8100 Problem Set 5.

November 3, 2021

1. Find the Marshallian demand for the utility function: (assume $\alpha, \beta, \gamma > 0$ and $a, b, c \geq 0$.) *Mind the corners.*

$$(x_1 + a)^{\alpha} (x_2 + b)^{\beta} (x_3 + c)^{\gamma}$$

2. Consider an environment of choice under uncertainty. There are finite outcomes $A = \{a_1, a_2, ..., a_n\}$ and you can assume $a_i \succ a_j$ for $i < j$.

Let $p_g(a)$ be the probability that outcome a occurs under compound gamble g. Let $b(g)$ be the best outcome according to $\succ$ such that there is a non-zero probability of that outcome under g: $p_g(a) > 0$.

A consumer's preferences over compound gambles are such that $g \succ g'$ if and only if $b(g) \succ b(g')$ or $b(g) \sim b(g')$ and $p(b(g)) > p(b(g'))$.

Let $\succsim$ be the preference relation on $\mathcal{G}$ (the set of compound gambles).
Axiom 1. ***Complete:*** *$\succsim$ is complete.*
Axiom 2. ***Transitive:*** *$\succsim$ is transitive.*
Axiom 3. ***Monotonic:*** For all $(\alpha \circ a_1, (1-\alpha) \circ a_n) \succsim (\beta \circ a_1, (1-\beta) \circ a_n)$ iff $\alpha \geq \beta$,
Axiom 4. ***Continuous:*** For all g $\exists p \in [0,1]$ such that $g \sim (p \circ a_1, (1-p) \circ a_n)$
Axiom 5. ***Substitution:*** If $g = (p_1 \circ g_1, ..., p_k \circ g_k)$ and $h = (p_1 \circ h_1, ..., p_k \circ h_k)$ and if $g_i \sim h_i$ for all $i \in \{1, ..., k\}$ then $g \sim h$.
Axiom 6. ***Reduction:*** For any gamble g and the simple gamble it induces g_s, $g \sim g_s$.

A) Among **completeness, transitivity, monotonicity, continuity, substitution, reduction**. Which assumptions are met by these preferences?

B) Can you construct a utility function that represents these preferences?

3. A consumer is an expected utility maximizer and has a utility function for wealth equal to $v(w) = \sqrt{w}$.

A) If the consumer starts with \$0, what is their certainty equivelent for game that pays \$$x$ with $p = 0.5$ and \$0 with $p = 0.5$.

B) If the consumer starts with w_0, what is their certainty equivelent for the same gamble?

C) As the consumer becomes more wealthy (w increases) how does their certainty equivalent for this gamble compare to the certainty equivalent for a risk-neutral consumer?

4. Consider the production function:

$$f(x_1, x_2) = (x_1^r + x_2^r)^{\frac{1}{2r}}$$

A) Find the conditional factor demands.

B) What is the cost function?

C) Show the cost function can be decomposed into the cost of producing one unit and a power function of output y.

D) What is the profit function when output price of y is p?

5. Consider the production function:

$$f(x_1, x_2) = (x_1 + x_2)^{\frac{1}{4}} + x_3^{\frac{1}{2}}$$

A) Show that the ratio of marginal products of x_1 and x_2 do not depend on the level of x_3.

B) What is the cost of producing y_1 units of output using only x_1 and x_2.

C) What is the cost of producing y_2 units of output using only x_3?

D) What is the cost of producing y units of output from x_1, x_2, x_3 when $w_1 = w_2 = w_3 = 1$?

E) What is firms profit when output price of y is p and $w_1 = w_2 = w_3 = 1$?