

# Notes on Evans & Didelez (2023)

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This document collects my notes on [Evans and Didelez \(2023\)](#). They propose a new parameterization for causal problems, termed **frugal parameterization**, which consists of three pieces: the joint distribution of the treatment and covariates  $p_{ZX}(z, x)$  (the 'past'), the causal distribution of interest  $p_{Y|X}^*(y|x)$ , and a dependence measure between the outcome and the covariates conditional on the treatment  $\phi_{YZ|X}^*$ . In sequential treatment models, this parameterization circumvents the so-called **g-null paradox** ([Robins and Wasserman, 1997](#)). Their main result shows that a frugal parametrization  $\theta = (\theta_{ZX}, \theta_{Y|X}, \phi_{YZ|X})$  of the observational distribution induces a corresponding parameterization  $\theta^* = (\theta_{ZX}, \theta_{Y|X}^*, \phi_{YZ|X}^*)$  that is also frugal. Replacing  $\theta_{ZX}$  in  $\theta^*$  with  $\eta_{ZX}(\theta_{ZX})$ , where  $\eta_{ZX}$  is a twice differentiable function with a Jacobian of constant rank, yields a parameterization of the causal joint distribution  $p_{ZXY}^*$ . Using this, they propose a rejection sampling algorithm to sample from  $p_{ZXY}$  (implemented in the R-package [causl](#)). Furthermore, they show that under certain assumptions we can obtain consistent parameter estimates for the model  $p_{Y|X}^*$  by maximizing the likelihood with respect to the observational data from  $p_{ZXY}$ .

## Comments/Questions:

- The parameterization is not unique: Given  $p_{Y|X}^*$ ,  $p_{ZX}$  and the dependence measure  $\phi_{YZ|X}^*$  need to be chosen. How do we choose them? Can this choice introduce uncertainty that is not accounted for, maybe like selective inference type problems?
- Similarly, what happens if the causal model  $p_{Y|X}^*$  is misspecified? Then, the causal and observational joint distributions will be misspecified as well, leading to false simulation results, right?

## Further reading:

- [Robins and Wasserman \(1997\)](#) and [McGrath et al. \(2022\)](#) to better understand the g-null paradox. Also, problem 29.1 in [Ding \(2023\)](#) is similar to their example R2.

## References

- Ding, P. (2023). A first course in causal inference.
- Evans, R. J. and Didelez, V. (2023). Parameterizing and Simulating from Causal Models. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, page qkad058.
- McGrath, S., Young, J. G., and Hernán, M. A. (2022). Revisiting the g-null paradox. *Epidemiology (Cambridge, Mass.)*, 33(1):114.
- Robins, J. M. and Wasserman, L. (1997). Estimation of effects of sequential treatments by reparameterizing directed acyclic graphs. In *Proceedings of the Thirteenth Conference on Uncertainty in Artificial Intelligence, UAI'97*, page 409–420, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.