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A New Look at Multiple-Point Geostatistics for Geological Modelling

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ABSTRACT

Multiple-point simulation is a family of methods used to generate conditional facies simulations that present the same spatial dependence as a user-defined training image. The main advantage of using a training image is the easy integration of geological concepts in numerical subsurface models. The strength of multiple-point methods is that it allows making the best use of geological expertise when data are too sparse for inferring a model of spatial continuity. However, despite being appealing to geologists, the multiple-point methods are often not used in practice due to the difficulty of building complex 3D training images that contain realistic features.

A new framework for multiple-point simulation is presented that involves small and simple training images. The main idea is to use random transformations to expand the range of structures available in the simple training image. The training image is not regarded any more as a global conceptual geological model, but rather a basic structural element of the subsurface. Complex geological structures are obtained, whose spatial structure can be parameterised by adjusting the statistics of the random transformations, based on field data or geological context. In most cases, such parameterisation is possible by adjusting two numbers only. One advantage is that the training images are so simple that they can be easily built even in 3D. The method allows building 3D models that reproduce shapes corresponding to a desired prior geological concept and are in phase with different types of field observations, such as orientation, facies or geophysical measurements.

The flexibility of the approach allows for naturally integrating different types of data. This is demonstrated on an example involving seismic data assimilation. Examples where complex 3D folded structures are built from a simple training image and a few orientation data are shown.

INTRODUCTION

Multiple-point simulations (MPS) is a geostatistical method using prior spatial models in the form of a non-parametric conceptual image, called training images. The use of a training image is appealing to geologists because it is a direct representation of a geological concept. It has been shown that MPS realisations are able to produce geologically realistic structures by using high-order spatial statistics that can represent a wide range of spatially structured, low entropy phenomena, and because of this, have been used in a variety of geological settings.

Although multiple point statistics (MPS) has been widely used in the petroleum sector for a decade, the methodology has not been extensively applied in the mining sector. Boisvert *et al* (2008) demonstrated a method to map vein deposits, and Bastante *et al* (2008) showed that MPS performed better than indicator kriging or conditional indicator simulation for mapping slate deposits. Given an appropriate training image MPS could be used in a wide variety of mining applications including mapping the thickness of coal seams, predicting the form of river meanders associated with placer deposits, mapping the dispersion of hydrothermal epigenetic deposits, or predicting the structures associated with volcanogenic massive sulfide ore and metamorphically reworked deposits.

However, MPS is not widely used mainly due to the problem of deciding which training image to use (De Almeida, 2010; Pyrcz, Boisvert and Deutsch, 2008) and the difficulties related to building the necessary 3D training image. Another difficulty is related to the parameterisation of the training image. It would be highly desirable to have a few continuous parameters able to parameterise a multiple-point prior, in a similar way as 'multiGaussian' priors that are controlled by parameters such

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as the range and the sill of a variogram. Therefore, even if robust tools exist for MPS, the practical difficulties related to building and parameterising the training image often lead to the adoption of parametric alternatives for the inferences of spatial continuity, such as traditional variogram-based techniques. Nonetheless, there are cases where more prior information is available than a two-point model of spatial correlation (eg connectivity or disconnectivity of the high values). In these cases, it would be preferable to have methods that integrate such prior spatial characteristics in stochastic models.

A new concept and algorithm for MPS that considers elementary/simplistic training images that are an expression of prior spatial continuity is presented. During the simulation, the diversity of patterns is enriched by applying random transformations. This enables the generation of complex geological images whose spatial structure can be parameterised by adjusting the statistics of the random transformations. In most cases, such parameterisation is possible by adjusting only two parameters. The approach presented resolves the problem of building a training image because creating elementary images is easy, even in 3D. The training image is no longer regarded as a global conceptual geological model, but rather a basic structural element of the subsurface, like the shapes used in object-based simulations.

TRANSFORM-INVARIANT DISTANCES

The direct sampling (DS) method is described in detail in Mariethoz, Renard and Straubhaar (2010) and Mariethoz and Renard (2010). The basic principle of DS is to identify a location $\mathbf{y}$ in the training image with a neighbourhood $\mathbf{N}_y$ matching the neighbourhood $\mathbf{N}_x$ observed at the location to simulate $\mathbf{x}$, and then to assign $Z(\mathbf{x})=Z(\mathbf{y})$. A match between two neighbourhoods is defined as an occurrence when the distance between these neighbourhoods, $d(\mathbf{N}_x,\mathbf{N}_y)$, is below a given threshold t.

Let us now define a geometrical transformation T_p applied on $\mathbf{N}_x$, where p is a parameter affecting the transformation. Several types of geometrical transformations can be considered. Here we consider two types of transformations, which are rotations centred on $\mathbf{x}$, denoted $R(.)$, and affinity (or homothetic) transforms, denoted $A(.)$. For rotations, the parameter p corresponds to the rotation angle and for affinity transforms, it corresponds to the affinity ratio.

For DS simulations, consideration of the distance between the ensemble of possible neighbourhoods resulting from geometric transforms of $\mathbf{N}_x$ and a neighbourhood $\mathbf{N}_y$ in the training image is proposed. By doing this, the neighbourhoods $\mathbf{N}_x$ and $\mathbf{N}_y$ are compared up to a transformation (ie there exists a transformation $T(.)$ that makes $\mathrm{T}(\mathbf{N}_x)$ identical to $\mathbf{N}_y$). A transform-invariant distance is defined as:

$$d^T(\mathbf{N}_x,\mathbf{N}_y) = \mathrm{argmin}\left\{d(T_p(\mathbf{N}_x),\mathbf{N}_y)\right\}: \forall\ p \text{ in } [a,b] \quad (1)$$

where a and b are bounds specified for parameter p. A transform-invariant distance can be computed by finding a value of p (in the interval $[a,b]$) such that the distance $d(T_p(\mathbf{N}_x),\mathbf{N}_y)$ is minimal. Once the value of p is found, the corresponding distance is the transform-invariant distance. Note that the parameter p is not prescribed in advance, but determined by a sampling of p values in $[a,b]$ that finds one occurrence corresponding to an acceptable distance (or mismatch) between both neighbourhoods. As soon as $d(T_p(\mathbf{N}_x),\mathbf{N}_y)\leq t$, one assigns $Z(\mathbf{x})=Z(\mathbf{y})$ and then the process is repeated at the next node to be estimated.

When using transform-invariant distances in MPS, one does not only consider the patterns found in the training image, but all their possible transformations in the range $[a,b]$. This extra degree of freedom allows for complex spatial patterns to be generated, even from simple training images. By adjusting the values of the transformation parameter bounds (either constant or spatially variable), it is possible to parameterise the ensemble of patterns available for the simulation.

To illustrate the method, the simplest possible training image that consists of a binary variable depicting parallel horizontal lines is used. Based on it, spatial patterns are generated by using different transform-invariant distances. The elementary training image is displayed in Figure 1a, and realisations corresponding to different transformation parameters are shown in Figure 1b to Figure 1j. The size of the training image is 100 by 100 pixels and the realisations are 500 by 500 pixels. Note that using such a small training image does not adhere to the general principle that recommends using a training image larger than the simulations (Journel and Zhang, 2006; Strebelle, 2002; Straubhaar *et al,* 2011).

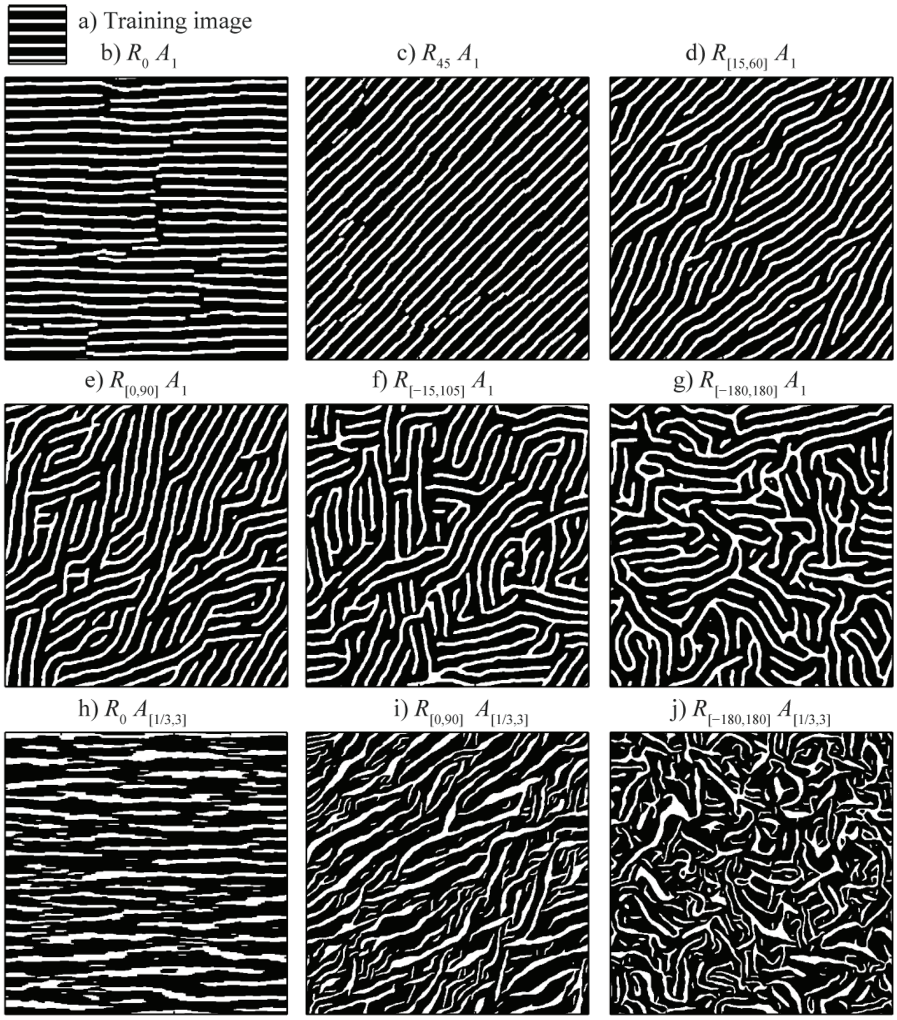

FIG 1 - Examples of textures obtained with transform-invariant distances: (A) lines training image, (B) to (J) corresponding realisations with various transformation parameters.

Since the patterns of the training image are reproduced up to a rotation, the pool of acceptable patterns is larger, resulting in increased continuity of the simulated lines. The diversity of patterns is not contained in the training image itself, but in the large amount of possible transformations. When rotations and affinities are used simultaneously, the added diversity of patterns creates shapes that can resemble geological structures such as clay drapes seen in cross-section (Figure 1h) or anastomosed channels seen in plan view (Figure 1i). In these examples, the training image only carries a concept of 'elongated and connected bodies' that can materialise in various ways under the different transformations applied.

USING VARYING TRANSFORMATION PARAMETERS FOR STRUCTURAL MODELLING

The examples shown in Figure 1 use homogeneous transformation parameters (ie identical bounds for transformation parameters are applied at all locations of the domain). By contrast, it is proposed in this section to model geological folding by imposing varying transformation parameters on the simulation domain. Consider M angle measurements m_i , i=1...M. Interpolation is needed in order to obtain the transformation parameters (orientations) over the entire domain. Therefore, it is more convenient to express the bounds $[a,b]$ for the rotations as a median value $\alpha(\mathbf{x})$ and a tolerance $\varepsilon(\mathbf{x})$. For example, using α=30 and ε=10 is equivalent to using the bounds [20,40].

At the location $\mathbf{x}_i$ of an orientation measurement, assuming no measurement error, one can safely define $\alpha(\mathbf{x}_i)=m_i$ and $\varepsilon(\mathbf{x}_i)=0$. At locations further away from the measurement, the bounds defining the orientation should be enlarged and become $[a,b]=[\alpha(\mathbf{x})-\varepsilon(\mathbf{x}), \alpha(\mathbf{x})+\varepsilon(\mathbf{x})]$. From the orientation and tolerance known at a limited number of locations, the inverse square distances is used to obtain the transformation parameter α on the entire domain, and distance to closest neighbour to infer the tolerance ε.

Figure 2 illustrates the proposed methodology. Seven orientation measurements are displayed by vectors in Figure 2a. Figure 2b and Figure 2c to represent respectively the result of the interpolated angles (in degrees), and tolerance. For the simulation, two different training images are considered (Figure 2d). The first one is continuous and represents hydraulic conductivity. The second training image, consisting of horizontal lines representing hydrofacies, is the same as used in Figure 1a. Two realisations, one with each training image, are shown in Figure 2e and Figure 2f. In both cases, the structures of the training image are oriented in a way that is consistent with the initial measured orientations $m(\mathbf{x}_i)$. Moreover, the continuities are well preserved, with elongated bodies of high hydraulic conductivity in the continuous case and parallel channels of constant thickness in the categorical case.

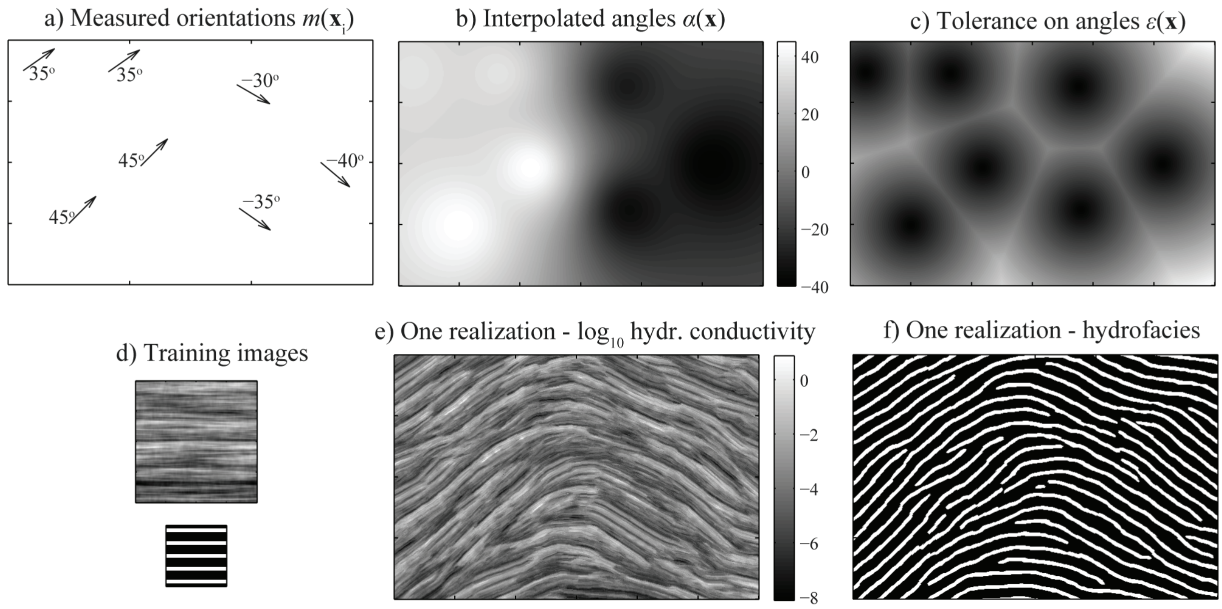

FIG 2 - Application of non-stationary transformation parameters to model a fold in 2D: (A) orientation measurements, (B) and (C) interpolated rotation parameters, (D) two different training images, (E) and (F) corresponding realisations.

To demonstrate the value and the practicality of the methodology, a 3D synthetic aquifer modelled with elementary 3D training images and rotation-invariant distances is presented. Six orientation measurements are available (Figure 3a), which correspond to a relatively complex geological structure consisting of an anticline on one side of the domain and a syncline on the other side. The training image (Figure 3b) consists of horizontal equally spaced layers populated with 'multiGaussian' realisations. Figure 3c shows one resulting realisation, displaying the expected folded structure and preserving the spatial continuity given by the training image. This example demonstrates that it is suitable to use elementary training images in complex geological environments where the classical approach would be difficult to put into practice.

DISCUSSION AND CONCLUSION

A new framework for MPS involving the use of elementary training images and transform-invariant distances is presented. Transformations (rotations and/or affinity in the cases presented) are randomly applied to the patterns of the training image resulting in a vast family of geological structures that all display a type of geological continuity related to the training image. Examples where realistic 2D and 3D structures are built from simplistic training images, with transformation parameters inferred using a small number of data points are shown.

One fundamental issue raised by the use of elementary training images is that the training image is no longer a direct reflection of the structures deemed to exist in the subsurface, but rather a vehicle for imposing primary geological structures or properties. Often MPS is seen as a process which, given a training image, produces other images (simulations) with identical statistical properties and possibly anchored to conditioning data. However, it has been noted that one cannot expect a complete statistical similarity between the training image and the corresponding simulations. In fact, an algorithmic transfer function lies between the training image and the simulation results. In this

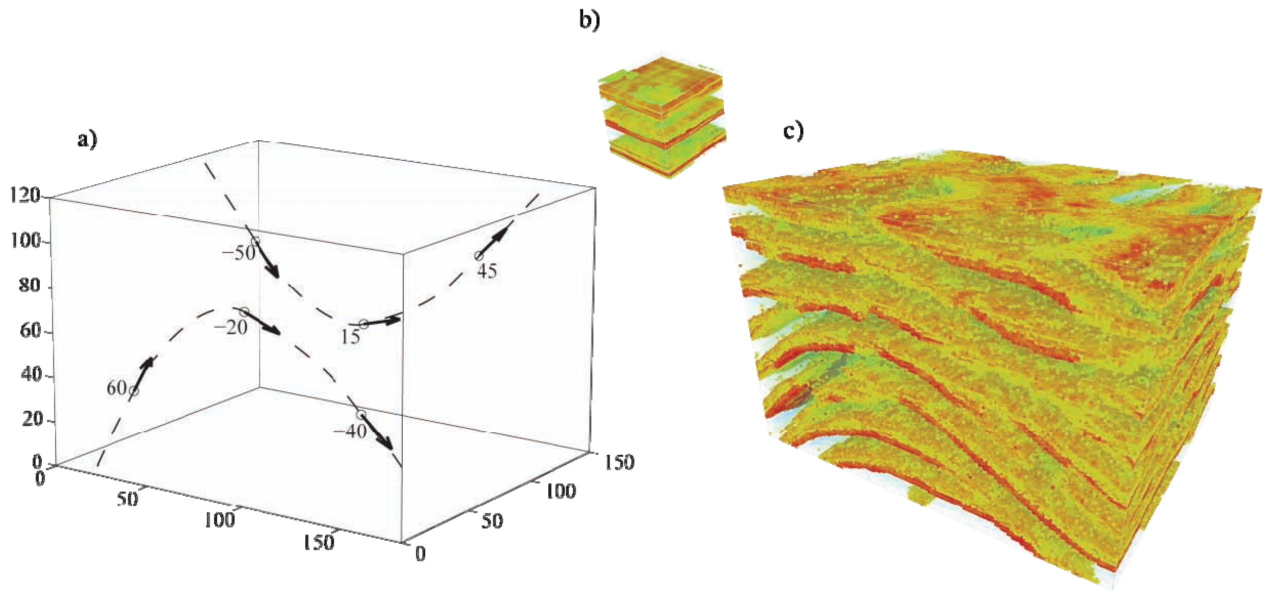

FIG 3 - A high-resolution 3D example of the use of elementary training images with transform-invariant distances: (A) six orientation data depicting an anticline on one side of the domain and a syncline of the other side, (B) elementary training images, (C) one realisation.

study we acknowledge that the training image is only an initial representation of the subsurface, which is further processed by the simulation algorithm and represents a broad spatial concept rather than a specific geological reality. Instead of forcing the reproduction of the patterns found in the training image (Mariethoz, Renard and Straubhaar, 2010; Suzuki and Strebelle, 2007), very simple training images are used and adapted with random transformations that are guided by conditioning data or by local orientation information.

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