Helmholtz-Hodge Decomposition

The Helmholtz-Hodge decomposition says that a twice-differentiable vector field $\mathbf{F(r)}$ on a bounded domain $V \subset \mathbb{R}^3$ can be expressed as the sum of a curl-free component $\mathbf{F_l}(\mathbf{r})$ and divergence-free component $\mathbf{F_t}(\mathbf{r})$ component:

$$\mathbf{F}(\mathbf{r}) = \mathbf{F_l}(\mathbf{r}) + \mathbf{F_t}(\mathbf{r})$$

Namely, $\mathbf{F_l}(\mathbf{r})$ is the longitudinal or irrotational (curl-free) part of vector:

$$\nabla \times \mathbf{F_l}(\mathbf{r}) = \mathbf{0}.$$

and $\mathbf{F_t}(\mathbf{r})$ is the solenoidal part (divergence-free, i.e. no sources or sinks):

$$\nabla \cdot \mathbf{F_t}(\mathbf{r}) = 0.$$

The irrotational part can be expressed as the negative gradient of a scalar potential function $\phi(\mathbf{r})$:

$$\mathbf{F_l}(\mathbf{r}) = -\nabla\phi(\mathbf{r}).$$

The solenoidal part can be expressed as the curl of a vector potential $\mathbf{a}(\mathbf{r})$

$$\mathbf{F_t}(\mathbf{r}) = \nabla \times \mathbf{a}(\mathbf{r})$$

Fuselier and Wright, 2016 propose a radial basis function method using matrix-valued kernels to compute the Helmholtz-Hodge decomposition on a bounded domain using samples of the field at a collection of nodes.

This repository applies their technique to bounded domains constructed from intersecting polygons in 2-D. This approach may be useful for doing spatial inference on irregularly sampled solenoidal vector fields (e.g. magnetic fields or the velocity field of an incompressible fluid) with boundary constraints

Example Simulation