Notes on Evans & Didelez (2023)

Gregor Steiner

July 19, 2023

This document collects my notes on Evans and Didelez (2023). They propose a new parameterization of the distributions of interest, termed **frugal parameterization**, which consists of three pieces: the joint distribution of the treatment and covariates $p_{ZX}(z, x)$ (the 'past'), the causal distribution of interest $p^*_{Y|X}(y|x)$, and a dependence measure between the outcome and the covariates conditional on the treatment $\phi^*_{YZ|X}$. In sequential treatment models, this parameterization circumvents the so-called **g-null paradox** (Robins and Wasserman, 1997). Their main result shows that a frugal parametrization $\theta = (\theta_{ZX}, \theta_{Y|X}, \phi_{YZ|X})$ of the observational distribution induces a corresponding parameterization $\theta^* = (\theta_{ZX}, \theta^*_{Y|X}, \phi^*_{YZ|X})$ that is also frugal. Replacing θ_{ZX} in θ^* with $\eta_{ZX}(\theta_{ZX})$, where η_{ZX} is a twice differentiable function with a Jacobian of constant rank, yields a parameterization of the causal joint distribution p^*_{ZXY}. Using this, they propose a rejection sampling algorithm to sample from p_{ZXY} (implemented in the R-package causl). Furthermore, they show that under certain assumptions we can obtain consistent parameter estimates for the model $p^*_{Y|X}(y|x)$ by maximizing the likelihood with respect to the observational data from p_{ZXY}.

Comments/Questions:

- The parameterization is not unique: Given $p^*_{Y|X}$, p_{ZX} and the dependence measure $\phi^*_{YZ|X}$ need to be chosen. How do we choose them? And how sensitive is the joint distribution p_{ZXY} to the choice?

Further reading:

- Robins and Wasserman (1997) and McGrath et al. (2022) to better understand the g-null paradox. Also, problem 29.1 in Ding (2023) is similar to their example R2.

References

Ding, P. (2023). A first course in causal inference.

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Robins, J. M. and Wasserman, L. (1997). Estimation of effects of sequential treatments by reparameterizing directed acyclic graphs. In *Proceedings of the Thirteenth Conference on Uncertainty in Artificial Intelligence*, UAI'97, page 409–420, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.