

node *centrality*

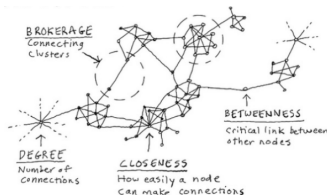
introduction to *network science in Python* (*NetPy*)

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18th Jan 2022

centrality *measures*

which *nodes* are most *important*?

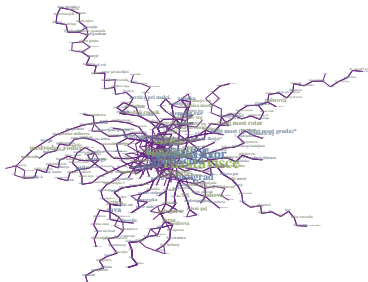
- *node centrality measures* for (*un*)*directed* networks
 - *clustering coefficients* [WS98, SV05, dNMB05]
 - *distance-based* centrality [Fre77, FBW91, New05]
 - *spectral analysis* centrality [Kat53, Bon87, BP98]



- *link analysis algorithms* for *directed* networks

networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 5.62$ connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “*scale-free*” with $\gamma = 2.62$ for cutoff $k_{min} = 5$



* reduced to largest connected component

centrality *clustering*

important *nodes* are *strongly embedded*

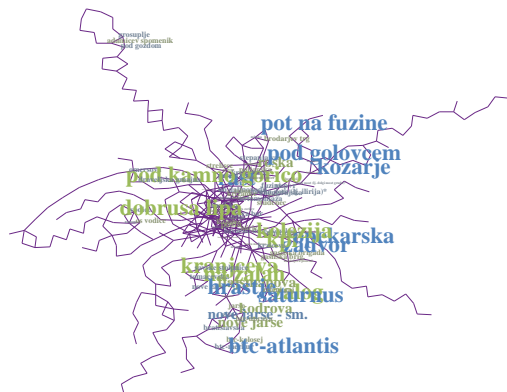
- for *undirected* G *clustering coefficient* C [WS98] of i is
 - t_i is number of *linked neighbors* or *triangles* of i

$$C_i = \frac{2t_i}{k_i(k_i-1)} \quad C_i = 0 \text{ for } k_i \leq 1$$

- C fails for *hub nodes* in *scale-free* networks [dNMB05]

networkology *clustering*

- *clustering coefficient* C in partial LPP network[†]
- *highest* $C_i = 1.0$ nodes are *Na Žalah etc.* with $k_i = 2$



[†] reduced to simple undirected graph

centrality *closeness*

important *nodes* are *close to other* nodes

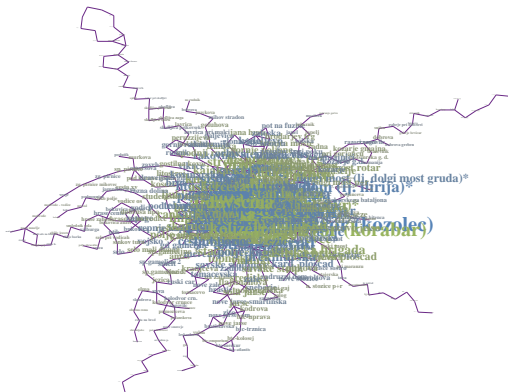
- for (*un*)*directed* G *closeness centrality* ℓ^{-1} [New10] of i is
 - d_{ij} is (*un*)*directed distance* between i and j
 - $d_{ij} = \infty$ for nodes in *different components*

$$\ell_i^{-1} = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

- ℓ^{-1} spans *small range* in *small-world* networks

networkology *closeness*

- *closeness centrality* ℓ^{-1} in partial LPP network[‡]
- *highest* $\ell_i^{-1} = 0.208$ node is *Gospodsvetska* with $k_i = 14$



[‡] reduced to simple undirected graph

centrality *betweenness*

important *nodes* are *bridges btw other* nodes

- for (*un*)*directed* G *betweenness centrality* σ [Fre77] of i is
 - g_{st} is number of *shortest paths between* s and t
 - g_{st}^i is number of *such shortest paths through* i

$$\sigma_i = \frac{1}{n^2} \sum_{st} \frac{g_{st}^i}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]

centrality *degrees*

important *nodes* are *linked by many* nodes

- for *undirected* G *degree centrality* d of i is

$$d_i = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i}{n-1}$$

- in *directed* G *in-degree centrality* d^{in} of i is

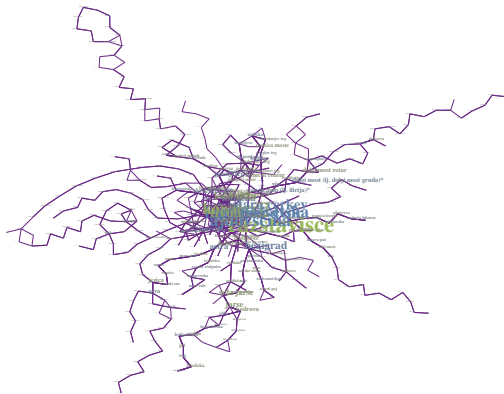
$$d_i^{in} = \frac{1}{n-1} \sum_{j \neq i} A_{ij} = \frac{k_i^{in}}{n-1}$$

- in *directed* G *out-degree centrality* d^{out} of i is

$$d_i^{out} = \frac{1}{n-1} \sum_{j \neq i} A_{ji} = \frac{k_i^{out}}{n-1}$$

networkology *degrees*

- *degree centrality* d in partial LPP network
- *highest* $d_i = 0.099$ node is *Razstavišče* with $k_i = 41$
- *highest* d_i node is *Razstavišče* with $k_i^{in} = 20$ and $k_i^{out} = 21$



centrality *eigenvector*

important *nodes* are *linked by important* nodes

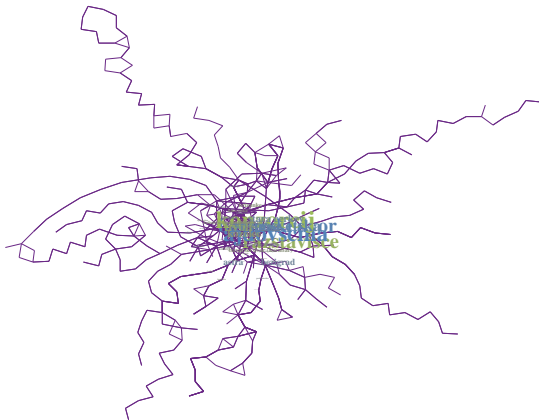
- for (*un*)*directed* G *eigenvector centrality* e [Bon87] of i is
 - e is *leading eigenvector* v_1 of A with *eigenvalue* λ_1^{-1}

$$e_i = \lambda_1^{-1} \sum_j A_{ij} e_j$$

- in *directed* G $e = 0$ for $k^{in} = 0$ *nodes etc.*

networkology *eigenvector*

- *eigenvector centrality* e in partial LPP network
- *highest* $e_i = 0.082$ node is *Konzorcij* with $k_i = 30$



centrality *overview*

which *nodes* are most *important*?

1 IA												18 VIIIA																							
1	DC																							2	EC										
	Degree Centrality																								Eigenvector Centrality										
2	BC	CC																							3	PR									
	Betweenness Centrality	Closeness Centrality																								PageRank									
3	RL	IC																							10	LR									
	Range-Linked Betweenness	Information Centrality																								LeaderRank									
4	BN	RC	IG	DCox	BCox	CCox	ECox	KSx	PRx	IGx	RCx	DCox	BCox	SCx	KL	COCox	PECox	KS																	
	Betweenness Centrality	Radiality Centrality	Integration											new Subgraph Centrality	Clique Level	cooper. weight COCOF	PCC+ECox	KatzRank																	
5	RWBC	RWCC	CC _{2,3,4}	ECox	PRox	KSox	COCox	RCox	IGox	DCox	BCox	SCox	KL	β	SC ₂	SC ₃	NC	EC ₂																	
	RandomWalk Betweenness	RandomWalk Closeness	2,3,4-localized CC											Bipartivity	2-localized SC	Neighborhood Centrality	2-localized EC																		
6	σ	ECC	WDC	DCox	CCox	KSx	PRox	IGox	DCox	BCox	SCox	KL	β	SC ₂	SC ₃	LI	EC ₃																		
	sigma Centrality	Eccentricity	Weighted Degree											3-localized SC	Lobby Index	3-localized EC																			
7	BC _{2,3,4}	ECC ⁻¹	SDC	DCRC	CCRC	BCRC	KSRC	PRRC	IGRC	DCRC	BCRC	CCRC	DCRC	BCRC	CCRC	SC ₄	EC ₄																		
	2,3,4-localized-BC	Inverse Eccentricity	Sphere Degree Centrality											4-localized SC	4-localized EC																				
Z																																			
C																																			
Name																																			
Hybrid																																			
22	FC	FD	US	DIS	ASS	DAM	UC																												
	Functional Centrality	Functional Diversity	UniScore	Pairwise Dis-connectivity	Assortative Mixing	Damage	United compl. Centrality																												
28	EI	CM	NαC	MC	HGI	HYP	HC																												
	Essentiality Index	Complexity Measure	Normalized α Centrality	Modular Centrality	Harary Graph Information	Hyperbolic Index	Harmonic Centrality																												

Betweenness-based

Distance-based

Linear Combinations

Subgraph-based

Clustering Coefficient-based

Edge Clustering Coefficient-based

Spectral-based

Miscellaneous

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centrality *references*



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centrality *references*



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