Chaos and irregularity in karst percolation

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Received 17 October 2012; accepted 7 November 2012; published XX Month 2012.

[1] This paper focuses on analyzing chaos in cave percolation water drip rates, which has implications for flow routing in fractured media and on the use of speleothems for paleoclimate reconstructions. It has been shown that the physics of dripping faucets involve a set of non-linear equations leading to chaotic drip rate, meaning that, for a given drip rate, the interval between individual drops can vary greatly. It can be expected that drip waters supplying stalagmites show similar properties, and consequently the dependency between water flux and stalagmite growth rate or geochemistry could be more complicated than usually assumed. We used high-frequency monitoring of two contrasting drips in a cave in Australia, and identified chaos in cave drip rate. Our findings also indicate that the occurrence of chaos can give insights into flow routing in fractured media. **Citation:** Mariethoz, G., A. Baker, B. Sivakumar, A. Hartland, and P. Graham (2012), Chaos and irregularity in karst percolation, *Geophys. Res. Lett.*, *39*, LXXXXX, doi:10.1029/2012GL054270.

1. Introduction

[2] Stalagmite dripping is influenced by a range of hydrologic, climatic and physical processes [e.g., *Jex et al.*, 2012; *Mariethoz et al.*, 2012; *Miorandi et al.*, 2010]. However, very few studies have looked quantitatively at drip rate variability on short timescales (<10 minutes). In contrast, several studies have identified chaos in dripping faucets and described the process both experimentally and theoretically [*Coullet et al.*, 2005; *D'Innocenzo and Renna*, 1996; *Martien et al.*, 1985; *Pinto et al.*, 2001]. The mechanism is seen as a continuous flow of water that builds up until the drop becomes too heavy to be held by surface tension. When it reaches this threshold point, the drop detaches and falls. This induces a rebound mechanism and vibrations in the residual water, affecting the time of formation of the next drop. Since each falling drop influences the motion of the next forming drop, the time intervals T_n between successive drop detachments become irregular or chaotic at certain discharges. It has been shown that this process can be represented as an oscillation according to a mass-spring model [*Dreyer and Hickey*, 1991; *Sartorelli et al.*, 1994], with a threshold associated to the drop detachment. These physics involve a set of non-linear equations leading to chaotic drip intervals. Such a chaotic behavior can be described as a system where a small perturbation in the input parameters can result in disproportionate consequences on the outcome [see *Lorenz*, 1963]. It has been observed that depending on the flow rate, the interval between two drops can either be a constant value (constant regular dripping), a superposition of discrete frequencies (irregular but somewhat predictable dripping over a reasonably long time horizon) or alternatively have a completely chaotic structure [*Fuchikami et al.*, 1999]. Minute differences in flow rate can trigger state transitions in this continuum from regularity to chaos.

[3] It is expected that similar processes can occur within fracture-fed cave drip waters. The only experiments aimed at identifying non-linear cave drip rates were measuring drip rates averaged over periods of 10 minutes [*Baker and Brunsdon*, 2003; *Genty and Deflandre*, 1998], and therefore were not able to clearly identify or quantify chaos. To the best of our knowledge, the present study is the first to draw a parallel between the fields of non-linear physics and karst hydrogeology. The major difference between a leaky faucet and a cave drip is that the cave drip is not a controlled laboratory experiment and, therefore, several external factors intervene. In particular, two main factors may influence the transition between stages of regular/irregular/chaotic dripping rates:

[4] 1. The flow rate is controlled by hydrologic processes, such as recharge, groundwater flow, evaporation, etc., which necessarily induce variations in a karst system on different timescales (e.g., daily air pressure changes, seasonal recharge variability). In particular, rainfall has been shown to exhibit chaotic behavior [e.g., *Rodriguez-Iturbe et al.*, 1989] (see *Sivakumar* [2004] for a review).

[5] 2. A drip is fed by a series of interconnected fractures and stores of various sizes which can be conceptualized as a series of buckets that exchange water under unsaturated conditions [*Bradley et al.*, 2010]. Although these local processes are difficult to investigate in detail, they are known to be non-linear [*Baker and Brunsdon*, 2003; *Genty and Deflandre*, 1998] and could cause irregular flow rate feeding the drip.

[6] To date, all experimental dripping faucet studies have identified chaos for drip rates of at least 2 drops per second. Cave drip rates are often much lower, of the order of one drop per second to two drops per minute or less, especially for stalagmite-forming drips. Such low drip rates have not been studied previously, and the possibility of chaos to occur in such environments remains to be investigated.

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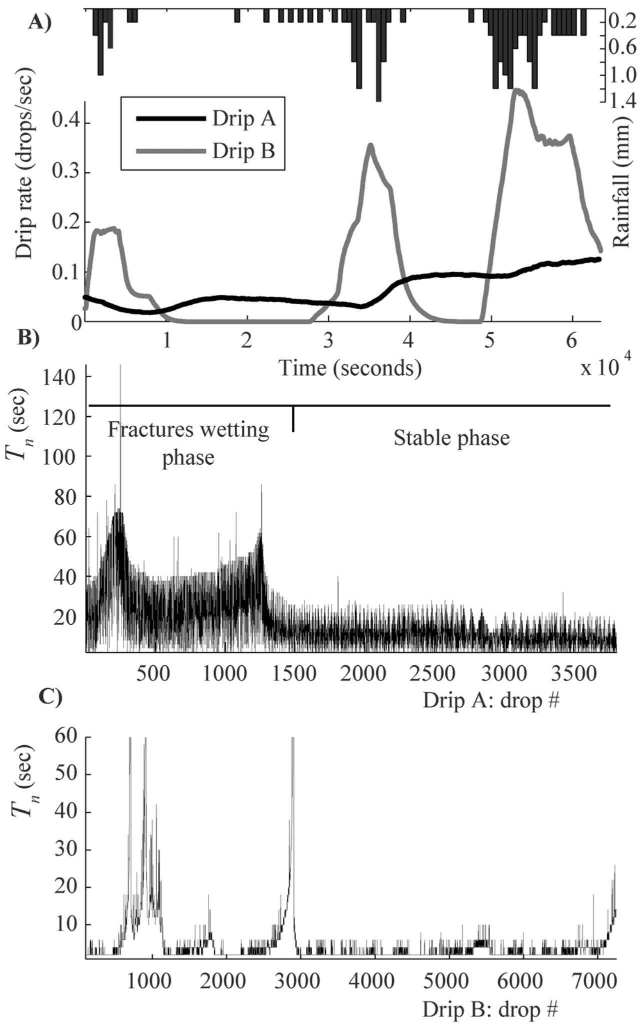

Figure 1. (a) Hydrograph of drips A and B, and rainfall averaged over a 10-minute interval. T_n for the entire recorded period at drips (b) A and (c) B.

[7] In this paper, we seek to establish whether chaos and irregularity can occur in cave percolation water drip rates, and, if it does, what insights it can provide into the related hydrogeological processes. This is accomplished using high-frequency monitoring of two drips in a cave in Australia during a recharge event. Not only have we identified the occurrence of irregular and chaotic behavior, but we also observed that the degree of disorder in the drip rate is related to the degree of connectivity to the recharge and the influence of a groundwater store. This dependence on the hydrogeological setting gives insights into flow routing in fractures. Another immediate consequence is that the interval between drops can vary greatly, depending on the flow rate feeding the drip. The importance of such non-linearity is that the resultant hydrologic variability may imprint on the signal of stalagmite proxies which show a strong dependency on water supply. For example, both δ^{13}C and δ^{18}O undergo within-cave fractionation, which can be dependent on the rate of water supply vs. magnitude of within-cave processes, such as ventilation [*Hendy*, 1971; *Spötl et al.*, 2005], and speleothem growth rate itself is in part dependent on water supply [*Dreybrodt*, 1999]. Fractionation effects resulting from hydrologic variations can presumably introduce a second order variability in proxies, reducing the climatic signal retained.

2. Site and Setting

[8] To test the occurrence of chaotic behavior in cave drip rates, we chose two specific drips near the main entrance of the Cathedral Cave, part of the Wellington Caves complex in New South Wales, Australia. Cathedral Cave, one of many caves that form part of the larger Wellington Caves Reserve (32°37′S; 148°56′E) is located west of the Blue Mountains, part of the Great Dividing Range mountain belt that runs approximately North-South along the Eastern sea-board of Australia, approximately 7 km south of the town of Wellington, New South Wales. The caves have developed in folded Devonian limestone, at the boundary between two distinct facies: a massive, marmorised limestone and a thinly bedded limestone [*Johnson*, 1975]. Orogenetic development during the mid-Devonian and early Carboniferous resulted in widespread folding and the Lachlan fold belt in which the caves are situated [*Osborne*, 2007]. Caves have developed along faults and vertical joints, with the widespread presence of paleokarst suggesting multiphase cave development [*Osborne*, 2007].

[9] The two monitored drips are located in the massive limestone. In dry periods, these drips are usually inactive. Both drip locations are in the shallowest part of the cave, approximately 2 to 5 meters below ground surface, therefore they are well connected to the surface and responsive to rainfall events. Drip A originates from a stalactite that is located on a fracture in the cave ceiling, and which is actively forming a stalagmite on the show-cave path. Since the drip is located about 5 meters below ground level and has reached super-saturation with respect to calcite, it can be assumed that the drip is not directly fed by direct runoff from the surface and that it is representative of karst drip water flow. Dripping is continuous despite the intermittence of rainfall, confirming that some storage is involved. Drip B is close to the surface (about 2 meters), with no overlying soil cover, therefore it is directly fed by surface runoff. It can be assumed to directly reflect rainfall, with very little storage involved. The dripping stops whenever rainfall stops, confirming a direct connection with the surface.

[10] Both drips were recorded with Stalagmate acoustic drip logger devices set to count the number of drops at every 2 second interval, meaning that individual drops can be captured if they are more than 2 seconds apart, or if drops occur with a frequency lower than 0.5 drops/sec. Measurement time was limited by the storage capacity of the loggers, which at 2 s logging intervals allowed continuous measurement for less than 24 hours. Therefore, we deployed the loggers during a rainfall event, and the drips were recorded between the 22nd of November 2011 at 2:30 PM and the 23rd of November 2011 at 8:10 AM, for a total 17 hours and 40 minutes.

[11] The hydrographs of drips A and B, computed using a moving average of drop occurrence, are shown in Figure 1a.

3. Quantifying Irregularity and Chaos

[12] In most leaky faucet studies, a quantity of interest is the interval between two successive drops, $T_n = t_{n+1} - t_n$, with t_n representing the moment when drop n occurred. Figures 1b and 1c show the record of T_n for drips A and B.

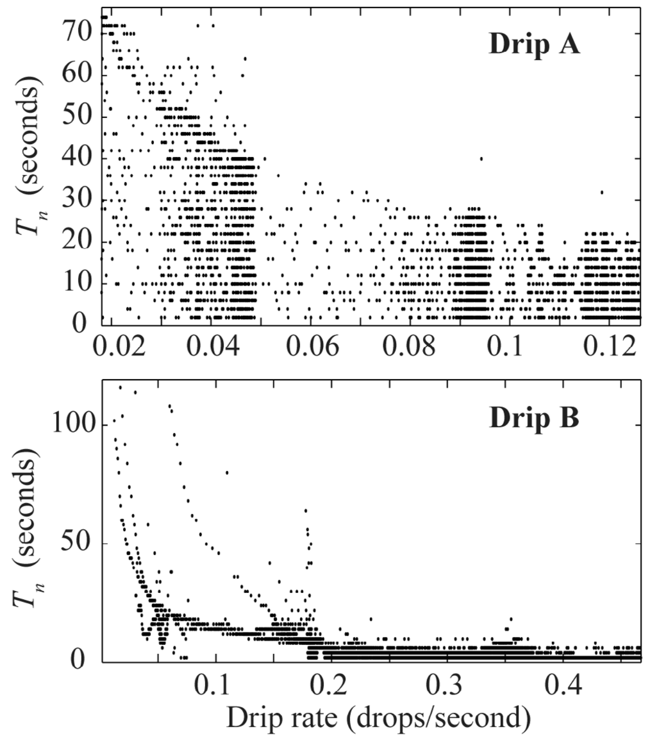

Figure 2. Dripping spectra of drops A and B (note the different axes).

When representing T_n, one no longer deals with a time series, since the X axis is not an absolute measure of time, but instead represents the number of drops since the beginning of the record. The Y axis is the time elapsed between the previous and the current drop. The flow rate at both drips was not constant during the measurement period, showing an overall increasing trend, which translated in intervals between drips being globally larger at the beginning of the sequence than at the end. Studying variations of T_n allows the determination of behaviors where, for example, there is a systematic pattern of two drops coming after each other, then a pause. Such an irregular pattern is clearly visible at the end of the sequence in Figure 1b.

[13] A useful measure of variability is to represent the dripping spectrum, which is the occurrence of intervals T_n as a function of the averaged drip rate over a ten minute period. The dripping spectra of both drips are shown in Figure 2. Each point in the graph represents the interval measured between two successive drops, and under which average flow rate this interval occurred. Dense areas of the spectrum correspond to rates for which a large array of values of T_n are observed. Empty areas denote intervals that are never observed for a given drip rate. Dripping spectra [*D'Innocenzo and Renna*, 1997; *Martien et al.*, 1985] are a way to distinguish regions of stability and chaos, certain rates resulting in regular dripping while others cause the emergence of several distinguishable intervals between drops.

[14] Since the experimental setting does not allow measuring T_n with a greater precision than 2 seconds, points are disposed along horizontal lines 2 seconds apart. However, it is clear that certain flow rates result in more variability in T_n. For drip A, it is noted that specific drip rates occurred more frequently (0.035–0.049 drops/second, 0.089–0.096 drops/second, and above 0.114 drops/second), during which a wide range of dripping intervals T_n is observed, indicating irregularity. The dripping spectrum of drip B is very different and indicates a limited number of specific intervals for a wide range of drip rates between 0.010 and 0.20 drops/second. It displays clear bifurcations, with T_n values taking two or more distinct modes, corresponding to a typical pattern observed in leaky faucet experiments.

[15] The succession of intervals between drops is further analyzed by considering the joint distribution $f(T_n, T_{n+1})$, which is represented by plotting a point for each drop n, whose X coordinate is the interval between T_{n-1} and T_n, and the Y coordinate is the interval between T_n and T_{n+1}. Figure 3 (top) shows such joint distributions for both drips. Although both drips show irregularity, one sees that drip B presents certain recurrent patterns of intervals. Such features can be a sign of non-linear, chaotic behavior.

[16] Since the data were collected during the initial stage of a recharge event, temporal analysis of the dripping irregularity can give insights into the fractured rock wetting processes. The time-varying joint distribution of $f(T_n, T_{n+1})$ for both drips for intervals of 500 drops are displayed in Figures S1 and S2 in the auxiliary material.[1] Drip A shows that the first 500 drops exhibit the most variability, with no definite pattern in the succession of intervals. Between the 501st and the 1000th drop, a pattern seems to take place involving mostly successions of 20 s after 20 s intervals, 5 s after 40 s intervals, and 20 s after 40 s intervals. The next period, between drops 1001 and 1500, seems chaotic again, but less than the initial one. The last three periods considered, consisting of drops 1501–2000, 2001–2500 and 2501–3000, seem to settle in a pattern involving a constant irregular pattern of large intervals followed by short intervals, also visible as oscillations at the end of the series in Figure 1b. This suggests that from the 1500th drop onwards, the system has settled into a stable state. Drip B shows a very different temporal behavior, with no specific wetting pattern at the initial stage of the rain event, and an irregular succession of stable stages (drops 1–500, 2001–2500, 3001–5000, 5501–6500) and unstable ones (drops 501–2000, 2501–3000, 5001–6000, 6501–7000).

[17] Following up on our examination of the irregularity in drip intervals, we investigated the presence of chaos using the surrogate data method [*Theiler et al.*, 1992]. This method consists of generating synthetic time series that possess similar statistical characteristics as the data (e.g., distribution, autocorrelation, frequency spectrum), but are nevertheless the outcomes of a linear process. The identification of significant differences between the original data and the surrogates is an indication of chaotic behavior. The method of *Schreiber and Schmitz* [1996] was used to generate surrogates for both drips. Figure 3 (bottom) shows the joint distributions of the surrogates. While the surrogate for drip A can reasonably well reproduce the irregularity of drip A, the surrogate of drip B fails to present the specific recurrent frequencies observed.

[18] As in the work by *Martien et al.* [1985], we used entropy to further characterize the joint distributions. A distribution with high entropy means that a wide variety of drop intervals is observed, and is a sign of disorder and lack of

[1]Auxiliary materials are available in the HTML. doi:10.1029/2012GL054270.

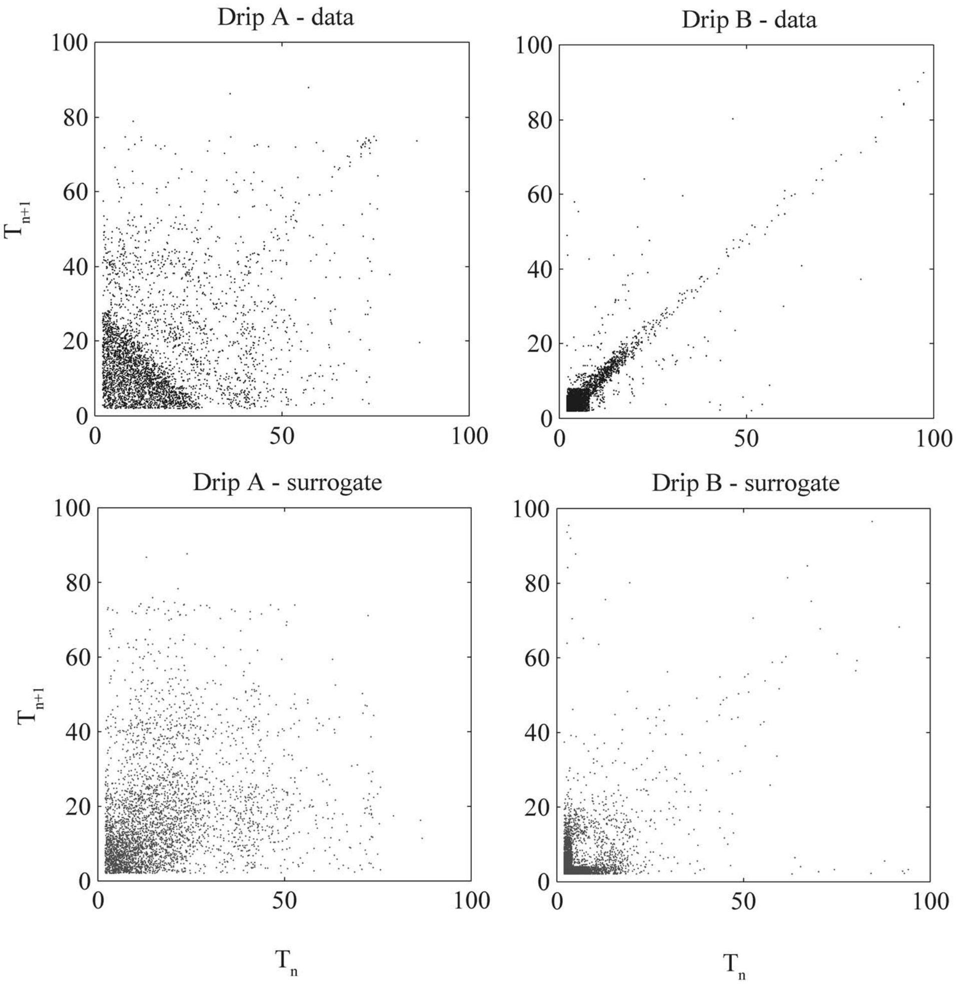

Figure 3. Joint distributions for drips (left) A and (right) B, for (top) data and (bottom) surrogates.

definite dripping patterns. Entropy was computed for each sequence of 100 drops for both drips A and B using the formulation of *Shannon* [1948]. Figure 4 shows these entropy values plotted against the mean drip rate for each 100 drop sequence. Triangles correspond to drip A and squares to drip B. Blue symbols represent the data and red symbols the surrogates. This figure shows a consistent trend of higher entropy with lower drip rates. Although both data and surrogates seem to indicate a similar relationship for drip A, the entropy of drip B notably deviates from the surrogate. This indicates a non-linear behavior in drip B that cannot be replicated by the linear process at the origin of the surrogate.

[19] As a final test for chaotic behavior, we used the Lyapunov spectrum method of *Sano and Sawada* [1985] to determine the maximum Lyapunov exponents of both drips (data and surrogates). A positive maximal Lyaponuv exponent is generally a sign of an unstable chaotic system. For drip A, the maximum exponent was found to be -0.0097 for the original data and 0.0136 for the surrogate data. Values close to 0 indicate that the series is quasi-periodic and that the surrogate is able to reproduce such characteristics. For drip B, the original data yielded a maximal Lyaponuv exponent of 0.9488, indicating chaos, whereas the surrogate data had an

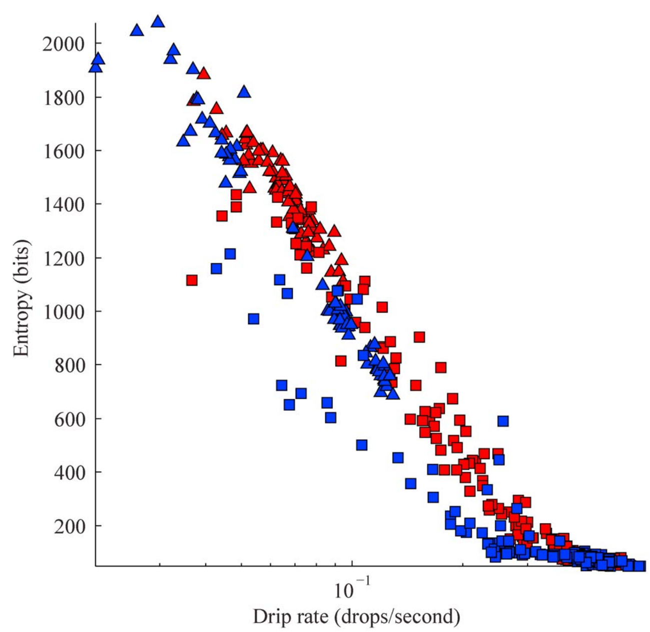

Figure 4. Entropy graph. Triangles: drip A; Squares: drip B. Blue: data; red: surrogate.

exponent of only 0.1565, indicating once again that a linear process is unable to reproduce the characteristics.

4. Implications for Karst Hydrology and Stalagmite Growth

[20] Dripping spectra, entropy of joint distributions and Lyapunov exponents have been used to characterize drip intervals from two percolation drips in Cathedral Cave, Australia. All methods consistently indicate that the pattern of drip intervals involves irregularity for both drips. While drip A only shows irregularity, drip B also displays a chaotic behavior, which is for the first time rigorously quantified in a stalactite drip.

[21] In drip A, which is located slightly deeper and not directly connected to the surface, irregular behavior is more pronounced at the beginning of the recharge event. It can be conceptualized that rainfall flows through multiple, solutionally-widened fractures, which combine to supply the drip. At the start of the recharge event, water flows through the system from multiple sources at different rates, causing high irregularity. Eventually, the fractures fill and the water source becomes more homogenous, causing entropy to diminish while remaining irregular. This point is reached after about 1500 drops. Such varying irregularity during the fractures' wetting phase is remarkable given the presence of a groundwater store above the drip. However, despite causing irregularity, the fracture wetting phenomenon does not seem to lead to chaos, possibly because the resulting drip rates are too low to incur the non-linear drop rebound mechanism observed in leaky faucets and because of the effect of a groundwater store above the drip.

[22] Drip B, which is known to be directly connected to the surface water source, has a completely different behavior, without initial pattern related to wetting. While it displays drip rates less irregular than drip A, with a set of defined recurring intervals, it is shown to be more chaotic than drip A. It is hypothesized that the regularity of the drip rate allows for the oscillatory effect observed in dripping faucet experiments to take place. Another compounding effect may be related to the dominant influence of rainfall, which has been identified to exhibit chaotic behavior [*Puente and Obregón*, 1996].

[23] This contrasting behavior of drips located in different hydrogeological settings, but subject to the same hydrologic forcings, gives insights into percolation processes. One implication of this research is that chaos can be used as a marker of surface-subsurface connectivity. Observation of chaos is likely to be an indication of a direct routing between surface recharge and cave drip waters. We argue that high, irregular drip intervals in the absence of chaos are a sign of progressive wetting of fractures, with reduced irregularity as the wetting progresses.

[24] Our work also has implications on the validity of paleoclimate reconstructions based on cave stalagmites. One of the controlling factors of many stalagmite proxies is the drip rate feeding a stalagmite. For example, the optimal growth rate occurs when the drip rate is between $\sim$1 and $\sim$5 minutes per drip [*Dreybrodt*, 1999]. At very high drip rates, degassing is not complete, and at slow drip rates, degassing occurs to equilibrium and water supply is limiting. Kinetically-enhanced degassing of CO_2 has been demonstrated to affect stalagmite $\delta^{13}C$ with a dependency on drip rate and cave ventilation [*Baker et al.*, 2011], with increasing fractionation observed at slower drip rates. Modeling studies of the relationship between stalagmite $\delta^{13}C$ and $\delta^{18}O$ and drip rate provide further insights into the discharge dependency [*Fohlmeister et al.*, 2011; *Scholz et al.*, 2009]. Our results suggest that the relationship between water flux and stalagmite climate proxies could be more complicated than usually assumed, with strong non-linearity occurring. One implication of this research is that drip points that are well connected with surface recharge are likely to exhibit chaotic behavior over an undefined but specific range of flow rates. Such intermittent discharge is not captured by classic time-averaged monitoring of discharge behavior. Because of this variability, it follows that climate proxies which show a strong growth rate effect (e.g., $\delta^{13}C$, extension rate) will have a degree of variance imposed by chaotic discharge behavior. Because natural variations occur at all scales, it is probable that chaotic discharge is a second order effect on proxy variance, but will nevertheless impact on the information encoded.

[25] Our work shows that drip rate averaging fails to record important information about hydrogeological processes. It is, therefore, recommended for future studies to use long-term, high-frequency recording of individual drip counts.

[26] **Acknowledgments.** This work was supported by the Australian Research Council, the National Water Commission, and the Groundwater Education Investment Fund. We thank Mike Augee and the Wellington Caves Management Committee for kindly letting us access the Wellington site, and Matthew McCabe for installation and maintenance of the weather station. We also thank two anonymous reviewers for comments leading to improvements of the paper.

[27] The Editor thanks Carlo Camporeale for his assistance in evaluating this paper.

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