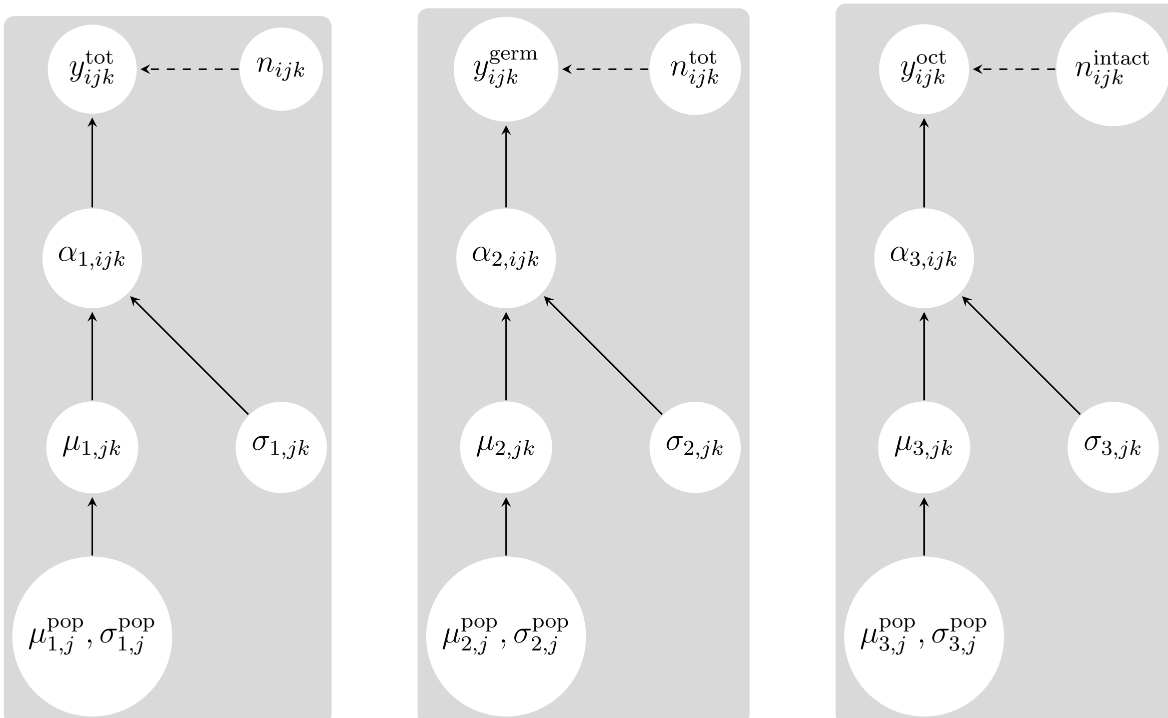
The models below represent the joint likelihood for data from the seed bag experiments. All data from seed bags and viability trials is in the form of binomial trials: we have counts of seeds at the start and end of an experimental window of time. All models for the parameters $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ have the same structure for seeds in bag i in year j in population k. If the number of seeds starting the trial (trials) is n_{ijk} and the number of seeds at the end of the trial (successes) is y_{ijk}, we write a model that has a population-level mean and year-level means drawn from the population-level distribution. The probability of success for each bag is drawn from this year- and population-level distribution:

I compared convergence diagnostics (R-hat, effective sample size) for centered and non-centered parameterizations of the model. Here, I use the centered parameterization because this led to improved convergence. In each model, we obtain the population-level posterior distribution probability of success (the θs) by marginalizing across years and taking the inverse logit.

Seed survival and germination rates for 1-year old seeds



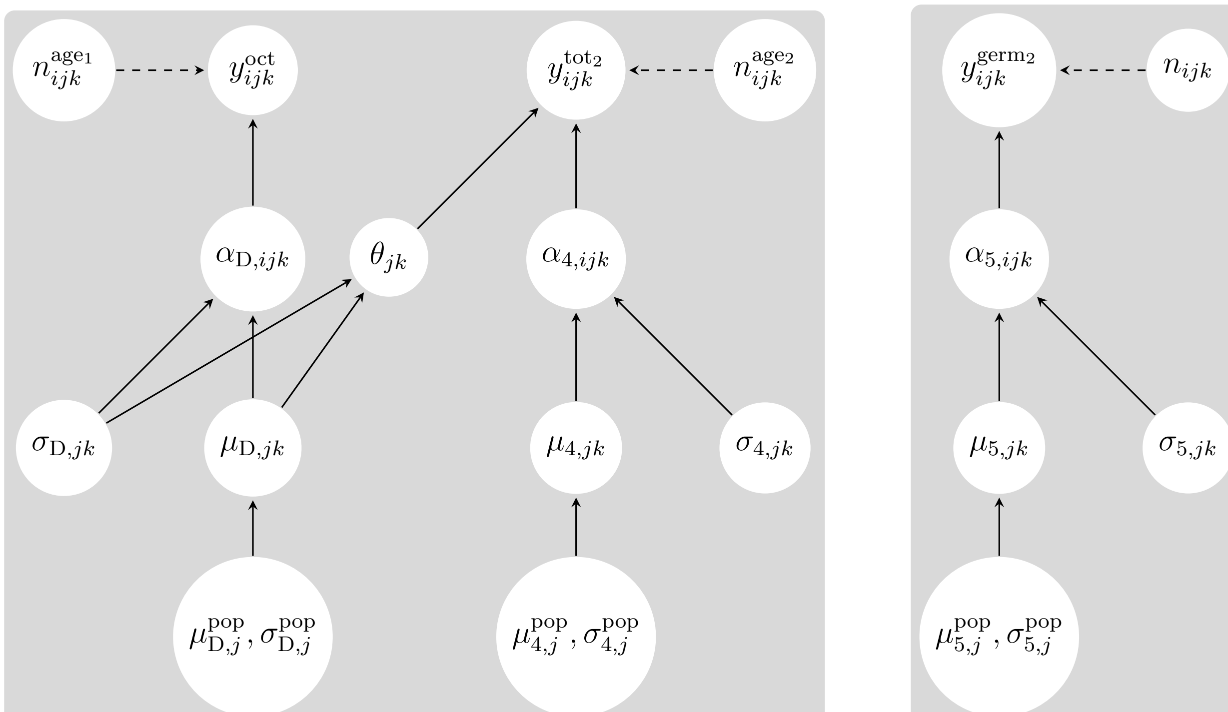
(a) Directed acyclic graphs for the hierarchical models for seed bag rates. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
[\boldsymbol{\alpha_1}, \boldsymbol{\mu_1}, \boldsymbol{\sigma_1}, \boldsymbol{\mu_1^{\text{pop}}}, \boldsymbol{\sigma_1^{\text{pop}}} | \boldsymbol{y^{\text{tot}}}] \propto \prod_{i=1}^{I}\prod_{j=1}^{J}\prod_{k=1}^{K} & \text{binomial}(y^{\text{tot}}_{ijk} | n_{ijk}, \text{logit}^{-1}(\alpha_{1,ijk})) \\
& \times \text{normal}(\alpha_{1,ijk} | \mu_{1,jk}, \sigma_{1,jk}) \\
& \times \text{normal}(\mu_{1,jk} | \mu^{\text{pop}}_{1,j}, \sigma^{\text{pop}}_{1,j}) \\
& \times \text{half-normal}(\sigma_{1,jk} | 0, 1) \\
& \times \text{normal}(\mu^{\text{pop}}_{1,j} | 0, 1000)\text{half-normal}(\sigma^{\text{pop}}_{1,j} | 0, 1).
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
[\boldsymbol{\alpha_2}, \boldsymbol{\mu_2}, \boldsymbol{\sigma_2}, \boldsymbol{\mu_2^{\mathbf{pop}}}, \boldsymbol{\sigma_2^{\mathbf{pop}}} | \boldsymbol{y^{\mathbf{total}}}] \propto \prod_{i=1}^{I}\prod_{j=1}^{J}\prod_{k=1}^{K} & \text{binomial}(y_{ijk}^{\text{tot}} | n_{ijk}^{\text{tot}}, \text{logit}^{-1}(\alpha_{2,ijk})) \\
& \times \text{normal}(\alpha_{2,ijk} | \mu_{2,jk}, \sigma_{2,jk}) \\
& \times \text{normal}(\mu_{2,jk} | \mu_{2,j}^{\text{pop}}, \sigma_{2,j}^{\text{pop}}) \\
& \times \text{half-normal}(\sigma_{2,jk} | 0, 1) \\
& \times \text{normal}(\mu_{2,j}^{\text{pop}} | 0, 1000)\text{half-normal}(\sigma_{1,j}^{\text{pop}} | 0, 1).
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
[\boldsymbol{\alpha_3}, \boldsymbol{\mu_3}, \boldsymbol{\sigma_3}, \boldsymbol{\mu_3^{\mathbf{pop}}}, \boldsymbol{\sigma_3^{\mathbf{pop}}} | \boldsymbol{y^{\mathbf{germ}}}] \propto \prod_{i=1}^{I}\prod_{j=1}^{J}\prod_{k=1}^{K} & \text{binomial}(y_{ijk}^{\text{germ}} | n_{ijk}^{\text{intact}}, \text{logit}^{-1}(\alpha_{3,ijk})) \\
& \times \text{normal}(\alpha_{3,ijk} | \mu_{3,jk}, \sigma_{3,jk}) \\
& \times \text{normal}(\mu_{3,jk} | \mu_{3,j}^{\text{pop}}, \sigma_{3,j}^{\text{pop}}) \\
& \times \text{half-normal}(\sigma_{3,jk} | 0, 1) \\
& \times \text{normal}(\mu_{3,j}^{\text{pop}} | 0, 1000)\text{half-normal}(\sigma_{1,j}^{\text{pop}} | 0, 1).
\end{aligned} \tag{3}
$$

Seed survival and germination rates for 2-year old seeds

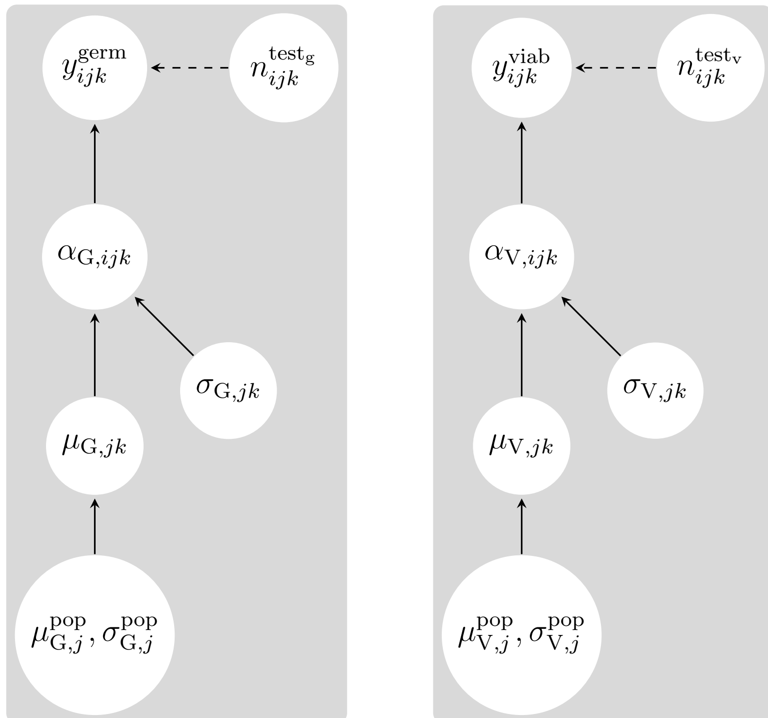


(a) Directed acyclic graphs for the hierarchical models for seed bag rates (age 2). Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
[\boldsymbol{\alpha_{\mathrm{D}}}, & \boldsymbol{\alpha_4}, \boldsymbol{\theta}, \boldsymbol{\mu_{\mathrm{D}}}, \boldsymbol{\sigma_{\mathrm{D}}}, \boldsymbol{\mu_4}, \boldsymbol{\sigma_4}, \boldsymbol{\mu_{\mathrm{D}}^{\mathrm{pop}}}, \boldsymbol{\sigma_{\mathrm{D}}^{\mathrm{pop}}}, \boldsymbol{\mu_4^{\mathrm{pop}}}, \boldsymbol{\sigma_4^{\mathrm{pop}}} | \boldsymbol{y^{\mathrm{oct}}}, \boldsymbol{y^{\mathrm{tot_2}}}] \propto \\
& \prod_{j=1}^{J} \Bigg\{ \bigg\{ \prod_{i=1}^{N_1} \prod_{k=1}^{K_1} \mathrm{binomial}\big(y_{ijk}^{\mathrm{oct}} | n_{ijk}^{\mathrm{age_1}}, \mathrm{logit}^{-1}(\alpha_{\mathrm{D},ijk})\big) \\
& \times \mathrm{normal}(\alpha_{\mathrm{D},ijk} | \mu_{\mathrm{D},jk}, \sigma_{\mathrm{D},jk}) \mathrm{normal}(\mu_{\mathrm{D},jk} | \mu_{\mathrm{D},j}^{\mathrm{pop}}, \sigma_{\mathrm{D},j}^{\mathrm{pop}}) \text{half-normal}(\sigma_{\mathrm{D},jk} | 0, 1) \\
& \times \mathrm{normal}(\mu_{\mathrm{D},j}^{\mathrm{pop}} | 0, 1000) \text{half-normal}(\sigma_{\mathrm{D},j}^{\mathrm{pop}} | 0, 1) \bigg\} \\
& \times \bigg\{ \prod_{i=1}^{N_2} \prod_{k=1}^{K_2} \mathrm{binomial}\big(y_{ijk}^{\mathrm{tot_2}} | n_{ijk}^{\mathrm{age_2}}, \mathrm{logit}^{-1}(\alpha_{4,ijk}) \times \theta_{jk}\big) \\
& \times \mathrm{normal}(\theta_{jk} | \mu_{\mathrm{D},jk}, \sigma_{\mathrm{D},jk}) \\
& \times \mathrm{normal}(\alpha_{4,ijk} | \mu_{4,jk}, \sigma_{4,jk}) \mathrm{normal}(\mu_{4,jk} | \mu_{4,j}^{\mathrm{pop}}, \sigma_{4,j}^{\mathrm{pop}}) \text{half-normal}(\sigma_{4,jk} | 0, 1) \\
& \times \mathrm{normal}(\mu_{4,j}^{\mathrm{pop}} | 0, 1000) \text{half-normal}(\sigma_{4,j}^{\mathrm{pop}} | 0, 1) \bigg\} \Bigg\}.
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
[\boldsymbol{\alpha_5}, \boldsymbol{\mu_5}, \boldsymbol{\sigma_5}, \boldsymbol{\mu_5^{\mathrm{pop}}}, \boldsymbol{\sigma_5^{\mathrm{pop}}} | \boldsymbol{y^{\mathrm{total}}}] \propto & \prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} \mathrm{binomial}(y_{ijk}^{\mathrm{tot}} | n_{ijk}^{\mathrm{tot}}, \mathrm{logit}^{-1}(\alpha_{5,ijk})) \\
& \times \mathrm{normal}(\alpha_{5,ijk} | \mu_{5,jk}, \sigma_{5,jk}) \\
& \times \mathrm{normal}(\mu_{5,jk} | \mu_{5,j}^{\mathrm{pop}}, \sigma_{5,j}^{\mathrm{pop}}) \\
& \times \text{half-normal}(\sigma_{5,jk} | 0, 1) \\
& \times \mathrm{normal}(\mu_{5,j}^{\mathrm{pop}} | 0, 1000) \text{half-normal}(\sigma_{5,j}^{\mathrm{pop}} | 0, 1).
\end{aligned} \tag{5}
$$

Viability trials

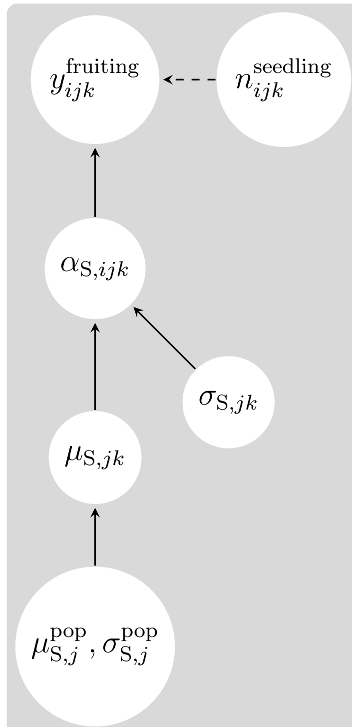


(a) Directed acyclic graphs for the hierarchical models for viability trials. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$\begin{aligned}[\boldsymbol{\alpha_G}, \boldsymbol{\mu_G}, \boldsymbol{\sigma_G}, \boldsymbol{\mu_G^{pop}}, \boldsymbol{\sigma_G^{pop}} | \boldsymbol{y^{tot}}] \propto \prod_{i=1}^{I}\prod_{j=1}^{J}\prod_{k=1}^{K} & \text{binomial}(y_{ijk}^{\text{germ}} | n_{ijk}^{\text{test}_\text{g}}, \text{logit}^{-1}(\alpha_{G,ijk})) \\ & \times \text{normal}(\alpha_{G,ijk} | \mu_{G,jk}, \sigma_{G,jk}) \\ & \times \text{normal}(\mu_{G,jk} | \mu_{G,j}^{\text{pop}}, \sigma_{G,j}^{\text{pop}}) \\ & \times \text{half-normal}(\sigma_{G,jk} | 0, 1) \\ & \times \text{normal}(\mu_{G,j}^{\text{pop}} | 0, 1000) \text{half-normal}(\sigma_{G,j}^{\text{pop}} | 0, 1).\end{aligned} \tag{6}$$

$$
\begin{aligned}
[\boldsymbol{\alpha_V}, \boldsymbol{\mu_V}, \boldsymbol{\sigma_V}, \boldsymbol{\mu_V^{\mathbf{pop}}}, \boldsymbol{\sigma_V^{\mathbf{pop}}} | \boldsymbol{y^{\mathbf{tot}}}] \propto \prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} & \mathrm{binomial}(y_{ijk}^{\mathrm{viab}} | n_{ijk}^{\mathrm{test_v}}, \mathrm{logit}^{-1}(\alpha_{V,ijk})) \\
& \times \mathrm{normal}(\alpha_{V,ijk} | \mu_{V,jk}, \sigma_{V,jk}) \\
& \times \mathrm{normal}(\mu_{V,jk} | \mu_{V,j}^{\mathrm{pop}}, \sigma_{V,j}^{\mathrm{pop}}) \\
& \times \text{half-normal}(\sigma_{V,jk} | 0, 1) \\
& \times \mathrm{normal}(\mu_{V,j}^{\mathrm{pop}} | 0, 1000) \text{half-normal}(\sigma_{V,j}^{\mathrm{pop}} | 0, 1).
\end{aligned}
\tag{7}
$$

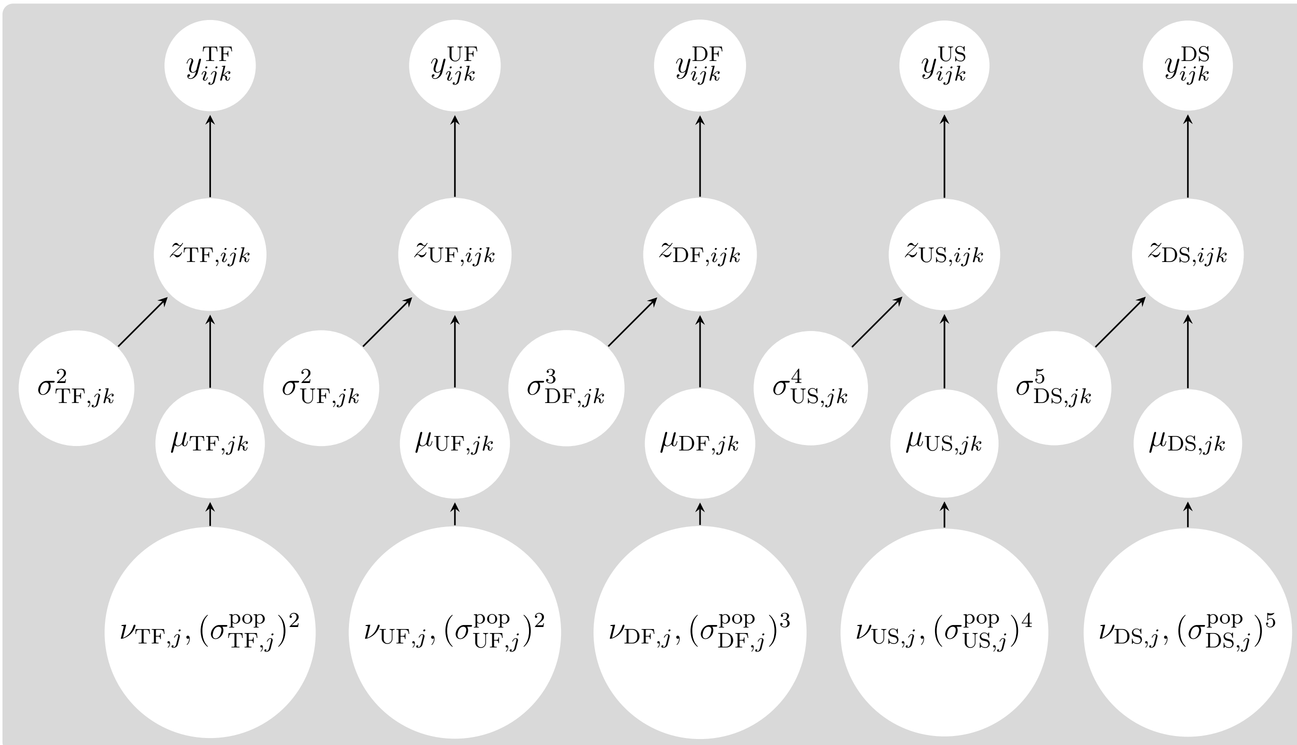
Survival of seedlings to fruiting plants



(a) Directed acyclic graphs for the hierarchical models for seed survival. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
[\boldsymbol{\alpha_S}, \boldsymbol{\mu_S}, \boldsymbol{\sigma_S}, \boldsymbol{\mu_S^{\text{pop}}}, \boldsymbol{\sigma_S^{\text{pop}}} | \boldsymbol{y^{\text{fruiting}}}] \propto \prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} & \text{binomial}(y_{ijk}^{\text{fruiting}} | n_{ijk}^{\text{seedling}}, \text{logit}^{-1}(\alpha_{S,ijk})) \\
& \times \text{normal}(\alpha_{S,ijk} | \mu_{S,jk}, \sigma_{S,jk}) \\
& \times \text{normal}(\mu_{S,jk} | \mu_{S,j}^{\text{pop}}, \sigma_{S,j}^{\text{pop}}) \\
& \times \text{half-normal}(\sigma_{S,jk} | 0, 1) \\
& \times \text{normal}(\mu_{S,j}^{\text{pop}} | 0, 1000) \text{half-normal}(\sigma_{S,j}^{\text{pop}} | 0, 1).
\end{aligned}
\tag{8}
$$

Fruits per plant and seeds per fruit



(a) Directed acyclic graphs for the hierarchical models fruits per plant and seeds per fruit. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
&[\boldsymbol{z}_{\mathbf{TF}}, \boldsymbol{z}_{\mathbf{UF}}, \boldsymbol{z}_{\mathbf{DF}}, \boldsymbol{z}_{\mathbf{US}}, \boldsymbol{z}_{\mathbf{DS}}, \boldsymbol{\mu}_{\mathbf{TF}}, \boldsymbol{\mu}_{\mathbf{UF}}, \boldsymbol{\mu}_{\mathbf{DF}}, \boldsymbol{\mu}_{\mathbf{US}}, \boldsymbol{\mu}_{\mathbf{DS}}, \boldsymbol{\sigma}^2_{\mathbf{TF}}, \boldsymbol{\sigma}^2_{\mathbf{UF}}, \boldsymbol{\sigma}^2_{\mathbf{DF}}, \boldsymbol{\sigma}^2_{\mathbf{US}}, \boldsymbol{\sigma}^2_{\mathbf{DS}},\\
&(\boldsymbol{\sigma}^{\mathbf{pop}}_{\mathbf{TF}})^2, (\boldsymbol{\sigma}^{\mathbf{pop}}_{\mathbf{UF}})^2, (\boldsymbol{\sigma}^{\mathbf{pop}}_{\mathbf{DF}})^2, (\boldsymbol{\sigma}^{\mathbf{pop}}_{\mathbf{US}})^2, (\boldsymbol{\sigma}^{\mathbf{pop}}_{\mathbf{DS}})^2, \boldsymbol{\nu}_{\mathbf{TF}}, \boldsymbol{\nu}_{\mathbf{UF}}, \boldsymbol{\nu}_{\mathbf{DF}}, \boldsymbol{\nu}_{\mathbf{US}}, \boldsymbol{\nu}_{\mathbf{DS}}, |\boldsymbol{y}^{\mathbf{TF}}, \boldsymbol{y}^{\mathbf{UF}}, \boldsymbol{y}^{\mathbf{DF}}, \boldsymbol{y}^{\mathbf{US}}, \boldsymbol{y}^{\mathbf{DS}}] \propto\\
&\prod_{j=1}^{J}\Big\{\prod_{i=1}^{N_1}\prod_{k=1}^{K_1}\text{Poisson}(y^{\text{TF}}_{ijk}|z_{\text{TF},ijk})\ \text{lognormal}(z_{\text{TF},ijk}|\log(\mu_{\text{TF},jk}), \sigma^2_{\text{TF},jk})\\
&\times \text{lognormal}(\mu_{\text{TF},jk}|\log(\text{g}(\nu_{\text{TF},j}), (\sigma^{\text{pop}}_{\text{TF},j})^2)\\
&\times \text{inverse gamma}(\sigma^2_{\text{TF},jk}|0.001, 0.001)\\
&\times \text{gamma}(\nu_{\text{TF},j}|1,1)\ \text{inverse gamma}((\sigma^{\text{pop}}_{\text{TF},j})^2|0.001, 0.001)\Big\}\\
\times&\Big\{\prod_{i=1}^{N_2}\prod_{k=1}^{K_2}\text{Poisson}(y^{\text{UF}}_{ijk}|z_{\text{UF},ijk})\ \text{Poisson}(y^{\text{DF}}_{ijk}|z_{\text{DF},ijk})\\
&\times \text{lognormal}(z_{\text{UF},ijk}|\log(\mu_{\text{UF},jk}), \sigma^2_{\text{UF},jk})\ \text{lognormal}(z_{\text{DF},ijk}|\log(\mu_{\text{DF},jk}), \sigma^2_{\text{DF},jk})\\
&\times \text{lognormal}(\mu_{\text{UF},jk}|\log(\text{g}(\nu_{\text{UF},j}), (\sigma^{\text{pop}}_{\text{UF},j})^2)\ \text{lognormal}(\mu_{\text{DF},jk}|\log(\text{g}(\nu_{\text{DF},j}), (\sigma^{\text{pop}}_{\text{DF},j})^2)\\
&\times \text{inverse gamma}(\sigma^2_{\text{UF},jk}|0.001, 0.001)\ \text{inverse gamma}(\sigma^2_{\text{DF},jk}|0.001, 0.001)\\
&\times \text{gamma}(\nu_{\text{UF},j}|1,1)\ \text{inverse gamma}((\sigma^{\text{pop}}_{\text{UF},j})^2|0.001, 0.001)\\
&\times \text{gamma}(\nu_{\text{DF},j}|1,1)\ \text{inverse gamma}((\sigma^{\text{pop}}_{\text{DF},j})^2|0.001, 0.001)\Big\}\\
&\times\Big\{\prod_{i=1}^{N_3}\prod_{k=1}^{K_3}\text{Poisson}(y^{\text{US}}_{ijk}|z_{\text{US},ijk})\ \text{lognormal}(z_{\text{US},ijk}|\log(\mu_{\text{UF},jk}), \sigma^2_{\text{US},jk})\\
&\times \text{lognormal}(\mu_{\text{US},jk}|\log(\text{g}(\nu_{\text{US},j}), (\sigma^{\text{pop}}_{\text{US},j})^2)\\
&\times \text{inverse gamma}(\sigma^2_{\text{US},jk}|0.001, 0.001)\\
&\times \text{gamma}(\nu_{\text{US},j}|1,1)\ \text{inverse gamma}((\sigma^{\text{pop}}_{\text{US},j})^2|0.001, 0.001)\Big\}\\
&\times\Big\{\prod_{i=1}^{N_4}\prod_{k=1}^{K_4}\text{Poisson}(y^{\text{DS}}_{ijk}|z_{\text{DS},ijk})\ \text{lognormal}(z_{\text{DS},ijk}|\log(\mu_{\text{UF},jk}), \sigma^2_{\text{DS},jk})\\
&\times \text{lognormal}(\mu_{\text{DS},jk}|\log(\text{g}(\nu_{\text{DS},j}), (\sigma^{\text{pop}}_{\text{DS},j})^2)\\
&\times \text{inverse gamma}(\sigma^2_{\text{DS},jk}|0.001, 0.001)\\
&\times \text{gamma}(\nu_{\text{DS},j}|1,1)\ \text{inverse gamma}((\sigma^{\text{pop}}_{\text{DS},j})^2|0.001, 0.001)\Big\}.
\end{aligned}
\tag{9}
$$