1. A consumer has utility function:

$$u(x_1, x_2, x_3) = x_1^{\alpha} x_2^{1-\alpha} + ln(x_3)$$

Assume $\alpha > 0$ *and* $p_1 = p_2 = p_3 = 1$.

A) How does the money spent on x_3 depend on α? Sketch a graph.

B) Referencing marginal utilities, say something that makes sense of why and how money spent on x_3 depends on α in this way.

2. Taylor produces fancy keyboards using machine time m, aluminum a, and brass b. Machine times costs 1, aluminum costs 4, and brass costs 9. The production function for fancy keyboards is:

$$f(m, a, b) = (a + b)^{\frac{1}{4}} m^{\frac{1}{4}}$$

A) What are Taylor's conditional input demands?

B) What is Taylor's cost function for keyboards?

Taylor is a monopolist for fancy keyboards, he assumes the inverse demand is $p(q) = 400 - q$*:*

C) how many keyboards does Taylor produce?

D) Prove Taylor operates in the elastic portion of the demand curve.

After some market research, Taylor finds out that the demand for keyboards depends on the proportion of brass used in production. Specifically, if he uses a *aluminum and* b *brass in production, he can sell* q *keyboards for* $p(q) = \left(\frac{b}{a+b}\right)(400 - q)$

E) Say something about why the cost-minimization approach to profit maximization will fail in this case.

F) What is the aluminum / brass composition of the keyboards Taylor produces to maximize profit.

G) What is Taylor's cost function for producing keyboards of this type?

H) What is the optimal number of keyboards for him to produce?

3. Firms have cost function y^2. There are J firms. Demand is $q = 100 - p$

A) The firms each assume price does not depend on their output. Find an equilibrium price p_{comp} for this market under this assumption as a function of J.

B) Find an expression for consumer surplus under perfect competition CS_{comp} as a function of J.

C) Find a symmetric Cournot equilibrium price $p_{cournot}$ of this market as a function of J.

D) Show that as $J \to \infty$, p_{comp} and $p_{cournot}$ both approach 0.

E) What is $lim_{J\to\infty} \frac{p_{comp}}{p_{cournot}}$?