

link *bridging*

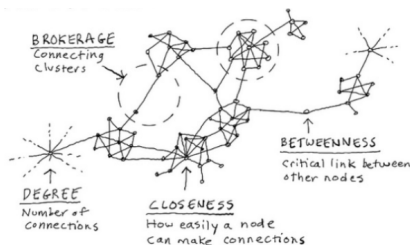
introduction to *network science in Python* (*NetPy*)

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bridging *measures*

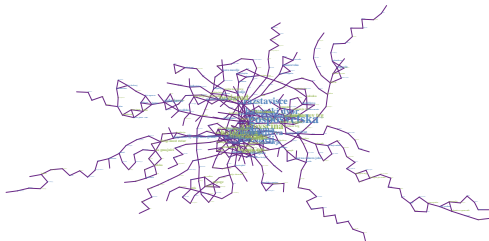
which *links* are most *important*?

- *link bridging measures* for (un)directed networks
 - *betweenness-based* centrality [Fre77, FBW91, New05]
- *link embeddedness measures* for (un)directed networks
 - *topological overlap* measures [RSM⁺02, OSH⁺07, dNMB11]



networkology *LPP*

- partial *LPP public bus transport network**
- $n = 416$ bus stops with $\langle k \rangle = 2.72$ connections
- *giant component* 95.4% nodes (6 components)
- “*small-world*” with $\langle C \rangle = 0.09$ and $\langle d \rangle = 14.26$
- “*scale-free*” with $\gamma = 2.43$ for cutoff $k_{min} = 2$



* reduced to largest connected component of simple undirected graph

bridging *betweenness*

important *links* are *between other nodes*

- for (*un*)*directed* *G* *link betweenness* σ [Fre77] of $\{i, j\}$ is
 - g_{st} is number of *shortest paths* *between* *s* and *t*
 - g_{st}^{ij} is number of *such shortest paths* *through* $\{i, j\}$

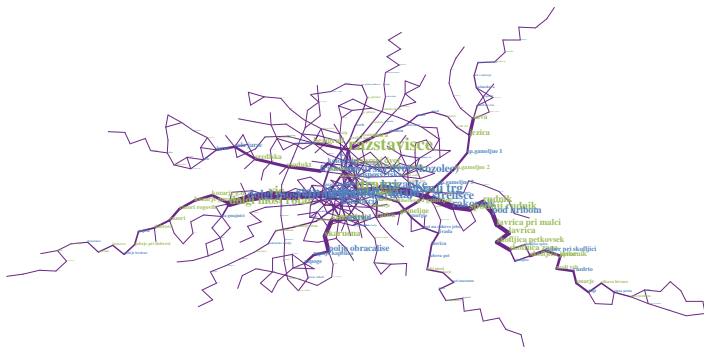
$$\sigma_{ij} = \sum_{st \notin \{i, j\}} \frac{g_{st}^{ij}}{g_{st}}$$

- σ considers *only shortest paths* [FBW91, New05]



networkology *betweenness*

- *link betweenness* σ in partial LPP network[†]
- *highest* $\sigma_{ij} = 0.176n^2$ link is {*Vič, Stan in dom*}



[†] reduced to largest connected component of simple undirected graph

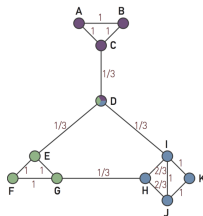
bridging *embeddedness*

important *links* are *embedded between nodes*

— for *undirected* G *link embeddedness*[‡] θ [OSH⁺07] of $\{i, j\}$ is

– Γ_i is set of *neighbors* or *neighborhood* of i

$$\theta_{ij} = \frac{|\Gamma_i \cap \Gamma_j|}{|\Gamma_i \cup \Gamma_j|} = \frac{|\Gamma_i \cap \Gamma_j|}{k_i - 1 + k_j - 1 - |\Gamma_i \cap \Gamma_j|} \quad \theta_{ij} = 0 \text{ for } k_i = k_j = 1$$



[‡] θ better known as topological overlap index/weight

networkology *embeddedness*

bridging *overview*

which *links* are most *important*?

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bridging *references*



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bridging *references*



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