Appendix X

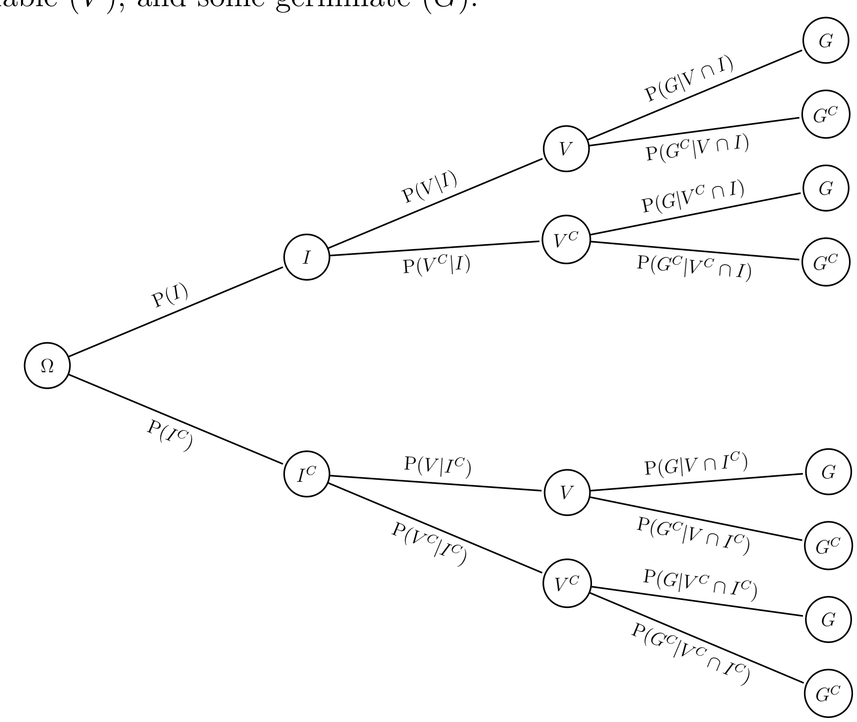
This appendix shows that the probability of germination, conditional on seeds being viable and intact, can be calculated from the probability of viability conditional on being intact and not germinating $v^{1/3}$ and the probability of being a seedling conditional on being intact, θ^g. In the model fitting process, we estimate the probabilities on the right hand side of the equation below, and use that to derive the probability of germination, conditional on seeds being viable and intact.

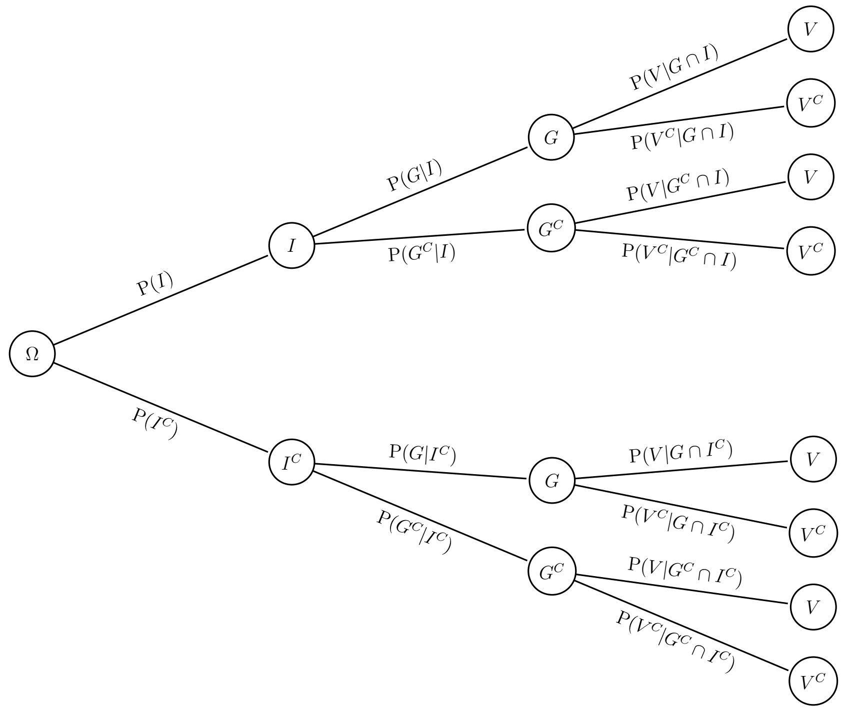
$$P(G|V \cap I) = g_1 = \frac{n^{\text{germinants}}}{n^{\text{intact}} \times v^{1/3} + n^{\text{germinants}}} = \frac{\theta^g}{1-(1-\theta^v)(1-\theta^g)} \tag{1}$$

The plot below shows that the two methods produce the same estimates for the probability of germination, a given number of intact seeds and number of seeds that are germinated.

Conditional probability tree

Here's a conditional probability tree to graphically represent the events in the seed bag experiments. The seed bags are buried, some are intact and/or germinate (I), some are viable (V), and some germinate (G).





We have these estimates

- P(I) = in January, the seeds that are intact and germinated
- P($G|I$) = in January, the seeds that germinated given that they were intact
- P($V|G \cap I$) = in January, the seeds that were viable given that they germinated and were intact (we assume this is 1; all seeds that germinate are viable)
- P($G^C|V^C \cap I$) = in January, the seeds that were intact but not viable did not germinate (we assume this is 1; any seed that is not viable can not germinate)
- P($V|G^C \cap I$) = in January, the seeds that viable given that they are intact and that not germinate (this is $v^{\frac{1}{3}}$)

The germination probability, g_1, is the probability that seeds germinate given that they are intact and viable. Put another way: of the viable seeds, how many germinate? In our probability tree, this is $g_1 = P(G|V \cap I)$. The conditional probability trees give the following equalities:

$$\begin{aligned} P(V \cap G \cap I) &= P(V|G \cap I)P(G|I)P(I) \\ P(G \cap V \cap I) &= P(G|V \cap I)P(V|I)P(I) \end{aligned} \tag{2}$$

To solve for this, we start by solving for P($V|I$). More accurately, we solve for P($V^C|I$) and use this to get P($V|I$) = 1-P($V^C|I$).

$$
\begin{aligned}
P(V^C \cap G^C \cap I) &= P(G^C \cap V^C \cap I) \\
P(V^C|G^C \cap I)P(G^C|I)P(I) &= P(G^C|V^C \cap I)P(V^C|I)P(I) \\
P(V^C|I) &= \frac{P(V^C|G^C \cap I)P(G^C|I)P(I)}{P(G^C|V^C \cap I)P(I)}
\end{aligned} \tag{3}
$$

We can then cancel out the probability of being intact. We also use P($V^C|G^C \cap I$) = 1-P($V|G^C \cap I$), P($G^C|I$) = 1-P($G|I$) and the assumption that P($G^C|V^C \cap I$) = 1 to rewrite the expression:

$$
P(V^C|I) = (1 - P(V|G^C \cap I))(1 - P(G|I)) \tag{4}
$$

We then substitute the probability of viability for seeds that are intact but did not germinate, P($V|G^C \cap I$), and the probability of germination for seeds that are intact, P($G|I$). We solve for P($V|I$) = 1-P($V^C|I$)

$$
P(V|I) = 1 - (1 - P(V|G^C \cap I))(1 - P(G|I)) \tag{5}
$$

We can set these expressions in equation (2) equal to each other, which allows us to solve for $g_1 = P(G|V \cap I)$:

$$
\begin{aligned}
P(V \cap G \cap I) &= P(G \cap V \cap I) \\
P(G|V \cap I)P(V|I)P(I) &= P(V|G \cap I)P(G|I)P(I) \\
P(G|V \cap I) &= \frac{P(V|G \cap I)P(G|I)P(I)}{P(V|I)P(I)} \\
P(G|V \cap I) &= \frac{P(G|I)P(I)}{P(V|I)P(I)}
\end{aligned} \tag{6}
$$

We use P($V|G \cap I$) = 1. We are then left with a statement saying that the probability of germination for seeds that are viable and intact is the probability of germination, conditional on being intact, divided by the probability of being viable, conditional on being intact. The probabilities of being intact cancel and we can use our solution for the probability of being viable conditional on being intact as well as the independently

$$
P(G|V \cap I) = \frac{P(G|I)P(I)}{P(V|I)P(I)} \tag{7}
$$

We use this relationship to calculate the probability of germination conditional on being viable and intact as a derived quantity of estimates for the probability of germination conditional on being intact, and of the probability of being viable conditional on being intact.

$$P(G|V \cap I) = \frac{\theta^g}{1 - (1 - \theta^v)(1 - \theta^g)} \tag{8}$$

Where θ^g is the probability of a bag having germinants and θ^v is the probability of viability in January.