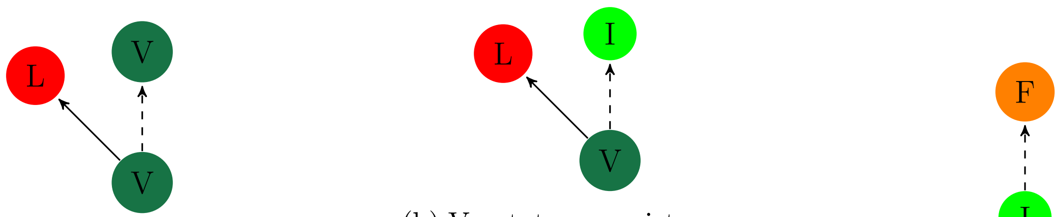
Unbranched plant with a terminal, determinate flower

The case we consider here is the one for a plant that does not branch and has a terminal, determinate flower. The unbranched, vegetative architecture is established by primary meristem divisions that generate a vegetative meristem and leaf (Figure 1A); because there is never more than one vegetative meristem, the plant does not branch. At the transition to flowering, the vegetative meristem transitions to an inflorescence meristem (Figure 1B). Finally, the terminal, determinate flower is represented in the model by the transition from the inflorescence meristem to a flower (Figure 1C).



(a) Vegetative meristem transitioning to one vegetative meristem and generating one leaf.

(b) Vegetatuve meristem transitioning to one vegetative meristem, and generating one inflorescence meristem.

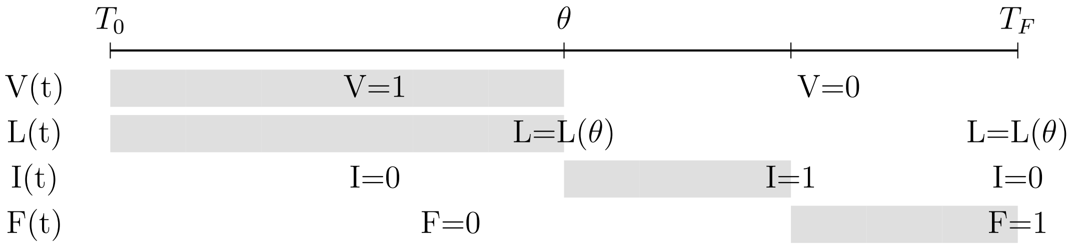
(c) Inflorescence meristem generating a flower.

Figure 1: State variable transitions in an unbranched plant with a terminal, determinate flower.

The table below summarizes the state and control variables in the model.

Symbol	Description	Units
State variables		
$V(t)$	Vegetative meristem population size	number of vegetative meristems
$L(t)$	Leaf population size	number of leaves
$I(t)$	Inflorescence meristem population size	number of inflorescence meristems
$F(t)$	Flower population size	number of flowers

In this case, there is 1 vegetative meristem that transitions to 1 inflorescence meristem at the transition to flowering. Because there there is only 1 vegetative meristem, there is a single transition to flowering (the solution is bang-bang) at the switch time θ. Before the transition to flowering, the plant accumulates leaf biomass; after the transition to flowering there is no additional accumulation of leaf biomass. At the transition to flowering, the plant has accumulated L(θ) leaves, which then remains constant to the end of the season. With a terminal, determinate flower, the plant can develop at most 1 flower. The figure below illustrates the assumptions described here:



The plant must make a flower before the end of the season, subject to resource constraints and constraints on meristem division rates. Because the solution is known to be bang-bang, the optimal strategy is the one that simultaneously minimizes the sum of the switch time (θ; ensuring the onset of reproduction), the time to produce the inflorescence meristem (τ_1), and the time to produce the flower (τ_2). The optimization problem is then:

$$\min_{\theta} \theta + \tau_1 + \tau_2$$

The meristem division rates M_0, M_1, M_2 are the fixed, upper limits on the per-capita rates at which the divisions in Figure 1A-C take place (units of meristems/(meristem time)). The conversion rate of standing biomass, α, describes the energy produced by a unit of leaf.

$$\begin{aligned}
\dot{L} &= \min(\alpha L(t), M_0) \\
\tau_1 &= \frac{1}{\min(M_1, \alpha L(\theta))} \\
\tau_2 &= \frac{1}{\min(M_2, \alpha L(\theta))}
\end{aligned}$$

For each combination of M_0, M_1, M_2, α, the objective function can be calculated by:

- Solve $\dot{L} = \min(\alpha L(t), M_0)$ to the switch time θ
- Calculate $L(\theta)$
- Use $L(\theta), M_1, M_2$ to calculate τ_1, τ_2
- Calculate $\theta + \tau_1 + \tau_2$

The optimal switch time is the one that minimizes the last line in the algorithm above.

0.0.1 Resource constraint only

When M_i approaches ∞ (case without meristem constraint), the problem becomes the following.

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \alpha L$$

$$\begin{aligned}
\int_0^\theta \frac{\mathrm{d}L}{\mathrm{d}t} &= \int_0^\theta \alpha L \\
\int_0^\theta \frac{\mathrm{d}L}{L} &= \int_0^\theta \alpha \mathrm{d}t \\
\log(L(\theta)) - \log(L(0)) &= \alpha\theta - \alpha 0 \\
\log\left(\frac{L(\theta)}{L(0)}\right) &= \alpha\theta \\
\frac{L(\theta)}{L(0)} &= \exp^{\alpha\theta} \\
L(\theta) = L(0) \exp^{\alpha\theta}
\end{aligned}$$

This means

$$\begin{aligned}
\tau_1 &= \frac{1}{\alpha L(0) \exp^{\alpha\theta}} \\
\tau_2 &= \frac{1}{\alpha L(0) \exp^{\alpha\theta}}
\end{aligned}$$

so the minimization is

$$\min_\theta \theta + \frac{2}{\alpha L(0) \exp^{\alpha\theta}}$$

0.0.2 Meristem constraint only

When α approaches ∞ (case without resource constraint), the problem becomes the following.

$$\frac{\mathrm{d}L}{\mathrm{d}t} = M_0 \tag{1}$$

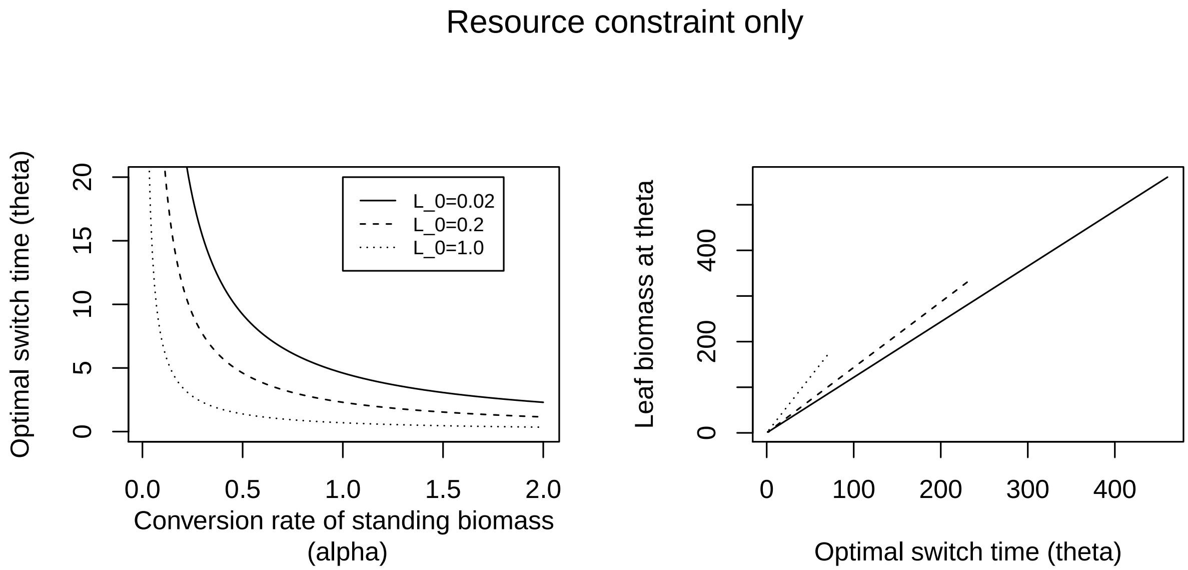


Figure 2: Properties of the optimal switch time for a 1D, unbranched plant without a meristem constraint. (A) If all per-capita rates of meristem division approach infinity, the optimal switch time decreases as the conversion rate of standing biomass (α) increases for a given initial amount of leaf biomass L_0. (B) The optimal switch time θ is linearly related to the leaf biomass at the transition to flowering.

$$\begin{aligned}\int_0^\theta \frac{\mathrm{d}L}{\mathrm{d}t} &= \int_0^\theta M_0 \\ \int_0^\theta \mathrm{d}L &= \int_0^\theta M_0 \mathrm{d}t \\ L(\theta) - L(0) &= M_0\theta - M_0 0 \\ L(\theta) &= L(0) + M_0\theta\end{aligned}$$

This means

$$\begin{aligned}\tau_1 &= \frac{1}{\min(M_1, \alpha(L(0) + M_0\theta)} \\ \tau_2 &= \frac{1}{\min(M_2, \alpha(L(0) + M_0\theta)}\end{aligned}$$

If M_1, M_2 are fast relative to M_0, the minimization becomes

$$\min_{\theta} \theta + \frac{2}{\alpha(L(0) + M_0\theta)}$$

$$\begin{aligned}
M_1 &= \alpha(L(0) + M_0\theta) \\
M_1 &= \alpha L(0) + \alpha M_0\theta \\
\alpha M_0\theta - M_1 &= -\alpha L(0) \\
\theta - \frac{M_1}{\alpha M_0} &= -\frac{L(0)}{M_0} \\
\theta &= \frac{1}{M_0}(\frac{M_1}{\alpha} - L(0))
\end{aligned}$$

Because θ is positive, $\frac{M_1}{\alpha} - L(0) \geq 0$. So $M_1 \geq \alpha L(0)$.