QQL Utility

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Sam, I'm struggling with the following. Duncan currently has the following QQL utility function in the chapter:

$$u^{QQL}(x, y) = y - \frac{s}{2}(x - \xi)^2 \quad (1)$$

He says that ξ measures the level of satiation for x (or what I want to call, $\bar{x}$).The thing I'm struggling with is that with his equation above, it looks to me that any positive consumption of x results in a reduction in utility regardless of the satiation level. The greater the distance, the greater the reduction in utility, to be sure, but it just seems weird to me that consuming x leads to reductions in total utility (unless I've missed something).

Perhaps it had the wrong sign?

Alternatively, as per our conversation, there should be a plus sign:

$$u^{QQL}(x, y) = y + \frac{s}{2}(x - \xi)^2 \quad (2)$$

But this multiplies out to:

$$u^{QQL}(x, y) = y + \frac{s}{2}(x)^2 - s\xi x + \frac{s}{2}\xi^2 \quad (3)$$

The indifference curves for this function look to me like they'd be a parabola perturbed to the right of the (0, 0) origin, so they'd start off upward-sloping, then be concave to the origin (I haven't tried to sketch them in R yet, but I think I'm right).

Corrected again

It should be the other way around:

$$u^{QQL}(x, y) = y - \frac{s}{2}(\bar{x} - x)^2 \quad (4)$$

This multiplies out to

$$u^{QQL}(x, y) = y - \frac{s}{2}(\bar{x})^2 + s\bar{x}x - \frac{s}{2}x^2 \quad (5)$$

The marginal rate of substitution for Equation 5 is:

$$mrs(x, y) = s\bar{x} - sx = s(\bar{x} - x) \quad (6)$$

So the marginal rate of substitution is decreasing as x approaches $\bar{x}$. The demand function is $p = s(\bar{x} - x)$. This reduces to the case we have below if $s = \frac{\bar{p}}{\bar{x}}$. But the utility function has a constant in it that we don't otherwise have. We also need to use something that isn't "s" for our symbol.

Simon's Old Alternative

The alternative, which is a version of the old one we've used is:

$$u^{QQL}(x,y) = y + \bar{p}x - \frac{\bar{p}}{\bar{x}}\frac{1}{2}x^2 \tag{7}$$

Here, consuming x results in increasing utility up to the point where $x = \bar{x}$. We can see this by looking at the marginal rate of substitution:

$$\frac{u_x}{u_y} = mrs(x,y) = \bar{p} - \frac{\bar{p}}{\bar{x}}x \tag{8}$$

If the price of x is zero, then the consumer will consume until $mrs(x,y) = 0$.

$$\begin{aligned} \bar{p} - \frac{\bar{p}}{\bar{x}}x &= 0 \\ \Rightarrow \frac{\bar{p}}{\bar{x}}x &= \bar{p} \\ x &= \bar{x} \end{aligned}$$

As far as I can tell, utility is increasing up to the point where $x = \bar{x}$ for Equation 7. I don't know what to say about Duncan's function.