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CONDITIONAL SIMULATION OF GEOLOGICAL TEXTURES BY IMAGE QUILTING

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ABSTRACT
Stochastic simulation is an invaluable tool for modelling three-dimensional geological characteristics. Stochastic simulation allows creating multiple realistic realizations of an unknown three-dimensional geological domain, with conditioning to any existing observations and reproducing a given spatial continuity. Some of the most successful methods in this field are multiple-point geostatistics, which define spatial continuity based on the concept of a training image. It was found that similar methods have been proposed in computer graphics. One such method, Image Quilting, is introduced in this paper.The main difficulty with it is that it was not originally designed to produce conditional simulations. In this paper, the existing MATLAB code developed for computer graphics has been modified for conditional 2D image quilting by using the rejection method and applied to geoscience. We have established the deviation due to the error tolerance to the conditioning data in the 2D conditional image quilting process by executing elapsed time for various number of conditioning points. The rejection method can be applied to image quilting for very few conditioning points with reasonable accuracy. The time required with this method increases enormously when moreconditioning points are considered. In such casesthe computational cost can be very high to obtaingeological texturesthat havethe correct data values at the conditioning locations. Herewe investigate another improved procedure (Selection Method)that allows accurate conditioning while performing with reasonable computing times.

Keywords: Geological texture synthesis,Multiple-point geostatistics, Conditioning, Image quilting, Sub-surface heterogeneity.

INTRODUCTION

Modelling subsurface heterogeneity is extremely important for predicting the behaviour of hydrogeological systems. Geological texture synthesis is a great challenge of subsurface modelling. For more than 50 years it has been extensively used for the management and the estimation of uncertain water resources. A broad variety of methods have been developed in geostatistics, aimed at characterizing the spatial structure, or texture, of the variable considered (e.g. hydraulic conductivity in hydrogeology).
In most cases, spatial covariance or variogram analysis is used to quantify the spatial geological continuity. Variograms only illustrate correlation among any two points in space and aredefined as a two-point statistic which cannot reproduce connected geological patterns. Boolean models are the

most commonly used algorithms for object-based models which work with deterministic shapes illustrated by stochastic constraints (Renard et al., 2006). The main critique towardstraditional geostatistical approaches is related to their failure to represent realistically some connected geological structures.

A solution to this criticism was suggested by Strebelle (2002), with an approach consisting ofthe assessment of the conditional probability distribution for a simulated value based on a training image (TI). Multiple-point (MP) geostatistics is able to produce images similar to those found with object-based models, with the benefit that it can be easily conditioned with field data, therefore exceeding the major difficulty of object-based model (Renard et al., 2006). The TI is a theoretical but explicit concept which can represent 2D or 3D spatial geological continuity. TI may originate from real data representative of the geology under consideration, or a large unconditional realization of a stochastic simulation technique. Strebelle recommends using TIs which can be developed from expert information, outcrops, or even a geologist's sketching. Therefore it is not necessary for a TI to be locally constrained to any data and be the equivalent size as the area under consideration, but can reproduce the spatial continuity inferred as same as the real geological character (Arpat and Caers, 2007). Guardiano and Srivastava (1993) initially used multiple-point statistics based on TIs in the field of geostatistical simulation. This was inspired by building upon the sequential indicator model that uses only two point statistics for generating facies geological models (Journel, 2005).

The concept of simulating models using Multiple-Point statistics from a TI seems easy, straightforward and smart for geologists (Hu and Chugunova, 2008). However, some limitations originate from the difficulty to find TIs, their reliability to characterize particular reservoir irregularities, the precision of the simulated forecast, the consistency among a particular TI and a particular data set, and computational issues. Therefore, more research is needed in this area.Our aim is to develop a method that better reproduces the TI patterns while reducing computational cost, inspired by the state-of-the-art in texture synthesis. To this end, we adaptthe method of image quilting that proceeds by stitching together geological texture patches based on training images to form the output realization.

In the field of texture synthesis, an early method was proposed by Efros and Leung (1999), which uses non-parametric sampling to "grow" texture by enforcing statistics locally, pixel by pixel. All the neighbours are synthesized by the conditional distribution of each pixel and this is done by searching the training image and searching all likely neighbourhoods. The algorithm gives decent outputs for an extensive variety of textures, but is computationally disadvantaged because every pixel has to be synthesized by thorough searching of the input image. This means that a lot of searching effort is wasted on pixels. Hence rather than taking a particular pixel as the unit of synthesis, a block or patch has to be considered (Efros and Freeman, 2001). Then the texture synthesis method will be nothing but matching a jigsaw puzzle, quilting together the blocks until they all match together.

Conditioning means that the resulting textures precisely replicates data values at the measured locations, and is an imperative requirement to integrate geological data. The image quilting method originates from the field of computer graphics and texture synthesis (Efros and Freeman, 2001), where most applications do not require conditioning. Therefore the method has never been adapted for conditioning. We are studying ways of implementing a conditional version of the method. Firstly, the simulated realization needs to be conditional to a range of data sets such as hard and soft data. The image quilting algorithm is mostly built from empirical arguments having neither a formal theory nor a certain explicit modelling. There may be possible conflict between the training image patterns and the patterns of the existing hard data, which is not resolved by the image quilting method as implemented in computer graphics. Such conflict will exist in most practical cases and therefore our final aim is to minimize it by accommodating both texture and data conditioning constraints.

In this paper, we first present an outline of the image quilting method, we then test the sensitivity of its parameters for the reproduction of structures observed in natural geological systems, and finally we propose ways of modifying the method to accommodate conditioning data. Two avenues are investigated to this end: conditioning by rejection, which is correct but very inefficient, and conditioning by selection that gives much better results.

IMAGE QUILTING FOR GEOLOGICAL TEXTURE SYNTHESIS

In computer graphicsthere is a need to generate realistic textures in applications such as animation movies or computer games. Texture synthesis methods should be capable of taking a sample of texture image and produce an unlimited amount of data which are not exactly similar to the original, but will be observed to be the similar texture by humans. Efros and Freeman (2001) presented a straightforward algorithm to synthesize larger similar texture (Figure 1,right side) by using a given geological texture example (Figure 1, left side). Their key idea was to generate similar texture by considering small pieces of existing texture and then sewing those pieces together in a coherent manner. This image quilting algorithm has reasonably good performance on semi-structured textures as well as stochastic textures. Extreme repetition and mismatched or distorted boundaries are the two most usual complications. Both these problems occur generally when the input texture does not cover adequate variability. The MATLAB code used by Efros and Freeman (2001) to generate single output image typically runs within few minutes. The simulation time is greatly governed bytwo parameters: the sizes of the input and output and the user defined block size.

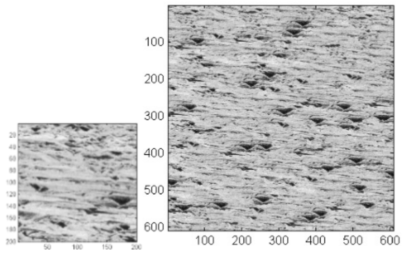


Figure 1: Texture synthesis by Image quilting process

Efros and Freeman (2001) developed this patch-based texture synthesis procedure having the following steps:

1) From the input texture image having the set of all overlapping patches, select the unit of synthesis as a square of user-specified size.
2) To synthesize a new realization, search the input image for such blocks that by some measure agrees with their neighbours along the section of overlaps.
3) Then for better representation of the features in the texture, let the blocks have ragged edges, and look at the error in the overlap area between it and the other block(s) before putting a selected block into the texture. Let, B_1 and $B_2$are two overlapping blocks along their vertical edges shown in Figure 2 with the overlapping regions of B_1^{ov} and B_2^{ov}, respectively. Then, the error surface is described as $e = (B_1^{ov} - B_2^{ov})^2$. To get the minimal vertical cut through the error surface, track *e(i = 2..N)* and calculate the cumulative minimum error E for all paths (Dijkstra, 1959) as shown in Eq.(1). Here *i* and *j* represent the corresponding row and column numbers respectively.

$$E_{i,j} = e_{i,j} + \min(E_{i-1,j-1}, E_{i-1,j}, E_{i-1,j+1}) \qquad (1)$$

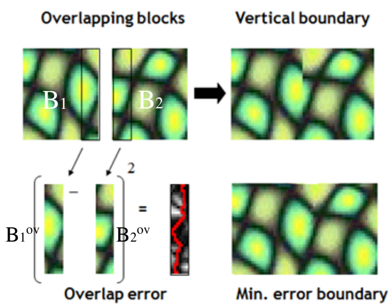


Figure 2: Minimum error boundary cut phenomena to find out the least error path between two overlapping blocks

4) Finally, one can find the end of the minimal vertical path through the surface by searching the minimum value of E in the last row. From there one can follow backward and get the path of the finest cut and state that to be the border of the new patch. For horizontal overlaps, asimilar

procedure can be followed and when there is both horizontal and vertical overlap, the best paths meet in the central and the global minimum is taken for choosing the appropriate cut. In this image quilting process, the only user-defined parameter is the size of the patch which depends on the characteristics of a particular training image. The patch must be big enough to capture the related properties from the image as well as small enough so that the interface between these structures is left up to the algorithm.

CONDITIONAL IMAGE QUILTING PROCESS

Rejection Method

The development of existing MATLAB code for Conditional 2D Image Quilting was performed by the Rejection Method. The elapsed time for various number of conditioning points was recorded. We have also established the time deviation due to the error tolerance in conditioning 2D image quilting process. For this conditioning:

1) Primarily we have assigned conditioning error tolerance in the mathematical model and then calculated absolute error as well as error percentage for each conditioning point.
2) After the first simulation of image quilting function we have checked whether conditioning is satisfied or not by rejection method.
3) If satisfied then go with that output image as the final realization, otherwise reject this image by changing it to a new one, and finally measuring the elapsed time and loop count i.e. the required number of simulation for conditioning through this procedure.

In this process, we have made the output realization conditioning by rejecting the simulated image until it satisfies with the particular data set at specific locations.

Selection Method

We are now in the process of developing another conditioning method named ‘Selection process’ where a compatible block has to be selected by considering the training image and forcing it to have similar related statistics as in the training image, as well as the relevant field data at certain specific locations of the final realization. In this process, we will search the input training image for blocks that by some measureagrees both with their neighbours along the section of overlaps and at the same time all hard data available within the realization as shown in Eq. 2. Therefore, total error (E_t) will be calculated by considering both the quilting error (E_q) and conditioning error (E_c) multiplied with weightage factorsw$_1$ and w$_2$ respectively. Then all the neighbours will be synthesized by the conditional distribution of each block and available field data.

$$E_t = w_1 E_q + w_2 E_c \qquad (2)$$

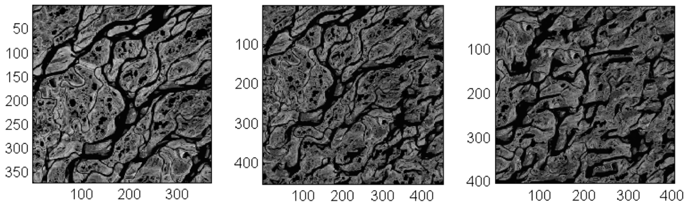
RESULTS AND DISCUSSIONS

Application of image quilting in hydrogeology

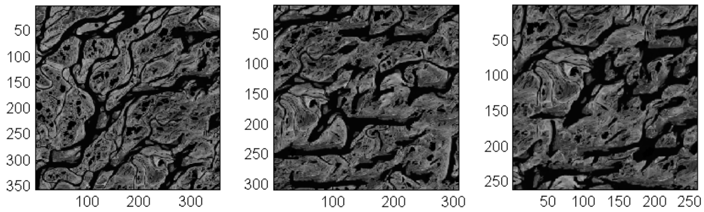
We have applied image quilting to different geological patterns and obtainednumerous variations in texture synthesis outputs. Here we have used colour values of a satellite image of the Lena Delta in Russian Federation shown in Figure 3 (a) (Landsat 7 image, USGS/EROS and NASA Landsat Project). With this particular image we performed a sensitivity analysis of the parameters of Image Quilting. The outputs are discussed here.

i) Variation in overlapping region between pixels:

Figure 3 shows different realizations found due to variation in the amount of overlap to allow between pixels while synthesizing the final texture. Sometimes we found ‘verbatim copy’ (Figure 3d, Figure 4d) of the training image as the final realization, where the realization is exactly the TI. This may occur when the patches are too big, and is not desirable (lack of variability between realizations). The elapsed timeshown in Table 1 rises with increasing overlapping region.



(a) Satellite image of the Lena Delta (b) 10% overlap (c) 20% overlap



(d) 30% overlap (e) 40% overlap (f) 50% overlap

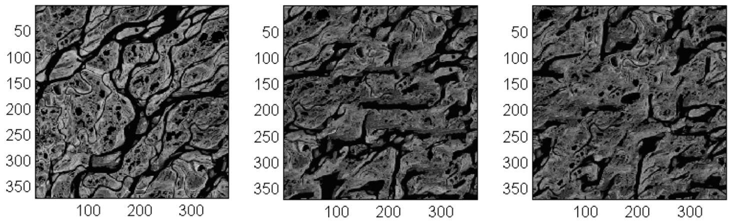
Figure 3: Different realizations due to the variation in overlapping amount to allow between pixels

Table 1:Elapsed times due to the variation in the amount of overlap to allow between pixels

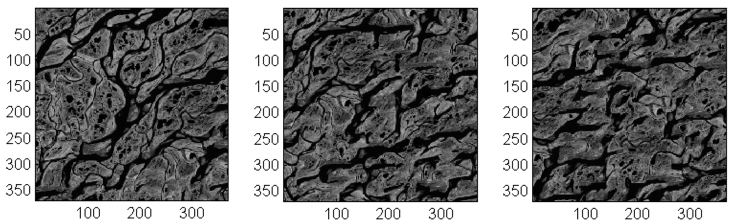
Overlap (%)	10	20	30	40	50
Elapsed time (sec)	34.9	38.1	42.6	47.5	63.7

ii) Variation in error tolerance:

Figure 4 shows different realizations found due to variation in the error tolerance used when computing list of compatible blocks. The elapsed times not varying significantly due to this deviation are shown in Table 2.



(a) Satellite image of the Lena Delta (b) 2% error tolerance(c) 4% error tolerance



(d) 6% error tolerance (e) 8% error tolerance (f) 10% error tolerance

Figure 4: Different realizations due to the variation in error tolerance

Table 2:Elapsed times due to the variation in error tolerance

Error tolerance (%)	2	4	6	8	10
Elapsed time (sec)	67.9	68.5	68.4	68.5	68.7

Conditional image quilting by rejection method

Elapsed time and loop count number are illustrated in Figure 5 for various number of conditioning points by Rejection Method. From the diagram, it is evident that as the number of conditioning points increases the required time rises enormously, making this method inappropriate. The deviation for various error tolerances is also shown in Figure 5. Here it is seen that the elapsed time and number of simulations needed increase with the accuracy of the conditioning to be done. There are few discrepancies present in this investigation as this process is fully a stochastic random process.

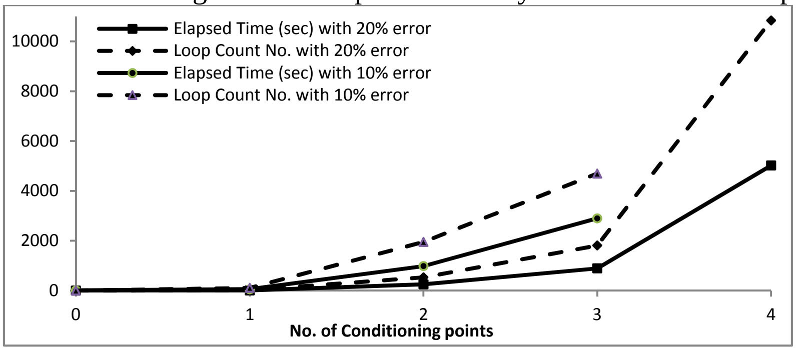


Figure 5: Elapsed Time and Loop Count No. for various number of conditioning points by Rejection Method

CONCLUSION

The image quilting process is mostly effective for semi-structured textures which were always the hardest for the geostatistician to synthesize.This image quilting method is extremely easy to apply in geostatistics and plenty of similar geological realizationscan be obtained through this algorithm very quickly.From this preliminary analysis, we found that the rejection method can be applied on image quilting for very few conditioning points with reasonable accuracy. The time required with this method increases enormously when moreconditioning points are considered. In such casesthe computational cost can be very high to obtaingeological textures having the data values at the conditioning locations.Here we investigate another improved procedure (Selection Method) that allows accurate conditioning while performing with reasonable computing times.

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