Iterative spatial resampling applied to seismic inverse modeling for lithofacies prediction

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Summary

Inverse modeling, which transforms obtained geophysical data into physical properties of the Earth, is an essential process for reservoir characterization. We propose a Markov chain Monte Carlo (McMC) workflow consistent with geology, well-logs, seismic data and rock-physics information. The workflow uses Direct Sampling (DS), a multiple-point geostatistical method, for generating realizations from the prior distribution and Iterative Spatial Resampling (ISR) for sampling from the posterior distribution conditioned to the geophysical data. We also show how adaptive sampling can improve efficiency of the method. The ISR algorithm perturbs realizations of a spatially dependent variable while preserving its spatial structure. The method is used as a transition kernel to produce a Markov chain of geostatistical realizations. These realizations are then used in a forward seismic model to compute the predicted data which are compared to the observed data. Depending on the acceptation/rejection criterion in the Markov process, it is possible to obtain a chain of realizations aimed either at characterizing the posterior distribution with Metropolis sampling or at calibrating a single realization until an optimum is reached. Thus the algorithm can be tuned to work either as an optimizer or as a sampler. The validity and applicability of the proposed method is demonstrated by results for seismic lithofacies inversion on a synthetic test set.

Introduction

Seismic data play a key role to reduce uncertainty in predictions of rocks and fluids away from well control points. However, in real applications it is nearly impossible to find a unique relationship between seismic response and reservoir properties. Seismic measurements are noisy and have larger scales of resolution than well data. Moreover, the relationships are non-unique due to the limited frequency of seismic waves, the forward modeling simplifications, and natural heterogeneity.

Statistical rock physics accounts for some of the uncertainty using multi-variate stochastic relations between elastic parameters and reservoir properties (Mukerji et al, 2001a; Mukerji et al, 2001b; Avseth et al, 2005). Many different workflows have been suggested to combine rock physics and geostatistical methods in seismic inversion. Bosch et al. (2010) classified these approaches into two groups, which are the sequential or cascaded approach and the joint or simultaneous workflow in a Bayesian formulation. The joint or simultaneous workflow accounts for the elastic parameters and the reservoir properties together and provides combined uncertainties. These Bayesian workflows include rock-physics relations to link reservoir properties and elastic properties and geostatistical models to provide geologically consistent prior models. Forward modeled synthetic data are compared with obtained seismic data to calculate the likelihood, and the final solutions are posterior models consistent with the expected geology, well data and seismic data. Gonzalez et al. (2008) combined multiple points geostatistics (MPS) and rock physics for seismic inversion. They generate multiple realizations of reservoir facies and saturations, conditioned to seismic and well data. MPS is used to characterize the geologic prior information, and statistical rock physics links reservoir properties to elastic properties. Thus their method provided multiple realizations, all consistent with the expected geology, well-log, seismic data and local rock-physics transformations. However, this workflow did not produce samples of the full posterior probability density function but generated multiple optimized models around the mode of the posterior. Also the MPS algorithm was inefficient for applying to 3-D and complicated actual field cases.

Since the conventional MPS algorithms such as SNESIM or SIMPAT store all data events from the training image (Strebelle, 2000; Arpat, 2005), computing load is dramatically increased according to the size of the template and the number of facies. On the other hand, in posterior sampling methods, rejection sampling method (Tarantola, 2005) is the only one to rigorously sample the posterior pdf. However, since it requires a large number of evaluations of forward model, rejection sampling is inefficient. Therefore, a key issue is to generate prior models and to find the most likely models honoring both spatial constraints and seismic data within limited computation time and cost.

We propose a Markov chain Monte Carlo (McMC) workflow consistent with geology, well-logs, seismic data and rock-physics information. The workflow uses Direct Sampling (DS), a multiple-point geostatistical method, for generating realizations from the prior distribution and Iterative Spatial Resampling (ISR) for sampling from the posterior distribution conditioned to the geophysical data. The DS algorithm (Mariethoz, 2009) characterizes the geological prior information and is used to condition to well data. The DS algorithm directly samples the training image for any given data event, without storing all patterns in a database. Therefore, less memory intensive DS can reproduce the structures of complex training images and deal with a wide range of non-stationary problems. The ISR algorithm perturbs realizations of a spatially dependent variable while preserving its spatial structure. The method is used as a transition kernel to produce a Markov chain of geostatistical realizations. These realizations are then used in a forward seismic model to compute the predicted data which are compared to the observed data. Depending on the acceptation/rejection criterion in the Markov process, it is possible to obtain a chain of realizations aimed either at characterizing the posterior distribution with Metropolis sampling or at calibrating a single realization until an optimum is reached. Thus the algorithm can be tuned to work either as an optimizer or as a sampler. The proposed method is demonstrated on a synthetic test dataset.

Methodology

Seismic inverse modeling

The transformation of geophysical data into reservoir properties can be posed as an inference problem involving the updating of prior knowledge with newly available data (Tarantola, 1987, 2005). It can be expressed as

$$\sigma_{post}(m) = c \times \gamma_{prior}(m) \times \gamma_{data}\big(d_{obs} - g(m)\big), \qquad (1)$$

where $\sigma_{post}(m)$ is the posterior probability density and $\gamma_{prior}(m)$ is the a priori probability density. In equation 1, c is a normalizing constant, and m is the earth model parameter configuration. The expression $\gamma_{data}\big(d_{obs} - g(m)\big)$ is the data-likelihood function; and it depends on the observations d_{obs} and their uncertainty, and the forward modeling operator **g** that maps the model space into the data space. The solutions of an inverse problem are the set of earth-model configurations that, when forward modeled into synthetic data, match the real data within some tolerance.

According to the chain rule (e.g., Bosch, 1999; Bosch et al., 2004), decomposing the model space into reservoir parameters (facies, porosity, etc.) and elastic parameters (Vp, Vs, density) the prior can be written as:

$$\gamma_{prior}(m_{res}, m_{elas}) = \gamma_{prior}(m_{elas}|m_{res}) \times \gamma_{prior}(m_{res}) \quad (2)$$

where $\gamma_{prior}(m_{res})$ is the prior pdf for the reservoir parameters (including their spatial distributions) and $\gamma_{prior}(m_{elas}|m_{res})$ is a conditional probability for the elastic parameters that summarizes the rock physics relationships between reservoir property and elastic property. Thus, the final posterior pdf can be drawn for the joint rock physics and seismic inversion as the following combination of equation 1 and 2 (Bosch et al., 2004):

$$\sigma_{post}(m_{res}, m_{elas}) = c\, \gamma_{prior}(m_{elas}|m_{res}) \gamma_{prior}(m_{res}) \gamma_{data}\big(d_{obs} - g(m_{elas})\big) \qquad (3)$$

The petrophysical conditional density $\gamma_{prior}(m_{elas}|m_{res})$ is the rock physics forward function that maps the reservoir model parameters to the elastic model parameters. Many different seismic inversion workflows combining elastic properties, geostatistics, and rock-physics models to predict reservoir properties can be presented in the shape of equation 3 (Figure 1) This workflow in a Bayesian formulation guarantees consistency between the elastic and reservoir properties.

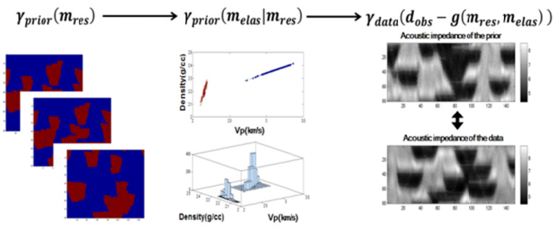

Figure 1: (left) Prior models are generated by multiple-points geostatistics; (middle) facies are converted to Vp and density according to the bivariate rock property pdfs; (right) To falsify incorrect models, the predicted seismic impedances are compared with inverted impedance of the data.

Iterative spatial resampling

Mariethoz et al. (2010a) suggested the Iterative Spatial Resampling (ISR) method to perturb realizations of a spatially dependent variable while preserving its spatial structure. This method is used as a transition kernel to produce Markov chains of geostatistical realizations. Depending on the acceptation/rejection criterion in the Markov process, it is possible to obtain a chain of realizations aimed to characterize a certain posterior distribution with Metropolis sampling. In the studied cases, ISR yields posterior distributions reasonably close to the ones obtained by rejection sampling, but with important reduction in CPU cost.

Creating a Markov chain using ISR is accomplished by performing the following steps at each iteration i:

a. Generate m_1 and evaluate its likelihood $L(m_1) = \gamma_{data}\big(d_{obs} - g(m_{1,elas})\big)$.

b. Select r_i as a subset of points from m_1.

c. Generate a proposal model m^* by conditional simulation using r_i as conditioning data. We use DS for multi-point geostatistical simulations.

d. Evaluate $L(m^*)$ and accept or reject m^* by the Metropolis acceptance criterion (Mosegaard and Tarantola, 1995).

Mosegaard and Tarantola (1995) indicate that their sampling method can also be used for optimization, creating a chain of ever-improving realizations by only accepting a proposed model if the likelihood improves.

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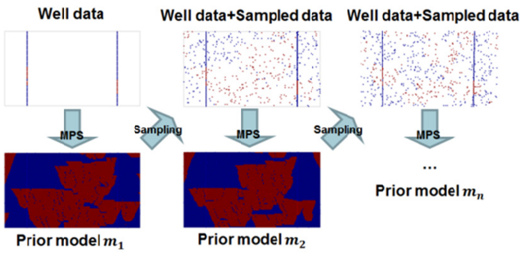

Figure 2: Sketch of the iterative spatial resampling method. An initial realization m_1 is randomly sampled to obtain a subset of points whsihc are used as conditioning data for the next simulated realization m_2. m_2 displays similar local features as m_1 due to the conditioning data, and represents a different realization from the prior multi-point geostatistical model.

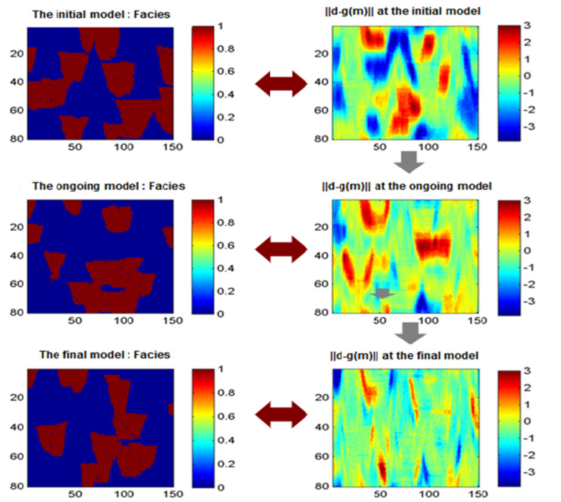

Figure 3: The iteration of a Markov chain using ISR. Left: generated facies models; Right: the residual between seismic data and predicted data from the earth model in each step. This misfit is getting smaller with iterations.

Adaptive spatial resampling

The search strategy of ISR performs successive steps in random directions for exploring various regions of the solution space. Since the search is stochastic, the global minimum will be reached after theoretically, an infinite number of iterations. However, in most practical applications, when the subset $\boldsymbol{r_i}$ is selected at random, it can get stuck in a non-optimal local minimum. In this work we improve the efficiency of ISR as an optimizer by adaptive sampling as described below.

At the every iteration, we always compare the predicted seismic data with the observed data and thus we have a spatial error map (right column images of Figure 3). We can use this information for generating the next step. Instead of just randomly sampling a subset of points to condition the next realization, we adaptively sample important points having lower residual error. Adaptive Spatial Resampling (ASR) accelerates to find an optimal model consistent with the given data, and in particular, it can be efficient for finding the most likely model.

Test case

Synthetic 2D facies and seismic dataset are presented to demonstrate the validity of the proposed inversion technique. The two-dimensional models used were extracted from a modified version of the top layer of the Stanford VI synthetic reservoir. The Stanford VI reservoir was created by the geostatistics group at Stanford University (Castro, et al., 2005) to test algorithms. All the information about the model relevant to this work is summarized in Figure 4. The reference facies model is a sand-shale channel system with 80 cells in the vertical (z) direction (dz; = 1m) and 150 cells in x (dx = 25m). Using appropriate rock physics models for the sand and shale, we compute acoustic impedance from the P-wave velocities and densities. We applied a frequency-domain Born filter for surface seismic reflection geometry with a 5~50 (Hz) bandwidth. This filtered version (bottom left) represents the seismic impedance data. For the inversion process, we assumed that we have only the seismic bandwidth acoustic impedance data, two wells with log information, and a training image.

First, we generated multiple prior models using MPS and it is used to find posteriors by rejection sampling. Rejection sampling method is the only way to represent perfect posterior pdf. However, since it requires a large number of evaluations of forward model, it is inefficient. We tested ISR as a sampling method by comparing its result with rejection sampling. Figure 6 shows some results as the reference and the e-types (ensemble averages) of models. Since the hard data comes from two wells, the e-type map of priors shows its limitation of lateral resolution. Both rejection sampling and ISR with Metropolis algorithm found channel distribution and these results look similar (second row of the Figure 6). However, the result of rejection sampling is the average of 551 accepted models after evaluating 100,000 prior models while ISR with Metropolis algorithm uses 10 Markov chains with 500 iterations (5000 evaluations). ISR shows better efficiency especially when compared to rejection sampling with 5000 evaluations (bottom left, Figure 5). The root mean square errors (RMSE) versus iterations for the 10 Markov chains are also shown (bottom right). Since the algorithm can accept a lower likelihood model using the Metropolis criterion, the chains jump up and down and thus it can escape from local minima.

As an optimizer the performance of ISR and ASR are compared in Figure 6. Here our objective is to find optimal models which match the given seismic data. The average of 30 chains as a thick line in Figure 6 shows the efficiency of ASR. Especially in the early stage of iterations, ASR moves quickly to find a better model while ISR goes down slowly with long flat sections. On average ASR can manage to find models with much lower rms error. The efficiency can be compared by the number of iterations required to reach the same logarithm RMSE; the value of ISR at the 500th iteration averaging over 30 chains was the same as the result of the 27th iteration for ASR.

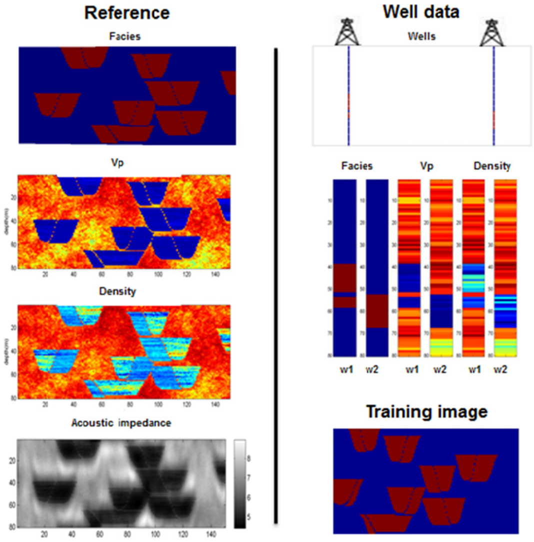

Figure 4: Left: The spatial distribution of the facies, P-wave velocities (Vp) and densities (ρ) are assumed as the reference. The filtered seismic band acoustic impedance is at bottom left. Right: The data of two wells are given as above and the wells are located at CDP25 and CDP125, respectively. Training image for MPS is shown at the bottom of the right column.

Conclusions

We presented the Iterative Spatial Resampling method (ISR) applied to seismic inverse modeling for lithofacies prediction. Iterative Spatial Resampling perturbs realizations of a spatially dependent variable while preserving its spatial structure. ISR is applied for seismic reservoir modeling to yield posterior samples efficiently. Depending on the acceptation/rejection criterion in the Markov process, it is possible to obtain a chain of realizations aimed either at characterizing a certain posterior distribution with Metropolis sampling or at calibrating one realization at a time. ISR can therefore be applied in both contexts of Bayesian inversion and optimization. In the studied cases, it yields posterior distributions reasonably close to the ones obtained by rejection sampling samplers, with important reduction in CPU cost. The efficiency of ISR can be significantly improved by Adaptive Spatial Resampling. As an optimization technique, ASR rapidly reduced the residual error and produced the most likely models. Thus both ISR and ASR are suitable for conditioning facies models to spatially distributed seismic data. Future study will be on adaptive ISR algorithms as a sampling method. To avoid underestimating the range of uncertainty while keeping its efficiency, various ideas such as ISR/ASR mixed algorithm, chain swapping and multiple starting points will be studied.

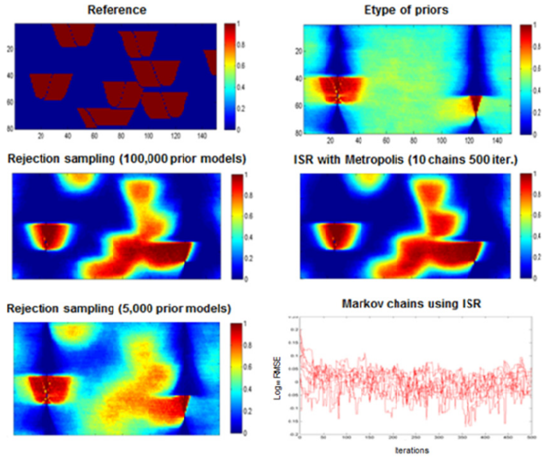

Figure 5: Representation of the averages of ensembles of models: (top) the reference facies and averaged prior models; (middle) rejection sampling and ISR results; (bottom) with limited number of priors, rejection sampling is insufficient to show the same results.

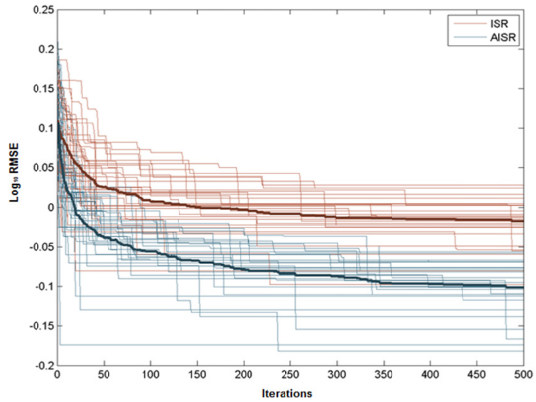

Figure 6: Adaptive spatial resampling (blue curves) and iterative spatial resampling (red curves) are compared for 30 Markov chains. ASR as an optimizer was faster to find out the most likely models, and found models with lower rms error. The average of 30 chains for each case is shown as a thick line. ASR rapidly reduced the residual error especially in the early stage of iterations.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2011 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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