Unfair Coin Tossing

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Outline

Outline

Coin Tossing Protocols

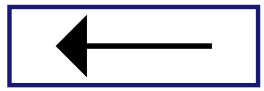
Setting

- 2 distrustful parties: one of them being potentially dishonest
- *Goal*: construct an ideal coin tossing resource

Ideal Coin Tossing Resource

Available Resources

Perfect Communication Channel



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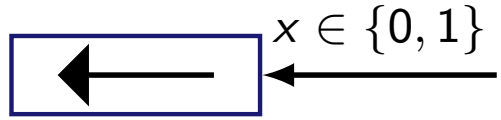

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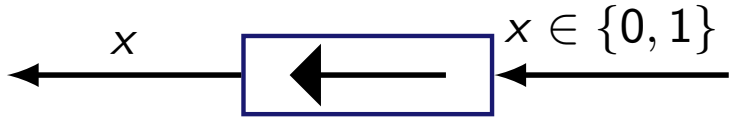

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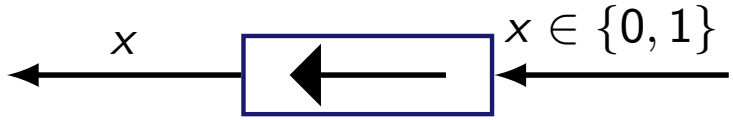

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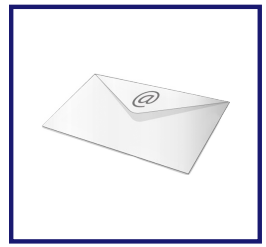

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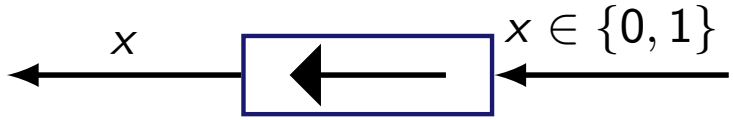

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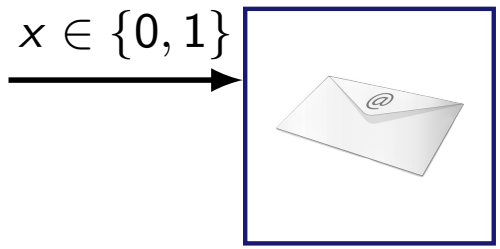

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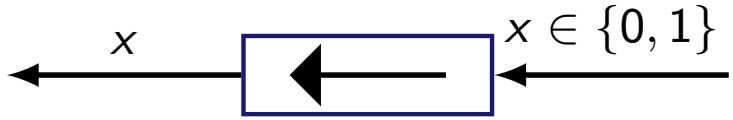

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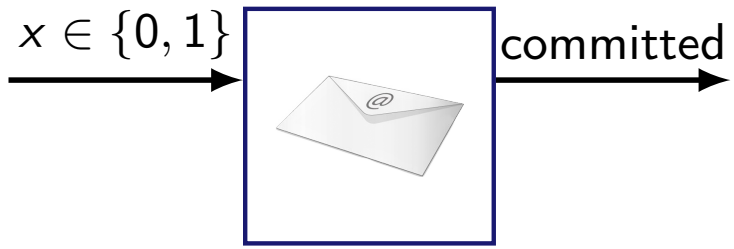

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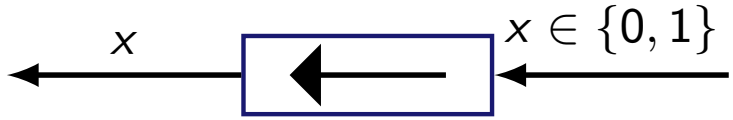

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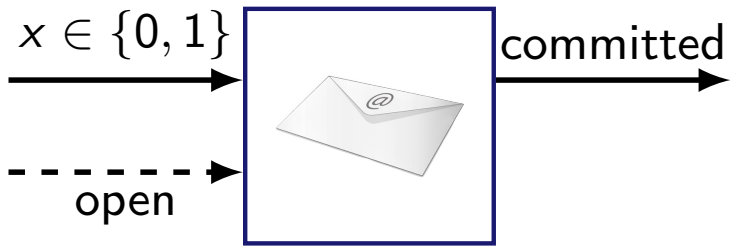

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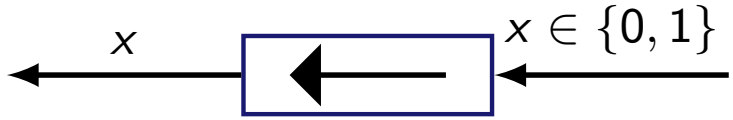

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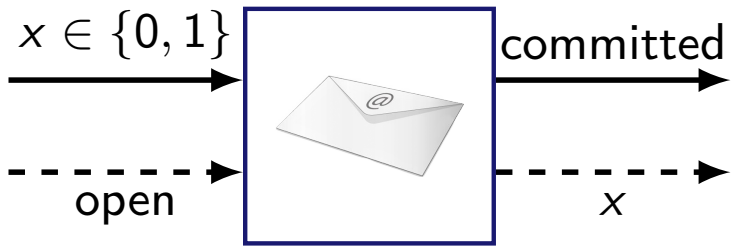
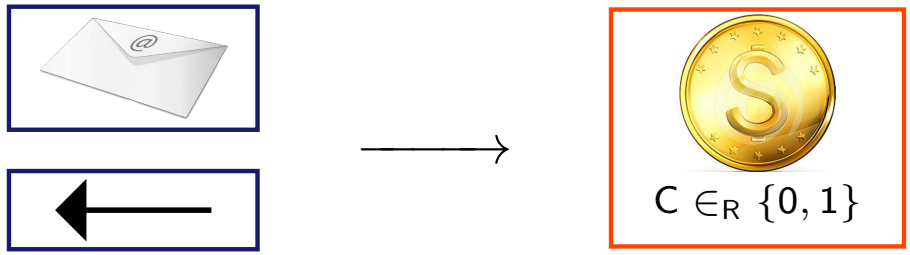


Motivation

Goal



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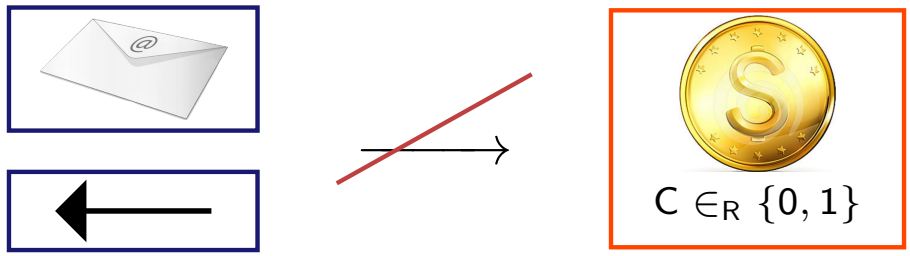
(Unachievable) Goal



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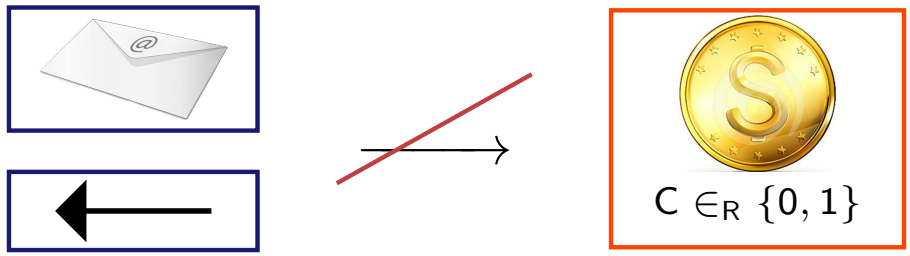

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Cl86 : every coin tossing protocol with r rounds has a bias of $\Omega\left(\frac{1}{r}\right)$

- No guarantee in case of abortion
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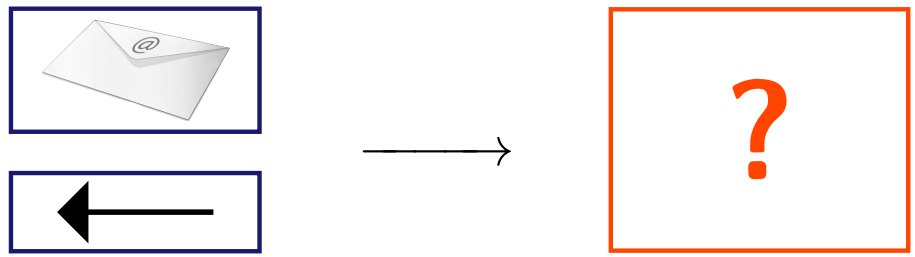

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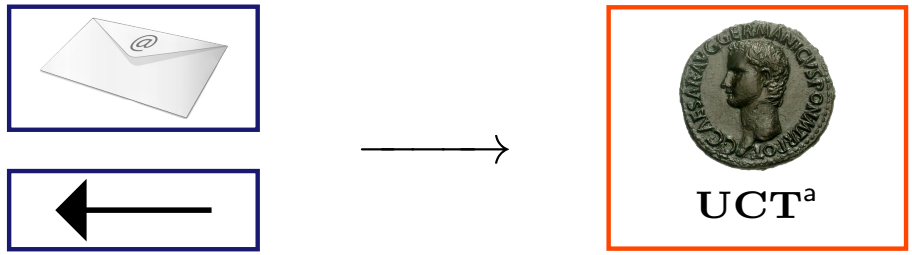

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The Construction Paradigm[1]

Construction Concept

real resource **R** $\xrightarrow{(\text{protocol } \pi, \varepsilon)}$ ideal resource **S**

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Generally Composable Constructions

- Sequential Composability

$$\mathbf{R} \xrightarrow{(\pi_1,\varepsilon_1)} \mathbf{S} \;\wedge\; \mathbf{S} \xrightarrow{(\pi_2,\varepsilon_2)} \mathbf{T} \;\Longrightarrow\; \mathbf{R} \xrightarrow{(\pi_1 \circ \pi_2, \varepsilon_1 + \varepsilon_2)} \mathbf{T}$$

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- Parallel Composability

$$\mathbf{R} \xrightarrow{(\pi,\varepsilon)} \mathbf{S} \;\Longrightarrow\; \mathbf{R}\|\mathbf{T} \xrightarrow{(\pi|\mathrm{id},\varepsilon)} \mathbf{S}\|\mathbf{T} \;\wedge\; \mathbf{T}\|\mathbf{R} \xrightarrow{(\mathrm{id}|\pi,\varepsilon)} \mathbf{T}\|\mathbf{S}$$

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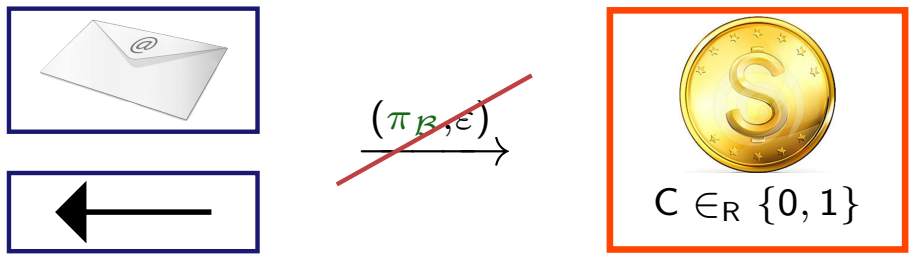
Blum's Coin Tossing Protocol[2]

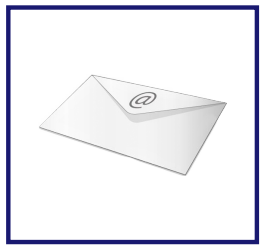
Unachievable Goal



[2] M. Blum: *Coin Flipping By Telephone A Protocol For Solving Impossible Problems*, 1983

Blum's Coin Tossing Protocol

Honest Execution



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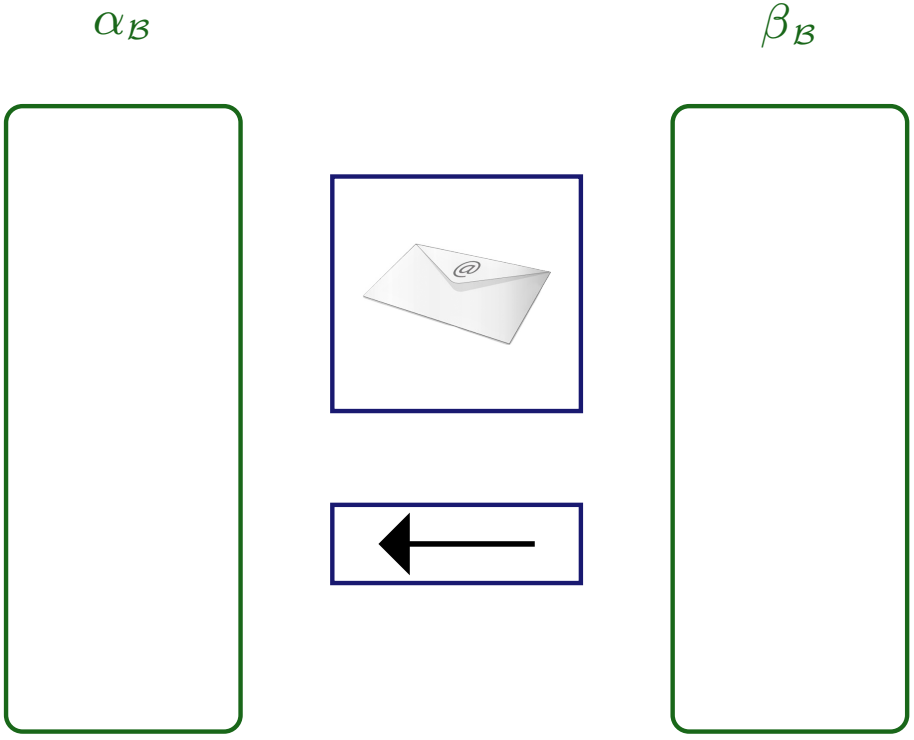

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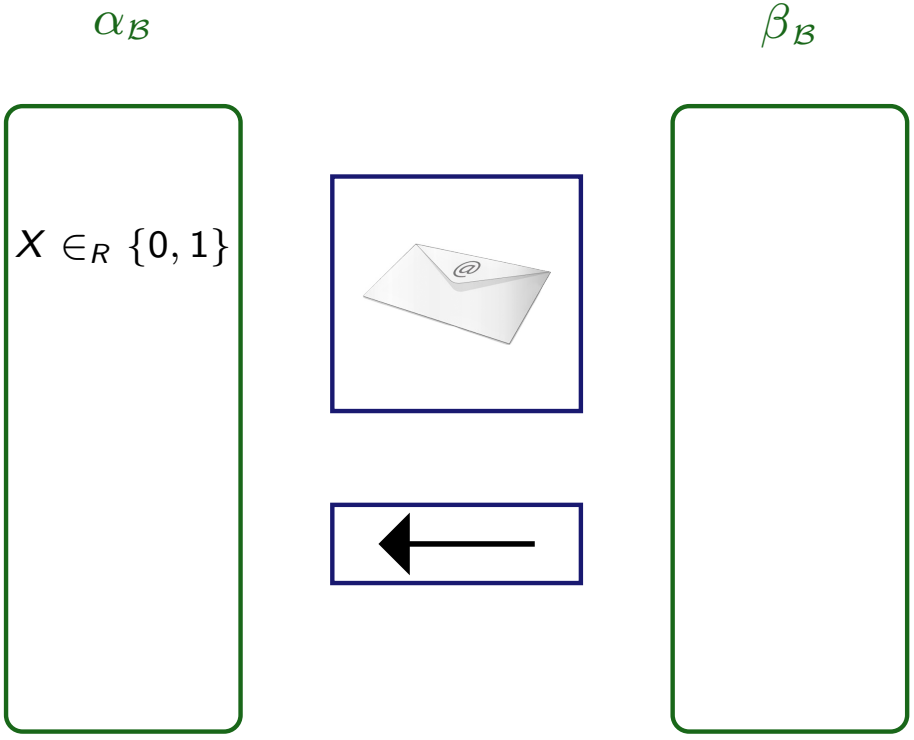

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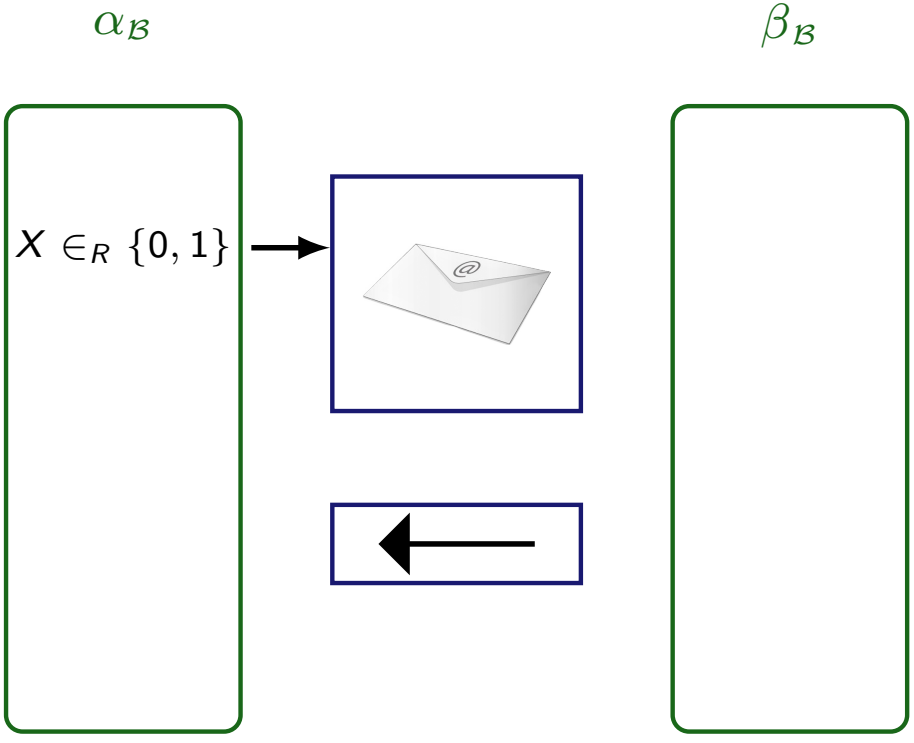

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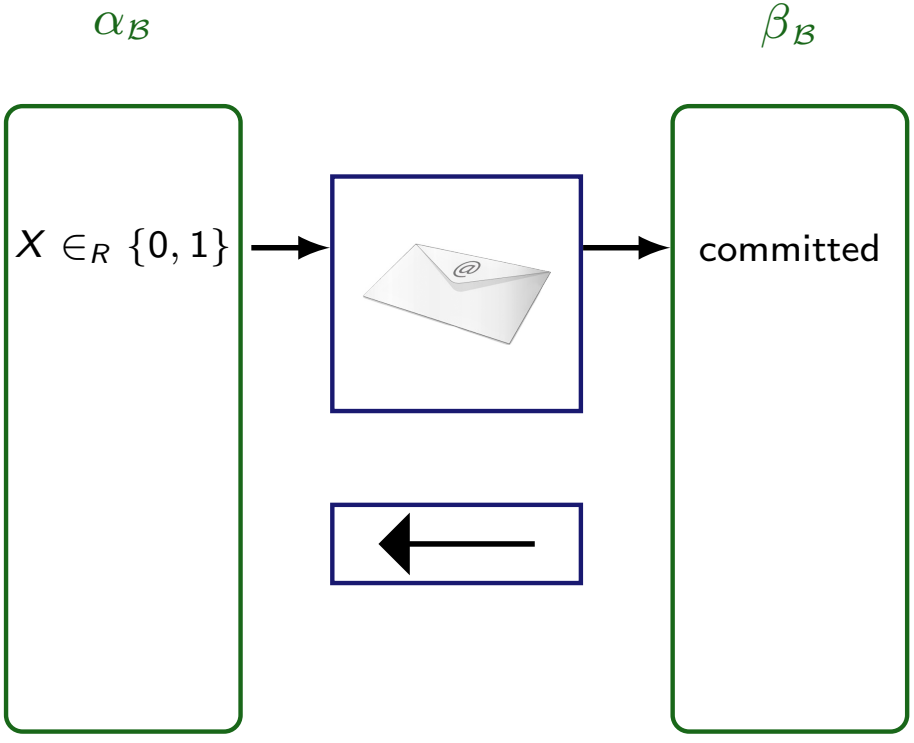

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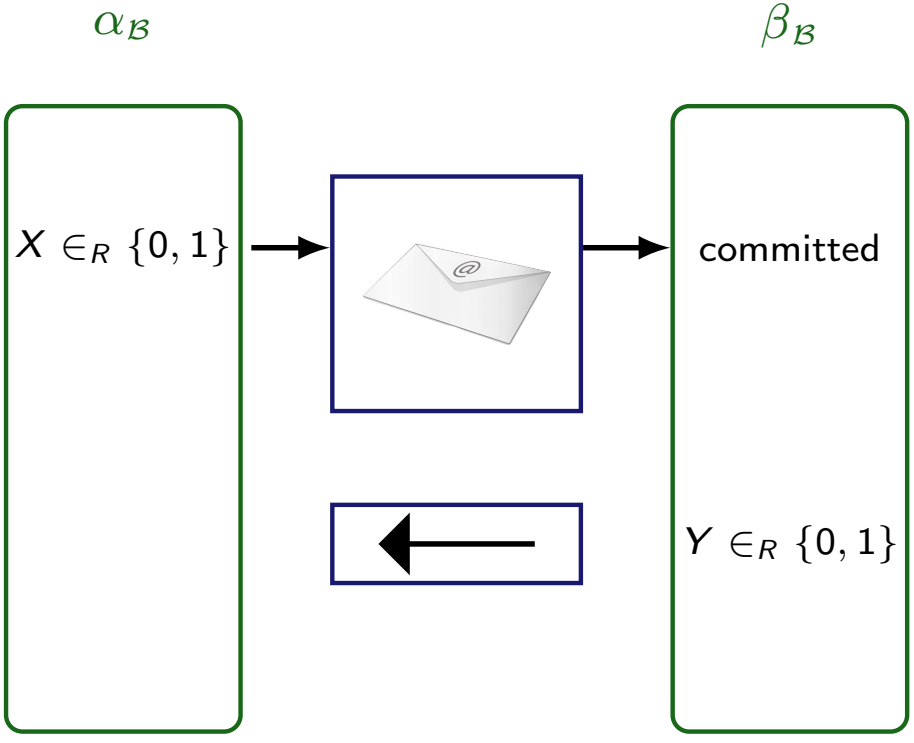

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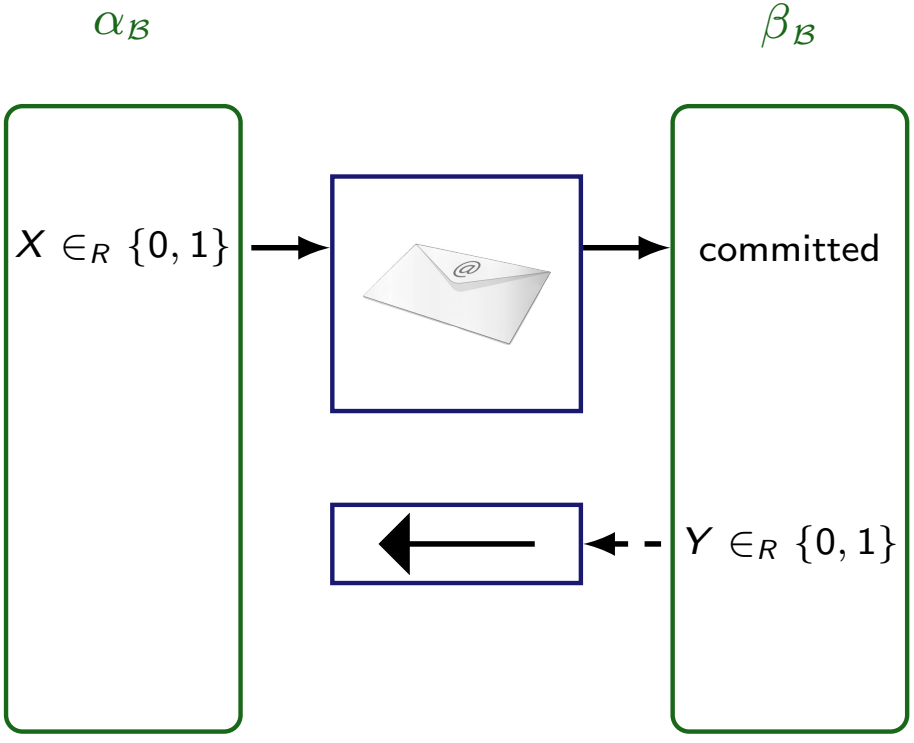

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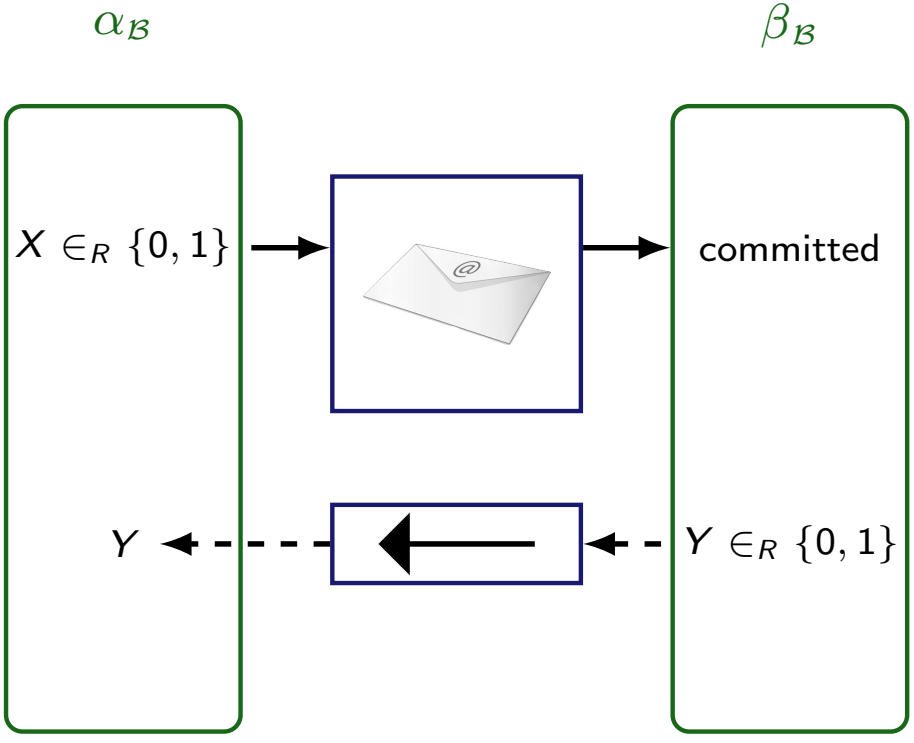

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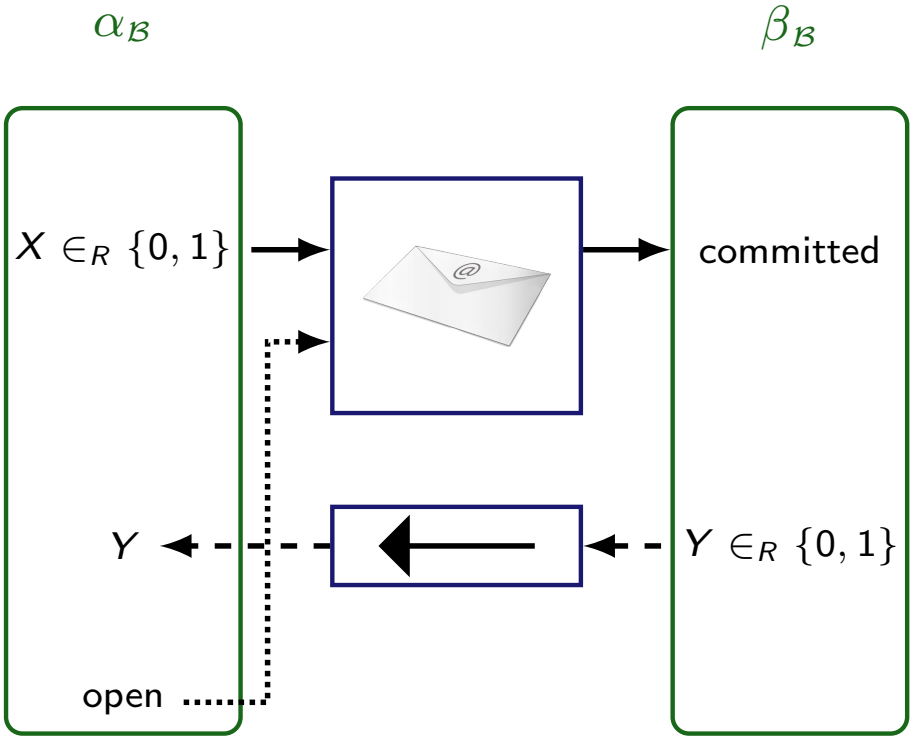

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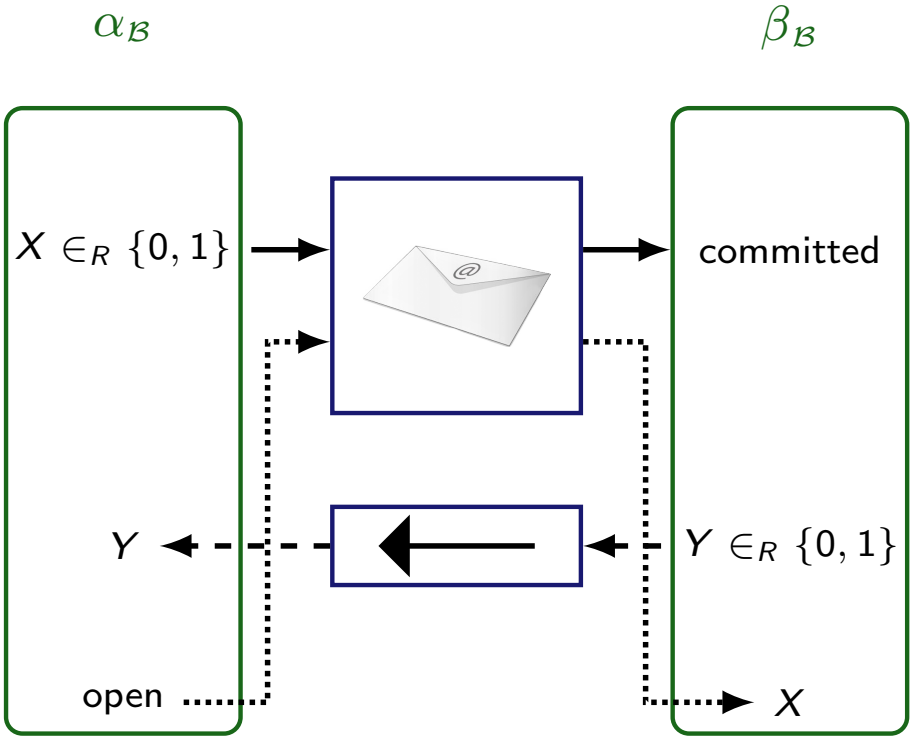

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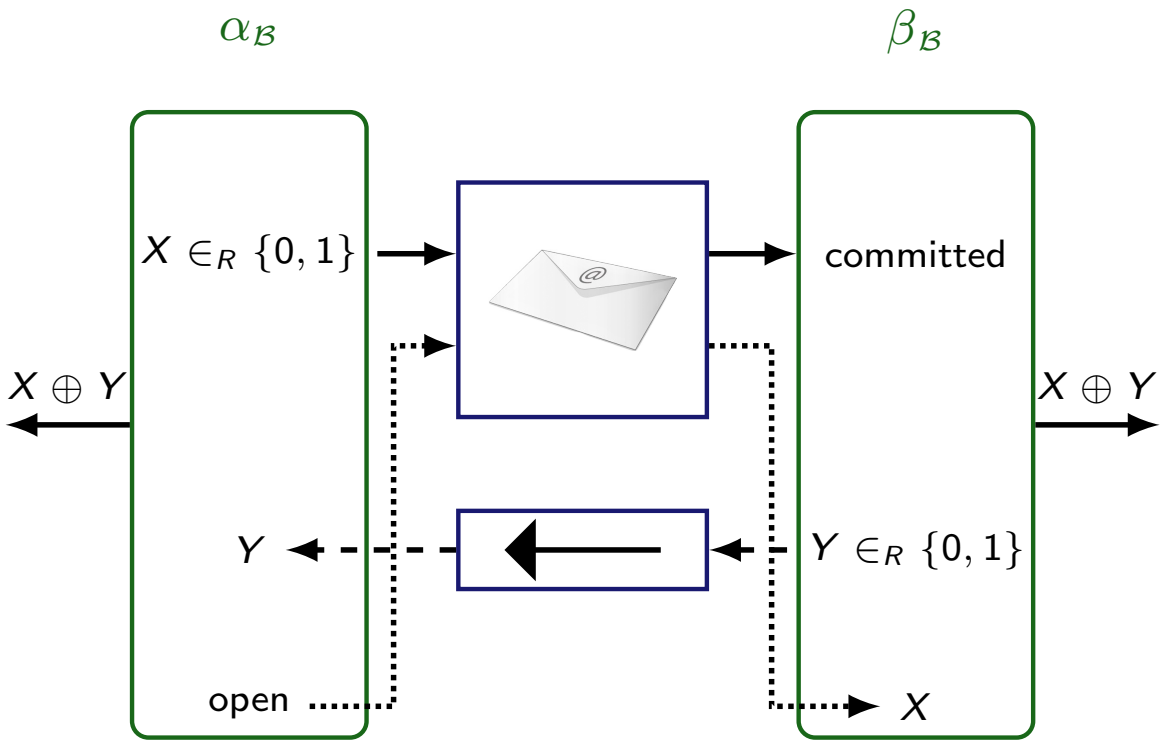

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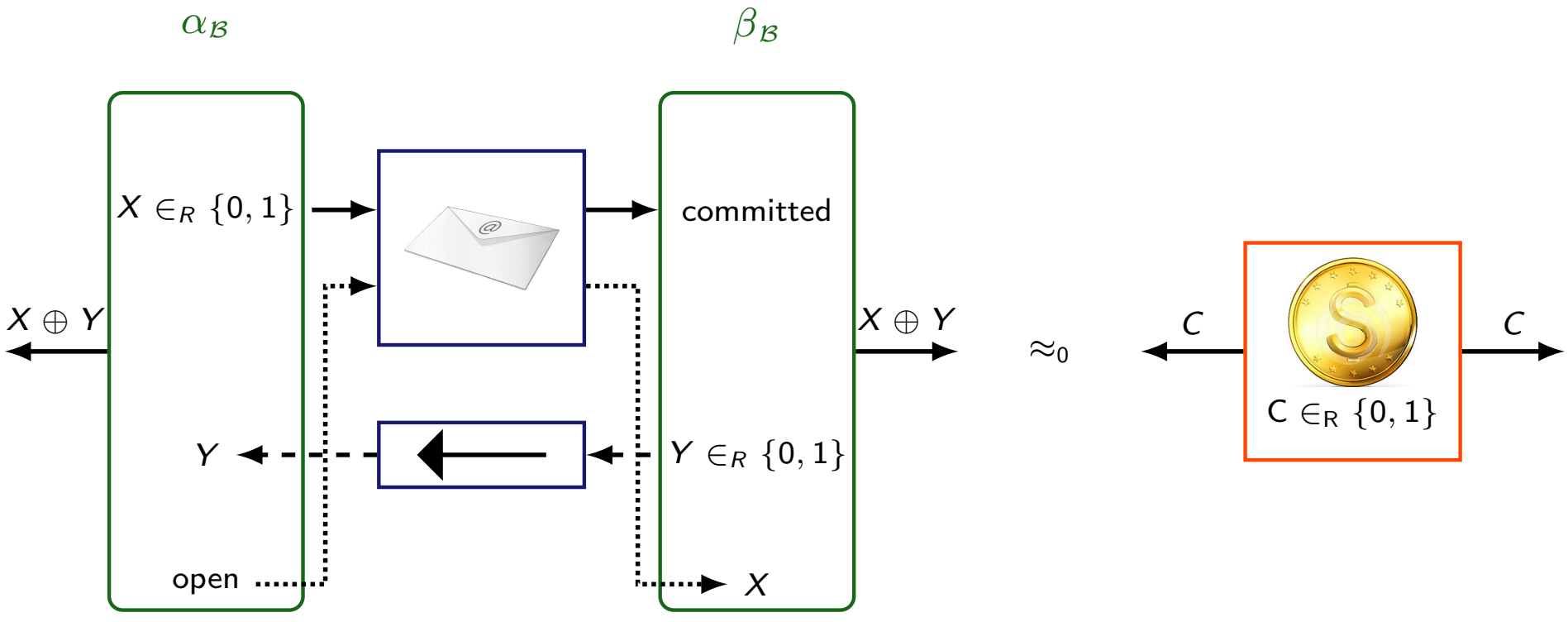
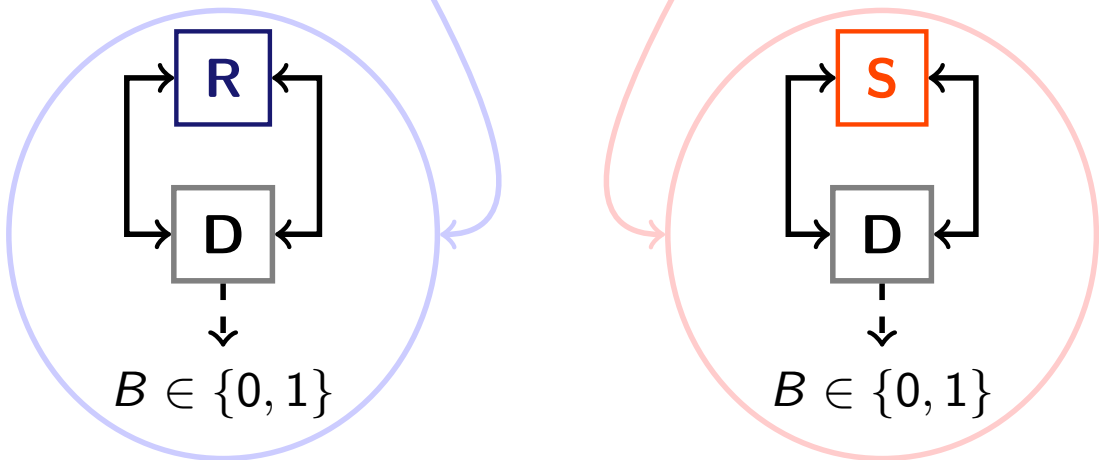


Comparing Resources

- Distinguishing advantage of a distinguisher **D**

$$\Delta^{\mathbf{D}}(\mathbf{R}, \mathbf{S}) := \left| \mathrm{P}^{\mathbf{D}\mathbf{R}}(B = 1) - \mathrm{P}^{\mathbf{D}\mathbf{S}}(B = 1) \right|$$



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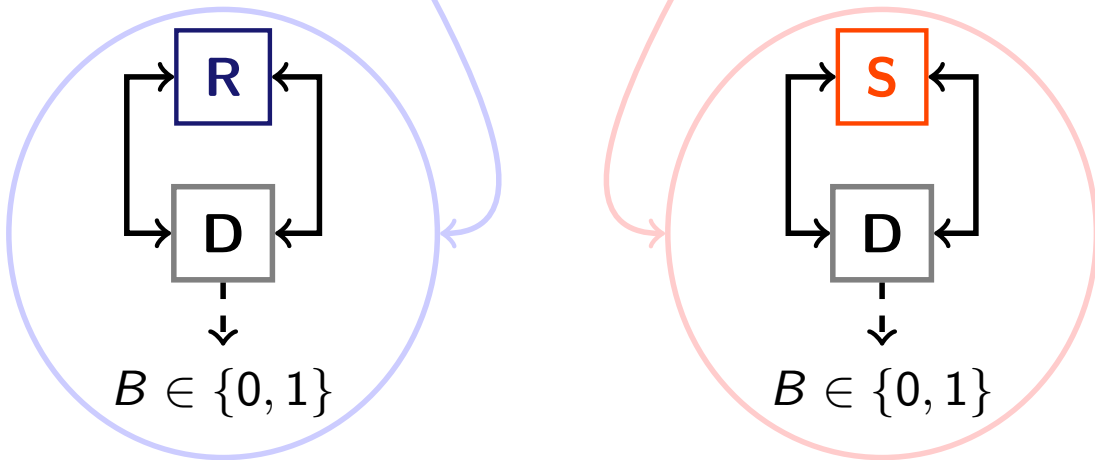

- Pseudo-metric induced

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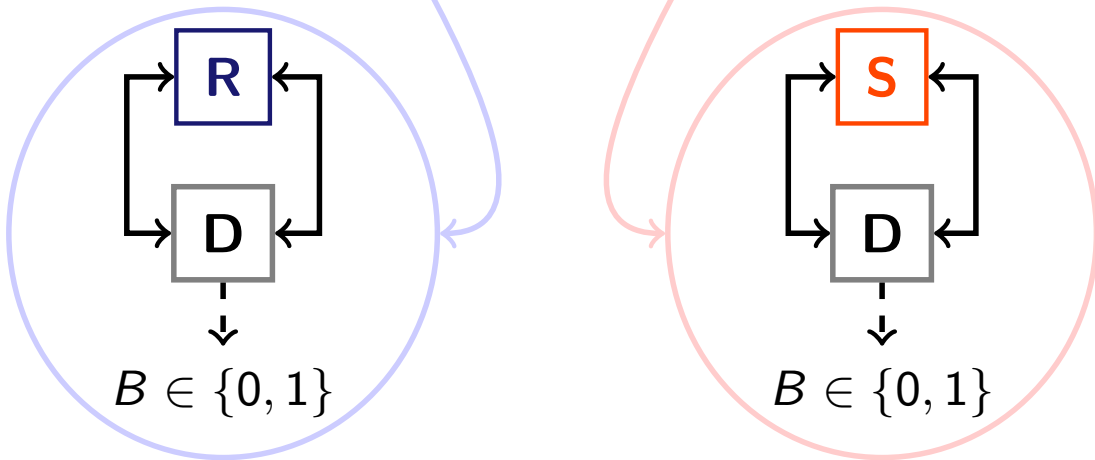

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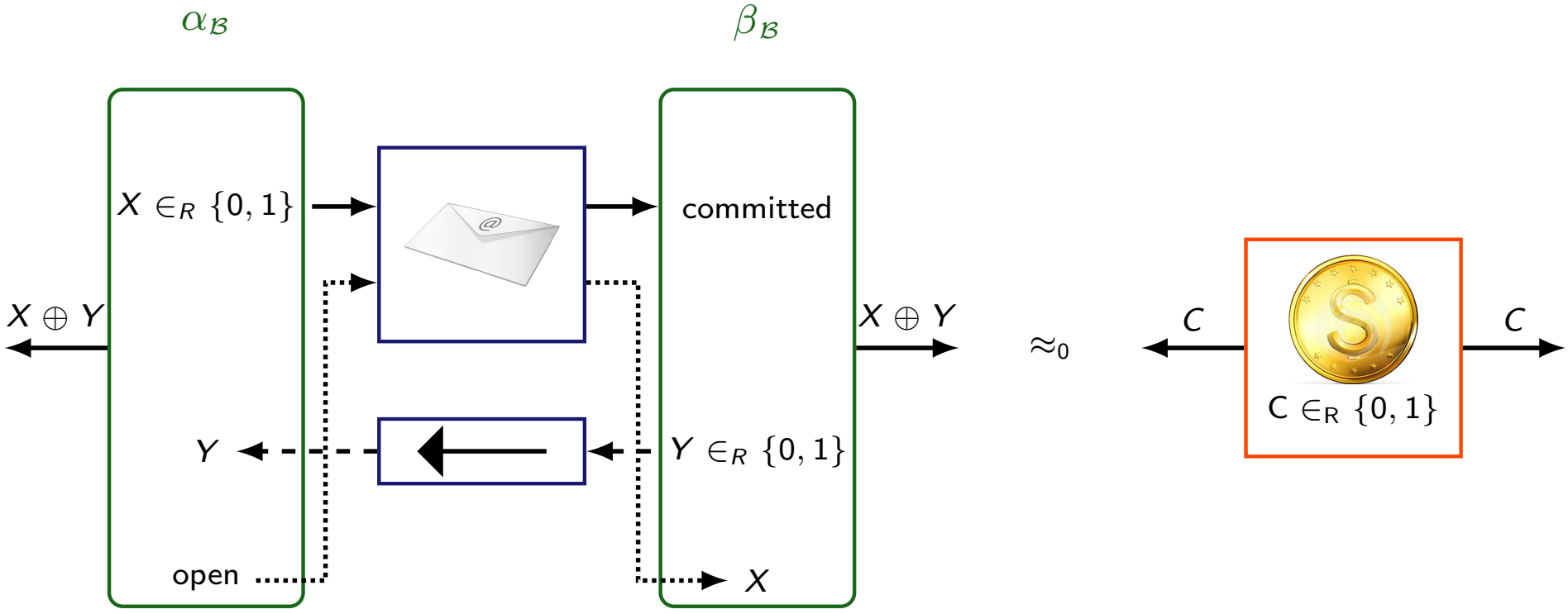
- *Information-theoretic* constructions $\quad \Longrightarrow \quad$ no restriction on $\mathcal{D}$

Blum's Coin Tossing Protocol

Honest Execution



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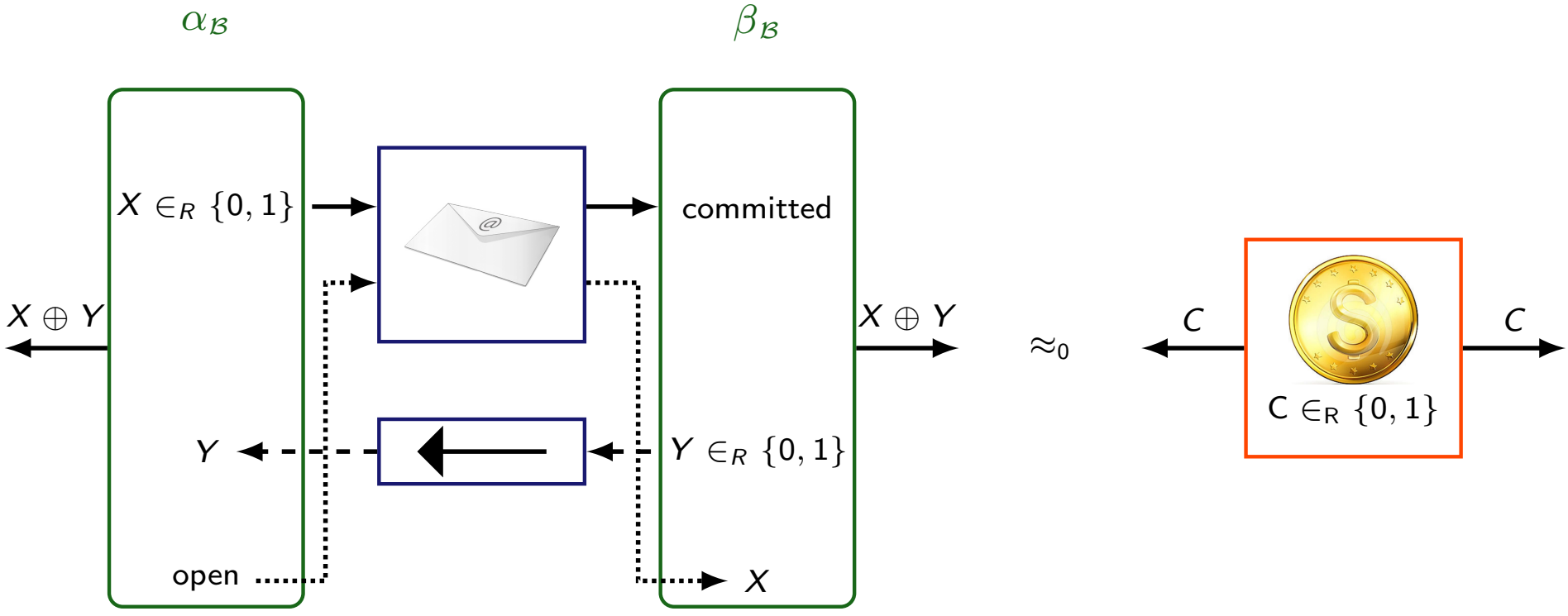
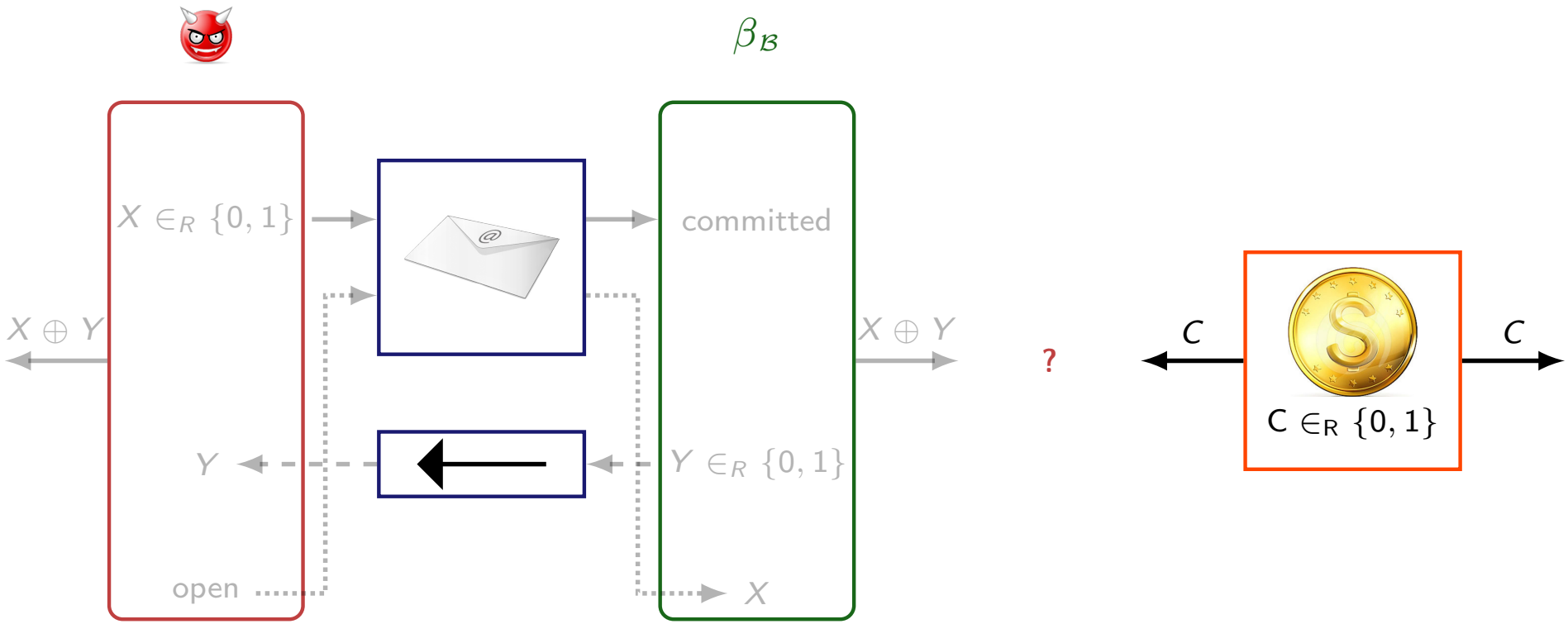


Setting

- 2 distrustful parties: one of them being potentially dishonest
- *Goal*: construct an ideal coin tossing resource

Blum's Coin Tossing Protocol - Security

Malicious Alice



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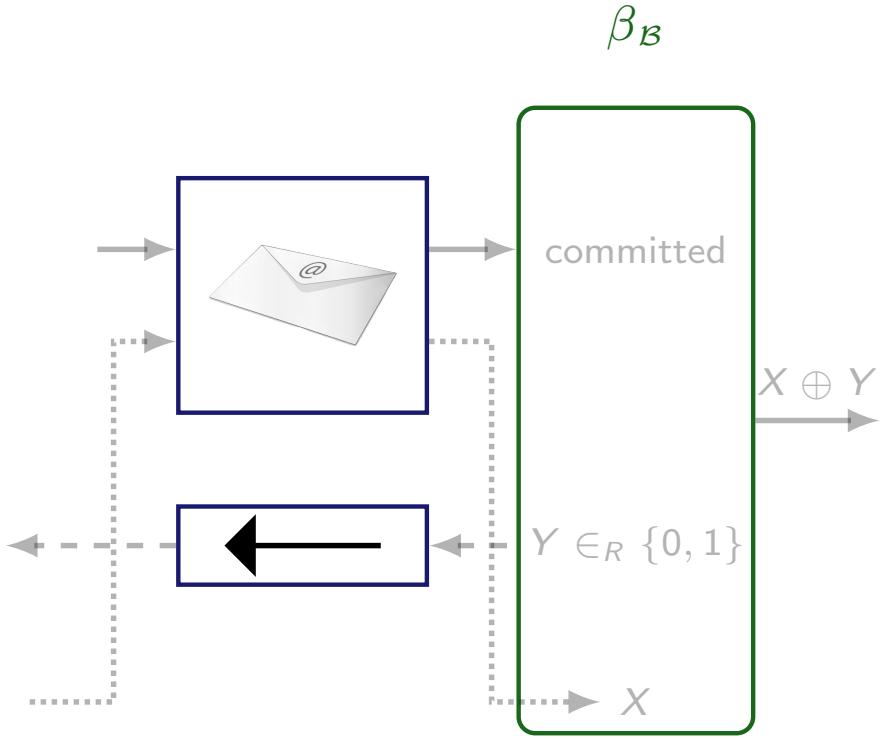

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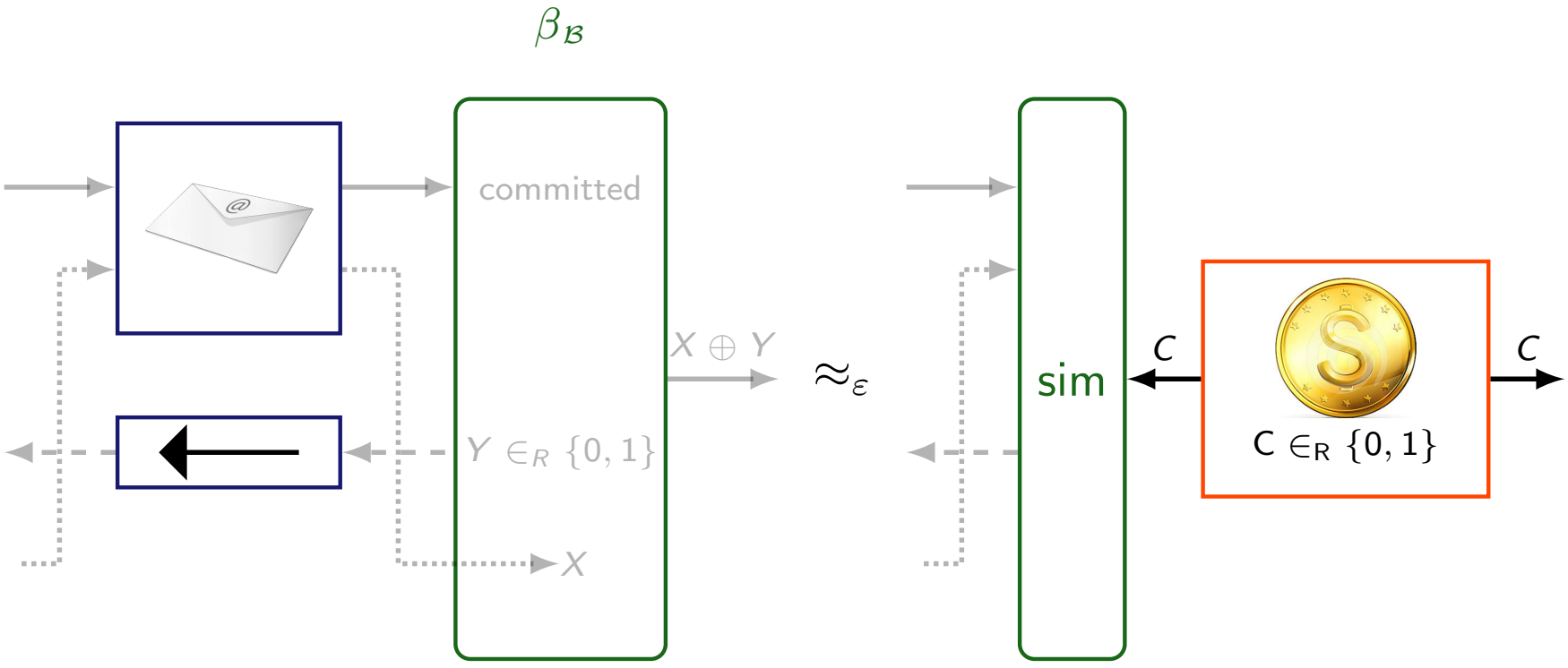
Simulator



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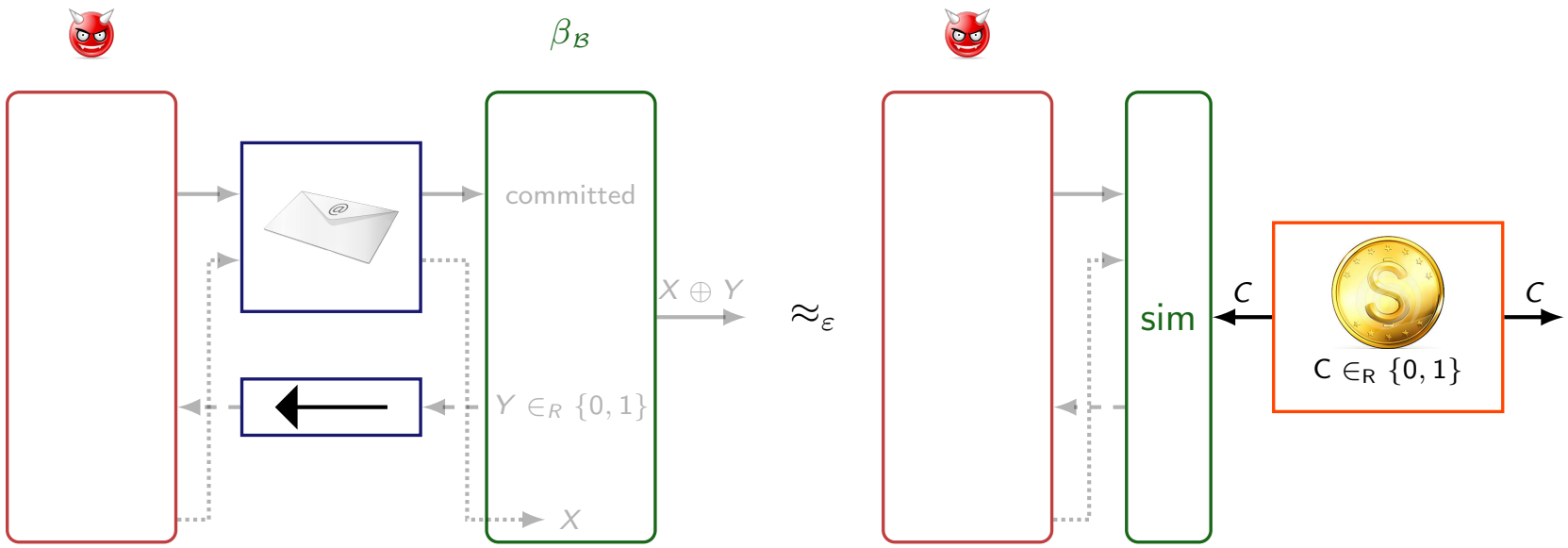
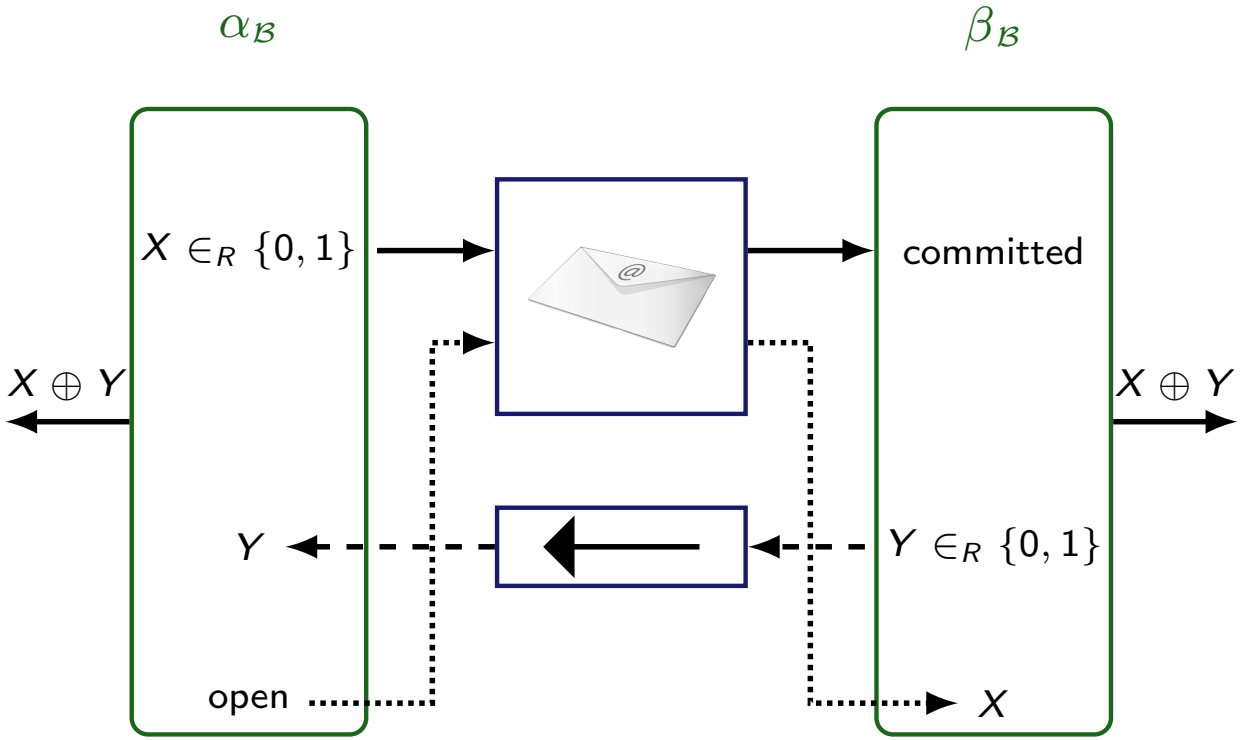


Blum's Coin Tossing Protocol

Malicious Alice - Real World



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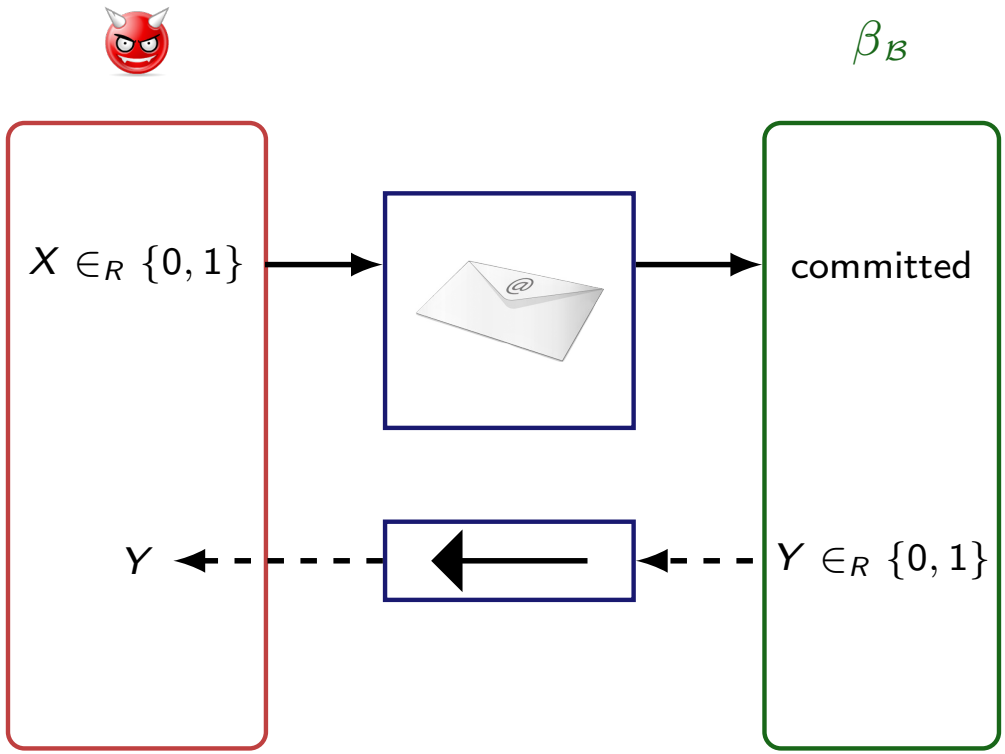

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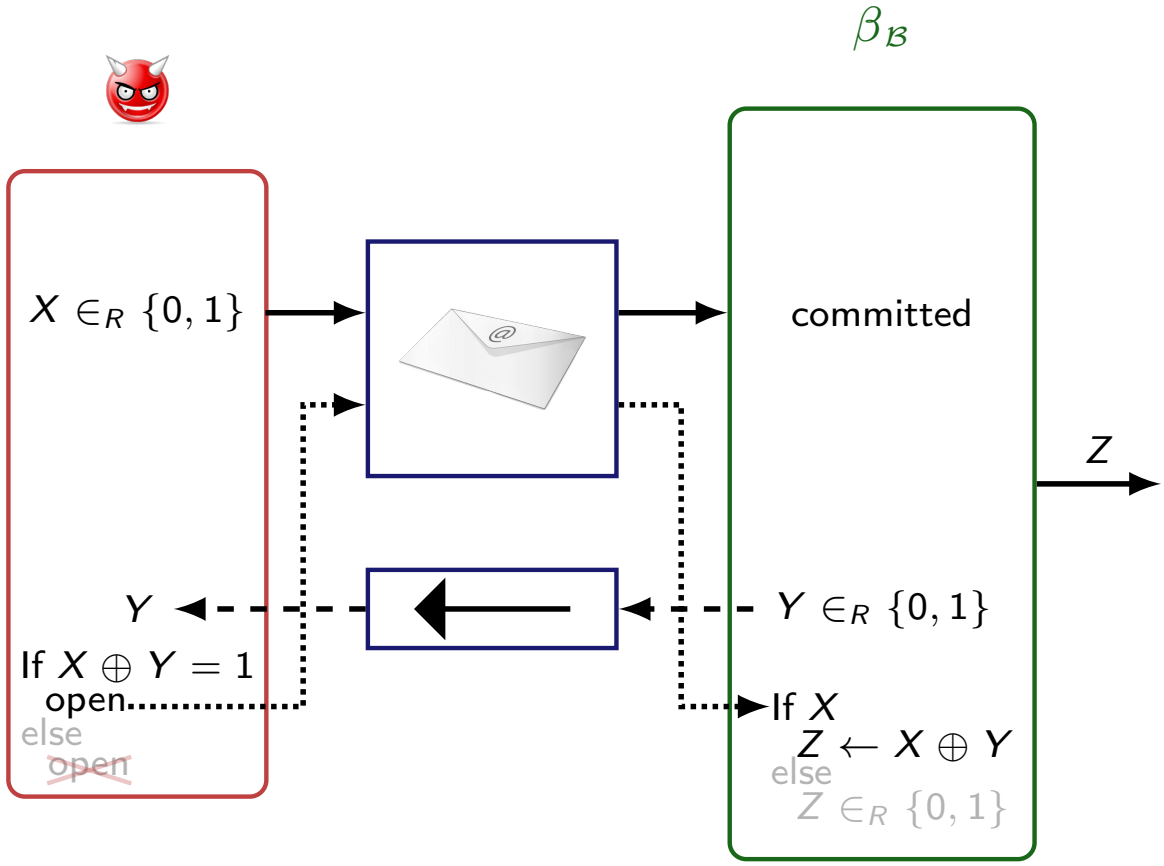

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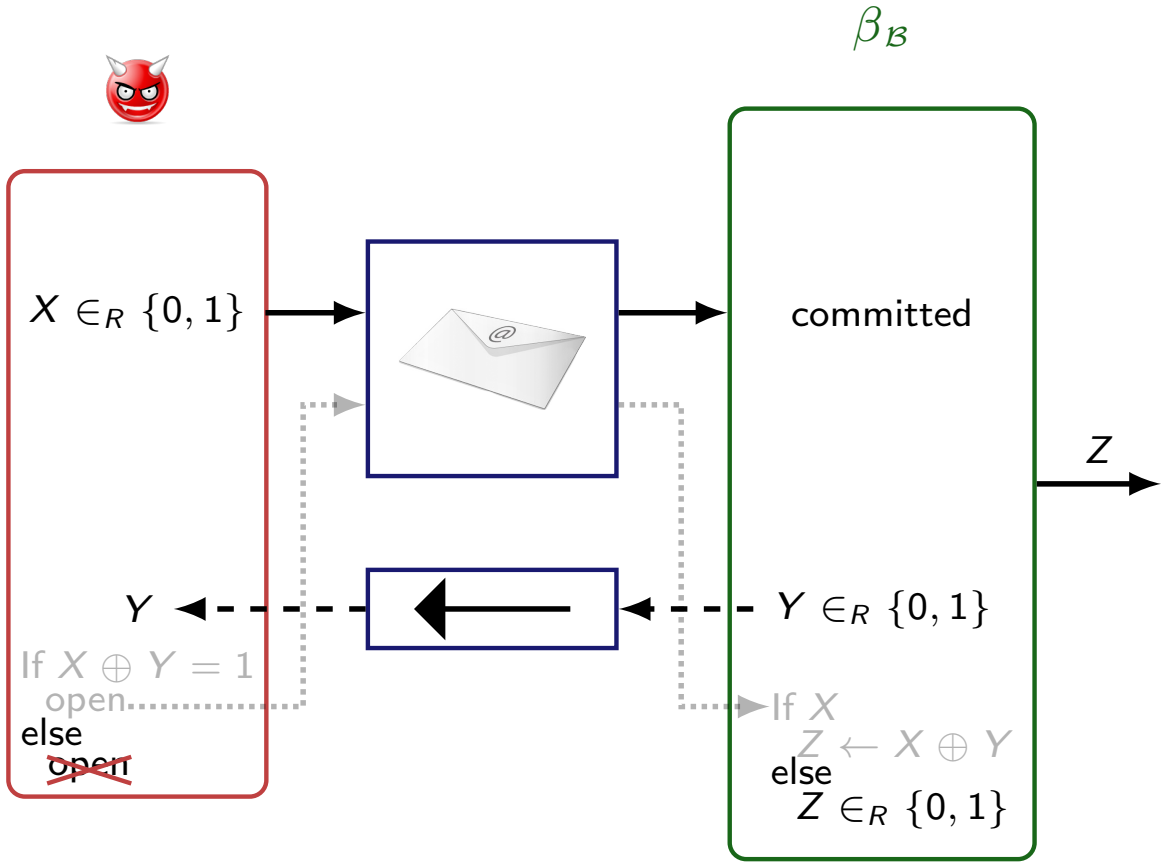

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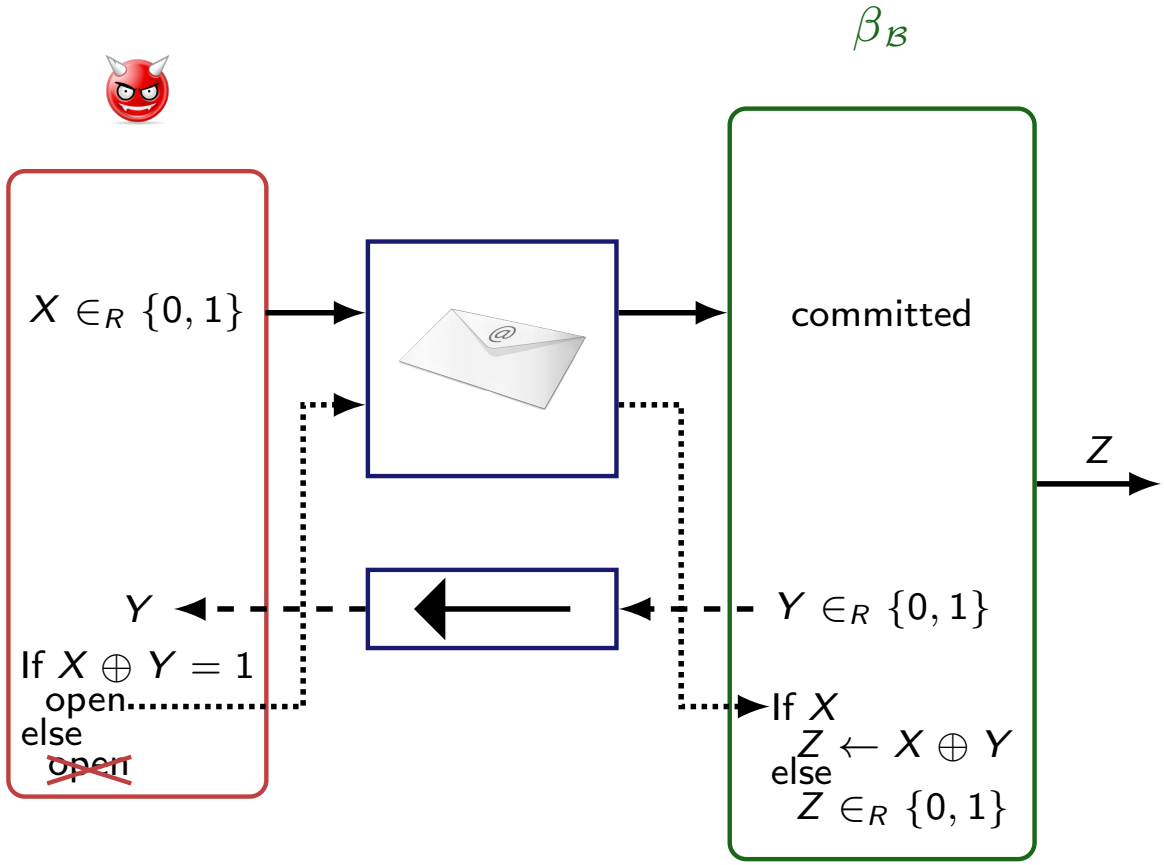

$$\mathrm{P}(Z = 1) = \frac{3}{4}$$

Blum's Coin Tossing Protocol

Malicious Alice - Ideal World

Blum's Coin Tossing Protocol

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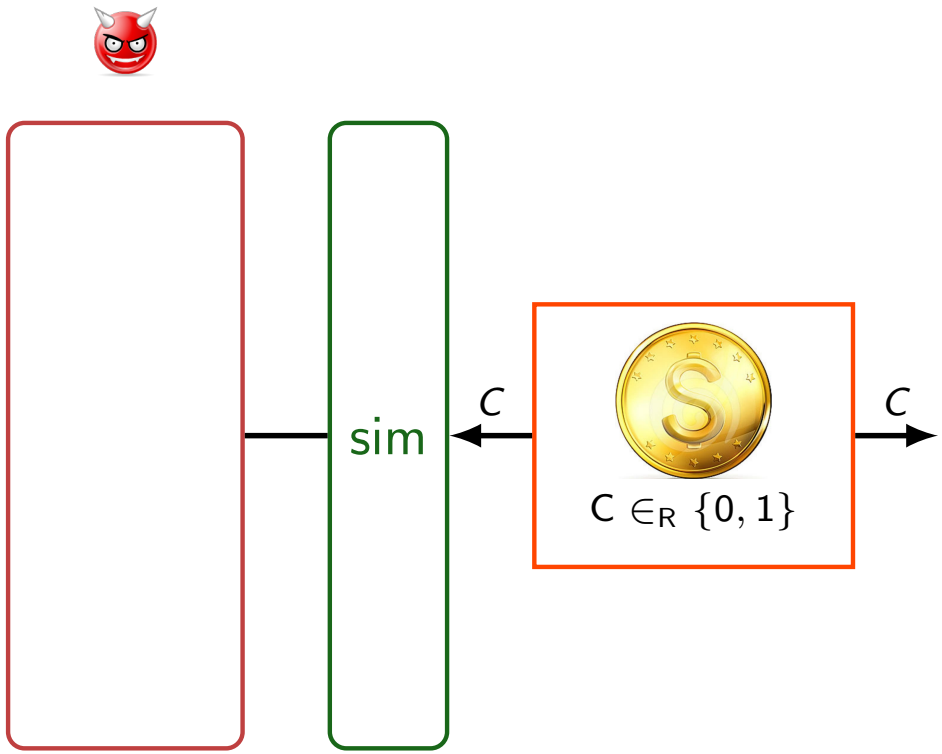

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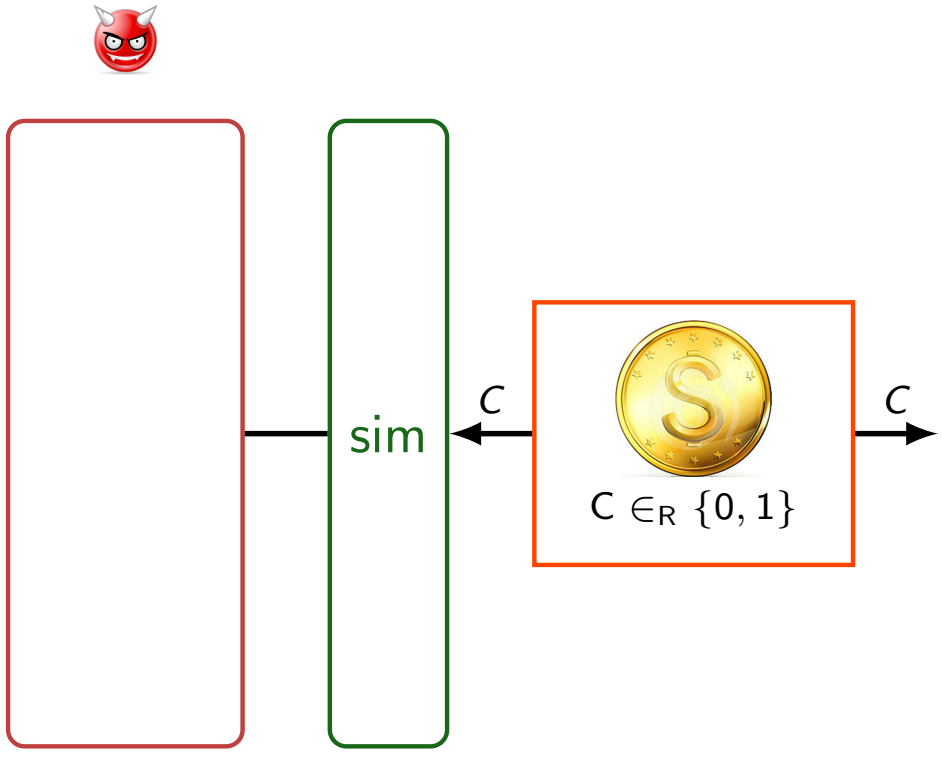
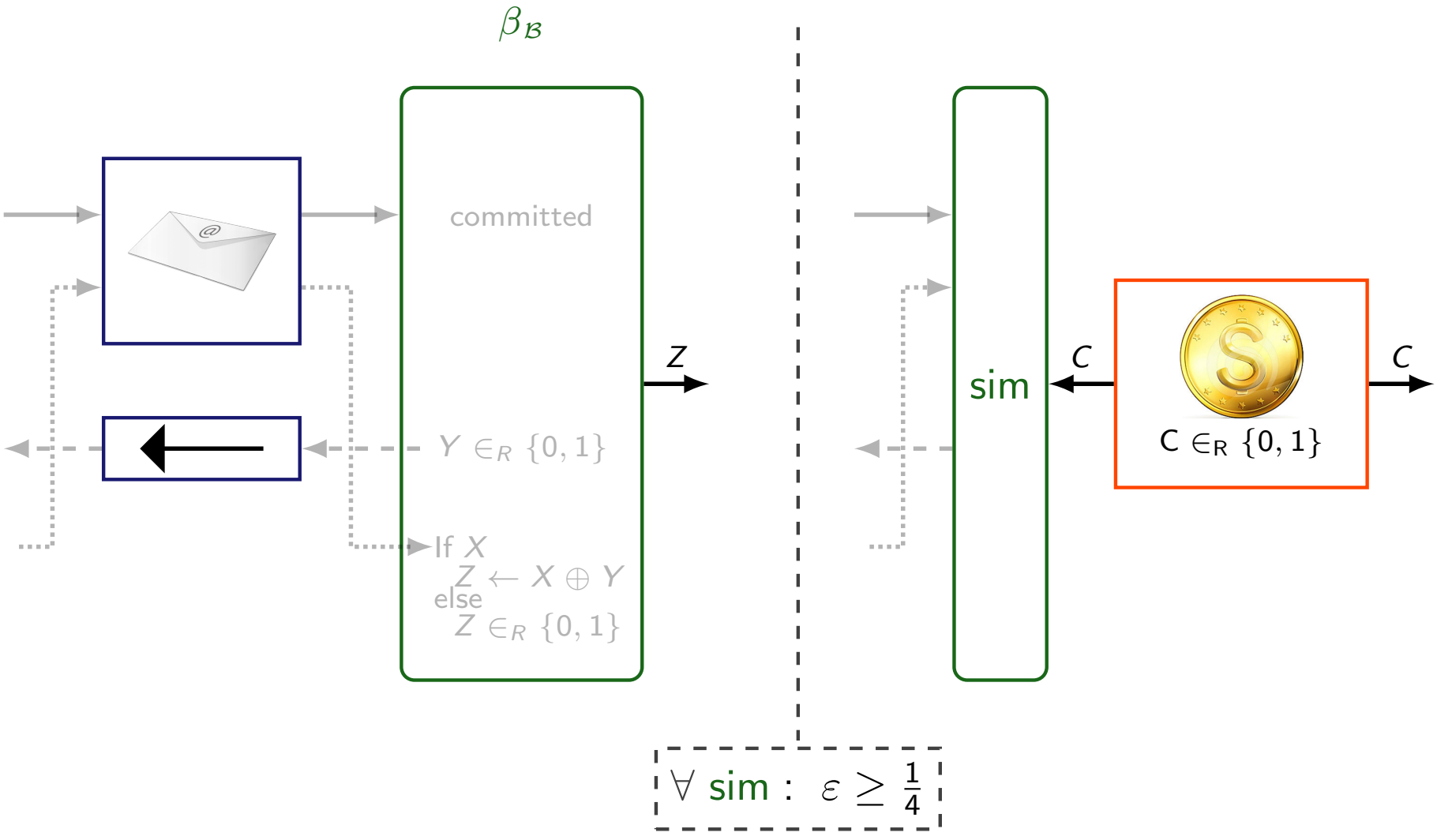


$$\forall \text{ } \forall \text{ sim}: \ \mathrm{P}(C = 1) = \frac{1}{2}$$

Blum's Coin Tossing Protocol

Malicious Alice



Outline

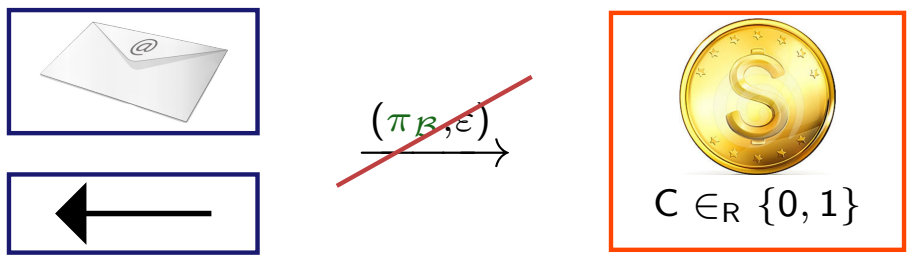
Unfair Coin Tossing

Goal

$\xrightarrow{(\pi_{\mathcal{B}},\varepsilon)}$

Unfair Coin Tossing

(Unachievable) Goal



Unfair Coin Tossing

(Unachievable) Goal

$\xrightarrow{(\pi_{\mathcal{B}},\varepsilon)}$

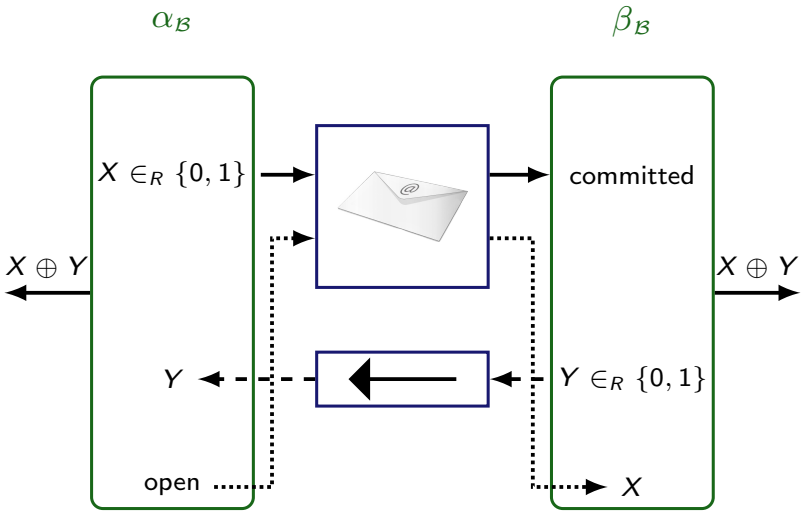
Unfair Coin Tossing

Goal

$\xrightarrow{(\pi_{\mathcal{B}},0)}$

Unfair Coin Tossing Resource

Unfair Coin Tossing Resource



Unfair Coin Tossing Resource

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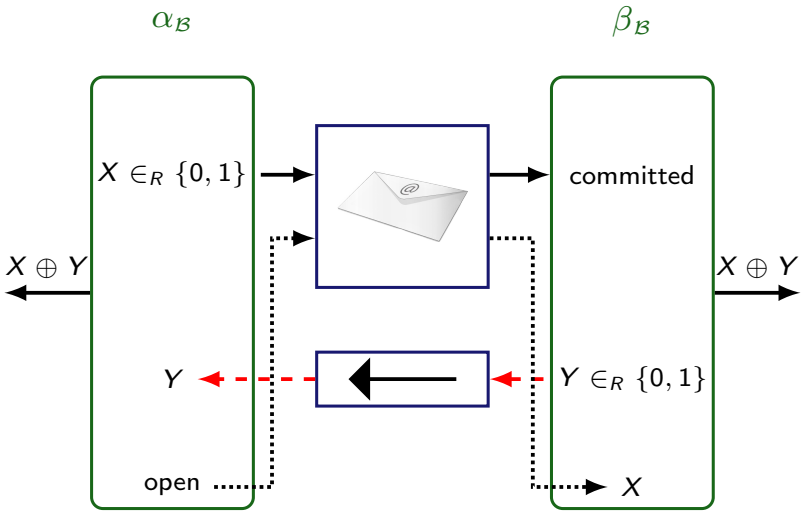

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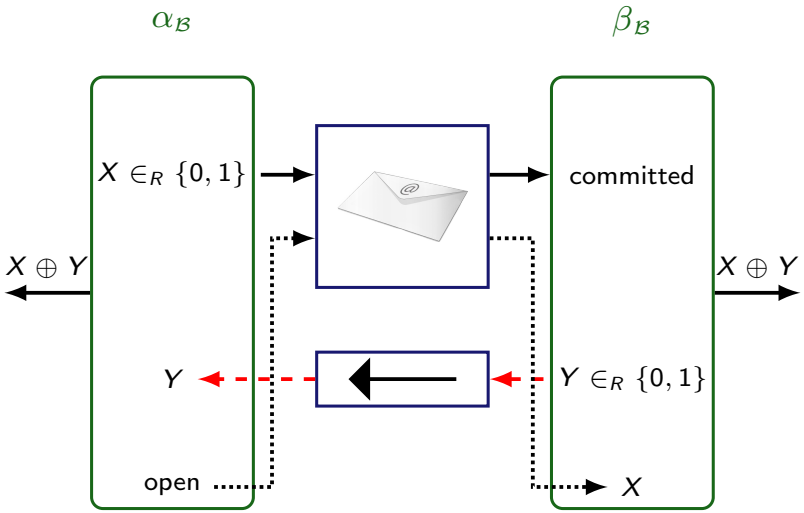

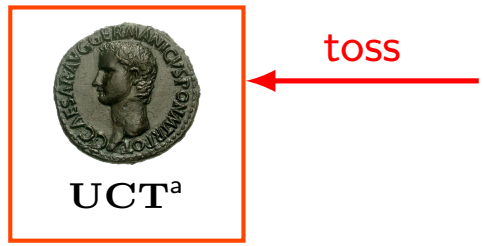

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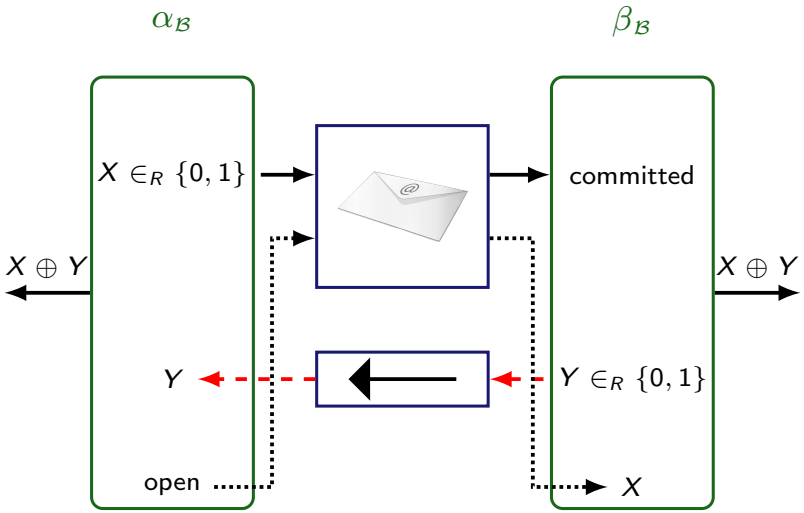

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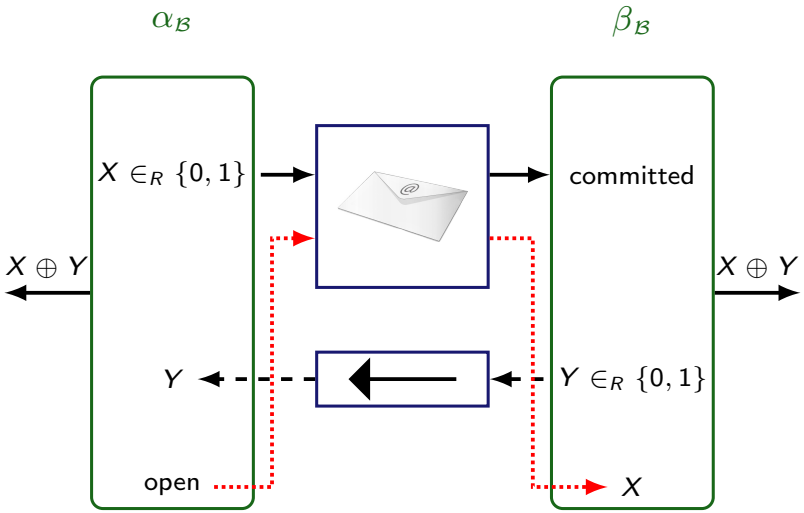

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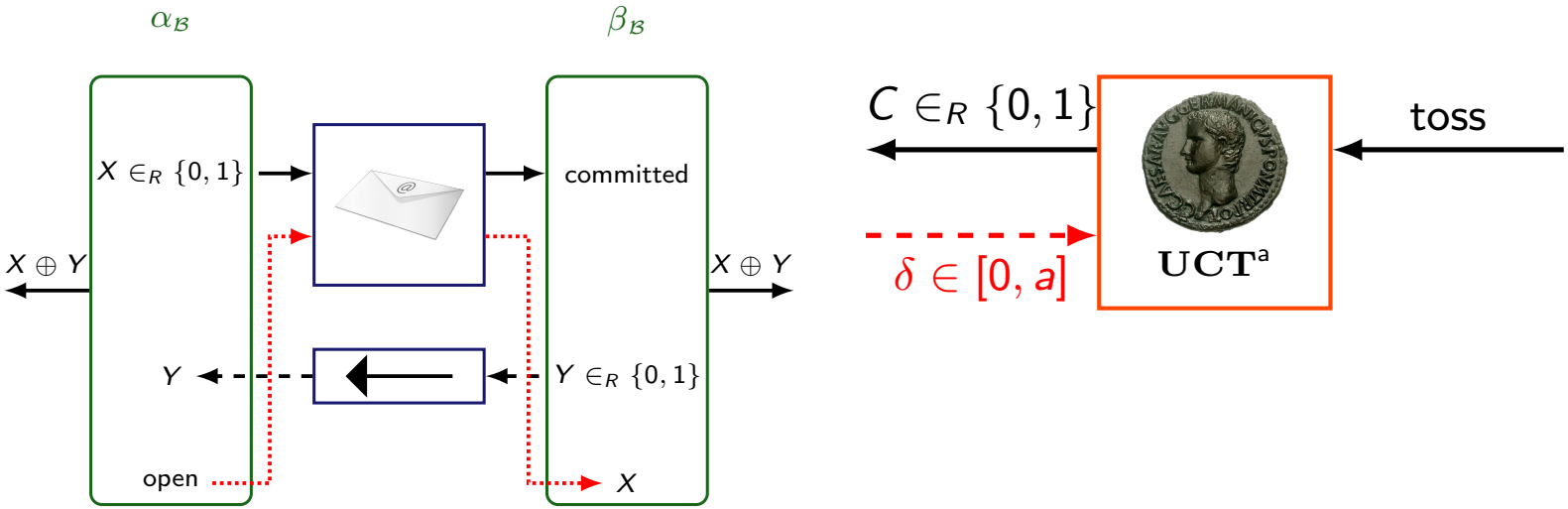

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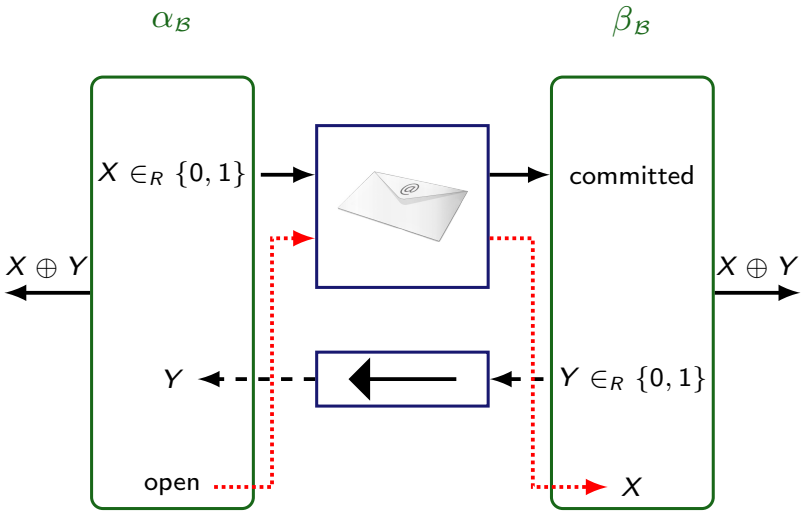

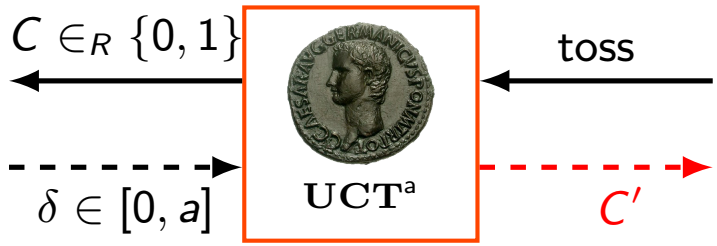

$C' := C \oplus N$, where $N \sim \mathrm{Ber}(\delta)$.

Unfair Coin Tossing Resource

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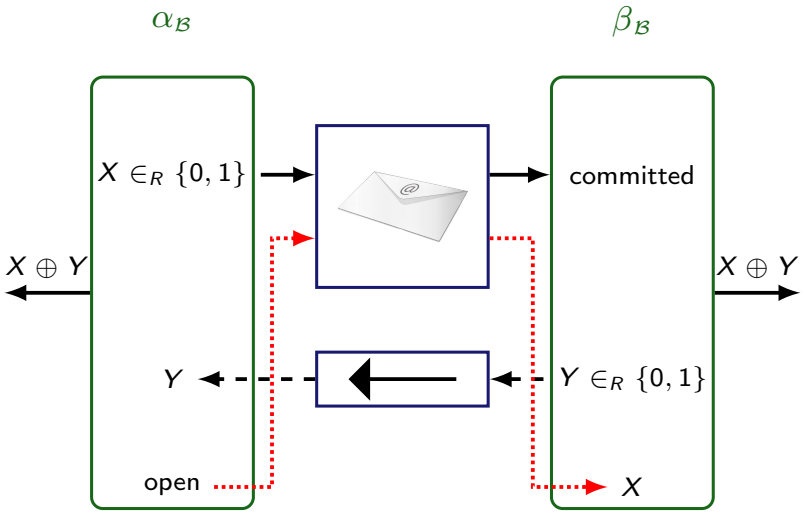
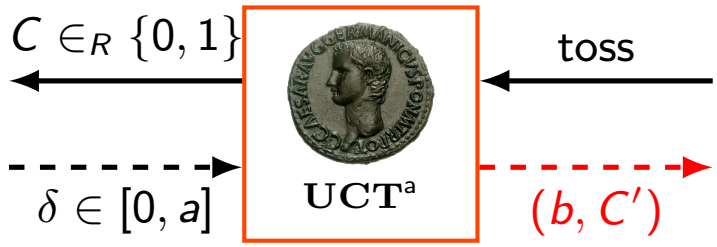




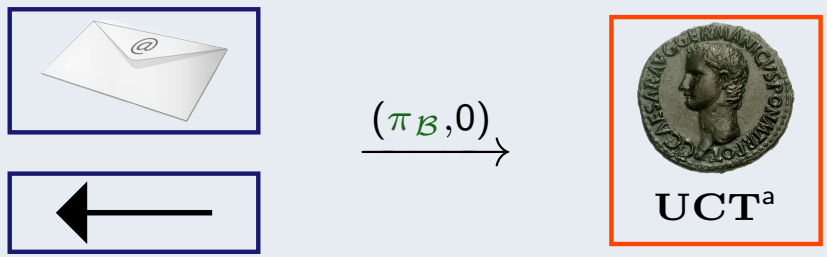
$$b := \mathbb{1}_{\delta \neq 0}$$

$$C' := C \oplus N, \text{ where } N \sim \mathrm{Ber}(\delta).$$

Blum's protocol

Theorem 1

Blum's protocol perfectly constructs a biased unfair coin tossing resource, i.e,

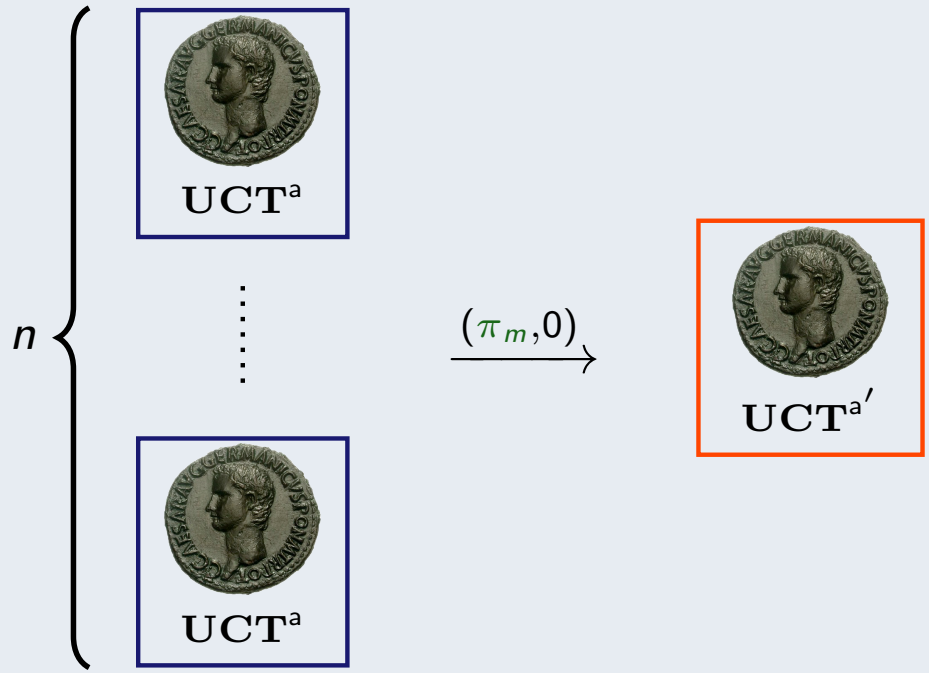


where $a \geq \frac{1}{2}$.

Multiple Unfair Coin Tossing Resources

Theorem 2

There exists a 2-party protocol π_m s.t. for an odd number n of unfair coin tossing resources,



where $a' \approx \frac{2}{\sqrt{2\pi n}} a$.

Outline

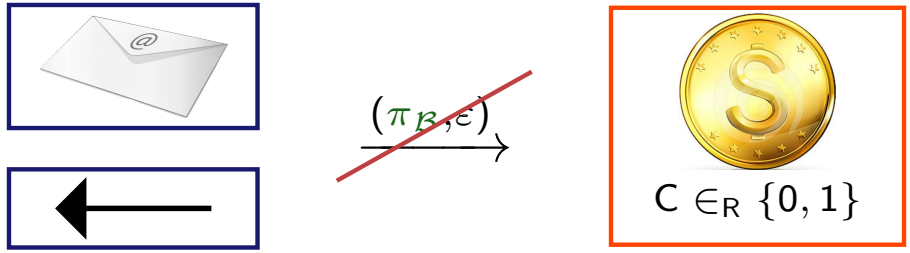
Summary

Unfair Coin Tossing Resource

Summary

Unfair Coin Tossing Resource

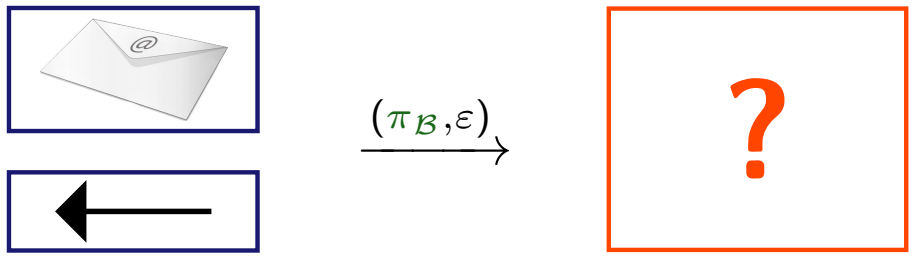
State the Exact Resource Constructed



Summary

Unfair Coin Tossing Resource

State the Exact Resource Constructed



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Unfair Coin Tossing Resource

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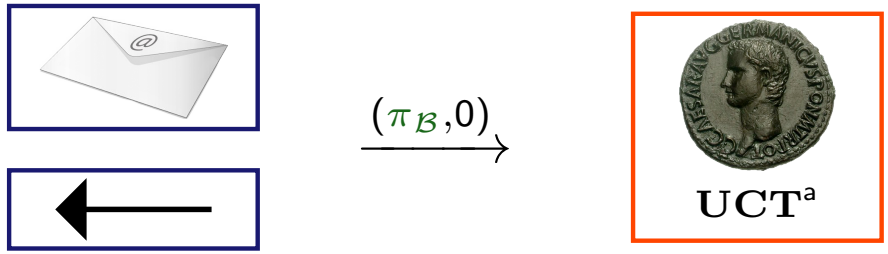

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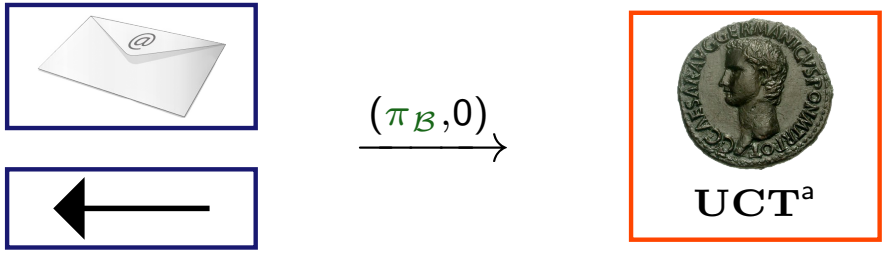
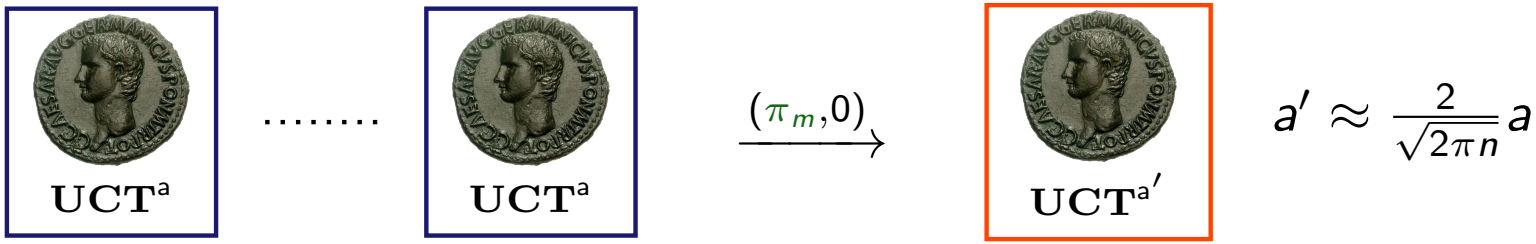


Majority Protocol



Thank You!

Construction Notion for 2-Party

Construction in (Alice, Bob)-setting

For a 2-party protocol $\pi = (\alpha, \beta)$,

$\mathbf{R} \xrightarrow{(\pi,\varepsilon)} \mathbf{S} \Leftrightarrow$

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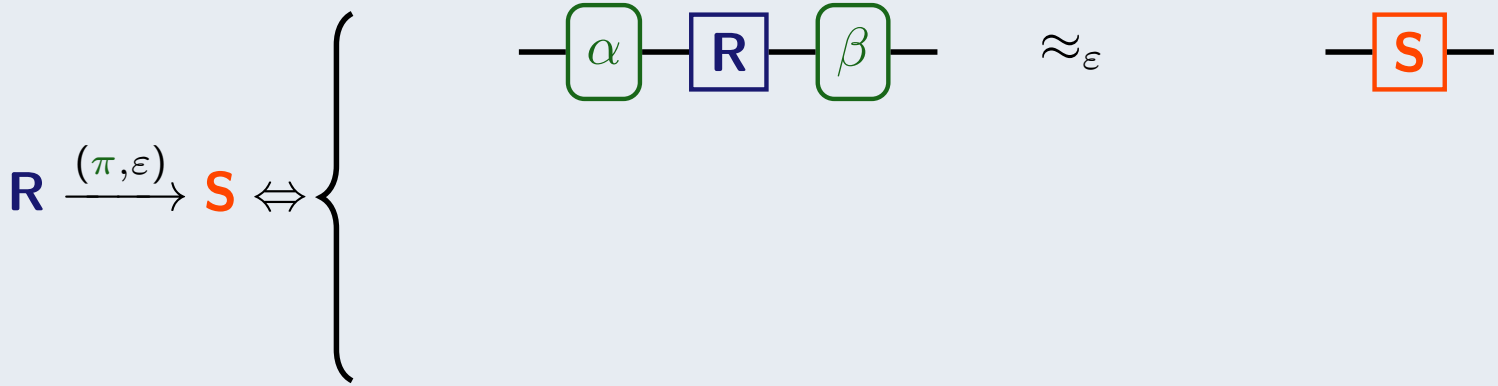

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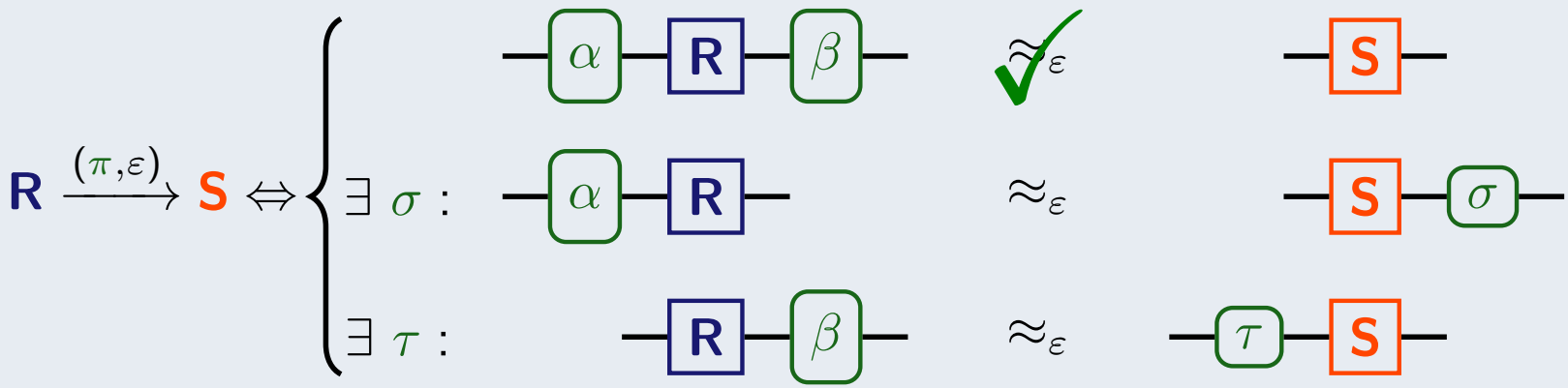

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Blum's Coin Tossing Protocol

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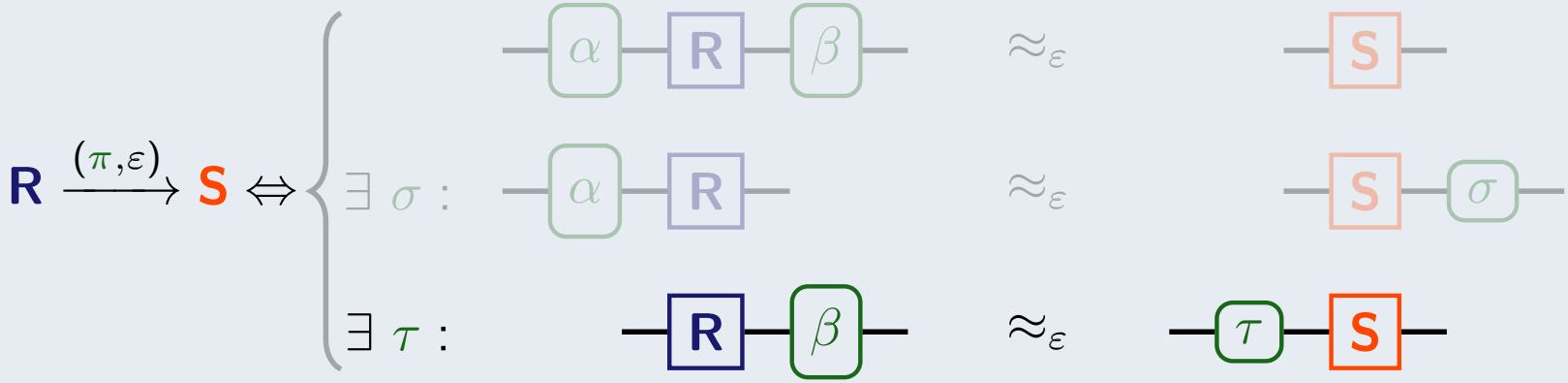

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