Advanced Review

Generating synthetic rainfall with geostatistical simulations

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Rainfall is an important driver of many Earth surface and subsurface processes such as floods, groundwater recharge, or plants growth. Models are used to investigate the physical response of different environmental aspects to a range of possible rainfall events. To provide meaningful outputs, such models require realistic inputs. However, a major challenge in these models is the representation of the chaotic behavior of rainfall as well as its high temporal and spatial variability. The primary sources of information about rainfall are rainfall measurements, numerical weather models and climate models. Because these sources of information are incomplete and uncertain, stochastic models have been developed to augment available data using statistical methods. Applications to rainfall modeling include interpolation of rainfall measurements, downscaling of numerical model outputs, or weather generators. In this study, we focus on geostatistical stochastic rainfall generation models, which aim at characterizing and reproducing the spatial structure of rainfall. To this end, the different steps of the geostatistical rainfall modeling are reviewed. The spatial structure of the rain is first characterized by variogram analysis of rainfall data. Then, geostatistical models are discussed that match the observed rain structure. Finally, geostatistical simulations are applied to the inferred models for the generation of synthetic rain fields.

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INTRODUCTION

Rainfall has many impacts on the environment, from plants growth and freshwater supply to devastating floods or landslides. However, the study of rainfall impacts on the environment is challenging, mainly due to the high variability of rainfall in space and in time, combined with the often nonlinear response of the environment to rainfall forcing.

Here we distinguish impact models that quantify the consequences of a given rainfall event, and rainfall generation models that provide a representation of the rainfall event itself. Impact models have been designed to study the complex impact of rainfall on different aspects of the environment. For instance, rainfall-runoff models investigate the influence of different rain inputs on peak flow generation at the outlet of a given catchment.[1,2] Applications range from flood prevention[3–5] to water system management.[6] When the interest is on water infiltration and storage, hydrological models focus upon groundwater recharge[7] or hazards related to soil saturation.[8] Since rainfall also affects living organisms, ecological models are used to assess rainfall impacts on either natural[9] or man-made[10] ecosystems. As a typical application, crop yield planning[11,12] allows quantification of the vulnerability of agricultural systems to rainfall or drought events.

Overall, these impact models aim at providing realistic physical responses of environmental state variables for a given rainfall input, but they do not model the rainfall itself. A correct rainfall input is therefore necessary, but is not always available. Indeed, rain measurements are often not sufficient, either because the duration of records is too short to include extreme events, or because the spatial

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coverage of measurements is incomplete. Furthermore, numerical weather and climate models do not yet accurately represent the physical processes responsible for fine scale rainfall patterns (e.g., clouds formation)[13,14] which may play an important role in shaping some key characteristics of the rainfall.

One solution to overcome the inherent incompleteness of information is to augment the available data statistically using stochastic rainfall generation. This is the aim of rainfall generation models, which provide rainfall inputs that are as realistic as possible, in terms of their spatial and temporal distribution. Depending on the application, inputs can be for instance a time series of rain amounts at a single location, a spatially distributed rainfall field, or a more complex space–time image of the rainfall. These rainfall inputs must represent the level of space–time variability that is important for the phenomenon being modeled. It may be necessary to capture not only simple features of the rain field like its mean or variance, but also more complex features such as the sequencing of dry and wet periods, the duration of rain intensity excess, or the average return period of extreme precipitations.

This review article focuses on geostatistical rainfall generation, which is one class of methods that considers rain fields as structured manifestations of random fields. It is based on the premise that rainfall has some kind of statistical structure in space and in time. If this structure is identified, it can provide a means of improved rainfall estimation. The need for stochastic rainfall generation is first highlighted and the different frameworks proposed in the literature are briefly reviewed. Next, geostatistics are introduced in regard to their application to rainfall modeling. Finally, the methods for stochastic rainfall generation based on geostatistics, namely geostatistical simulations, are reviewed in details with a special emphasis on rainfall model selection.

STOCHASTIC RAINFALL GENERATION

The rainfall information to be used in environmental models can be divided into two categories: rainfall measurements and physical models of the atmosphere.

The first device used historically for rainfall measurement was the rain gauge.[15,16] It is still widely in use today because it can provide accurate, high resolution (usually up to 0.1 mm of rain), continuous, and often long term rainfall measurements.[17–19] However, due to the high spatial variability of rainfall, the rain gauge representativeness is restricted to the few square meters surrounding the device.[20,21] Terrestrial weather radars were developed to image the rain field and thereby to complement rain gauge networks.[22–24] Radar data provide a high spatial resolution ranging from 5 km to 100 m per pixel, but suffer from numerous instrumental biases. In addition, their high cost restricts their coverage to wealthy countries. Tropical Rainfall Measuring Mission (TRMM)[25,26] and Global Precipitation Measurement (GPM)[27] satellite missions were developed to extend the coverage of rainfall measurements to the worldwide scale using infrared and radar measurements. As a counterpart to their global coverage, these satellite rainfall measurements provide only low spatial (around 5 km per pixel) and temporal (hourly to daily) resolutions, and only a short (<20 years) historical dataset.

Numerical weather and climate models attempt to reproduce to a large extent the physics of the atmosphere by solving a set of partial differential equations that govern the state of the atmosphere. However, some processes responsible for precipitation, e.g., cloud micro-physics or subgrid-scale convection,[14] are not explicitly resolved but are represented as functions of resolved variables, so called parametrization. Numerical weather and climate models primarily focus on weather forecast[13] and climatic studies.[28–30] Numerical models represent the atmosphere as a single process involving many variables (e.g., temperature, pressure, humidity, precipitation, etc.), and rainfall is only one particular component of this process. Rain fields can be extracted from such numerical model outputs for further impact studies. However, such rainfall estimates are known to be biased.[31,32] A first source of bias originates from the coarse resolution of numerical weather and climate models that leads to unresolved processes like cloud formation or convection. The other significant source of bias originates from the imperfect modeling of the micro-physics of rain generation processes.

As a consequence of the above, both rainfall measurements and numerical models outputs present limitations for use as inputs to impact models. In general, these data have to be augmented by statistical means to make them usable in impact studies. This is achieved by stochastic rainfall models. Different situations are possible depending on the application: (1) weather generators expanding a limited dataset by generating long duration synthetic rainfall fields that mimic the observed structure of rainfall; (2) interpolation of rain measurements based upon densification of point rainfall measurements by generating a synthetic rain field reflecting the structure of the data while honoring the measurements; and (3) downscaling of rainfall data by injecting fine scale structures into a dataset available only at a coarser scale.

Stochastic rainfall models are data-driven in the sense that they leave out the processes of the rain generation: the physics are replaced by the belief that some characteristic features of the rain field, quantified by means of statistics, remain constant in time and space over a given domain. Randomness is introduced in the modeling to account for the incomplete knowledge of the rainfall processes. This leads to the use of probability distributions instead of single values to represent model outputs, an approach useful to quantify uncertainty.

Stochastic rainfall models proceed in two steps. First, relevant statistical information is extracted from a calibration dataset. Second, several synthetic outputs sharing the same statistical properties as the input are generated stochastically. The result is an ensemble of realistic rain fields that samples the rainfall configurations compatible with the structure observed in the calibration dataset.

A variety of stochastic rainfall models have been developed, each focusing on different components of the rain structure and adapted for different types of available data (Table 1):

- Point process models focus on the organization of clusters within rain fields. They make assumptions about the physical processes responsible for rain generation to determine basic objects that form the rain field, for instance rain cells, rain bands or storms.
- Multifractal models focus on the similarity of the rain patterns across scales. They acknowledge that rainfall is not exactly fractal but rather multifractal, i.e., that rain patterns do not reproduce perfectly across scales, but rather evolve slowly with scale variation.
- Multisite temporal rainfall models focus on rainfall time series at a limited set of locations. The spatial dependency between locations is taken into account by defining a dependency between the statistical parameters that govern the statistical structure of each single time series.
- Generalized linear models (GLMs) focus on the statistical relationship between rainfall and a set of covariates. For rainfall generation, GLMs assume that the covariates are known (e.g., from outputs of numerical weather or climate models) and make use of the statistical relationship between rainfall and covariates to simulate rainfall.

TABLE 1 | Different Stochastic Frameworks Applied to Rainfall Modeling

Modeling Framework	Focus on	Statistical Tools	Applications
Point process	Constitutive objects: rain cells, rain bands, etc.	Poisson processes Neymann–Scott processes Barlet–Lewis processes	Rainfall generators[88–90] Downscaling of global climate models[91]
Multifractals	Persistence of rain patterns across scales	Fractals multifractals Random cascades	Rainfall generators[84] Analysis of scaling properties of rainfall[92] Downscaling of radar images[21,93]
Multisite temporal	Temporal structure	Markov chains Autoregressive models Censored Gaussian distributions	Rainfall generators[59,94,95] Analysis of spatial variability of rainfall time series[96]
Generalized linear model	Statistical link with covariates	Generalized linear models	Rainfall generators[97–99] Analysis of relationships between rainfall and covariates[100,101]
Multiple-points statistics	High-order statistics	Multiple-point statistics Analogs	Rainfall generators[84,102] Downscaling of global climate models[103]
Geostatistical simulations	Spatial structure	Multivariate gaussian random fields Variograms	Rainfall generators[51–54] Interpolation of rainfall measurements[55,79] Assessment of uncertainty in radar images[56,57]

- Multiple-points statistics focus on the reproduction of rain patterns characterized by high-order statistics and inferred from an analog training set. This nonparametric approach provides the advantage of simple modeling, but requires exhaustive training sets for inference of rainfall statistics.
- Geostatistical models focus on the spatial structure of the rain. They consider rainfall as a random field, and investigate its spatial correlation structure using only statistics related to pairs of points. As the main focus of this review, geostatistical rainfall modeling will be detailed in *Rainfall Generation by Geostatistical Simulations* section after a brief description of geostatistics in *Geostatistics for Rainfall Modeling* section.

GEOSTATISTICS FOR RAINFALL MODELING

Geostatistics are an ensemble of probabilistic methods concerned by problems related to spatial variability.[33,34] They focus not only on stochastic generation but rather form a broad framework to account for the spatial patterns and the correlation structure of spatially fluctuating phenomena such as rainfall among other applications. Three main types of applications of geostatistics to rainfall modeling can be identified:

1. *Change of support*. Most of rainfall data are spatial averages over fixed areas (e.g., pixel size in radar images) or temporal averages over intervals of fixed duration (e.g., time aggregation in rain gauge time series). The size of the domain over which data are averaged is termed in geostatistics the support of the data.[34] When rainfall varies at scales smaller than the support of the data, a part of the natural variability cannot be represented. As a result, spatial or temporal integration not only makes rainfall visually smoother (Figure 1(a) and(b)), but it also alters its statistical properties (Figure 1 (c) and (d)), and particularly the extreme values (Figure 1(d)). In the geostatistical framework, it is possible to deal with this modification of rainfall distribution using change of support methods. For example, change of support methods can be used to homogenize rainfall measurements originating from instruments with different supports (e.g., rain gauges and weather radar),[21,35–40] or for disaggregation or downscaling of satellite data,[41] radar images,[42,43] or rain rate time series.[44]
2. *Interpolation by local estimation*. Geostatistics may also be used to predict the expected value of rainfall at any point of the domain of interest, based on sparse rainfall measurements available at a restricted set of locations. Kriging allows interpolation of rain gauge measurements to generate rain maps.[45–47] The predicted value is pointwise optimal, but this optimal estimation leads to smoothed rain estimates in space or time (Figure 2(a)). Kriging can also be used in its multivariate version to incorporate information from linearly correlated covariates, such as for example elevation.[48–50]

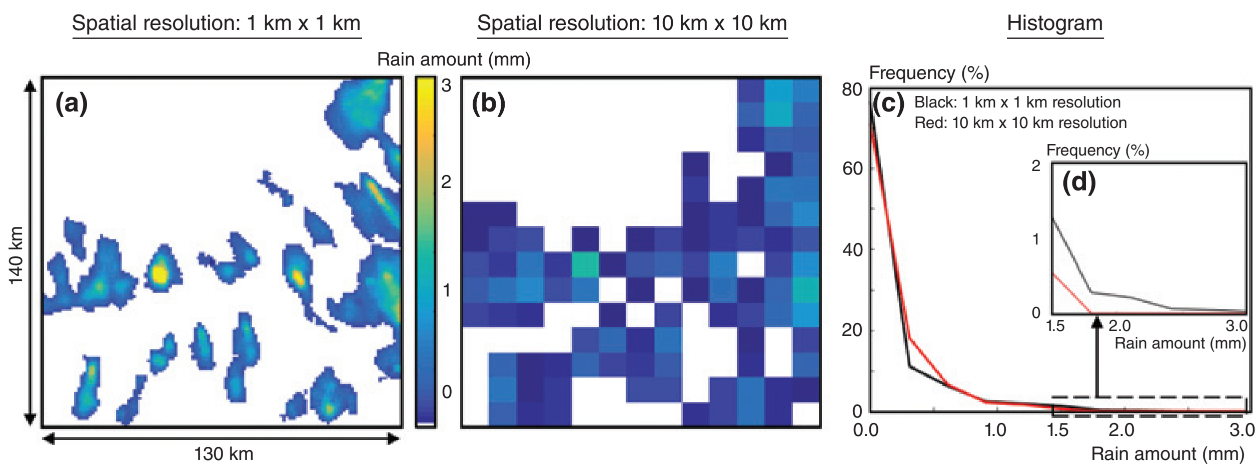


FIGURE 1 | Effect of change of support for an hourly rain field measured by weather radar on May 5, 2005, 10–11 h, over Northwestern Switzerland (Source: MeteoSwiss). (a) Aspect of the rain field for two different spatial supports: 1 km × 1 km (A) and 10 km × 10 km (B). (b) Histograms for the two spatial supports (C).

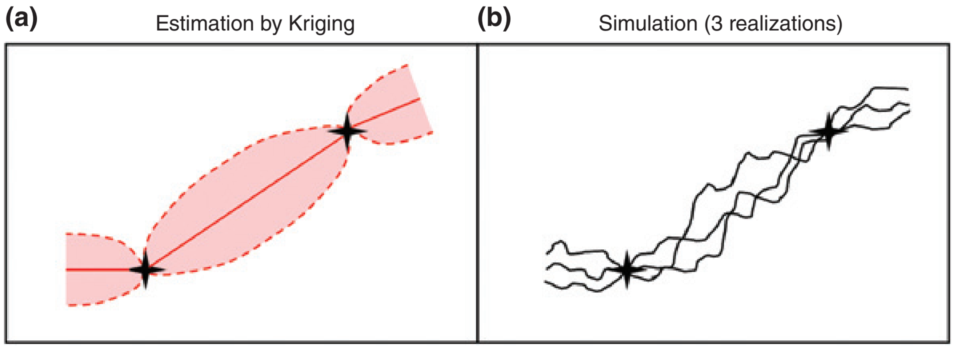


FIGURE 2 | Comparison between Kriging and geostatistical simulation (one-dimensional case). (a) Estimation (solid red line) and related confidence interval (dashed red lines) obtained by Kriging from two data points (black stars). (b) Set of three realizations obtained by geostatistical simulation (black curves) conditioned to the same observations than in (a).

3. *Simulation*. The aim of geostatistical simulation is to generate a set of synthetic rain fields which reproduce the properties of a given observation dataset, expressed as their spatial covariance. The output is an ensemble of equally probable rain fields matching the observed spatial rainfall structure, and optionally honoring observation data at some locations. Unlike Kriging, simulations produce rain fields which honor the structure of the rain (no smoothing effect) but in counterpart are stochastic. This means that an infinite number of possible rain fields can be generated, termed realizations (Figure 2 (b)). One of the main uses of geostatistical simulations in the context of rainfall modeling is to build stochastic rainfall generators[51–53] and to assess uncertainties of rain field reconstructions in case of complex rain structure (e.g., in the presence of rain intermittency)[54,55] or in case of complex measurement errors (e.g., uncertainty of radar images).[56,57]

RAINFALL GENERATION BY GEOSTATISTICAL SIMULATIONS

Among geostatistical methods, simulations are the most suited for rainfall generation because they produce an ensemble of nonsmooth synthetic rain fields. This is critical when the environmental response to rainfall is nonlinear,[5,58] as is the case in most impact models. Rainfall generation by geostatistical simulation will therefore be reviewed in details hereafter, following the workflow adopted in geostatistical studies: data analysis, model selection, and generation of synthetic rain fields.

Preliminary: Modeling Framework

Geostatistics model each observed rain field as a single realization of a spatialized random process.[33,34] At each point of the domain D of interest at location $u_i = (x_i, y_i)$, the observed amount of rain r_{u_i} is assumed to be the outcome of a random variable Y_i (Eq. (1)). The spatial coherence between locations is acknowledged by considering each local random variable as a member of a regional random function Y (Eq. (2)). In addition, two assumptions are made about this regional random function. As a first assumption, the regional random function is assumed to be fully characterized by its covariance structure. Therefore, the relevant statistics to study the rain field are its regional mean and variance as well as the correlation structure between pairs of locations. In turn, a multivariate Gaussian distribution is assumed for the regional random function because, conveniently, it is fully characterized by its mean vector and variance–covariance matrix (Eq. (3)). The second assumption is that the regional random function is second order stationary, meaning that the rain field is statistically homogeneous. Formally, it means that its covariance structure depends only on the distance separating locations and not on the locations themselves (Eq. (4)).

These modeling assumptions can be summarized as follows:

$$\forall u_i \in D,\ r_{u_i} = \text{outcome of } Y_i \tag{1}$$

$$Y = (Y_1, Y_2, \ldots, Y_i, \ldots, Y_n)^T \tag{2}$$

$$Y \sim N\left(\mu = \begin{pmatrix} \mu_1 \\ \ldots \\ \mu_N \end{pmatrix}, \Sigma = \begin{pmatrix} {\sigma_1}^2 & \ldots & Cov(Y_1, Y_N) \\ \ldots & \ldots & \ldots \\ Cov(Y_N, Y_1) & \ldots & {\sigma_N}^2 \end{pmatrix}\right) \tag{3}$$

$$\Sigma(i,j) = \Sigma(\|Y_j - Y_i\|) \quad (4)$$

Due to the above modeling assumptions, the covariance structure of the rain and its representation through a variance-covariance matrix are the main concerns in geostatistical modeling of rainfall. In practice, this modeling can be divided in three steps:

1. Geostatistical analysis where the input data are analyzed to characterize the statistical properties of the rain field. The univariate marginal distribution is usually studied by histogram analysis, while the covariance structure is analyzed using a geostatistical tool called the experimental variogram.
2. Geostatistical modeling where a covariance model is built according to the rain field covariance structure observed in the rain histogram and experimental variogram. The parameters of this model are inferred based on the experimental variogram.
3. Geostatistical rainfall generation. Several synthetic rain fields are stochastically produced such that they honor the covariance model and conditioning data (locations where rainfall is known).

The following sections explain these three steps in details.

Geostatistical Analysis of Rainfall

The objective of geostatistical analysis is to derive estimators for the first and second moments of the multivariate Gaussian random field of interest (Eq. (3)). The main tools used in geostatistics are histogram to characterize the marginal distribution of rainfall and experimental variogram to characterize the spatial covariance structure of rainfall.

The histogram (Eq. (5)) represents the distribution of measured rain amounts across the domain. Consider the N locations u_i, $i = 1 \dots N$ previously defined where the rain amounts r_{u_i} are measured. Let r be a given rain amount and $n(r)$ be the number of measurement points where $r_{u_i} = r$. The histogram is then defined by:

$$Hist(r) = \frac{n(r)}{N} \quad (5)$$

Data analysis shows that as temporal resolution increases, rainfall histograms present an increased proportion of zero values as well as a skewed distribution of the nonzero rain values[42,44,59,60] (Figure 4(b)). The large number of zero rain measurements across the domain corresponds to dry areas generated by the intermittent nature of rainfall, whereas the skewed distribution of positive rain amounts reflects the significant part of extreme rainfalls.

The experimental variogram (Eq. (6)) quantifies the dissimilarity between pairs of points separated by a given distance. Let $h = \|u_j - u_i\|$ be the separation distance between two locations u_i and u_j, and $n(h)$ be the number of pairs of points separated by distance h. The experimental variogram is then defined by:

$$\hat{\gamma}(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} \left(y(u_i) - y(u_i + h)\right)^2 \quad (6)$$

For practical reasons, pairs are often grouped by bins of similar separating distance, and only the bin-averaged variogram value is considered. This has the advantage of smoothing the actual experimental variogram and avoiding possible fluctuations linked to sparse data. Near the origin, the variogram describes the small-scale spatial roughness (steep variogram) or smoothness (near horizontal behavior) of rainfall (Figure 3). For large lags, the variogram stabilizes at a value called the sill, which is equal to the variance of the rainfall (Figure 3). The separation distance at which the sill is reached is called the range, or correlation distance (Figure 3).

Variogram analysis is generally used to quantify the spatial variability of rainfall. It shows that the rain field exhibits an increasing roughness for decreasing integration times (Figure 4(c)). This results in experimental variograms with shorter correlation distances and steeper tangents at the origin when the temporal resolution increases.[20,38,61] In addition, the variability of rainfall is higher for convective than for stratiform events.[62]

Geostatistical Models of Rainfall

Once the marginal distribution (histogram) and the covariance structure (experimental variogram) of the rain field have been identified thanks to data analysis, the next step of the geostatistical workflow is to design a geostatistical model that represents these observed statistical properties. Note here the fundamental difference between experimental statistical metrics (histogram and experimental variogram), which are solely a representation of the data, and a model, which is based on parameters and assumptions. The statistical model typically consists in three

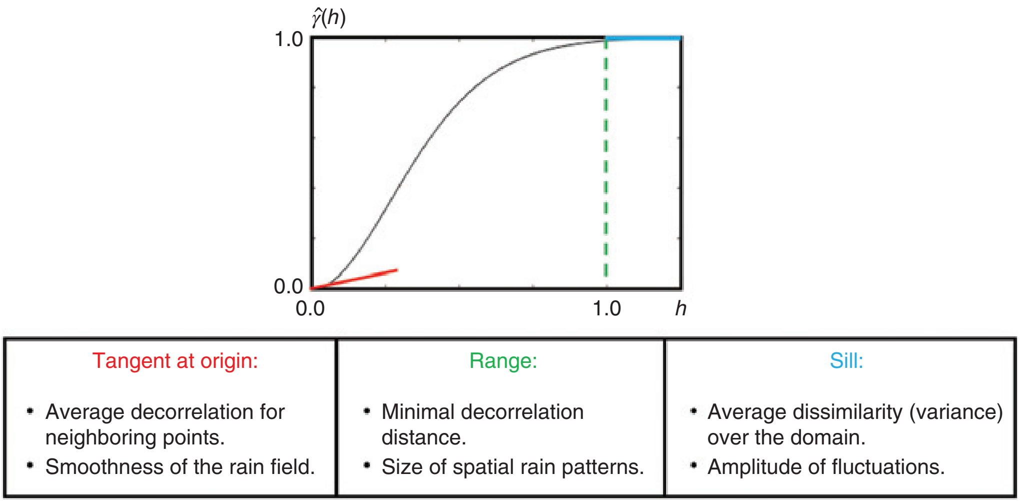


FIGURE 3 | The main components of a variogram and their relationships with rain field features.

components: (1) the construction of a type of random field to adequately represent the observed rain field heterogeneity; (2) a Gaussian anamorphosis, which is mathematical transformation applied to a non-Gaussian process so that it can be modeled as a Gaussian random field; and (3) a model of variogram to account for the covariance structure of the rain.

Component 1: Construction of a Type of Random Field

As mentioned above, the apparent distribution of rain closely depends on the temporal resolution of the data. Therefore, different temporal resolutions lead to different random field models (Table 2).

For integration times longer than 1 day, the empirical distribution of measurements is often close to the hypothesis of multivariate Gaussian distribution defined in *Preliminary: Modeling Framework* section. In such cases, the rain field can be modeled directly by a simple multivariate Gaussian field.[46–50]

The rain field becomes increasingly complex for subdaily integration times where rain intermittency generates a large number of locations with zero rainfall.[63] A first option to accommodate this large number of zero rainfall values is to model the rain field as a truncated multivariate Gaussian field.[42,53,54,64,65] In this model, a multivariate Gaussian field defined by Eqs (1)–(4) is truncated using a threshold (Figure 5). All the points of the multivariate Gaussian field under the truncation threshold are set to zero and are termed censored. They model the dry part of the rain field. The rest of the multivariate Gaussian field (above the truncation threshold) is kept unchanged and models the positive rainfall intensities. Rain occurrence and precipitation accumulation are modeled by a single truncated multivariate Gaussian random field. It can be seen as modeling a rain potential, which produces rain when the truncation threshold is exceeded and generates the highest rain rates for locations with the highest potential.

Another model dealing with rainfall intermittency considers two separated random fields. They account for rainfall occurrence and rain accumulation, respectively.[51,52,55,66,67] A binary random field is first generated to represent the rain occurrence. Then a multivariate Gaussian random field is pasted in the rainy areas to model the positive rain amounts.

For very short integration times (less than 5 min), it appears that the rain intensity decreases at the border of rainy areas.[68] The truncated multivariate Gaussian field model naturally accommodates this smooth transition between dry and wet areas.[53] This is not the case for the model with two independent random fields, although it can be refined to accommodate this rain decay by adding a deterministic drift to the multivariate Gaussian field that models rain accumulation (also called 'Dry Drift').[52]

Component 2: Gaussian Anamorphosis

The definition of a geostatistical model relies on the formulation of multivariate Gaussian distributions, which require Gaussian histograms. However, for subdaily integration times, the distribution of rainfall is often skewed even after modeling dry areas separately.[42,44,53,59] Therefore, common practice does not focus upon directly modeling raw rainfall R, but rather a transform Y of these raw measurements such that the transformed field Y exhibits a Gaussian marginal distribution (Eq. (3); Figure 5).

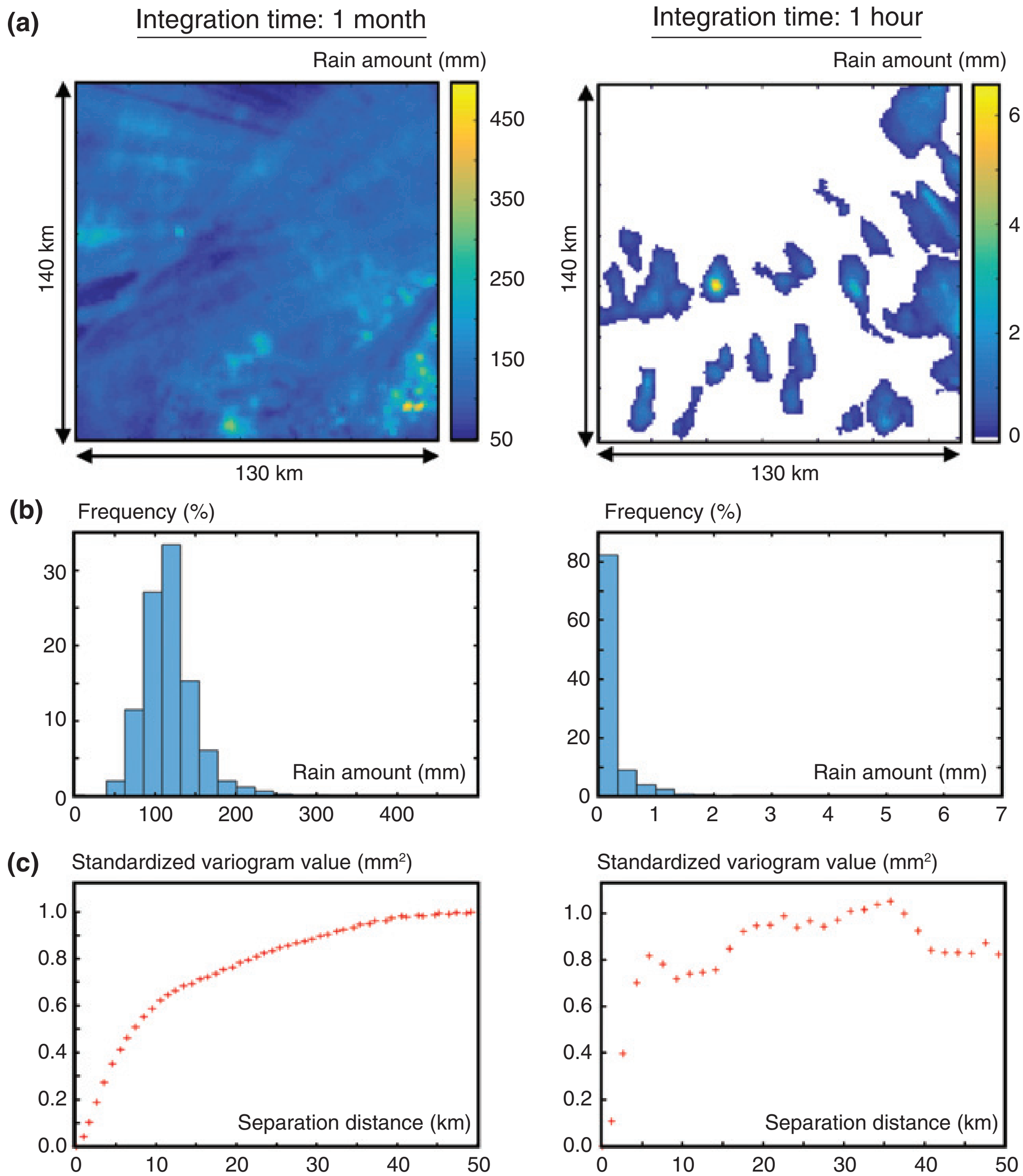


FIGURE 4 | Typical rain fields measured by weather radar (Source: MeteoSwiss) over northwestern Switzerland (a) and corresponding histograms (b) and experimental variograms (c) for monthly (May 2005, left) and hourly (May 5, 2005, 10–11 h, right) integration times. Artifacts observed in the monthly integrated rain field are due to instrumental biases in radar images.

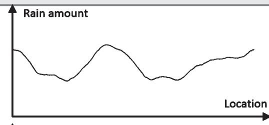
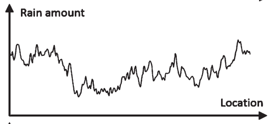
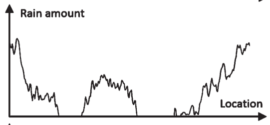
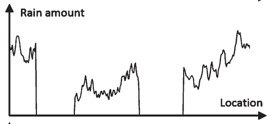
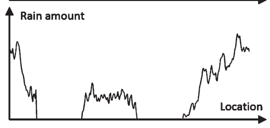
$$Y = F(R),\ Y \sim N(0,1) \tag{7}$$

The transformation, or anamorphosis, consists of applying a deterministic function to the empirical histogram in order to obtain a Gaussian histogram for the transformed field (Table 2). Note that in the case of a truncated multivariate Gaussian field, the Gaussian anamorphosis is applied only to the noncensored part of the random field. The covariance structure of the rain process is then modeled by a latent multivariate Gaussian random field (Figure 5), which is not directly observed, but is linked to the true rain field trough the Gaussian anamorphosis. Several transformations have been proposed such as normal-score transform,[67] box–cox transform,[60] power function,[59] gamma function,[64] quadratic form of the power transform,[42] or power exponential.[44]

Component 3: Variogram Model

A variogram model must be selected to represent the spatial correlation of rainfall. Because the form of the variogram is limited by the need to respect some properties of the covariance structure, it is usually chosen

TABLE 2 | Geostatistical Spatial Models of Rainfall and Examples of One-Dimensional (1D) Synthetic Rain Fields Generated by the Corresponding Models

Data Resolution	Random Field Model	Gaussian Anamorphosis	Variogram Model	1D Aspect of the Rain Field
>1 month	Single multivariate Gaussian	No	Cubic Gaussian	
1 day to 1 month	Single multivariate Gaussian	No	Spherical Exponential	
<1 day	Truncated multivariate Gaussian	Yes	Spherical Exponential	
<1 day	Independent superposed random fields	Yes	Spherical Exponential	
<1 day	Independent superposed random fields + deterministic drift	Yes	Spherical Exponential	

among a list of predefined types (see e.g., Ref 34 for a detailed description of usual models, and their analytical expression) known to have desired mathematical properties, such as positive-definitiveness to avoid the occurrence of negative variances.

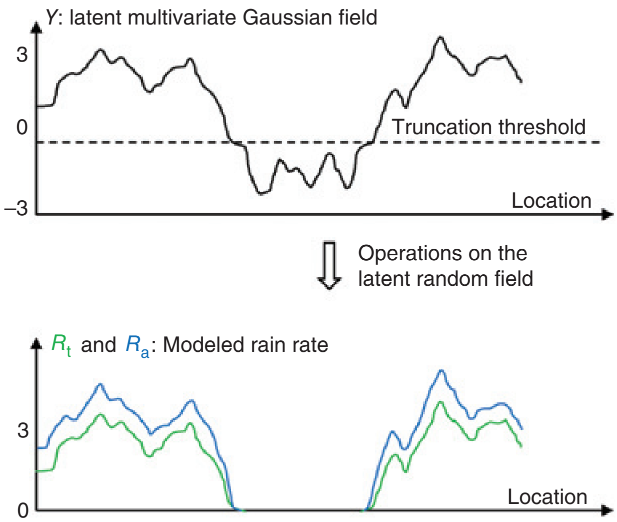


FIGURE 5 | Rainfall modeling by a latent multivariate Gaussian random field (Y). The green curve (R_t) represents a rain field modeled by a truncated field. The blue curve (R_a) represents a rain field modeled by a truncated field coupled with a Gaussian anamorphosis. The difference between R_t and R_a is the change in rain values incurred by the anamorphosis.

Due to support effects, the apparent rain variability depends on the temporal resolution of data, thereby leading to different types of variogram models being appropriate depending on the resolution (Table 2). For yearly to monthly integration times, the rain field is quite smooth and its covariance structure can be modeled by a cubic[48] or Gaussian variogram model. With increasing measurement resolution, the rain field often has a rough spatial structure. In this case, spherical[51,52,66,67] or exponential[46,50,54] variogram models are selected to accommodate the high spatial variability of rainfall. Finally, when observed at high temporal resolution (integration time under 5 min), the rain field sometimes exhibits experimental variograms with two nested structures.[69,70] This can be interpreted as the distinct signature of rain cells and mesoscale clusters, and modeled by the linear combination of two variogram models.

Once a variogram type has been selected, its parameters must be inferred from the available data. In case of single or superimposed multivariate Gaussian fields, the parameters of the variogram model are usually inferred from the experimental variogram either by visual or by weighted least squares fitting.[34] In case of truncated multivariate Gaussian fields, a maximum likelihood approach is preferred because this allows to take the censored values in consideration for variogram parameter estimation.[42,44]

Table 2 summarizes the different geostatistical models proposed to match the rainfall spatial patterns observed at several temporal resolutions.

Component 3b: Extension of Variogram Models to Space–Time Data

Taken in its full complexity, rainfall is a process that varies in both space and time. In this case, the geostatistical framework can be extended to investigate not only the spatial covariance structure of the rain, but also its temporal covariance structure and in turn the interactions between these spatial and temporal structures.[71] To this end, time is considered as a third dimension of the modeling domain with several consecutive times t_j. Locations u_i are reindexed to include time in their coordinates: $u_{i,j} = (x_i, y_i, t_j)$, $i = 1 \ldots N$, $j = 1 \ldots M$. The extension from spatial to space–time models then consists of adopting new variogram models (component 3) that are compatible with the covariance structure of the newly defined space–time measurements. Fortunately, the two other components (construction of a type of random field and Gaussian anamorphosis) of the geostatistical model extend directly to the space–time context.

For integration times longer than one day, the extension to space–time data consists of enabling the spatial patterns to evolve smoothly along time. In practice, this results in adding a temporal component to the previously defined spatial variogram models in order to create symmetric space–time variogram models.[71,72] The addition of this temporal correlation term into the variogram model leads to model the temporal morphing of the spatial rain patterns (Figure 6(a)).

For subdaily integration times, the effect of advection becomes significant in rain measurements (Figure 6(b)). Indeed the rain generation processes occur within the clouds which shift over the ground. This ‘moving rain’ influences the space–time covariance structure of the rain field, and space–time variogram analyses show that advection has a strong effect on the space–time covariance structure[73–75] that makes it asymmetric. Asymmetric space–time variogram models[76,77] should therefore be used to model the effect of advection.

Geostatistical Rainfall Generation

Once a geostatistical model has been designed and its parameters inferred, the multivariate Gaussian distribution used to mimic the rain structure is totally determined. This multivariate Gaussian distribution has then to be sampled in order to generate several synthetic rain fields which reproduce the rain structure: this is the principle of geostatistical simulation.

The standard method for sampling of a spatialized multivariate Gaussian distribution consists in multiplying a vector of independent and identically distributed (iid) normal random variables

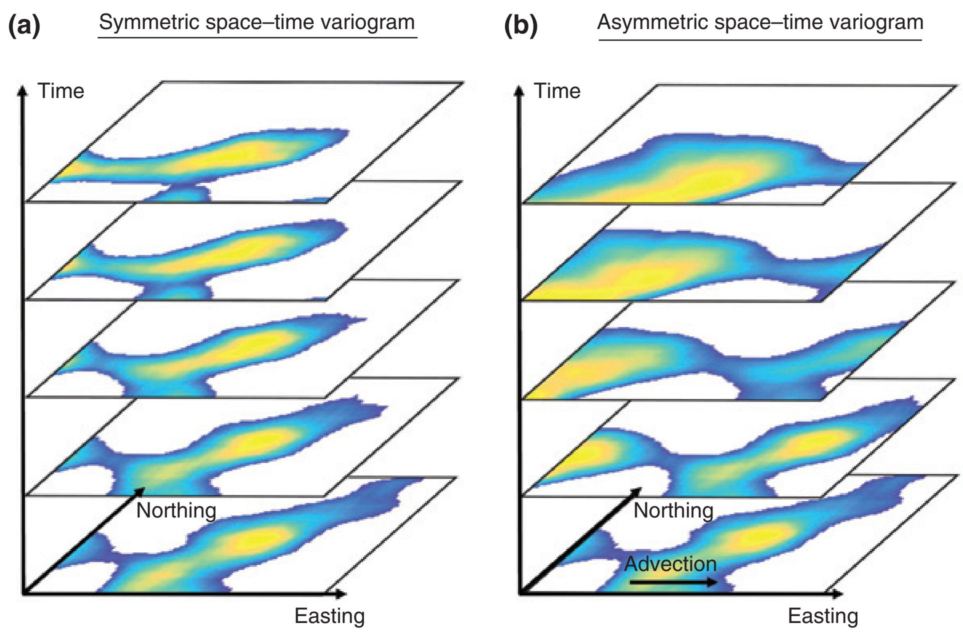


FIGURE 6 | Synthetic rain fields generated with a symmetric space–time variogram model (no advection) (a) and an asymmetric space–time variogram model (constant advection) (b).

TABLE 3 | Geostatistical Simulations for Rainfall Generation

Simulation Method	Advantages	Drawbacks	References
Choleski decomposition	Exact method	Computational cost	34
Sequential Gaussian simulation	Easy to implement	Questionable reproduction of the covariance structure when a local neighborhood is used[104]	105,106
Simulated annealing	Allows for imposing additional constraints	Computational cost	107
Spectral simulation	Good reproduction of the prescribed covariance Fast if FFT is used	Higher computational cost for rough structures	78,108
Turning bands	Good reproduction of the prescribed covariance Fast	Complex to implement	78,51

located at each point of the simulation domain by the 'square root' of the variance-covariance matrix of the model obtained by Choleski decomposition. This method gives rigorous results but is unfortunately restricted to small datasets (grid size under 100×100 cells) because it requires the Choleski decomposition of a matrix whose size is the square of the number of simulation locations, a computationally expensive operation.

In practice, approximate methods must be used to perform the sampling of the multivariate Gaussian distribution. The choice of the simulation method depends of the application of interest. Table 3 summarizes the different geostatistical simulation methods available for rainfall generation and outlines their main advantages and drawbacks.

All these simulation methods generate a set of synthetic rain fields that reproduce the covariance structure prescribed by the variogram model independently of any sample data. Such simulations are called nonconditional simulations.

However, many applications (e.g., interpolation of sparse rainfall measurements) require the simulated rain fields to agree with observed rainfall values at some locations. An additional step, called conditioning, is then needed to pass from the nonconditional simulation that only honors the covariance structure of the calibration dataset to a conditional simulation that also matches the measured rainfall values at given locations. The general procedure to achieve this is conditioning by Kriging.[34,78] It considers the conditional simulation as the superposition of a nonconditional simulation and a kriging estimate of the mismatch between the nonconditional simulation and observations (Figure 7). This approach allows conditioning the simulation obtained by all the methods mentioned above, except for Sequential Gaussian Simulation which produces realizations that are already conditioned by construction.

CONCLUSION

Geostatistics have proved invaluable to deal with the reproduction of the covariance structure of rainfall. Thanks to steady improvement in geostatistical

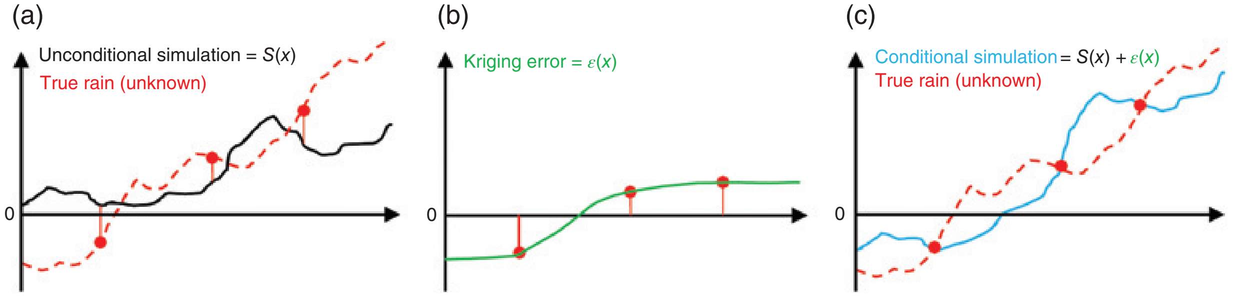


FIGURE 7 | Conditioning a simulation. (a) One-dimensional unconditional simulation (black curve) compared to the true rain field (dashed red curve) which is unknown except at observation locations (red dots). (b) Kriging error (green curve) obtained by kriging the mismatch between unconditional simulation and observations at observation locations (red bars). (c) Conditional simulation (blue curve) obtained by adding the kriging error to the unconditional simulation.

models, even complex space–time rain fields observed at high resolution have been successfully modeled and reproduced.

Geostatistical simulations are then relevant to interpolate sparse rainfall measurements,[55,79] to generate nonconditional rain fields with prescribed spatial covariance structure,[51,53,54] and to deal with measurement errors having complex spatial properties.[56,57] In addition, recent advances in modeling the space–time behavior of rainfall[74,75,80] have led to improved rainfall simulations at the subdaily time scale. This is particularly useful to generate synthetic rain fields that mimic features of rainfall such as advection over the ground or morphing of spatial rain patterns along time. Due to these characteristics, applications of synthetic rain fields generated by geostatistical simulation are important for representing processes driven by the spatial distribution of rainfall and its temporal evolution, such as flash floods,[5] landslide triggering[8] or erosion.[81]

However, geostatistical simulations are also subject to limitations. In the first instance they strongly rely on multivariate Gaussian random fields. This modeling assumption, unavoidable for ensuring the reproduction of the covariance structure at a large number of locations, makes geostatistical simulations blind to structures governed by statistics above order two. Where such high order statistics explain a significant part of the process of interest, geostatistical simulations are only able to take into account the limited part of the structure explained by two-point statistics, which could lead to unrealistic results.[82] Fortunately, currently available rainfall data suggest that geostatistical models are often adequate, with the notable exception of local scale orographic effects and strong convective events for which geostatistical simulations can fail to reproduce the occurrence of extreme rainfalls.

In the future, the development of geostatistical simulations seems promising for stochastic generation of subdaily distributed rainfall. To capitalize on recent advances in this domain,[51,53,83] efforts should continue to provide more flexible models of rain occurrence, advection, and space–time covariance structure. However, regardless of the improvements in geostatistical simulations, it is acknowledged that some features of the rain field will never be captured by this framework. This is the case for extreme rain events and for rain fields made of spatial patterns characterized by high order statistics. In that case, it looks reasonable to restrict the use of geostatistical simulations to what they are really able to reproduce, i.e., distributed rain fields. To model and reproduce the rain field in all its diversity, an interesting approach that should be pursued consists in combining geostatistical simulations with other stochastic methods such as fractals, point processes or multiple-points statistics in order to build hierarchical models which capitalize on the strengths of each involved methods.[83–85]

A last and challenging topic for the future is the integration of the current geostatistical simulation methods for rainfall generation into weather generators that simultaneously account for several components of the climate system (e.g., temperature, pressure, precipitation, etc.) and their interactions. A first possibility is to switch from simulations which account only for rainfall, towards multivariate simulations that take into account correlations with climatic covariates.[86] A second option is to constrain the geostatistical parameters to external variables such as synoptic conditions or weather types. This could allow rainfall to be linked to other components of the weather generator by conditioning every member to the same external variables. This solution is already used to embed precipitations in weather generators in case of multisite temporal models,[87] and an extension to geostatistical models would be attractive.

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REFERENCES

1. Morin E, Goodrich DC, Maddox RA, Gao X, Gupta HV, Sorooshian S. Spatial patterns in thunderstorm rainfall events and their coupling with watershed hydrological response. *Adv Water Resour* 2006, 29:843–860.
2. Mandapaka PV, Krajewski WF, Mantilla R, Gupta VK. Dissecting the effect of rainfall variability on the statistical structure of peak flows. *Adv Water Resour* 2009, 32:1508–1525.

3. Zoccatelli D, Borga M, Zanon F, Antonescu B, Stancalie G. Which rainfall spatial information for flash flood response modelling? A numerical investigation based on data from the Carpathian range, Romania. *J Hydrol* 2010, 394:148–161.

4. Nokolopoulos I, Anagnostou EN, Borga M, Vivoni ER, Papadopoulos A. Sensitivity of a mountain basin flash flood to initial wetness condition and rainfall variability. *J Hydrol* 2011, 402:165–178.

5. Paschalis A, Fatichi S, Molnar P, Rimkus S, Burlando P. On the effects of small scale space–time variability of rainfall on basin flood response. *J Hydrol* 2014, 514:313–327.

6. Bruni G, Reinoso R, van de Giesen NC, Clemens FHLR, ten Veldhuis JAE. On the sensitivity of urban hydrodynamic modelling to rainfall spatial and temporal resolution. *Hydrol Earth Syst Sci* 2015, 19:691–709.

7. Owor M, Taylor RG, Tindimugaya C, Mwesigwa D. Rainfall intensity and groundwater recharge: empirical evidence from the Upper Nile Basin. *Environ Res Lett* 2009, 4:035009.

8. von Ruette J, Lehmann P, Or D. Effects of rainfall spatial variability and intermittency on shallow landslide triggering patterns at a catchment scale. *Water Resour Res* 2014, 50:7780–7799.

9. Schlabing D, Frassl MA, Eder MM, Rinke K, Bárdossy A. Use of a weather generator for simulating climate change effects on ecosystems: a case study on Lake Constance. *Environ Model Software* 2014, 61:326–338.

10. Mavromatis T, Hansen JW. Interannual variability characteristics and simulated crop response of four stochastic weather generators. *Agric For Meteorol* 2001, 109:283–296.

11. Qian B, De Jong R, Yang J, Wang H, Gameda S. Comparing simulated crop yields with observed and synthetic weather data. *Agric For Meteorol* 2011, 151:1781–1791.

12. Supit I, van Diepen CA, de Wit AJW, Wolf J, Kabat P, Baruth B, Ludwig F. Assessing climate change effects on European crop yields using the Crop Growth Monitoring System and a weather generator. *Agric For Meteorol* 2012, 164:96–111.

13. Bauer P, Thorpe A, Brunet G. The quiet revolution of numerical weather prediction. *Nature* 2015, 525:47–55.

14. Bony S, Stevens B, Frierson D, Jakob C, Kageyama M, Pincus R, Shepherd TG, Sherwood S, Siebesma A, Sobel AH, et al. Clouds, circulation and climate sensitivity. *Nat Geosci* 2015, 8:261–268.

15. Strangeways I. A history of rain gauges. *Weather* 2010, 65:133–138.

16. Tapiador FJ, Turk FJ, Petersen W, Hou AY, García-Ortega E, Machado LA, Angelis CF, Salio P, Kidd C, Huffman GJ. Global precipitation measurement: methods, datasets and applications. *Atmos Res* 2012, 104:70–97.

17. Ciach G. Local random errors in tipping-bucket rain gauge measurements. *J Atmos Oceanic Tech* 2003, 20:752–759.

18. Wang J, Fischer BL, Wolff DB. Estimating rain rates from tipping-bucket rain gauge measurements. *J Atmos Oceanic Tech* 2008, 25:43–56.

19. Lanza LG, Vuerich E. The WMO field intercomparison of rain intensity gauges. *Atmos Res* 2009, 94:534–543.

20. Villarini G, Mandapaka PV, Krajewski K, Moore RJ. Rainfall and sampling uncertainties: a rain gauge perspective. *J Geophys Res* 2008, 113, D11102. doi:10.1029/2007JD009214.

21. Gires A, Tchiguirinskaia I, Schertzer D, Schellart A, Berne A, Lovejoy S. Influence of small scale rainfall variability on standard comparison tools between radar and rain gauge data. *Atmos Res* 2014, 138:125–138.

22. Germann U, Galli G, Boscacci M, Bolliger M. Radar precipitation measurement in a mountainous region. *Q J R Meteorol Soc* 2006, 132:1669–1692.

23. Berne A, Krajewski WJ. Radar for hydrology: unfulfilled promise or unrecognized potential? *Adv Water Resour* 2013, 51:357–366.

24. Lengfeld K, Clemens M, Münster H, Ament F. Performance of high-resolution X-band weather radar networks—the PATTERN example. *Atmos Meas Tech* 2014, 7:4151–4166.

25. Kummerow C, Barnes W, Kozu T, Shiue J, Simpson J. The Tropical Rainfall Measuring Mission (TRMM) sensor package. *J Atmos Oceanic Tech* 1998, 15:809–817.

26. Biasutti M, Yuter SE, Burleyson CD, Sobel AH. Very high resolution rainfall patterns measured by TRMM precipitation radar: seasonal and diurnal cycles. *Clim Dyn* 2012, 39:239–258.

27. Hou AY, Kakar RK, Neeck S, Azarbarzin AA, Kummerow CD, Kojima M, Oki R, Nakamura K, Iguchi T. The global precipitation measurement mission. *Bull Am Meteorol Soc* 2014, 95:701–722.

28. Evans JP, McCabe MF. Regional climate simulation over Australia's Murray–Darling basin: a multitemporal assessment. *J Geophys Res* 2010, 115:D14114.

29. Braconnot P, Harrison SP, Kageyama M, Bartlein PJ, Masson-Delmotte V, Abe-Ouchi O-BAB, Zhao Y. Evaluation of climate models using palaeoclimatic data. *Nat Clim Change* 2012, 2:417–424.

30. Shepherd TG. Atmospheric circulation as a source of uncertainty in climate change projections. *Nat Geosci* 2014, 2014:703–708.

31. Stensrud DJ, Yussouf N. Reliable probabilistic quantitative precipitation forecasts from a short-range

ensemble forecasting system. *Weather Forecast* 2007, 22:2–17.

32. Willems P, Vrac M. Statistical precipitation downscaling for small-scale hydrological impact investigations of climate change. *J Hydrol* 2011, 402:193–205.
33. Goovaerts P. *Geostatistics for Natural Resources Evaluation*. New York: Oxford University Press; 1997.
34. Chilès JP, Delfiner P. *Modeling Spatial Uncertainty*. 2nd ed. John Wiley & Sons New Jersey 2012.
35. Morin E, Krajewski WF, Goodrich DC, Gao X, Sorooshian S. Estimating rainfall intensities from weather radar data: the scale-dependency problem. *J Hydrometeorol* 2003, 4:782–797.
36. Jensen NE, Pedersen L. Spatial variability of rainfall: variations within a single radar pixel. *Atmos Res* 2005, 77:269–277.
37. Chappell A, Renzullo L, Raupach TH, Haylock M. Evaluating geostatistical methods of blending satellite and gauge data to estimate near real-time daily rainfall for Australia. *J Hydrol* 2013, 493:105–114.
38. Peleg N, Ben-Ascher M, Morin E. Radar subpixel-scale rainfall variability and uncertainty: lessons learned from observations of a dense rain-gauge network. *Hydrol Earth Syst Sci* 2013, 17:2195–2208.
39. Berndt C, Rabiei E, Haberlandt U. Geostatistical merging of rain gauge and radar data for high temporal resolutions and various station density scenarios. *J Hydrol* 2014, 508:88–101.
40. Sideris IV, Gabella M, Erdin R, Germann U. Real-time radar–rain-gauge merging using spatio-temporal co-kriging with external drift in the alpine terrain of Switzerland. *Q J Roy Meteorol Soc* 2014, 140:1097–1111.
41. Park N. Spatial downscaling of TRMM precipitation using geostatistics and fine scale environmental variables. *Adv Meteorol* 2013, 2013. http://dx.doi.org/10.1155/2013/237126
42. Allcroft DJ, Glasbey CA. A latent Gaussian Markov random-field model for spatiotemporal rainfall disaggregation. *Appl Stat* 2003, 52:487–498.
43. Onibon H, Lebel T, Afoufa A, Guillot G. Gibbs sampling for conditional spatial disaggregation of rain fields. *Water Resour Res* 2004, 40:W08401. doi:10.1029/2003WR002009.
44. Allard D, Bourotte M. Disaggregating daily precipitations into hourly values with a transformed censored latent Gaussian process. *Stoch Environ Res Risk Assess* 2015, 29:453–462.
45. Gandin L. Objective analysis of meteorological fields, Gidro-meteorologichoskoe Isdatel'stvo, Leningrad, translated from Russian, Israel Program for Scientific Translation, Jerusalem, *Q J R Meteorol Soc* 1963, 92. doi:10.1002/qj.49709239320.
46. Creutin JD, Obled C. Objective analyses and mapping techniques for rainfall fields': an objective comparison. *Water Resour Res* 1982, 18:413–431.
47. Lebel T, Bastin G, Obled C, Creutin JD. On the accuracy of areal rainfall estimation: a case study. *Water Resour Res* 1987, 23:2123–2134.
48. Goovaerts P. Geostatistical approaches for incorporating elevation into the spatial interpolation of rainfall. *J Hydrol* 2000, 228:113–129.
49. Lloyd CD. Assessing the effect of integrating elevation data into the estimation of monthly precipitation in Great Britain. *J Hydrol* 2005, 308:128–150.
50. Moral FJ. Comparison of different geostatistical approaches to map climate variables: application to precipitation. *Int J Climatol* 2010, 30:620–631.
51. Leblois E, Creutin JD. Space-time simulation of intermittent rainfall with prescribed advection field: adaptation of the turning band method. *Water Resour Res* 2013, 49:3375–3387.
52. Schleiss M, Chamon S, Berne A. Stochastic simulation of intermittent rainfall using the concept of "dry drift". *Water Resour Res* 2014, 50:2329–2349.
53. Baxevani A, Lennartsson J. A spatiotemporal precipitation generator based on a censored latent Gaussian field. *Water Resour Res* 2015, 51:4338–4358.
54. Kleiber W, Katz RW, Rajagopalan B. Daily spatiotemporal precipitation simulation using latent and transformed Gaussian processes. *Water Resour Res* 2012, 48:W01523. doi:10.1029/2011WR011105.
55. Pardo-Igúzquiza E, Grimes DIF, Teo CK. Assessing the uncertainty associated with intermittent rainfall fields. *Water Resour Res* 2006, 42, W01412. doi:10.1029/2004WR003740.
56. Ahrens B, Jaun S. On evaluation of ensemble precipitation forecasts with observation-based ensembles. *Adv Geosci* 2007, 10:139–144.
57. Germann U, Berenguer M, Sempere-Torres D, Massimiliano Z. REAL – Ensemble radar precipitation estimation for hydrology in a mountainous region. *Q J R Meteorol Soc* 2009, 135:445–456.
58. Peleg N, Marra F, Fatichi S, Paschalis A, Molnar P, Burlando P. Spatial variability of extreme rainfall at radar subpixel scale. *J Hydrol* 2016. doi:10.1016/j.jhydrol.2016.05.033.
59. Ailliot P, Thompson C, Thomson P. Space–time modelling of precipitation by using a hidden Markov model and censored Gaussian distributions. *Appl Stat* 2006, 58:405–426.
60. Erdin R, Frei C, Kunsch HR. Data transformation and uncertainty in geostatistical combination of radar and rain gauges. *J Hydrometeorol* 2012, 13:1332–1346.
61. Sun Y, Bowman KP, Genton M, Tokay A. A Mate´rn model of the spatial covariance structure of point rain

rates. *Stoch Environ Res Risk Assess* 2015, 29:411–416.

62. Krajewski WF, Ciach G, Habib E. An analysis of small-scale rainfall variability in different climatic regimes. *Hydrol Sci J* 2003, 48:151–162.
63. Schleiss M, Jaffrain J, Berne A. Statistical analysis of rainfall intermittency at small spatial and temporal scales. *Geophys Res Lett* 2011, 38:L18403. doi:10.1029/2011GL049000.
64. Guillot G. Approximation of Sahelian rainfall fields with meta-Gaussian random functions; Part 1: model definition and methodology. *Stoch Environ Res Risk Assess* 1999, 13:1000–1112.
65. Sansó B. L. G. Venezuelan rainfall data analysed by using a Bayesian space-time model. *Appl Stat* 1999, 48:345–362.
66. Barancourt C, Creutin JD, Rivoirard J. A method for delineating and estimating rainfall fields. *Water Resour Res* 1992, 28:1133–1144.
67. Grimes IFG, Pardo-Igúzquiza E. Geostatistical analysis of rainfall. *Geograph Anal* 2010, 42:136–160.
68. Schleiss M, Chamon S, Berne A. Nonstationarity in intermittent rainfall: the "dry drift". *J Hydrometeorol* 2014, 15:1189–1204.
69. Berne A, Delrieu G, Creutin JD, Obled C. Temporal and spatial resolution of rainfall measurements required for urban hydrology. *J Hydrol* 2004, 299:166–179.
70. Emmanuel I, Andrieu H, Leblois E, Flahaut B. Temporal and spatial variability of rainfall at the urban hydrological scale. *J Hydrol* 2012, 430–431:162–172.
71. Kyriakidis PC, Journel AG. Geostatistical space-time models: a review. *Math Geol* 1999, 31:651–684.
72. Gneiting T. Nonseparable, stationary covariance functions for space-time data. *J Am Stat Assoc* 2002, 97:590–600.
73. Zawadski II. Statistical properties of precipitation patterns. *J Appl Meteorol* 1973, 12:459–472.
74. Lepioufle JM, Leblois E, Creutin JD. Variography of rainfall accumulation in presence of advection. *J Hydrol* 2012, 464–465:494–504.
75. Creutin JD, Leblois E, Lepioufle JM. Unfreezing Taylor's hypothesis for precipitation. *J Hydrometeorol* 2015. doi:10.1175/JHM-D-14-0120.1.
76. Stein ML. Space-time covariance functions. *J Am Stat Assoc* 2005, 100:310–320.
77. De Iaco S, Posa D, Myers DE. Characteristics of some classes of space–time covariance functions. *J Stat Plan Inference* 2013, 143:2002–2015.
78. Lantuéjoul C. *Geostatistical Simulation: Models and Algorithms*. Springer-Verlag, Berlin 2002.
79. Chappell A, Renzullo L, Haylock M. Spatial uncertainty to determine reliable daily precipitation maps. *J Geophys Res* 2012, 117:D17115. doi:10.1029/2012JD017718.
80. Li B, Murthi A, Bowman KP, North GR, Genton MG, Sherman M. Statistical tests of taylor's hypothesis: an application to precipitation fields. *J Hydrometeorol* 2009, 10:254–265.
81. Fiener P, Auerswald K. Spatial variability of rainfall on a sub-kilometre scale. *Earth Surf Process Landf* 2009, 34:848–859.
82. Zinn B, Harvey CF. When good statistical models of aquifer heterogeneity go bad: a comparison of flow, dispersion, and mass transfer in connected and multivariate Gaussian hydraulic conductivity fields. *Water Resour Res* 2003, 39:1051, doi:10.1029/2001WR001146, 3.
83. Paschalis A, Molnar P, Fatichi S, Burlando P. A stochastic model for high-resolution space-time precipitation simulation. *Water Resour Res* 2013, 49:8400–8417. doi:10.1002/2013WR014437.
84. Wojcik R, McLaughling D, Konings AG, Entekhabi D. Conditioning stochastic rainfall replicates on remote sensing data. *IEEE Trans Goesci Remote Sens* 2009, 48:2436–2449.
85. Peleg N, Morin E. Stochastic convective rain-field simulation using a high-resolution synoptically conditioned weather generator (HiReS-WG). *Water Resour Res* 2014, 50:2124–2139.
86. Bourotte M, Allard D, Porcu E. A flexible class of non-separable cross-covariance functions for multivariate space–time data. *Spat Stat* 2016. doi: 10.1016/j.spasta.2016.02.004.
87. Ailliot P, Allard D, Monbet V, Naveau P. Stochastic weather generators: an overview of weather type models. *J Soc Fr Stat* 2015, 156:101–113.
88. Onov C, Chandler RE, Kakou A, Northrop A, Wheater HS, Isham V. Rainfall modelling using Poisson-cluster processes: a review of developments. *Stoch Environ Res Risk Assess* 2000, 14:384–411.
89. Zhang Z, Switzer P. Stochastic space-time regional rainfall modeling adapted to historical rain gauge data. *Water Resour Res* 2007, 43:W03441. doi:10.1029/2005WR004654.
90. Cowperwait PSP. A spatial-temporal point process model with a continuous distribution of storm types. *Water Resour Res* 2010, 46:W12507. doi:10.1029/2010WR009728.
91. Willems P. A spatial rainfall generator for small spatial scales. *J Hydrol* 2001, 252:126–144.
92. Schertzer D, Lovejoy S. Physical modeling and analysis of rain and clouds by anisotropic scaling multiplicative processes. *J Geophys Res* 1987, 92:9693–9714.
93. Molnar P, Burlando P. Preservation of rainfall properties in stochastic disaggregation by a simple random cascade model. *Atmos Res* 2005, 77:137–151.

94. Hughes JP, Guttorp P. A non-homogeneous hidden Markov model for precipitation occurence. *Appl Stat* 1999, 48:15–30.
95. Bárdossy A, Pegram GGS. Copula based multisite model for daily precipitation simulation. *Hydrol Earth Syst Sci* 2009, 13:2299–2314.
96. Johnson GL, Daly C, Taylor GH, Hanson CL. Spatial variability and interpolation of stochastic weather simulation model parameters. *J Appl Meteorol* 2000, 39:778–796.
97. Yang C, Chandler RE, Isham VS, Wheater HS. Spatial-temporal rainfall simulation using generalized linear models. *Water Resour Res* 2005, 41:W11415. doi:10.1029/2004WR003739.
98. Furrer EM, Katz RW. Generalized linear modeling approach to stochastic weather generators. *Climate Res* 2007, 34:129–144.
99. Verdin A, Rajagopalan B, Kleiber W, Katz RW. Coupled stochastic weather generation using spatial and generalized linear models. *Stoch Environ Res Risk Assess* 2015, 29:347–356.
100. Ambrosino C, Chandler RE, Todd MC. Southern African monthly rainfall variability: an analysis based on generalized linear models. *J Clim* 2011, 26:4600–4617.
101. Rust HW, Vrac M, Sultan B, Lengaigne M. Mapping weather-type influence on senegal precipitation based on a spatial–temporal statistical model. *J Clim* 2013, 26:8189–8209.
102. Oriani F, Straubhaar J, Renard P, Mariethoz G. Simulation of rainfall time series from different climatic regions using the direct sampling technique. *Hydrol Earth Syst Sci* 2014, 18:3015–3031.
103. Jha SK, Mariethoz G, Evans J, McCabe MF, Sharma A. A space and time scale-dependent nonlinear geostatistical approach for downscaling daily precipitation and temperature. *Water Resour Res* 2014, 51:6244–6261.
104. Lantuéjoul C, Dessassis N. Simulation of a Gaussian random vector: a propagative version of the Gibbs sampler. In: *Proceedings of the 9th International Geostatistics Congress*, Oslo, Norway, June, 2012, 174–181.
105. Gómez-Hernández JJ, Journel AG. Joint sequential simulation of multigaussian fields. In: *Geostatistics Troia'92*. Netherlands: Springer; 1993, 85–94.
106. Emery X. Testing the correctness of the sequential algorithm for simulating Gaussian random fields. *Stoch Environ Res Risk Assess* 2004, 18:401–413.
107. Deutsch CV, Cockerham PW. Practical considerations in the application of simulated annealing to stochastic simulation. *Math Geol* 1994, 26:67–82.
108. Le Ravalec M, Noetinger B, Hu LY. The FFT moving average (FFT-MA) generator: an efficient numerical method for generating and conditioning Gaussian simulations. *Math Geol* 2000, 32:701–722.