ECONOMICS 8100

GREG LEO

Part 1. **Budget**

1. Consumption Set X

Assumptions: (Universe of Choice Objects): X
Bundles: Elements of X. $x \in X$

Assumptions about X. 1. $\emptyset \neq X \subseteq \mathbb{R}^n_+$. 2. X is closed. 3. X is convex. 4. $0 \in X$.

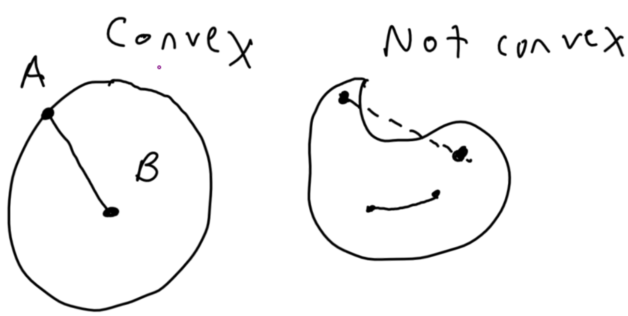

Figure 1.1. Examples of a Convex/Non-Convex Set.

2. Budget Set B

Budget Set: $B \subseteq X$

X defines the scope of the model. B is what an *individual consumer* chooses among.

Example. Budget Set with Prices and Income
$B = \{x | x \in X \,\&\, x_1 p_1 + x_2 p_2 \leq m\}$

Example. Ice Cream Bowls
Every ice cream bowl x has some non-negative number of scoops of Vanilla, Chocolate, Strawberry.

$$X = \mathbb{R}^3_+$$

Budget B is the set of bowls with *no more than one scoop of ice cream.*

$$B = \left\{ x | x \in R^3_+ \& \sum_{i=1}^{3} x_i \leq 1 \right\}$$

This is the unit-simplex in $\mathbb{R}_3$.
$(1, 0, 0) \in B$. (On the boundary.)
$(0.5, 0.5, 0) \in B$. (On the boundary.)
$(0.25, 0.25, 0.25) \in B$. (In the interior.)
$(2, 0, 0) \notin B$

Part 2. **Preference**

3. The Preference Relation

Preference Relation is a **Binary Relation**.
Formally, a binary relation on set X is a subset of the Cartesian product X with itself.

$$\succeq \subseteq X \times X$$

Another way to denote an ordered pair is "in" the relation:
If $(x, y) \in \succeq$ we can also write $x \succeq y$.
Informally we say "x" is at least as good as "y", or "x" preferred "y".

Axioms of $\succeq$.
Axiom 0 *(reflexive):* $\forall x \in X$, $x \succeq x$. This is implied by *axiom 1.*

Axiom 1 *(complete):* $\forall x, x' \in X$, either $x \succeq x'$ or $x' \succeq x$ (or both).
The consumer has "some" preference over every pair of objects.

Axiom 2 *(transitive):* $\forall x, x', x'' \in X$ *if* $x \succeq x'$ *and* $x' \succeq x'' \Rightarrow x \succeq x''$.

$\succeq$ is a "weak order" if it is complete and transitive.

4. Relations and Sets Related to $\succeq$

Subrelations:
$\sim$ is the indifference relation. $x \succeq y$ and $y \succeq x \Leftrightarrow x \sim y$.
$\succ$ is the strict relation. $x \succeq y$ and not $y \succeq x \Leftrightarrow x \succ y$.
Related Sets:
$\succeq (x)$ "upper contour set"

5. From Preferences to Choice

Choice Correspondence.

We will assume that from a budget set B a consumer "chooses" *choice set* C according to their preference $\succeq$. $C = \{x | x \in B \,\&\, \forall x' \in B, x \succeq x'\}$.

Informally, C is the set of objects that are at least as good as anything else in the set.

Example With Transitive Preferences

$X = \{a, b, c\}$. $a \succeq b, c \succeq a, c \succeq b$.

$$C(\{a\}) = a, C(\{b\}) = b, C(\{c\}) = c$$

$$C(\{a, b\}) = a, C(\{a, c\}) = c, C(\{b, c\}) = c$$

$$C(\{a, b, c\}) = c$$

6. Cycles Lead to Empty Choice Sets

6.1. **The Problem with Intransitive Preferences.** $X = \{a, b, c\}$. $a \succeq b, c \succeq a, b \succeq c$. *This is intransitive!*

Choice correspondence:

$$C : P(X) / \emptyset \rightarrow X$$

$$C(\{a\}) = a, C(\{b\}) = b, C(\{c\}) = c$$

$$C(\{a, b\}) = a, C(\{a, c\}) = c, C(\{b, c\}) = b$$

$$C(\{a, b, c\}) = \emptyset$$

This consumer cannot make a choice from the set $\{a, b, c\}$.

6.2. **Cycles and Empty Choices.** Notice in the previous example, $a \succ b, a \succ c, c \succ a$. We have proved (essentially) that if there is a cycle, there is an empty choice set.

In fact, suppose, there is an empty choice set **and** X is finite. There must be a cycle.

$$\forall x \in B, \#\left(\succsim(x)\right) < \#(B)$$

By completeness, $\forall x \exists x' \in X : x' \succ x$. Choose an x_1, let x_2 be any element of $\succ(x_1)$. We have $x_2 \succ x_1$. If there is an $x_3 \in \succ(x_2)$ such that $x_1 \succ x_3$ we have identified a cycle. Otherwise, we continue with an inductive step. Suppose we have $x_n \succ ... \succ x_1$. $\succ(x_n)$ is non-empty. Either it contains an element x_{n+1} such that there is an $x_i \succ x_{n+1}$ in which case we have identified a cycle or it does not and we continue with another inductive step. Either we find a cycle or reach the N_{th} step with $x_N \succ x_{n-1} \succ ... \succ x_1$. $\succ (x_N)$ is non-empty.

So, the cycle condition is equivalence to a non-empty choice set. Transitivity of $\succsim$ implies transitivity of $\succ$ which implies no cycles (try this last step at home). But do no-cycles imply transitivity of $\succsim$? No. Here is a counter-example:

$$x \succ y, y \sim z, z \succ x$$

7. Intransitivity: Empty Choices, Incoherent Choices: Pick One.

So if no-cycles of the strict preference is equivalent to non-empty choice (in finite sets), and transitvity of $\succsim$ is not equivelent to no-cycles, why do we assume it?

Finite non-emptyness: *For any B with $\#(B) \in \mathbb{I}$, $C(B) \neq \emptyset$*

Coherence: *For every x, y and B, B' such that $x, y \in B \cap B'$, $x \in C(B) \wedge y \notin C(B) \Rightarrow y \notin C(B')$.*

Suppose there is an intransitive $\succsim$. There exists either a B where $C(B) = \emptyset$ or there exists a x, y, B, B' where the choice correspondence is incoherent.

By intransitivity:

$$1)\, x \succ y, y \succ z, z \succ x$$

$$C(\{x, y, z\}) = \emptyset$$

$$2)\, x \sim y, y \sim z, z \succ x$$

$$3)\, x \sim y, y \succ z, z \succ x$$

For both of these we have incoherent choice from the following sets:

$$x \notin C(\{x, y, z\})$$

$$y \in C(\{x, y, z\})$$

$$x \in C(\{x, y\})$$

$$4)\, x \succ y, y \sim z, z \succ x$$

Can you find the incoherent choice?