(1) $H = \frac{1}{2} m L^2 \omega^2 + \frac{1}{2} m g L \theta^2 + \cancel{\text{const}}$

who cares.

The forces on the pendulum are

$$\ddot{\theta} = -\frac{g}{L}\theta \qquad (q = \theta,\ p = mL^2\omega)$$

The Euler method ~~for~~ solves this via

$$\begin{pmatrix} \theta_{n+1} \\ \omega_{n+1} \end{pmatrix} = \begin{pmatrix} \theta_n \\ \omega_n \end{pmatrix} + \Delta t \begin{pmatrix} \omega_n \\ -\frac{g}{L}\theta_n \end{pmatrix}.$$

$$\frac{1}{2} m L^2 \omega_{n+1}^2 = \frac{1}{2} m L^2 \left(\omega_n - \frac{\Delta t g}{L}\theta_n\right)^2$$

$$= \frac{1}{2} m L^2 \omega_n^2 - \Delta t\, m g L\, \theta_n \omega_n + \frac{\Delta t^2}{2} m g^2 \theta_n^2$$

$$\frac{1}{2} m g L \theta_{n+1}^2 = \frac{1}{2} m g L \left(\theta_n - \Delta t\, \omega_n\right)^2$$

$$= \frac{1}{2} mgL\,\theta_n^2 + \Delta t\, mgL\,\theta_n\,\omega_n + \frac{1}{2}\Delta t^2\, mgL\,\omega_n^2$$

So $$E_{n+1} = E_n + \frac{\Delta t^2}{2}\frac{g}{L}\left(mL^2\omega_n^2 + mgL\theta_n^2\right)$$

$$= E_n \underbrace{\left(1 + \frac{\Delta t^2 g}{L}\right)}_{\text{positive \#.}}$$

Note this implies

$$E_n = E_0 \cdot \left(1 + \frac{\Delta t\, g}{L}\right)^n,$$

monotonic exp. growth.

(2) (a) $$\frac{d^2 x}{dt^2} = \frac{d^2 x}{d(\tau/\omega)^2} = \omega^2 \frac{d^2 x}{d\tau}.$$

Then $$\frac{d^2 x}{dt^2} = -\omega^2 x \quad \text{implies}$$

$$\frac{dx}{d\tau} = -x,$$

Init. velocity is $\frac{dx}{dt} = \omega \frac{dx}{d\tau}$,

so in our dimensionless units, the init vel is v_0/ω.

2(b) The general trends are that g_2 increases faster than g_1, roughly linearly on log-log axes. This means the error is a power law in Δt. If we had plotted Δt^{-2} & Δt^{-1}, we could see the agreement.

(3) a with $y = x - ct$,

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial y}(-c)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$$

so

$$-cu' + 6uu' + u''' = 0.$$

(b) If $y = z/\sqrt{c}$, $\frac{\partial}{\partial y} = \sqrt{c}\,\frac{\partial}{\partial z}$, so,

If $v = cu$, then

$$-c\frac{\partial u}{\partial y} = -c^{5/2}\frac{\partial v}{\partial z}$$

$$6u\cdot\frac{\partial u}{\partial y} = 6c^{5/2}\,v\frac{\partial v}{\partial z}$$

$$\frac{\partial^3 u}{\partial y^3} = c^{5/2} \frac{\partial^3 v}{\partial z^3},$$

Divide through by $c^{5/2}$ to find

$$-v' + 6vv' + v''' = 0.$$

(c) There is good agreement.

(d) repeated Peaks are observed. Their location depends on the integration timestep.

Because the solutions are time invariant,

$$w(z) = \frac{1}{2} \operatorname{sech}^2 \left(\frac{1}{2} t + a \right)$$

is a solution for all. a.

Solitons will be observed far from each other, where the numerical methods cant distinguish btwn 0 & a small number.

$w(z) + w(z+a)$ is <u>close</u> to a solution so long as a is "large enough".

large enough dep. on

the alg.