Poisson: $p(k) = \frac{\mu^{k}}{k!} e^{-\mu}$

small μ, $p(k) \sim \frac{1}{k!}$

large μ, $p(k) \sim k^{\mu}$ initially

$\frac{1}{k!}$ finally

Gaussian $P_G \propto e^{-x^2/2\sigma^2}$

cauchy $P_c \propto \dfrac{1}{x^2+1}$

Previously :

- Gaussian vs cauchy.

- Generating random #'s.

$p(x)$ = pdf.

$$P(x) = \int_{x_{min}}^{x} dx' \, p(x')$$

$$= \text{c.d.f.}$$

$$x_{sim} = P^{-1}(r) \quad \text{with } r \in [0,1) \text{ uniform.}$$

has pdf $p(x)$

Binning and cont. dist's.

$$\int_{x_{min}}^{x_{max}} dx \; p(x) = 1 = \sum_n p_n \Delta x$$

Discrete binning

$$\sum_n p_n^{discrete} = 1$$

code: – generated rand. #'s

- converted using F^{-1}

- binned

- plotted $p(x)$

What if we don't know $p(x)$?

If you have normally distributed data, how do you determine σ?

Many ways to fit function to data. A common method is by minimizing error

$$e = \left(\sum_n |sim_n - pred_n|^p \right)^{1/p}$$

$p = 2$ is "least squares".

If $x = r^2$, $\quad p(x) = \frac{1}{2\sqrt{x}}$

Take data, and fit to

$$p(x) = a x^n$$

Expect $n = -\frac{1}{2}$

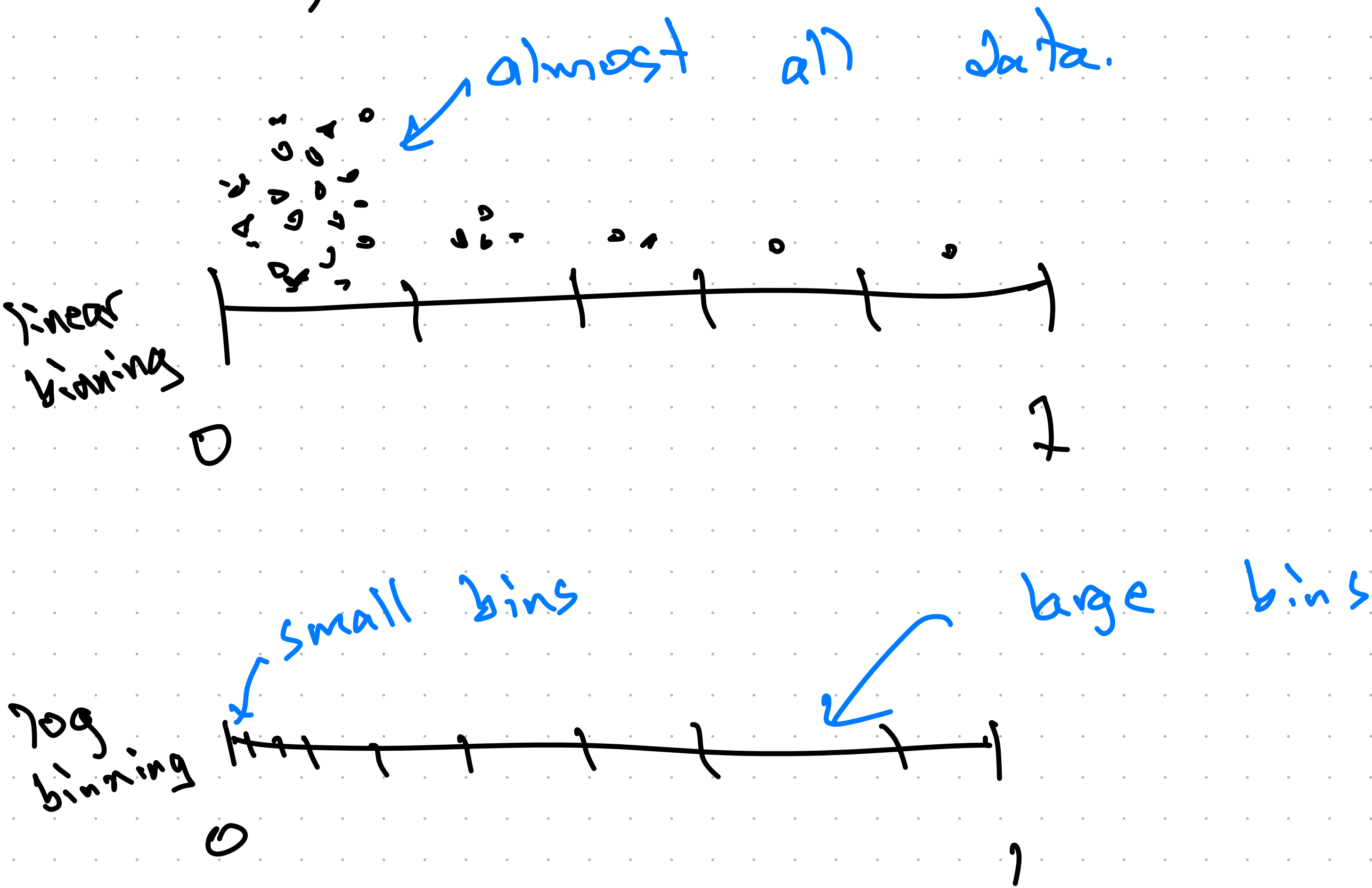
$$p(x) = a x^n$$

$$\underbrace{\log_b(p(x))}_{\log y} = \log_b(a x^n)$$

$$= n \underbrace{\log_b(x)}_{\log x} + \log_b(a)$$

depends on b, a

Our problem



almost all data.
linear binning
0
1
small bins
large bins
log binning
0
1

Linear binning:

$$p_n^{\text{discrete}} = \int_{x_n - \frac{\Delta x}{2}}^{x_n + \frac{\Delta x}{2}} dx \; p(x)$$

$$\lim_{\Delta x \to 0} \int_{x_n - \frac{\Delta x}{2}}^{x_n + \frac{\Delta x}{2}} dx \; p(x) = \Delta x \cdot p(x_n) + O(\Delta x^2)$$

Log binning. $y = \log x$

$$p_n^{\text{discrete}} = \int_{e^{y_n - \frac{\Delta y}{2}}}^{e^{y_n + \frac{\Delta y}{2}}} dx \quad p(x)$$

$$\lim_{\Delta y \to 0} \int_{e^{y_n - \Delta y/2}}^{e^{y_n + \Delta y/2}} dx \quad p(x)$$

$$\lim_{\Delta y \to 0} \int_{x_n e^{-\Delta y/2}}^{x_n e^{+\Delta y/2}} dx \, p(x)$$

$$= x_n \cdot p(x_n) \cdot \Delta y$$

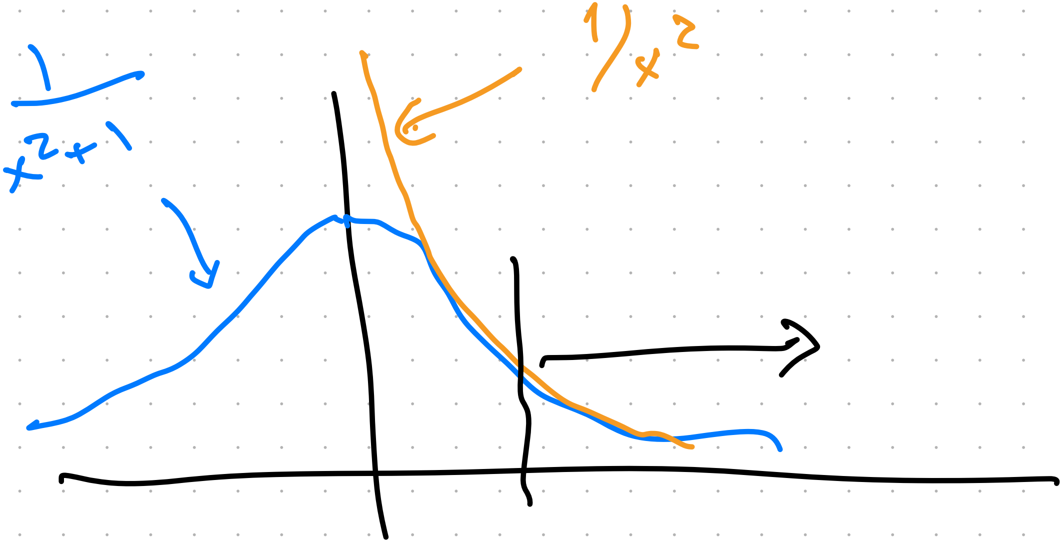
comes from log binning.

For log binned data, if

$$p(x) = \frac{1}{\sqrt{x}},$$

then

$$p_n^{\text{discrete}} \propto \sqrt{x}$$ for log binned data


$\frac{1}{x^2+1}$
$1/x^2$

$x = \frac{1}{r}$ $r = \frac{1}{x}$ $p(x) = \frac{1}{x}$ $p(x) = \frac{1}{x^2}$

$x\,p(x) = \frac{1}{x}$

```
def fcn(x, p1, p2, p3 ....):
    return p1 * x^n1 + p2 x^n2
```