Support Vector Machines & Kernels

Doing *really* well with linear decision surfaces

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Adapted from slides by Tim Oates

Outline

- Prediction
  - Why might predictions be wrong?
- Support vector machines
  - Doing really well with linear models
- Kernels
  - Making the non-linear linear

Why Might Predictions be Wrong?

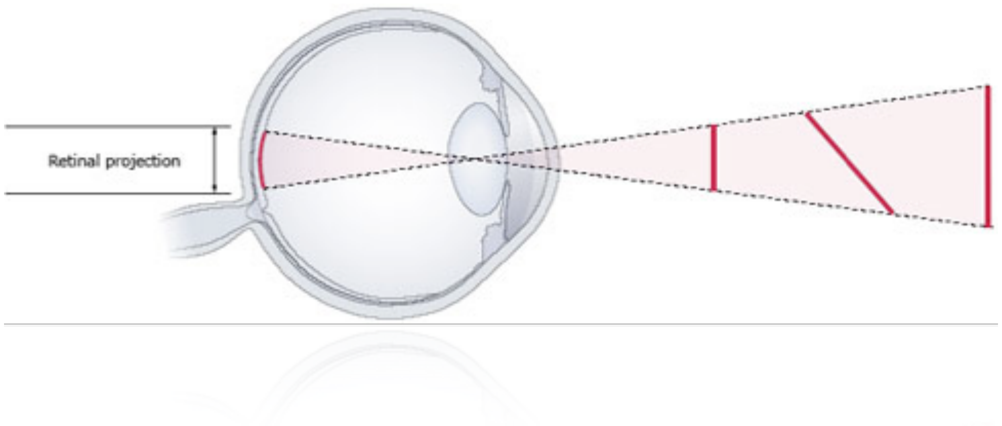
- True non-determinism
  - Flip a biased coin
  - p(heads) = θ
  - Estimate θ
  - If $\theta > 0.5$ predict ‘heads’, else ‘tails’

Lots of ML research on problems like this:

- Learn a model
- Do the best you can in expectation

Why Might Predictions be Wrong?

- Partial observability
  - Something needed to predict y is missing from observation x



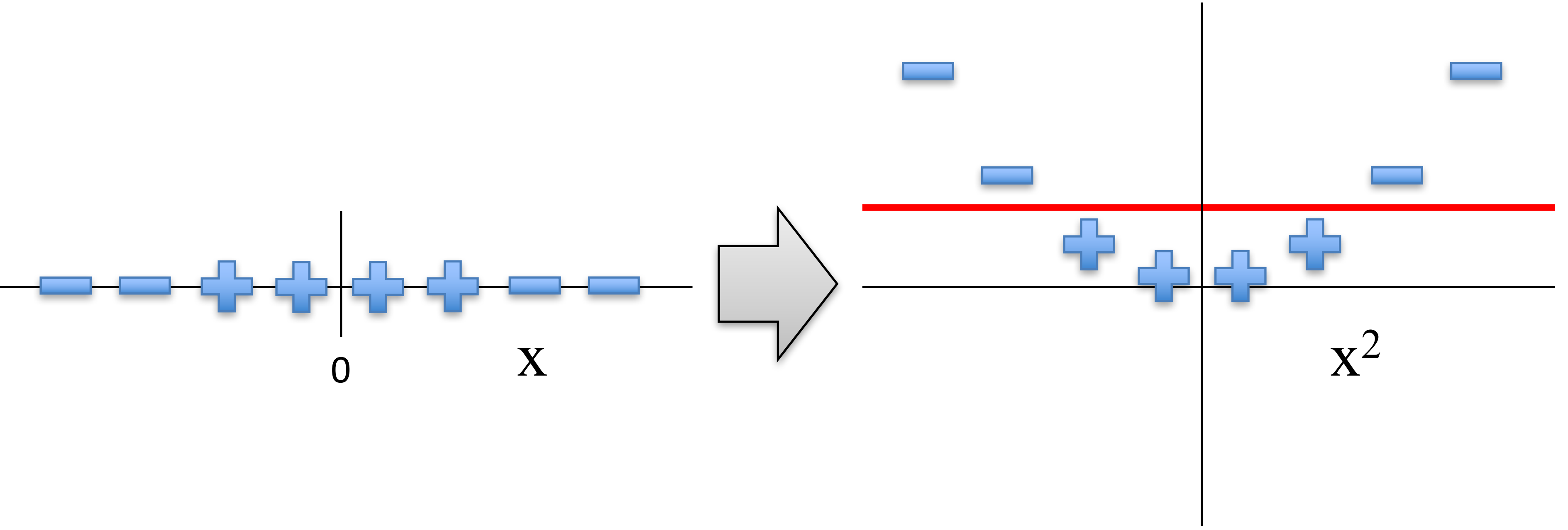
- Noise in the observation x
  - Measurement error
  - Instrument limitations

Why Might Predictions be Wrong?

- True non-determinism
- Partial observability
  - hard, soft
- Representational bias
- Algorithmic bias
- Bounded resources

Representational Bias

- Having the right features (x) is crucial



Support Vector Machines

Doing ***Really*** Well with Linear Decision Surfaces

Strengths of SVMs

- Good generalization
  - in theory
  - in practice
- Works well with few training instances
- Find globally best model
- Efficient algorithms
- Amenable to the kernel trick

Minor Notation Change

To better match notation used in SVMs

...and to make matrix formulas simpler

We will drop using superscripts for the i th instance

i th instance	$\boldsymbol{x}^{(i)}$	$\rightarrow$	$\mathbf{x}_i$
i th instance label	$y^{(i)}$	$\rightarrow$	y_i
j th feature of i th instance	$x_j^{(i)}$	$\rightarrow$	x_{ij}

Bold denotes vector

Non-bold denotes scalar

Linear Separators

- Training instances

$$\mathbf{x} \in \mathbb{R}^{d+1}, x_0 = 1$$

$$y \in \{-1, 1\}$$

- Model parameters

$$\boldsymbol{\theta} \in \mathbb{R}^{d+1}$$

- Hyperplane

$$\boldsymbol{\theta}^\intercal \mathbf{x} = \langle \boldsymbol{\theta}, \mathbf{x} \rangle = 0$$

- Decision function

$$h(\mathbf{x}) = \text{sign}(\boldsymbol{\theta}^\intercal \mathbf{x}) = \text{sign}(\langle \boldsymbol{\theta}, \mathbf{x} \rangle)$$

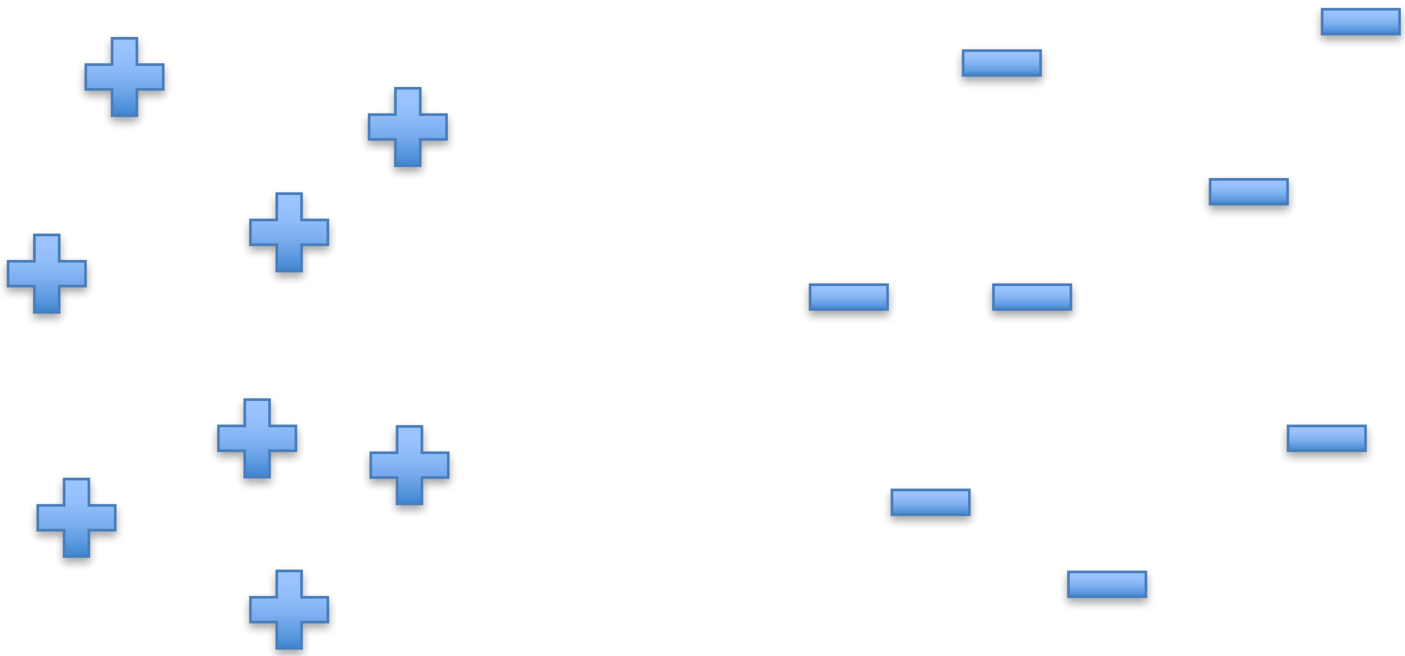
<u>Recall:</u>

Inner (dot) product:

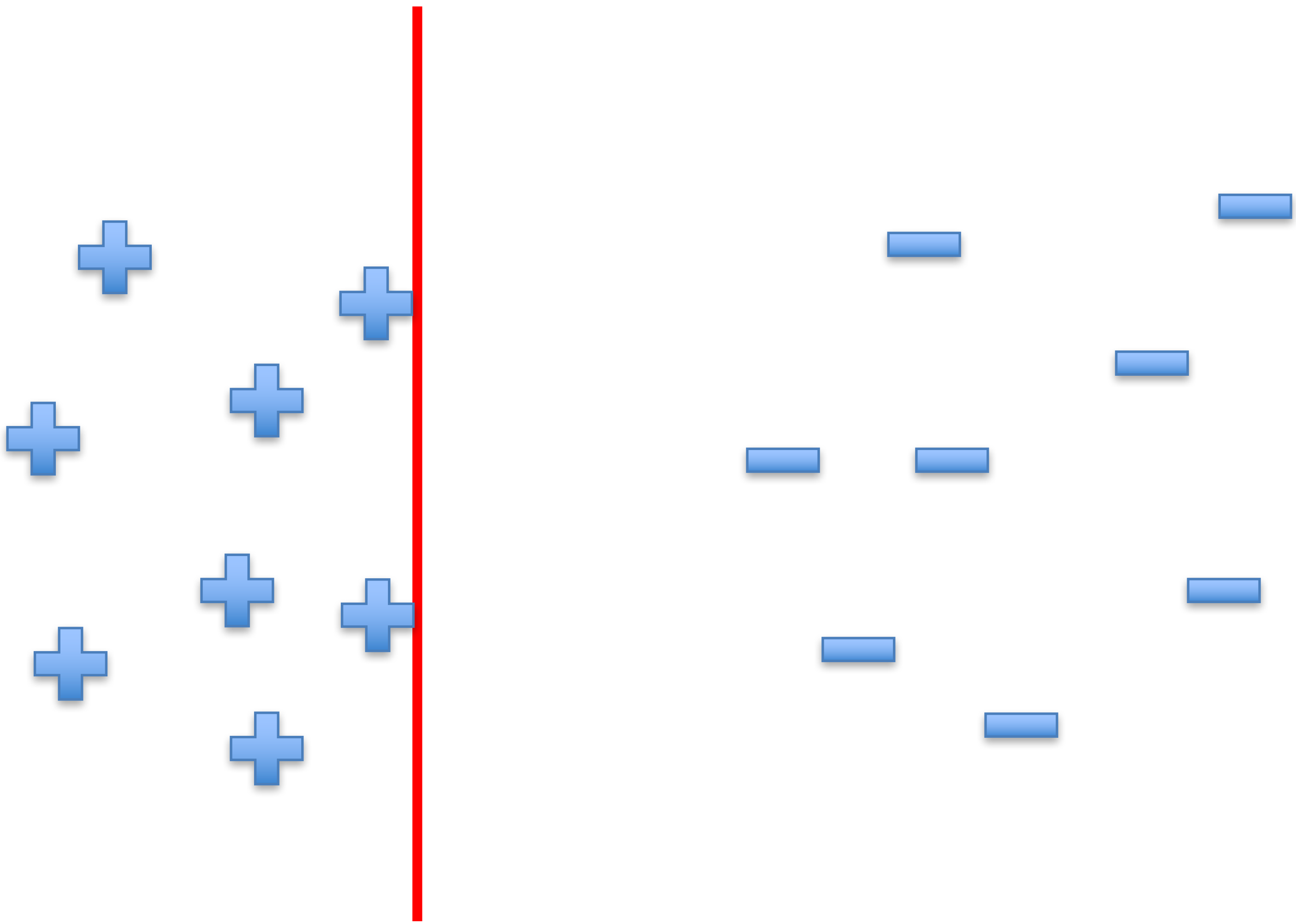
$$\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u} \cdot \mathbf{v} = \mathbf{u}^\intercal \mathbf{v}$$

$$= \sum_i u_i v_i$$

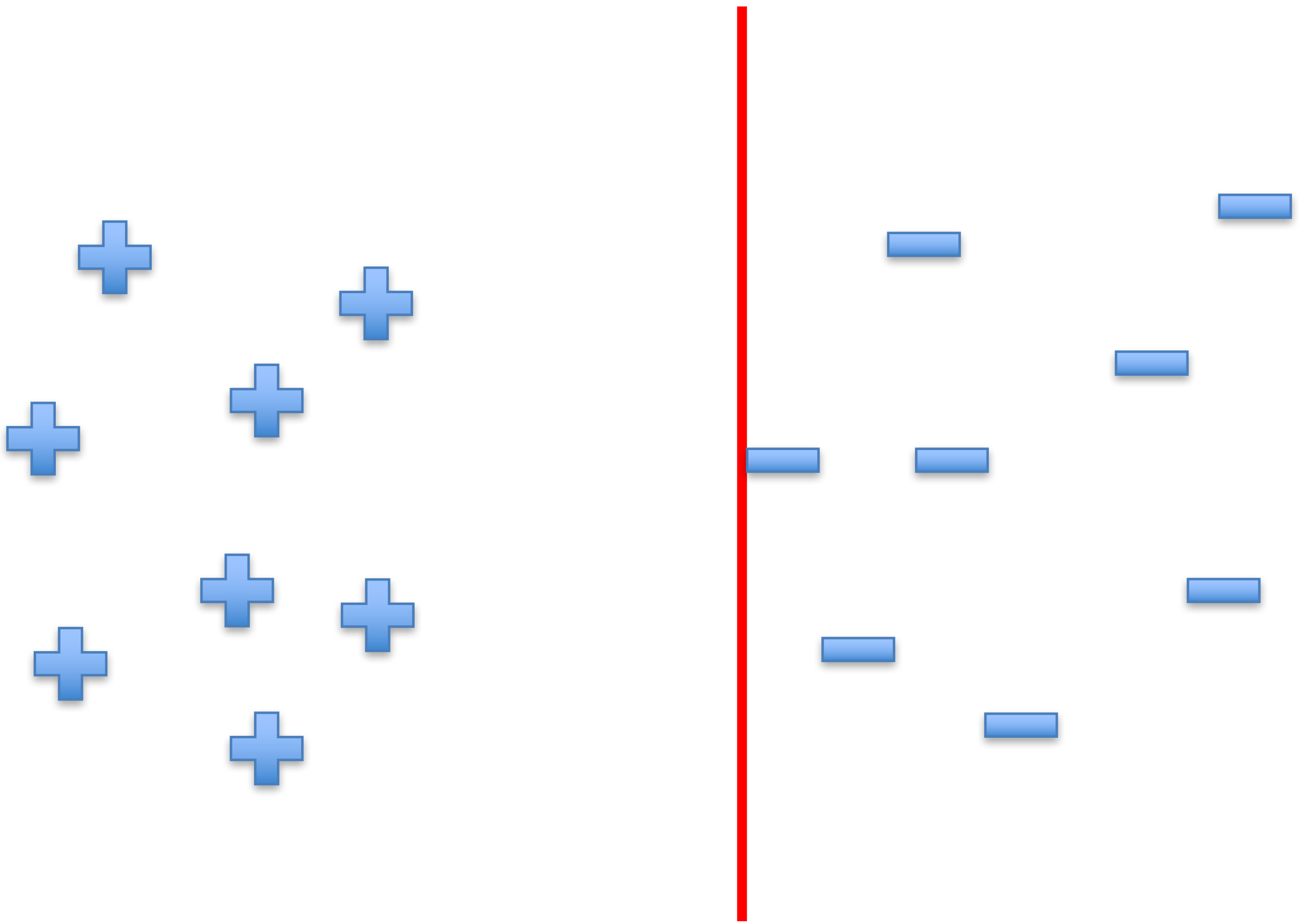
Intuitions



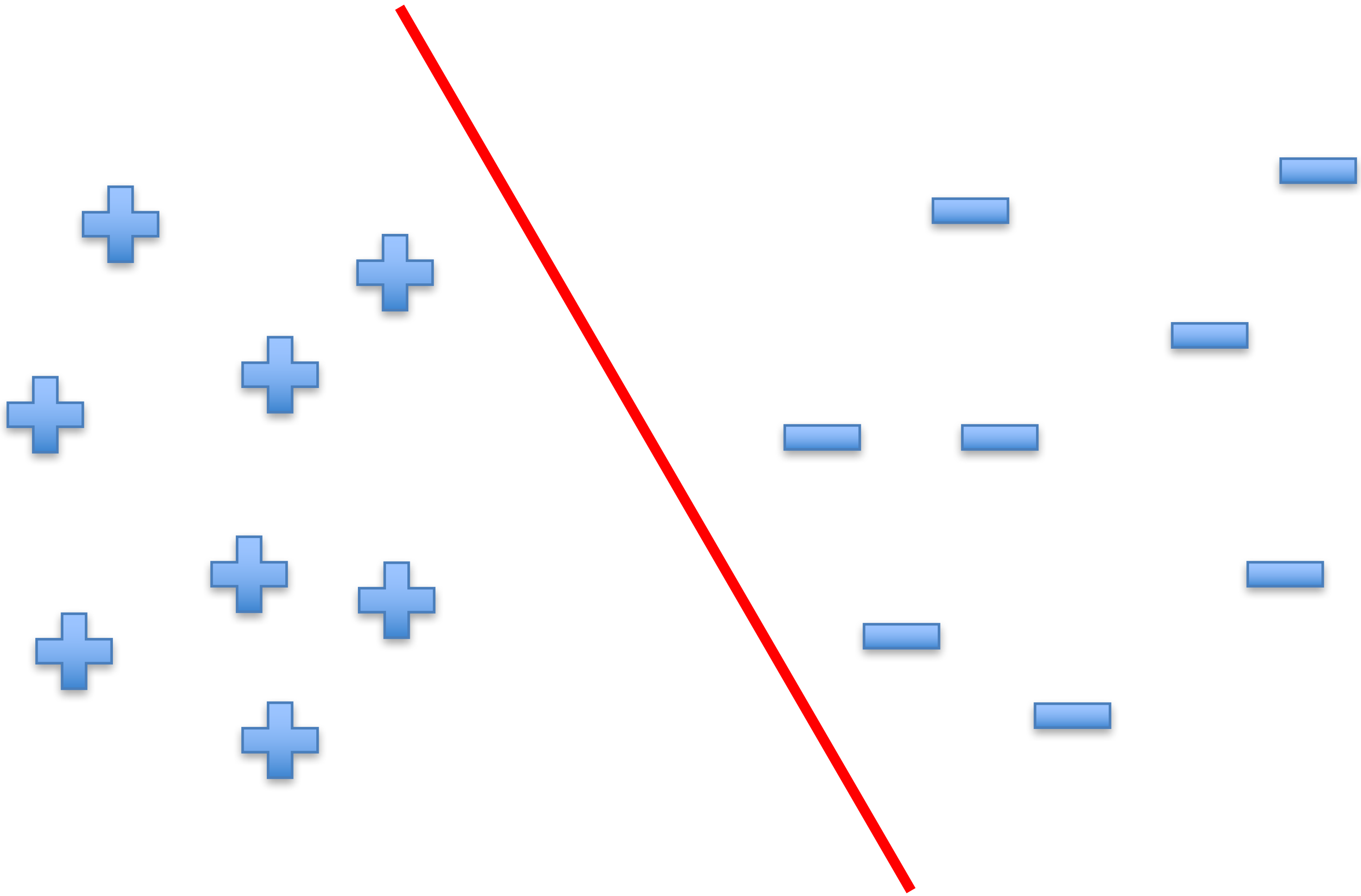
Intuitions



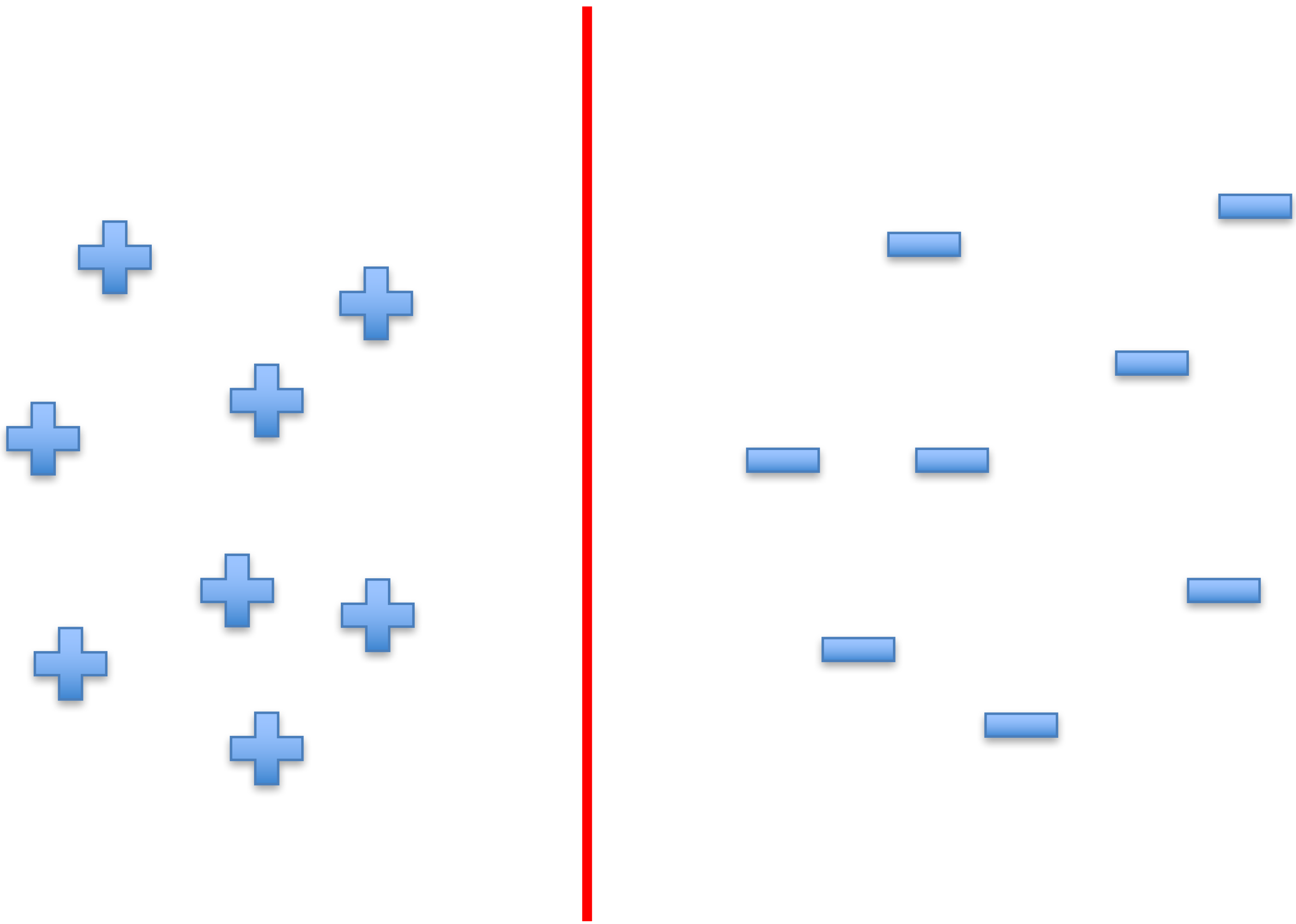
Intuitions



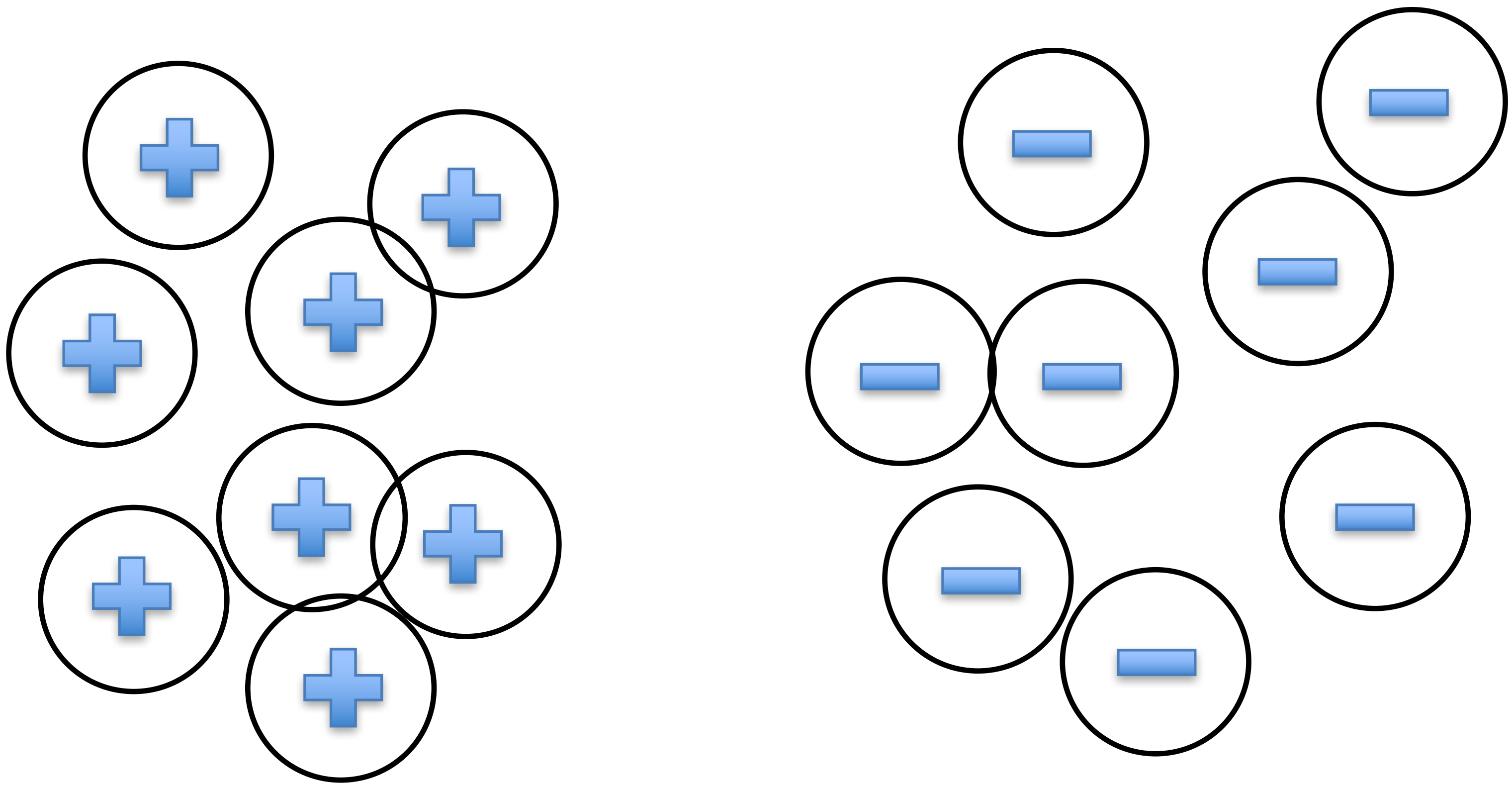
Intuitions



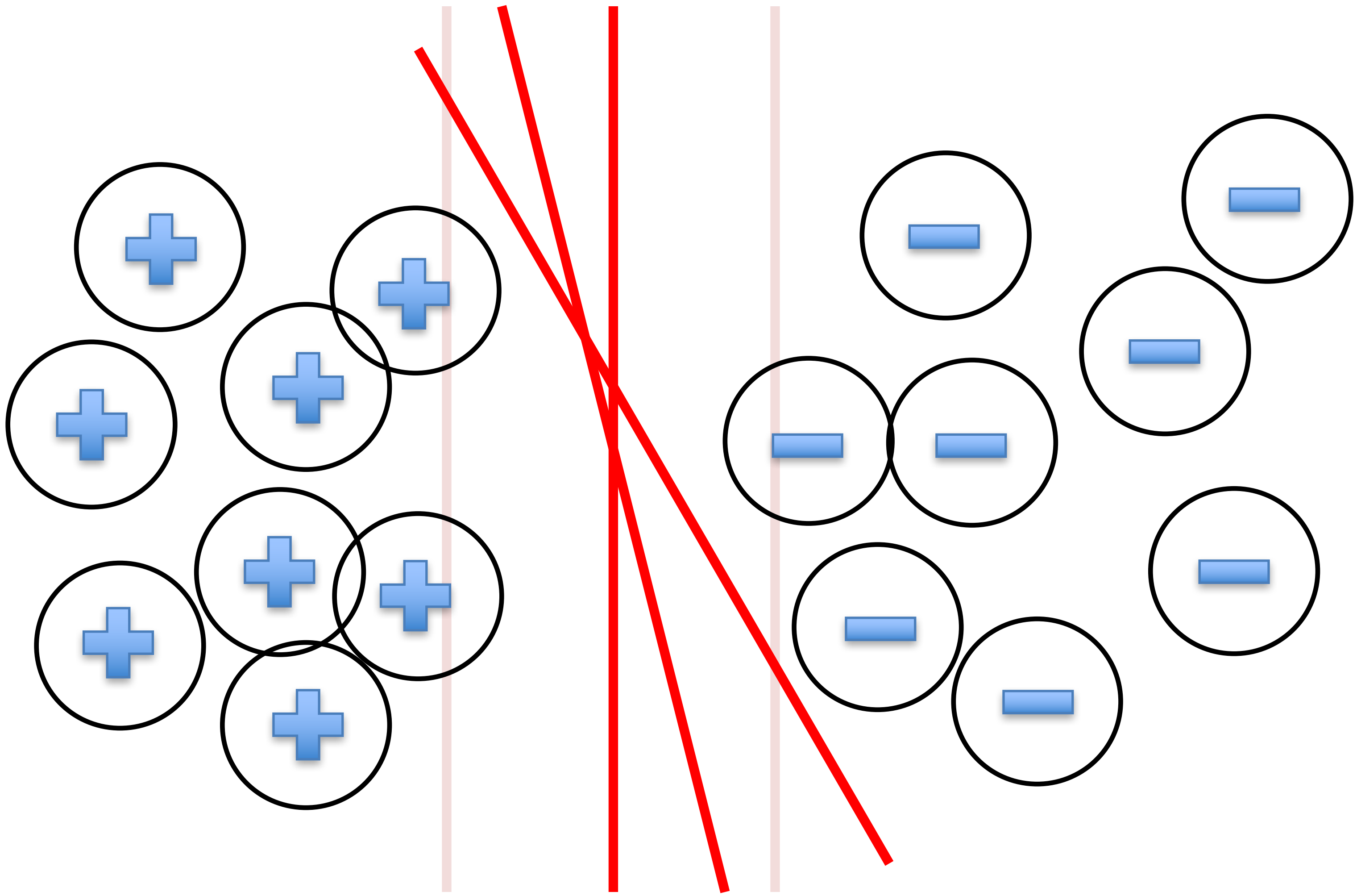
A “Good” Separator



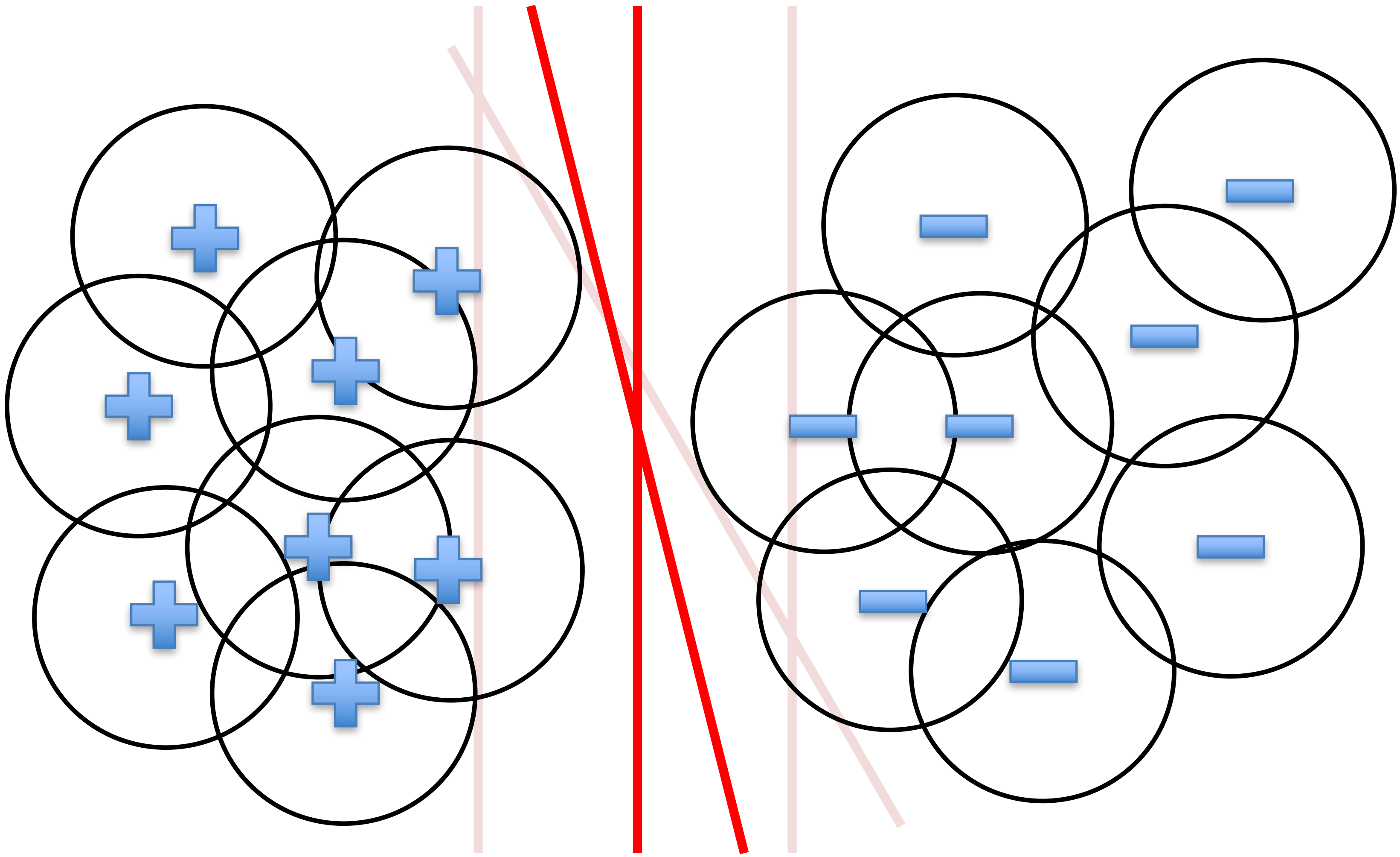
Noise in the Observations



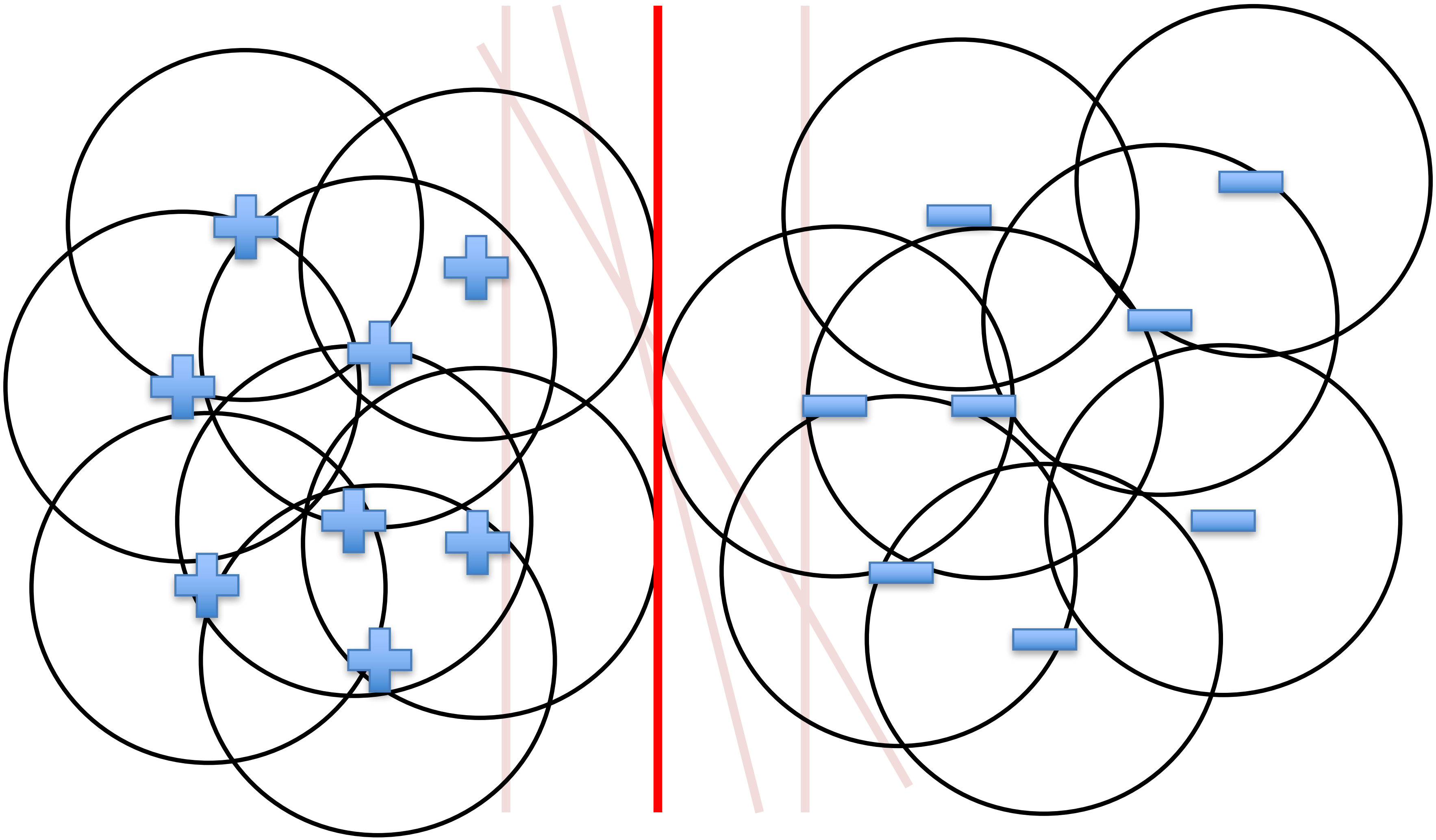
Ruling Out Some Separators



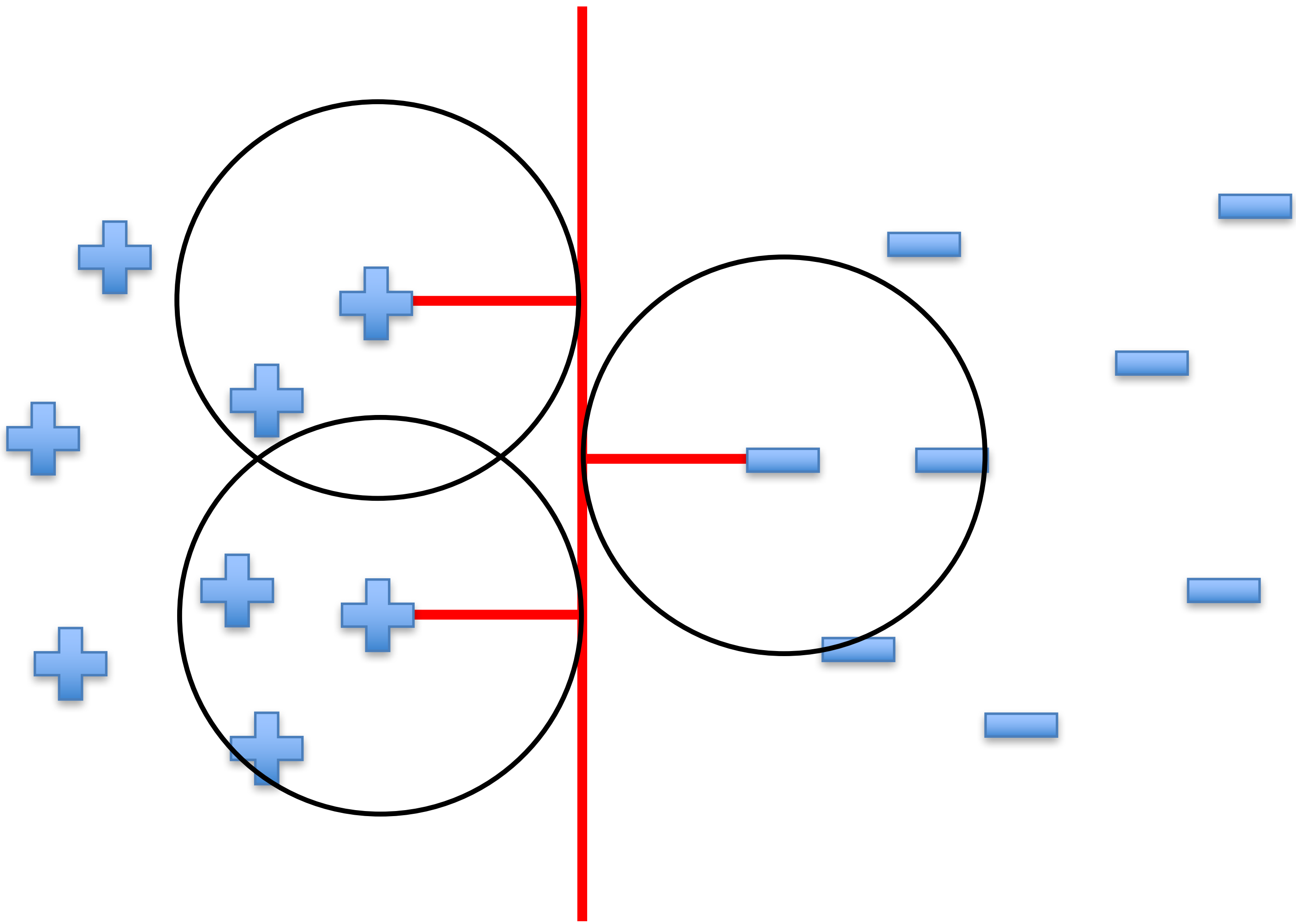
Lots of Noise



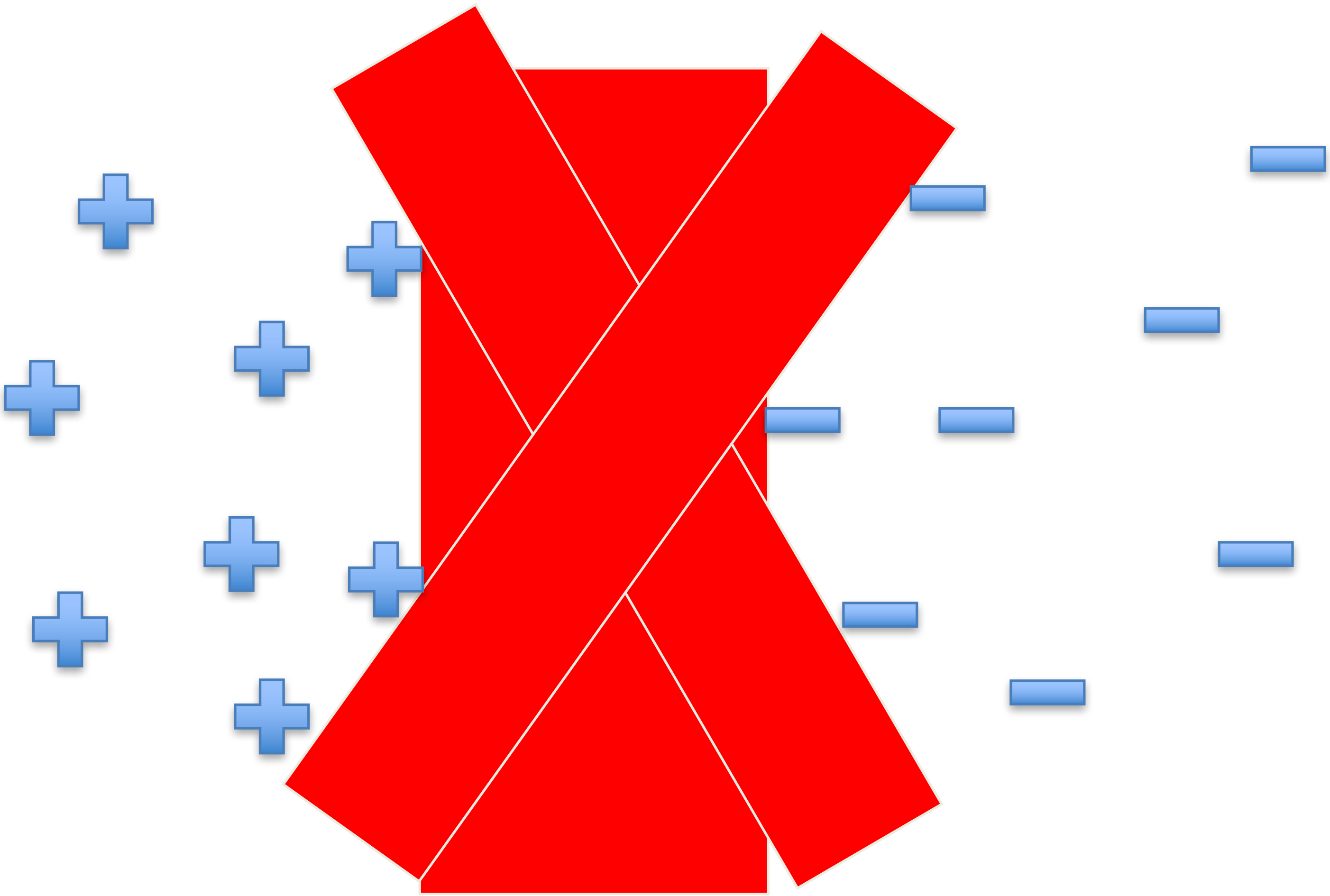
Only One Separator Remains



Maximizing the Margin



“Fat” Separators



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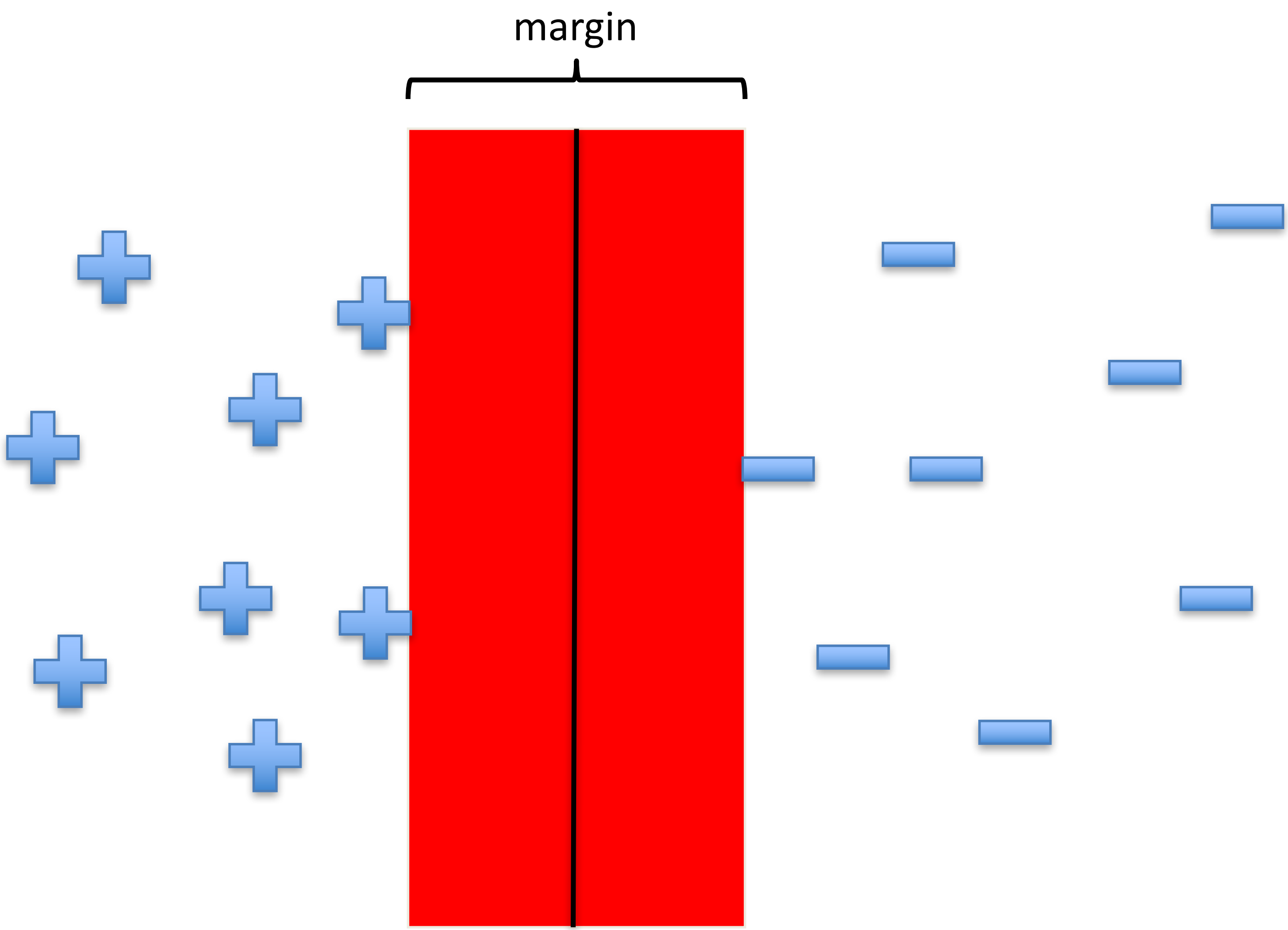

Why Maximize Margin

Increasing margin reduces *capacity*

- i.e., fewer possible models

Remember Lesson from Learning Theory:

- If the following holds:
  - H is sufficiently constrained in size
  - and/or the size of the training data set n is large,

  then low training error is likely to be evidence of low generalization error