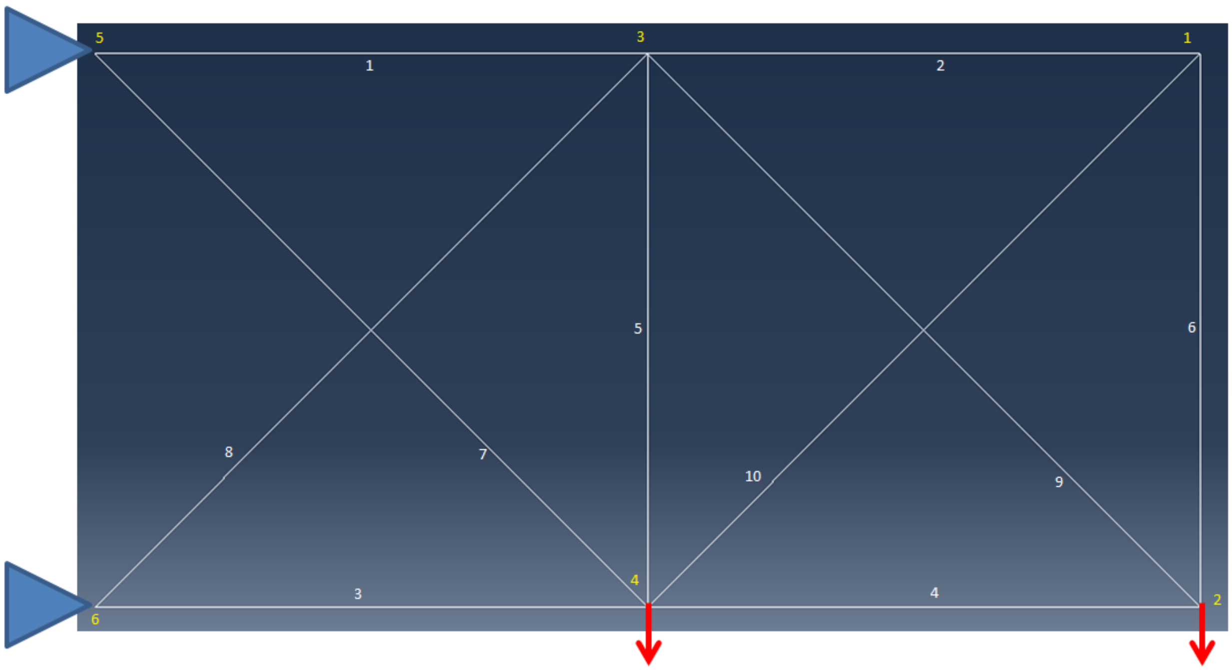
Problem Statement

The 10-bar truss structure shown below was analyzed and optimized for minimum weight subject to various constraints.

- Force P = 100,000 lbf
- Length L = 360"
- Elastic Modulus E = 10^7 psi (Aluminum truss members)
- Density ρ = 0.1/(32.174*12) lbm/in^3



The optimization problems solved in this work can be broken up into three parts:

1. Optimize the areas of the structure due to stress constraints (linear approximations) to minimize weight
    a. Table 6.4.1
    b. Table 6.7.2
2. Optimize areas of the structure due to stress constraints (linear approximations) to minimize weight (linear approximation)
    a. Table 6.4.2
3. Optimize areas of the structure due to stress and displacement constraints (linear approximations) to minimize weight
    a. Table 6.7.4

In order to develop the function approximations, sensitivity analysis was completed using the Adjoint method along with linear function approximations. These concepts have been outlined in previous homework assignments and details will be omitted here.

Table 6.4.1: Stress Constraints Only

The optimization problem statement is given below

Minimize

$$W = \rho A L$$

Subject to

$$g_{1-8,10}(x) = \sigma_{1-8,10} \leq \pm 25\ ksi$$

$$g_1(x) = \sigma_9 \leq \pm 75\ ksi$$

Side Bounds

$$0.1 \leq A_{1}, \dots, A_{10} \leq 10$$

Design Variables

$$A_{1}, \dots, A_{10}$$

Note that g(x) function imply approximations

The final results of Table 6.4.1 have been matched as shown below:

Member	Initial Area (in^2)	Optimum Area (in^2), Haftka	Optimum Area (in^2), Loughnane
1	5	7.900	7.900
2	5	0.100	0.100
3	5	8.100	8.100
4	5	3.900	3.900
5	5	0.100	0.100
6	5	0.100	0.100
7	5	5.800	5.798
8	5	5.510	5.515
9	5	3.680	3.677
10	5	0.140	0.141

Convergence of the results of Table 6.4.1 was achieved in 11 cycles, using an initial starting design of x0 = [5 5 5 5 5 5 5 5 5 5]. Initial move limits were 75% of the initial design variable. Then, for each cycle the move limits were decreased by 5% and convergence was achieved in exactly 11 cycles. During each cycle, a comparison of the move limit and the side bounds was done. If the move limit violated a side bound, a new move limit exactly equal to the side bound violated was imposed.

Note that the word *cycle* is taken in Haftka, et al. to mean the iterations of actual design changes, not the iterations used to find the solution to each approximate problem developed. The latter are simply referred to as iterations, which occur within the fmincon function in MATLAB for this work.

Note that the convergence criterion for this problem was given by the percent change in the weight of the structure. The optimization stopped when the percent change in the weight was < 1e-6 for 3 consecutive iterations. The iteration history for the 11 cycles is shown below, and it can easily be seen that a less strict convergence criterion could have been used here.

Cycle	Weight	Stress constraints satisfied?
Initial	2098	Y
1	1423	Y
2	1528	Y
3	1524.2	Y
4	1507.2	Y
5	1500.2	Y
6	1497.5	Y
7	1497.6	Y
8	1497.6	Y
9	1497.6	Y
10	1497.6	Y
11	1497.6	Y

Table 6.4.2, Column 1: Stress Constraints with Weight Approximation

The optimization problem statement is given below

Minimize

$$f_{weight,linear}(x)$$

Subject to

$$g_{1-8,10}(x) = \sigma_{1-8,10} \leq \pm 25\ ksi$$

$$g_1(x) = \sigma_9 \leq \pm 75\ ksi$$

Side Bounds

$$0.1 \leq A_1, \ldots, A_{10} \leq 10$$

Design Variables

$$A_1, \ldots, A_{10}$$

For the comparison of this table, it must be noted that move limits play a large role in the iteration history. The move limit scheme that was used to find the optimum weight based on a linear approximation of the structures weight was relatively complicated and is described below:

- For the first 11 cycles

  - 75% change in design variable was allowed, with a 5% decrease in allowable design changes each iteration
- For cycles 12-15
  - 10% changes in design variables were allowed
- For cycles 15-20
  - 5% changes in design variables were allowed
- For cycles > 20
  - 2% changes in design variables were allowed

The direct comparison of table 6.4.2 results is shown below:

Cycle	Linear Approximation, Haftka	Linear Approximation, Loughnane
Initial	2098	2098
1	1845	501.26
2	1637	1529.8
3	1601	1489.4
4	1558	1510.2
5	1531	1464
6	1514	1526.5
7	1507	1472.2
8	1502	1529.9
9	1500	1481.4
10	1500	1528.5
11	1500	1486.4
12	1499	1525.7

Only the first 12 cycles are shown, because that is all that appears in Haftka. However, it can easily be seen that the move limit schemes used in both approximations are very different. This is most apparent in the first cycle, where the Haftka and Loughnane approximations are much different.

However, despite the move limit schemes, ultimately both approximations do converge very close to the actual solution. After 100 cycles, the function approximations predicted a weight of 1509.5, with areas very similar to those given in Table 6.4.1 as the optimum design.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Actual Optimum Design	7.900	0.100	8.100	3.900	0.100	0.100	5.798	5.515	3.677	0.141
Linear Approximation	7.917	0.100	8.083	3.900	0.100	0.100	5.775	6.044	3.482	0.141

Given these results, perhaps allowing the move limits to go to 1% or less than 1% would likely have been a good idea for this problem. Again the convergence criterion for this problem was given by the percent change (< 1e-6 for 3 consecutive iterations) in the weight approximation of the structure, which again could have probably been less restrictive. Convergence to the tolerance defined was not achieved after 100 cycles using the move limits scheme presented, although playing with move limits and convergence criteria could easily results in faster convergence of the approximations.

Table 6.7.2/Table 6.7.4: Stress and Displacement Constraints

The optimization problem statement is given below

Minimize

$$W = \rho A L$$

Subject to

CASE A	**CASE B**
$\sigma_{1-10} \leq \pm 25\ ksi$	$\sigma_{1-10} \leq \pm 25\ ksi$
$-2 \leq v_1 \leq 2$	$v_1 = -2\ in$
$-2 \leq v_2 \leq 2$	$v_3 = -1\ in$
$-2 \leq v_3 \leq 2$	
$-2 \leq v_4 \leq 2$	

Side Bounds

$$0.1 \leq A_1, \dots, A_{10} \leq 40$$

Design Variables

$$A_1, \dots, A_{10}$$

For this problem, there are two cases investigated from Table 6.7.4, and one case from Table 6.7.2.

The stresses in every member are constrained to be ± 25 ksi. That is, member 9 does not have any special requirements. Additionally, for these designs, the starting design is taken to be x0 = [20 20 20 20 20 20 20 20 20 20]. Also, as a result of the initial design, it was chosen to nominally increase the side bounds on the upper limit of areas to 40.

The move limit scheme used for these problems is described as follows:

- For the first 8 cycles
  - 90% change in design variable was allowed, with a 10% decrease in allowable design changes each iteration
- For cycles 8-12
  - 10% changes in design variables were allowed
- For cycles 12-50
  - 5% changes in design variables were allowed
- For cycles 50-100
  - 1% changes in design variables were allowed

- For cycles > 100
    - 0.1% changes in design variables were allowed

First the results for the optimum areas of the fully stressed design are compared from Table 6.7.2. The fully stressed design was not subjected to displacement constraints, and so is the same problem as that solved for Table 6.4.1, aside from the starting design and the lack of the additional stress constraint in member 9. The solution shown below required 11 cycles for convergence, just as the initial design described by Table 6.4.1.

Member	Initial Area (in^2)	FSD and optimum areas (in^2), Haftka	FSD and optimum areas (in^2), Loughnane
1	20	7.940	7.938
2	20	0.100	0.100
3	20	8.060	8.062
4	20	3.940	3.938
5	20	0.100	0.100
6	20	0.100	0.100
7	20	5.740	5.745
8	20	5.570	5.569
9	20	5.570	5.569
10	20	0.100	0.100
Total Weight		1593.200	1593.181

The final two optimization problems considered added an additional degree of complexity by defining displacement constraints. Displacement constraints were approximated using the linear function approximation. However, since the A, B, Aeq, and Beq matrices typically used as input to MATLAB's fmincon function did not seem to work no matter which way the problems were formulated all constraints were included inequality constraints. This means that the equality constraints that were supposed to have been imposed for Case A were instead posed as inequality constraints. (i.e. instead of v1 = -1, v2 = -2, I used v1 ≥ -2and v2 ≥ -1) This had a significant effect on the results of the problem.

The results of the Case A, which included only the replacement inequality constraints for displacements at nodes 1 and 3 are shown below:

CASE A	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Weight
Optimum Design, Haftka	22.66	1.40	21.58	8.43	0.10	0.10	12.69	14.54	11.93	1.98	4048.96
Optimum Design, Loughnane	29.62	0.42	25.01	13.65	0.19	0.10	11.33	19.41	19.33	0.10	5037.77

In Case A convergence as defined above was achieved in 104 iterations, however the displacements were v1 = -2 and v3 = -0.77. Clearly this is not the same solution that would have been achieved using equality constraints. The plot of convergence is excluded here, as the wrong problem was solved anyways.

Case B results are shown below, including constraints for the displacements at nodes 1, 2, 3, and 4:

CASE B	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Weight
Optimum Design, Haftka	30.52	0.10	23.20	15.22	0.10	0.55	7.46	21.04	21.53	0.10	5060.85
Optimum Design, Loughnane	30.74	0.10	23.93	14.73	0.10	0.10	8.54	20.94	20.85	0.10	5076.67

Convergence to the tolerance defined (<1e-6) was not achieved after 200 cycles using the move limits scheme presented. However, convergence can easily be seen from the plot below. This implies that the convergence criteria defined was too strict.

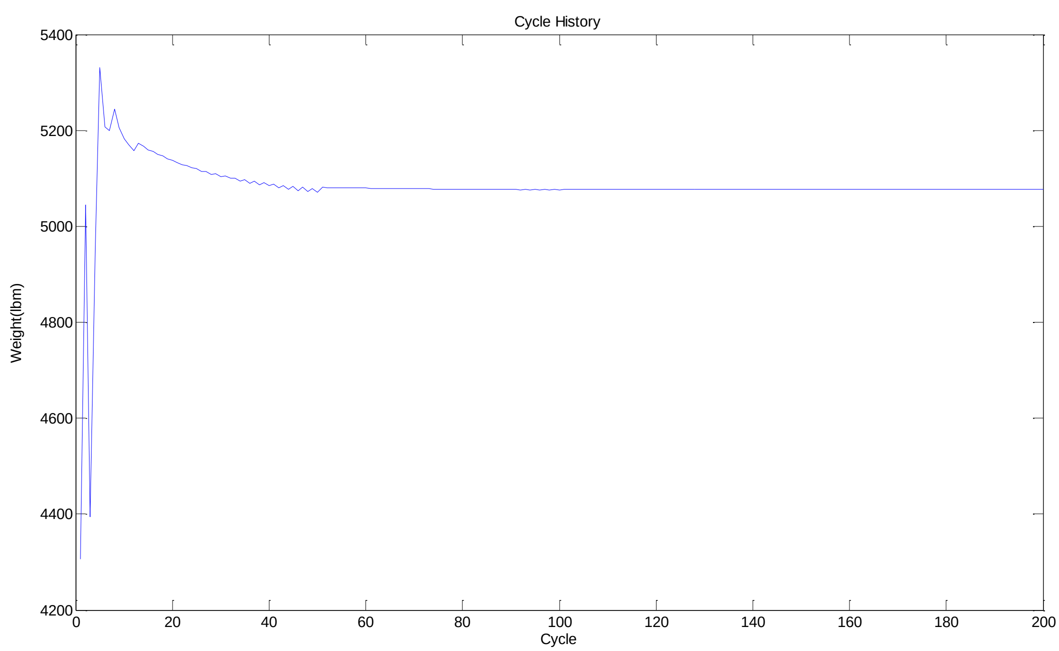


Table 6.7.2/Table 6.7.4: Stress and Displacement Constraints

Table 6.4.1 results were matched exactly.

Table 6.4.2 results were replicated; however they were highly dependent on move limits.

Table 6.7.2 results were replicated; however they were highly dependent on move limits.

Table 6.7.4 results were replicated exactly for Case B, and were unable to be replicated for Case A due to the complication of using equality constraints with MATLAB’s fmincon function.