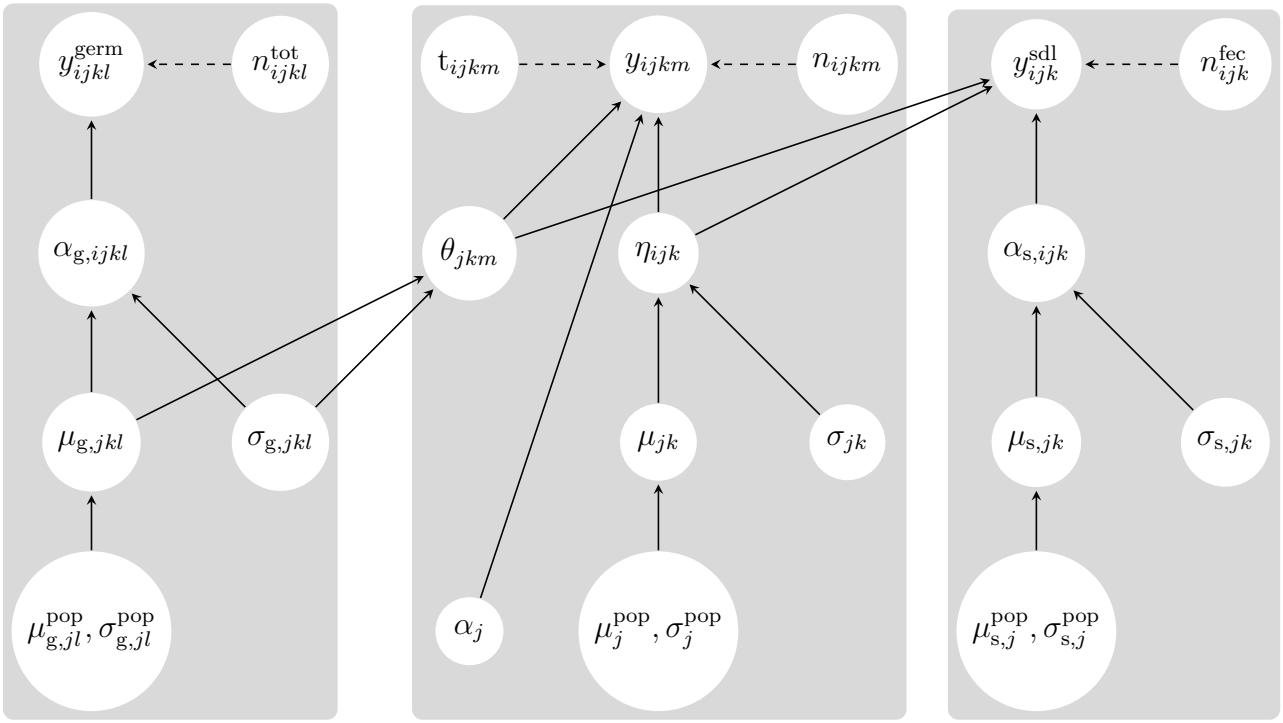
Last updated: April 5, 2021

1 Appendix: Posteriors and joint likelihoods

Add description of this appendix. The appendix includes directed acyclic graphs, and posterior and proportion joint distributions for all models.

2 Full model: germination, seed survivorship, initial seed survival

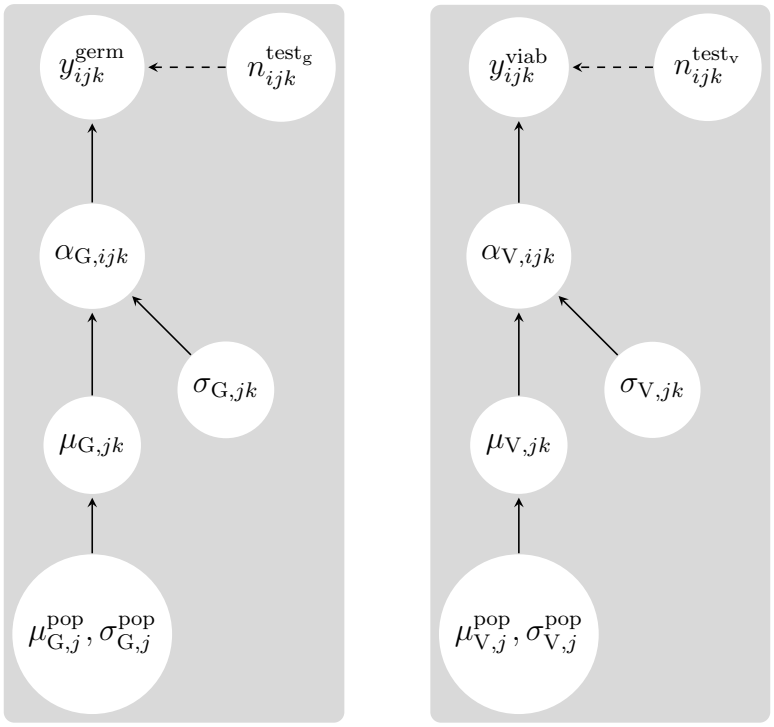


(a) Directed acyclic graphs for the joint models for seed germination, persistence, and survival from seed production to the first October. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$g(\eta_{ijk}, \theta_{jkm}, t_{ijkm}) = \theta_{jkm} \times \exp(-(\frac{t_{ijkm}}{\exp(-\frac{\eta_{ijk}}{\alpha_j})})^{\alpha_j})$$

$$
\begin{aligned}
&[\boldsymbol{\mu}_{\mathrm{g}}, \boldsymbol{\sigma}_{\mathrm{g}}, \boldsymbol{\mu}_{\mathrm{g}}^{\mathrm{pop}}, \boldsymbol{\sigma}_{\mathrm{g}}^{\mathrm{pop}}, \boldsymbol{\eta}, \boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\mu}^{\mathrm{pop}}, \boldsymbol{\sigma}^{\mathrm{pop}} | \mathbf{y}_{\mathrm{g}}, \mathbf{y}] \propto \\
&\prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} \Bigg[\Big[\prod_{m=1}^{M} \mathrm{binomial}(y_{ijkm} | n_{ijkm}, g(\theta_{jkm}, \eta_{ijk}, \alpha_j, t_{ijkm})) \\
&\times \mathrm{normal}(\eta_{ijk} | \mu_{jk}, \sigma_{jk}) \\
&\times \mathrm{normal}(\mu_{jk} | \mu_j^{\mathrm{pop}}, \sigma_j^{\mathrm{pop}}) \mathrm{half\text{-}normal}(\sigma_{jk} | 0, 1) \\
&\times \mathrm{normal}(\mu_j^{\mathrm{pop}} | 0, 1) \mathrm{half\text{-}normal}(\sigma_j^{\mathrm{pop}} | 0, 1) \\
&\times \mathrm{gamma}(\alpha_j | 2, 2) \Big] \\
&\Big[\times \prod_{l=1}^{L} \mathrm{binomial}(y_{\mathrm{g},ijkl} | n_{\mathrm{g},ijkl}, \mathrm{logit}^{-1}(\alpha_{\mathrm{g},ijkl})) \\
&\times \mathrm{normal}(\alpha_{\mathrm{g},ijkl} | \mu_{\mathrm{g},jkl}, \sigma_{\mathrm{g},jkl}) \\
&\times \mathrm{normal}(\mu_{\mathrm{g},jkl} | \mu_{\mathrm{g},jl}^{\mathrm{pop}}, \sigma_{\mathrm{g},jl}^{\mathrm{pop}}) \mathrm{half\text{-}normal}(\sigma_{\mathrm{g},jkl} | 0, 1) \\
&\times \mathrm{normal}(\mu_{\mathrm{g},jl}^{\mathrm{pop}} | 0, 1) \mathrm{half\text{-}normal}(\sigma_{\mathrm{g},jl}^{\mathrm{pop}} | 0, 1). \Big] \Bigg] \\
&\prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=2}^{K} \mathrm{binomial}(y_{\mathrm{p},ijk} | n_{\mathrm{p},ijk}, g(\theta_{jk,m=1}, \eta_{ijk}, \alpha_j, t_{ijk,m=1}) \mathrm{logit}^{-1}(\alpha_{\mathrm{s},ijk})) \\
&\times \mathrm{normal}(\alpha_{\mathrm{s},ijk} | \mu_{\mathrm{g},jk}, \sigma_{\mathrm{s},jk}) \\
&\times \mathrm{normal}(\mu_{\mathrm{s},jk} | \mu_{\mathrm{s},j}^{\mathrm{pop}}, \sigma_{\mathrm{s},j}^{\mathrm{pop}}) \\
&\times \mathrm{half\text{-}normal}(\sigma_{\mathrm{s},jk} | 0, 1) \\
&\times \mathrm{normal}(\mu_{\mathrm{s},j}^{\mathrm{pop}} | 0, 1) \mathrm{half\text{-}normal}(\sigma_{\mathrm{s},j}^{\mathrm{pop}} | 0, 1)
\end{aligned}
\tag{1}
$$

Viability trials

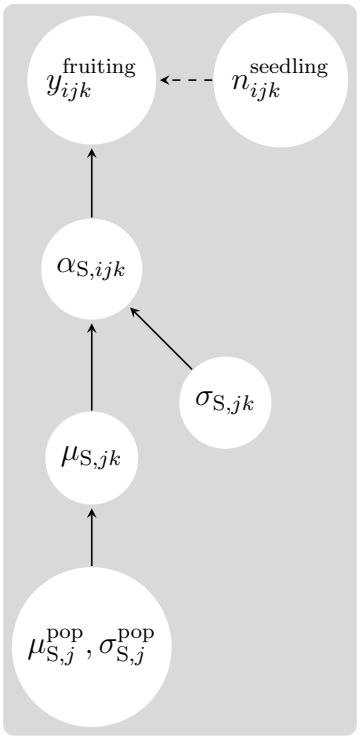


(a) Directed acyclic graphs for the hierarchical models for viability trials. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
[\boldsymbol{\alpha_G}, \boldsymbol{\mu_G}, \boldsymbol{\sigma_G}, \boldsymbol{\mu_G^{\text{pop}}}, \boldsymbol{\sigma_G^{\text{pop}}} | \boldsymbol{y^{\text{tot}}}] \propto & \prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} \text{binomial}(y_{ijk}^{\text{germ}} | n_{ijk}^{\text{test}_\text{g}}, \text{logit}^{-1}(\alpha_{G,ijk})) \\
& \times \text{normal}(\alpha_{G,ijk} | \mu_{G,jk}, \sigma_{G,jk}) \\
& \times \text{normal}(\mu_{G,jk} | \mu_{G,j}^{\text{pop}}, \sigma_{G,j}^{\text{pop}}) \\
& \times \text{half-normal}(\sigma_{G,jk} | 0, 1) \\
& \times \text{normal}(\mu_{G,j}^{\text{pop}} | 0, 1) \text{half-normal}(\sigma_{G,j}^{\text{pop}} | 0, 1).
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
[\boldsymbol{\alpha_V}, \boldsymbol{\mu_V}, \boldsymbol{\sigma_V}, \boldsymbol{\mu_V^{\text{pop}}}, \boldsymbol{\sigma_V^{\text{pop}}} | \boldsymbol{y^{\text{tot}}}] \propto & \prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} \text{binomial}(y_{ijk}^{\text{viab}} | n_{ijk}^{\text{test}_\text{v}}, \text{logit}^{-1}(\alpha_{V,ijk})) \\
& \times \text{normal}(\alpha_{V,ijk} | \mu_{V,jk}, \sigma_{V,jk}) \\
& \times \text{normal}(\mu_{V,jk} | \mu_{V,j}^{\text{pop}}, \sigma_{V,j}^{\text{pop}}) \\
& \times \text{half-normal}(\sigma_{V,jk} | 0, 1) \\
& \times \text{normal}(\mu_{V,j}^{\text{pop}} | 0, 1) \text{half-normal}(\sigma_{V,j}^{\text{pop}} | 0, 1).
\end{aligned} \tag{3}
$$

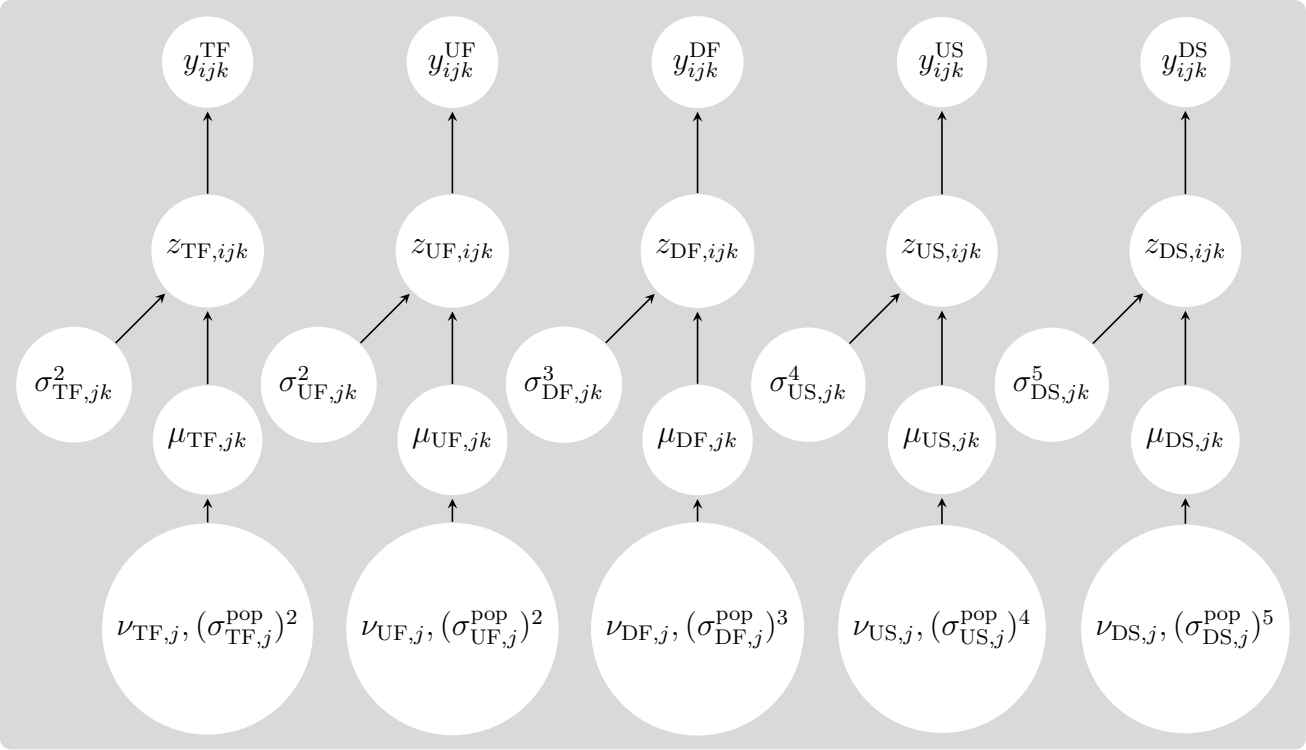
Survival of seedlings to fruiting plants



(a) Directed acyclic graphs for the hierarchical models for seedling survival to fruiting. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
[\boldsymbol{\alpha_S}, \boldsymbol{\mu_S}, \boldsymbol{\sigma_S}, \boldsymbol{\mu_S^{\text{pop}}}, \boldsymbol{\sigma_S^{\text{pop}}} | \boldsymbol{y^{\text{fruiting}}}] \propto & \prod_{i=1}^{I} \prod_{j=1}^{J} \prod_{k=1}^{K} \text{binomial}(y_{ijk}^{\text{fruiting}} | n_{ijk}^{\text{seedling}}, \text{logit}^{-1}(\alpha_{S,ijk})) \\
& \times \text{normal}(\alpha_{S,ijk} | \mu_{S,jk}, \sigma_{S,jk}) \\
& \times \text{normal}(\mu_{S,jk} | \mu_{S,j}^{\text{pop}}, \sigma_{S,j}^{\text{pop}}) \\
& \times \text{half-normal}(\sigma_{S,jk} | 0, 1) \\
& \times \text{normal}(\mu_{S,j}^{\text{pop}} | 0, 1) \text{half-normal}(\sigma_{S,j}^{\text{pop}} | 0, 1).
\end{aligned}
\tag{4}
$$

Fruits per plant and seeds per fruit



(a) Directed acyclic graphs for the hierarchical models fruits per plant and seeds per fruit. Solid arrows depict the relationships among random variables, and dashed arrows depict the deterministic relationships.

$$
\begin{aligned}
&g(\theta) = \exp\theta \\
&[\boldsymbol{z_{\mathbf{TF}}}, \boldsymbol{z_{\mathbf{UF}}}, \boldsymbol{z_{\mathbf{DF}}}, \boldsymbol{z_{\mathbf{US}}}, \boldsymbol{z_{\mathbf{DS}}}, \boldsymbol{\mu_{\mathbf{TF}}}, \boldsymbol{\mu_{\mathbf{UF}}}, \boldsymbol{\mu_{\mathbf{DF}}}, \boldsymbol{\mu_{\mathbf{US}}}, \boldsymbol{\mu_{\mathbf{DS}}}, \boldsymbol{\sigma^2_{\mathbf{TF}}}, \boldsymbol{\sigma^2_{\mathbf{UF}}}, \boldsymbol{\sigma^2_{\mathbf{DF}}}, \boldsymbol{\sigma^2_{\mathbf{US}}}, \boldsymbol{\sigma^2_{\mathbf{DS}}}, \\
&(\boldsymbol{\sigma^{\mathbf{pop}}_{\mathbf{TF}}})^2, (\boldsymbol{\sigma^{\mathbf{pop}}_{\mathbf{UF}}})^2, (\boldsymbol{\sigma^{\mathbf{pop}}_{\mathbf{DF}}})^2, (\boldsymbol{\sigma^{\mathbf{pop}}_{\mathbf{US}}})^2, (\boldsymbol{\sigma^{\mathbf{pop}}_{\mathbf{DS}}})^2, \boldsymbol{\nu_{\mathbf{TF}}}, \boldsymbol{\nu_{\mathbf{UF}}}, \boldsymbol{\nu_{\mathbf{DF}}}, \boldsymbol{\nu_{\mathbf{US}}}, \boldsymbol{\nu_{\mathbf{DS}}}, |\boldsymbol{y^{\mathbf{TF}}}, \boldsymbol{y^{\mathbf{UF}}}, \boldsymbol{y^{\mathbf{DF}}}, \boldsymbol{y^{\mathbf{US}}}, \boldsymbol{y^{\mathbf{DS}}}] \propto \\
&\prod_{j=1}^{J} \Big\{ \prod_{i=1}^{N_1} \prod_{k=1}^{K_1} \text{Poisson}(y^{\text{TF}}_{ijk} | z_{\text{TF},ijk}) \ \text{lognormal}(z_{\text{TF},ijk} | \log(\mu_{\text{TF},jk}), \sigma^2_{\text{TF},jk}) \\
&\quad \times \text{lognormal}(\mu_{\text{TF},jk} | \log(\text{g}(\nu_{\text{TF},j}), (\sigma^{\text{pop}}_{\text{TF},j})^2) \\
&\quad \times \text{half-normal}(\sigma^2_{\text{TF},jk} | 0, 1) \\
&\quad \times \text{gamma}(\nu_{\text{TF},j} | 1, 1) \ \text{half-normal}((\sigma^{\text{pop}}_{\text{TF},j})^2 | 0, 1) \Big\} \\
&\times \Big\{ \prod_{i=1}^{N_2} \prod_{k=1}^{K_2} \text{Poisson}(y^{\text{UF}}_{ijk} | z_{\text{UF},ijk}) \ \text{Poisson}(y^{\text{DF}}_{ijk} | z_{\text{DF},ijk}) \\
&\quad \times \text{lognormal}(z_{\text{UF},ijk} | \log(\mu_{\text{UF},jk}), \sigma^2_{\text{UF},jk}) \ \text{lognormal}(z_{\text{DF},ijk} | \log(\mu_{\text{DF},jk}), \sigma^2_{\text{DF},jk}) \\
&\quad \times \text{lognormal}(\mu_{\text{UF},jk} | \log(\text{g}(\nu_{\text{UF},j}), (\sigma^{\text{pop}}_{\text{UF},j})^2) \ \text{lognormal}(\mu_{\text{DF},jk} | \log(\text{g}(\nu_{\text{DF},j}), (\sigma^{\text{pop}}_{\text{DF},j})^2) \\
&\quad \times \text{half-normal}(\sigma^2_{\text{UF},jk} | 0, 1) \ \text{half-normal}(\sigma^2_{\text{DF},jk} | 0, 1) \\
&\quad \times \text{gamma}(\nu_{\text{UF},j} | 1, 1) \ \text{half-normal}((\sigma^{\text{pop}}_{\text{UF},j})^2 | 0, 1) \\
&\quad \times \text{gamma}(\nu_{\text{DF},j} | 1, 1) \ \text{half-normal}((\sigma^{\text{pop}}_{\text{DF},j})^2 | 0, 1) \Big\} \\
&\quad \times \Big\{ \prod_{i=1}^{N_3} \prod_{k=1}^{K_3} \text{Poisson}(y^{\text{US}}_{ijk} | z_{\text{US},ijk}) \ \text{lognormal}(z_{\text{US},ijk} | \log(\mu_{\text{UF},jk}), \sigma^2_{\text{US},jk}) \\
&\quad \times \text{lognormal}(\mu_{\text{US},jk} | \log(\text{g}(\nu_{\text{US},j}), (\sigma^{\text{pop}}_{\text{US},j})^2) \\
&\quad \times \text{half-normal}(\sigma^2_{\text{US},jk} | 0, 1) \\
&\quad \times \text{gamma}(\nu_{\text{US},j} | 1, 1) \ \text{half-normal}((\sigma^{\text{pop}}_{\text{US},j})^2 | 0, 1) \Big\} \\
&\quad \times \Big\{ \prod_{i=1}^{N_4} \prod_{k=1}^{K_4} \text{Poisson}(y^{\text{DS}}_{ijk} | z_{\text{DS},ijk}) \ \text{lognormal}(z_{\text{DS},ijk} | \log(\mu_{\text{UF},jk}), \sigma^2_{\text{DS},jk}) \\
&\quad \times \text{lognormal}(\mu_{\text{DS},jk} | \log(\text{g}(\nu_{\text{DS},j}), (\sigma^{\text{pop}}_{\text{DS},j})^2) \\
&\quad \times \text{half-normal}(\sigma^2_{\text{DS},jk} | 0, 1) \\
&\quad \times \text{gamma}(\nu_{\text{DS},j} | 1, 1) \ \text{half-normal}((\sigma^{\text{pop}}_{\text{DS},j})^2 | 0, 1) \Big\}. \qquad (5)
\end{aligned}
$$