8100 Problem Set 3.

September 22, 2021

1. Find the bundle that maximizes $U(x) = x_1 x_2$ subject to the two constraints: **(1)** $\left(x_1^2 + x_2^2\right)^{\frac{1}{2}} \leq 10$ and **(2)** $2x_1 + x_2 \leq 20$. What constraints bind at this point? What is the value of the Langrange multiplier at the optimum for the constraint(s) that bind?

2. When maximizing $U(x) = x_1 x_2$ subject to the two constraints: **(1)** $\left(x_1^2 + x_2^2\right)^{\frac{1}{2}} \leq 10$ and **(2)** $2x_1 + x_2 \leq m$. Find a condition on m, such that only constraint 1 binds at the optimum.

3. Prove that for two budgets $p_1 x_1 + p_2 x_2 \leq m$ and $\tilde{p}_1 x_1 + \tilde{p}_2 x_2 \leq \tilde{m}$ and bundle available in the convex combination: $(tp_1 + (1-t)\tilde{p}_1) x_1 + (tp_2 + (1-t)\tilde{p}_2) x_2 \leq tm + (1-t)\tilde{m}$ is affordable in one of the two original budgets.

4. Maximize $u(x_1, x_2) = x_1 + ln(x_2)$ subject to $p_1 x_1 + p_2 x_2 = m$. Set up the Lagrange function and find the first order condition. Interpret this condition in economic terms. At what price income combinations is the slope of the indifference curve at the optimum bundle not equal to the slope of the boundary of the budget set? Using your interpretation of the first order condition, say something about why these price/income combinations lead to a situation where the equal slop condition cannot be met.