ME 7060 Structural Reliability

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Final Project: Reliability Analysis of a Regulation Basketball Hoop

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**** This final project is worth 8 homework assignments ****

1 Introduction

Basketball backboards have historically been made of many different materials, including wood, aluminum, steel, acrylic, and tempered glass. Additionally, while professional and collegiate backboards are rectangular, outdoor backboards are often rectangular or fan-shaped. Originally, they were constructed of wood, but a decision was made to transition to glass backboards as basketball gained popularity and spectators behind the backboard had a hard time seeing the game. This was first done by the Indiana Hoosiers at Indiana University [0]. Since that time, basketball hoops have been notorious for having glass backboards that shatter when players slam dunk on them.

Currently, the National Collegiate Athletic Association (NCAA) and National Basketball Association (NBA) use so-called "unbreakable backboards" in which springs are built in to the rim and the design is such that the stress is almost entirely taken off the glass. Recently, Sports Science tested these unbreakable backboards with a current NBA star, and although the player was unable to break the glass, he was able to generate a maximum force of 1000 lbs during his 40 attempts [2]. However, on older regulation backboards, the force imparted on the rim by the player was not absorbed by springs, but rather transferred to the bracket that mounts the rim to the backboard, and thus to the backboard itself. My analysis in this report is concerned with modeling of this older-style backboard.

2 Problem Description

In this study, structural reliability analysis of an older-style regulation (e.g. National Collegiate Athletic Association (NCAA) or National Basketball Association (NBA)) basketball backboard is performed to investigate the probability of a catastrophic backboard-shattering failure.

The probability of failure is computed first using the Hasofer-Lind method with only normal distributions and this result is then compared to the Hasofer Lind-Rackwitz Fiessler (HL-RF) method with non-normal distributions. Additionally, the investigation probes different distribution ranges (using HL-RF) for two key parameters: the force on the rim and the thickness of the backboard. This is because it was difficult to define distributions for these variables, and it was of interest to discover how much of an effect these two distributions and their parameter definitions have on the convergence properties of β and thus p_f.

The backboard geometry was modeled in Abaqus/CAE (see Fig. 1) and Abaqus/Standard was used for stress solutions. Scripting for reliability analysis was done in MATLAB & Python scripting languages.

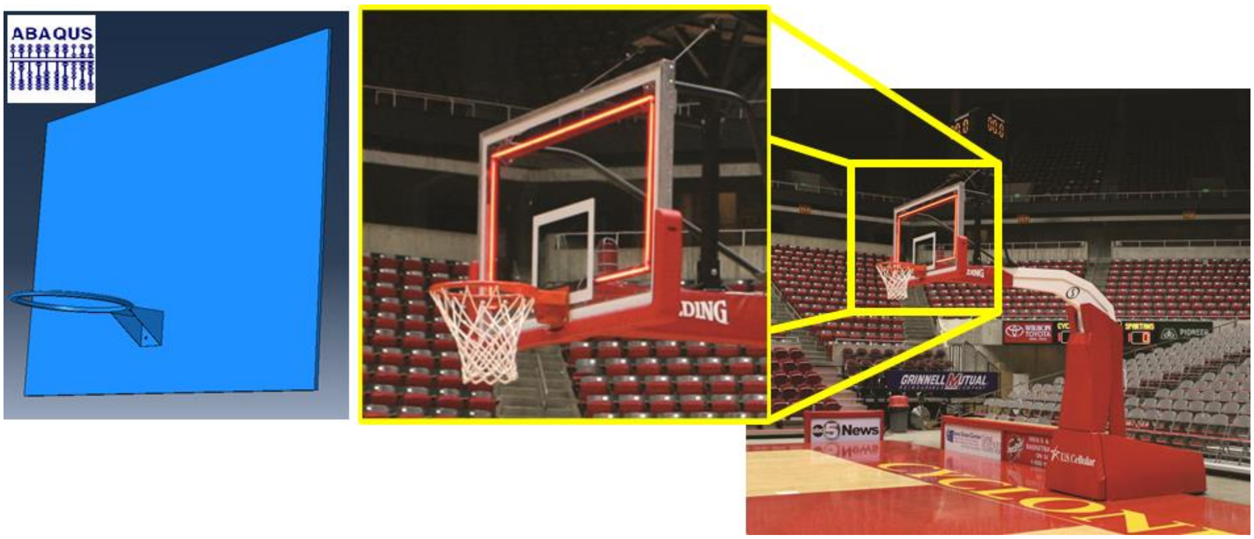

Figure 1: NCAA basketball Hoop (right) and associated model created in Abaqus/CAE (left)

2.1 Limit State Function

The limit state function associated with this problem is defined in terms of an allowable stress for the backboard, because presumably once the backboard feels stress above its' breaking strength, catastrophic failure will occur. This is due to the brittle nature of tempered glass; it will break rather than yield. It is well known that the strength of glass is dependent upon the cracks and micro-cracks present in the material [3], but the approximate tensile strength of any tempered glass can still be bounded as in Fig. 2 below. It can be seen from Fig. 2 that tempered glass has a breaking strength somewhere between 120 and 250 MPa (17.4 - 36.3 ksi).

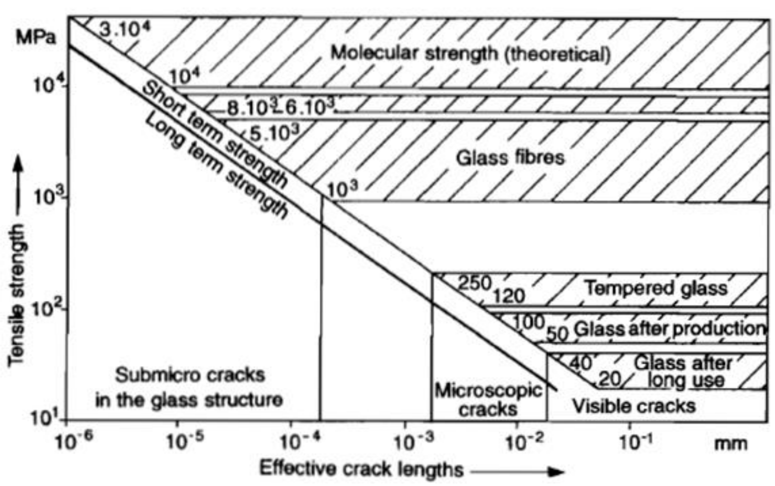

Figure 2: Plot showing the relationship between effective crack lengths and tensile strength for various types of glass, including tempered glass used for basketball backboards

Due to the wide range available for breaking strength of glass, and since this is a reliability analysis attempting to get a number that represents the "on-average" probability of failure of all old-style backboards, I chose to use a tensile strength of σ_{median} = 185 MPa = 26.8 ksi as my limit state. Therefore the failure condition and limit state are given by the following equations below, whereby if g(x) ≤ 0, a catastrophic backboard failure is assumed to occur.

$$\sigma_{backboard,max} \leq 26.8\ ksi$$

$$g(x) = 26.8\ -\ \sigma_{backboard,max}$$

It should be noted here that it will be the maximum Von Mises (equivalent tensile) stress in each case that is being compared to the median tensile strength of tempered glass.

2.2 Random Variables

2.2.1 Force on the rim

In the study done by Sports Science [2], in addition to the 1000 pound maximum loading they found, it was stated that the force required to break an old-style regulation backboard is known to be around 625 lbs. Additionally, the average NBA player weighs 221 pounds, and in one study it was computed that Dominque Wilkins' Windmill dunk generated 76.88 N = 17.28 pounds of force [4].

The fact of the matter is that the forces that a basketball rim experience can vary significantly. Thus, I chose a range of values to cover the feasible region of forces that could do some damage (while people are not really trying to, as in the study by Sports Science), each with equal probability.

$$F_{\text{slam dunk}} = F_{\text{basketball player}} \sim \text{Unif}\ (300{,}700)\ \text{lbs}$$

2.2.2 Backboard Geometry & Material Properties

The backboard geometry of a regulation backboard is fixed at 7' length and 4' height. There is no specific thickness requirement for the boards, however they are often about 0.5" thick, so part of my investigation is to allow the thickness to vary between 0.25 and 0.75" to see what effect this may have on the maximum stress.

$$\text{thickness}_{\text{backboard}} \sim \text{Unif}\ (0.25{,}0.75)\ \text{in}$$

Regulation backboards are constructed from tempered glass. Since such a large range of breaking strengths exist for tempered glass, I chose very large standard deviations for the material properties. Although perhaps slightly unreasonable large for these cases, I think that most estimates of material properties have much uncertainty that is not considered, and I'm interested for this investigation to see if this type of deviation has a significant effect, or if we could potentially make backboards out of very different materials reliably.

Additionally, since the mean value of Poisson's ratio is so small, it was found that using a typical (say, 10%) coefficient of variation to define the standard deviation occasionally results in zero stress derivatives. Therefore a large standard deviation was chosen to overcome this numerical issue.

$$E_{backboard} = E_{tempered\ glass} \sim N\ (10,1)\ \text{Mpsi}$$

$$\nu_{backboard} = \nu_{tempered\ glass} \sim N\ (0.22,0.1)$$

2.2.3 Rim Material Properties

Rims in regulation play are typically of a fixed diameter (5/8") and the inner diameter of the rim is located 6" from the front of the backboard. Additionally, the geometry of the rim is not assumed to greatly affect the stresses that the backboard feels, as it is likely more about the moment generated by loading on the rim, while the rim is assumed relatively rigid throughout the event where a force is applied.

However, it is of interest to see if the material that the rim is made out of has any effect on the reliability of a backboard too, so I've chosen to investigate the Elastic Modulus and Poisson's Ratio of the steel used for construction. Again, I'm assuming heavy variation for the material properties, in this case for the rim steel. A similar issue was found with the Poisson's Ratio for the steel, so again a large standard deviation was chosen.

$$E_{rim\ assembly} = E_{steel} \sim N\ (29,0.7225)\ \text{Mpsi}$$

$$\nu_{rim\ assembly} = \nu_{steel} \sim N\ (0.29,0.1)$$

3 Simulation Model

3.1 Model Assembly

The simulation model consists of 5 parts pieced together in a single assembly as shown in Fig. 3 below; the rim, the mounting plate, and two side brackets for the rim assembly.

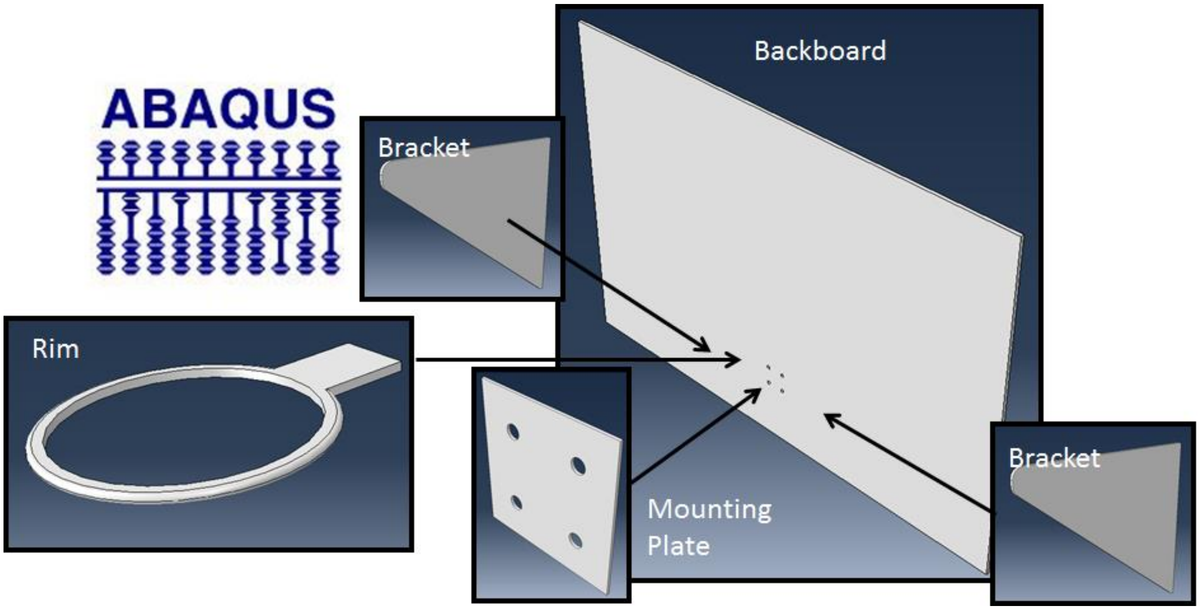

Figure 3: Assembly of basketball hoop build in Abaqus/CAE. Parts of the assembly include the rim, the backboard, the front mounting plate, and two side brackets.

The rim is of regulation diameter (5/8”) and distance from the backboard (6”). Additionally, the backboard is of regulation length (72”) and height (48”).

3.2 Structural Constraints

The next step in building the model was to define how each piece moves relative to one another. I chose to model the rim assembly with simple tie constraints between the rim, brackets, and front mounting plate. Additionally, I placed a tie constraint between the back of the front mounting plate and the backboard (see Fig. 4). However, this was insufficient to give me the output I was expecting to see.

The essential constraints that gave me what I was looking for were the rigid body tie constraints I placed in the holes through the mounting bracket and backboard (see Fig. 4). I defined a reference point in the center of each hole and told the backboard and the front mounting plate to move together as if a 4 bolts, washers, and nuts firmly fixed the rim to the backboard.

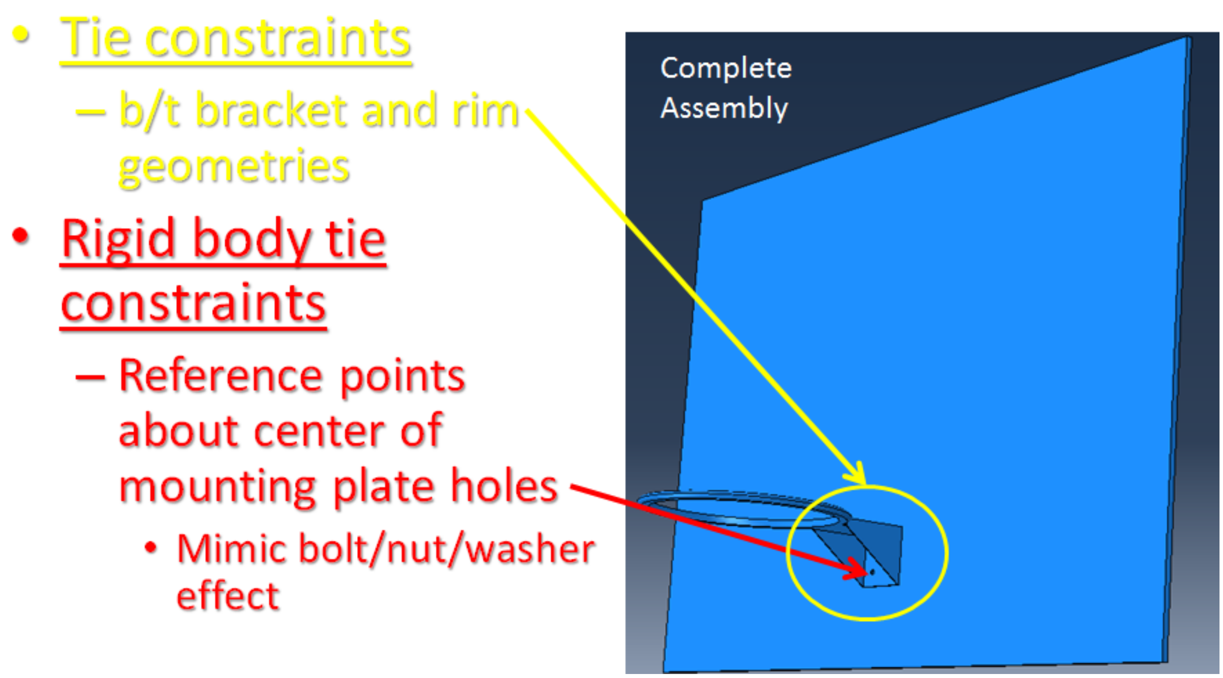

Figure 4: Outline of structural constraint types and locations used for this FE model.

3.3 Loads and Boundary Conditions

After the definition of structural constraints, I had to choose where to apply the load and how to apply boundary conditions to mimic a regulation backboard. I chose to apply the load in the center of the front of the rim (see Fig. 5), which is the decision that consequently also forced me into doing a full model rather than a half-symmetric model (about the center of the rim and backboard).

After playing with a variety of different boundary conditions for quite a while, I finally decided upon the fixed edges shown in Fig. 5 below.

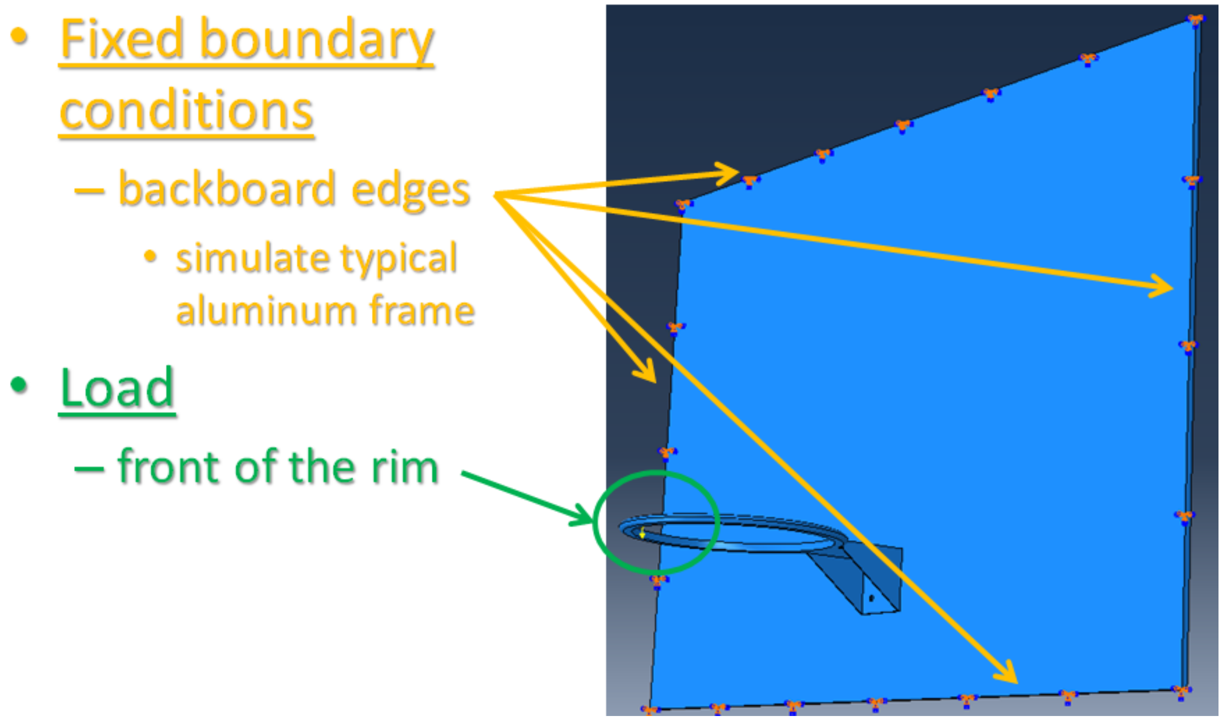

Figure 5: Outline of loads and boundary conditions used for this FE model.

3.4 Mesh Convergence

I played around with a variety of different ways to mesh the structure, but in the end I decided on a relatively fine mesh, using seed points given by a defined approximate global size of 0.5". When I chose to space seed points out more for a larger mesh on the backboard, I ended up with non-symmetric results every time. Thus, it turned out that the best way to get consistent results in this case was to keep the mesh sizes relatively consistent throughout the model. Seeding parts in this way resulted in the following number of elements in each simulation used for analysis:

- Rim ~ 801 elements
- Backboard ~ 19,526 elements
- Bracket (x2) ~ 91 elements each
- Front Mounting Plate ~ 814 elements

The final mesh can be seen in its' deformed state in Fig. 6, where it is clear that my analysis is mimicking the catastrophic backboard failures observed in reality. Note here that each simulation, with constraint, loads, boundary conditions, and mesh sizing as specified takes around 1 minute to complete. Additionally, writing and reading the stress file for each simulation takes about 5 seconds.

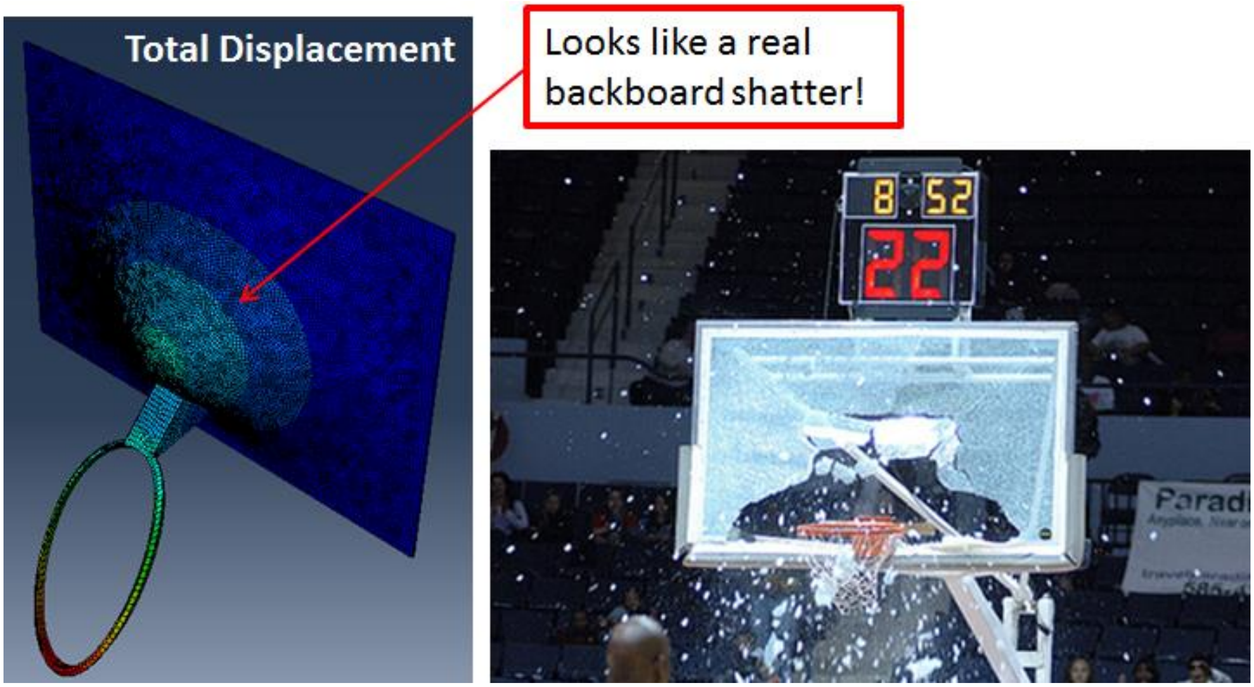

Figure 6: FE model results showing (as expected) how backboard failures are occurring. Note that this model shows the total displacement for visualization purposes, not the stress.

4 Simulation Results & Analysis

4.1 Limit-State Derivatives

After establishing the initial FE model, I had to converge on a step size for the max stress derivatives and make sure the signs of each derivative were making sense. Table 1 below outlines the study completed on Limit-State derivatives.

Table 1: Stress derivative convergence study

	Limit-State Derivatives					
Perturbation	t	F	Eb	vb	Es	vs
0.10%	4.41E+05	-8.30E+01	1.50E-04	0	-4.86E-05	-1.03E+03
0.20%	2.94E+05	-4.15E+01	1.55E-04	0	-5.20E-05	-1.20E+03
0.30%	1.96E+05	-5.53E+01	1.57E-04	2.67E+03	-5.32E-05	-7.99E+02
0.50%	1.47E+05	-4.98E+01	1.56E-04	1.60E+03	-5.48E-05	-4.79E+02
1%	1.13E+05	-4.15E+01	1.56E-04	-1.69E+02	-5.34E-05	-5.48E+02
2%	7.88E+04	-4.15E+01	1.56E-04	-2.19E+03	-5.34E-05	-4.79E+02
3%	5.25E+04	-4.15E+01	1.45E-04	-2.41E+03	-5.31E-05	-4.68E+02
5%	5.74E+04	-4.15E+01	1.37E-04	-2.86E+03	-5.25E-05	-4.59E+02
10%	4.84E+04	-4.15E+01	1.29E-04	-3.14E+03	-5.05E-05	-4.38E+02

As is shown highlighted in green, I chose to use a perturbation step of 1% for forward finite difference gradient calculations. After this point, some of the derivatives appear to diverge.

It is also important to consider the information contained in each of these derivatives. I will look at each parameter individually to conceptually think about if each number makes sense.

- **Thickness**:
  positive → limit-state increases → maximum stress decreases.
  A thicker backboard can handle more stress.
- **Force**:
  negative → limit-state decreases → maximum stress increases.
  A higher force tends toward higher stress.
- **Elastic Modulus (backboard)**:
  positive → limit-state increases → maximum stress decreases.
  A more elastic backboard can handle more stress.
- **Poisson's Ratio (backboard)**:
  negative → limit-state decreases → maximum stress increases.
  Less possible expansion of the backboard tends toward higher stress.
- **Elastic Modulus (rim)**:
  negative → limit-state decreases → maximum stress increases.
  Less stiffness in the rim makes it give more when load applied, so the backboard feels more stress.
- **Poisson's Ratio (rim)**:
  negative → limit-state decreases → maximum stress increases.
  Less possible expansion of rim leads to less force absorbed by the rim and more force transmitted to backboard, causing more stress.

4.2 Reliability Analysis

4.2.1 Hasofer-Lind (HL) Method

In my initial analysis, I chose to use all normally-distributed random variables to investigate a baseline probability of failure for my model. To do this, I re-defined the load and thickness variables as:

$$F_{\text{slam dunk}} = F_{\text{basketball player}} \sim \mathbf{N\ (500,100)\ lbs}\ (= \text{Unif}\ (375,625)\ \text{lbs},\ X_F = 500\ \text{lbs})$$

$$\text{thickness}_{\text{backboard}} \sim \mathbf{N(0.5,0.0798)\ in}\ (= \text{Unif}\ (0.4,0.6)\ \text{in},\ \ X_t = 0.5\ \text{in})$$

**Note that these decisions for normal random variables were based on equivalent normal means and standard deviations for the uniform distribution ranges and parameter values shown in parentheses above.

Using these inputs along with the other normally-distributed random variables defined earlier, converge results for the safety index β can be seen in Table 2 below.

Table 2: HL Method results

	Iteration #					
	1	2	3	4	5	6
Limit State	6071.9	1290	-64.5	6.8	10.1	1.3
dLimit/dt	112600	101854	109245.8	109086.2	109057.6	109143.5
dLimit/dF	-41.46	-46.1291	-46.5796	-46.8087	-46.772	-46.8064
dLimit/dEb	0.000156	0.000147	0.000156	0.000155	0.000155	0.000155
dLimit/dvb	-181.818	-3452.78	-3501.46	-3503.62	-3503.56	-3503.48
dLimit/Es	-5.3E-05	-5E-05	-5.2E-05	-5.2E-05	-5.2E-05	-5.2E-05
dLimit/dvs	-551.724	482.19	517.9308	517.9353	517.936	517.9372
beta	0.613488	0.748836	0.742072	0.742757	0.743778	0.743909
alpha t	-0.90787	-0.86898	-0.88132	-0.88007	-0.88017	-0.88018
alpha F	0.4189	0.493177	0.47089	0.473226	0.473034	0.473015
alpha Eb	-0.01576	-0.01573	-0.01579	-0.01569	-0.01569	-0.01568
alpha vb	0.001837	0.036914	0.035398	0.035421	0.035434	0.035405
alpha Es	0.003877	0.003889	0.003803	0.003803	0.003804	0.003802
alpha vs	0.005574	-0.00516	-0.00524	-0.00524	-0.00524	-0.00523
t	0.455554	0.455554	0.448072	0.447811	0.447837	0.447759
F	525.699	525.699	536.9309	534.9434	535.1492	535.1832
Eb	9990330	9990330	9988220	9988283	9988347	9988327
vb	0.220113	0.220113	0.222764	0.222627	0.222631	0.222635
Es	29001718	29001718	29002104	29002039	29002041	29002044
vs	0.290342	0.290342	0.289614	0.289611	0.289611	0.28961
u t	-0.55697	-0.55697	-0.65072	-0.654	-0.65367	-0.65465
u F	0.25699	0.25699	0.369309	0.349434	0.351492	0.351832
u Eb	-0.00967	-0.00967	-0.01178	-0.01172	-0.01165	-0.01167
u vb	0.001127	0.001127	0.027643	0.026268	0.026309	0.026355
u Es	0.002378	0.002378	0.002912	0.002822	0.002825	0.00283
u vs	0.00342	0.00342	-0.00386	-0.00389	-0.00389	-0.0039
beta error	0	0.220621	0.009033	0.000923	0.001375	0.000177

In the cases shown in this report, since there are more than 3 design variables, we cannot picture the iterations through normal space. The relative error ε converges to below 0.001 after 5 iterations, and thus using the final converged β value a probability of failure can be computed as:

$$p_f = \Phi(-\beta) = .22847 = 22.85\%$$

This probability of failure is very high, however it should again be noted that I chose 500 lbs as the initial design point for the force, and it is claimed that these backboards will shatter at 625 lbs [2].

4.2.1 Hasofer Lind-Rackwitz-Fiessler(HL) Method

In the following 4 cases, I will show that convergence for a wide range of variability in the system is achieved in a relatively small number of iterations. Due to this, I decided that there would be limited benefit from introducing adaptive approximations during iterations.

4.2.1.1 Case I: F ~ Unif (375 , 625) lbs , t ~ Unif (0.4 , 0.6)

In this analysis, I replaced the normally-distributed force and thickness with the uniformly-distributed variables I used to create the equivalent normal distributions in the previous analysis.

This time, equivalent normal means and standard deviations are computed at each point within the loop after the first iteration, and from the table below this can be seen to converge in only 5 iterations, rather than 6.

Table 3: HL-RF Method results, Case I, F ~ Unif (375,625) lbs, t ~ Unif (0.4,0.6)

	Iteration #				
	1	2	3	4	5
Limit State	6071.9	1286.9	188.1	41.4	2.9
dLimit/dt	112600	101884.7	107015.8	108302.5	108412.8
dLimit/dF	-41.46	-46.1384	-45.4912	-46.0362	-45.927
dLimit/dEb	0.000156	0.000147	0.000158	0.000145	0.000159
dLimit/dvb	-181.818	-3452.77	-3538.88	-3490.95	-3491.19
dLimit/Es	-5.3E-05	-5E-05	-5.2E-05	-5.2E-05	-5.2E-05
dLimit/dvs	-551.724	482.1893	518.0483	518.1397	518.1483
beta	0.613846	0.764475	0.787086	0.792135	0.792486
alpha t	-0.90827	-0.837	-0.85758	-0.85071	-0.8528
alpha F	0.418036	0.545218	0.51207	0.523496	0.520008
alpha Eb	-0.01577	-0.01803	-0.01951	-0.01796	-0.0197
alpha vb	0.001838	0.042311	0.043639	0.043171	0.043193
alpha Es	0.003879	0.004457	0.004669	0.004683	0.004685
alpha vs	0.005578	-0.00591	-0.00639	-0.00641	-0.00641
t	0.455515	0.455515	0.452176	0.449944	0.450039
F	525.5931	525.5931	540.7634	539.1414	540.2055
Eb	9990319	9990319	9986216	9984644	9985774
vb	0.220113	0.220113	0.223235	0.223435	0.22342
Es	29001720	29001720	29002462	29002655	29002680
vs	0.290342	0.290342	0.289548	0.289497	0.289492
u t	-0.55754	-0.55754	-0.63986	-0.67499	-0.67388
u F	0.25661	0.25661	0.416806	0.403043	0.41468
u Eb	-0.00968	-0.00968	-0.01378	-0.01536	-0.01423
u vb	0.001128	0.001128	0.032345	0.034347	0.034197
u Es	0.002381	0.002381	0.003407	0.003675	0.003709
u vs	0.003424	0.003424	-0.00452	-0.00503	-0.00508
beta error	0	0.245386	0.029578	0.006415	0.000443

Using the final converged β value a probability of failure can be computed as:

$$p_f = \Phi(-\beta) = .21404 = 21.4\%$$

4.2.1.2 <u>Case II:</u> F ~ Unif (375 , 625) lbs , t ~ Unif (0.25 , 0.75)

In this investigation, I wanted to span the entire range of thicknesses I defined originally, while keeping all other random variables the same from Case I. Again, per the HL-RF method, equivalent normal means and standard deviations are computed at each point within the loop after the first iteration.

Table 4: HL-RF Method results, Case II, F ~ Unif (375,625) lbs, t ~ Unif (0.25,0.75)

	Iteration #			
	1	2	3	4
Limit State	6071.9	1525.9	-9.1	2.4
dLimit/dt	112600	102914.8	112736.9	112620.2
dLimit/dF	-41.46	-49.5839	-51.9292	-51.9323
dLimit/dEb	0.000156	0.000146	0.000152	0.000151
dLimit/dvb	-181.818	-3363.31	-3445.78	-3446.18
dLimit/Es	-5.3E-05	-4.9E-05	-5.1E-05	-5.1E-05
dLimit/dvs	-551.724	482.6517	482.8924	482.8832
beta	0.265862	0.339994	0.339571	0.339681
alpha t	-0.98344	-0.97017	-0.9717	-0.97164
alpha F	0.181055	0.241752	0.235557	0.235804
alpha Eb	-0.00683	-0.00715	-0.00694	-0.0069
alpha vb	0.000796	0.016461	0.015725	0.01574
alpha Es	0.00168	0.001744	0.001683	0.001695
alpha vs	0.002416	-0.00236	-0.0022	-0.00221
t	0.447846	0.447846	0.435261	0.435357
F	504.8008	504.8008	508.1919	507.9692
Eb	9998184	9998184	9997570	9997644
vb	0.220021	0.220021	0.22056	0.220534
Es	29000323	29000323	29000428	29000413
vs	0.290064	0.290064	0.28992	0.289925
u t	-0.26146	-0.26146	-0.32985	-0.32996
u F	0.048136	0.048136	0.082194	0.079988
u Eb	-0.00182	-0.00182	-0.00243	-0.00236
u vb	0.000212	0.000212	0.005597	0.00534
u Es	0.000447	0.000447	0.000593	0.000571
u vs	0.000642	0.000642	-0.0008	-0.00075
beta error	0	0.278836	0.001242	0.000323

Using the final converged β value a probability of failure can be computed as:

$$p_f = \Phi(-\beta) = .3673 = 36.73\%$$

This probability of failure is much higher (as expected) due to the extremely wide range of thicknesses available. A 0.25" thick is going to break a lot sooner than one that is 0.4" thick!

4.2.1.3 <u>Case III:</u> F ~ Unif (300 ,700) lbs , t ~ Unif (0.4 , 0.6)

In Case III, rather than increasing the range of the thickness I wanted to increase the range of the load, so I chose to extend it 75 lbs either way. Thus, we can compare Case III to Case I to investigate the effect of the range of forces chosen.

Table 5: HL-RF Method results, Case III, F ~ Unif (300,700) lbs, t ~ Unif (0.4,0.6)

	Iteration #						
	1	2	3	4	5	6	7
Limit State	6071.9	1073.1	219.9	20.4	18	-9.6	2.1
dLimit/dt	112600	100076.4	102973.2	104684.4	104696.4	104711.4	104601
dLimit/dF	-41.46	-50.8084	-48.743	-49.6019	-49.4248	-49.4935	-49.4735
dLimit/dEb	0.000156	0.000148	0.000154	0.000157	0.000157	0.000157	0.000157
dLimit/dvb	-181.818	-3544.03	-3646.34	-3598.08	-3598.86	-3643.66	-3643.25
dLimit/Es	-5.3E-05	-5.2E-05	-5.3E-05	-5.3E-05	-5.3E-05	-5.3E-05	-5.3E-05
dLimit/dvs	-551.724	516.762	517.7987	552.3573	552.3946	552.3977	552.3967
beta	0.54414	0.636605	0.656691	0.658588	0.660324	0.659392	0.659596
alpha t	-0.80513	-0.68367	-0.73061	-0.72012	-0.7237	-0.72247	-0.72252
alpha F	0.592905	0.728859	0.681672	0.692782	0.689031	0.690304	0.690249
alpha Eb	-0.01398	-0.01405	-0.01507	-0.01521	-0.01524	-0.01523	-0.01524
alpha vb	0.001629	0.033625	0.035647	0.034812	0.03489	0.035319	0.03533
alpha Es	0.003438	0.003545	0.003775	0.003736	0.003743	0.003743	0.003745
alpha vs	0.004944	-0.0049	-0.00506	-0.00534	-0.00536	-0.00535	-0.00536
t	0.465045	0.465045	0.466335	0.463106	0.463531	0.463274	0.46338
F	551.4832	551.4832	571.9799	569.1317	570.3614	570.1758	570.2042
Eb	9992393	9992393	9991054	9990105	9989986	9989939	9989955
vb	0.220089	0.220089	0.222141	0.222341	0.222293	0.222304	0.222329
Es	29001352	29001352	29001631	29001791	29001778	29001786	29001783
vs	0.290269	0.290269	0.289688	0.289668	0.289648	0.289646	0.289647
u t	-0.4381	-0.4381	-0.43523	-0.47978	-0.47426	-0.47788	-0.47639
u F	0.322623	0.322623	0.463995	0.447648	0.456258	0.454984	0.455181
u Eb	-0.00761	-0.00761	-0.00895	-0.0099	-0.01001	-0.01006	-0.01005
u vb	0.000887	0.000887	0.021406	0.023409	0.022927	0.023038	0.023289
u Es	0.001871	0.001871	0.002257	0.002479	0.00246	0.002472	0.002468
u vs	0.00269	0.00269	-0.00312	-0.00332	-0.00352	-0.00354	-0.00353
beta error	0	0.169929	0.031552	0.002888	0.002636	0.001411	0.000309

Using the final converged β value a probability of failure can be computed as:

$$p_f = \Phi(-\beta) = .2547 = 24.47\%$$

This probability of failure is higher in this case, but not as much higher as when I extended the thickness range. This shows that indeed both variables have a significant effect on the probability of failure, and in the ranges investigated the variability in thickness does much more damage than variability in the force applied. However, this is a positive thing, because in reality it is much easier to control the thickness of the backboard than how hard a player is slamming the basketball through the hoop.

4.2.1.3 <u>Case III:</u> F ~ Unif (300 ,700) lbs , t ~ Unif (0.25 , 0.75)

In Case IV, I wanted to see how much the probability of failure increased when I combined both effects; of increasing the range of thickness and increasing the range of force.

Table 5: HL-RF Method results, Case IV, F ~ Unif (300,700) lbs, t ~ Unif (0.25,0.75)

	Iteration #				
	1	2	3	4	5
Limit State	6071.9	1441.2	-19.2	26.1	4.7
dLimit/dt	112600	102446	111329.7	111195.3	111377.6
dLimit/dF	-41.46	-48.408	-49.8032	-49.8777	-49.9243
dLimit/dEb	0.000156	0.000148	0.000154	0.000154	0.000153
dLimit/dvb	-181.818	-3408.78	-3446.35	-3446.78	-3492.1
dLimit/Es	-5.3E-05	-5E-05	-5.1E-05	-5.1E-05	-5.2E-05
dLimit/dvs	-551.724	482.6569	517.3736	517.3741	482.883
beta	0.259314	0.326178	0.325294	0.326448	0.326656
alpha t	-0.95922	-0.93179	-0.93705	-0.93666	-0.93671
alpha F	0.282554	0.362562	0.348791	0.349841	0.349687
alpha Eb	-0.00666	-0.00697	-0.00681	-0.00681	-0.00676
alpha vb	0.000776	0.016042	0.015231	0.015248	0.015428
alpha Es	0.001639	0.001688	0.001641	0.001642	0.001651
alpha vs	0.002356	-0.00227	-0.00229	-0.00229	-0.00213
t	0.450384	0.450384	0.440224	0.440127	0.439945
F	511.6922	511.6922	518.8418	518.067	518.185
Eb	9998272	9998272	9997728	9997786	9997776
vb	0.22002	0.22002	0.220523	0.220495	0.220498
Es	29000307	29000307	29000398	29000386	29000387
vs	0.290061	0.290061	0.289926	0.289926	0.289925
u t	-0.24874	-0.24874	-0.30393	-0.30482	-0.30577
u F	0.07327	0.07327	0.11826	0.11346	0.114205
u Eb	-0.00761	-0.00761	-0.00895	-0.0099	-0.01001
u vb	0.000201	0.000201	0.005233	0.004955	0.004978
u Es	0.000425	0.000425	0.000551	0.000534	0.000536
u vs	0.000611	0.000611	-0.00074	-0.00074	-0.00075
beta error	0	0.25785	0.00271	0.003549	0.000636

Using the final converged β value a probability of failure can be computed as:

$$p_f = \Phi(-\beta) = .3719 = 37.19\%$$

This is the highest probability of failure computed yet, although if I combine the information gathered from Cases II, III, and IV, I can say that it is likely the thickness range does indeed have a much more significant effect on the probability of failure, because the converged β value for Cases II and IV are very close to one another, despite the addition of the force variability.

5 Summary

This report has demonstrated that the reliability of a regulation NCAA/NBA backboard against a catastrophic shattering failure is highly dependent upon what aspects of the system you choose to give variation, and additionally on the amount of variation given. Specifically, each case investigated provided the following results:

- **HL Method** (Normal R.V.s) $p_f = 22.85\%$
- **HL-RF Method Case I** ((Combination Normal/Non R.V.s) $p_f = 21.4\%$
- **HL-RF Method Case II** (wider thickness range) $p_f = 36.73\%$
- **HL-RF Method Case III** (wider force range) $p_f = 24.47\%$
- **HL-RF Method Case IV** (wider thickness & force ranges) $p_f = 37.19\%$

As stated previously, since fast convergence was observed for all the HL and HL-RF algorithms implemented here, adaptive approximations were altogether unnecessary in this project.

In closing I set out on this project aiming to investigate what the chances are that an old-style regulation backboard would shatter. In my estimation, the only answer I can really provide in the end is that the chances depend on your model assumptions; namely what kinds of forces are being applied and what things about the basketball hoop actually contain variability.

Based on the results shown above and how extremely high the computed failure probabilities are, and considering that backboards have never shattered roughly 25% of the time, if I were to do this analysis over I would likely fix the random material properties and make the thickness a deterministic value of 0.5”. I think this would provide the most appropriate “real-world” analog, however the issue of how much force is applied and with what variability is highly, highly speculative and would remain a problematic issue.

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