Optimality of Non-Adaptive Strategies: The Case of Parallel Games

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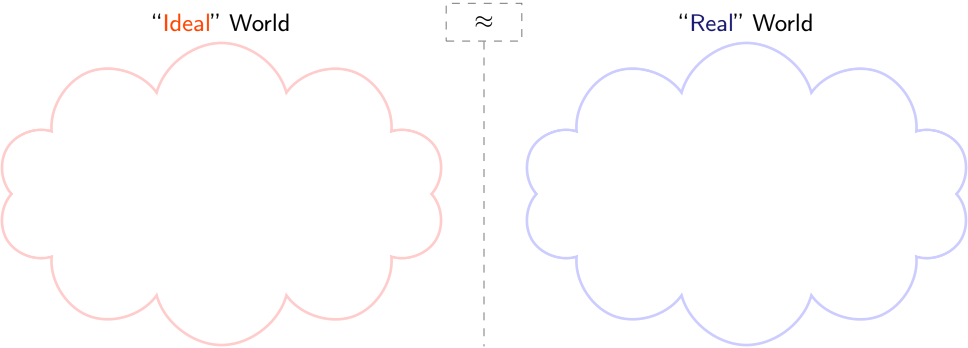
ISIT 2014

Outline

Indistinguishability Proofs



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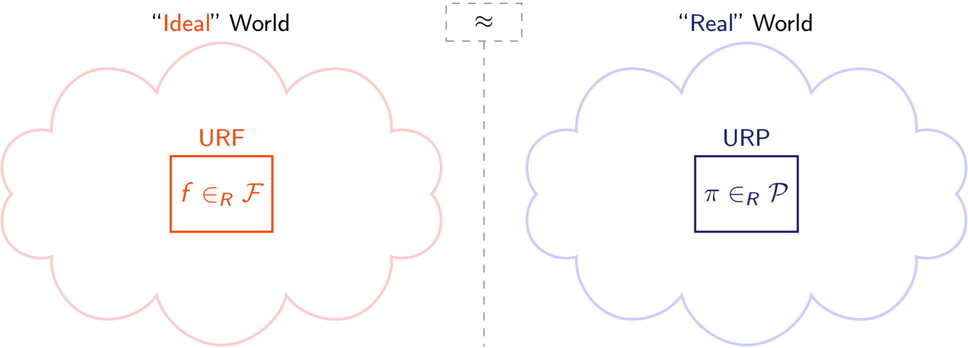

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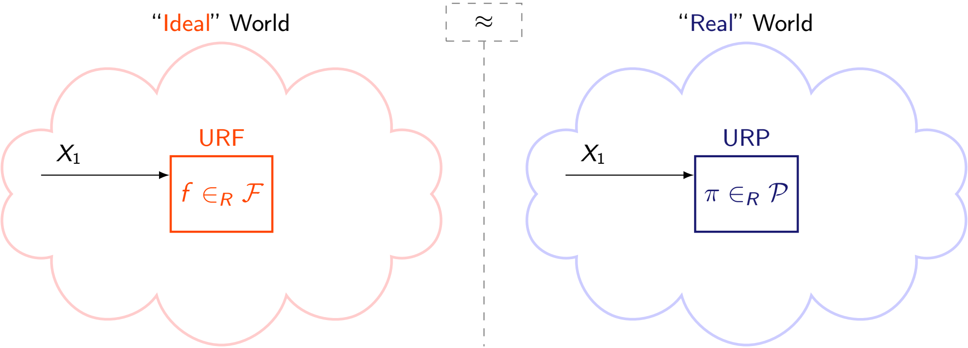

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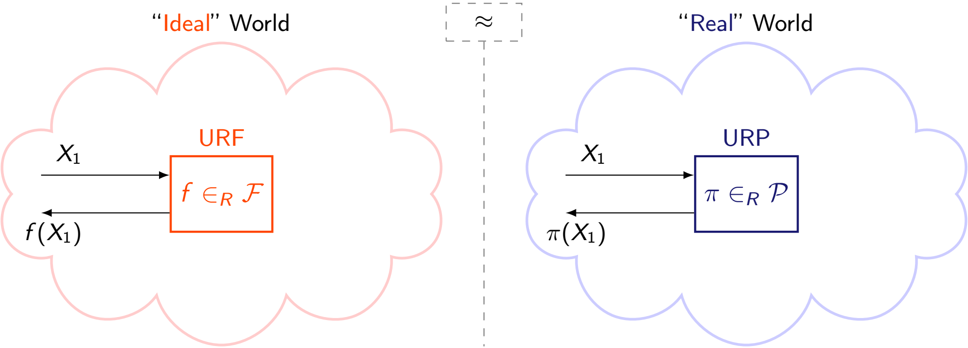

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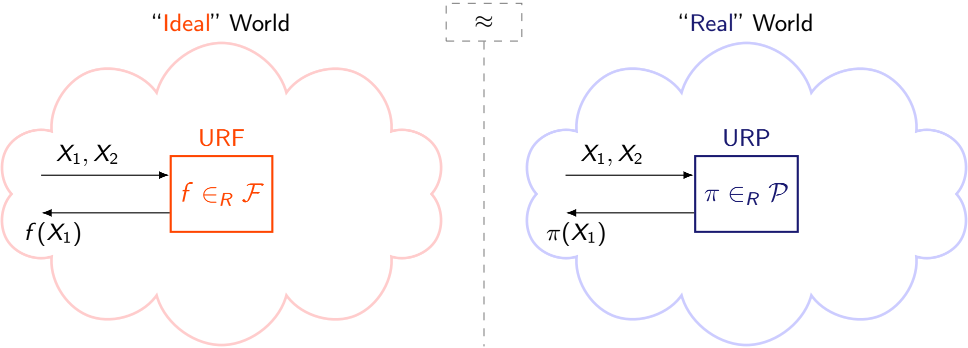

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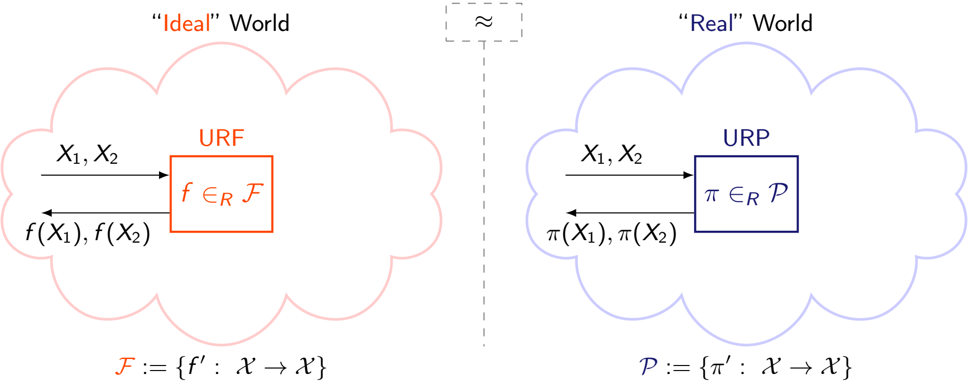

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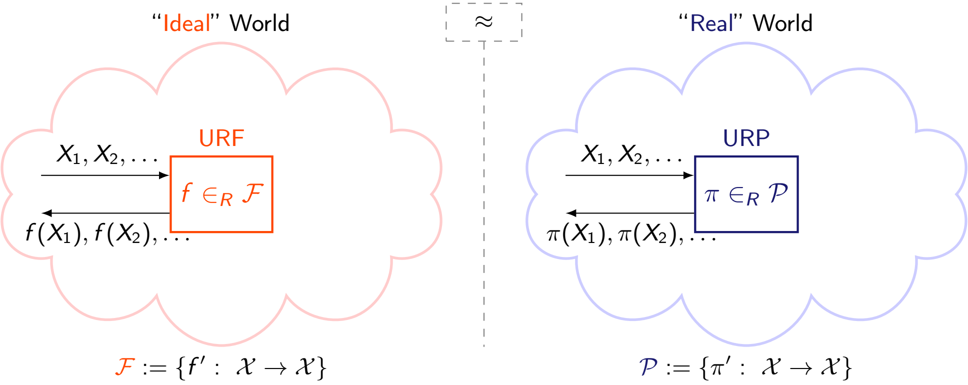

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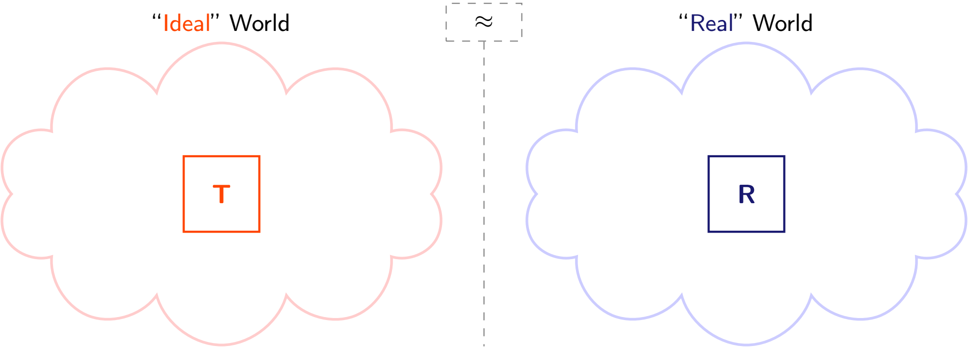

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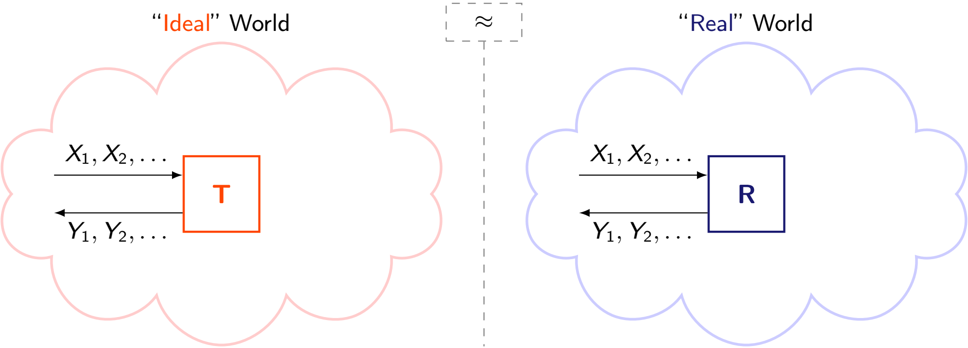

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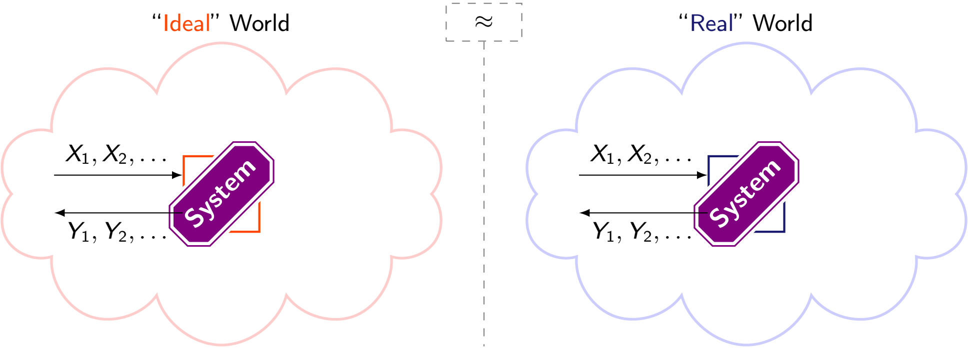

Discrete Random System [Mau02]

An $(\mathcal{X}, \mathcal{Y})$-system **S** is a sequence of $\left\{\mathrm{p}^{\mathbf{S}}{}_{Y_k|X^kY^{k-1}}\right\}_{k\geq 1}$

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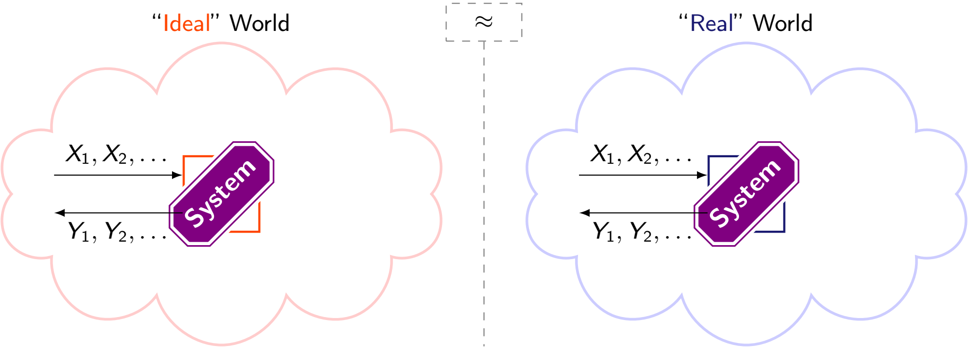

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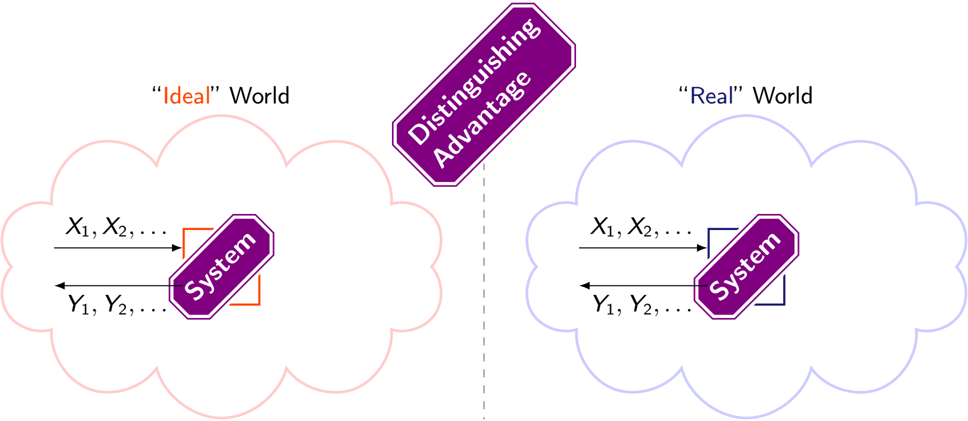

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Comparing Systems

- Distinguishing advantage of a distinguisher **D**

$$\Delta_k^{\mathbf{D}}(\mathbf{T}, \mathbf{R}) := \left| \mathrm{P}^{\mathbf{DT}}(B = 1) - \mathrm{P}^{\mathbf{DR}}(B = 1) \right|$$

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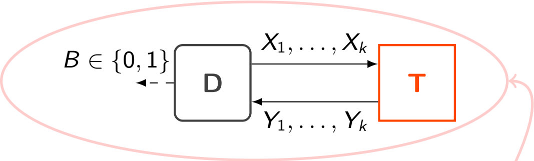

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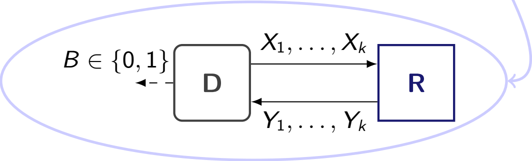
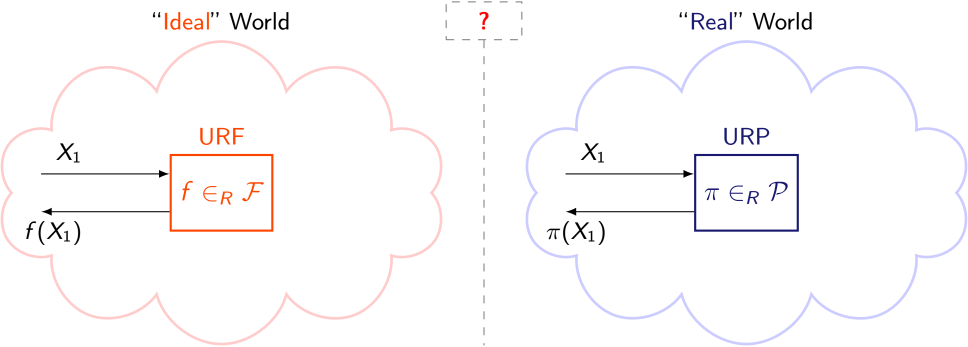


Proving Indistinguishability



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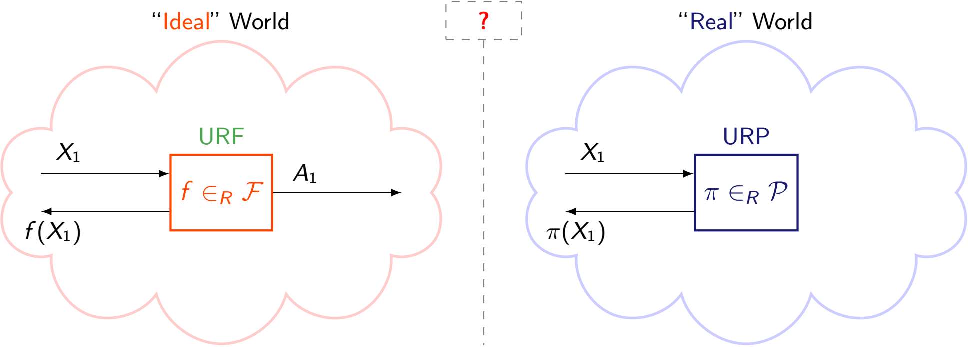

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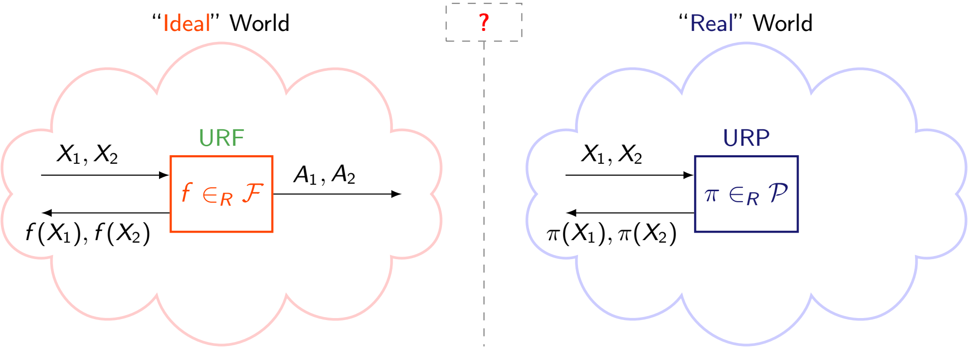

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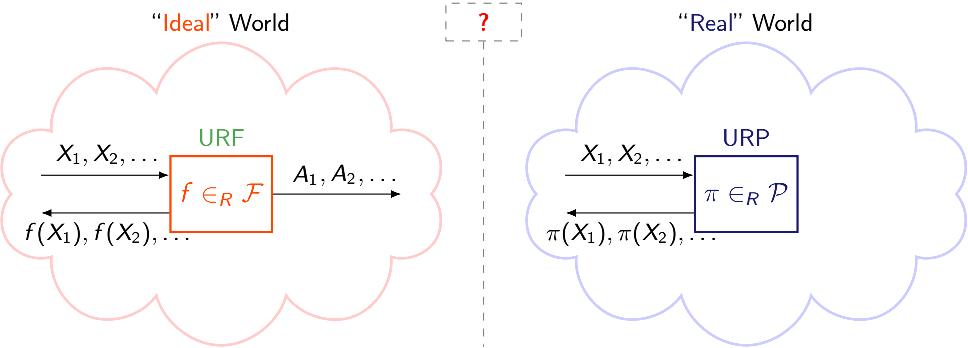

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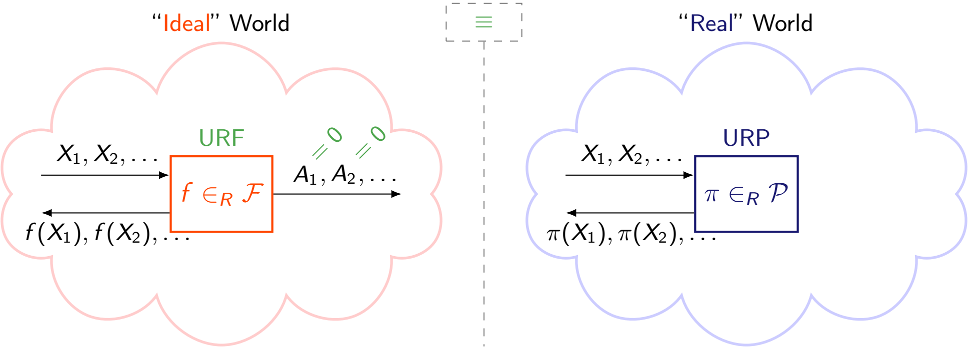

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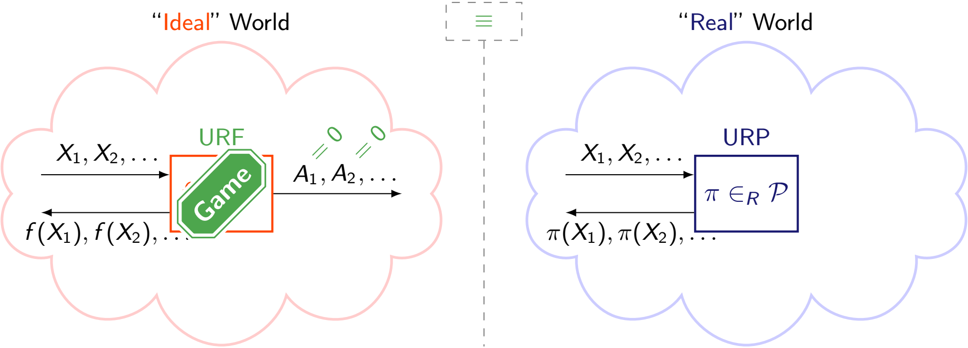

$(\mathcal{X}, \mathcal{Y})$-Game [MPR07]

$(\mathcal{X}, \mathcal{Y})$-game **G** is an $(\mathcal{X}, \mathcal{Y} \times \{0,1\})$-system with a *monotone binary output* (MBO) $A_1, A_2, \ldots$, where $\forall k \geq 1 : \; A_k = 1 \implies A_{k+1} = 1$

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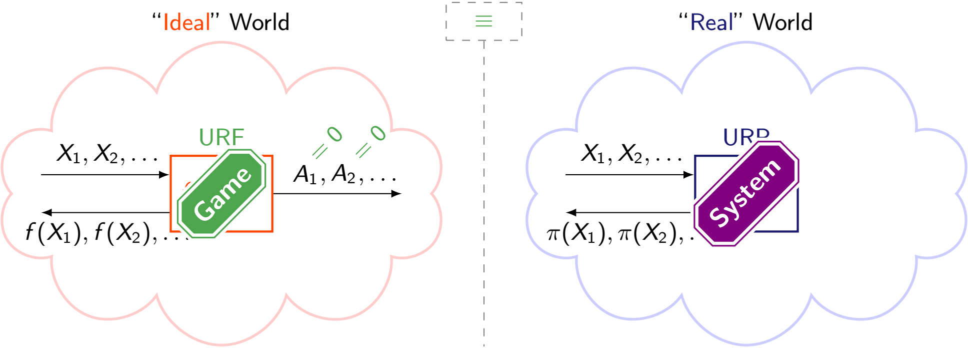

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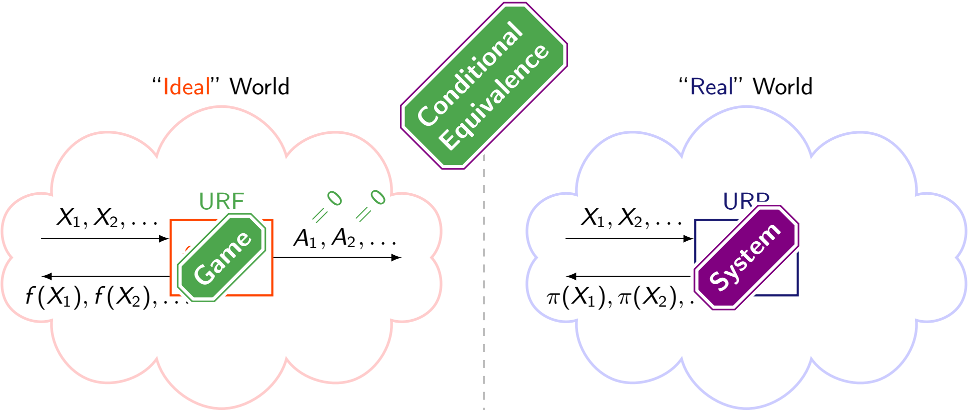
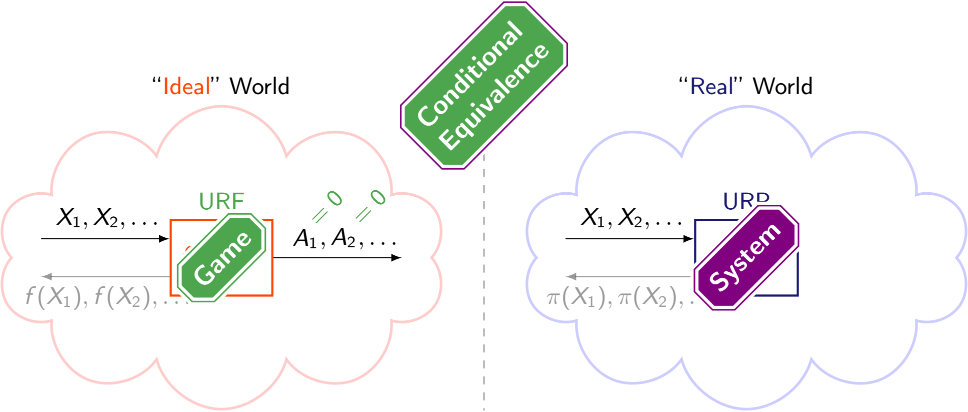


Conditional Equivalence [Mau02]

$$\mathbf{G} \models \mathbf{S} :\Leftrightarrow \mathrm{p}^{\mathbf{G}}_{Y^j | X^j A_j = 0} = \mathrm{p}^{\mathbf{S}}_{Y^j | X^j}, \quad \text{for all } j \geq 1.$$

Proving Indistinguishability



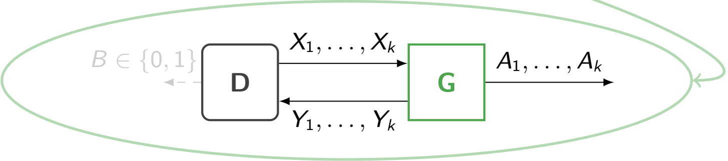
Particular Case of [Mau02]

URF $\models$ URP $\implies$ "adaptivity does not help in distinguishing URF from URP"

Winning a Game

- Probability of winning a game

$$\Gamma_k^{\mathsf{D}}(\mathsf{G}) := \underbrace{\mathsf{P}^{\mathsf{DG}}(A_k)(1)}$$



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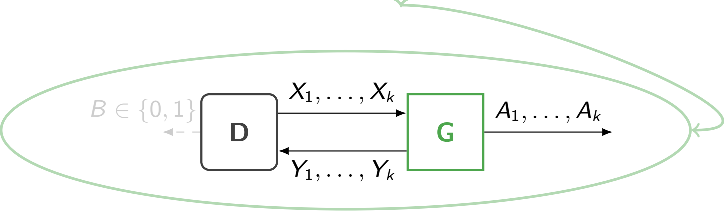

- **D** is non-adaptive $:\Longleftrightarrow \mathsf{p}^{\mathsf{D}}_{X_k|Y^{k-1}X^{k-1}} = \mathsf{p}^{\mathsf{D}}_{X_k|X^{k-1}}, \quad$ for all $k \geq 1$

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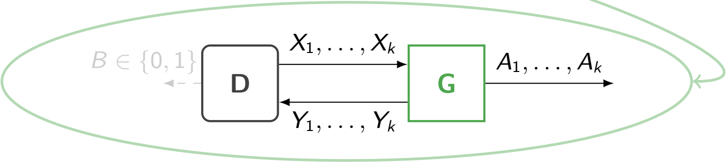

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- $\mathcal{D}_{\mathrm{na}}$ set of non-adaptive game winners.

Results Overview

$$\mathrm{NA}(\mathbf{G}) \quad :\Longleftrightarrow \quad \forall k \in \mathbb{N} : \max_{\mathbf{D} \in \mathcal{D}} \Gamma_k^{\mathbf{D}}(\mathbf{G}) = \max_{\mathbf{D} \in \mathcal{D}_{\mathrm{na}}} \Gamma_k^{\mathbf{D}}(\mathbf{G})$$

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Theorem 1

$$\mathrm{NA}\left(\mathbf{G}_1\right) \text{ and } \mathrm{NA}\left(\mathbf{G}_2\right)$$

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[Mau02]: $\mathrm{CE}(\mathbf{G}) \Longrightarrow \mathrm{NA}(\mathbf{G})$

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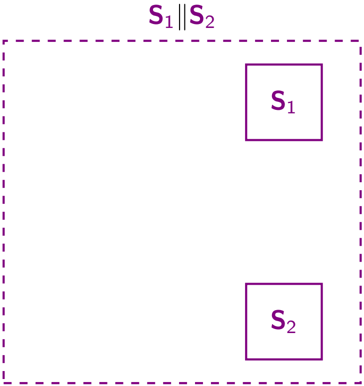

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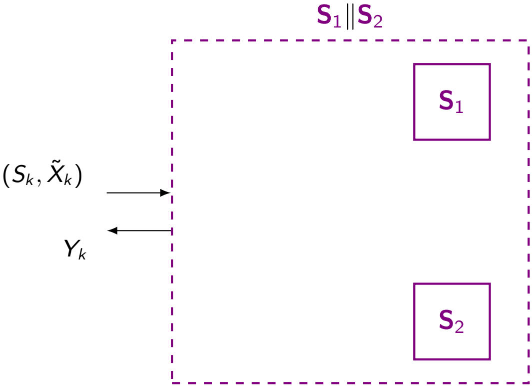

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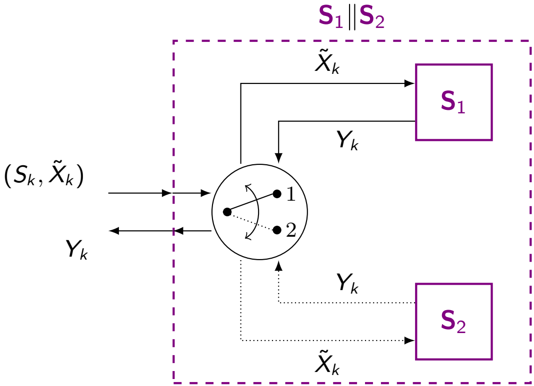

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$\boxed{\mathbf{G}_1}$

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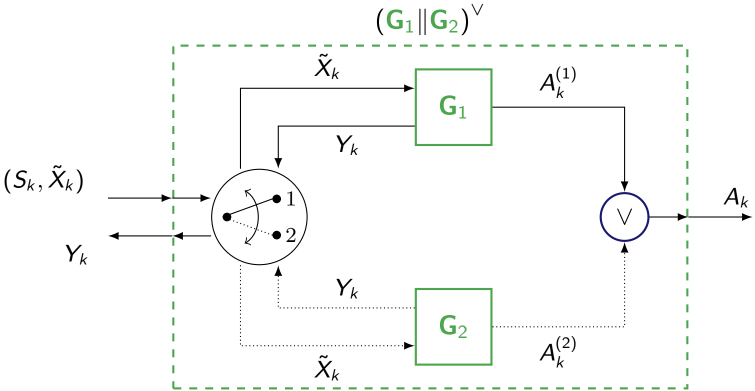

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$\mathrm{NA}\left(\mathbf{H}\right) \wedge \neg\mathrm{NA}\left(\left(\mathbf{H} \| \mathbf{H}\right)^{\vee}\right)$

Counter-Example

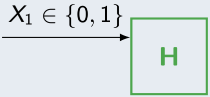

Figure : 1^{st} query

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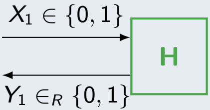

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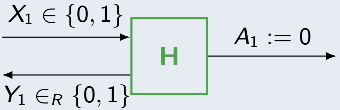

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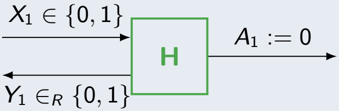

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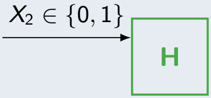

Figure : 2^{nd} query

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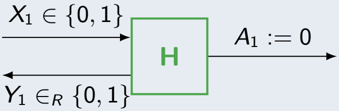

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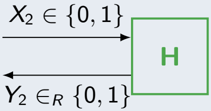

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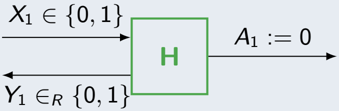

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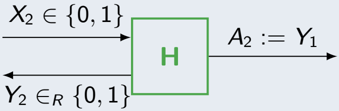

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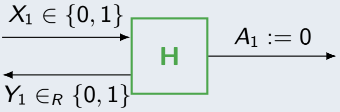

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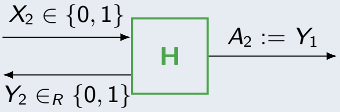

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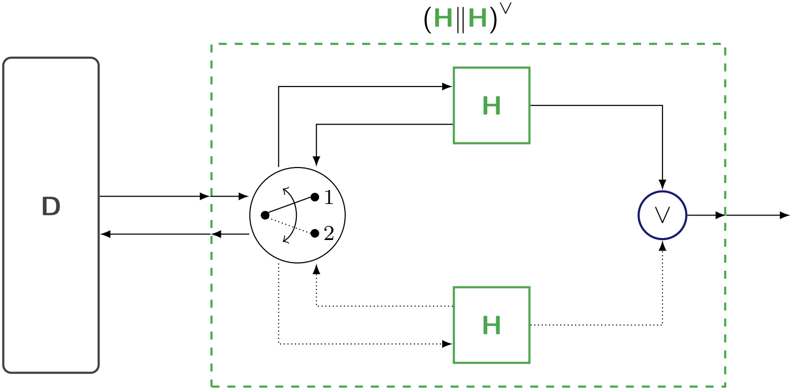
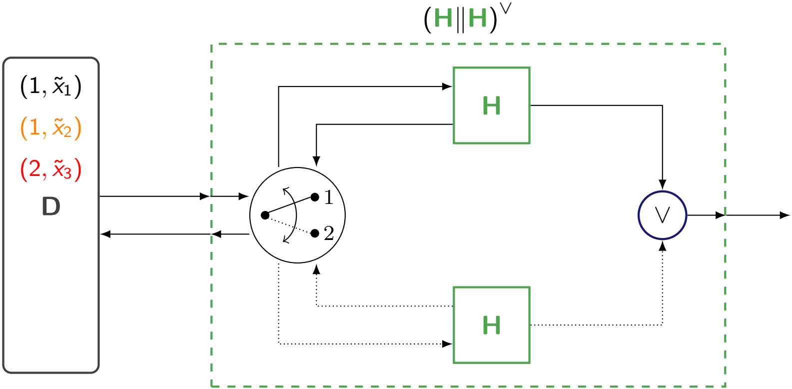
$\mathrm{NA}(\mathsf{H})$

$$\max_{\mathsf{D} \in \mathcal{D}} \Gamma_k^{\mathsf{D}}(\mathsf{H}) = \max_{\mathsf{D} \in \mathcal{D}_{\text{na}}} \Gamma_k^{\mathsf{D}}(\mathsf{H}) = \begin{cases} 0 & \text{if } k \leq 1, \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

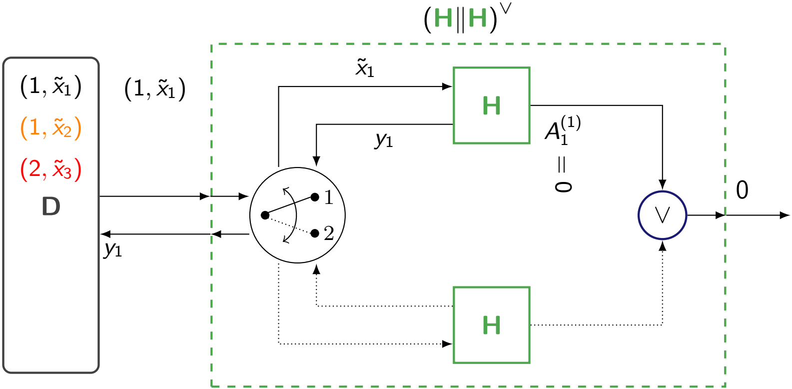
$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Non-Adaptive Game Winner



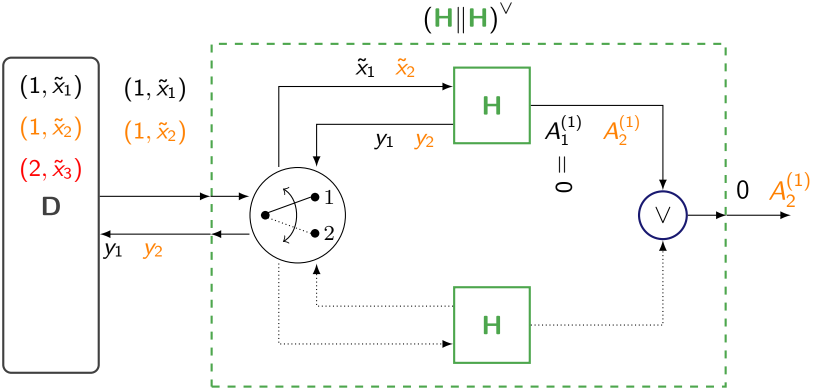
$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Non-Adaptive Game Winner



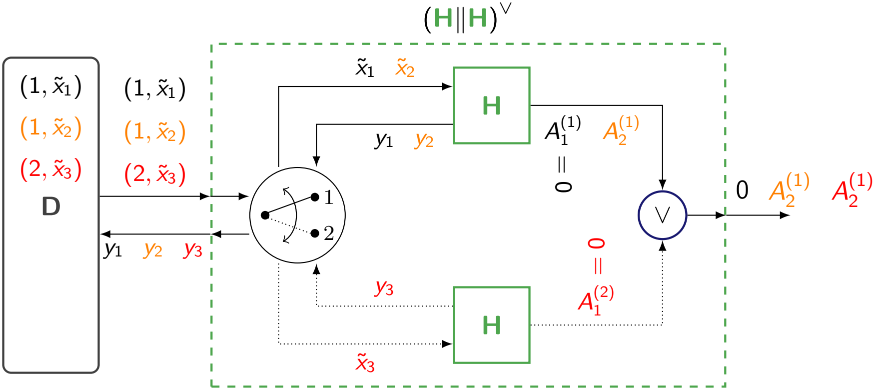
$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Non-Adaptive Game Winner



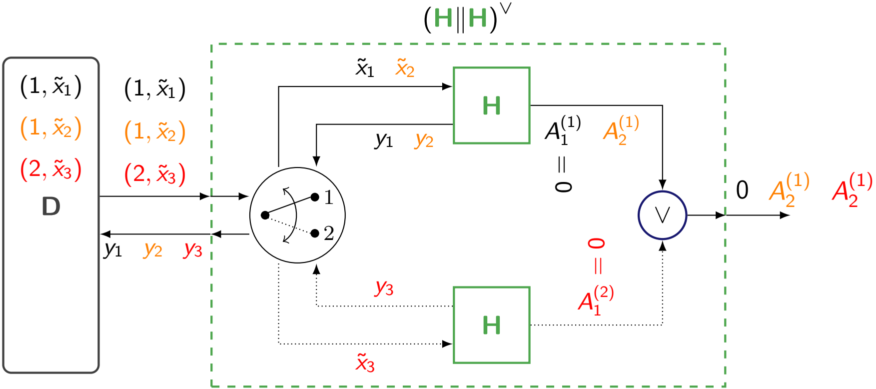
$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Non-Adaptive Game Winner



$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Non-Adaptive Game Winner

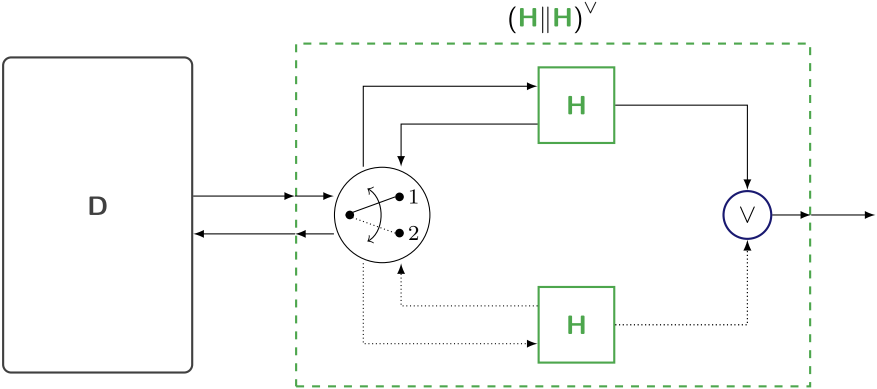


$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Non-Adaptive Game Winner



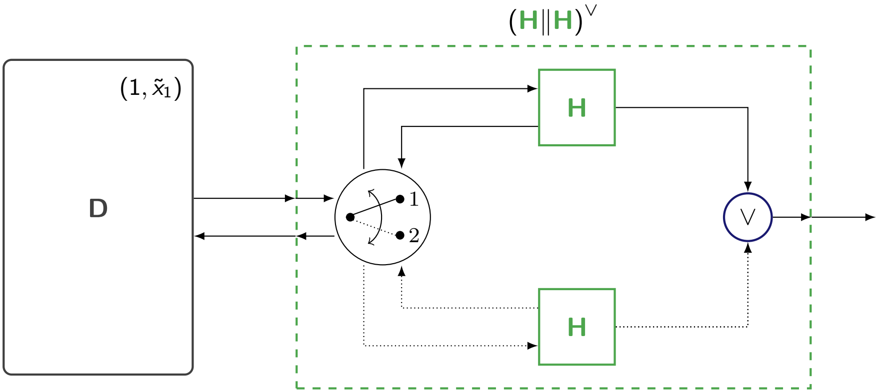
$$\max_{\mathbf{D}\in\mathcal{D}_{\mathrm{na}}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) = \frac{1}{2}$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



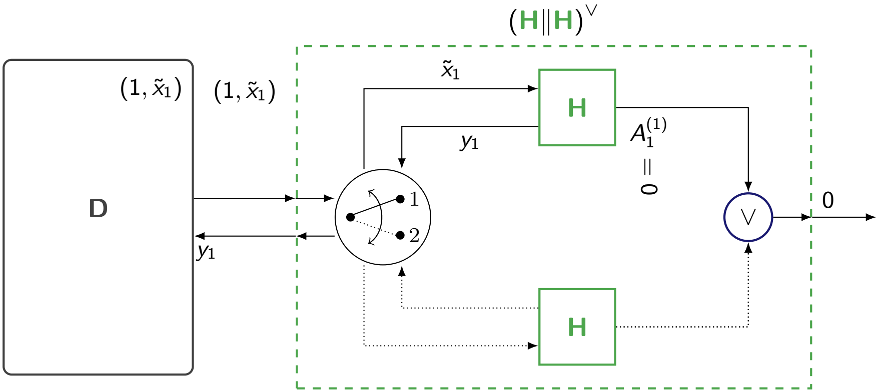
$$\max_{\mathsf{D}\in\mathcal{D}} \Gamma_3^{\mathsf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



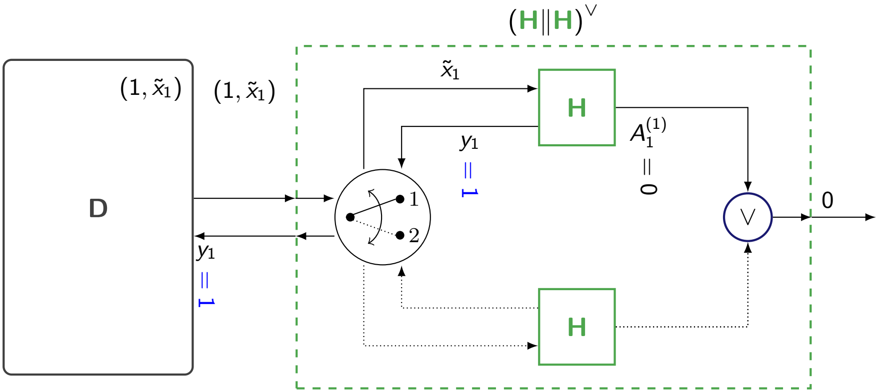
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq$$

$\neg$NA $\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



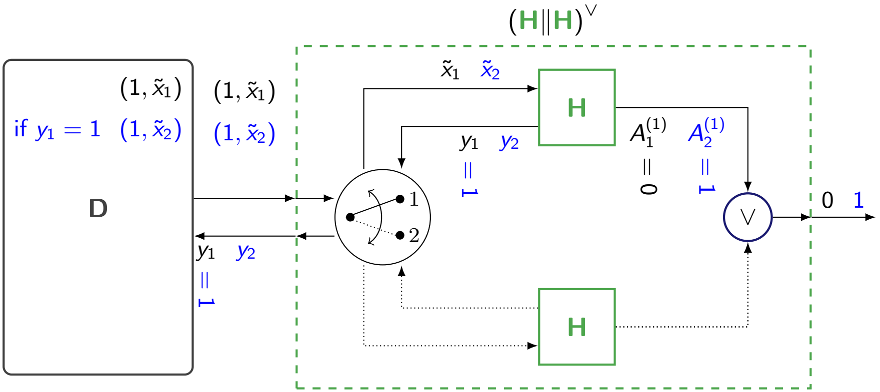
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}} \left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



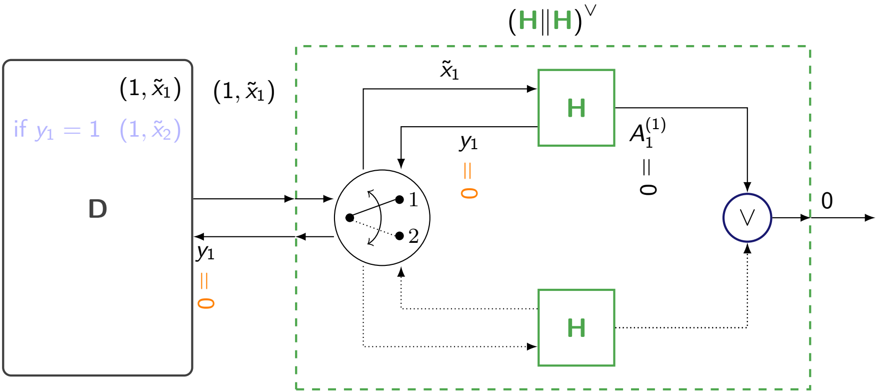
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2}$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



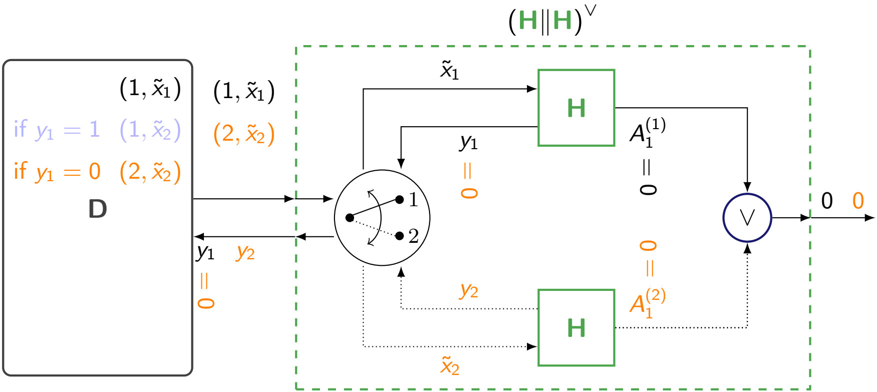
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2} \cdot 1$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



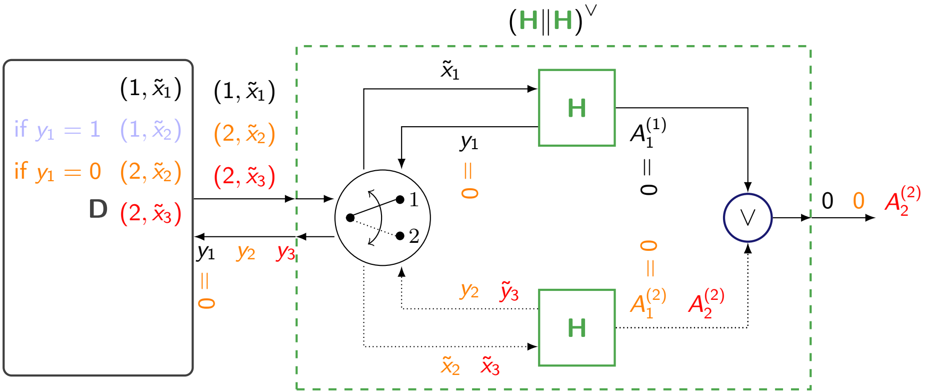
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2}\cdot 1 + \frac{1}{2}$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



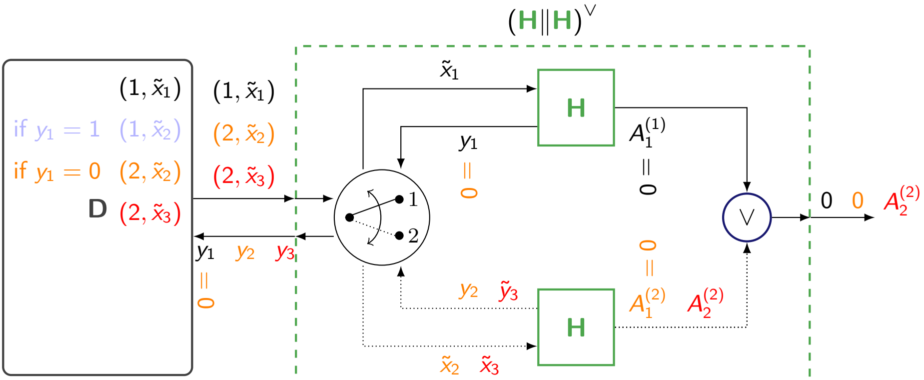
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2}\cdot 1 + \frac{1}{2}$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



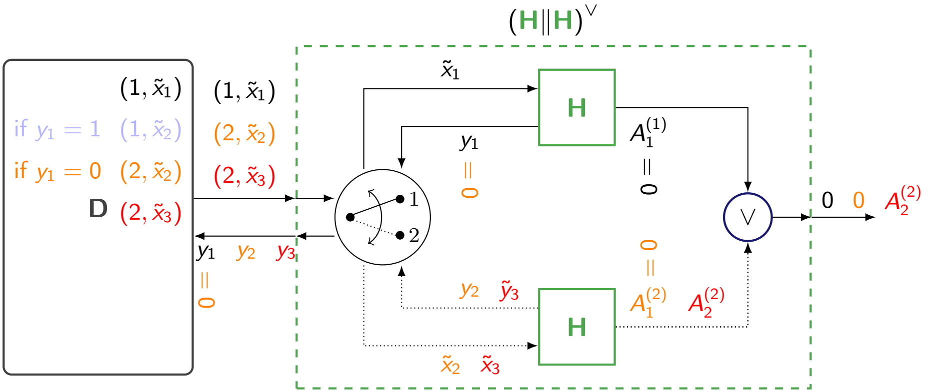
$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{1}{2}$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4}$$

$\neg\mathrm{NA}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$ - Adaptive Game Winner



$$\max_{\mathbf{D}\in\mathcal{D}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right) \geq \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{4} > \frac{1}{2} = \max_{\mathbf{D}\in\mathcal{D}_{\mathrm{na}}} \Gamma_3^{\mathbf{D}}\left((\mathbf{H}\|\mathbf{H})^{\vee}\right)$$

Results Overview

$$\mathrm{NA}\left(\mathbf{G}\right) \quad :\Longleftrightarrow \quad \forall k \in \mathbb{N}: \quad \max_{\mathbf{D}\in\mathcal{D}} \Gamma_k^{\mathbf{D}}\left(\mathbf{G}\right) = \max_{\mathbf{D}\in\mathcal{D}_{\mathsf{na}}} \Gamma_k^{\mathbf{D}}\left(\mathbf{G}\right)$$

$$\mathrm{CE}\left(\mathbf{G}\right) \quad :\Longleftrightarrow \quad \exists \mathbf{S}: \quad \mathbf{G} \models \mathbf{S}$$

Overview

[Mau02]: $\mathrm{CE}\left(\mathbf{G}\right) \Longrightarrow \mathrm{NA}\left(\mathbf{G}\right)$

[DGMT14]: $\mathrm{CE}\left(\mathbf{G}\right) \cancel{\Longleftarrow} \mathrm{NA}\left(\mathbf{G}\right)$

Theorem 1

$\mathrm{NA}\left(\mathbf{G}_1\right)$ and $\mathrm{NA}\left(\mathbf{G}_2\right)$

$$\cancel{\Downarrow}$$

$$\mathrm{NA}\left(\left(\mathbf{G}_1 \| \mathbf{G}_2\right)^{\vee}\right)$$

Theorem 2

$\mathrm{CE}\left(\mathbf{G}_1\right)$ and $\mathrm{CE}\left(\mathbf{G}_2\right)$

$$\Downarrow$$

$$\mathrm{CE}\left(\left(\mathbf{G}_1 \| \mathbf{G}_2\right)^{\vee}\right)$$

$$\mathrm{CE}\left(\mathbf{G}_1\right) \wedge \mathrm{CE}\left(\mathbf{G}_2\right) \implies \mathrm{CE}\left(\left(\mathbf{G}_1 \| \mathbf{G}_2\right)^{\vee}\right)$$

Lemma

$$\mathbf{G}_1 \models \mathbf{S}_1 \text{ and } \mathbf{G}_2 \models \mathbf{S}_2 \quad \implies \quad \left(\mathbf{G}_1 \| \mathbf{G}_2\right)^{\vee} \models \mathbf{S}_1 \| \mathbf{S}_2$$

Results Overview

$$\mathrm{NA}\left(\mathbf{G}\right) \quad :\Longleftrightarrow \quad \forall k \in \mathbb{N}: \ \max_{\mathbf{D}\in\mathcal{D}} \Gamma_k^{\mathbf{D}}\left(\mathbf{G}\right) = \max_{\mathbf{D}\in\mathcal{D}_{\mathsf{na}}} \Gamma_k^{\mathbf{D}}\left(\mathbf{G}\right)$$

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Overview

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$$\Downarrow$$

$\mathrm{CE}\left(\left(\mathbf{G}_1 \| \mathbf{G}_2\right)^{\vee}\right)$