Notes on Evans & Didelez (2023)

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This document collects my notes on Evans and Didelez (2023). They propose a new parameterization for causal problems, termed **frugal parameterization**, which consists of three pieces: the joint distribution of the treatment and covariates $p_{ZX}(z,x)$ (the 'past'), the causal distribution of interest $p^*_{Y|X}(y|x)$, and a dependence measure between the outcome and the covariates conditional on the treatment $\phi^*_{YZ|X}$ (could be a conditional odds-ratio or a copula). In sequential treatment models, this parameterization circumvents the so-called **g-null paradox** (Robins and Wasserman, 1997). Their main result shows that a frugal parametrization $\theta = (\theta_{ZX}, \theta_{Y|X}, \phi_{YZ|X})$ of the observational distribution induces a corresponding parameterization $\theta^* = (\theta_{ZX}, \theta^*_{Y|X}, \phi^*_{YZ|X})$ that is also frugal. Replacing θ_{ZX} in θ^* with $\eta_{ZX}(\theta_{ZX})$, where η_{ZX} is a twice differentiable function with a Jacobian of constant rank, yields a parameterization of the causal joint distribution p^*_{ZXY}. Using this, they propose a rejection sampling algorithm to sample from p_{ZXY} (implemented in the R-package causl). Furthermore, they show that under certain assumptions we can obtain consistent parameter estimates for the model $p^*_{Y|X}$ by maximizing the likelihood with respect to the observational data from p_{ZXY}.

As far as I understand, the main advantage of the frugal parameterization is that it provides a convenient characterization of the joint distribution p_{ZXY}, which is usually unknown or difficult to handle. Instead, we can specify a model for $p_{ZX}(z,x)$ and $p^*_{Y|X}(y|x)$ and choose a dependence measure of Y and Z conditional on X. Then, we can use these three to perform likelihood-based inference or simulate from the joint distribution. Consider the following example corresponding to Figure 1: The joint distribution can be factorized into the frugal parameterization:

$$
\begin{aligned}
p_{ZXY}(z,x,y|\theta^*) &= p_{ZX}(z,x|\theta_{ZX})p^*_{Y|ZX}(y|z,x) && (1)\\
&= p_{ZX}(z,x|\theta_{ZX})p^*_{X}(y|x;\theta^*_{Y|X})c(y,z|x), && (2)
\end{aligned}
$$

where $c(y,z|x) =: \phi^*_{YZ|X}$ is a copula density for continuous Y and Z given X. Given observations for (Y,X,Z), we can estimate θ^* by maximizing the expression above. Or we can even obtain a posterior distribution

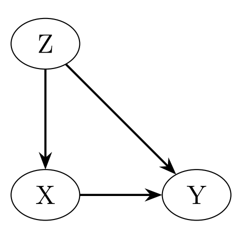
$$
p(\theta^*|z,x,y) \propto p_{ZXY}(z,x,y|\theta^*)p(\theta^*), \tag{3}
$$

where $p(\theta^*)$ is some prior distribution.

Comments/Questions:

- It's not clear to me what the role of η_{ZX} is and how we obtain it. Is it chosen by the analyst?
- The parameterization is not unique: Given $p^*_{Y|X}$, p_{ZX} and the dependence measure $\phi^*_{YZ|X}$ need to be chosen. How do we choose them? Can this choice introduce uncertainty that is not accounted for, maybe like selective inference type problems?

Figure 1: Causal diagram



- Similarly, what happens if the causal model $p^*_{Y|X}$ is misspecified? Then, the causal and observational joint distributions will be misspecified as well, leading to false simulation results, right?

Further reading:

- Robins and Wasserman (1997) and McGrath et al. (2022) to better understand the g-null paradox. Also, problem 29.1 in Ding (2023) is similar to their example R2.
- Bergsma and Rudas (2002) to better understand the discrete case and for some of the proofs.

References

Bergsma, W. P. and Rudas, T. (2002). Marginal models for categorical data. *The Annals of Statistics*, 30(1):140 – 159.

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Evans, R. J. and Didelez, V. (2023). Parameterizing and Simulating from Causal Models. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, page qkad058.

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