CSE 2321: Homework 2

Due Tuesday, May 22. You may turn in the homework during class or in the envelope outside my office (Caldwell 411) before class.

Before completing the homework, carefully read the homework directions posted on Piazza in Resources > Homework.

1. Determine if each of the following propositions is true or false. You do not need to justify your answer; simply write the word True or False. (3 pts each)

   (a) $\exists x \in \mathbb{R}, x^2 + 9 = 0$

   (b) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{N}), x < y$

   (c) $\emptyset \subseteq \emptyset$

   (d) $(\exists x \in \mathbb{N}), ((x < -42) \rightarrow (x^2 < 0))$

2. Recall that, using equivalence relations, the negation of $\forall x, p(x) \rightarrow q(x)$ can be expressed as $\exists x, p(x) \wedge \neg q(x)$.

   In a similar manner, express the negation of each of the following propositions so that all negation signs that exist are directly on the predicates (e.g. $\neg p(x)$). Your solutions should not have any double negations. Not all predicates necessarily need to be negated. You do not need to show any work for this question. (4 pts each.)

   (a) $\forall x, p(x) \wedge \neg q(x)$

   (b) $\exists x \forall y, q(x) \rightarrow p(y)$

   (c) $\exists x, (q(x) \vee r(x)) \rightarrow p(x)$

3.
   - Let $D(x)$ be the predicate "x is a dragon."
   - Let $F(x)$ be the predicate "x breathes fire."

   Translate the following English statements into predicate logic using universal ($\forall$) and existential ($\exists$) quantifiers. Do not use any quantifiers that we have not discussed in class. (4 pts each)

   (a) There are no dragons.

   (b) There are at least two dragons.

   (c) There is at most one dragon.

   (d) There is a fire-breathing dragon.

   (e) All dragons breathe fire.

4. Determine if each statement below is true or false. If the statement is true, simply write the word *True* for your answer; no other justification is needed. If the statement is false, you should write the word *False* and **also give a counter-example to the statement to justify your answer**.

   For example, if the statement is "For all sets A and B, $A \subseteq A \cap B$", a correct answer would be:

   False. If $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \cap B = \{2\}$, and $A \not\subseteq B$.

   Assume in all cases that that the domain of the given sets is $\mathbb{N}$. In other words, A, B and C are subsets of the natural numbers. (4 pts each)

   (a) For all sets A and B, $B \subseteq (A \cup B)$.

   (b) For all sets A and B, $(A \cup B) \subseteq A$.

   (c) For all sets A and B, $(A \cup B) - B = A$.

   (d) For all sets A and B, $[A - (B - A) = A]$.

   (e) For all sets A, B and C, if $A \neq B$ and $B \neq C$, then $A \neq C$.

5. Let $A = \{a, b, c, d\}$ and $B = \{d, e, f, g, h, i\}$. $P(S)$ denotes the power set of S. (4 pts each)

   (a) What is $P(A) \cap P(B)$?

   (b) What is $|P(A \cup B)|$?

   (c) What is $|P(A) \cup P(B)|$?

   (d) What is $|P(P(A \times B))|$?

6. Consider the set S = {1, 2, {1}, {1, 2}}. Determine if each statement below is True or False. No justification is needed; just write the word True or False for your answer. (3 pts each)

   (a) $1 \in S$

   (b) $1 \subseteq S$

   (c) $\{1, 2\} \in S$

   (d) $\{1, 2\} \subseteq S$

   (e) $\{\{1, 2\}\} \in S$

   (f) $\{\{1, 2\}\} \subseteq S$