

Notes on Evans & Didelez (2023)

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This document collects my notes on [Evans and Didelez \(2023\)](#). They propose a new parameterization for causal problems, termed **frugal parameterization**, which consists of three pieces: the joint distribution of the treatment and covariates $p_{ZX}(z, x)$ (the 'past'), the causal distribution of interest $p_{Y|X}^*(y|x)$, and a dependence measure between the outcome and the covariates conditional on the treatment $\phi_{YZ|X}^*$. In sequential treatment models, this parameterization circumvents the so-called **g-null paradox** ([Robins and Wasserman, 1997](#)). Their main result shows that a frugal parametrization $\theta = (\theta_{ZX}, \theta_{Y|X}, \phi_{YZ|X})$ of the observational distribution induces a corresponding parameterization $\theta^* = (\theta_{ZX}, \theta_{Y|X}^*, \phi_{YZ|X}^*)$ that is also frugal. Replacing θ_{ZX} in θ^* with $\eta_{ZX}(\theta_{ZX})$, where η_{ZX} is a twice differentiable function with a Jacobian of constant rank, yields a parameterization of the causal joint distribution p_{ZXY}^* . Using this, they propose a rejection sampling algorithm to sample from p_{ZXY} (implemented in the R-package [causl](#)). Furthermore, they show that under certain assumptions we can obtain consistent parameter estimates for the model $p_{Y|X}^*$ by maximizing the likelihood with respect to the observational data from p_{ZXY} .

Comments/Questions:

- The parameterization is not unique: Given $p_{Y|X}^*$, p_{ZX} and the dependence measure $\phi_{YZ|X}^*$ need to be chosen. How do we choose them? And how sensitive is the joint distribution p_{ZXY} to the choice?

Further reading:

- [Robins and Wasserman \(1997\)](#) and [McGrath et al. \(2022\)](#) to better understand the g-null paradox. Also, problem 29.1 in [Ding \(2023\)](#) is similar to their example R2.

References

- Ding, P. (2023). A first course in causal inference.
- Evans, R. J. and Didelez, V. (2023). Parameterizing and Simulating from Causal Models. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, page qkad058.
- McGrath, S., Young, J. G., and Hernán, M. A. (2022). Revisiting the g-null paradox. *Epidemiology (Cambridge, Mass.)*, 33(1):114.
- Robins, J. M. and Wasserman, L. (1997). Estimation of effects of sequential treatments by reparameterizing directed acyclic graphs. In *Proceedings of the Thirteenth Conference on Uncertainty in Artificial Intelligence, UAI'97*, page 409–420, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.