8100 Problem Set 2.

September 8, 2021

1. *Cars have any real number* $[0, \infty)$ *of horsepower and any integer number of cup holders* $\{0, 1, ...\}$.

a) Suppose $\succsim$ is such that a car is preferred to another if it has strictly more cup holders or the same cup holders but more horsepower. Find a utility function that represents these preferences.

b) Show that $\succsim (x)$ and $\precsim (x)$ are closed for these preferences.

c) Show that one of these sets is not closed if the preferences are such that horsepower is the priority, rather than than cup holders.

2. X is a finite set. For every combination $(x, x') \in X$ $(x \neq x')$, the pair (x, x') is included in the set $\succ$ with 50% chance.

a) When $\#(X) = n$, what is the probability that $\succ$ is asymmetric?

b) When $\#(X) = n$, what is the probability that $\succ$ is asymmetric, and complete?

c[1]) When $\#(X) = n$, what is the probability that $\succ$ is asymmetric, complete, and transitive?

3. Prove that if $U(x)$ represents preference relation $\succsim$, and $V(y)$ is a *strictly* increasing function that maps $\mathbb{R} \to \mathbb{R}$, then $V(U(x))$ represents $\succsim$.

4. Let $A_i, i \in I$ be a collection (not necessarily finite) of convex sets. Prove that $\bigcap_{i \in I} A_i$ is convex.

[1] This might be a bit tricky.