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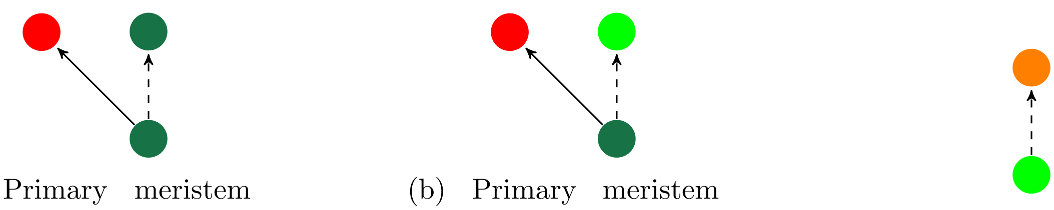
Unbranched, determinate case

The basic case we consider here is one in which the plant is unbranched and the inflorescence is determinate. Plants with an unbranched structure are defined by primary meristem divisions that generate either a vegetative and primary meristem (Figure 1A) or a vegetative and inflorescence meristem (Figure 1B) but do not branch. A determinate inflorescence is represented in the model as an inflorescence meristem division transitioning to a floral meristem (Figure 1C).



(a) Primary meristem transitioning to one vegetative meristem and generating one primary meristem.

(b) Primary meristem transitioning to one vegetative meristem, and generating one inflorescence meristem.

(c) Inflorescence meristem generating a floral meristem

Figure 1: Meristem transitions in an unbranched plant with a determinate inflorescence.

The table below summarizes the state and control variables in models based on these meristem transitions.

Term	**Description**
P	Primary meristems
V	Vegetative biomass
I	Inflorescence meristems
F	Floral meristems
$u(t)$	The probability that a primary meristem division produces a primary and vegetative meristem (Figure 1A).
$1 - u(t)$	The probability that a primary meristem division produces an inflorescence and a vegetative meristem (Figure 1B).
$\beta_1(t)$	The per-capita rate of division by primary meristems.
$\beta_2(t)$	The per-capita rate of division by inflorescence meristems.

11 Unbranched, determinate case with resource constraint

We rewrite the optimal control problem to include a state variable inequality constraint, which here is the resource constraint

$$
\begin{aligned}
&\max_u \int_0^T \log(F(t))\mathrm{d}t && \text{Objective function}\\
\text{subject to } &\dot{P} = \gamma[\beta_1(t)][(u(t))P] - [\beta_1(t)][(1-u(t))P] && \text{System of ODEs}\\
&\dot{V} = [\beta_1(t)][u(t)P] + [\beta_1(t)][(1-u(t))P] = \beta_1[P] &&\\
&\dot{I} = [\beta_1(t)][(1-u(t))P] - [\beta_2(t)]I && (1)\\
&\dot{F} = [\beta_2(t)]I &&\\
&P(0) > 0;\ V(0),\ I(0),\ F(0) \geq 0 && \text{Initial conditions}\\
&0 \leq \beta_1(t), \beta_2(t) \leq M && \text{Meristem constraint}\\
&0 \leq \beta_1(t)P + \beta_2(t)I \leq \alpha V && \text{Resource constraint}\\
&0 \leq u(t) \leq 1 && \text{Control constraint}
\end{aligned}
$$

We write the Hamiltonian as

$$
H = \log F + \lambda_1[-\beta_1(1-u)P] + \lambda_2[\beta_1 P] + \lambda_3[\beta_1(1-u)P - \beta_2 I] + \lambda_4[\beta_2 I] + \eta[V - (\beta_1 P + \beta_2 I)] \tag{2}
$$

To obtain the switching functions, we take the derivative of the Hamiltonian with respect to each control

$$
\frac{\partial H}{\partial u} = (\lambda_1 - \lambda_3)\beta_1 P = 0 \text{ at } \beta_1^*, \beta_2^* \tag{3}
$$

$$
\frac{\partial H}{\partial \beta_1} = (\lambda_3(1-u) - \lambda_1(1-u) + \lambda_2 - \eta)P = 0 \text{ at } u^*, \beta_2^* \tag{4}
$$

$$
\frac{\partial H}{\partial \beta_2} = (\lambda_4 - \lambda_3 - \eta)I = 0 \text{ at } u^*, \beta_1^* \tag{5}
$$

The adjoint equations are

$$
\begin{aligned}
-\frac{\partial H}{\partial P} &= \dot{\lambda_1} = \beta_1(\lambda_1(1-u) - \lambda_2 - \lambda_3(1-u) + \eta\beta_1) \\
-\frac{\partial H}{\partial V} &= \dot{\lambda_2} = -\eta \\
-\frac{\partial H}{\partial I} &= \dot{\lambda_3} = \beta_2(\eta + \lambda_3 - \lambda_4) \\
-\frac{\partial H}{\partial F} &= \dot{\lambda_4} = -\frac{1}{F}
\end{aligned} \tag{6}
$$

The transversality condition is

$$
\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = 0. \tag{7}
$$

The complementary slackness condition is

$$
\eta \geq 0,\ \eta[V - (\beta_1 P + \beta_2 I)] = 0 \tag{8}
$$

From the above equation, $\eta = 0$ when $V - (\beta_1 P + \beta_2 I) > 0$ (off the constraint). On the other hand, $\eta > 0$ when $V - (\beta_1 P + \beta_2 I) \leq 0$ (on the constraint).

When we are not on the boundary, $(V - (\beta_1 P + \beta_2 I) > 0)$.

We do the following on the boundary

$$
\phi = \frac{(P(\lambda_3 - \lambda_1)(1-u) + \lambda_2) - (I(\lambda_4 - \lambda_3)}{P - I} \tag{9}
$$