Homework 4
Computer Science
B351 Spring 2017
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All work herein is mine

Introduction

The aim of this homework is to get you well-acquainted with FOL and refutation. You will turn-in one file, a *pdf with the written answers called `h4.pdf`. I am providing this LaTeX document for you to freely use as well. Please enjoy this homework and ask yourself what interests you and then how can you add that interest to it! All problems are worth 100 pts. each. Include thet statement, "All the work herein is mine."

Homework Questions

1. Convert the following logical sentences to clausal form:

   (a) $\exists\ y\ p(y) \vee [\exists\ y\ (q(y) \rightarrow (\exists\ x\ (p(x) \vee\ q(x, y, C)))]$

   $[[p(bob()), \neg q(y), p(steve()), q(steve(), y, C)]]$

   (b) $\forall x \forall y \forall x\ d(x, y) \wedge d(y, z) \rightarrow d(x, z)$

   (c) $(P \vee Q) \wedge (\neg P \rightarrow (Q \vee R))$

   $[[P, Q]]$

2. Let $\mathcal{U} = \{1, 2, 3\}$, $p = \{1, 3\}$, $m = \{(1, 1), (2, 1), (3, 2)\}$

   (a) Determine $\models \forall\ x\ \exists\ y\ m(x, y)$

   True

   (b) Determine $\models \forall\ y\ \exists\ x\ m(x, y)$

   False

   (c) Determine $\models \forall\ x\ \exists\ x\ m(x, x)$

   True

   (d) Determine $\models \exists\ x\ \forall\ y\ m(x, y)$

   False

   (e) Determine $\models \exists\ x\ \forall\ y\ m(y, x)$

   True

   (f) Determine $\models \exists\ x\ \forall\ x\ m(x, x)$

   True

3. You've decided to add a new quantifier: M that takes one variable. The syntax is $M\ x\ f(x)$ for some sentence f. The meaning of $M\ x\ f(x)$ is that the number of times $\sigma(u, x)f(x)$ is true, where $\sigma(u, x)$ is substituting a value from the domain $u \in \mathcal{U}$ is at least 1.5 times more than when it is false. We can assume $\mathcal{U}$ is finite too. Use the model in the previous problem.

   (a) Determine $\models Mx\ p(x)$

   True

   (b) Determine $\models \forall\ x\ My\ m(y, x) \rightarrow p(x)$

   True

4. Ursala, Kaiser, and Shilah are dogs. We know the following:

   (a) Ursala is silver.

   (b) Shilah is gray and loves Kaiser.

   (c) Kaiser is either gray or silver (but not both) and loves Ursala.

   What does this sentence mean? $\exists x \exists y (gray(x) \wedge silver(y) \wedge loves(x, y)$. Use resolution refutation to prove this.

   At least one grey dog loves a silver dog.

5. Consider a robot that works in a mine – it has to push some objects and not push others depending on a colored tag that is either green or red. Here are the facts:

   - If pushable objects are green, the non-pushable are red.

FOL: $green(p(x)) \rightarrow red(\neg p(y))$
Python: $[[\neg green(p(x)), red(\neg p(y))]]$

- All objects are either green or red.

  FOL: $green(x) \vee red(x)$
  Python: $[[green(x), red(x)]]$

- If there is a non-pushable object, then all pushable objects are green.

  FOL: $\neg p(bob()) \rightarrow (p(x) \wedge green(x))$
  Python: $[[p(bob()), p(x)], [p(bob()), green(x)]]$

- Object 1, a cart, is pushable.

  FOL: $p(Cart)$
  Python: $[p(Cart)]$

- Object 2, a pile of ore, is not pushable.

  FOL: $\neg p(Ore)$
  Python: $[\neg p(Ore)]$

Assume you're trying to prove that there is a red object.

- Rewrite the statements into FOL (formal) and show their robotic equivalent (Python).
- Convert to clausal form.
- Use refutation to prove the there is a red object, by working *only* on the robotic equivalent. Clearly indicate the process.

6. Assume $\mathcal{U} = \{Alex, Bob, Cathy\}$, $M(x)$ means x is a mechanic, $N(x)$ means x works at NASA, $W(x, y)$ means x worked with y, $I(x, y, z)$ means x introduced y to z. Write constants A, B, C to mean $Alex, Bob, Cathy$, respectively. Write the following in FOL:

   - Cathy is a mechanic. Example: $M(C)$
   - Bob is not a mechanic.

     $\neg M(B)$
   - Either Alex is a mechanic or Bob is, but I know Cathy works at NASA.

     $(M(A) \vee M(B)) \wedge N(C)$
   - Bob introduced Alex to Cathy, since Cathy works at NASA.

     $N(C) \rightarrow I(B, A, C)$
   - Someone is a mechanic, but everyone works at NASA.

     $\exists\ x\ M(x) \wedge \forall\ y\ N(y)$
   - Bob introduced himself to Cathy.

     $I(B, B, C)$
   - Nobody as been introduced to Alex.

     $\forall\ x\ \forall\ y\ \neg I(x, y, A)$
   - If someone introduced Bob to Alex, then Bob isn't a mechanic.

     $\exists\ x\ I(x, B, A) \rightarrow \neg M(B)$
   - Nobody works with anyone here.

     $\forall\ x\ \forall\ y\ \neg W(x, y)$
   - Somebody works with Cathy, but it's not a mechanic, because Cathy works at NASA.

     $N(C) \rightarrow \exists\ x\ (W(x, C) \wedge \neg M(x))$