The Effect of Temperature on the Spring Constant of a Spring

1. Research Question:

How does the spring constant of an initial 39.6±0.3 N/m spring depend on the temperature of the spring (50°C, 45°C, 40°C, 35°C, 30°C, 25°C, 20°C) as measured by analyzing the displacement of the spring caused by a 1kg mass?

1.1 Introduction:

My interest in this subject stemmed from when I was young when my parents bought a trampoline in my backyard and I would spend countless hours jumping around on the trampoline as a pass time. However, growing up in Canada, the temperature drastically shifted over the course of the year and I noticed that my trampoline would become stiffer/harder to jump on when it was cold and springier/easier to jump on when it was hotter. Upon learning about the spring constant being what determines the stiffness of a material in the IB Physics Syllabus, my head immediately went back to my childhood as I recollected the days when I would be confused as to why I was not jumping as high as I normally would. Therefore, I decided to investigate this idea in this internal assessment, determining how temperature may affect the spring constant of a spring.

2. Background Information:

2.1 Hooke's Law

The Spring Constant of a Spring and Temperature are the focus of this IA. First, to understand the Spring Constant, we need to understand Hooke's law which states that the displacement of a spring is proportional to the force applied to the spring.
This relationship is visually represented in the diagram below.

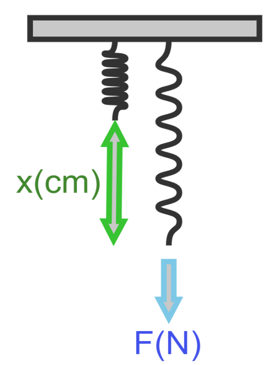


Figure 1: A visual representation of displacement as a function of Force according to Hooke's Law (chemix)

Hooke's law can also be represented mathematically using the equation below where F is the restoring force of the spring, k is the spring constant and x is the displacement of the spring's length as a result of the extension caused by the force.

$$F = - kx \quad (1)$$

The spring constant (k) in Equation 1 is different for every object since by definition it is the amount of force required to stretch an object by one meter. The larger the spring constant, the stiffer it is and the smaller the spring constant, the more stretchy it would be.

2.2 Measuring Spring Constant

To measure the Spring Constant, I need to control the Force Applied. To do such, I will use the force of gravity accompanied by a 1kg mass. By replacing the Force in the Hooke's Law equation and rearranging the variables, we derive the equation below.

$$k = \frac{mg}{x} \quad (2)$$

The negative sign in front of the kx will be removed since the force of gravity is the applied force on the spring while the force in the equation is the opposite. By definition, it is the restoring force of the spring, acting in the opposite direction of the applied force, hence why the negative sign is being removed.

2.3 Effect of Temperature

When considering the effect of temperature, it is important to note what temperature normally does to an object. Every object is composed of molecules. The temperature of an object in Kelvin is directly proportional to the average kinetic energy of molecules. Therefore, when I heat the spring, the inner molecules of the spring will move at a more rapid pace, and hence the spring will be easier to stretch out resulting in a lower spring constant. When spring constant vs temperature is graphed, this would look like a negative linear graph as greater temperatures would make the spring constant even lower. It is important to note that there is no theoretical equation to model this relationship which will make these results unable to determine the complete accuracy. In this investigation, I was limited by the range of temperatures I was able to use since I didn't have a heat gun and using a hair dryer did not produce my desired temperatures. Hence, I decided to use a hot water bath which heated the spring to a maximum of 50°C as shown in my independent variable range. The minimum at 20°C was chosen to be used as a base since normally spring constant is measured at room temperature and all subsequent measures were compared to this base.

2.4 Hypothesis

If the temperature of the spring is increased, then the spring constant of the spring will decrease since with higher temperature, particles in the spring move at a more rapid pace, allowing the spring to stretch out further and lowering the spring constant since it is inversely proportional to the spring's displacement as shown in equation 2.

3. Methodology

3.1 Variables

Independent Variable	Temperature of the metal spring with a ±0.1°C uncertainty (50°C, 45°C, 40°C, 35°C, 30°C, 25°C, 20°C). This range was chosen due to the hot water bath being only able to reach a maximum of 50°C in the spring and room temperature being the opposite extreme to serve as a basis for comparison
Dependant Variable	The displacement of the spring caused by the weight of a 1 kg mass, which will be used to determine the spring constant.

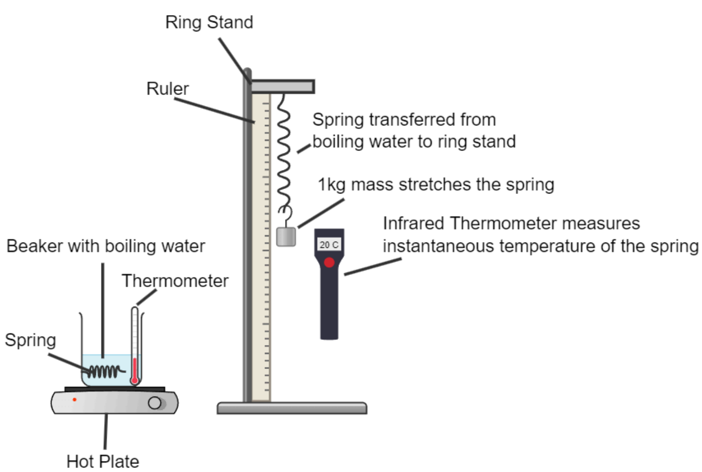
Control Variables	**Relevance**	**Method of control**
Apparatus	A shifting apparatus, especially the meter stick will result in greater random uncertainty.	Secure the ruler and the entire apparatus together using tape and ensure nothing is able to shift.
Spring	Each spring has a different spring constant.	Use the same spring for every trial.
Mass of the weight	Using different weights might have slightly different masses which would result in greater random error.	Use the same weight for every trial.
Temperature of spring	The top, middle and bottom of the spring can have different temperatures so it's hard to precisely measure the temperature of the entire spring.	Measure the top, middle, and bottom of the spring and average the values to gain a reasonable value for the temperature of the entire spring.

3.2 Apparatus

● Meter stick (±0.1cm) ● Hot plate ● Mercury thermometer	● Beaker tong ● Ring stand + ring ● Mass (±0.01 kg)

• 1 Infrared Thermometer • Beaker (1 L)	• Metal spring • Water (~300 mL) • Safety goggles

3.3 Diagram



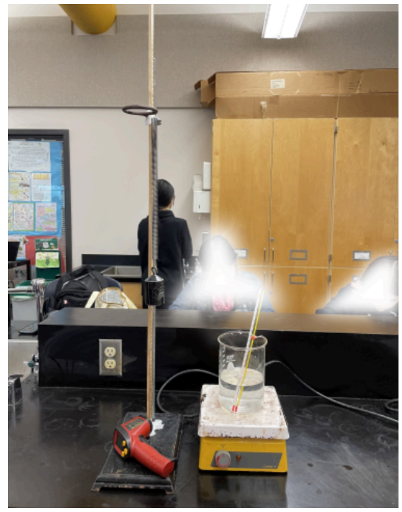


Figure 2: Apparatus used in this investigation

3.4 Methodology

1. Boil 300 mL of water inside a 1 L beaker and place a spring inside the beaker. Monitor the temperature with a thermometer.
2. Once the water starts to boil, remove the spring using beaker tongs, hook it onto the ring stand and wait for the spring to cool to around 50°C.
3. Start a voice recording to say the temperatures and heights immediately after they are recorded to minimize time for the spring to cool.
4. Record out loud the temperature of the top, middle and bottom of the spring using the infrared thermometer to ensure accurate temperature distribution.
5. Record out loud the initial height of the bottom of the spring with the meter stick.
6. Place a 1 kg mass on the spring and record out loud the final height of the bottom of the spring with the meter stick to determine the spring's displacement.
7. Reanalyze the voice recording and record the data points onto a data table.
8. Repeat steps 1-5, but wait for the spring to cool to around 45°C, 40°C, 35°C, 30°C, 25°C, 20°C.
9. Repeat the experiment 5 more times to increase the reliability of the data

3.5 Risk Assessment:

This experiment does not entail any environmental or ethical concerns as no harmful chemicals or wasted materials were used. However, since the experiment involved boiling water using a hot plate, safety goggles were used as a precaution. Beaker tongs were also used to get the spring in and out of the boiling water. The spring also reaches high temperatures (around 50°C) so the beaker tongs were also used to transfer the spring onto the ring stand. The electrical socket the hot plate is plugged into may be a hazard as it may cause a shock if exposed to water. To avoid any risk of shock, the socket was placed far from where water was used.

4. Data and Observations:

Table 1: Raw data of the temperature of the top, middle and bottom of the spring and displacement

Trial	Temperature at top (°C±0.1)	Temperature at middle (°C±0.1)	Temperature at bottom (°C±0.1)	Start Point (cm±0.1)	End Point (cm±0.1)	Spring Displacement (cm±0.2)
1	49.7	52.0	51.3	48.3	73.5	25.2
	45.8	45.7	44.9	48.3	73.3	25
	40.4	40.3	39.6	48.3	73.3	25
	36.2	34.9	35.1	48.3	73.2	24.9
	31.7	32.0	29.7	48.3	73.2	24.9
	25.3	24.2	24.6	48.2	73	24.8
	20.9	21.0	21.3	48.3	73.1	24.8
2	53.7	52.0	51.7	48.3	73.5	25.2
	45.2	45.0	44.9	48.3	73.4	25.1
	42.1	41.9	40.2	48.4	73.4	25
	35.4	35.3	34.9	48.4	73.3	24.9
	29.8	30.4	30.1	48.2	73.1	24.9
	26.6	25.4	24.8	48.3	73.2	24.9
	20.8	21.4	21.5	48.3	73.1	24.8
3	55.0	50.3	51.7	48.3	73.7	25.4
	45.6	46.4	43.9	48.2	73.4	25.2
	40.6	40.5	41.9	48.2	73.3	25.1
	35.5	35.7	35.2	48.3	73.2	24.9

	31.4	30.4	29.3	48.3	73	24.7
	25.6	25.9	24.7	48.2	72.9	24.7
	19.3	19.5	20.4	48.4	73.2	24.8
4	50.4	48.1	51.3	48.2	73.6	25.4
	45.4	46.5	44.4	48.3	73.6	25.3
	41.6	39.8	39.2	48.3	73.4	25.1
	35.2	34.9	34.3	48.2	73.3	25.1
	30.8	29.2	29.0	48.3	73.2	24.9
	25.1	25.3	24.9	48.3	73.2	24.9
	20.5	19.8	20.1	48.3	73.1	24.8
5	50.8	50.5	49.7	48.4	73.7	25.3
	45.6	44.9	44.3	48.3	73.5	25.2
	42.1	41.8	40.7	48.2	73.3	25.1
	35.5	35.4	34.4	48.2	73.3	25.1
	30.9	30.1	29.6	48.3	73.2	24.9
	25.4	24.5	23.7	48.2	73.1	24.9
	19.8	20.2	20.7	48.2	72.9	24.7

The infrared thermometer has a rated $\pm 0.1°C$ uncertainty associated with it. The meter stick's uncertainty is 0.5 mm, but due to the nature of the spring bouncing up and down, I have adjusted the uncertainty to 1 mm to account for a greater margin of error.

5. Data Analysis:

Table 3: Processed data of correlation between Temperature, Displacement and Spring Constant

Desired Temperature	Average Temperature of top, middle and bottom of the spring (°C±0.1)	Spring Displacement (m±0.002)	Spring Constant (N/m±0.3)
50.0°C	51.0	0.252	38.9
	52.5	0.252	38.9
	52.3	0.254	38.6
	49.9	0.254	38.6
	50.3	0.253	38.8
45.0°C	45.5	0.25	39.2
	45.0	0.251	39.1

	45.3	0.252	38.9
	45.4	0.253	38.8
	44.9	0.252	38.9
40.0°C	40.1	0.25	39.2
	41.4	0.25	39.2
	41.0	0.251	39.1
	40.2	0.251	39.1
	41.5	0.251	39.1
35.0°C	35.4	0.249	39.4
	35.2	0.249	39.4
	35.5	0.249	39.4
	34.8	0.251	39.1
	35.1	0.251	39.1
30.0°C	31.1	0.249	39.4
	30.1	0.249	39.4
	30.4	0.247	39.7
	29.7	0.249	39.4
	30.2	0.249	39.4
25.0°C	24.7	0.248	39.6
	25.6	0.249	39.4
	25.4	0.247	39.7
	25.1	0.249	39.4
	24.5	0.249	39.4
20.0°C	21.1	0.248	39.6
	21.2	0.248	39.6
	19.8	0.248	39.6
	20.1	0.248	39.6
	20.2	0.247	39.7

The displacement was converted from centimeters to meters by dividing by 100 as the equation to determine spring constant requires the displacement to be in meters. The uncertainty was also shifted by 2 decimal places to accompany this change. Below are the Sample Calculations for the Average Temperature, Spring Displacement, and Spring Constant with their uncertainties.

5.1 Sample Calculations

Sample Calculation for Average Temperature	**Sample Calculation for Average Temperature uncertainty**
Using the formula below, we can determine the average temperature of the spring $\frac{x_1+x_2+x_3}{3} = x_{average}$ Ex. $\frac{49.7°C + 52.0°C + 51.3°C}{3} = 51.0°C$ The final answer is rounded to 3 decimal places to keep track of significant figures, though subsequent calculations will use the full answer to increase precision.	The average temperature uncertainty is determined by adding the 3 temperature uncertainties together, converting to percent uncertainty then converting back to absolute uncertainty with the calculated average temperature. Multiplying by 100% is unnecessary as it would be immediately converted back. The final uncertainty is rounded to 1 significant figure. Ex. $\frac{\pm 0.1°C + \pm 0.1°C + \pm 0.1°C}{49.7°C + 52.0°C + 51.3°C} \times 51.0°C = \pm 0.1°C$
Sample Calculation for Spring Displacement	**Sample Calculation for Spring Displacement Uncertainty**
Subtracting the end point from the start point will give the Spring's displacement Ex. 73.5 cm - 48.3 cm = 25.2 cm	Since we are subtracting the values to determine the displacement, the uncertainty is added together Ex. ±0.1 cm + ±0.1 cm = ±0.2 cm
Sample Calculation for the Spring Constant	**Sample Calculation for the Spring Constant uncertainty**
The equation to determine spring constant, as determined in the hypothesis in Equation 2, is shown below. The final answer is rounded to 3 decimal places to keep track of significant figures, though the graph will accompany all decimal places to increase precision. $k = \frac{mg}{x}$ Ex. $k = \frac{(1kg)(9.81m/s^2)}{0.248m} = 39.6\ N/m$	The spring constant uncertainty is determined by converting the absolute uncertainty of the displacement to percent uncertainty, then multiplying by the calculated spring constant to convert back to absolute uncertainty. Again, multiplying by 100% would be redundant so it is not shown in the sample calculation. The uncertainty is rounded to 1 significant figure. Ex. $\frac{\pm 0.002\ m}{0.248\ m} \times 39.6\ N/m = \pm 0.3\ N/m$

5.2 Determining a relationship:

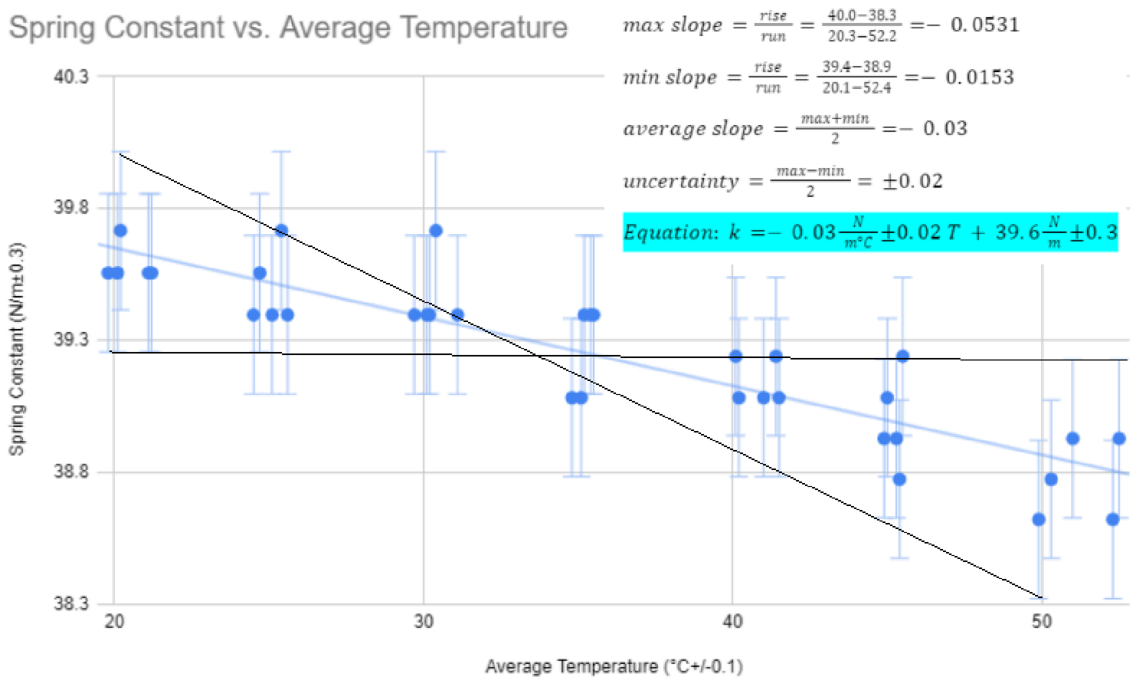


Figure 3: The Spring Constant as a function of Temperature

The data collected on the graph represents a linear function, having a slope which correlates the 2 variables. The equation of a linear function is shown below

$$y = mx + c \tag{3}$$

Where y is the independent variable, m is the slope, which gives the relationship between the x and y values, x is the independent variable and c is a constant. In the case of my data, the spring constant is the independent variable, and the temperature is the dependent variable. Therefore, the data from the graph yields the equation below.

$$k =- 0.03\frac{N}{m°C} \pm 0.02\, T + 39.6\frac{N}{m} \pm 0.3 \tag{4}$$

The spring constant is proportional to the temperature by a factor of -0.0263$\frac{N}{m°C}$ and therefore, changes in temperatures do not constitute great changes in spring constant as the slope is incredibly small. Of course, since there is uncertainty in the slope of the graph by approximately ±0.02$\frac{N}{m°C}$, the relationship may of course differ, but still when considering the maximum slope which is -0.0531$\frac{N}{m°C}$, the slope remains extremely small demonstrating the minimal relationship between temperature and spring constant.

The same data was predicted by the initial hypothesis saying that the spring constant would be inversely proportional to the temperature as demonstrated through the negative slope of the graph. This was due to the fact that temperature by definition is the average kinetic energy of the molecules in a substance. Therefore, when I increased the temperature, the average kinetic energy of the molecules inside the spring also increased which allowed for a greater stretch of the spring during the experiment and therefore resulted in a decrease in the spring constant.

To put into context, the slope of the graph displays that for every increase in temperature of 1°C, the spring constant would decrease by 0.0263N/m, meaning it will require less force to stretch the spring. It is also important to note that this value only applies to the spring that I used in my experiment as other springs would have completely different spring constant values. Additionally, since the spring constant is a vastly different variable than temperature, the line of best fit does not pass through the origin since there needs to be a constant that relates the 2 variables in some way, which I determined to be 39.6 N/m.

Of course, there are uncertainties in the data I collected which hinders its reliability. The uncertainty in the slope I determined to be ±0.02N/m which is a considerably large uncertainty as the actual slope is 0.03N/m. As determined by the minimum slope, the uncertainty of the slope has enough effect to disprove the hypothesis as the minimum slope is -0.0153 which when looking at the slope created on the graph, shows no relationship between the variables since the line is completely flat.

In addition, the uncertainty bars look massive on the graph displaying the large random uncertainty present. The scale of the y-axis of the graph is extremely small and the change in spring constant is minimal. Having a random spread of points on a small scale may lead to misinterpretation of the data, obscuring subtle trends and potentially undermining the significance of the findings. The randomness in the graph may in fact correlate to a different type of function that is not linear which would completely disprove the hypothesis.

6. Conclusion

The correlation between temperature and the spring constant of a spring is negatively linear as demonstrated in the graph above. The line of best fit demonstrates the linear relationship between temperature and the spring constant, although it is extremely small, demonstrating that the relationship is minimal.

The relationship between temperature and spring constant also does not have a theoretical equation since the relationship is so small. Therefore there is no theoretical value for me to compare my data to so I cannot determine the percent error for this experiment. This therefore also limits the reliability of the data as without a theoretical equation, I cannot fully prove the hypothesis was supported.

7. Evaluation

There were several sources of error in the methodology of this experiment. It is worth noting the accuracy of the data collected cannot be determined since there is no theoretical equation to compare the data to. Below are major random and systematic errors present in my methodology. It is important to note I cannot determine the precise effect of each systematic error since there is no theoretical equation to compare it to.

Random Errors	**Significance**	**How to minimize**
Uneven temperature distributions	The temperature of the spring was not even throughout the spring. An effort was made to address this uncertainty by measuring the top, middle, and bottom temperature, but it still remains an uncertainty. However this uncertainty is minimal as the graph displayed minimal outliers except for at the 50°C point and the uncertainty is extremely small (±0.1)	Having access to a thermal camera would allow me to record the temperature at every point of the spring, but this technology was unavailable to me.
Displacement uncertainty	Spring bounces up and down and mass is far away from the ruler forcing me to “eyeball” some values. This uncertainty is large as shown through the scattering of the points on the vertical axis and the ±0.3 uncertainty.	Have a camera setup to the side which would record the initial and final position of the mass, which can be then fully analyzed using computer software.

Systematic Errors	**Significance**	**How to minimize**
Loss of temperature	The temperature of the spring would decrease drastically when it was not in the water bath. I removed part of the error by recording myself saying the data points out loud to work faster but it still results in	Use a thermal camera to have temperature readings at every point of the experiment or use a larger water bath and measured displacement by a certain force applied while inside this water bath, but this would pose major

	a large uncertainty which would shift the graph to the left to a significant extent.	safety risks.
Weight of spring	The calculations using Hooke's law only considered the weight of the 1 kg mass, but there is also a force caused by the weight of the spring that was not considered as it is difficult to know which portion of the spring exerts force. This would shift the graph upwards to a small portion as usually the weight of the spring is negligible.	Use a force scale on top of the 1 kg mass to have a precise measurement of the force caused by the weight of the mass on the spring and use it for all the calculations.

Weaknesses	Significance
Small range of temperature	The range of temperatures was extremely small which did not help in determining a relationship between variables as the relationship was also really small. I chose this range since I was limited by the equipment I had access to. I could not go lower than 20°C since that was the room temperature and I couldn't go higher than 50°C since once I removed the spring from the boiling water, I noticed that 50°C was the highest that the spring could reach since the air cooled the spring down so rapidly. I thought of other methodologies since using an oven to bake the spring, but was highly risky since the spring would reach extremely high temperatures. Additionally, I thought of using a different liquid with a higher boiling point such as peanut oil which has a boiling point of 230°C, but I determined that the risk involved with heating a metal spring to such temperatures was too high.
Only using one spring	As mentioned throughout this investigation, the spring constant differs for every spring. Therefore the relationship between the spring constant and temperature may be different for every spring so the conclusion to this experiment of a negative linear relationship may possibly be different for different springs. To further validate the hypothesis, different springs of different materials can undergo the same experimentation to see if they exhibit the same results.

7.1 Extension

If given the opportunity to investigate this topic again, I would use a different apparatus to measure temperature and displacement since this produced the most errors. By using something like a heat gun to reach higher temperatures and a thermal camera to measure such, I can have a larger temperature scale and reduce uncertainty. Similarly, by using a camera and software to measure the displacement of the spring, I can obtain a more accurate reading of the spring constant which can be used to better determine the relationship between temperature and spring constant.

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