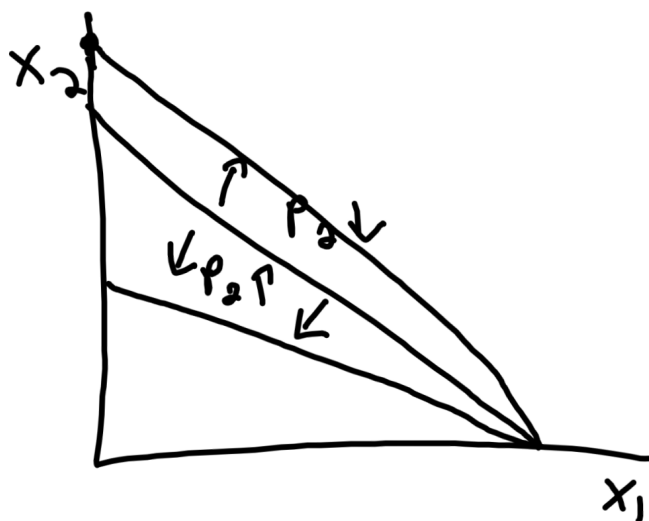
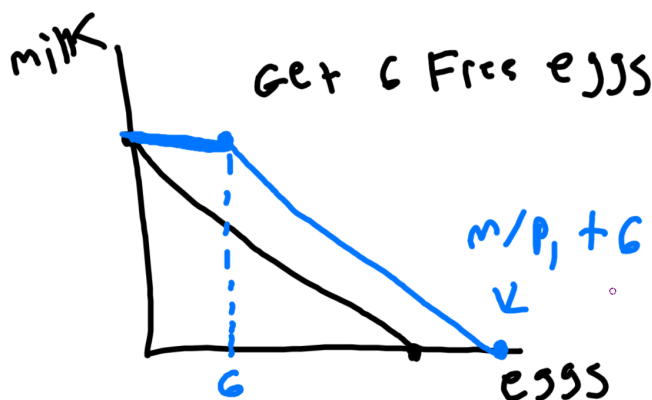
FIGURE 3.4. How Budget Changes with Changes to  $p_2$ 

FIGURE 3.5. Six Free Eggs

We will focus on quantity taxes.

3.4. More Complex Scenarios.

3.5. Price Depends on Quantity.

## Part 2. Preferences (3.1-3.8)

### 4. THE PREFERENCE RELATION

4.1. **Definitions.** The preference relation is a set of statements about **pairs** of bundles. The statement  $x$  is preferred to bundle  $x'$  is shorted to:

$$x \succsim x'$$

### Ice Cream Example:

Suppose a consumer eats bowls of ice cream. The bundles (bowls) are written with the vanilla scoops first and chocolate second. For example:  $(2, 0)$  is two scoops of vanilla and zero of chocolate.

*A consumer who likes vanilla ice cream might have these preferences:*

$$(2, 0) \succsim (0, 2)$$

$$(1, 0) \succsim (0, 1)$$

*A consumer who like more ice cream to less might have these preferences:*

$$(2, 0) \succsim (1, 0)$$

$$(2, 2) \succsim (1, 1)$$

*For someone who gets sick of ice cream: (who wants to eat 100 scoops of ice cream?)*

$$(1, 0) \succsim (100, 0)$$

*For someone who does not care about flavor:*

$$(1, 0) \succsim (0, 1) \text{ \& } (0, 1) \succsim (1, 0)$$

### Indifference Relation: $\sim$

When the following is true:  $x \succsim y$  and  $y \succsim x$  we say “x is indifferent to y” and write  $x \sim y$ .

### Strict Preference Relation: $\succ$

When the following is true:  $x \succsim y$  and **not**  $y \succsim x$  we say “x is strictly preferred to y” and write  $x \succ y$ .

**4.2. Assumptions on  $\succsim$ . Axiom 1. Reflexive.** *For all bundles. The bundle is at least as good as itself.*

In set notation:

$$\forall x \in X : x \succsim x$$

**Axiom 2. Complete.** *For every pair of distinct bundles. Either one is at least as good as the other or the consumer is indifferent.*

In set notation:

$$\forall x, y \in X \text{ \& } x \neq y : x \succsim y \text{ or } y \succsim x \text{ or both}$$

A consumer can say “I’m indifferent.” but not “I don’t know”.

**Axiom 3. Transitivity.**

$$x \succsim y, y \succsim z \text{ implies } x \succsim z$$

Transitivity (along with the other assumptions) implies we can always put a set of objects into a ranking.

**4.3. Example of Violating Transitivity.** *Example:* Suppose there are three people on a dating app.

Person 1. Rich, Very Intelligent, Average Looking

Person 2. Financially Constrained, Genius, Good Looking

Person 3. Moderately Well Off, Average Intelligence, Best Looking

Suppose you prefer a person who is better in two aspects than another. We have a cycle:

$$1 \succ 3 \succ 2 \succ 1$$

This kind of multi-dimensional preference can easily cause intransitivity.

**4.4. From Preference to Choice.** Choice Function:

$$C : B \rightarrow B$$

$$C(B) \subseteq B$$

The choice function takes a budget set as input and returns the things the consumer would like to have from that set.

$C(B)$  is all the objects in  $B$  such that those objects are at least as good as everything else in the set.

$$C(B) = \{x | x \in B : \forall x' \in B, x \succsim x'\}$$

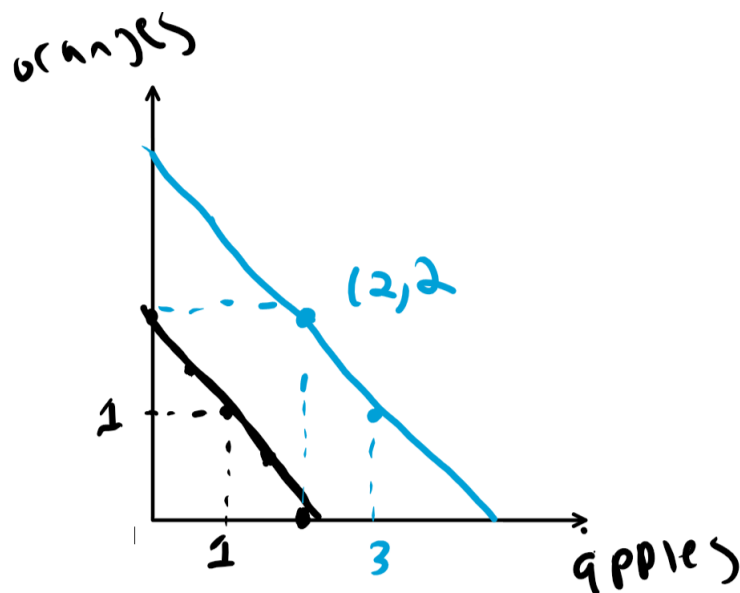


FIGURE 4.1. Indifference curves through the points  $(1,1)$  and  $(2,2)$  for a consumer who will always give up one orange to get one apple.

**4.5. Indifference Curves and the Weakly Preferred Set.** *An indifference curve is a set bundles such that the consumer is indifferent between all of the bundles on the curve.*

**Note:** There are many indifference curves. We only sketch a few to get an idea of the “shape” of preferences.

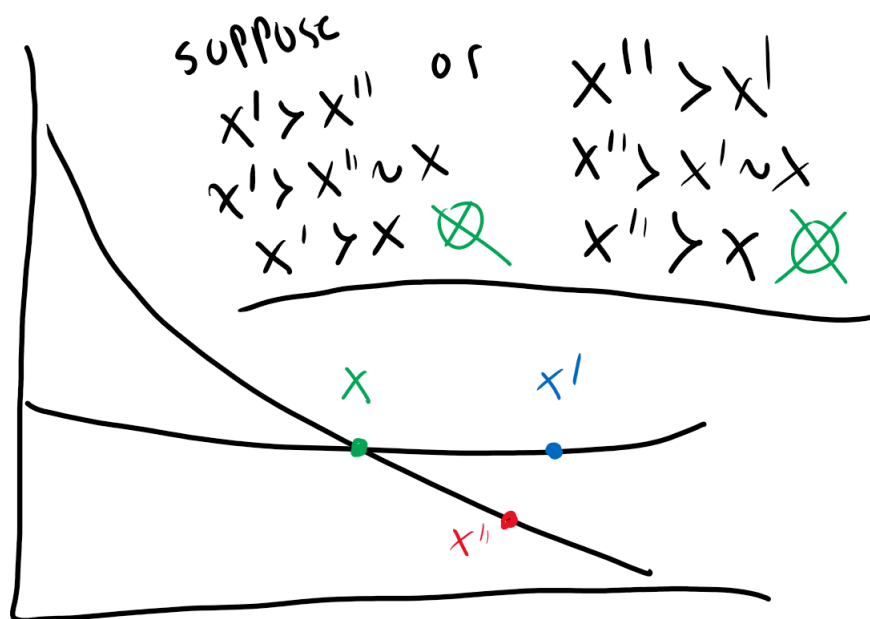


FIGURE 4.2. Indifference curves cannot cross if preferences are transitive.

#### 4.6. Indifference Curves Cannot Cross.

#### 4.7. Examples of Preferences.

4.7.1. *Perfect Substitutes*. These preferences are such that my willingness to trade-off between the goods is the same everywhere.

The indifference curves are always downward sloping lines with the same slope. **The slope measures the amount of  $x_2$  you are willing to give up to get 1 more unit of  $x_1$ .**

Steep slope: stronger preference for  $x_1$ .

Shallow slope: stronger preference for  $x_2$ .





FIGURE 4.3. Indifference curves for perfect substitutes preferences. This consumer would be willing to give up 2 units of  $x_2$  in exchange for 1 unit of  $x_1$ .

4.7.2. *Perfect Complements*. Preferences where one good cannot substitute for the other.

*Example:* Left and right shoes.

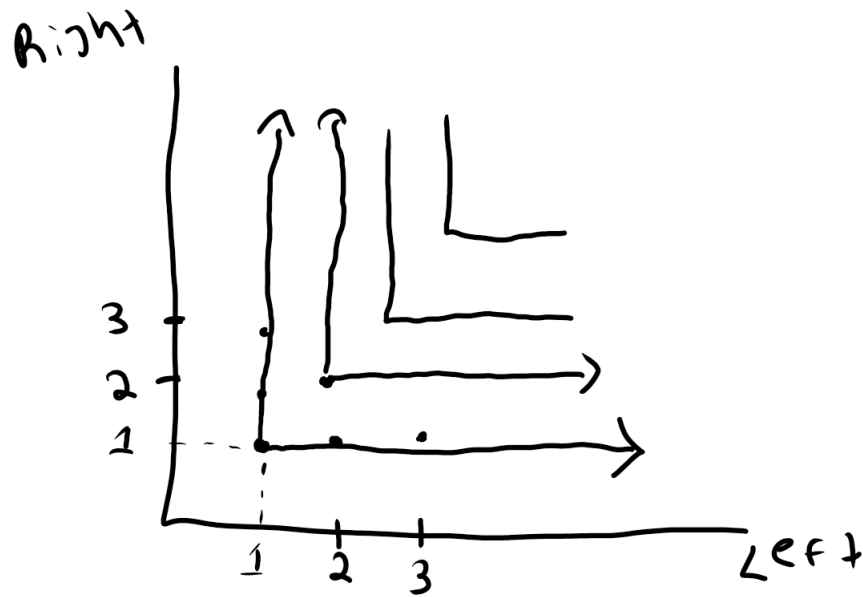


FIGURE 4.4. Indifference curves for perfect complements preferences where Left/Right shoes must be consumed in a 1-to-1 one ratio.

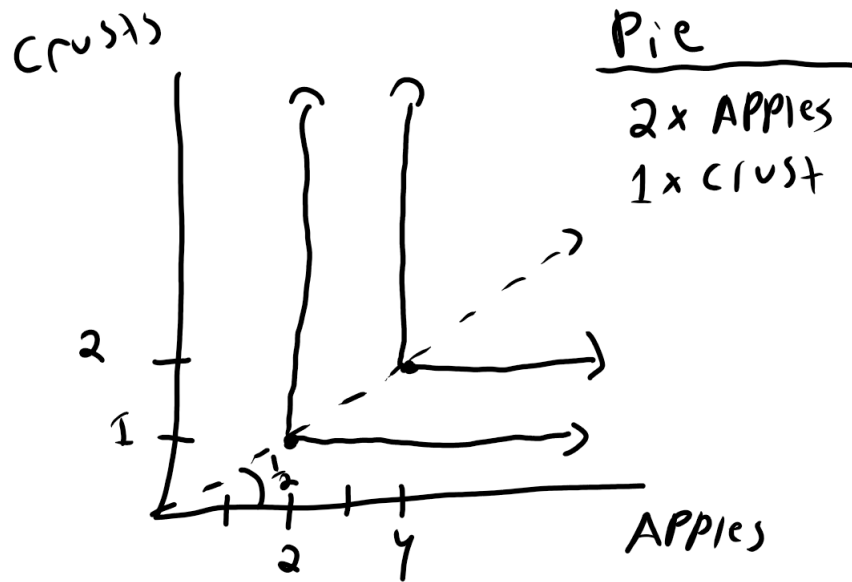


FIGURE 4.5. Indifference curves for perfect complements preferences where the goods are consumed in a 2-to-1 ratio. In this case, 2 apple and 1 crust make a pie.



FIGURE 4.6. When one good is a “bad”, indifference curves slope upward!

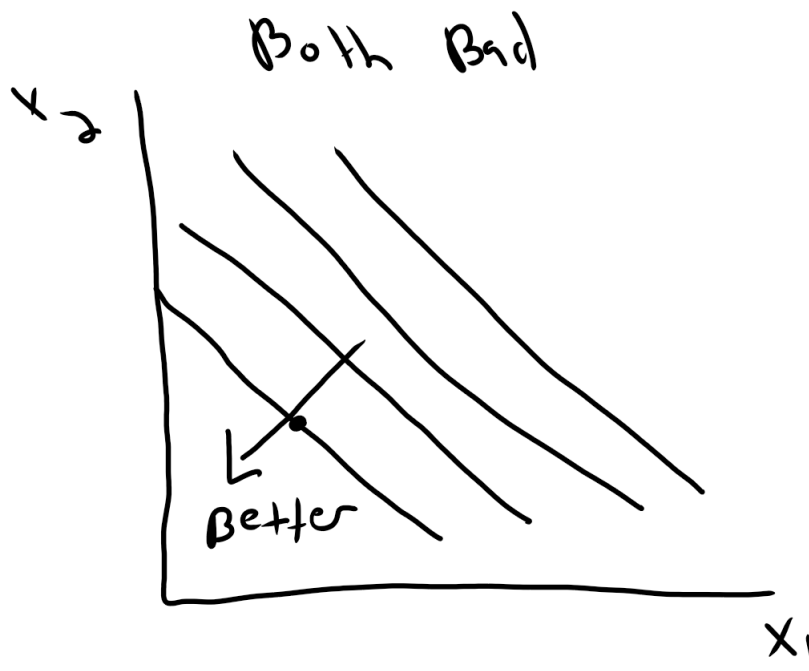


FIGURE 4.7. When both goods are bad, indifference curves slope down, but preference “increases” towards the origin (to the south west).

#### 4.7.3. Bads.

#### 4.8. Further Assumptions: “Well Behaved Preferences”.

4.8.1. *Monotonicity.* The assumption that everything is a “good”.

There are two forms of this assumption. *Strict* and *Weak*.

**Strict Monotonicity:** For two bundles  $(x_1, x_2)$  and  $(y_1, y_2)$ ,  $(x_1, x_2) \succ (y_1, y_2)$  if  $x_1 \geq y_1$  and  $x_2 \geq y_2$ .  $(x_1, x_2) \succ (y_1, y_2)$  if either  $x_1 > y_1$  or  $x_2 > y_2$

*Perfect substitutes are strictly monotonic.*

*Perfect complements are **not** strictly monotonic.*

**Weak Monotonicity. (AKA “Monotonic”):** For two bundles  $(x_1, x_2)$  and  $(y_1, y_2)$ ,  $(x_1, x_2) \succeq (y_1, y_2)$  if  $x_1 \geq y_1$  and  $x_2 \geq y_2$ .  $(x_1, x_2) \succ (y_1, y_2)$  if **both**  $x_1 > y_1$  **and**  $x_2 > y_2$

*Perfect substitutes are weakly monotonic.*

*Perfect complements **are** weakly monotonic.*

4.8.2. *What does monotonicity imply about the indifference curves?* 1. Weakly downward sloping. We cannot have upward sloping indifference curves.

2. Preference increases to the north east (away from the origin).

4.8.3. *Convexity and Strict Convexity.* The assumption that mixtures are better than extremes.

There are two forms of this assumption. *Strict* and *Weak*.

**Strictly Convex:** For two indifferent bundles  $(x_1, x_2) \sim (y_1, y_2)$ , for any  $t \in (0, 1)$ , the mixture given by  $(tx_1 + (1 - t)y_1, tx_2 + (1 - t)y_2) \succ (x_1, x_2)$  and  $(tx_1 + (1 - t)y_1, tx_2 + (1 - t)y_2) \succ (y_1, y_2)$ .

**Weakly Convex:** For two indifferent bundles  $(x_1, x_2) \sim (y_1, y_2)$ , for any  $t \in [0, 1]$ , the mixture given by  $(tx_1 + (1 - t)y_1, tx_2 + (1 - t)y_2) \succeq (x_1, x_2)$  and  $(tx_1 + (1 - t)y_1, tx_2 + (1 - t)y_2) \succeq (y_1, y_2)$ .

#### 4.9. Marginal Rates of Substitution and Slope of the Indifference Curve.

### 5. UTILITY (4.1-4.5)

#### 5.1. Definition.

#### 5.2. Monotonic Transformations.

#### 5.3. MRS from Utility Function.

#### 5.4. Examples of Utility Functions.

##### 5.4.1. Perfect Substitutes.

##### 5.4.2. Quasi-Linear.

##### 5.4.3. Cobb-Douglass.

##### 5.4.4. Perfect Complements.