Intermediate Microeconomics*

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These notes are based on my Vanderbilt Economics Course 3012. **They are preliminary.** If you find any typos or errors in this text, please e-mail me at g.leo@vanderbilt.edu.

Contents

Part I

Bundles & Budget

1 Bundles

Bundles are the fundamental object of study in microeconomics. In our models, when a consumer makes a choice, they choose a **bundle** from the set of bundles available to them (the **budget set**). Bundles can be anything or combination of things you can think of. In this course, however, bundles are *usually* going to be amounts of some things we call **goods** and very often we will just look at two goods.

Bundle: $x = (x_1, x_2)$

Example. Ice Cream Bowls (the bundles) are made of up two goods: scoops of vanilla ice cream and scoops of chocolate ice cream. x_1 is the amount of vanilla. x_2 is the amount of chocolate. $(1, 1)$ represents one scoop of each flavor, $(2, 2)$ two scoops of each flavor, and $(0.28, 100)$ a lot of chocolate (100 scoops) and a little vanilla (0.28 scoops).

Since bundles with two goods are represented by ordered pairs, we can plot bundles on and x_1, x_2 axis. An example of this is shown below.

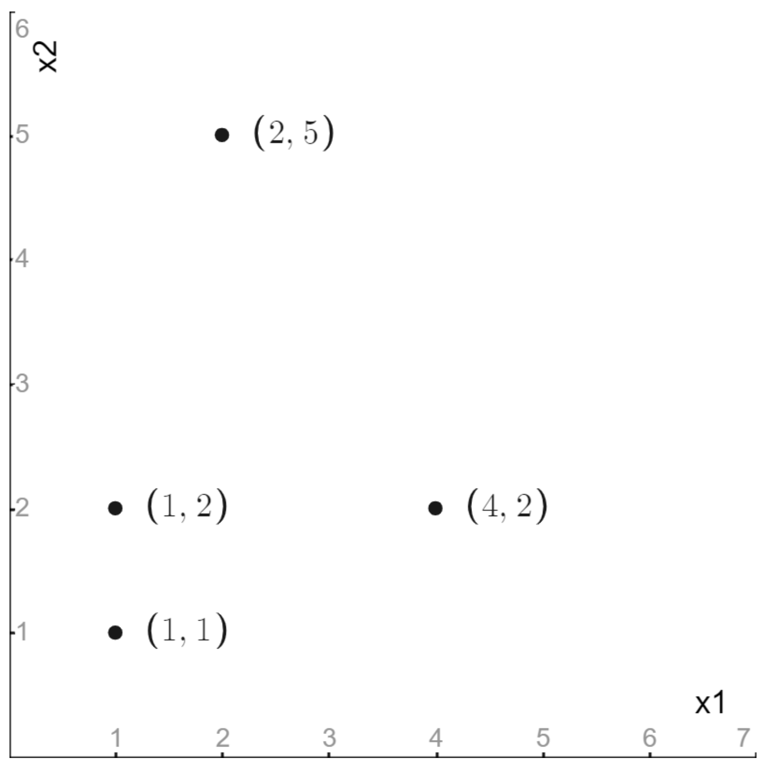

Figure 1.1: A Few Bundles on The Cartesian Plane.

2 Feasible Set

The set of all bundles relevant to a model is called the **Feasible Set**. The feasible set defines the scope of a model.

The Feasible Set: X is the "feasible" set of bundles.

Example. The feasible set for a model about choosing ice cream bowls is the set of all ordered pairs possible ice cream bowls: (x_1, x_2). Of course, it does not make sense to have a negative amount of ice cream, so in this case we might say $X = \mathbb{R}^2_+$. (This notation says that the feasible set is made up of 2 real numbers that are non-negative.)

3 Budget Set

Budget Set: B

The budget set is the set of bundles *available* to a particular consumer. The budget set must be a subset of the feasible set. In set notation we write: $B \subseteq X$

3.1 Budget Sets from Prices and Income

Not everything in the feasible set is going to be achievable for every consumer. Some bundles are affordable and others are not. The set of bundles that a consumer can *actually choose from* is called the **budget set**. Our budget sets will be constructed by assuming consumers have some income and that each good has a price.

Prices: p_1, p_2: Price units of good 1 and good 2.
Income: m.

With these, we can define the cost of a bundle:

Cost of a bundle: $p_1 x_1 + p_2 x_2$

The set of all bundles that a consumer can afford is called the **Budget Set.** We can define if formally this way:

Budget set: $B = \{x | x \in X \,\&\, x_1 p_1 + x_2 p_2 \leq m\}$ **.**[a]

[a] In "normal" language, this says the budget set is the set of bundles such that the price of the bundle is less than income.

Since we are able to plot bundles, we can also plot the budget set. To do this, it is easiest to first, we draw the **Budget Line**. This is the set of bundles that are "just affordable".

Budget Line: $x_1 p_1 + x_2 p_2 = m$

Now we can plot this on an x_1, x_2 plane. Let's put x_2 of the vertical axis. In this case, it is useful to rewrite the budget line into a form we are more familiar with:

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

This is now clearly an equation for a line with intercept $\frac{m}{p_2}$ and slope $-\frac{p_1}{p_2}$. Before we plot it, let's interpret it a little. Notice that if $x_1 = 0$ we get $x_2 = \frac{m}{p_2}$. This says "If I were only to buy x_2, I could afford $\frac{m}{p_2}$ units of x_2. Furthermore, for every unit that we increase x_1 by, x_2 goes down by $-\frac{p_1}{p_2}$. This says "If I am spending all my money, if I want to buy one more unit of x_1, I have to give up $-\frac{p_1}{p_2}$ units of x_2. This is a very important thing to know about the slope of the budget line. **The slope of the budget line represents the trade-off between x_1 and x_2 at the market prices.** We are now ready to plot the budget set. It is the budget line and all of the bundles "below" the budget line.

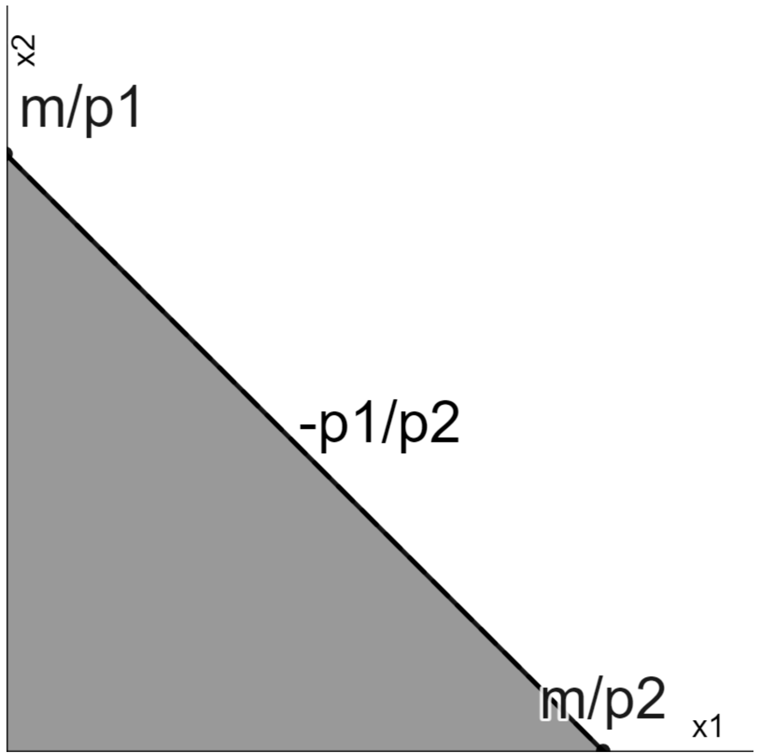

Figure 3.1: Graphical Representation of the Budget Set with slope $-\frac{p_1}{p_2}$ and intercepts $\frac{m}{p_1}$ and $\frac{m}{p_2}$.

3.2 Changing Prices and Income

We are often interested in looking at how the budget set changes when we change on of the parameters of the model: m, p_1, or p_2.

We can work out how the budget set changes by looking at changes in the budget line. There are three key elements to the budget line: the slope $-\frac{p_1}{p_2}$ and the intercepts $\frac{m}{p_1}$ and $\frac{m}{p_2}$.

When income changes, notice that only the intercepts change. If m increases, both intercepts increase. This should be intuitive. Since the intercepts represent how much of a good we can buy if we only buy that good, then if income increases, we can afford more. When income decreases, the opposite happens.

Importantly, when income changes, the slope of the budget line does not change. This is because the trade-off between the goods stays the same regardless of income (as long as the price remain the same).

When a price changes on the other hand, the slop of the budget line changes and **one** of the intercepts changes. For instance, if p_1 goes up, the slope of the budget line becomes steeper (because more x_2 has to be given up to get an extra unit of x_1). Furthermore, the x_1 intercept decreases because less x_1 can be afforded if we only buy x_1.

Some of the possible changes are demonstrated in the graphs below.

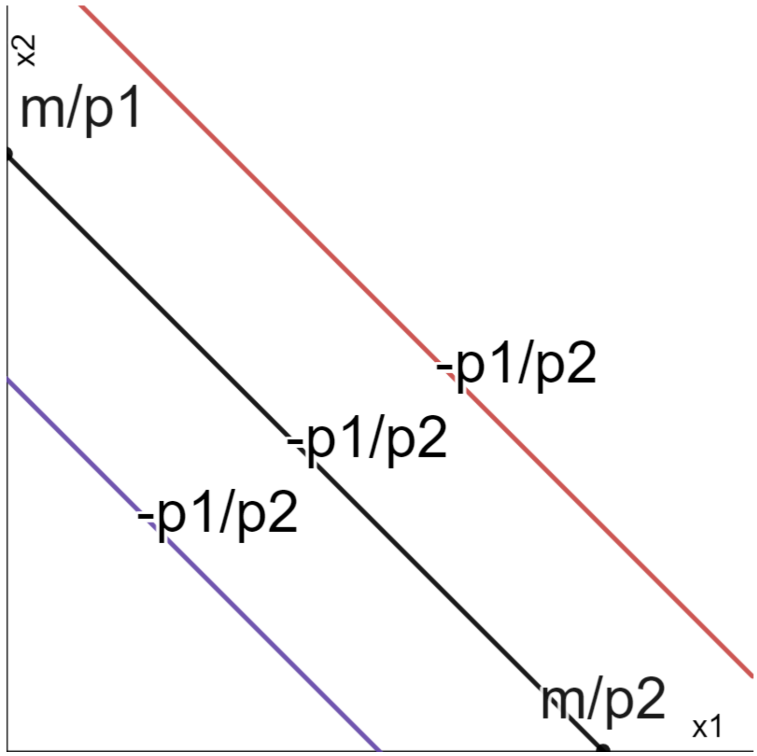

Figure 3.2: How Budget Changes with Income.

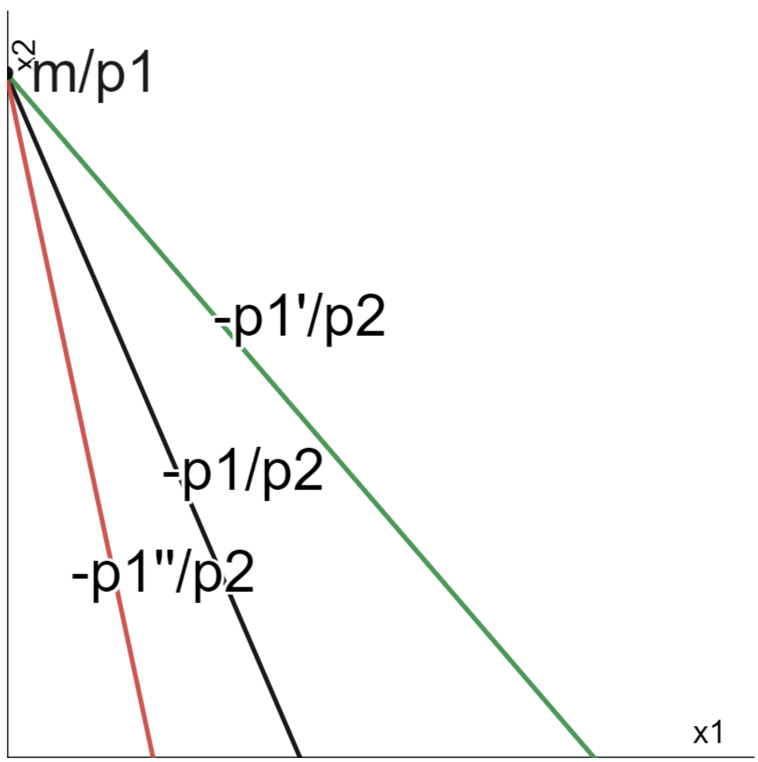

Figure 3.3: How Budget Changes with change in p_1.

In summary:

m **changes:**

Both endpoints change. If m increases, $\frac{m}{p_1}$ (the amount I can buy of good 1 changes) increases and $\frac{m}{p_2}$ (maximum affordable x_2) increases. The slope does not change. If m decreases, the opposite happens.

p_1 **changes:**

p_1. If p_1 goes up, the slope decreases (more negative). If p_1 goes down, the slope increases. The x_2 intercept stays the same.

p_2 **changes:**

p_2. If p_2 goes up, the slope increases and the x_2 intercept decreases. If p_2 goes down the slope decreases (becomes more negative) and the x_2 intercept increases. The x_1 intercept stays the same.

3.3 Taxes

Taxes represent a certain kind of price change. There are two kinds of taxes that are used frequently: *quantity* and *ad valorem* taxes.

A **quantity tax** is determined by **number of units** (x_i) purchased where an **ad valorem** tax is determined by the **value** of the good purchased ($x_i p_i$).

With a quantity tax of t dollars on good i, the amount paid in tax is tx_i. With an ad valorem tax of percentage τ on good i, the amount pain in tax is $\tau\left(p_i x_i\right)$. The key difference is that as price of a good changes, the amount collected by the government does not change with a quantity tax (assuming the amount purchased does not change), but it does with an ad valorem tax. Most sales taxes are ad valorem. However, there are quantity taxes we encounter frequently. Pay close attention next time you are pumping gas, there is usually a sticker showing how much you pay in tax *per gallon*. That's a quantity tax.

Here's what happens to the budget line when we ad a quantity tax and ad valorem tax on good 1.

Quantity tax on good 1:

$$p_1 x_1 + t x_1 + p_2 x_2 = m$$

$$\left(p_1 + t\right) x_1 + p_2 x_2 = m$$

Ad valorem Tax on good 1:

$$\left(p_1 x_1\right) + \tau\left(p_1 x_1\right) + p_2 x_2 = m$$

$$[\left(1+\tau\right) p_1] x_1 + p_2 x_2 = m$$

Notice that in both cases, the tax effectively just increases the price of the good. This makes taxes easy to plot, they have the same effect as a price increase. However, there are some complex scenarios you should think about. What if a quantity tax only kicked in after buying a certain amount of some good? What if instead of a tax, a subsidy (a decrease in price) was put on a good? What if that subsidy only held for the first k units of the good? We will talk about many of these scenarios in class and work with them in practice problems.

Part II

Preferences

4 The Preference Relation $\succsim$

4.1 Definitions

Now that we know how to model what a consumer can have, we should talk about what the prefer. We represent preferences with a mathematical tool called a **relation**.

Preference Relation
The preference relation denoted $\succsim$ is a set of statements about **pairs** of bundles. The statement *"bundle x is preferred to bundle x'"* is shortened to:

$$x \succsim x'$$

Example: Ice Cream
Suppose a consumer eats bowls of ice cream. The bundles (bowls) are written with the vanilla scoops first and chocolate second. For example: $(2, 0)$ is two scoops of vanilla and zero of chocolate.
A consumer who likes vanilla more than chocolate might have these preferences:

$$(1, 0) \succsim (0, 1), (2, 0) \succsim (0, 2)$$

A consumer who like more ice cream to less might have these preferences:

$$(2, 0) \succsim (1, 0), (2, 2) \succsim (1, 1)$$

A consumer who gets sick of ice cream: (does anyone want to eat 100 scoops of ice cream?)

$$(1, 0) \succsim (100, 0)$$

A consumer who does not care about flavor might have:

$$(1, 0) \succsim (0, 1), (0, 1) \succsim (1, 0)$$

In the case of the consumer who does not care about flavor above, notice that we have both $(1, 0) \succsim (0, 1)$ and $(0, 1) \succsim (1, 0)$. That is, a scoop of vanilla is just as good as a scoop of chocolate and a scoop of chocolate is just as good as a scoop of vanilla. When this is the case, we say the consumer is **indifferent.**

Indifference Relation: $\sim$
When $x \succsim y$ and $y \succsim x$ we say "x is indifferent to y" and write $x \sim y$.

When a consumer is not indifferent, we say they have strict preference for some bundle.

Strict Preference Relation: $\succ$
When $x \succsim y$ and **not** $y \succsim x$ we say "x is strictly preferred to y" and write $x \succ y$.

4.2 Assumptions on $\succsim$

In economics, we like to make as few assumptions about consumer's preferences as we can. There's a surprising amount we can say about consumer choice with just a few assumptions about the structure of preferences.

The first three assumptions or **axioms** we will look at ensure that for any budget set, consumers will have some favorite or set of favorite bundles. That is, given any set of bundles, they will actually be able to choose *something*. We will talk more about why these assumptions assure that fact in class.

> ***Axiom 1. Reflexive.***
> *For all bundles. The bundle is at least as good as itself.*
> In set notation:
>
> $$\forall x \in X : x \succsim x$$

This is what we call a *technical* assumption. It does not carry a lot of content for us to talk about, but it helps ensure some minimal structure. After all, if a bundle was not "as least as good as itself", we'd have some trouble since that would imply that either it cannot be compared to itself or that it is both strictly better than itself and at the same time strictly worse than itself.

> ***Axiom 2. Complete.***
> *For every pair of distinct bundles. Either one is at least as good as the other or the consumer is indifferent.*
> In set notation:
>
> $$\forall x, y \in X \& x \neq y : x \succsim y \, or \, y \succsim x \, or \, both$$

This axiom is a little more interesting. It says that for every pair of bundles, the consumer has *some* preference. The consumer can say "I'm indifferent." but not "I don't know". That is, everything is comparable.

> ***Axiom 3. Transitivity.***
> If x is at least as good as y and y is at least as good as z then x is at least as good as z.
>
> $$x \succsim y, y \succsim z \,\textbf{implies}\, x \succsim z$$

Transitivity lets us chain together preferences. It is really the **key** and most powerful assumption here. Transitivity ensures (along with the other assumptions) implies we can always put a set of objects into a **ranking** (possible with ties). Once we have a ranking, there's always going to be some things that are at the top of that ranking. Those are the things our consumers will choose.

4.3 Example of Violating Transitivity

In many circumstances, transitivity is an uncontroversial assumption. However, it is possible to construct perfectly reasonable decision processes where transitivity fails. Here is one of those examples:

Suppose there are three people on a dating app:

Person 1. Rich, Very Intelligent, Average Looking

Person 2. Financially Constrained, Genius, Good Looking

Person 3. Moderately Well Off, Average Intelligence, Best Looking

Now let's compare every pair of people. Person 2 is both more intelligent and better looking than person 1. Person 3 is wealthier and better looking than person 2. Person 1 is wealthier and more intelligent than person 3.

From this, we can construct a preference ordering: $2 \succ 1, 3 \succ 2, 1 \succ 2$. Notice, this is intransitive. It is clear who is better in any pair, but who would is is best from the set of all three? This kind of multi-dimensional comparison can easily cause intransitivity.

4.4 From Preference to Choice

So far, we have a pretty satisfying model of preferences, but economics is about *choice*. How do we model choice? Intuitively, we want to write down formal that, from any budget set, the consumer will choose the best thing (according to their preferences). To do this, let's define a **Choice Function.** We can write:

$$C : B \to B$$

This says that C is a function that maps the set B (a budget set) into itself. That is, from the set B, the function C returns some objects from the set B. This statement ensures that the set of "choices" will always be a subset of the budget set. In set notation, that would be expressed as: $C(B) \subseteq B$.

That's good, but there's no structure here involving the preference relation. What we really want is that $C(B)$ (the potential choices from the set budget set B) is the set of all bundles in B that are at **least as good as everything else in B.** We can express that formal as follows.

$$C(B) = \{x | x \in B : \forall x' \in B, x \succsim x'\}$$

This says, $C(B)$ is defined to be all the bundles (x) in the budget set (B) such that (:) for all ($\forall$) other bundles (x') in ($\in$) the budget set, we have that x is at least as good as x'. This is not the easiest statement to read if you are not familiar with this kind of formal expression, but I hope that you will agree that it is a rather elegant, and efficient way of expressing an otherwise rather complicated idea.

Notice that in the example in the last section of choosing a partner on a dating app, there is no partner that is at least as good as all the other partners. In that case, **the choice set is empty!** Having empty choice sets is potentially problematic for a mathematical model of choice. So, when can we be sure that there is always some bundle that a consumer will choose from any budget set in our models.

Fortunately for us, our three assumptions: reflexivity, completeness, and transitivity are enough to ensure that the consumer will always have some favorite things in any budget and will be able to make a choice. As an aside, transitivity is even a little stronger than we need for this, as it also ensures a form of consistency of choice called "coherence". We will talk a little about that in class.

4.5 Indifference Curves and the Weakly Preferred Set

At this point, we have spent a good amount of time looking at how to formally express preferences. In practice, it is hard to work with these formal statements. Like anything else, it is nice to be able to visualize preferences. We can achieve this through **indifference curves.**

Indifference curve: an indifference curve is a set bundles such that a consumer is indifferent between every pair of bundles in the set.

In mathematics terms, an indifference curve is called an *equivalence class.* That is, it is some set that are "equivalent" in terms of preferences. This term is not necessary to know, but it may come up in future courses.

Note: There are many indifference curves. We only sketch a few to get an idea of the "shape" of preferences. Every bundle has an indifference curve passing through it.

Let's look at an example. Suppose we have a consumer who likes apples just as much as oranges. They are indifferent between the bundle "two apple" $(2, 0)$ and the bundle "two orange" $(0, 2)$. These two bundles are on the same indifference curve in the graph below. The consumer is also indifferent between $(4, 0)$, $(2, 2)$ and $(0, 4)$ they are on the same indifference curve in the graph below.

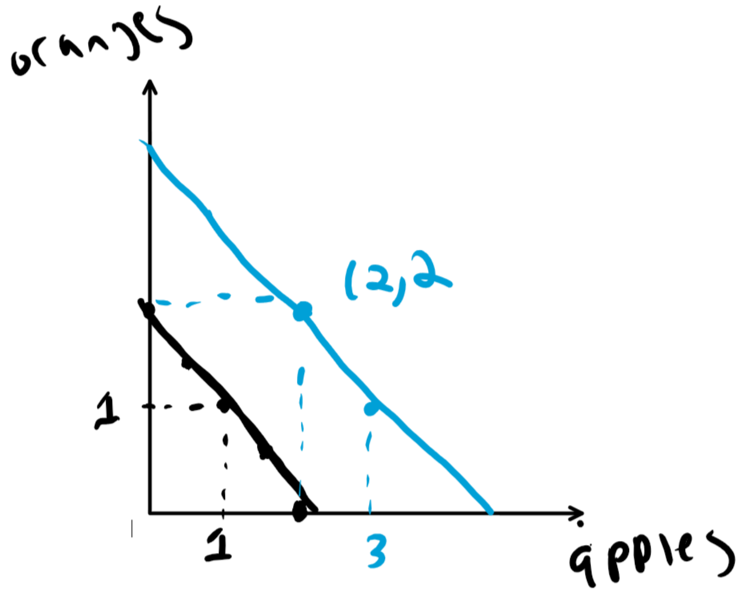

Figure 4.1: Indifference curves when apples are just as good as oranges.

4.6 Indifference Curves Cannot Cross

What can we say about the shape of indifference curves? It turns out, with only the assumptions of reflexivity, completeness, and transitivity: not much. Indifference curves could have some wild shapes. But under these assumptions there is one thing we know: **two distinct indifference curves cannot cross.**

Below is a proof of this claim. You are not responsible for knowing this proof, but you may be interested to see the logic. Understanding the logic might help you understand the way our axioms are used in proving formal statements about preferences.

Proof the two indifference curves cannot cross.
Look at the graph below. Here I have drawn two distinct indifference curves that cross each other. Notice that if two curves cross, they have to cross *somewhere*. I have labeled that somewhere x in the graph. This is a bundle that is on **both** indifference curves. However, since these are distinct indifference curves, there must be some bundle x' and x'' that are respectively on the different indifference curves and thus not indifferent to each other. However, since x is on both indifference curves, we must have $x' \sim x$ and $x'' \sim x$. Let's derive a contradiction to prove this scenario can never happen.
Since it is not the case that $x' \sim x''$ if preferences are **complete**, it must be that either $x' \succ x''$ or $x'' \succ x'$. If we take the first possibility $x' \succ x''$ we have $x' \succ x''$ and $x'' \sim x$. By **transitivity,** it must be that $x' \succ x$ but we already know that $x' \sim x$. If we take the second possibility $x'' \succ x'$ we have $x'' \succ x'$ and $x' \sim x$. By **transitivity,** it must be that $x'' \succ x$ but we already know that $x'' \sim x$. Thus, no matter what, we have found a contradiction.

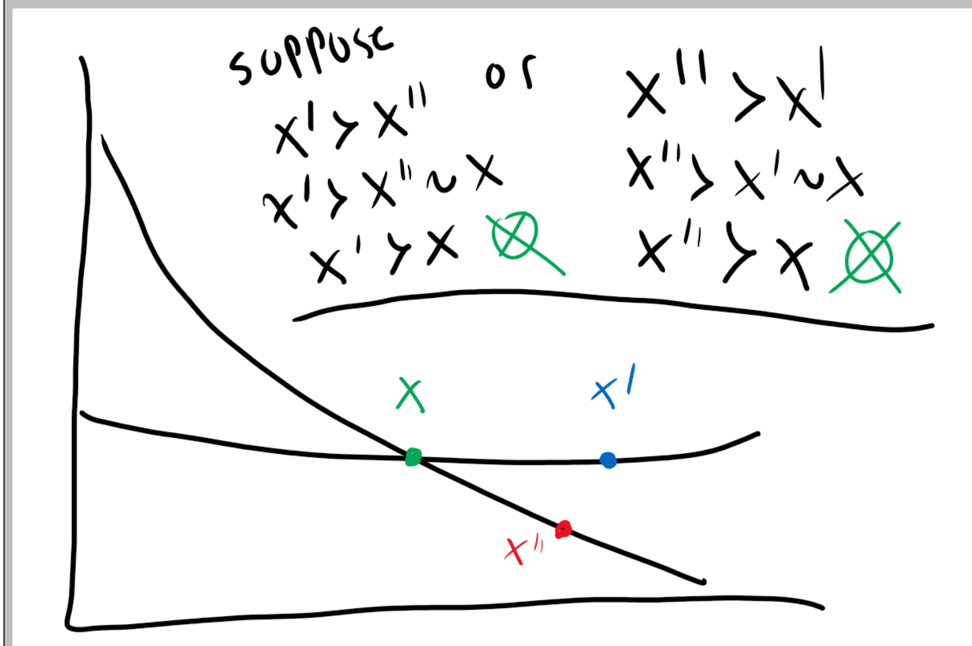

Figure 4.2: Indifference curves cannot cross if preferences are transitive.

4.7 Common Types of Preferences

There are a few "families" of preferences you should know about. These different families represent different types of trade-offs consumers are willing to make between two goods.

4.7.1 Perfect Substitutes

Perfect Substitutes preferences are such that a consumer's willingness to trade-off between the goods is the **same everywhere**.

The indifference curves are always downward sloping lines with the same slope. **The slope measures the amount of x_2 the consumer is willing to give up to get 1 more unit of x_1.**

Steep slope: stronger preference for x_1.

Shallow slope: stronger preference for x_2.

Figure 4.3: Indifference curves for perfect substitutes preferences. This consumer would be willing to give up 2 units of x_2 in exchange for 1 unit of x_1.

4.7.2 Perfect Complements

Perfect Complements preferences are such that a consumer must consume the goods in a **fixed proportion**.

An example of this is left and right shoes. You always consume left and right shoes in a 1-to-1 proportion. That is, you want one left shoe for every right shoe. If you have the same number of left and right shoes, you are not willing to give up any left shoes to get more right shoes, because that would reduce the number of usable pairs you have.

Another example is ingredients in a recipe. Suppose you bake pies and a pie always needs two apples and one crust. If you have two apples and one crust, or four apples and two crusts, or six apples and three crusts, you would not be willing to give up apples to get more crusts or give up crusts to get more apples, it would reduce the number of pies you can make.

The indifference curves for these preferences are **L-shaped**. The kinks of these L-shaped curves pass along a line through the origin where the points on that line are the points where the goods are consumed in the "correct" proportion. That is, where there is not too much of either good. For left and right shoes, if left shoes are x_1 and right shoes are x_2, that the line $x_2 = x_1$ (the 45-degree line). For pies, if apples are x_1 and crusts are x_2 then the line through the kink points is where $2x_2 = x_1$. *I have plotted these below.*

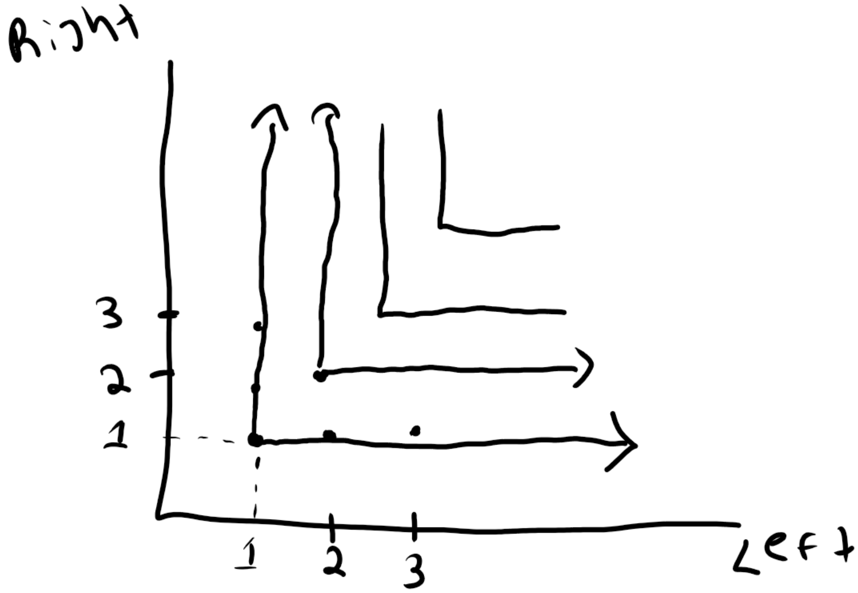

Figure 4.4: Indifference curves for perfect complements preferences where Left/Right shoes must be consumed in a 1-to-1 one ratio.

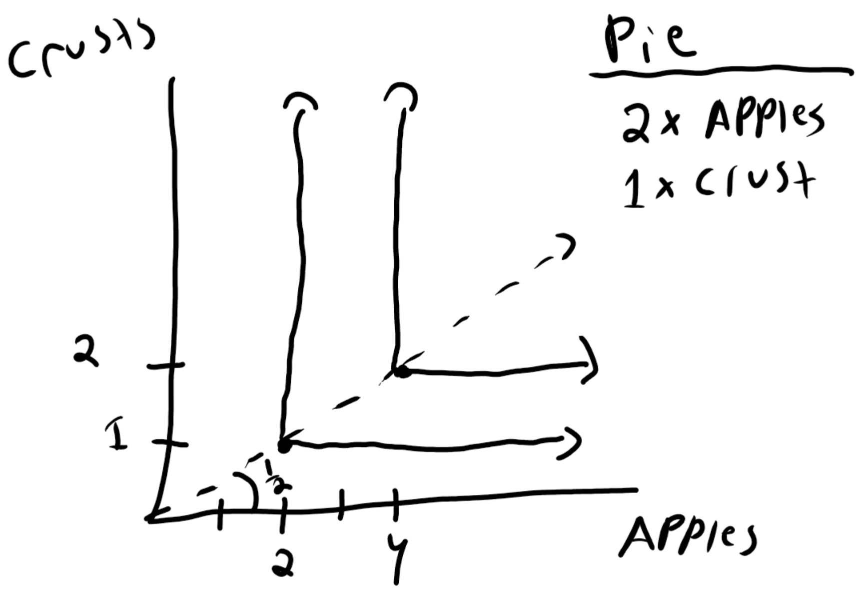

Figure 4.5: Indifference curves for perfect complements preferences where the goods are consumed in a 2-to-1 ratio. In this case, 2 apple and 1 crust make a pie.