Random-Parameter Gravity Models and Transit Ridership: TRB data analysis competition

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Abstract

Transfers are an integral element of urban transit systems because they enable increased network coverage. Despite their necessity, passengers often dislike transfers due to various factors such as increased travel time, decreased reliability, and lack of infrastructure at transfer facilities, among others. In this paper, we aim to estimate the transfer volume at the hub in a small radial transit network, and we take intoaccount these various factors to better understand their potential impacts on transfer behavior. Our overall approach is based on the well-known gravity model from the urban transportation literature. The variables used in this analysis are grounded in literature pertaining to transit network design, transfer penalties, and friction factors in public transportation. These factors are analyzed in a novel approach that uses random parameters. The results of the analysis show a range of transfer volumes from XX to YY, depending on the random conditions.

Keywords: TRB data analysis competition, trip distribution, gravity model, transfer penalty, public transit

1. Introduction

The purpose of this project is to estimate the transfer volumes in a given transit network, shown in Figure # and Table #.

The approach uses an adapted gravity model, using randomly generated parameters. The gravity model is not the only alternative for trip distribution; alternatives include the entropy model (Bierlaire, ????). The gravity model was chosen for this analysis because it is simple and most commonly used.

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2. Methodology

2.1. Adapted Gravity Model

The gravity model has been in existence for over 100 years, and it is one of the most common models used to model trip distributions used in urban transportation planning (Meyer and Miller, 2000). The general gravity model has the following formula:

$$T_{ij} = \frac{P_i[A_j f_{ij} k_{ij}]}{\sum_{n=1}^{m} A_j f_{ij} k_{ij}} \tag{1}$$

where T_{ij} are the estimated trips between discrete zones i and j, P_i are the trips produced at i, A_j are the trips attracted to zone j, f is a function of the travel disutility between i and j, and k_{ij} is a *post-hoc* adjustment factor.

Based on the literature pertaining to trip distribution and public transportation origin destination estimation, the f function in the basic gravity model depends on travel time and a friction factor. Therefore, an adapted gravity model is formulated for this analysis, as follows:

$$T_{ij} = \frac{P_i A_j t_{ij}^{-b}}{\sum_{j=1}^{N} A_j t_{ij}^{-b}} \tag{2}$$

In the adapted model, f has been replaced by the travel time, t between i and j, with an exponential friction factor b. Placing the friction factor as a negative exponent ensures that the marginal utility of travel decreases non-linearly as travel time increases. In the adapted model, the adjustment factor k is replaced with an iterative process that incrementally balances the model's predicted attractions with the starting values.

Since the friction factor and the travel time between stations are not known for the given system, we generate them using assumptions based in the trip distribution literature, as discussed in the following sections.

2.2. Friction Factor

One of the variables in the gravity model is the friction factor, which can assume various functional forms. This analysis uses an inverse costs function, which pertains to the cost of travel between zones (or in this case, between stations) (Meyer and Miller, 2000). In practice, a constant friction factor, b is empirically calculated for a given transit system based on characteristics of the network, and the travelers. For this analysis, a value of $b = 1$ is assumed, because no information is available to estimate another value.

2.3. Travel Time Function

The travel time is an aggregate function of several parameters including the distance between stations, the speed of the rail system, the potential transfer wait time, and the perceived disutility of that wait time as shown in the following equation:

[*insertequation*]

We create these parameters randomly, based on assumptions derived from real-world empirics and discussed below. Probability density functions of our assumed distributions are given in Figure 1.

2.4. Distance between Stations

Vuchic (2005) ranks the average stop spacing of urban metro systems in the world. Of these, Athens has the shortest average stop spacing of 595m (0.37 miles) and Mexico City has the longest of 1222m (0.76 miles) (**?**). The arithmetic mean of average stop spacing in urban networks is 0.594 miles. It should be noted that Vuchic (2005) also lists the average stop spacing for regional rail systems, which can range up to 1770m (1.10 miles) for St. Petersburg.

For this analysis, a random station spacing is generated based on the empirical information collected by Vuchic (2005). This analysis assumes that stations are spaced according to a lognormal distribution with a mode of 0.6 miles, a mean of 0 and a variance of 1.

2.4.1. Travel Speed

Speeds are normally distributed with a mean of 21.4461 and a standard deviation of 5.2957 miles per hour. These assumptions ar built on values published in the National Transit Database for heavy rail systems (**?**).

2.5. Transfer Time

Transfer time depends on the timing of train arrivals on intersecting rail lines, which is in turn dependent upon train headways. Rail networks with short headways (less than 10 minutes) do not tend to coordinate arrival times on intersecting lines, since transfer time will always be less than 10 minutes. Rail networks with longer headways (greater than 10 minutes) tend to coordinate their train arrival times in order to create convenient transfer times. (**?**) Therefore, this study assumes that transfer times vary between 0 and 10 minutes according to a truncated normal distribution, with a mean of 5 minutes and a standard deviation of 2.

2.6. Transfer Time Multiplier

Transit passengers perceive out-of-vehicle time differently, more onerous, than in-vehicle time. Reasons for this include out-of-vehicle transfers, at grade transfer, being exposed to the elements, and uncertainty, among others (**?**). According to the 2nd edition of Transit Capacity Quality Service Manual (TCQSM), passengers perceive one minute of wait time during transfers to be 2.5 times more onerous than one minute of in-vehicle travel time, on average, for work trips. The range for this transfer time multiplier is from 1.1 to 4.4 (**?**).

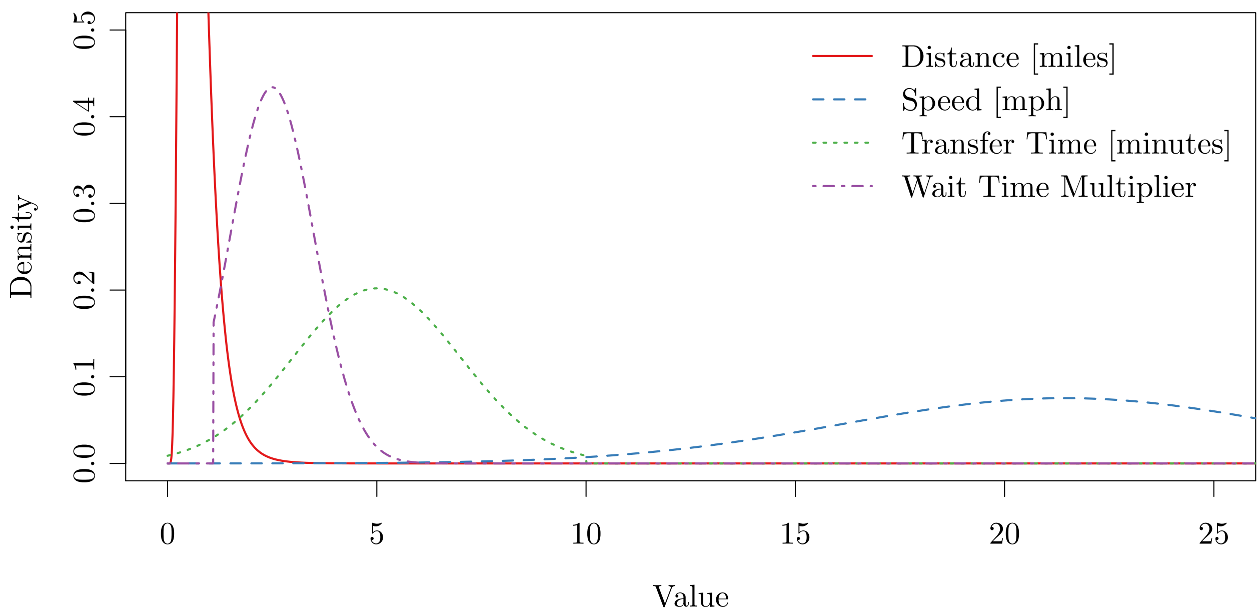

Figure 1: Probability density functions of random parameters used in the analysis.

This analysis assumes a randomly distributed transfer time multiplier based on the details provided by the TCQSM: we randomly draw from a normal distribution with a mean of 2.5 and a standard deviation of 1.

2.6.1. Time Multiplier

3. Results

Given the distributions for the three variables discussed above, they were then combined into a gravity model and implemented in the open source software R. XXX draws were conducted for each probabilistic variable.

The final product of our analysis the shown in Figure 2 and Table XXX. As can be seen below, XX is the most likely number of transfers.

4. Discussion

We presented a probabilistic approach to determining the number of transfers from boardings and alightings in a given network. This approach is grounded in values found in practice for transfer penalty, station spacing, and

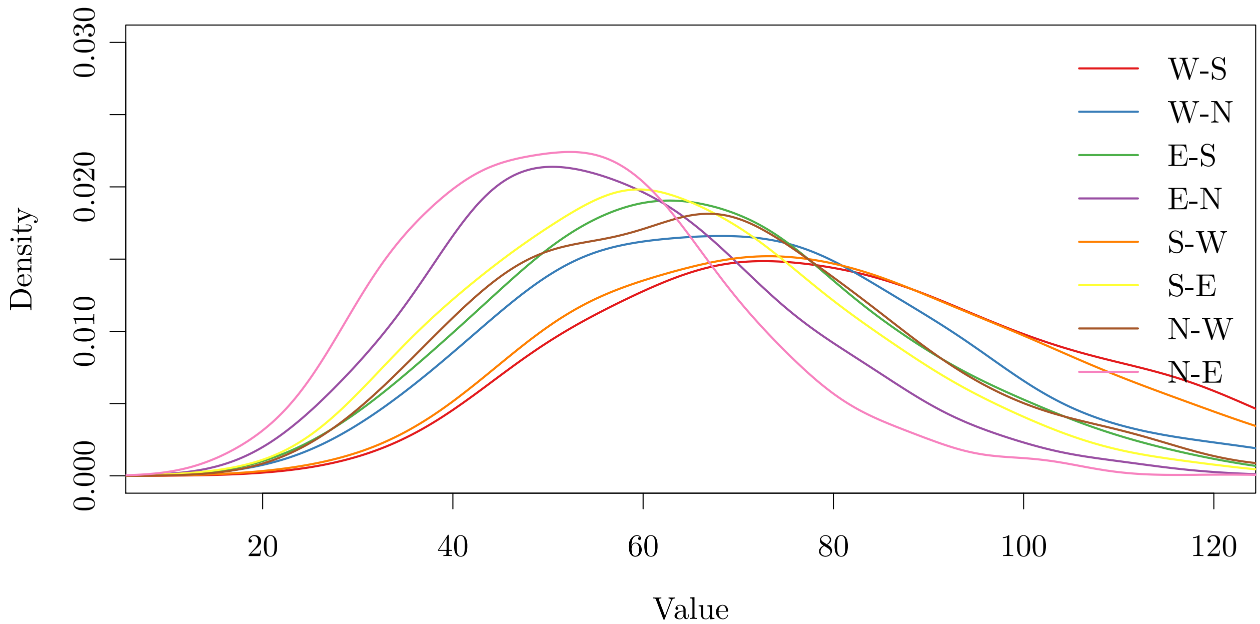

Figure 2: Probability densities of predicted transfer volume.

Table 1: Transfers by Direction

	W-S	W-N	E-S	E-N	S-W	S-E	N-W	N-E
Minimum	25.04	20.56	18.70	14.55	22.22	16.90	22.60	14.70
Maximum	183.21	163.87	150.70	121.34	181.54	148.44	144.95	121.51
Std. Dev.	25.78	22.60	20.36	18.18	25.45	19.63	21.26	16.65
Mean	82.41	70.86	66.37	57.75	80.10	63.26	66.87	52.59
Most Likely	72.60	68.30	62.80	50.49	73.02	59.70	67.08	52.14

A word on execution

This project was executed as a training exercise on literate programming using R (R Development Core Team, 2011), `knitr` (Xie, 2012), and LaTeX. The source code is available on GitHub as the `GT_TranspoComp` project.

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