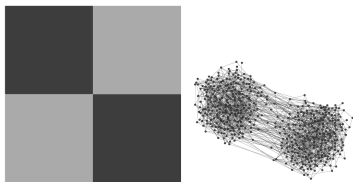


*core-periphery* structure

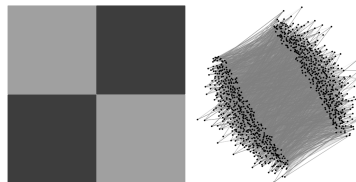
introduction to *network science in Python* (*NetPy*)

Lovro Šubelj  
University of Ljubljana  
18th Jan 2022

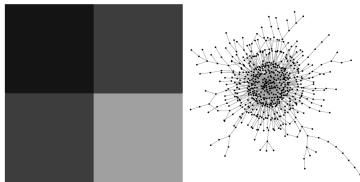
# core-periphery *block model*



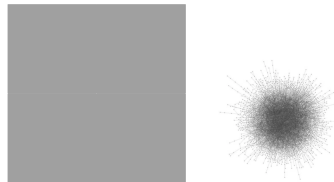
*community block model* [GN02]



*disassortative block model* [NL07]



*core-periphery block model* [Sei83]



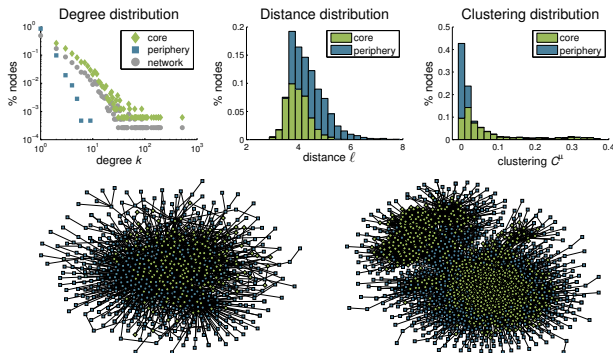
*random graph* [ER59]

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\* origin of core-periphery structure in international relations

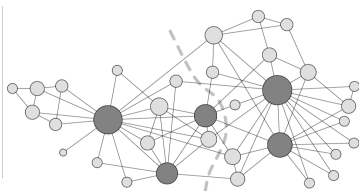
# core-periphery *structure*

- *core/periphery nodes* have *higher/lower degrees*  $k$
- *core/periphery nodes* are on *shorter/longer distances*  $\ell$
- *core/periphery nodes* have *higher/lower clustering*  $C^i$

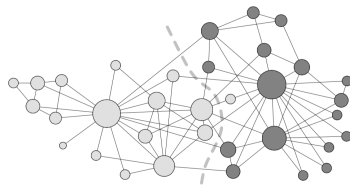


## core-periphery *stochastic*

- $G(\{C_1, C_2\}, \{p_{11}, p_{12}, p_{22}\})$  *stochastic block model* [HLL83]
  - $n_i$  is *size* of *cluster*  $C_i$  &  $p_{ij}$  is *link density* between  $C_i$  and  $C_j$
- *density-based core-periphery* structure for  $p_{11} \gg p_{12} \gg p_{22}$
- *lookalike core-periphery* for  $n_1 p_{11} \gg 1, n_1 p_{12} \ll 1, n_2 p_{22} \approx 1$



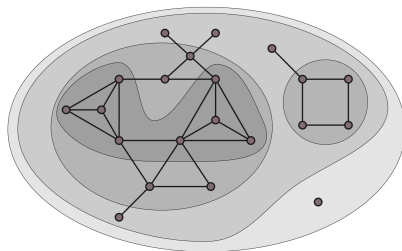
*non-corrected block model*  $p_{11} > p_{12} > p_{22}$



*degree-corrected block model*  $p_{11} \approx p_{22} > p_{12}$

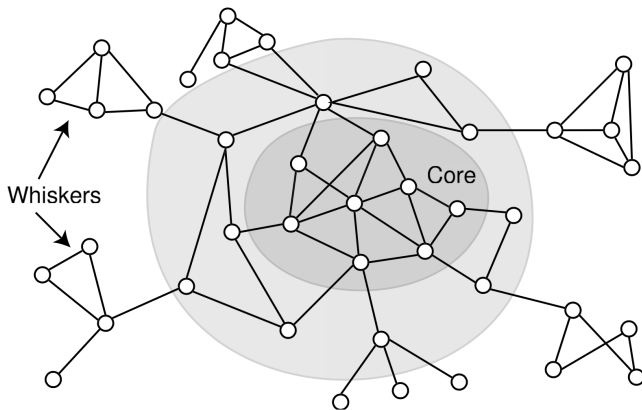
## core-periphery *k*-cores

- *k*-cores are subgraphs of nodes with  $\geq k$  neighbors [Sei83]  
remove nodes with degree  $< k$  until no such node remains [BZ11]
- *k*-shells are nodes of *k*-cores that are not in  $k + 1$ -cores
- *k*-cores are nested while *k*-shells form decomposition



0-cores are connected components & *k*-cores can be disconnected

## core-periphery *nestedness*



*nested cores & whiskers communities* [LLDM09, YL13]

# core-periphery *references*



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## core-periphery *references*



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