8100 Problem Set 1.

September 1, 2021

Prove the following results. Unless otherwise stated, assume $\succeq$ is complete and transitive *and* $\succ$, $\sim$ *are as defined in the notes.*

1. *If* $\succsim$ *is complete but there is a cycle such that* $x \succ y, y \succ z, z \succ x$*, every budget* $B \subset X$ *with* $\#(B) \leq 2$*,* $C(B) \neq \emptyset$ *but there is some budget* $B \subseteq X$ *with* $\#(B) \geq 3$ *such that* $C(B) = \emptyset$*.*

2. *If* $\succeq$ *is complete and transitive, then* $\forall x, x' \in X$ *either* $\succeq(x) \subseteq \succeq(x')$ *or* $\succeq(x') \subseteq \succeq(x)$*.*

3. $\succ \cup \sim = \succeq$ & $\succ \cap \sim = \emptyset$**.**

4. $\sim(x) = \succeq(x) \cap \preceq(x)$**.**

5. $\succeq(x) = \sim(x) \cup \succ(x)$.

6. $\sim$ is transitive but not necessarily complete.

7. $\succ$ is transitive but not necessarily complete.