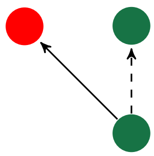
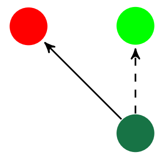
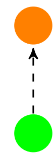
Unbranched, determinate case



(a) Primary meristem transitioning to one vegetative meristem and generating one primary meristem.



(b) Primary meristem transitioning to one vegetative meristem, and generating one inflorescence meristem.



(c) Inflorescence meristem generating a floral meristem

Figure 1: Meristem transitions in an unbranched plant with a determinate inflorescence.

The optimal control problem we are interested in is

$$\max_u \int_0^T \log(F(t))dt$$

$$\begin{aligned}
\text{subject to } \dot{P} &= -[q(t)V][(1-p(t))P], \\
\dot{V} &= [q(t)V][p(t)P] + [q(t)V][(1-p(t))P], \\
\dot{I} &= [q(t)V][(1-p(t))P], \\
\dot{F} &= [(1-q(t))V]I, \\
& P(0) > 0;\ V(0),\ I(0),\ F(0) \geq 0, \\
& 0 \leq p(t), q(t) \leq 1. \qquad (1)
\end{aligned}$$

notation here is confusing; either could be 0->1

Term	**Description**
P	Primary meristems
V	Vegetative biomass
I	Inflorescence meristems
F	Floral meristems
p	The probability that a primary meristem division produces a primary and vegetative meristem. A primary meristem division either produces a primary and vegetative meristem (Figure 1A) or an inflorescence and vegetative meristem (Figure 1B).
q	The fraction of photosynthate that is allocated to vegetative growth. Here, vegetative growth consists of primary meristem divisions. Any photosynthate not allocated to primary meristem divisions is allocated to inflorescence meristem divisions.

The Hamiltonian is

$$H = \log F + ((PV\lambda_1 - PV\lambda_3)p - IV\lambda_4 + PV\lambda_3 + PV\lambda_2 - PV\lambda_1)q + IV\lambda_4 \tag{2}$$

The optimality conditions are

$$\frac{\partial H}{\partial p} = (PV)(\lambda_1 - \lambda_3)q = 0 \text{ at } p^* \tag{3}$$

$$\frac{\partial H}{\partial q} = (PV)(\lambda_1 - \lambda_3)p - IV\lambda_4 + PV\lambda_3 + PV\lambda_2 - PV\lambda_1 = 0 \text{ at } q^* \tag{4}$$

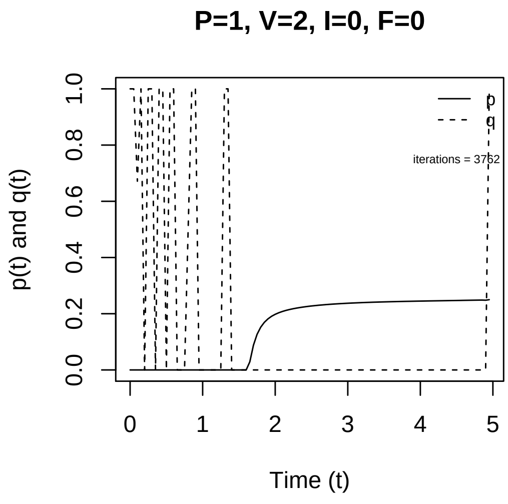
The transversality condition is

$$\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0. \tag{5}$$

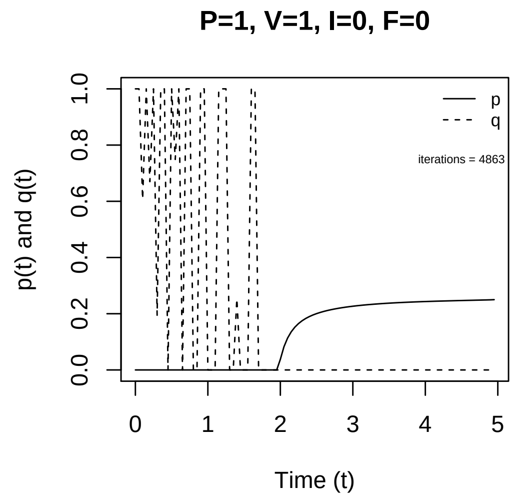
The adjoint equations are

$$\begin{aligned}
-\frac{\partial H}{\partial P} &= \dot{\lambda_1} = -((V\lambda_1 - V\lambda_3)p + V\lambda_3 + V\lambda_2 - V\lambda_1)q \\
-\frac{\partial H}{\partial V} &= \dot{\lambda_2} = -((P\lambda_1 - P\lambda_3)p - I\lambda_4 + P\lambda_3 + P\lambda_2 - P\lambda_1)q - I\lambda_4 \\
-\frac{\partial H}{\partial I} &= \dot{\lambda_3} = V\lambda_4 q - V\lambda_4 \\
-\frac{\partial H}{\partial L} &= \dot{\lambda_4} = -\frac{1}{F}
\end{aligned} \tag{6}$$

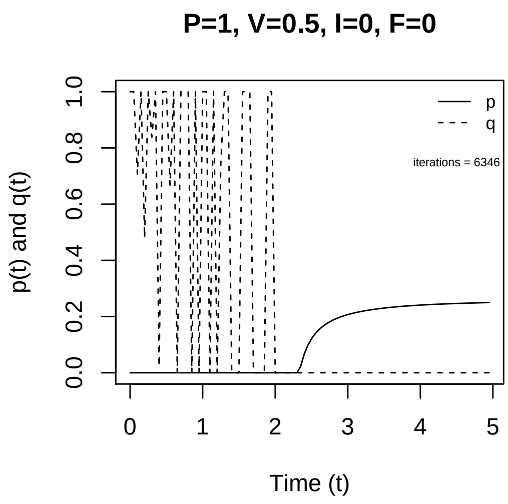
I used an adapted forward-backward sweep. The figures on the following pages summarize some solutions for different initial conditions and show the general trajectory of state variables and adjoint variables.



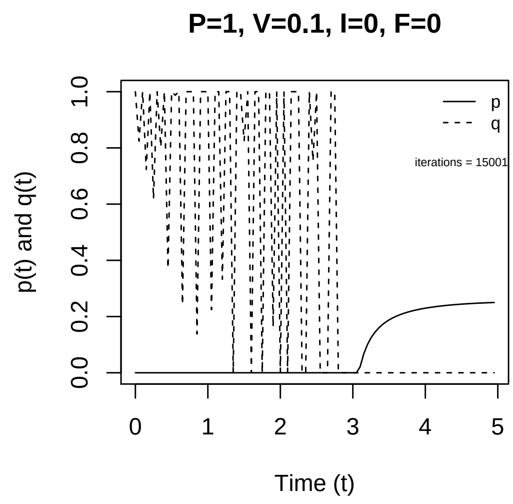
(a) ...



(b) ...



(c) ...



(d) ...

Figure 2: Trajectories of controls p (solid line) and q (dashed line) for a range of initial conditions. The optimal control suggests no division of primary meristems ($p(t) = 0$) initially with oscillating allocation of photosynthate to vegetative growth. I cut off the solutions at 15000 iterations, so the controls in Panel d have not converged. See next two pages for figures corresponding to state trajectories and adjoint variables for Panel d.

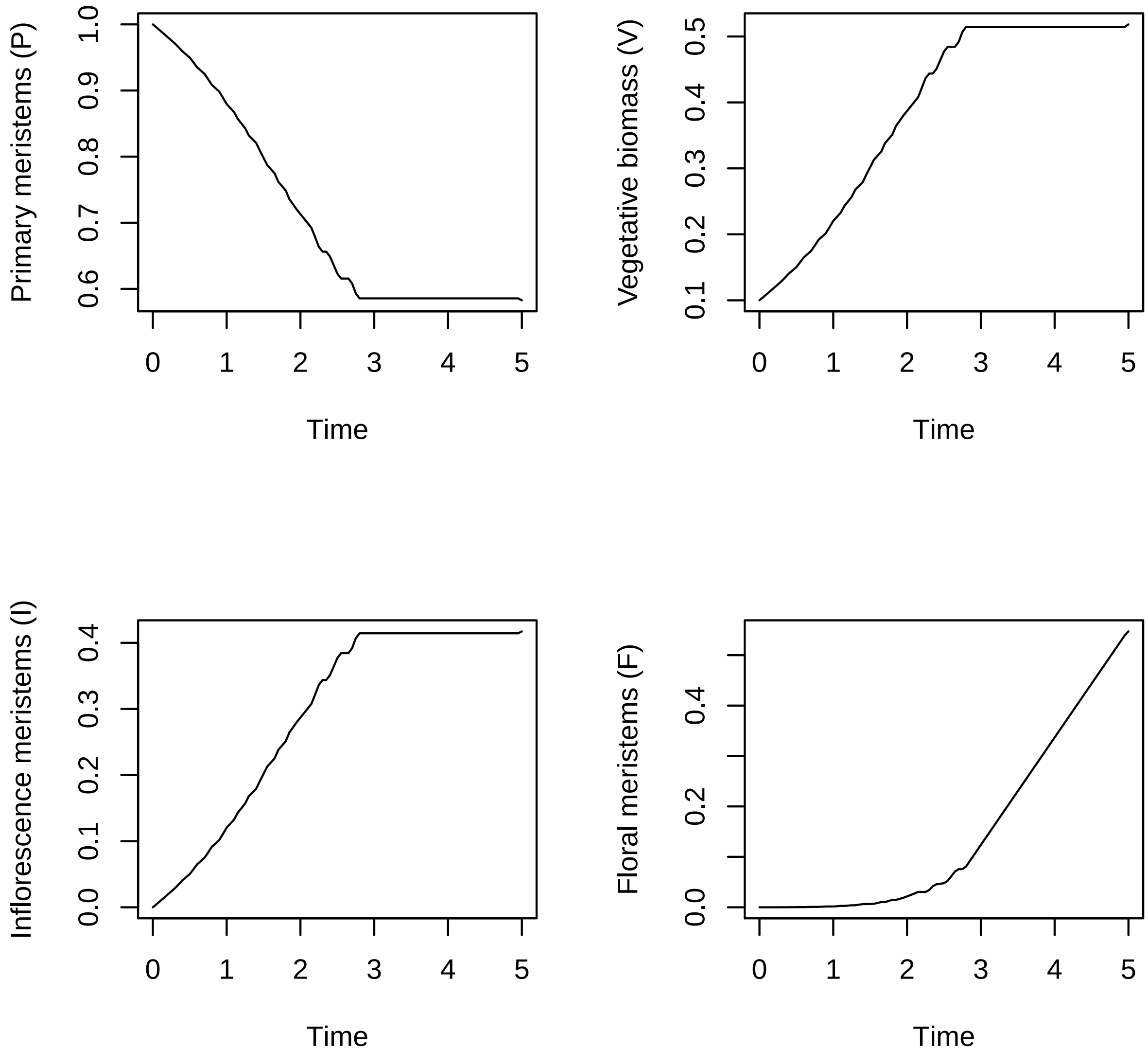


Figure 3: Trajectories of state variables for the case where the initial conditions are $P(0) = 1,\ V(0) = 0.1,\ I(0) = 0,\ F(0) = 0$.

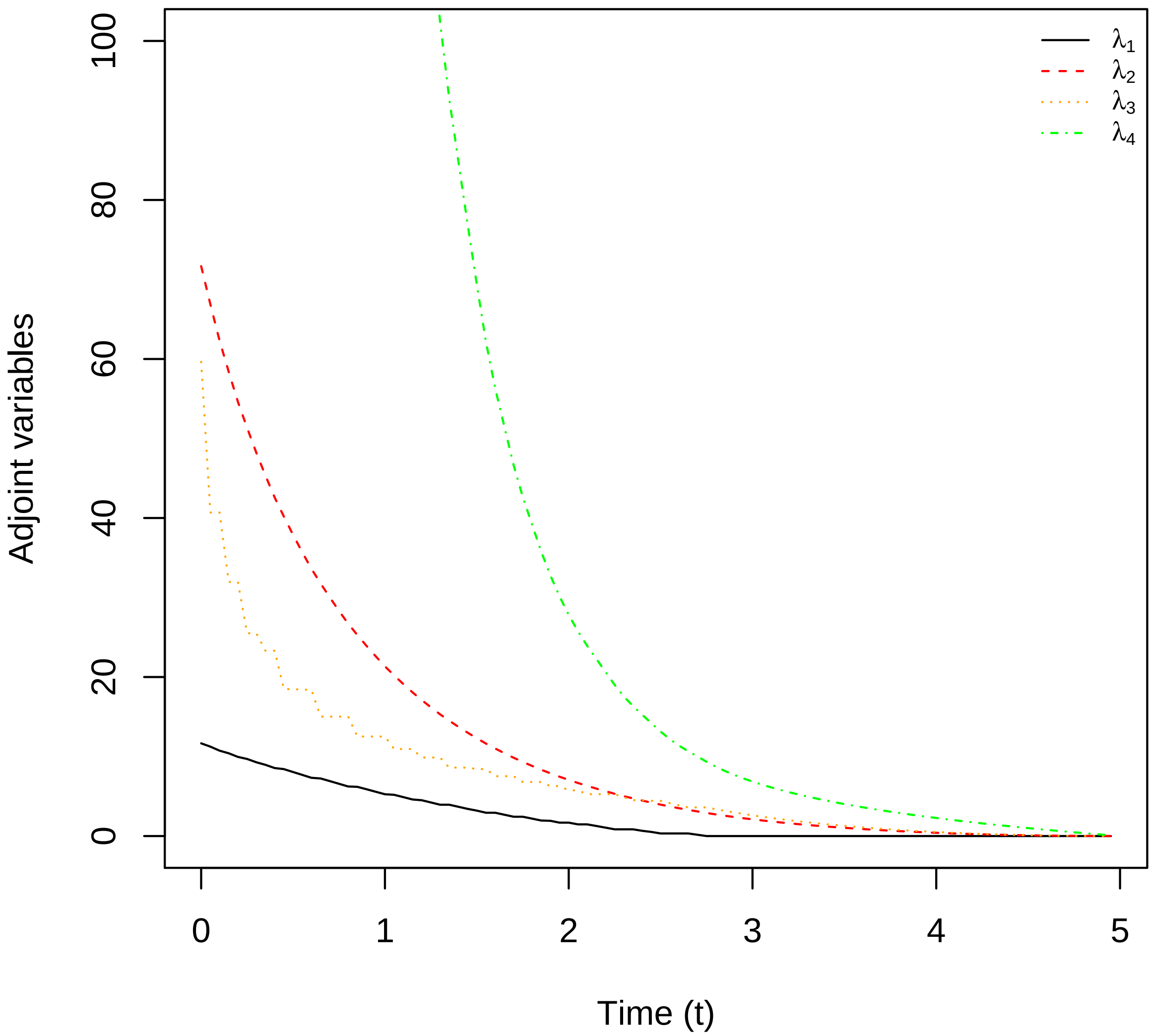
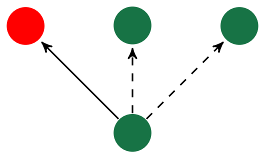
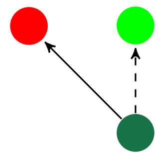


Figure 4: Trajectories of adjoint variables for the case where the initial conditions are $P(0) = 1,\ V(0) = 0.1,\ I(0) = 0,\ F(0) = 0$.

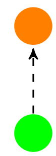
6 Branched, determinate case



(a) Primary meristem transitioning to one vegetative meristem and generating two primary meristems.



(b) Primary meristem transitioning to one vegetative meristem, and generating one inflorescence meristem.



(c) Inflorescence meristem generating a floral meristem

Figure 5: Meristem transitions in an unbranched plant with a determinate inflorescence.

The optimal control problem we are interested in is

$$\max_u \int_0^T \log(F(t))dt$$

$$\begin{aligned}
\text{subject to } \dot{P} &= [q(t)V][p(t)P] - [q(t)V][(1-p(t))P], \\
\dot{V} &= [q(t)V][p(t)P] + [q(t)V][(1-p(t))P], \\
\dot{I} &= [q(t)V][(1-p(t))P], \\
\dot{F} &= [(1-q(t))V]I, \\
& P(0) > 0;\ V(0),\ I(0),\ F(0) \geq 0, \\
& 0 \leq p(t), q(t) \leq 1. \qquad (7)
\end{aligned}$$

The Hamiltonian is

Term	Description
P	Primary meristems
V	Vegetative biomass
I	Inflorescence meristems
F	Floral meristems
p	The probability that a primary meristem division produces a primary and vegetative meristem. A primary meristem division either produces a primary and vegetative meristem (Figure 1A) or an inflorescence and vegetative meristem (Figure 1B).
q	The fraction of photosynthate that is allocated to vegetative growth. Here, vegetative growth consists of primary meristem divisions. Any photosynthate not allocated to primary meristem divisions is allocated to inflorescence meristem divisions.

$$H = \log F + ((2PV\lambda_1 - PV\lambda_3)p - IV\lambda_4 + PV\lambda_3 + PV\lambda_2 - PV\lambda_1)q + IV\lambda_4 \tag{8}$$

The optimality conditions are

$$\frac{\partial H}{\partial p} = (PV)(2\lambda_1 - \lambda_3)q = 0 \text{ at } p^* \tag{9}$$

$$\frac{\partial H}{\partial q} = (PV)(2\lambda_1 - \lambda_3)p - IV\lambda_4 + PV\lambda_3 + PV\lambda_2 - PV\lambda_1 = 0 \text{ at } q^* \tag{10}$$

The transversality condition is

$$\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0. \tag{11}$$

The adjoint equations are

$$\begin{aligned}
-\frac{\partial H}{\partial P} &= \dot{\lambda_1} = -((2V\lambda_1 - V\lambda_3)p + V\lambda_3 + V\lambda_2 - V\lambda_1)q \\
-\frac{\partial H}{\partial V} &= \dot{\lambda_2} = -((2P\lambda_1 - P\lambda_3)p - I\lambda_4 + P\lambda_3 + P\lambda_2 - P\lambda_1)q - I\lambda_4 \\
-\frac{\partial H}{\partial I} &= \dot{\lambda_3} = V\lambda_4 q - V\lambda_4 \\
-\frac{\partial H}{\partial L} &= \dot{\lambda_4} = -\frac{1}{F} \qquad (12)
\end{aligned}$$