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September 30, 2021

Part I

Budget (2.1-2.7)

1 Bundles

Bundle: $x = (x_1, x_2)$

Example. Ice Cream Bowls. x_1 is the amount of vanilla. x_2 is the amount of chocolate.

$(1, 1)$ one scoop of each flavor.

$(2, 2)$ two scoops of each flavor.

$(0.28, 100)$ a lot of chocolate and a little vanilla.

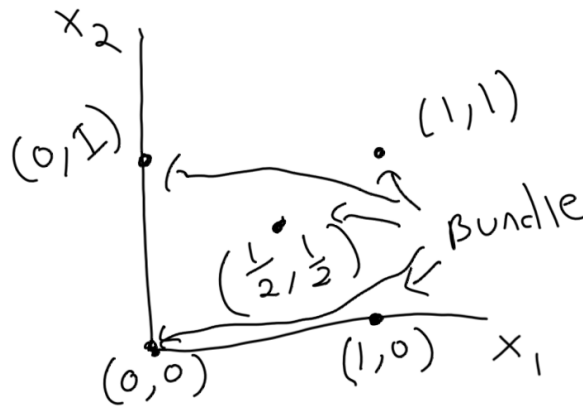
Figure 1.1: Bundles on Cartesian Plane.

2 Feasible Set

The Feasible Set: X is the "feasible" set of bundles.

The feasible set is the universe of bundles that might be relevant in a model. The feasible set defines the scope of a model.

3 Budget Set

Budget Set: B

The budget set is the set of bundles *available* to a particular consumer.

The budget set must be a subset of the feasible set.

In set notation: $B \subseteq X$

3.1 Budget Sets from Prices and Income

Prices: p_1, p_2: Price of good 1 and price of good 2.

Cost of a bundle: $p_1 x_1 + p_2 x_2$.

Income: m.

Budget set: $B = \{x | x \in X \ \& \ x_1 p_1 + x_2 p_2 \leq m\}$.

In non-math language, this says the budget set is the set of bundles such that the price of the bundle is less than income.
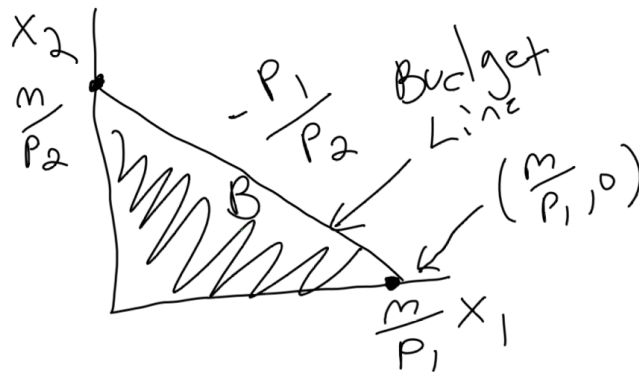

Figure 3.1: Graphical Representation of the Budget Set

3.2 Changing Prices and Income
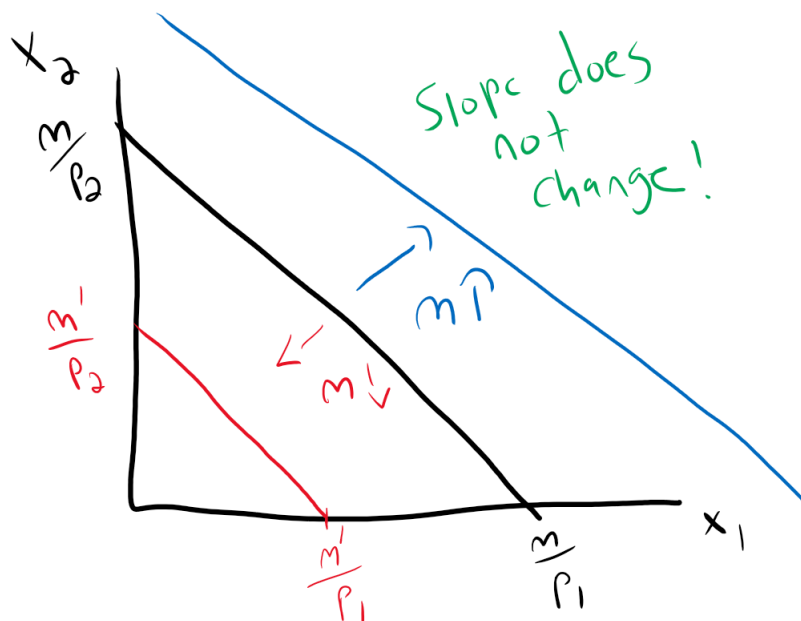
Suppose income increases. m changes.
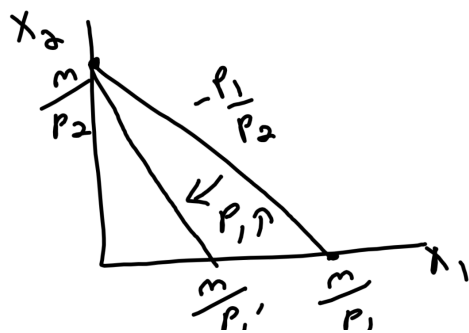
Figure 3.2: How Budget Changes with Income

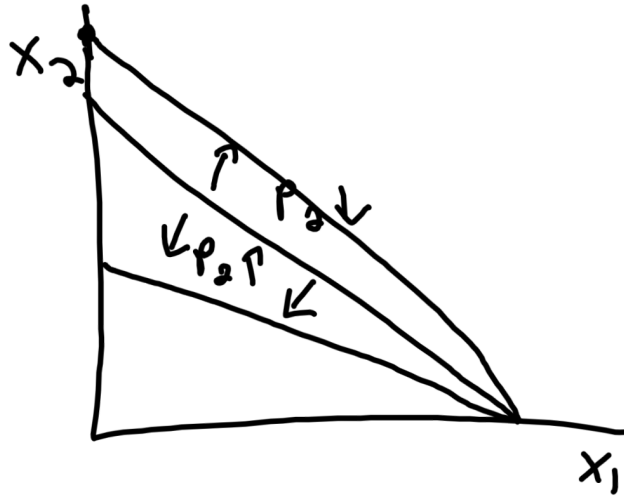Figure 3.3: How Budget Changes with and increase in p_1.

Figure 3.4: How Budget Changes with Changes to p_2

Both endpoints change. If m increaes, $\frac{m}{p_1}$ (the amount I can buy of good 1 changes) increases and $\frac{m}{p_2}$ (maximum affordable x_2) increases. The slope does not change. If m decreases, the opposite happens.

Suppose one of the prices changes.

p_1. If p_1 goes up, the slope decreases (more negative). If p_1 goes down, the slope increases. The x_2 intercept stays the same.

p_2. If p_2 goes up, the slope increases. In p_2 goes down the slope decreases (more negative). The x_1 intercept stays the same.

3.3 Taxes

Quantity tax on good 1:

$$p_1 x_1 + t x_1 + p_2 x_2 = m$$

$$(p_1 + t) x_1 + p_2 x_2 = m$$

Ad Valorem Tax on good 1:

$$(p_1 x_1) + \tau (p_1 x_1) + p_2 x_2 = m$$

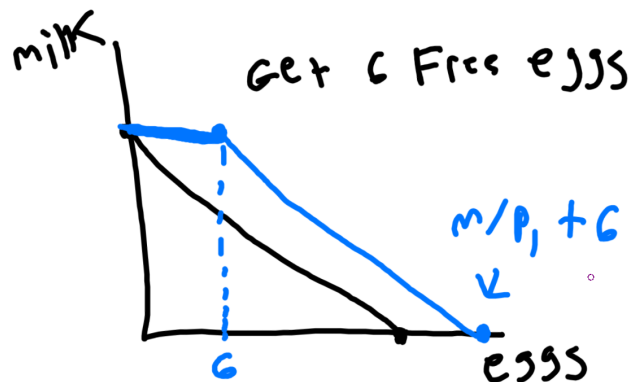$$(1 + \tau)(p_1 x_1) + p_2 x_2 = m$$

We will focus on quantity taxes.

Figure 3.5: Six Free Eggs

3.4   More Complex Scenarios

3.5   Price Depends on Quantity

Part II

Preferences (3.1-3.8)

4   The Preference Relation

4.1   Definitions

The preference relation is a set of statements about **pairs** of bundles. The statement x is preferred to bundle x' is shorted to:

$$x \succsim x'$$

Ice Cream Example:

Suppose a consumer eats bowls of ice cream. The bundles (bowls) are written with the vanilla scoops first and chocolate second. For example: $(2, 0)$ is two scoops of vanilla and zero of chocolate.

A consumer who likes vanilla ice cream might have these preferences:

$$(2, 0) \succsim (0, 2)$$

$$(1, 0) \succsim (0, 1)$$

A consumer who like more ice cream to less might have these preferences:

$$(2,0) \succsim (1,0)$$

$$(2,2) \succsim (1,1)$$

For someone who gets sick of ice cream: (who wants to eat 100 scoops of ice cream?)

$$(1,0) \succsim (100,0)$$

For someone who does not care about flavor:

$$(1,0) \succsim (0,1) \,\&\, (0,1) \succsim (1,0)$$

Indifference Relation: $\sim$

When the following is true: $x \succsim y$ and $y \succsim x$ we say "x is indifferent to y" and write $x \sim y$.

Strict Preference Relation: $\succ$

When the following is true: $x \succsim y$ and **not** $y \succsim x$ we say "x is strictly preferred to y" and write $x \succ y$.

4.2   Assumptions on $\succsim$

Axiom 1. Reflexive. *For all bundles. The bundle is at least as good as itself.*

In set notation:

$$\forall x \in X : x \succsim x$$

Axiom 2. Complete. *For every pair of distinct bundles. Either one is at least as good as the other or the consumer is indifferent.*

In set notation:

$$\forall x, y \in X \,\&\, x \neq y : x \succsim y \,or\, y \succsim x \,or\, both$$

A consumer can say "I'm indifferent." but not "I don't know".

Axiom 3. Transitivity.

$$x \succsim y, y \succsim z \,\textbf{implies}\, x \succsim z$$

Transitivity (along with the other assumptions) implies we can always put a set of objects into a ranking.

4.3 Example of Violating Transitivity

Example: Suppose there are three people on a dating app.

Person 1. Rich, Very Intelligent, Average Looking

Person 2. Financially Constrained, Genius, Good Looking

Person 3. Moderately Well Off, Average Intelligence, Best Looking

Suppose you prefer a person who is better in two aspects than another. We have a cycle:

$$1 \succ 3 \succ 2 \succ 1$$

This kind of multi-dimensional preference can easily cause intransitivity.

4.4 From Preference to Choice

Choice Function:

$$C : B \to B$$

$$C(B) \subseteq B$$

The choice function takes a budget set as input and returns the things the consumer would like to have from that set.

$C(B)$ is all the objects in B such that those objects are at least as good as everything else in the set.

$$C(B) = \{x | x \in B : \forall x' \in B, x \succsim x'\}$$

4.5   Indifference Curves and the Weakly Preferred Set
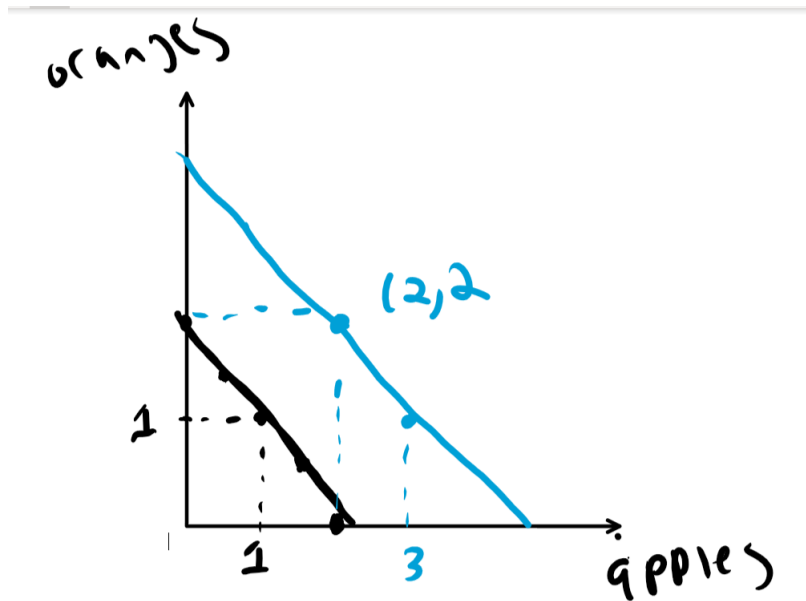
oranges

(2,2

1

1   3

apples

Figure 4.1: Indifference curves through the points $(1, 1)$ and $(2, 2)$ for a consumer who will always give up one orange to get one apple.

An indifference curve is a set bundles such that the consumer is indifferent between all of the bundles on the curve.

Note: There are many indifference curves. We only sketch a few to get an idea of the "shape" of preferences.

4.6   Indifference Curves Cannot Cross
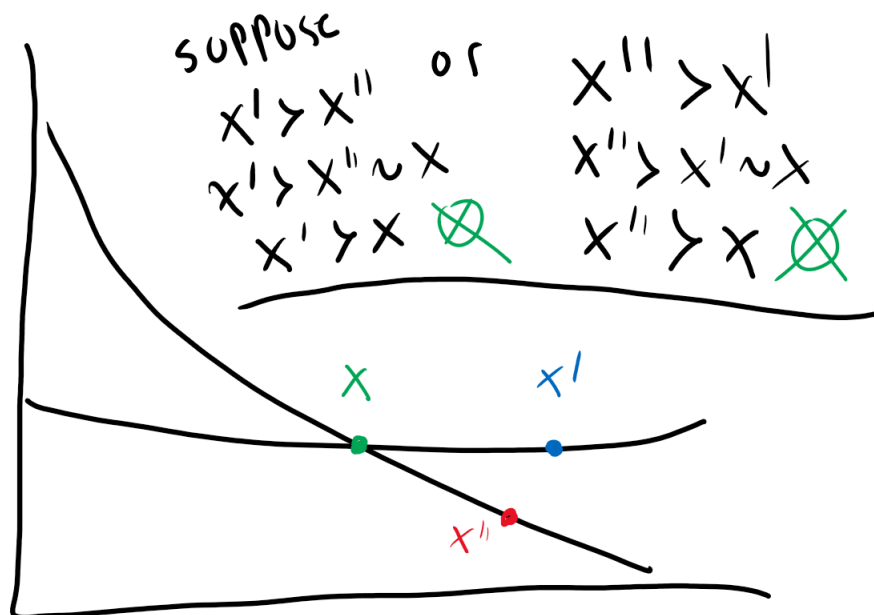

Figure 4.2: Indifference curves cannot cross if preferences are transitive.

4.7   Examples of Preferences

4.7.1   Perfect Substitutes

These preferences are such that my willingness to trade-off between the goods is the same everywhere.

The indifference curves are always downward sloping lines with the same slope. **The slope measures the amount of x_2 you are willing to give up to get 1 more unit of x_1.**

Steep slope: stronger preference for x_1.

Shallow slope: stronger preference for x_2.

Figure 4.3: Indifference curves for perfect substitutes preferences. This consumer would be willing to give up 2 units of x_2 in exchange for 1 unit of x_1.

4.7.2 Perfect Complements

Preferences where one good cannot substitute for the other.
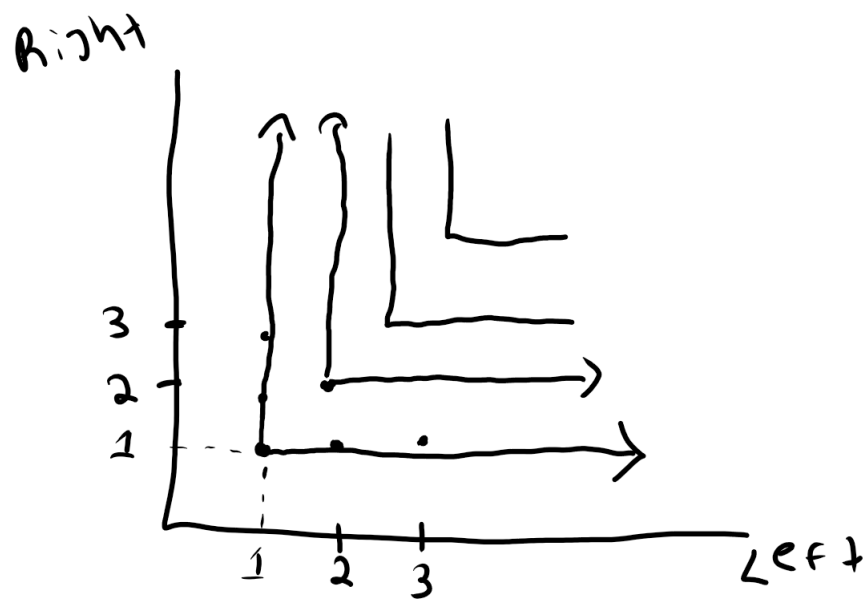
Example: Left and right shoes.

Figure 4.4: Indifference curves for perfect complements preferences where Left/Right shoes must be consumed in a 1-to-1 one ratio.
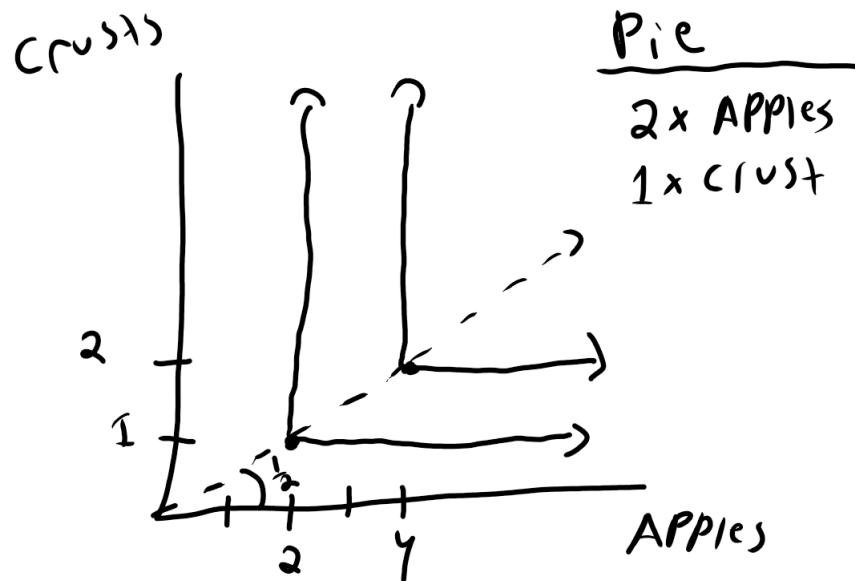
Figure 4.5: Indifference curves for perfect complements preferences where the goods are consumed in a 2-to-1 ratio. In this case, 2 apple and 1 crust make a pie.

4.7.3 Bads

one x_1 good / one x_2 bad

x_2

upward
sloping!

x_1

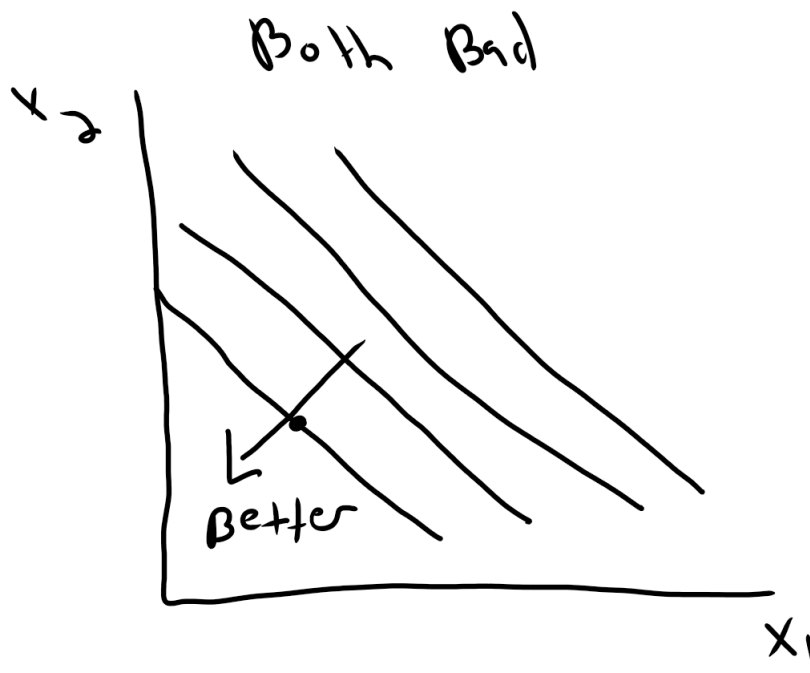Figure 4.6: When one good is a "bad", indifference curves slope upward!

Figure 4.7: When both goods are bad, indifference curves slope down, but preference "increases" towards the origin (to the south west).

4.8 Further Assumptions: "Well Behaved Preferences"

4.8.1 Monotonicity

The assumption that everything is a "good".

There are two forms of this assumption. *Strict* and *Weak*.

Strict Monotonicity: For two bundles (x_1, x_2) and (y_1, y_2), $(x_1, x_2) \succsim (y_1, y_2)$ if $x_1 \geq y_1$ and $x_2 \geq y_2$. $(x_1, x_2) \succ (y_1, y_2)$ if either $x_1 > y_1$ or $x_2 > y_2$

Perfect substitutes are strictly monotonic.

*Perfect complements are **not** strictly monotonic.*

Weak Monotonicity. (AKA "Monotonic"): For two bundles (x_1, x_2) and (y_1, y_2), $(x_1, x_2) \succsim (y_1, y_2)$ if $x_1 \geq y_1$ and $x_2 \geq y_2$. $(x_1, x_2) \succ (y_1, y_2)$ if **both** $x_1 > y_1$ **and** $x_2 > y_2$

Perfect substitutes are weakly monotonic.

*Perfect complements **are** weakly monotonic.*

4.8.2   What does monotonicity imply about the indifference curves?

1. Weakly downward sloping. We cannot have upward sloping indifference curves.

2. Preference increases to the north east (away from the origin).

4.8.3   Convexity and Strict Convexity

The assumption that mixtures are better than extremes.

There are two forms of this assumption. *Strict* and *Weak.*

Strictly Convex: For two indifferent bundles $(x_1, x_2) \sim (y_1, y_2)$, for any $t \in (0,1)$, the mixture given by $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succ (x_1, x_2)$ and $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succ (y_1, y_2)$.

Weakly Convex: For two indifferent bundles $(x_1, x_2) \sim (y_1, y_2)$, for any $t \in [0,1]$, the mixture given by $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succsim (x_1, x_2)$ and $(tx_1 + (1-t)\, y_1, tx_2 + (1-t)\, y_2) \succsim (y_1, y_2)$.

The geometry of convex indifference curves. Assume preferences are monotonic:

If preferences are strictly convex, then the indifference curve always lies strictly below a line between any two points on that indifference curve.

If preferences are weakly convex, then the indifference curve always lies weakly below a line between any two points on that indifference curve.
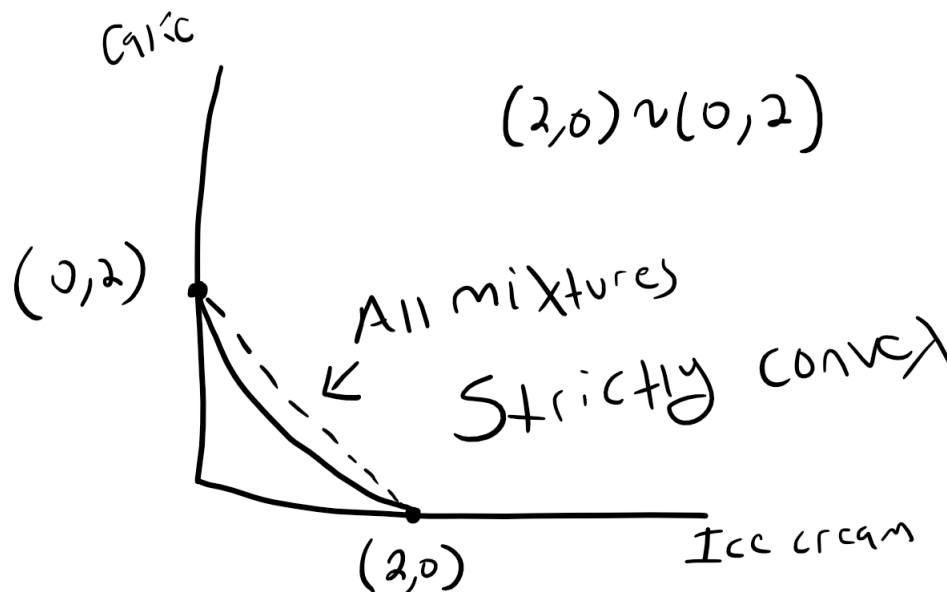

Figure 4.8: Example of Convex Indifference Curves

Perfect substitutes, and perfect complements are both **weakly** convex because their indifference curves include "flat" portions.

4.9 Marginal Rates of Substitution and Slope of the Indifference Curve

The MRS is defined as the slope of the indifference curve. We need this because optimal bundles will often occur where the slope of the indifference curve is the same as the slope of the budget line.

The MRS measures my willingness to trade off between good 1 and good 2. Specifically, it's how much x_2 I would give up to get one more unit of x_1.

5 Utility (4.1-4.5)

5.1 Definition

A utility function is a way of assigning "scores" to bundles, such that better bundles according to $\succsim$ get a higher score. For example, suppose a consumer's preferences are:

$$A \succ B \succ C \sim D$$

Some utility functions that represent these preferences:

$U(A) = 10, U(B) = 5, U(C) = U(D) = 0$

$U(A) = 12, U(B) = 1, U(C) = U(D) = -100$

Definition. A utility function $U(x)$ represents preferences $\succsim$ when for every pair of bundles x and y, $U(x) \geq U(y)$ if and only if $x \succsim y$.

That is, if x is better than y according to $\succsim$ it gets a higher utility according to $U()$.

Utility is Ordinal:

Because the magnitude of the numbers are meaningless, and only the relationships matter, we say that these utility functions are "**ordinal**" rather than "**cardinal**". There is no sense in which two times higher utility means that the preference is two times stronger. We can only infer the ranking of things, but not how strong the preferences are from $\succsim$ and a utility function that represents $\succsim$.

5.2 Monotonic Transformations

Because utility is ordinal, we are free transform one utility function into another, as long as it maintains the same preferences. Any **strictly increasing** function of a utility function represents the same preferences as the original utility function. For example, suppose:

$$U(x_1, x_2) = x_1 + x_2$$

This represents the preferences of someone who only cares about the total amount of stuff, but not the composition. In fact, this utility function represents *perfect substitutes preferences*. Here are some other utility functions that represent the same preferences:

$$U'(x_1, x_2) = x_1 + x_2 + 100 = U(x_1, x_2) + 100$$

$$U'(x_1, x_2) = (x_1 + x_2)^2 = (U(x_1, x_2))^2$$

Since the functions $f(u) = u + 100$ and $f(u) = u^2$ are strictly increasing for $u \geq 0$ (which is always true for the original utility function). These are monotonic transformations of the original utility function.

5.3 MRS from Utility Function

*The **Marginal Rate of Substitution** (MRS) is the slope of the indifference curve.*

The **Marginal Utility** of good i is $mu_i = \frac{\partial u(x_1, x_2)}{\partial x_i}$.

$$MRS = -\frac{mu_1}{mu_2} = -\frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}}$$

Note that MRS is an **ordinal** property since it represents the slope of indifference curves. Because two preferences that are the same have the same indifference curves, they will also have the same MRS. **Same MRS, same preferences**.

5.4 Examples of Utility Functions

5.4.1 Perfect Substitutes

$$u(x_1, x_2) = ax_1 + bx_2$$

$$MRS = -\frac{a}{b}$$

Constant MRS implies a constant willingness to trade off between the two goods.

$$u(x_1, x_2) = (ax_1 + bx_2)^2$$

$$-\frac{a}{b}$$

5.4.2 Quasi-Linear

Consumer only gets tired of one of the two goods.

One common quasi-linear utility function:

$$u\left(x_1, x_2\right) = x_1 + ln\left(x_2\right)$$

$$MRS = -\frac{\frac{\partial(x_1 + ln(x_2))}{\partial x_1}}{\frac{\partial(x_1 + ln(x_2))}{\partial x_2}} = -x_2$$

Another example of a quasi-linear utility function:

$$u\left(x_1, x_2\right) = 10x_1 + \sqrt{x_2}$$

Practice taking the MRS of this function. Notice that it only depends on the amount of x_2!

5.4.3 Cobb-Douglass

Consumer Gets Tired of Both Goods.

$$u\left(x_1, x_2\right) = x_1^\alpha x_2^\beta$$

$$mu_1 = \frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^\beta$$

$$mu_2 = \frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_2} = \beta x_1^\alpha x_2^{\beta-1}$$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{\alpha x_1^{\alpha-1} x_2^\beta}{\beta x_1^\alpha x_2^{\beta-1}}$$

$$= -\frac{\alpha x_1^\alpha x_1^{-1} x_2^\beta}{\beta x_1^\alpha x_2^\beta x_2^{-1}} = -\frac{\alpha x_1^{-1}}{\beta x_2^{-1}} = -\frac{\alpha}{\beta}\frac{x_2}{x_1}$$

Let's compare two CD Functions:

Increasing the exponent on either good will increase the consumers desire for that good. They will still get tired of it, but between two consumers, one with a larger exponent on a good, that consumer will have a stronger desire for the good at the same bundle.

$$u\left(x_1, x_2\right) = x_1 x_2$$

$$MRS = -\frac{x_2}{x_1}$$

At the point $(1,1)$: $MRS = -1$.

$$\tilde{u}(x_1, x_2) = x_1^{10} x_2$$

$$MRS = -10\frac{x_2}{x_1}$$

At the point $(1,1)$: $MRS = -10$

Notice that the consumer with $\tilde{u}(x_1, x_2) = x_1^{10} x_2$ would be willing to give up ten-times more x_2 to get the same amount of x_1 as the consumer with utility function $u(x_1, x_2) = x_1 x_2$.

5.4.4 Perfect Complements

$$u(x_1, x_2) = min\{ax_1, bx_2\}$$

Examples.

Left\Right Shoes

$$u(l, r) = min\{l, r\}$$

Notice that $min\{1,1\} = min\{2,1\} = min\{1,2\} = 1$. That is, the bundles $(1,1), (2,1), (1,2)$ are all indifferent. For this consumer, indifference curves are "L-shaped" and the kinks of these curves follow the path $l = r$.

Apple pies. 2 Apples, 1 Crust makes a pie.

$$u(a, c) = min\left\{\frac{1}{2}a, c\right\}$$

Notice that $min\{\frac{1}{2}2, 1\} = min\{\frac{1}{2}2, 2\} = min\{\frac{1}{2}3, 1\} = 1$. That is, the bundles $(2,1), (2,2), (3,1)$ are all indifferent. For this consumer, indifference curves are "L-shaped" but the "kinks" of these curves follow the line $\frac{1}{2}a = c$.

Notice $\frac{1}{2}a$ is the most pies we could possible make with a apples and c is the most possible pies we could make with c apples.

5.4.5 Max Preferences

$$u(x_1, x_2) = max\{x_1, x_2\}$$

Notice here, the consumer is indifferent between $(2,2), (1,2), (2,1)$. The indifference curves are *backwards L-shaped* curves. The bend the opposite direction to prefect complements indifference curves.

6 Choice (Chapter 5.1-5.3,5.5)

6.1 Three Possibilities

We are going to find the optimal bundles in a budget set. We are going to look for bundles that are all at least as good as everything else in the budget set.

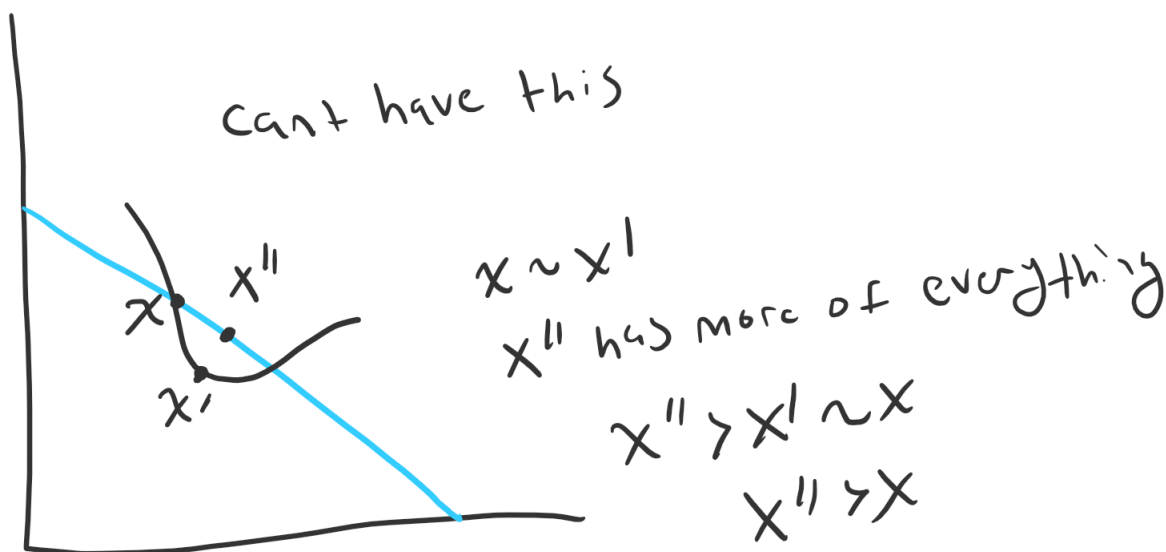Assume $\succsim$ is reflexive, complete, transitive and $\succsim$ **monotonic**.

Figure 6.1: An optimal bundle cannot be on an indifference curve that passes "into" the budget set.

There are only three possibilities for an optimal bundle:

1. (Tangent) It is at bundle where the indifference curve at that bundle had the same slope as the budget line.

2. (Touching but not tangent) The bundle is a "non-smooth" point on the indifference curve, but the that point just touches the budget line.

3. (Boundary) We are at one of the boundaries ($x_1 = 0$ or $x_2 = 0$) in this case the slope of the indifference curve and the slope of the budget need not be equal.
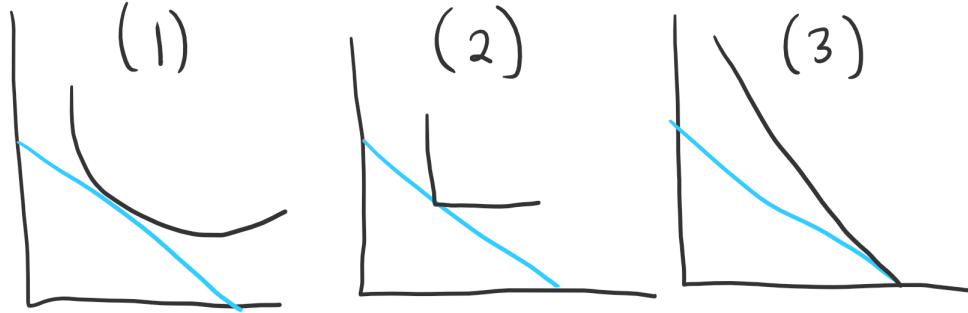
Figure 6.2: Graphical Examples of the Three Possibilities

Under some weak conditions (we can take derivatives of the utility function). The tangency condition is necessary for an *interior* optimum (involves consuming some of both things).

That is, if there is an optimal bundle that involves consuming some of both goods, it must have the property that the slope of the indifference curve at that optimal bundle is the same as the slope of the budget line. T

This condition is formalized by the familiar equation:

$$MRS = -\frac{p_1}{p_2}$$

6.2 Examples

6.2.1 Cobb Douglass:

$$u(x_1, x_2) = x_1 x_2$$

$$p_1 x_1 + p_2 x_2 = m$$

Tangency condition:

$$MRS = -\frac{\frac{\partial (x_1 x_2)}{\partial x_1}}{\frac{\partial (x_1 x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

Tangency Condition:

$$* x_1 p_1 = x_2 p_2$$

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Budget Condition:

$$** \; x_1 p_1 + x_2 p_2 = m$$

Plug Tangency Condition into Budget Condition:

$$x_1 p_1 + x_1 p_1 = m$$

$$2 x_1 p_1 = m$$

$$x_1^* = \frac{1}{2} \frac{m}{p_1}$$

Plug this back into one of the two equations:

$$x_1 p_1 = x_2 p_2$$

$$\frac{1}{2} \frac{m}{p_1} p_1 = x_2 p_2$$

$$\frac{1}{2} \frac{m}{p_1} \frac{p_1}{p_2} = x_2$$

$$x_2^* = \frac{1}{2} \frac{m}{p_2}$$

The optimal bundle:

$$\left(\frac{\frac{1}{2} m}{p_1}, \frac{\frac{1}{2} m}{p_2} \right)$$

6.2.2 Perfect Substitutes

$$u(x_1, x_2) = 2 x_1 + x_2$$

$p_1 = 1$, $p_2 = 1$, $m = 10$

$$1 x_1 + 1 x_2 = 10$$

Tangency Condition:

$$-\frac{2}{1} = -\frac{1}{1}$$

$$-2 = -1$$

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This is never true. There can't be an interior solution. There has to be a boundary solution. Let's check the utility of both.

$$u(x_1, x_2) = 2x_1 + x_2$$

Only consume x_1:

$$\left(\frac{m}{p_1}, 0\right) = (m, 0)$$

$$u(m, 0) = 2m = 20$$

Only consume x_2:

$$\left(0, \frac{m}{p_2}\right) = (0, m)$$

$$u(0, m) = m = 10$$

Since consuming only x_1 gives me more utility, that is the optimal bundle:

$$(m, 0)$$

6.2.3 Anything is Optimal

$$u(x_1, x_2) = 2x_1 + x_2$$

$p_1 = 2$, $p_2 = 1$, $m = 10$

$$2x_1 + 1x_2 = 10$$

$$-\frac{2}{1} = -\frac{2}{1}$$

$$-2 = -2$$

As long as I spend all of my money, any bundle is optimal.
All of the bundles such that:

$$p_1 x_1 + p_2 x_2 = m$$

6.2.4 Perfect Complements

$$u(x_1, x_2) = min\{x_1, x_2\}$$

$$2x_1 + x_2 = 15$$

We still know the budget condition must be true:

$$** \; 2x_1 + x_2 = 15$$

What is the other condition?

"No Waste Condition". (Equation for the "kink" points).

$$* \; x_1 = x_2$$

Plug one condition into the other:

$$2x_1 + x_2 = 15$$

$$2x_1 + x_1 = 15$$

$$3x_1 = 15$$

$$x_1 = 5$$

Plug back into one of the equations:

$$x_2 = 5$$

6.2.5 Perfect Complements (2 Apples, 1 Crust)

$$u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

$$2x_1 + x_2 = 15$$

We still know the budget condition must be true:

$$** \; 2x_1 + x_2 = 15$$

What is the other condition?

"No Waste Condition". (Equation for the "kink" points).

$$\frac{1}{2}x_1 = x_2$$

Combine the conditions:

$$2x_1 + x_2 = 15$$

$$2x_1 + \frac{1}{2}x_1 = 15$$

$$\frac{5}{2}x_1 = 15$$

$$x_1 = 6$$

$$x_2 = 3$$

6.2.6   Max Preferences

$$u(x_1, x_2) = max\{x_1, x_2\}$$

$$2x_1 + x_2 = 15$$

Try this one at home: what is the optimal bundle?

Solution:

$$x_1 = 0, x_2 = 15$$

7   Demand (6.1-6.8)

$$x_1 x_2$$

$$p_1, p_2, m$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$\frac{x_2}{p_1} = \frac{x_1}{p_2}$$

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

In this form, we have "the marginal utility of a dollar spent on both goods is the same"

$$x_1 p_1 = x_2 p_2$$

$$x_1^* = \frac{\frac{1}{2}m}{p_1}$$

$$x_2^* = \frac{\frac{1}{2}m}{p_2}$$

7.1 Marshallian Demand

The Mashallian demand is the optimal amount of a good, given prices and income.

$$x_1^* (p_1, p_2, m)$$

$$x_2^* (p_1, p_2, m)$$

7.2 Changes in Income

$$\frac{\partial x_1^* (p_1, p_2, m)}{\partial (m)}?$$

How does demand change with income?

7.2.1 Normal/Inferior

If demand *increases* when income increases, we say the good is **"Normal".**

If demand *decreases* when income increases, we say the good is **"Inferior".**

Example: Cobb-Douglass $x_1 x_2$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Example: Inferior.

$$x_1 = \frac{10}{m p_1}$$

7.2.2   Income Offer Curve

Income offer curve is a plot of optimal bundles (x_1^*, x_2^*) as income changes but prices are fixed.

Example: $u(x_1, x_2) = x_1 x_2$. $p_1 = 2, p_2 = 1$:

$x_1 = \frac{\frac{1}{2}m}{2} = \frac{1}{4}m$, $x_1 = \frac{\frac{1}{2}m}{1} = \frac{1}{2}m$
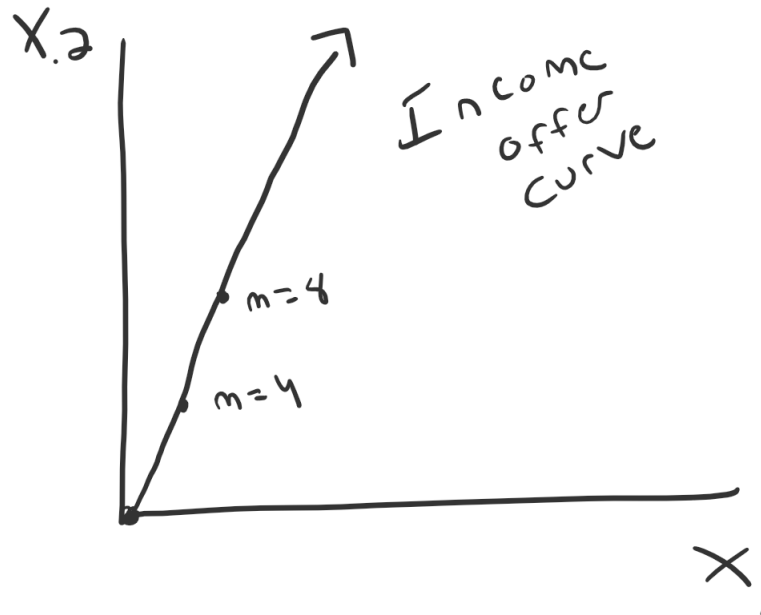

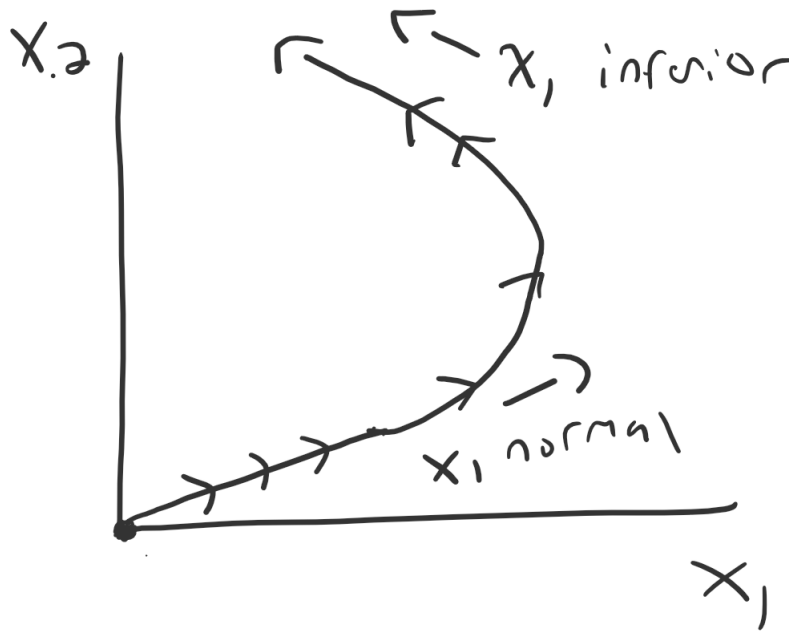Figure 7.1: Income offer curve for Cobb Douglass preference example.

Figure 7.2: Example of an income offer curve where x_1 is initially normal but becomes inferior as m grows.

7.2.3 Engle Curve

The relationship between income and a **single** good. Plotting m on the y-axis against x_i on the x-axis.

$$x_1 = \frac{1}{4}m$$

$$m = 4x_1$$

Really what we are plotting is the amount of income a consumer would need to have to demand some amount x_1 of good 1.
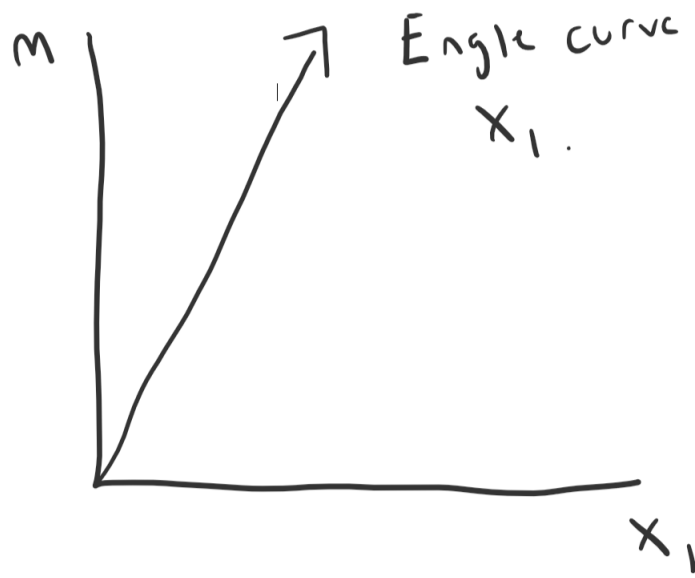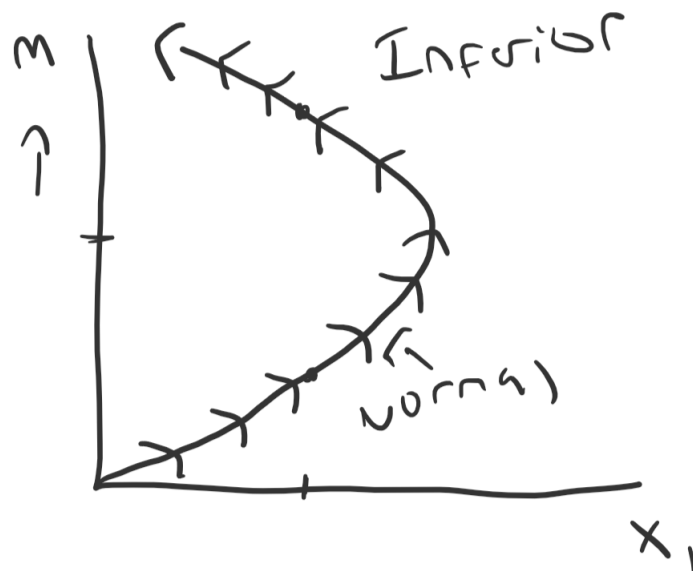
Figure 7.3: Engle curve for $x_1 = \frac{1}{4}m$.

Figure 7.4: Engle curve for a good that is normal for low income and inferior for high income.

7.2.4   Example: Perfect Complements

$U\left(x_1, x_2\right) = min\left\{x_1, x_2\right\}$. $p_1 = 2$, $p_2 = 1$.
At the optimum, $x_1 = x_2$ (no waste condition).
$2x_1 + 1x_2 = m$ (budget condition)

$$2x_1 + x_1 = m$$

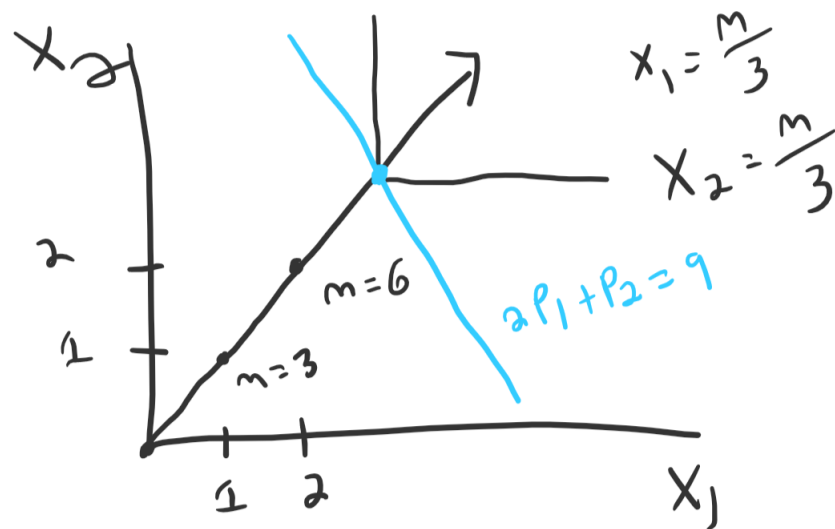$$x_1 = \frac{m}{3}$$

$$x_2 = \frac{m}{3}$$

Income Offer Curve:

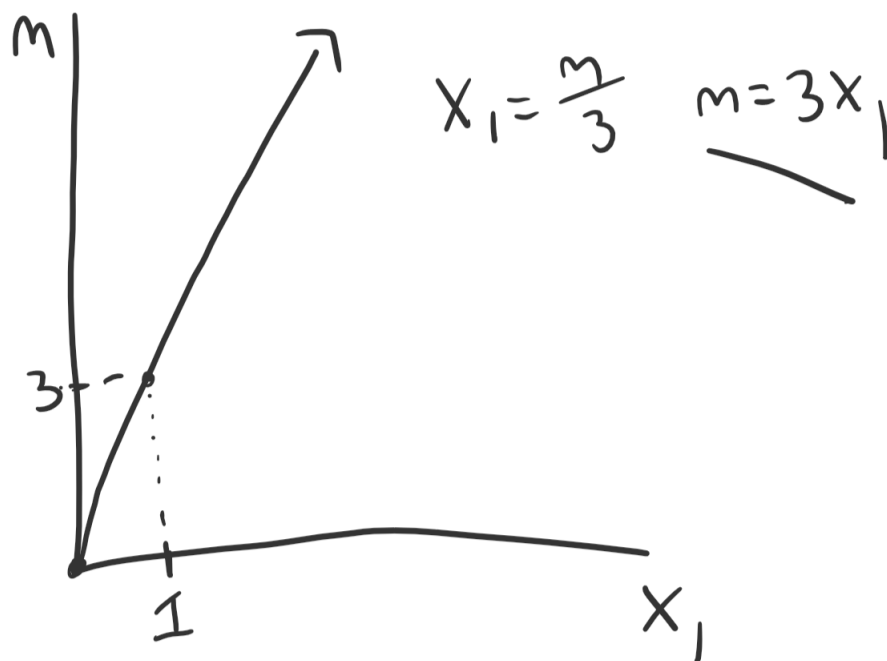Figure 7.5: Income offer curve for $min\{x_1, x_2\}$ with $p_1 = 2$ and $p_2 = 1$

$$X_1 = \frac{m}{3} \qquad m = 3X_1$$

Figure 7.6: Engle Curve of x_1 for $min\{x_1, x_2\}$ with $p_1 = 2$ and $p_2 = 1$

7.3 Changes in "Own" Price

What happens to demand for a good when the price for that good changes.

$$\frac{\partial x_1(p_1, p_2, m)}{\partial p_1}?$$

7.3.1 Ordinary/Giffen

When the price of a good goes up, and demand goes *down*, we say the good is **ordinary.**

When the price of a good goes up, and demand goes *up*, we say the good is **giffen.**

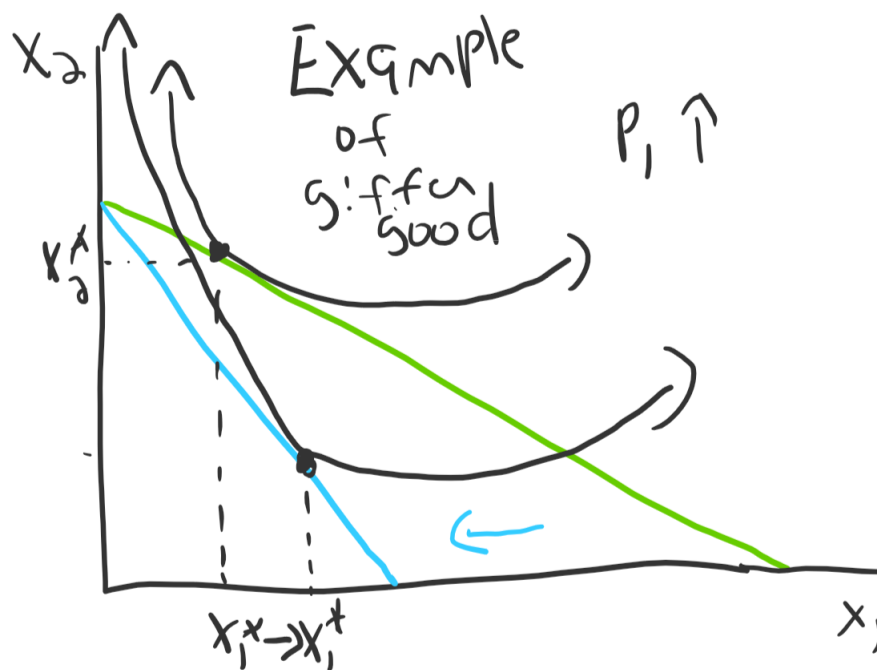(We will see this more later). A giffen good has to be inferior.

Figure 7.7: Indifference curves and budget for a "giffen" good. Note that as the price p_1 increases from the green to blue budget, the optimal amount of x_1 increases.

7.3.2 Price Offer Curve

Hold income and one of the prices fixed, the price offer curve is the set of bundles x_1^*, x_2^* that are optimal at each level of the other price.
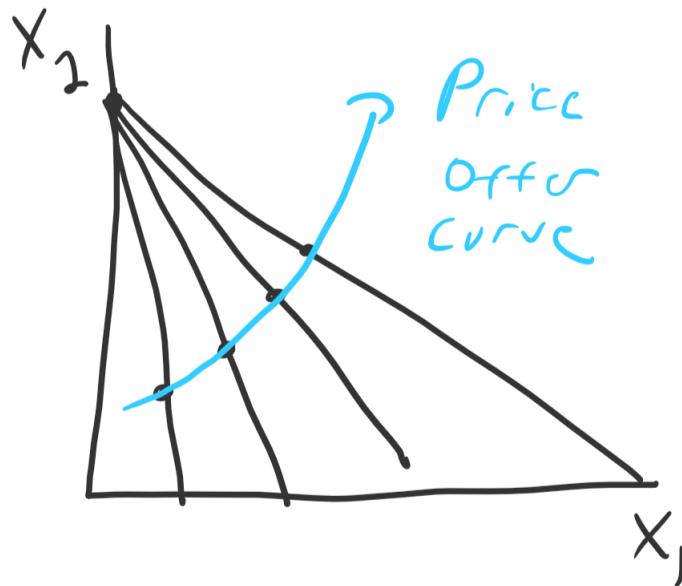
Figure 7.8: Example of a price offer curve (blue). The price offer curve plots the bundles that are optimal along each of the budget lines (black) as p_1 changes.

7.3.3 Plotting the Demand Curve

The **demand** for a good is $x_i(p_1, p_2, m)$ that is, the optimal amount that a consumer chooses given the prices and income. When we talk about "plotting" the demand curve of x_1 we usually mean holding p_2 and m fixed and plotting how the demand for x_1 changes as p_1 changes. For this, we put p_1 on the vertical axis and x_1 on the horizontal axis.

For example, suppose demand for x_1 is:

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Let's plug in an income $m = 10$ and hold that fixed. We get $x_1 = \frac{5}{p_1}$. To plot this with p_1 on the vertcal axis, it is useful to solve for p_1. When we do this we get $p_1 = \frac{5}{x_1}$ this is what we call the **inverse demand.** It is the price that would be responsible for the consumer buying some amount x_1 of the good. This is actually what we plot when we are asked to plot the "demand".
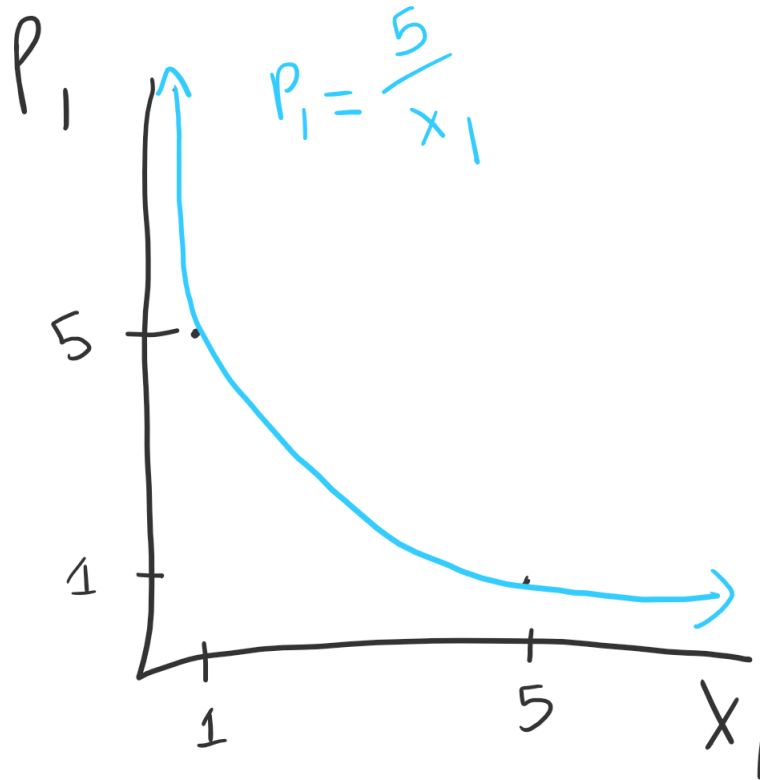
Figure 7.9: Plotting demand for $x_1 = \frac{5}{p_1}$.

7.4 Changes in "Other" Price

So far we have looked at what happens to a good when we change income and it's own price. We might also be interested in how demand changes for a good when the change the price of another good.

7.4.1 Complements/Substitutes

If the demand for a good goes **down** when the price of the other good goes up, we say the goods are **complements**.

If the demand for a good goes **up** when the price of the other good goes up, we say the goods are **substitutes**.

If demand for a good does not change when the price of the other good goes up, we say the goods are neither complements nor substitutes.

7.4.2 Examples Perfect Complements

$u = min\{x_1, x_2\}$ *has* demand $x_1 = \frac{m}{p_1 + p_2}$ and $x_2 = \frac{m}{p_1 + p_2}$. For both goods, as you increase the price of the other good, the demand goes down. They are **complements** (*hopefully this is not a surprise*).

7.4.3 Examples Perfect Substitutes

$u = x_1 + x_2$ has demand $x_1 = \frac{m}{p_1}$ $x_2 = 0$ if $p_1 < p_2$ and $x_1 = 0$ $x_2 = \frac{m}{p_2}$ if $p_1 > p_2$. Let's look at the change in p_1. If $p_1 < p_2$ and p_1 increases, then if it increases enough to that $p_1 > p_2$ the demand for x_2 increases from 0 to $x_2 = \frac{m}{p_2}$. So, as long as the change in price p_1 has any effect on the demand for p_2 (it might not if it does not change which price is higher in this example) then the goods are **substitutes.**

7.4.4 Examples Cobb Douglass

Suppose $u = x_1 x_2$. Demand is $x_1 = \frac{\frac{1}{2}m}{p_1}$ and $x_2 = \frac{\frac{1}{2}m}{p_2}$. Neither good's demand depends on the price of the other good. **They are neither complements nor substitutes.**

8 Slutsky Decomposition

Decomposes the change in demand for a good into two parts:

Substitution Effect: *Price went up, so I will demand less because I buy other things instead. This will always lead to a decrease in demand.*

Income Effect: *Price went up, so what I continue to buy is now more expensive. My effective income is now lower and my demand will change. May be positive or negative.*

> **Law of Demand:**
> For a change in price of good i the substitution effect (on good i) will always lead to a decrease or no change in demand x_i.
> Thus, if price of a **normal** good increases, demand will decrease.

Three things that can happen.

Ordinary/Normal- *Both effects decrease demand.*

Ordinary/Inferior- *Substitution decreases demand (it always does) and income effect increases demand, but not enough to overcome the decrease due to substitution.*

Giffen/Inferior- *Substitution decreases demand (it always does) and income effect increases demand so much that it overcomes the decrease due to substitution and increases demand overall.*

8.1 The Slutsky Decomposition.

This decomposition is a thought experiment. Suppose price of a good increases, we go from the budget:

$$p_1 x_1 + p_2 x_2 = m$$

To a new budget:

$$p_1' x_1 + p_2 x_2 = m$$

The **total effect** is:

$$x_1^* (p_1, p_2, m) - x_1^* \left(p_1', p_2, m \right)$$

How can we decompose demand? To study substitution effect only, we need to know what the consumer would choose if the price had changed, but their demand could not change due to income. Thus, we think about how much income would they need at the new prices to afford the old bundle? If we were to give the consumer this extra income and ask what they buy at the new prices, however their demand is different than the original bundle could not be due to income effect! It is due only to substitution.

To find this, we calculate the:

compensating income: cost of the original bundle under the new prices.

If we are analyzing a change in p_1 this would be:

$$\tilde{m} = p_1' x_1^* (p_1, p_2, m) + p_2 x_2^* (p_1, p_2, m)$$

Now we construct a new budget:

$$p_1' x_1 + p_2 x_2 = \tilde{m}$$

We ask: what does the consumer choose on this budget?

$$x_1^* \left(p_1', p_2, \tilde{m} \right)$$

The **substitution effect** is:

$$x_1^* (p_1, p_2, m) - x_1^* \left(p_1', p_2, \tilde{m} \right)$$

That is the difference between the original demand and the demand on this thought experiment budget (new prices, extra income).

The **income effect** is the remainder:

$$x_1^* \left(p_1', p_2, \tilde{m} \right) - x_1^* \left(p_1', p_2, m \right)$$

That is, the difference between what they choose on the thought experiment budget (new prices, extra income) and what they choose under the new prices with their actual income.
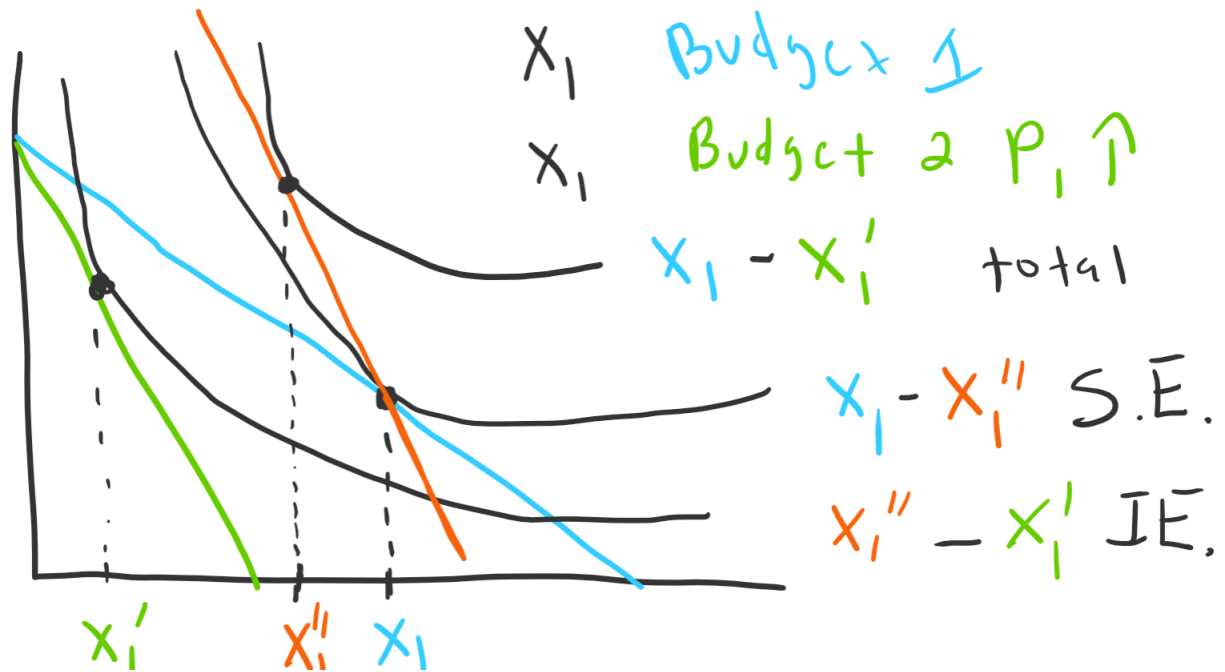
8.2   Graphically Decomposing Demand

Figure 8.1: The Slutsky Decomposition for an Ordinary/Normal Good.

Figure 8.2: Slutsky Decomposition for an inferior good that is not Giffen. Notice that, while the income effect increases demand (orange to green) after the price increases, it does not increase enough to overcome the decrease in demand due to substitution (blue to orange).

Giffen.

$P_1 \uparrow$
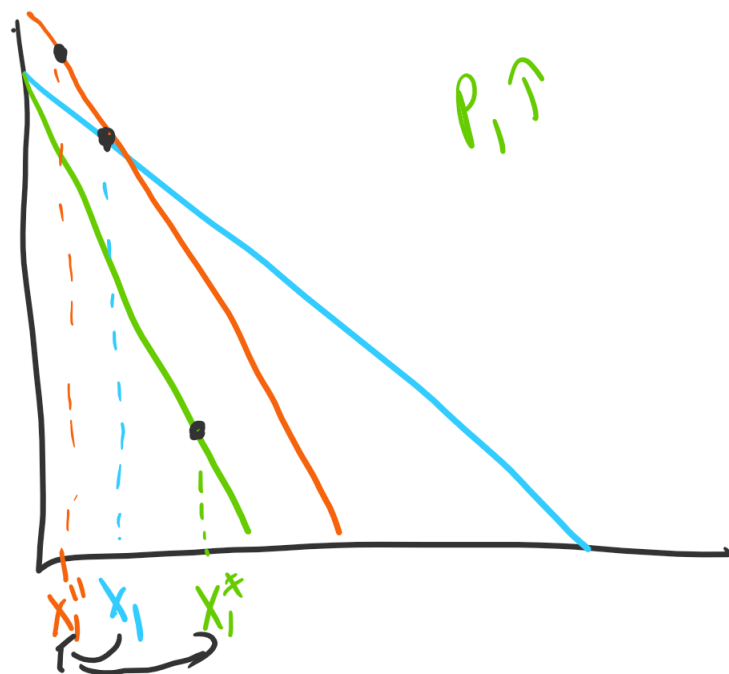
$X_1 - X_1^{\bar{x}}$ (Increase)

$X_1 - X_1''$ SE.

$X_1'' - X_1'$ IE

(Incres

Figure 8.3: The Slutsky decomposition for a Giffen good. Notice the total effect is positive (demand increases when price increases). Demand decreases (blue to orange demand) due to substitution, but increases enough due to income (orange to green) to overcome this and lead to an overall increase in demand.

40

$$v = \min\{x_1, x_2\}$$

$x_1 = x_2$

$x_1 - x_1'$

$x_1'' = x_1$

$x_1 - x'' = 0$
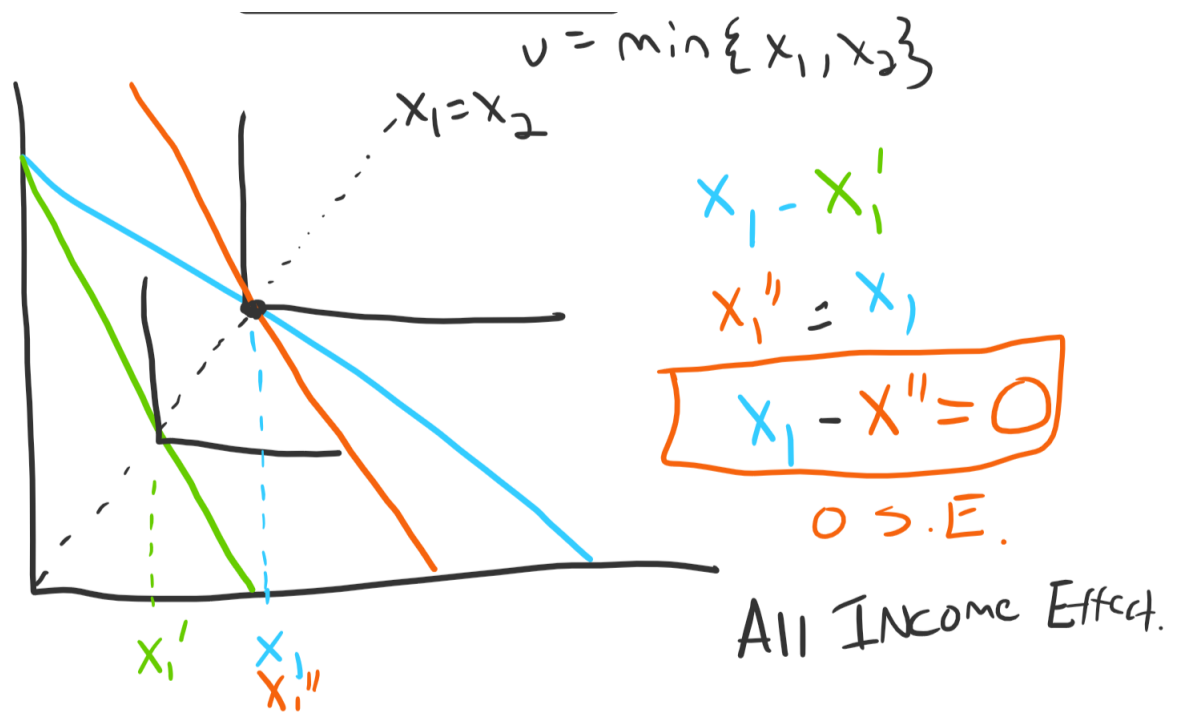
0 S.E.

All Income Effect.

x_1'

x_1''

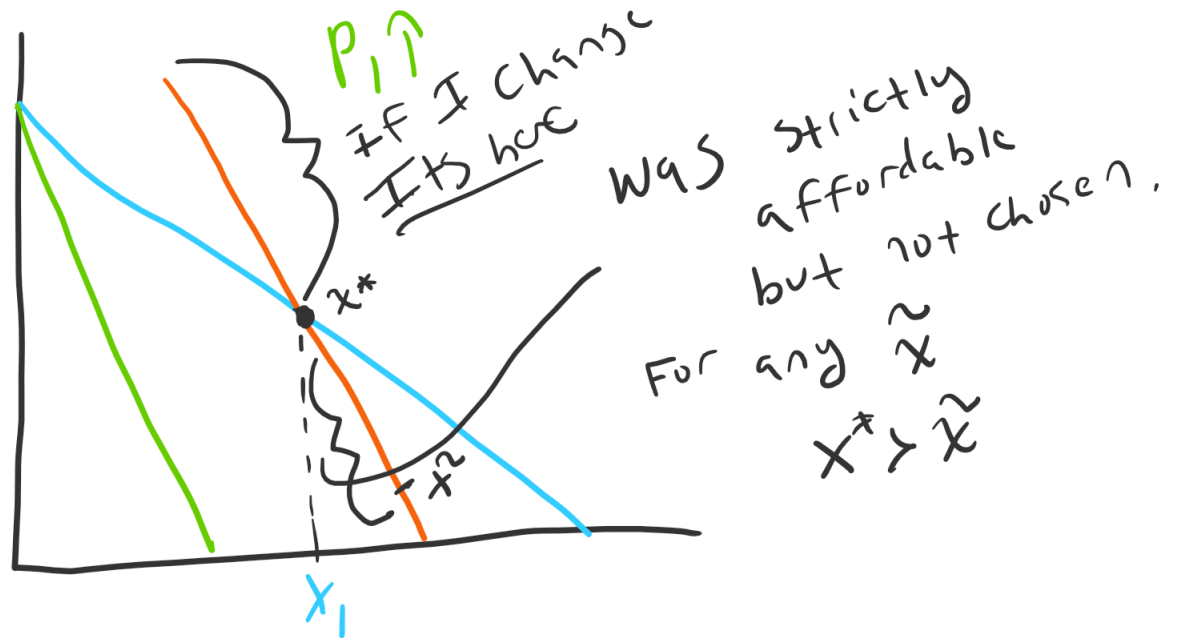Figure 8.4: The Slutsky decomposition for perfect complements. There is **only** substitution effect.

Figure 8.5: **The substitution effect must be negative.** Everything on the lower portion of the orange budget (which determines the substituion effect) was available under the original (blue) budget and was **strictly affordable**. Thus, the point chosen on the blue budget x^* must be strictly better than any of those points. Thus, no point on the lower point of the orange budget can be chosen. This implies demand must decrease due to substitution.